SymTFT for continuous spacetime symmetries.

Nicola Andrea Dondi (ICTP & INFN Trieste) - DESY Theory Workshop - 25/09/2025

Based on:
[2509.07965] with F.Apruzzi, I. García Etxebarria, H. T. Lam, S. Schäfer-Nameki

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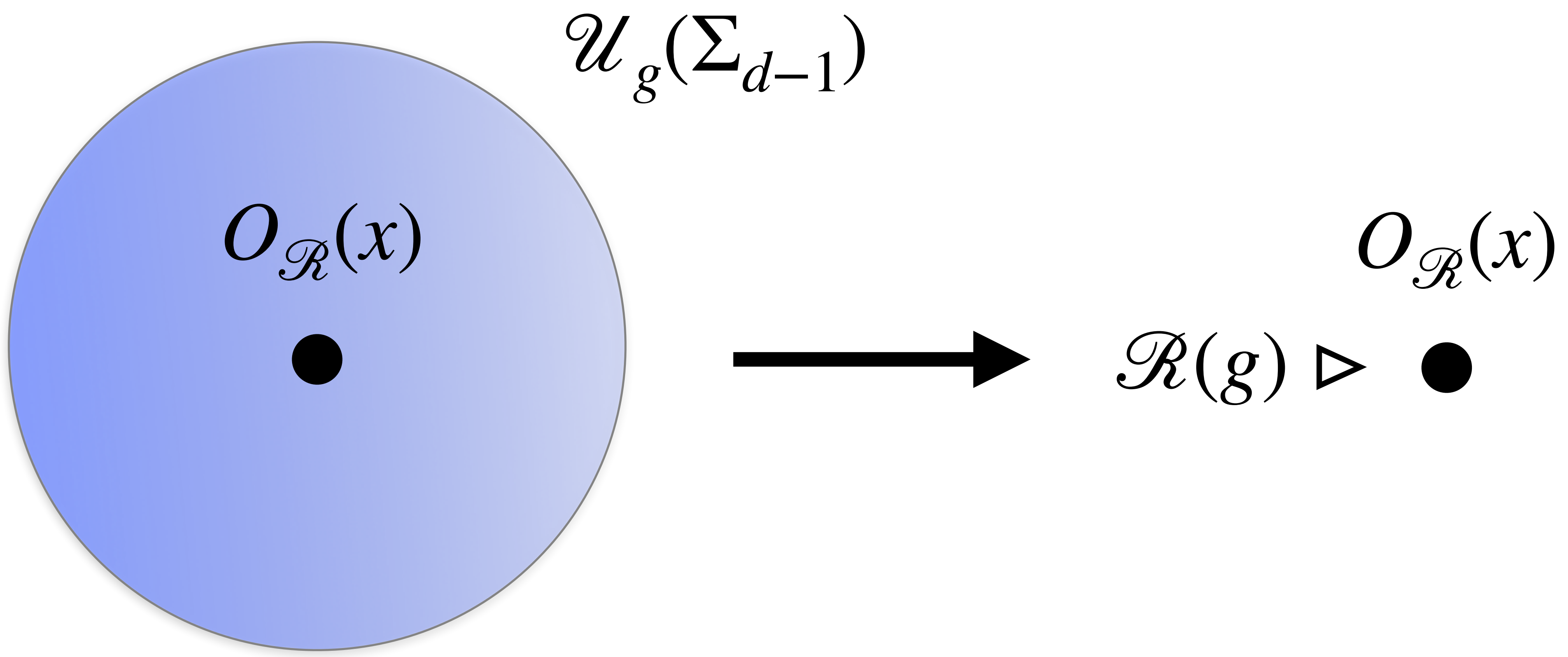
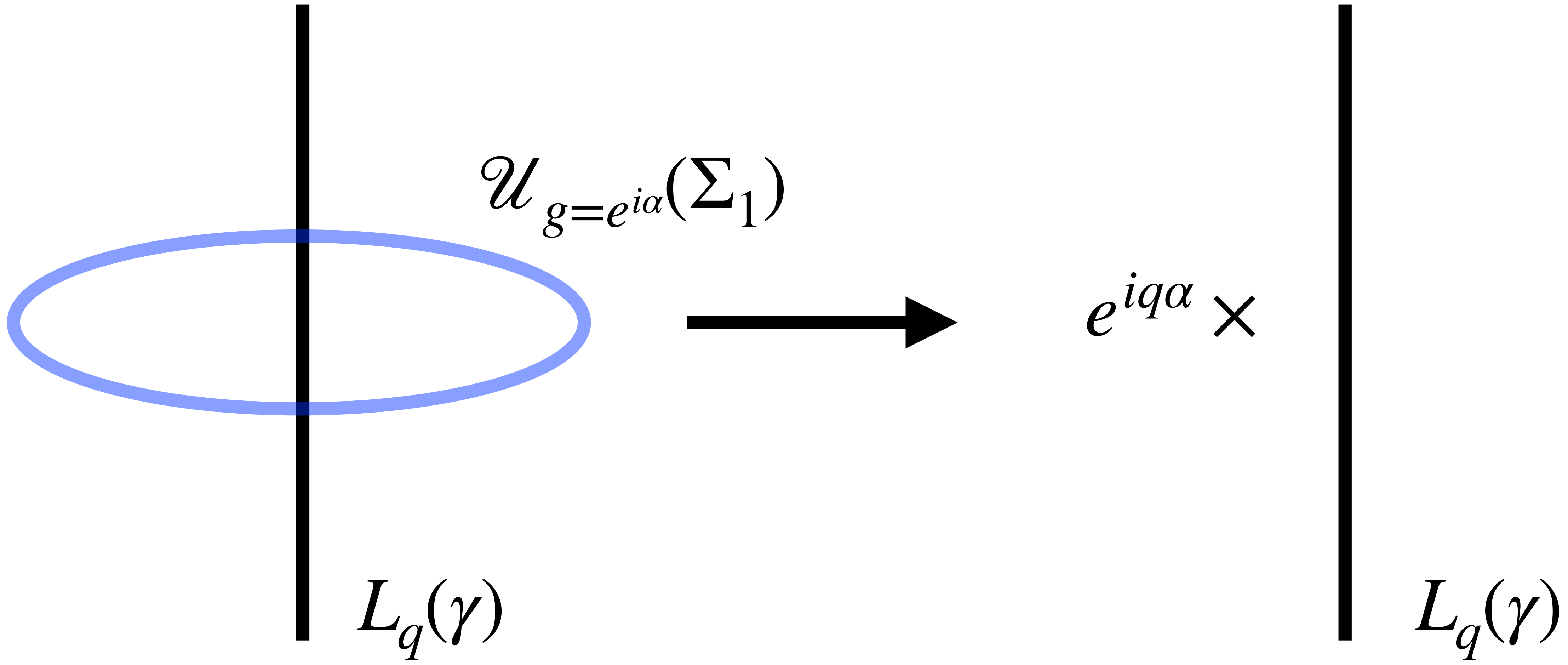
Symmetries and topological operators





