Renormalization of Chiral Gauge Theories with non-anticommuting γ_5 in the BMHV Scheme at the Multi-Loop Level

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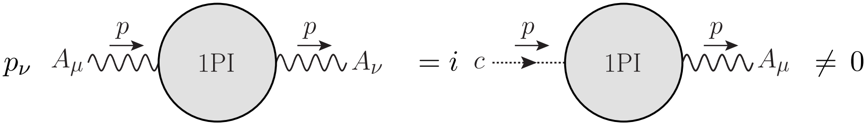
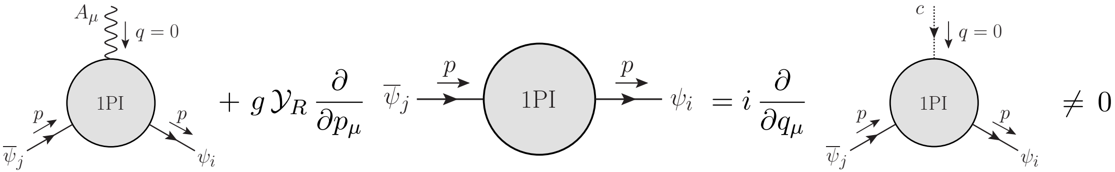
Preview — Breaking of Gauge Invariance in the BMHV Scheme

Spurious symmetry breakings induced by the BMHV algebra, e.g.:

- Violation of the transversality of the gauge boson self energy



- Violation of the fermion self energy to fermion gauge boson interaction current relation



The γ_5-Problem

Chiral Gauge Theories and the γ_5-Problem

- γ_5 is manifestly 4-dimensional $\implies D \neq 4$: cannot retain

$$\{\gamma_5, \gamma^\mu\} = 0 \tag{1}$$

$$\mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1) \tag{2}$$

$$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4i\varepsilon^{\mu\nu\rho\sigma} \tag{3}$$

- BMHV scheme: $\{\gamma^\mu, \gamma_5\} \neq 0$

$$\mathbb{M} = \mathbb{M}_4 \oplus \mathbb{M}_{-2\epsilon}, \qquad \eta_{\mu\nu} = \overline{\eta}_{\mu\nu} + \hat{\eta}_{\mu\nu}, \qquad X^\mu = \overline{X}^\mu + \hat{X}^\mu$$

$$\{\gamma^\mu, \gamma_5\} = \{\hat{\gamma}^\mu, \gamma_5\} = 2\hat{\gamma}^\mu\gamma_5, \qquad \{\overline{\gamma}^\mu, \gamma_5\} = 0, \qquad [\hat{\gamma}^\mu, \gamma_5] = 0$$

- Regularization induced symmetry breaking: $\mathcal{S}(\Gamma_{\mathrm{reg}}) \neq 0$
  $\implies$ include symmetry-restoring counterterms, satisfying $\mathcal{S}(\Gamma^{(n)}_{\mathrm{subren}}) = -bS^{(n)}_{\mathrm{ct}}$
  $\implies$ such that $\mathcal{S}(\Gamma_{\mathrm{ren}}) \stackrel{!}{=} 0$

Chiral Gauge Theories in D Dimensions and the BMHV Scheme

Chiral gauge theory

$$\mathcal{L} = i\overline{\psi}_i\gamma^\mu\partial_\mu\psi_i - g\mathcal{Y}_{Rij}B_\mu\overline{\psi}_i\mathbb{P}_\mathrm{L}\gamma^\mu\mathbb{P}_\mathrm{R}\psi_j + \ldots$$

Challenges in D dimensions:

1. Kinetic term, i.e. γ^μ, must be D-dimensional

$$\overline{\psi}\partial\!\!\!/\psi = \overline{\psi_L}\overline{\partial\!\!\!/}\psi_L + \overline{\psi_R}\overline{\partial\!\!\!/}\psi_R + \overline{\psi_L}\widehat{\partial\!\!\!/}\psi_R + \overline{\psi_R}\widehat{\partial\!\!\!/}\psi_L$$

