# Non-perturbative results in CFTs at finite temperature

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*Synergies Towards the Future Standard Model*

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QUANTUM UNIVERSE

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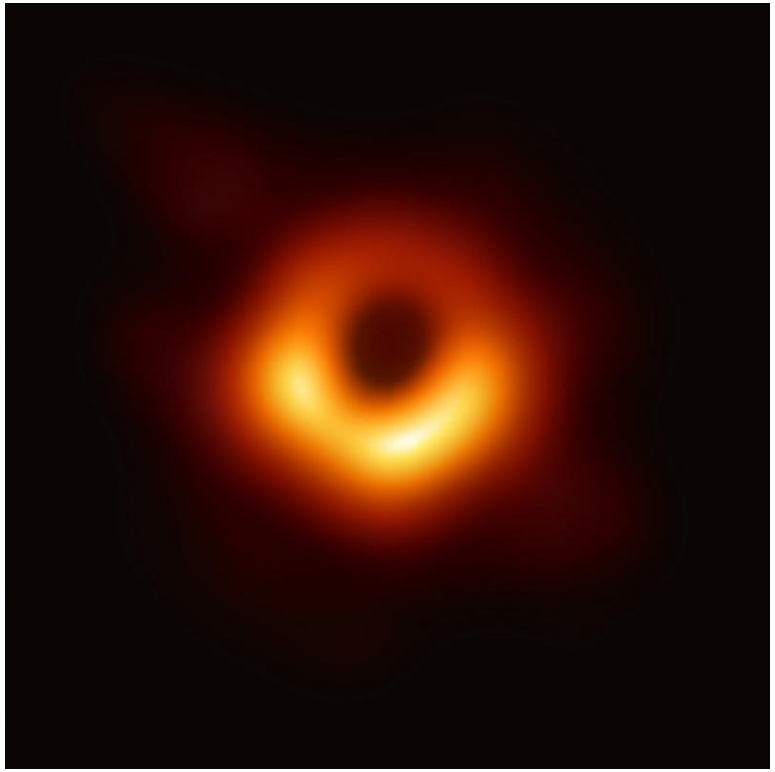
We want to compute Quantum Gravity (QG) observables in a black hole background.



[Event Horizon Telescope '19]

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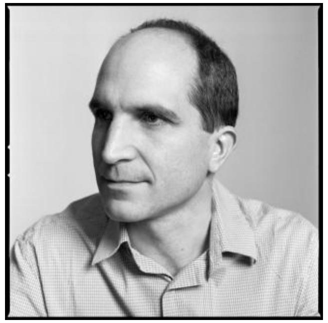

- We can consider QG in Anti-de Sitter (AdS) space.
- QG theories in AdS are dual to Conformal Field Theories (CFTs). [Maldacena '97]
- CFTs are perfect labs for non-perturbative computations, given their high symmetry.

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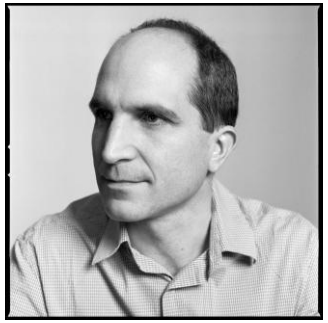

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Can we study black holes in Anti-de Sitter space?

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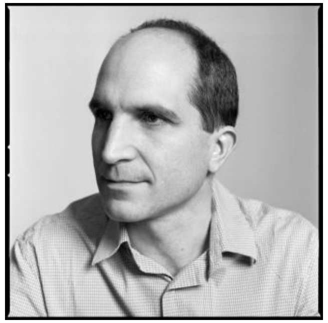

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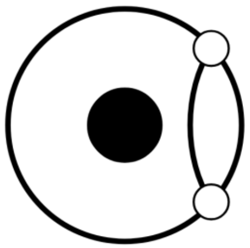
Can we study black holes in Anti-de Sitter space?

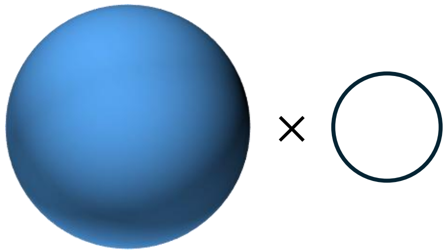
- Schwarzschild black holes in AdS are dual to CFTs on $S_R^{d-1} \times S_\beta^1$ . [Witten '98]
- The thermal circle $S_\beta^1$ encodes the temperature of the black hole, i.e., $\beta = 1/T$ .