[Gaiotto, Kapustin, Seiberg, Willet '14]

[Kapustin, Seiberg '14]

Topological Defect Operators (TDO)

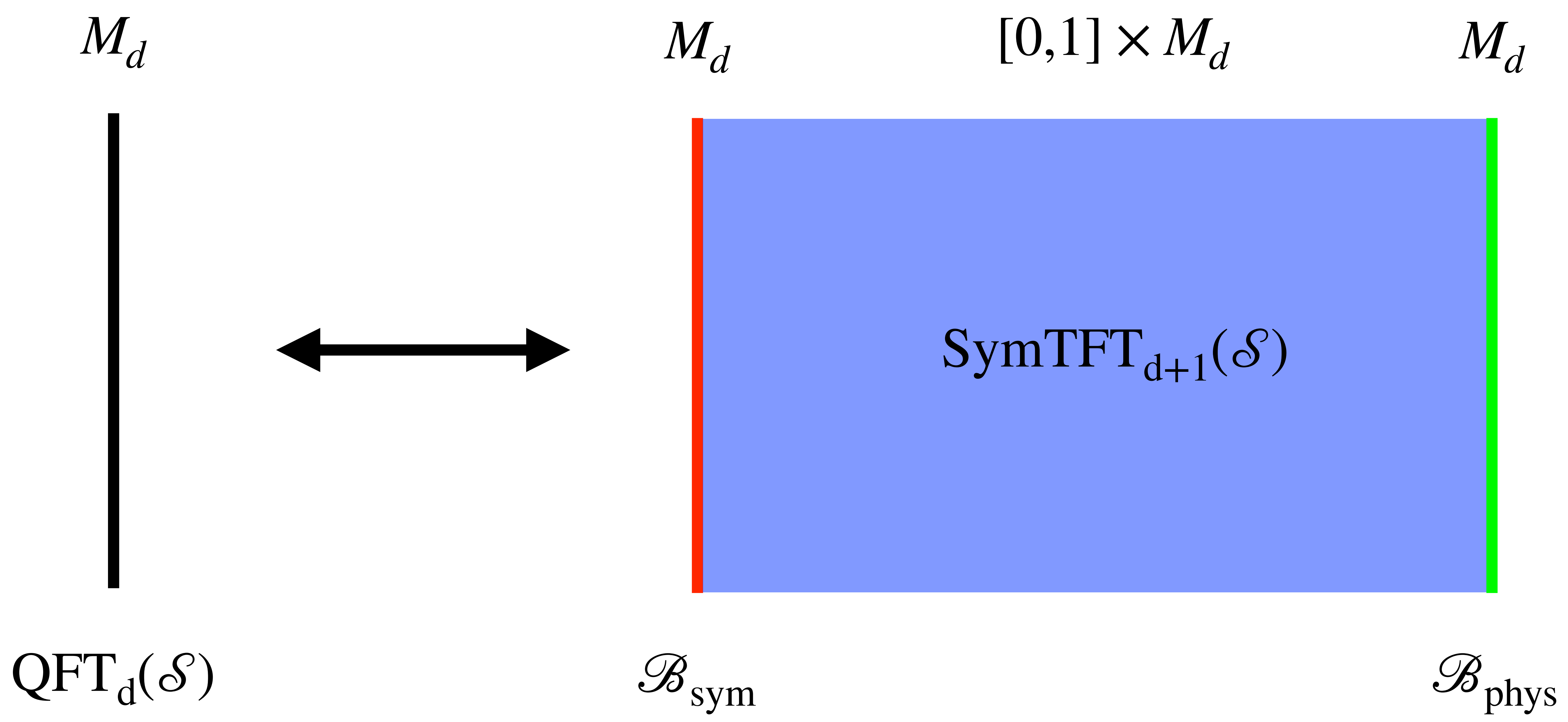
$\Updownarrow$

Symmetries

- Extension to non-invertible / categorical symmetries
- Can we incorporate spacetime symmetries?

[see also discussion in Harlow, Ooguri '19]

SymTFTs: overview



$\mathrm{SymTFT}_{\mathrm{d+1}}(\mathcal{S})$: TQFT flat-gauging $\mathcal{S}$ in one higher dimension.

- $\mathscr{B}_{\mathrm{phys}}$: gapless / gapped boundary condition of $\mathrm{SymTFT}_{\mathrm{d+1}}(\mathcal{S})$ containing dynamical information on the original theory $\mathrm{QFT}_{\mathrm{d}}(\mathcal{S})$
- $\mathscr{B}_{\mathrm{sym}}$: gapped boundary condition of $\mathrm{SymTFT}_{\mathrm{d+1}}(\mathcal{S})$.
- Bulk topological operator corresponds to symmetry operators / charged objects depending on their endability.

Recent progress

- Finite group symmetries: symmetry theory is Dijkgraaf-Witten-type.

  [Freed, Moore, Teleman 22], [Apruzzi, Bonetti, García Etxebarria, Hosseini, Schäfer-Nameki ’21], [Gaiotto, Kulp ’20]…

- Continuous group symmetries: typically BF-type theories.

  [Brennan, Sun 24], [Bonetti, Del Zotto, Minasian ‘24], [Apruzzi, Bedogna, Dondi ’24], [Antinucci, Benini ’24]…

Useful for studying various symmetry properties in a theory-independent ways:

- Encode different symmetry global variants

  [Apruzzi, Bonetti, García Etxebarria, Hosseini, Schäfer-Nameki ’21]…

- Classify anomalies and higher-charges.

  [Bhardwaj, Schafer-Nameki 23], [Kaidi, Nardoni, Zafrir, Zheng ’23]…

- Construct gapped and gapless phases

  [Bhardwaj, Bottini, Pajer, Schafer-Nameki, 24], [Bonetti, Del Zotto, Minasian ’25]…

Non-abelian BF theories: Generalities

$$S = \frac{i}{2\pi}\int_{M_{d+1}} \langle B_{d-1}, F\rangle\,, \quad F = \mathrm{d}A + A\wedge A\,.$$

EOM: $\mathrm{d}_A B_{d-1} = F = 0.$

- A : G-connection.
- B_{d-1}: $\mathfrak{g}$-valued $(d-1)$-form.
- $\langle \ldots, \ldots \rangle$: Adjoyint-invariant bilinear on the algebra.

This theory is topological and has an extended gauge symmetry:

$$\delta_{(\epsilon,\sigma)}A = \mathrm{d}_A\epsilon$$
$$\delta_{(\epsilon,\sigma)}B_{d-1} = [B_{d-1},\epsilon] + \mathrm{d}_A\sigma_{d-2}$$

$$\mathscr{L}_\xi A = \iota_\xi(F) + \mathrm{d}_A(\iota_\xi A)$$
$$\mathscr{L}_\xi B_{d-1} = \iota_\xi(\mathrm{d}_A B_{d-1}) + [\iota_\xi A, B_{d-1}] + \mathrm{d}_A(\iota_\xi B_{d-1})$$