$\Longrightarrow$ kinetic term necessarily mixes chiralities

2. Analytic continuation to D dimensions not unique
   - $\mathbb{P}_\mathrm{L}\gamma^\mu\mathbb{P}_\mathrm{R}$ admits many inequivalent but equally correct extensions
   - use the most symmetric option $\Longrightarrow$ most natural and symmetric choice

$$\overline{\psi}\mathbb{P}_\mathrm{L}\gamma^\mu\mathbb{P}_\mathrm{R}\psi = \overline{\psi}\mathbb{P}_\mathrm{L}\overline{\gamma}^\mu\mathbb{P}_\mathrm{R}\psi = \overline{\psi_R}\overline{\gamma}^\mu\psi_R$$

Always: Mismatch D versus 4 $\Longrightarrow$ breaks gauge invariance

Symmetry Restoration

Symmetry Restoration — Quantum Action Principle

- The ultimate symmetry requirement is the Slavnov-Taylor identity

$$\underset{D\to 4}{\mathrm{LIM}}\left(\mathcal{S}_D(\Gamma_{\mathrm{DRen}})\right)=0$$

- Regularized Quantum Action Principle of Dimensional Regularization

DReg:[BM'77], DRed:[DS'05]
Rev:[OP,SS'95, HB et al.'23]

$$\mathcal{S}_D(\Gamma_{\mathrm{DRen}})=(\widehat{\Delta}+\Delta_{\mathrm{ct}})\cdot\Gamma_{\mathrm{DRen}}$$

- Possible symmetry breaking can be rewritten as a composite operator insertion

$$\widehat{\Delta}=\mathcal{S}_D(S_0),\qquad \widehat{\Delta}+\Delta_{\mathrm{ct}}=\mathcal{S}_D(S_0+S_{\mathrm{ct}})$$

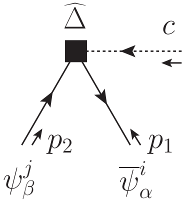
Symmetry Restoration — Practical Application

- Perturbative requirement from the Slavnov-Taylor identity

$$\underset{D\to 4}{\mathrm{LIM}} \left(\widehat{\Delta} \cdot \Gamma^{n}_{\mathrm{DRen}} + \sum_{k=1}^{n-1} \Delta^{k}_{\mathrm{ct}} \cdot \Gamma^{n-k}_{\mathrm{DRen}} + \Delta^{n}_{\mathrm{ct}} \right) = 0$$

- Tree-level breaking: $\widehat{\Delta}$-operator reflects the breaking of chiral gauge invariance

$$\mathcal{S}_D(S_0) = \widehat{\Delta} = -\int d^D x\, g\, \mathcal{Y}_{Rij}\, c \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\partial\!\!\!/}} \mathbb{P}_{\mathrm{R}} + \overrightarrow{\widehat{\partial\!\!\!/}} \mathbb{P}_{\mathrm{L}} \right) \psi_j \right\}$$

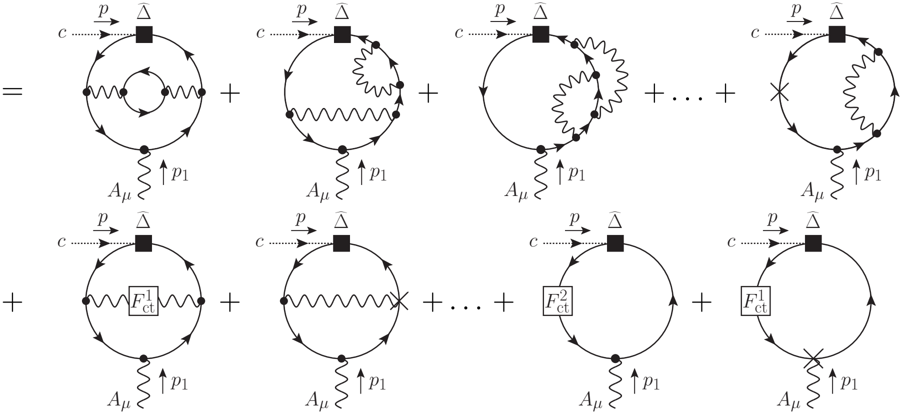


$$= -g\, \mathcal{Y}_{Rij} \left(\widehat{p\!\!\!/}_1 \mathbb{P}_{\mathrm{R}} + \widehat{p\!\!\!/}_2 \mathbb{P}_{\mathrm{L}} \right)_{\alpha\beta}$$

- Compute Green functions involving an insertion of the composite operator $\widehat{\Delta} + \Delta_{\mathrm{ct}}$