QG observables in an AdS black hole background

$\updownarrow$

CFT observables on $S_R^{d-1} \times S_\beta^1$



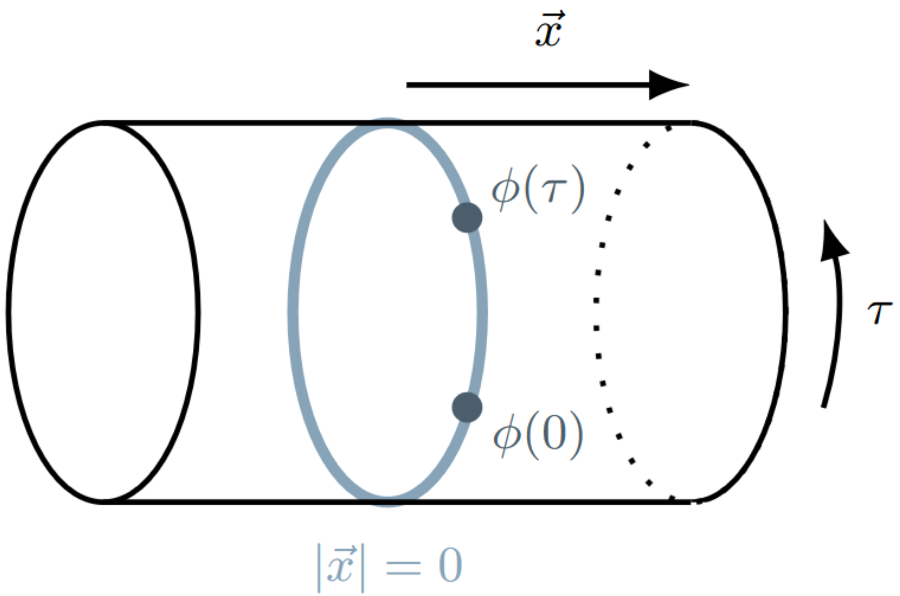
- We focus on the limit $R \to \infty$ (black brane).
- This corresponds to an infinite temperature limit: no phase transitions!
- We will study CFT observables on $\mathbb{R}^{d-1} \times S_\beta^1$ .

- We defined the geometry of the problem.
- Now we need to define the field content of the model.

- In AdS, we consider a theory of Einstein gravity coupled to a scalar field $\phi$ of squared mass $m^2 = \Delta_\phi(\Delta_\phi - d)$.
- On the boundary, this is dual to a strongly coupled CFT with a large number of colours $N$.

$$
\begin{aligned}
4\pi\lambda &= \left(\frac{L_{\mathrm{AdS}}}{\ell_s}\right)^4 \\
\frac{\lambda}{N} &= g_s
\end{aligned}
$$

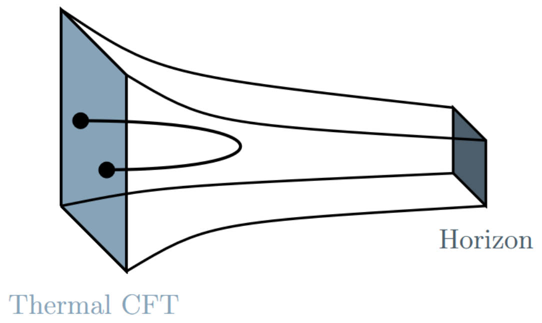
- We choose an observable: the scalar two-point function on $\mathbb{R}^{d-1} \times S^1_\beta$ with zero spatial separation.




$= \langle \phi(\tau)\phi(0) \rangle_\beta$

- This is related to the close-to-boundary expansion of the massive scalar's wavefunction.

$$\Phi(z, \tau, \vec{x}) = z^{d-\Delta_\phi} \phi_{(0)}(\tau, \vec{x}) \left( 1 + \cdots + \frac{z^{2\Delta_\phi - d}}{2\Delta_\phi - d} \langle \phi(\tau, \vec{x}) \phi(0,0) \rangle_\beta + \cdots \right)$$