$$\boxed{\text{On flat connections Diff} \simeq \text{Gauge}}$$

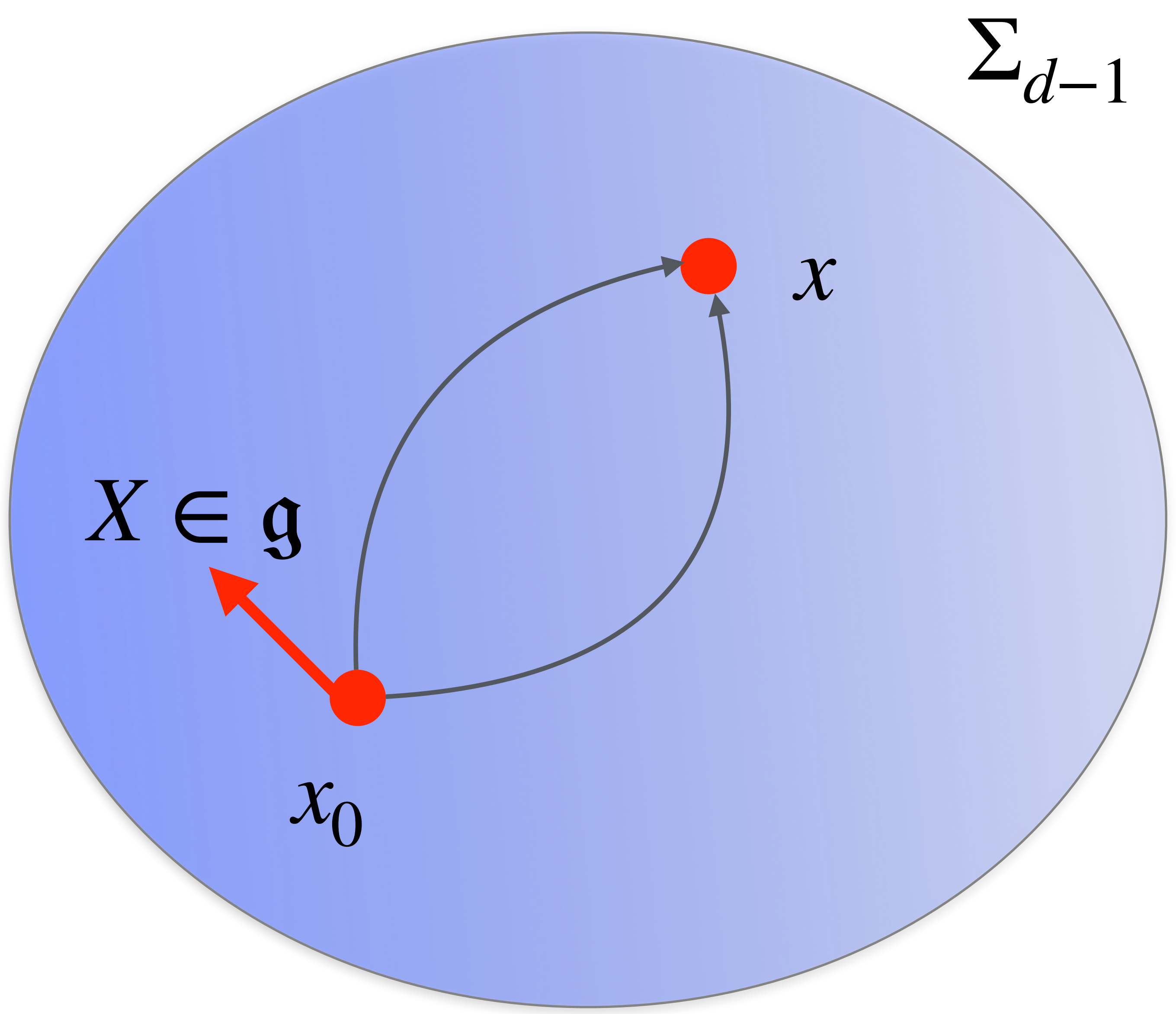
[Blau, Thompson '91 (review)]

Non-abelian BF theories: TDOs

Spectra of topological defects:

- $$\mathcal{U}_{\mathcal{R}}\left(\Sigma_1\right) = \mathrm{tr}_{\mathcal{R}}\mathcal{P}\exp\left\{i\int_{\Sigma_1} A\right\}$$
- $$\mathcal{U}_{[e^X]}\left(\Sigma_{d-1}\right) = \int_{[X]} \mathrm{dX}\exp\left\{\mathrm{i}\int_{\Sigma_{\mathrm{d}-1}} \langle \bar{\alpha}_0(\mathrm{x}), \mathrm{B}_{\mathrm{d}-1}\rangle\right\}$$

$$\bar{\alpha}_0 \in \Omega^0(\Sigma_{d-1}, \mathfrak{g}), \quad \mathrm{d}_A\bar{\alpha}_0 = 0, \quad \bar{\alpha}_0(x_0) = X \in \mathfrak{g}$$



Specific cases of a class of defects labeled by $([g], \mathcal{R}^{H_g})$ with H_g centralizer of $g \in G$.