Symmetry Restoration — 3-Loop Application

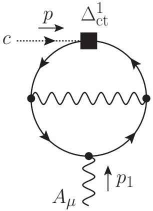
$$i\big(\widehat{\Delta}\cdot\Gamma^3\big)_{A_\mu c} =$$



$$+ \quad \dots$$

Symmetry Restoration — 3-Loop Application

$$i\left(\Delta^1_{\text{ct}}\cdot\Gamma^2\right)_{A_\mu c} =$$



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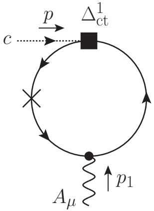
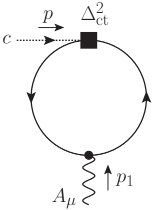


$+ \ldots$

$$i\left(\Delta^2_{\text{ct}}\cdot\Gamma^1\right)_{A_\mu c} =$$



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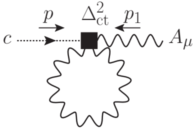
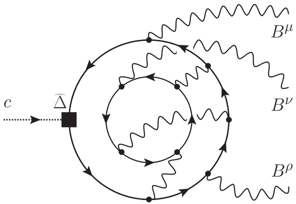


Symmetry Restoration — 4-Loop Application

[AM,DS,MW'25]

$$i\big(\widehat{\Delta}\cdot\Gamma^4\big)_{B_\rho B_\nu B_\mu c} =$$



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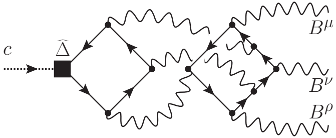
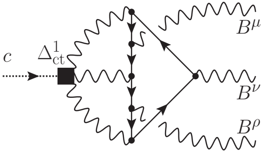


$+ \ \ldots$

$$i\big(\Delta^1_{\mathrm{ct}}\cdot\Gamma^3\big)_{B_\rho B_\nu B_\mu c} =$$



$+ \ \ldots$

Results

Right-Handed Model

Gauge Boson Self Energy in an Abelian Chiral Gauge Theory

1-loop result [HB et al. '21,'23]

$$i\Gamma^{\nu\mu}_{BB}(p)\Big|^{1}_{\text{div}} = \frac{ig^2}{16\pi^2}\frac{\mathsf{Tr}\left(\mathcal{Y}_R^2\right)}{3}\left[\frac{2}{\epsilon}\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \frac{1}{\epsilon}\widehat{p}^2\overline{\eta}^{\mu\nu}\right]$$

2-loop result

$$i\Gamma^{\nu\mu}_{BB}(p)\Big|^{2}_{\text{div}} = \frac{ig^4}{(16\pi^2)^2}\frac{\mathsf{Tr}\left(\mathcal{Y}_R^4\right)}{3}\left[\frac{2}{\epsilon}\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \left(\frac{1}{2\epsilon^2} - \frac{17}{24\epsilon}\right)\widehat{p}^2\overline{\eta}^{\mu\nu}\right]$$

3-loop result [DS,MW'23]

$$\begin{aligned}
i\Gamma^{\nu\mu}_{BB}(p)\Big|^{3}_{\text{div}} = &- \frac{ig^6}{(16\pi^2)^3}\left[\mathcal{B}^{3,\text{inv}}_{BB}\,\frac{1}{\epsilon^2} + \mathcal{A}^{3,\text{inv}}_{BB}\,\frac{1}{\epsilon}\right]\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right)\\
&- \frac{ig^6}{(16\pi^2)^3}\left[\widehat{\mathcal{C}}^{3,\text{break}}_{BB}\,\frac{1}{\epsilon^3} + \widehat{\mathcal{B}}^{3,\text{break}}_{BB}\,\frac{1}{\epsilon^2} + \widehat{\mathcal{A}}^{3,\text{break}}_{BB}\,\frac{1}{\epsilon}\right]\widehat{p}^2\,\overline{\eta}^{\mu\nu}\\
&+ \frac{ig^6}{(16\pi^2)^3}\,\overline{\mathcal{A}}^{3,\text{break}}_{BB}\,\frac{1}{\epsilon}\,\overline{p}^2\,\overline{\eta}^{\mu\nu},
\end{aligned}$$