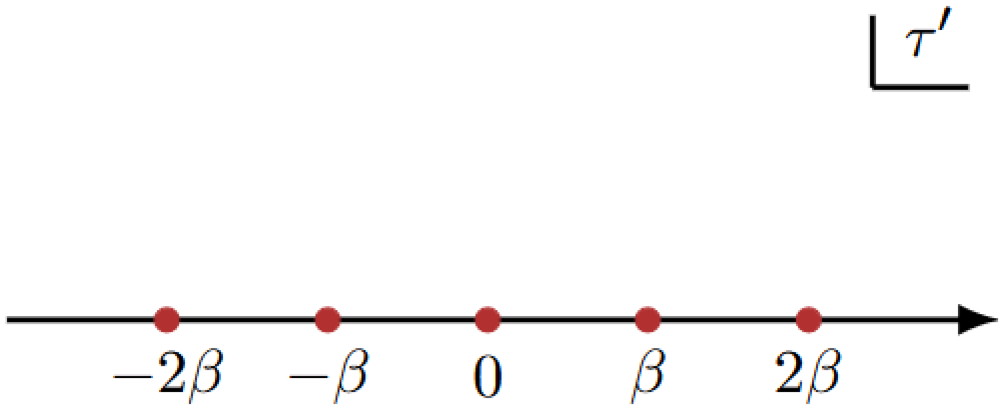
- The scalar two-point function admits an **OPE decomposition**:

$$\langle \phi(\tau)\phi(0)\rangle_\beta = \frac{1}{\tau^{2\Delta_\phi}} + \sum \underbrace{a_\Delta}_{\textit{Theory-dependent OPE data}} \left(\frac{\tau}{\beta}\right)^{\Delta - 2\Delta_\phi}$$

- In our holographic CFT, **large $N$** and **strong coupling** simplify the OPE decomposition:

$$\langle \phi(\tau)\phi(0)\rangle_\beta = \frac{1}{\tau^{2\Delta_\phi}} + \sum \underbrace{a_{[\phi\phi]_{n,\ell}}}_{\substack{\textit{Double twists}\\ \textit{(scalar-scalar)}}} \left(\frac{\tau}{\beta}\right)^{\Delta - 2\Delta_\phi} + \sum \underbrace{a_{[T^n]}}_{\substack{\textit{Multi stress-tensors}\\ \textit{(gravitons)}}} \left(\frac{\tau}{\beta}\right)^{\Delta - 2\Delta_\phi} + \mathcal{O}(1/N)$$

We compute $\langle \phi(\tau)\phi(0)\rangle_\beta$ using its analytic properties:




- The two-point function is a function of $\tau'$

$$g(\tau') = \langle \phi(\tau')\phi(0)\rangle_\beta$$

- By periodicity, it has an infinite number of real poles.

