Bootstrapping surface defects in $\mathcal{N} = 4$ SYM

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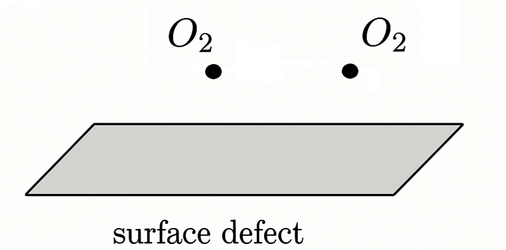
Based on work in progress with L. Bianchi, M. Ferragatta and V. Forini

Summary

Main focus: the Gukov-Witten surface defect in $\mathcal{N} = 4$ SYM.



Main results:

- We bootstrapped the 2pt function of the $20'$ operator $\mathcal{O}_2$ in presence of the defect, at large N and strong coupling λ.
- We extracted from it defect CFT data (e.g. 1pt functions) for protected and non-protected operators.

The Gukov-Witten surface defect in $\mathcal{N} = 4$ SYM

Defined by singular behavior in the $SU(N)$ gauge and scalar fields along its support. It is parametrized by $(\alpha_I, \beta_I, \gamma_I, \eta_I)$. [Gukov, Witten '06]

$$\Phi = \frac{1}{\sqrt{2}z} \operatorname{diag}\left((\beta_1 + \mathrm{i}\gamma_1)\, 1_{N_1}, \dots, (\beta_M + \mathrm{i}\gamma_M)\, 1_{N_M} \right) ,$$

$$A = \operatorname{diag}\left(\alpha_1 1_{N_1}, \alpha_2 1_{N_2}, \dots, \alpha_M 1_{N_M}\right) d\psi \, , \quad N = \sum_{I=1}^{M} N_I \, .$$

- The defect breaks the bulk symmetry group as

$$SO(4,2) \times SU(4)_R \to SO(2,2) \times SO(4)_R \times SO(2)_t \, .$$

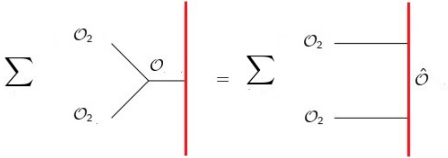
- At $N \gg 1$ and $\lambda \gg 1$, it is dual to probe D3 branes or "bubbling" geometries. [Drukker et al. '08]

Main observable: 2pt function in presence of a defect

- It depends on 5 invariant cross-ratios [Holguin, Kawai '25]

$$\langle \mathcal{O}_2\, \mathcal{O}_2 \rangle \cong \mathcal{F}(z, \bar{z}, \alpha, \bar{\alpha}, \theta)\,, \quad \mathcal{O}_2 \in 20' \text{ of } SU(4)_R .$$

- Expanding using either the bulk or defect OPE [Billo' et al. '16]



$$\langle \mathcal{O}_2\, \mathcal{O}_2 \rangle \cong \sum_{\mathcal{O}} \lambda_{22\mathcal{O}}\, a_{\mathcal{O}}\, \underbrace{\mathcal{G}_{\mathcal{O}}(z, \bar{z}, \alpha, \bar{\alpha}, \theta)}_{\text{superconformal block}} = \sum_{\widehat{\mathcal{O}}} b^2_{\mathcal{O}_2 \hat{\mathcal{O}}}\, \hat{\mathcal{G}}_{\hat{\mathcal{O}}}(z, \bar{z}, \alpha, \bar{\alpha}, \theta)\,,$$

- $\{a_{\mathcal{O}}, b_{\mathcal{O}_2 \hat{\mathcal{O}}}\}$ are (non-)protected bulk 1pt and bulk-defect 2pt couplings. Together with the spectrum of defect operators, they are the defect CFT data. Bulk spectrum is known.

Our strategy: Analytic Bootstrap

Main idea: extract CFT data and reconstruct correlators from their (Lorentzian) singularities using a conformal dispersion relation.

[Caron-Huot '17], [Carmi, Caron-Huot '19], [Lemos et al. '17], [Bianchi, DB '22], [Barrat et al. '22]

$$\langle \mathcal{O}_2\, \mathcal{O}_2 \rangle \cong \int_0^r \frac{dw'}{2\pi i} \frac{\text{Disc}_{w<r} \langle \mathcal{O}_2\, \mathcal{O}_2 \rangle}{w' - w} + \int_{1/r}^{\infty} \frac{dw'}{2\pi i} \frac{\text{Disc}_{w>1/r} \langle \mathcal{O}_2\, \mathcal{O}_2 \rangle}{w' - w} \, .$$

- Discontinuities are controlled by (a subset of) OPE data

$$\text{Disc}\, \langle \mathcal{O}_2\, \mathcal{O}_2 \rangle = \sum_{\mathcal{O}} \lambda_{22\mathcal{O}}\, a_{\mathcal{O}}\, \text{Disc}\left[\mathcal{G}_{\mathcal{O}}(rw, r/w, \alpha, \bar{\alpha}, \theta)\right] \, .$$

- Caveat: the dispersion relation may miss a contribution from low-spin defect operators. Can we fix it?

Constraints from SUSY

Superconformal Ward identities read

$$
\begin{aligned}
(\partial_z + \partial_\alpha)\, \mathcal{F}(z, \bar{z}, \alpha, \bar{\alpha}, \theta)\big|_{\alpha \to z} &= 0\,, \\
(\partial_{\bar{z}} + \partial_{\bar{\alpha}})\, \mathcal{F}(z, \bar{z}, \alpha, \bar{\alpha}, \theta)\big|_{\bar{\alpha} \to \bar{z}} &= 0\,.
\end{aligned}
$$

We solved them and found that:

- $\mathcal{F}(z, \bar{z}, \alpha, \bar{\alpha}, \theta) =$ combination of $\{f(z, \alpha, \theta), F(z, \bar{z}, \alpha, \bar{\alpha}, \theta)\}$.
- $\mathcal{F}(z, \bar{z}, \alpha, \bar{z}, \theta) = f(z, \alpha, \theta)$.
- $f(z, \alpha, \theta)$ is related to a chiral algebra 4pt function that can be bootstrapped exactly.