Gauge Boson Self Energy in an Abelian Chiral Gauge Theory

4-loop result [AM,DS,MW'25]

$$i\Gamma^{\nu\mu}_{BB}(p)\big|^4_{\mathrm{div}} = -\frac{ig^8}{(16\pi^2)^4}\,\delta Z_B^{(4)}\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) + \frac{ig^8}{(16\pi^2)^4}\left(\delta\widehat{X}^{(4)}_{BB}\,\widehat{p}^2\,\overline{\eta}^{\mu\nu} + \delta\overline{X}^{(4)}_{BB}\,\overline{p}^2\,\overline{\eta}^{\mu\nu}\right),$$

with

$$\begin{aligned}
\delta Z_B^{(4)} &= \mathcal{A}^{4,\mathrm{inv}}_{BB}\,\frac{1}{\epsilon} + \mathcal{B}^{4,\mathrm{inv}}_{BB}\,\frac{1}{\epsilon^2} + \mathcal{C}^{4,\mathrm{inv}}_{BB}\,\frac{1}{\epsilon^3},\\
\delta\widehat{X}^{(4)}_{BB} &= \widehat{\mathcal{A}}^{4,\mathrm{break}}_{BB}\,\frac{1}{\epsilon} + \widehat{\mathcal{B}}^{4,\mathrm{break}}_{BB}\,\frac{1}{\epsilon^2} + \widehat{\mathcal{C}}^{4,\mathrm{break}}_{BB}\,\frac{1}{\epsilon^3} + \widehat{\mathcal{D}}^{4,\mathrm{break}}_{BB}\,\frac{1}{\epsilon^4},\\
\delta\overline{X}^{(4)}_{BB} &= \overline{\mathcal{A}}^{4,\mathrm{break}}_{BB}\,\frac{1}{\epsilon} + \overline{\mathcal{B}}^{4,\mathrm{break}}_{BB}\,\frac{1}{\epsilon^2},
\end{aligned}$$

- Transversality of the gauge boson self energy is violated
- Violation is local
- Symmetry restoration necessary and possible

Violation of the Gauge Boson Transversality

4-loop result [AM,DS,MW'25]

$$\begin{aligned} i\Delta \cdot \Gamma\big|^{4}_{B_\mu c} &= i\widehat{\Delta} \cdot \Gamma^{4}_{B_\mu c} + i\Delta^{1}_{\text{ct}} \cdot \Gamma^{3}_{B_\mu c} + i\Delta^{2}_{\text{ct}} \cdot \Gamma^{2}_{B_\mu c} + i\Delta^{3}_{\text{ct}} \cdot \Gamma^{1}_{B_\mu c} \\ &= \frac{g^8}{(16\pi^2)^4} \left[\delta\widehat{X}^{(4)}_{BB}\, \widehat{p}^2\, \overline{p}^\mu + \left(\delta\overline{X}^{(4)}_{BB} + \mathcal{F}^{4,\text{break}}_{BB} \right) \overline{p}^2\, \overline{p}^\mu \right], \end{aligned}$$

with $\delta\widehat{X}^{(4)}_{BB}$ and $\delta\overline{X}^{(4)}_{BB}$ as before, as well as

$$\begin{aligned} \mathcal{F}^{4,\text{break}}_{BB} = &\left(\frac{403759}{25920} + \frac{\pi^4}{3375} + \frac{30113\zeta(3)}{4500} - \frac{403\zeta(5)}{45} \right) \mathsf{Tr}\big(\mathcal{Y}^8_R\big) \\ &+ \left(\frac{4525151}{11664000} - \frac{17\,\pi^4}{20250} - \frac{15569\zeta(3)}{40500} \right) \mathsf{Tr}\big(\mathcal{Y}^6_R\big)\, \mathsf{Tr}\big(\mathcal{Y}^2_R\big) \\ &+ \left(\frac{38768057}{7776000} + \frac{12061\zeta(3)}{900} - \frac{713\zeta(5)}{45} \right) \mathsf{Tr}\big(\mathcal{Y}^4_R\big)^2 - \frac{4240349}{69984000} \mathsf{Tr}\big(\mathcal{Y}^4_R\big)\, \mathsf{Tr}\big(\mathcal{Y}^2_R\big)^2 \end{aligned}$$