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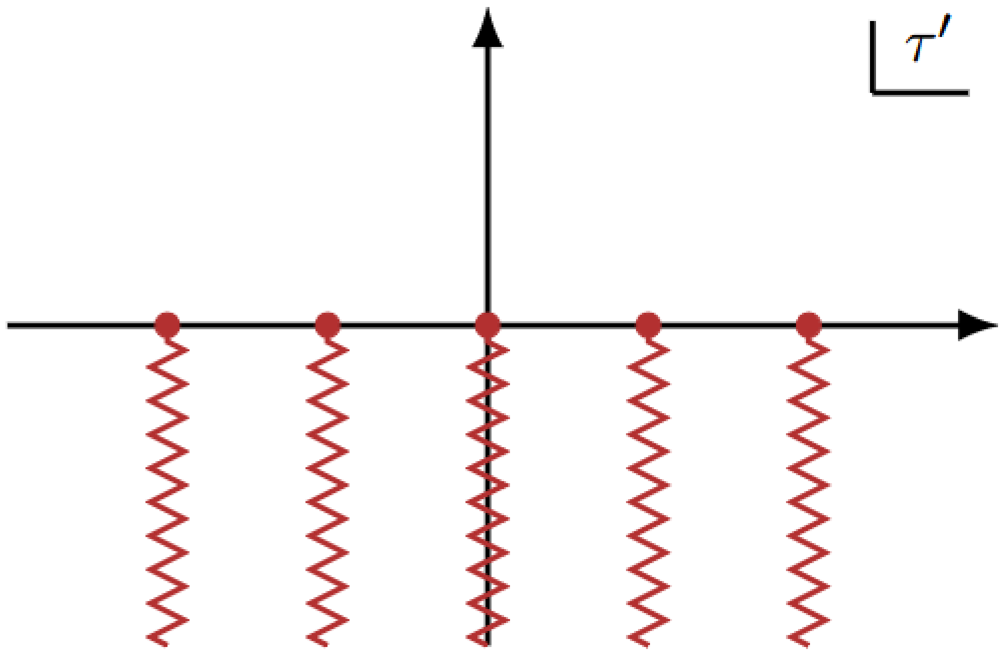
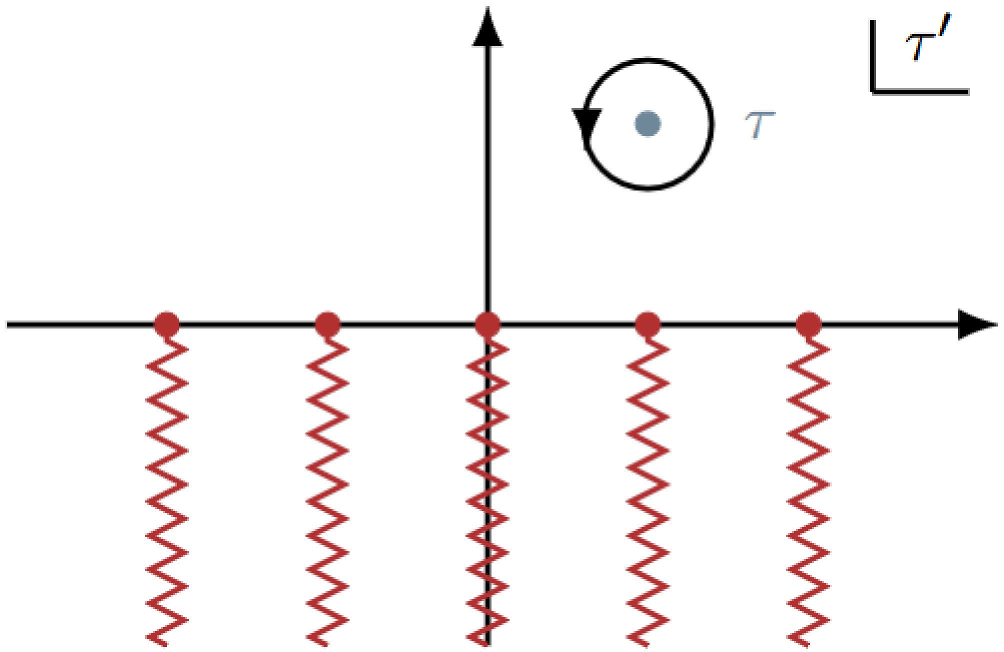



- The variable can be complexified.
- In the complex $\tau$-plane, branch cuts can appear.

We compute $\langle \phi(\tau)\phi(0)\rangle_\beta$ using its **analytic properties**:




- The two-point function can be rewritten using Cauchy formula:

$$g(\tau) = \frac{1}{2\pi i}\oint_{\mathcal{C}} \mathrm{d}\tau' \, \frac{g(\tau')}{\tau' - \tau}$$

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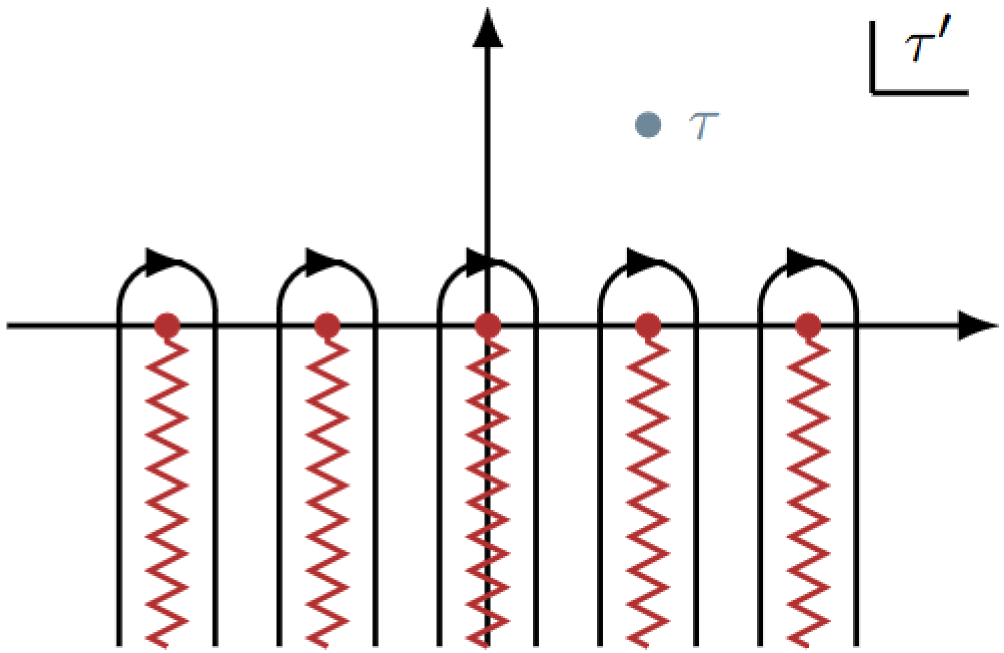