[Cattaneo, Rossi '05], [Jia, Luo, Tian, Wang, Zhang '25], [Bonetti, Del Zotto, Minasian '25]

A SymTFT for conformal symmetry

Proposal: BF theory for gauge group $SO(d+1,1)$. Gauge fields on boundaries acquire geometric interpretations

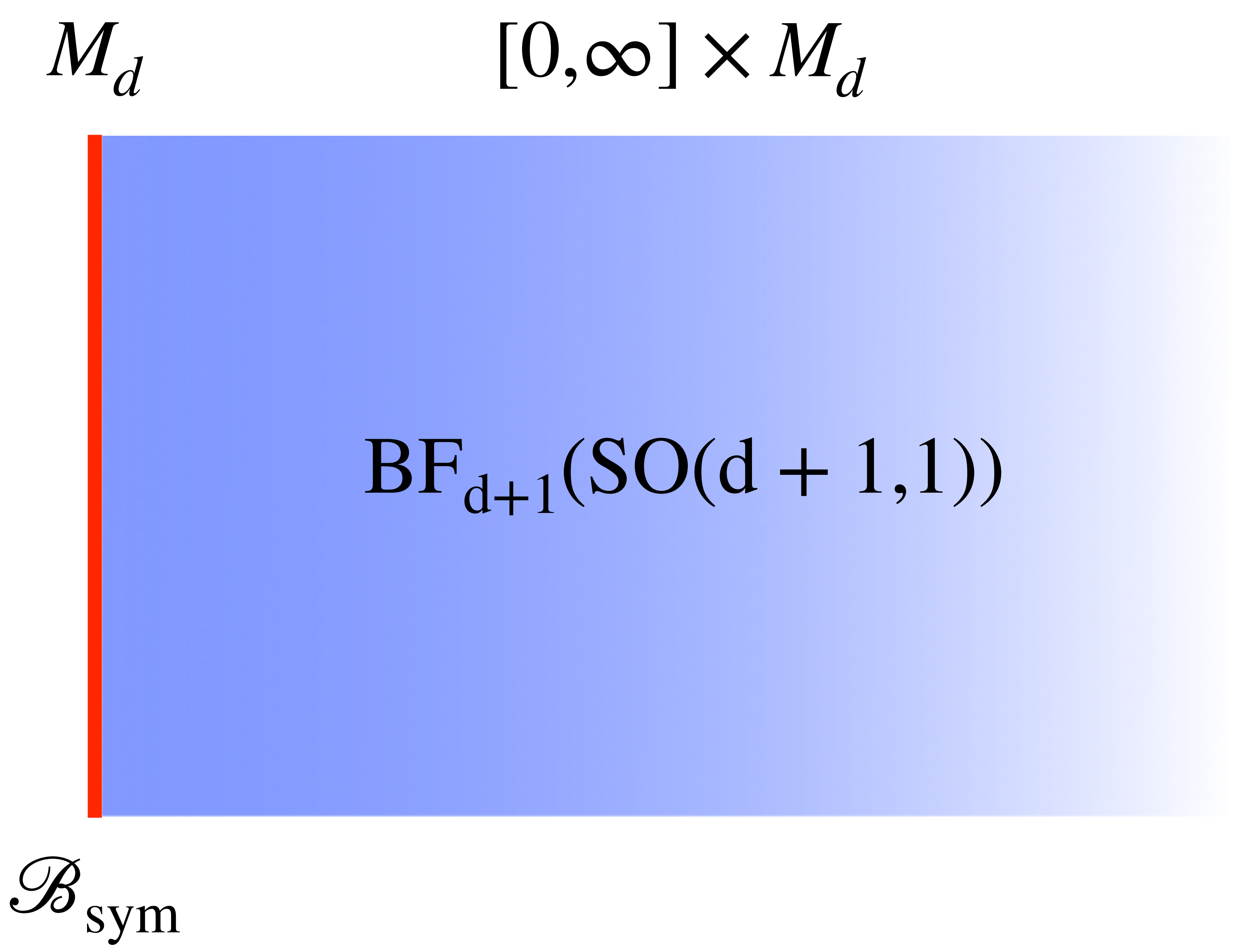
$$A\,|_{\mathscr{B}_{\text{sym}}} \equiv \mathscr{A} = e^a P_a + \frac{1}{2}\omega^{ab} L_{ab} + f^a K_a + \sigma D$$