Singular Counterterm Action $S_{\rm sct} = S_{\rm sct,inv} + S_{\rm sct,break}$

At the L-loop level (with $L \in \{1,2,3,4\}$): [AM,DS,MW'25]

$$S^{(L)}_{\rm sct,inv} = \frac{g^{2L}}{(16\pi^2)^L} \int d^Dx \left\{ \delta Z^{(L)}_B \Big(-\frac{1}{4} \overline{F}^{\mu\nu} \overline{F}_{\mu\nu} \Big) + \delta Z^{(L)}_{\psi,kj} \overline{\psi}_i i \overline{\not{D}}_{\mathrm{R},ik} \psi_j \right\},$$

and

$$\begin{aligned} S^{(L)}_{\rm sct,break} = \frac{g^{2L}}{(16\pi^2)^L} \int d^Dx \bigg\{ & \delta \widehat{X}^{(L)}_{BB} \frac{1}{2} \overline{B}_\mu \widehat{\partial}^2 \overline{B}^\mu + \delta \overline{X}^{(L)}_{BB} \frac{1}{2} \overline{B}_\mu \overline{\partial}^2 \overline{B}^\mu \\ & + g^2 \delta \overline{X}^{(L)}_{BBBB} \frac{1}{8} \overline{B}_\mu \overline{B}^\mu \overline{B}_\nu \overline{B}^\nu + \delta \overline{X}^{(L)}_{\overline{\psi}\psi,ij} \Big(\overline{\psi}_i i \overline{\not{\partial}} \mathbb{P}_{\mathrm{R}} \psi_j \Big) \bigg\}, \end{aligned}$$

with

$$\overline{D}^\mu_{R,ij} = \Big(\overline{\partial}^\mu \delta_{ij} + i g \mathcal{Y}_{R,ij} \overline{B}^\mu \Big) \mathbb{P}_{\mathrm{R}}.$$

Finite Counterterm Action S_{fct}

At the L-loop level (with $L \in \{1,2,3,4\}$): [AM,DS,MW'25]

$$S_{\text{fct}}^{(L)} = \frac{g^{2L}}{(16\pi^2)^L} \int d^4x \Big\{ \mathcal{F}_{BB}^{L,\text{break}} \frac{1}{2} \overline{B}_\mu \overline{\partial}^2 \overline{B}^\mu + g^2 \mathcal{F}_{BBBB}^{L,\text{break}} \frac{1}{8} \overline{B}_\mu \overline{B}^\mu \overline{B}_\nu \overline{B}^\nu + \mathcal{F}_{\overline{\psi}\psi,ij}^{L,\text{break}} \overline{\psi}_i i \overline{\not{\partial}} \mathbb{P}_{\mathrm{R}} \psi_j \Big\}$$

At the 4-loop level, for Standard Model hypercharges $\mathcal{Y}_R^{\text{SM}}$, we find:

$$\mathcal{F}_{BB}^{4,\text{SM}} \approx 140.38 \qquad \mathcal{F}_{\overline{e}e}^{4,\text{SM}} \approx -108.73 \qquad \mathcal{F}_{\overline{L}L}^{4,\text{SM}} \approx 70.35$$

$$\mathcal{F}_{BBBB}^{4,\text{SM}} \approx -456.91 \qquad \mathcal{F}_{\overline{u}u}^{4,\text{SM}} \approx 112.46 \qquad \mathcal{F}_{\overline{Q}Q}^{4,\text{SM}} \approx -0.84$$

$$\mathcal{F}_{\overline{d}d}^{4,\text{SM}} \approx 5.29$$

The Standard Model

Fermions in D Dimensions

In principal 2 options [PE,PK,DS,MW'24]

1. Option 1: $(\psi_L + \psi_R \longrightarrow \psi)$
   - electron: $e = e_L + e_R$

$$\mathcal{L}_{\mathrm{kin},e} = \bar{e} i \partial\!\!\!/ e = \overline{e_L} i \overline{\partial\!\!\!/} e_L + \overline{e_R} i \overline{\partial\!\!\!/} e_R + \overline{e_L} i \widehat{\partial\!\!\!/} e_R + \overline{e_R} i \widehat{\partial\!\!\!/} e_L$$