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- After reshaping the contour, we obtain a dispersion relation:

$$g_{dr}(\tau) = \sum_{m=-\infty}^{\infty} \int_{-i\infty}^{0} \frac{\mathrm{d}\tau'}{2\pi i} \frac{\mathrm{Disc}\, g(\tau')}{\tau' + m\beta - \tau}$$

We can **plug the OPE** in the dispersion relation to obtain our result!

$$g(\tau') = \frac{1}{\tau'^{2\Delta_\phi}} + \sum a_\Delta \left(\frac{\tau'}{\beta}\right)^{\Delta - 2\Delta_\phi}$$

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$$\boxed{g(\tau) = \sum_\Delta \frac{a_\Delta}{\beta^{2\Delta_\phi}} \left[\zeta_H\left(2\Delta_\phi - \Delta, \frac{\tau}{\beta}\right) + \zeta_H\left(2\Delta_\phi - \Delta, 1 - \frac{\tau}{\beta}\right)\right]}$$

The holographic application is immediate: we consider $\Delta_\phi = 3$ in 4D as an example.

$$\langle\phi(\tau)\phi(0)\rangle_\beta = \frac{1}{\tau^6} + \sum a_{[\phi\phi]_{n,\ell}} \left(\frac{\tau}{\beta}\right)^{\Delta-6} + \sum a_{[T^n]} \left(\frac{\tau}{\beta}\right)^{\Delta-6} + \mathcal{O}(1/N)$$

$$g(\tau) = \sum_{\Delta} \frac{a_\Delta}{\beta^6}\left[\zeta_H\left(6-\Delta, \frac{\tau}{\beta}\right) + \zeta_H\left(6-\Delta, 1-\frac{\tau}{\beta}\right)\right]$$

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$$g(\tau) = \frac{\pi^6}{60\beta^6}\left[26\cos\left(\frac{2\pi\tau}{\beta}\right) + \cos\left(\frac{4\pi\tau}{\beta}\right) + 33\right]\csc^6\left(\frac{\pi\tau}{\beta}\right)$$

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$[\phi\phi]_{n,\ell}$ don't contribute!