In this talk, we take $\mathscr{B}_{\text{sym}}$ to be a *flat space*

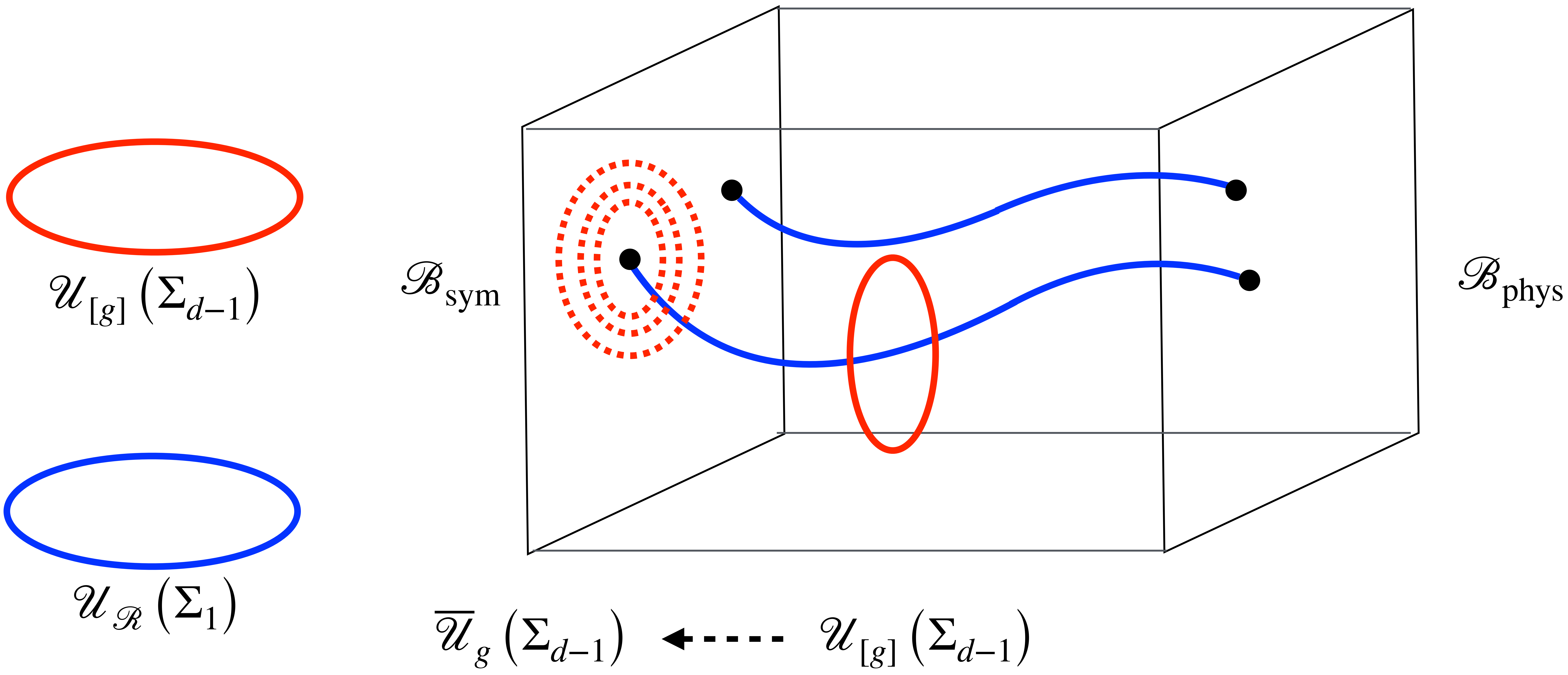
Dirichlet boundary condition:

$$S_\partial = \frac{i}{2\pi}\int_{\mathscr{B}_{\text{sym}}} \langle A - \mathscr{A}, B_{d-1}\rangle, \quad \mathscr{F} = 0$$