  [Beem et al. '13], [Cordova et al. '17], [Lemos, Bianchi '19]

Bootstrapping $\langle O_2 O_2 \rangle$

We can combine SUSY Ward identities with the analytic conformal bootstrap to obtain

$$\text{Disc}\langle \mathcal{O}_2 \mathcal{O}_2 \rangle \to \langle \mathcal{O}_2 \mathcal{O}_2 \rangle + \underbrace{\text{~~low-spin ambiguity~~}}_{\text{compare with } f(z,\alpha,\theta)} \to \text{defect CFT data.}$$

- The discontinuity is controlled by few superconformal blocks.
- The coefficients of these blocks are known from localization.
- $f(z, \alpha, \theta)$ can be determined exactly.
- Comparing this piece of the correlator with the results of the dispersion relation helps fixing the low-spin ambiguity.

Holomorphic piece of $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$ (for $\beta_I = \gamma_I = 0$)

Related to 4pt of modules σ and flavor currents J. [Lemos, Bianchi '19]

$$\langle \sigma(\infty) J(z) J(1) \sigma(0) \rangle \cong \frac{f(z,\alpha)}{z}\,.$$

We know:

- $J(z)J(1)$ OPE $\sim$ bulk OPE $\to$ singularity at $z=1$. [Rigatos '25]
- $J(z)\sigma(0)$ OPE + $z \leftrightarrow 1/z$ symmetry $\to$ sing. at $z = 0, \infty$.

Dispersion relation for 4pt functions gives exact result [Bissi et al. '19]

$$f(z,\alpha) = a_{\mathcal{O}_2}^2 + \frac{z(1-\alpha)^2}{\alpha(1-z)^2} + \frac{\lambda_{222} a_{\mathcal{O}_2} \left(z - \alpha^2\right)}{2\alpha(1-z)}\,.$$

where, for any N and λ, [Lee et al. '98] [Drukker et al. '08] [Choi et al. '24]

$$\lambda_{p_1 p_2 p} = \frac{\sqrt{p_1 p_2 p}}{N}\,, \quad a_{\mathcal{O}_2} = -\frac{2}{\sqrt{6}} \sum_{I=1}^{M} N_I \left(\frac{N - N_I}{2N}\right)\,.$$

Full $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$ at $N \gg 1$ and $\lambda \gg 1$ (for $\beta_l = \gamma_l = 0$)

The discontinuity is very simple at $N \gg 1$ and $\lambda \gg 1$,

$$\text{Disc}\,\langle \mathcal{O}_2 \mathcal{O}_2 \rangle = \text{Disc}\left[\mathcal{G}_1(r, w, \alpha, \bar{\alpha})\right] + \lambda_{222} a_{\mathcal{O}_2} \text{Disc}\left[\mathcal{G}_{[0,2,0]}(r, w, \alpha, \bar{\alpha})\right]$$

We compute:

- Non-holomorphic piece of $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$

$$F(z, \bar{z}, \alpha, \bar{\alpha}) = \frac{1}{\alpha\bar{\alpha}(1-z)^2(1-\bar{z})^2} + \frac{\lambda_{222} a_{\mathcal{O}_2}\left(1 - z^2\bar{z}^2 + 2z\bar{z}\log(z\bar{z})\right)}{2\alpha\bar{\alpha}(1-z)(1-\bar{z})(1-z\bar{z})^3}\,.$$

- new defect CFT data, e.g. non-protected 1pt functions,

$$\begin{aligned}
\lambda_{22(n,\ell)} a_{n,\ell} &= \frac{\pi\, 4^{-\ell-2n-5}\Gamma(n+3)\Gamma(\ell+n+4)}{\Gamma\left(n+\frac{5}{2}\right)\Gamma\left(\ell+n+\frac{7}{2}\right)}\left(2a_{\mathcal{O}_2}^2 - (-1)^n \lambda_{222} a_{\mathcal{O}_2}\right) \\
\Delta_{n,\ell} &= 4 + 2n + \ell\,.
\end{aligned}$$

Conclusions and outlook

In this talk:

- We discussed the structure of $\langle \mathcal{O}_2 \mathcal{O}_2 \rangle$ in presence of a Gukov-Witten surface defect in $\mathcal{N} = 4$ SYM.
- We determined the holomorphic piece of the correlator, exploiting its relation with a chiral algebra 4pt function.
- We bootstrapped the full 2pt function at large N and strong coupling λ, using minimal input from localization.

$$\{\lambda_{222}, a_{\mathcal{O}_2}\} \to \text{Disc}\langle \mathcal{O}_2 \mathcal{O}_2 \rangle \to \langle \mathcal{O}_2 \mathcal{O}_2 \rangle \to \text{defect CFT data.}$$

In the future:

- Extend the analysis to $\langle \mathcal{O}_p \mathcal{O}_p \rangle$ at any value of $(\alpha_I, \beta_I, \gamma_I, \eta_I)$.
- Holographic interpretation, flat space limit.
- Connection to integrability in the limit $\alpha, \beta, \gamma \to 0$.

  [Holguin, Kawai '25], [Chalabi et al. '25]

Thank you for your attention!