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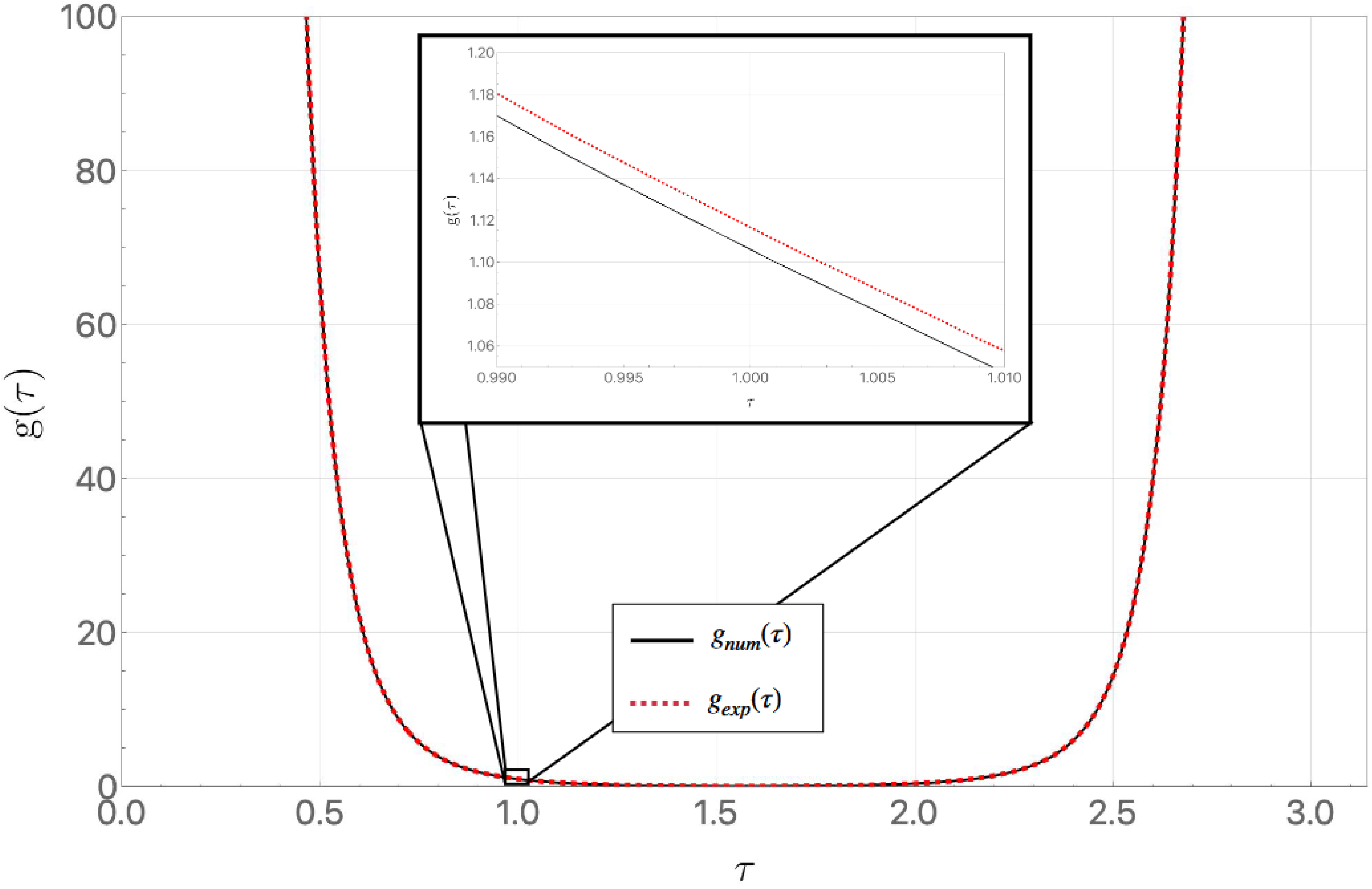
«Holographic» correction!

We plug the explicit 4D black brane value $a_T = \frac{\pi^4}{40}$ to obtain the exact result.

$4D, \Delta_\phi = 3$, black brane

$$\langle \phi(\tau)\phi(0)\rangle_\beta = \frac{\pi^6}{60\beta^6}\left[26\cos\left(\frac{2\pi\tau}{\beta}\right) + \cos\left(\frac{4\pi\tau}{\beta}\right) + 33\right]\csc^6\left(\frac{\pi\tau}{\beta}\right) + \frac{\pi^6}{40\beta^6}\csc^2\left(\frac{\pi\tau}{\beta}\right)$$

An explicit numerical check: 4D black brane holographic correlator, $\Delta_\phi = 3$