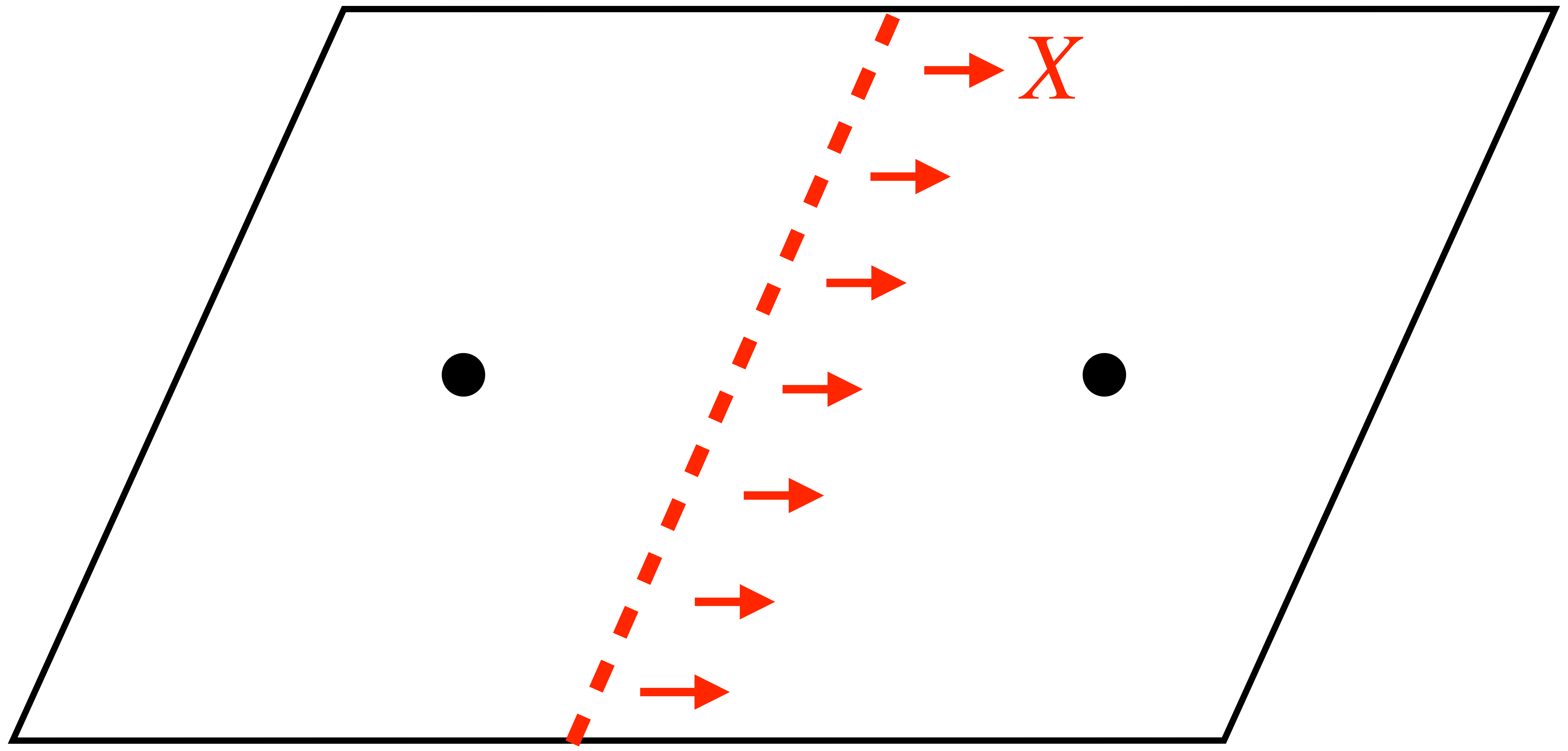
$$\mathscr{A} = \bar{e}^a \otimes P_a, \quad \bar{e}^a_{\ \mu} \in SO(d)$$