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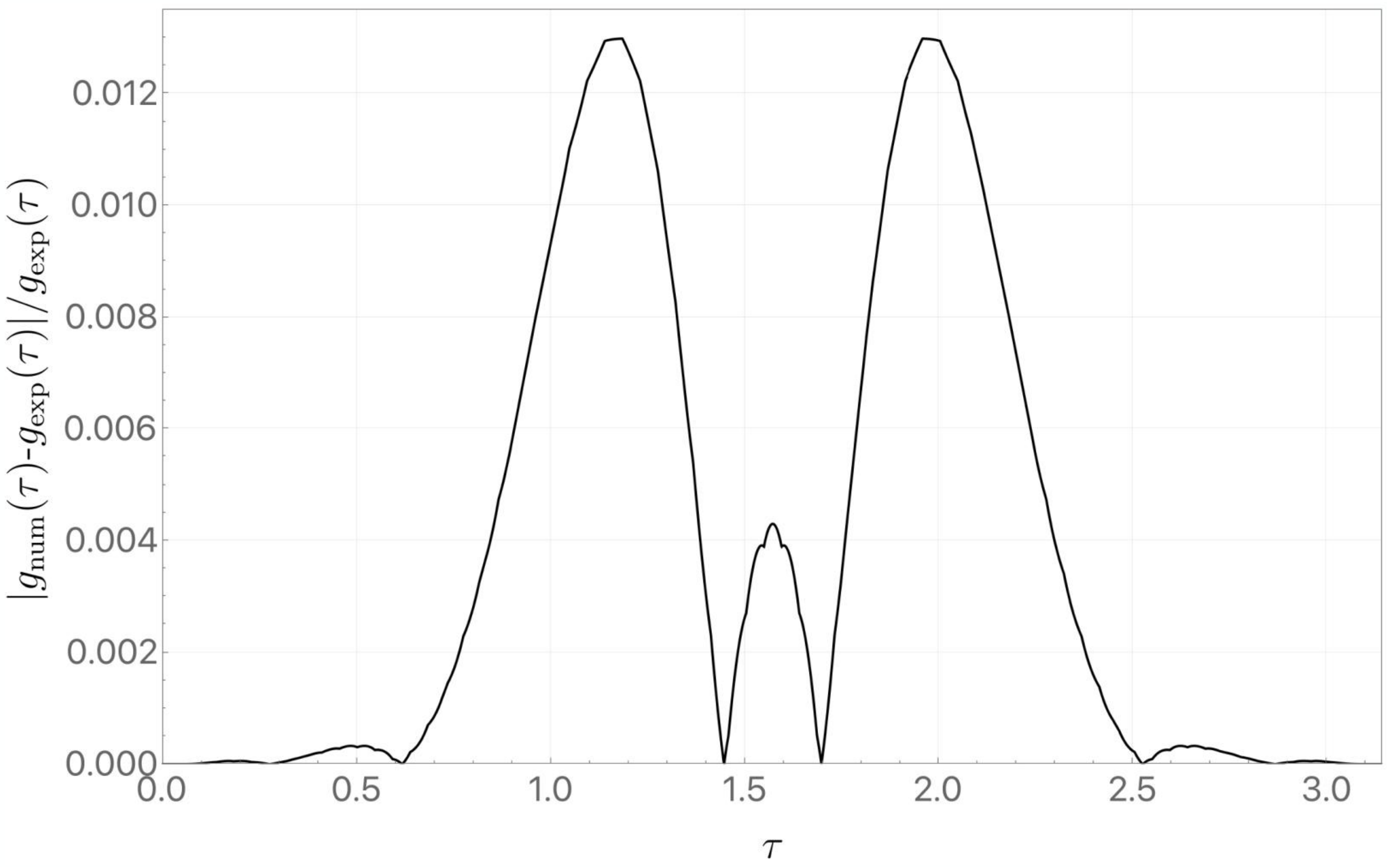


Can we apply our formalism to other areas of Physics?

Consider the 3D Ising model, of crucial relevance in condensed matter physics and conformal bootstrap

- The spectrum is more complicated, but we can truncate the sum:

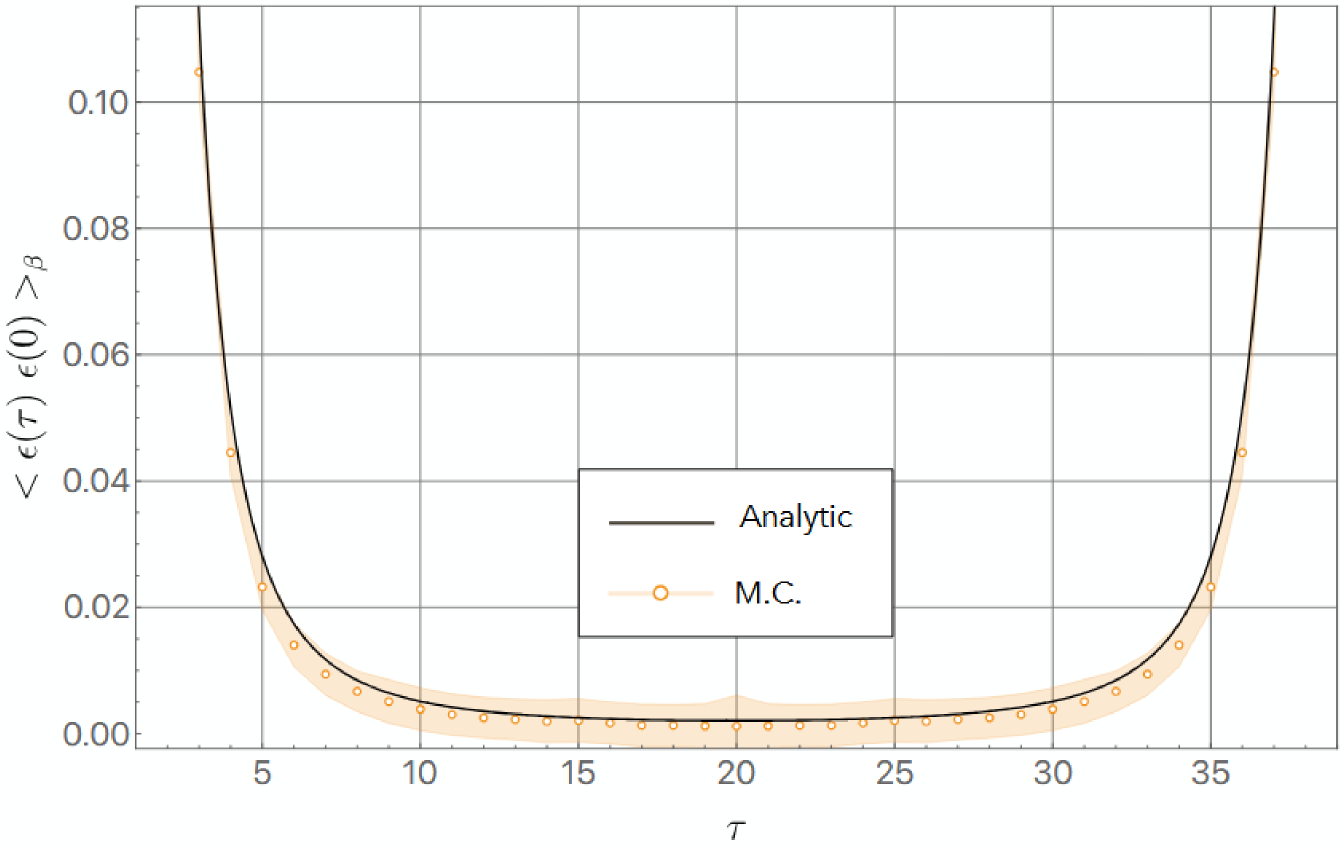
$$\langle \epsilon(\tau)\epsilon(0)\rangle_\beta \approx \sum_{1,\epsilon,T} \frac{a_\Delta}{\beta^{2\Delta_\epsilon}}\left[\zeta_H\left(2\Delta_\epsilon - \Delta, \frac{\tau}{\beta}\right) + \zeta_H\left(2\Delta_\epsilon - \Delta, 1 - \frac{\tau}{\beta}\right)\right] + \kappa$$

*Extra arc contribution*

- Dynamical OPE data can be numerically bootstrapped by using KMS sum rules:

$$a_{\Delta_\epsilon} = 1.09(22) \qquad a_{\Delta_T} = 5.37(19)$$ [Barrat, EM, Miscioscia, Pomoni, ‘24]

Is the approximation precise enough? We compare with a Monte Carlo simulation.




Only 3 operators $(1, \epsilon, T)$ are needed to lie inside MC uncertainty!

In conclusion:

- It is possible to obtain **analytic, non-perturbative results** in CFTs at finite temperature.
- These results can be succesfully applied to Holography to derive **analytic, non-perturbative results in Quantum Gravity.**
- These results work perfectly in tandem with previous (and future!) numerical bootstrap studies, see the 3D Ising model example.

In the future:

- Full study of the black hole dual geometry $S_R^{d-1} \times S_\beta^1$ . [Barrat, Bozkurt, EM, Miscioscia, Pomoni, *to appear*]
- Investigation of **phase transitions** as a function of $\beta/R$ .
- $\langle JJ \rangle_\beta$ and $\langle TT \rangle_\beta$ correlators and relevant deformations at finite temperature.

**Thank you!**