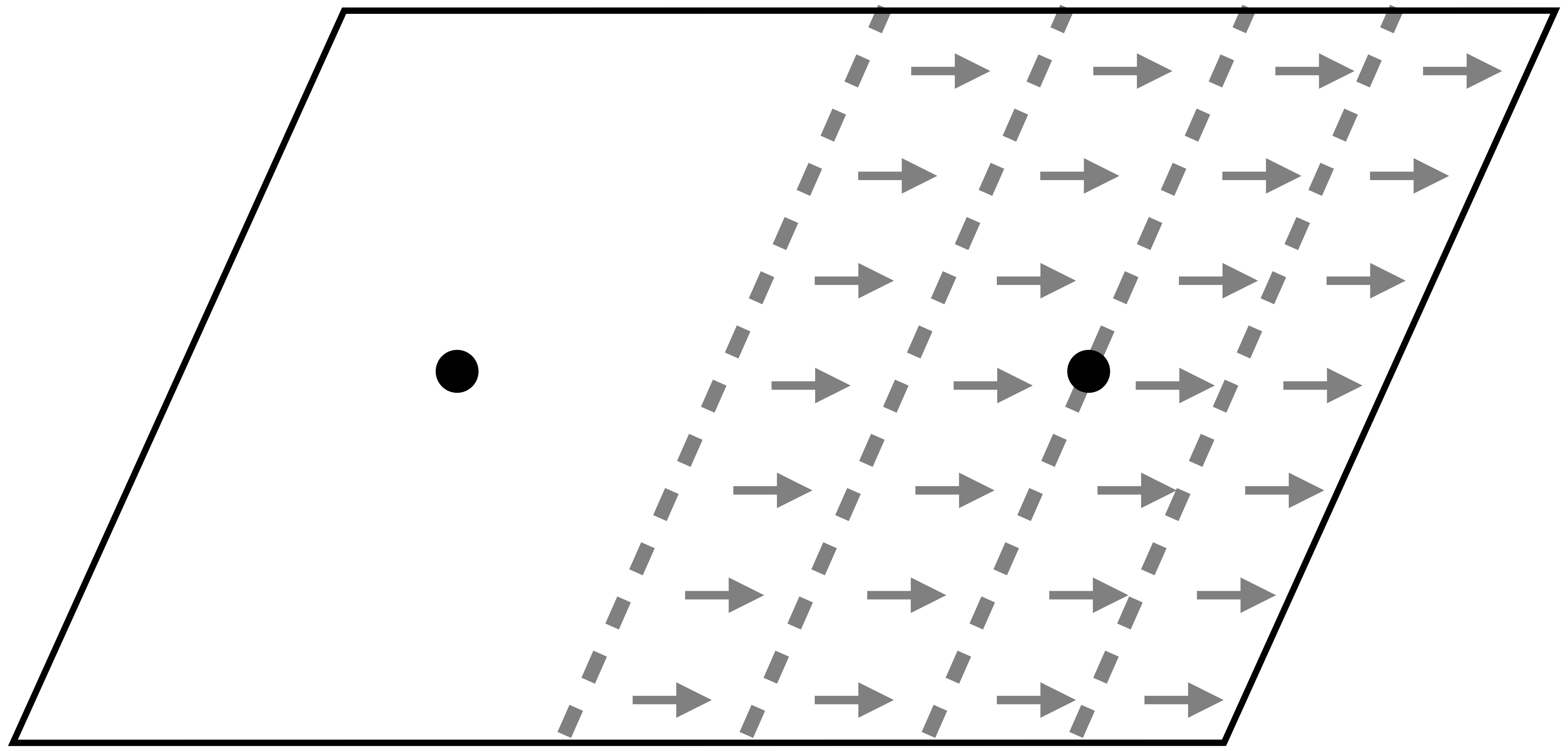


Linking action for TDOs



How to move a point





$\mathscr{B}_{\rm sym} : \mathscr{A} = \bar{e}^a \otimes P_a$

$\mathscr{B}_{\rm sym} : \mathscr{A}^{(X)} = [\bar{e}^a + X\,\delta^{(1)}(\Sigma_{d-1})] \otimes P_a$

The new background $\mathscr{A}^{(X)}$ is also flat $\Longrightarrow$ we can find a gauge transformation back to $\mathscr{A}$, or equivalently, a diffeomorphism $\xi(X)$!

$$\langle O(x)\,\overline{\mathscr{U}}_X(\Sigma_{d-1})\,O(y)\rangle_{\mathscr{A}} = \langle O(x)\,O(y)\rangle_{\mathscr{A}^{(X)}} = \langle O(x)\,e^{\mathscr{L}_{\xi(X)}}O(y)\rangle$$

Incorporating anomalies

In dimension $d+1=2n+1$, anomalies are incorporated by decorating the SymTFT with anomaly theories:

$$S^{(d=2)}=\frac{ik}{2\pi}\int_{I\times M_2}\left\{\langle A,F\rangle-\frac{1}{3}\langle A,A^2\rangle\right\}\qquad S^{(d=4)}=\frac{ik}{6(2\pi)^2}\int_{I\times M_4}\left\{\langle A,F,F\rangle-\frac{1}{2}\langle A,A^2,F\rangle+\frac{1}{10}\langle A,A^2,A^2\rangle\right\}$$

$$\langle\dots\rangle\,:\,\mathrm{Sym}^{\mathrm{n+1}}(\mathfrak{g})\to\mathbb{R}/\mathbb{C},\quad\langle \mathrm{gX_1g^{-1}},\dots,\mathrm{gX_{n+1}g^{-1}}\rangle=\langle\mathrm{X_1},\dots,\mathrm{X_{n+1}}\rangle$$

For $SO(2n+1,1)$ with generators M_{ab} there can be more than one multilinear products:

- For $n=2\ell+1,\ \ell\in\mathbb{Z}_{\geq0}$ there are two: symmetries version of $\sim(Tr[M_{ab}M^{ab}])^{\frac{n+1}{2}}$ and $\sim\epsilon_{a_1b_1\dots a_{n+1}b_{n+1}}M^{a_1b_1}\,..\,M^{a_{n+1}b_{n+1}}$ (gravitational anomaly + conformal type-A)
- For $n=2\ell,\ \ell\in\mathbb{Z}_{\geq0}$ there is only the ϵ-like (only conformal type-A).

Future directions

- Interplay between spacetime and internal (categorical) symmetries:

  - Extend the classification of gapless spacetime symmetry breaking phases.
  - Mixed gravitational - higher form anomalies? [Nicolis, Penco, Piazza, Rattazzi ’15]

- Can we realise spacetime supersymmetries?

  [Grassi, Penati, ’25], [Ambrosino, Luo, Wang, Zhang ’24], [Wang, Zhang ’23]

- Are type-B conformal anomalies realised by bulk topological couplings?

  [Imbimbo, Porro, ’25], [Imbimbo, Rovere, Warman, ’23]

Thank you for your attention!