   - $\widehat{\partial\!\!\!/}$-terms mix physical fields with different gauge quantum numbers
2. Option 2: $(\psi_L + \psi_R^{\mathrm{st}} \longrightarrow \psi_1)$ and $(\psi_L^{\mathrm{st}} + \psi_R \longrightarrow \psi_2)$
   - electron: $e_1 = e_L + e_R^{\mathrm{st}}$ and $e_2 = e_L^{\mathrm{st}} + e_R$

$$\begin{aligned}\mathcal{L}_{\mathrm{kin},e} = \bar{e}_1 i \partial\!\!\!/ e_1 + \bar{e}_2 i \partial\!\!\!/ e_2 = {} & \overline{e_L} i \overline{\partial\!\!\!/} e_L + \overline{e_R^{\mathrm{st}}} i \overline{\partial\!\!\!/} e_R^{\mathrm{st}} + \overline{e_L} i \widehat{\partial\!\!\!/} e_R^{\mathrm{st}} + \overline{e_R^{\mathrm{st}}} i \widehat{\partial\!\!\!/} e_L \\ & + \overline{e_L^{\mathrm{st}}} i \overline{\partial\!\!\!/} e_L^{\mathrm{st}} + \overline{e_R} i \overline{\partial\!\!\!/} e_R + \overline{e_L^{\mathrm{st}}} i \widehat{\partial\!\!\!/} e_R + \overline{e_R} i \widehat{\partial\!\!\!/} e_L^{\mathrm{st}}\end{aligned}$$

   - preserves global symmetry
   - proliferation of terms
   - complicated propagator in the massive case

Evanescent Interactions in the BMHV Scheme

Example: W-electron-neutrino interaction [PE,PK,DS,MW'24]

- 4 dimensions: $\overline{e}\,\overline{\gamma}^\mu \mathbb{P}_\mathrm{L} \nu W^-_\mu + \mathrm{h.c.}$
- D dimensions: $\overline{e}\,\mathbb{P}_\mathrm{R}\overline{\gamma}^\mu \mathbb{P}_\mathrm{L} \nu \overline{W}^-_\mu + \overline{e}\,\mathbb{P}_\mathrm{L}\widehat{\gamma}^\mu \mathbb{P}_\mathrm{L} \nu \widehat{W}^-_\mu + \mathrm{h.c.}$

  or: $\overline{e}\,\mathbb{P}_\mathrm{R}\overline{\gamma}^\mu \mathbb{P}_\mathrm{L} \nu \overline{W}^-_\mu + \mathrm{h.c.}$

In general:

$$\begin{aligned}\mathcal{L} \supset \overline{\psi}_i i\partial\!\!\!/ \psi_i &- g\mathcal{Y}_{Rij} B_\mu \overline{\psi}_i \mathbb{P}_\mathrm{L}\gamma^\mu \mathbb{P}_\mathrm{R}\psi_j - g\mathcal{Y}_{Lij} B_\mu \overline{\psi}_i \mathbb{P}_\mathrm{R}\gamma^\mu \mathbb{P}_\mathrm{L}\psi_j \\ &- g\mathcal{Y}_{LRij} B_\mu \overline{\psi}_i \mathbb{P}_\mathrm{R}\gamma^\mu \mathbb{P}_\mathrm{R}\psi_j - g\mathcal{Y}_{RLij} B_\mu \overline{\psi}_i \mathbb{P}_\mathrm{L}\gamma^\mu \mathbb{P}_\mathrm{L}\psi_j\end{aligned}$$

Evanescent interaction currents

- restricted by hermiticity, CPT-invariance and non-broken symmetries
- additional tree-level breaking

$$\mathcal{S}_D(S_0) = \widehat{\Delta} = \widehat{\Delta}_{c\overline{\psi}\psi} + \widehat{\Delta}_{cB\overline{\psi}\psi}$$

Application to the Standard Model

Standard Model quarks in the BMHV scheme [PK,AM,DS,MW] to be published

- photon interaction current

$$eQ_{ab}\overline{q}_I^{i,a}\not{A}q_I^{i,b} - (1-c_{\text{QED}})eQ_{ab}\overline{q}_I^{i,a}\widehat{\not{A}}q_I^{i,b}$$

- Z_μ interaction current

$$-\frac{s_W}{c_W}eQ_{ab}\overline{q}_I^{i,a}\not{Z}q_I^{i,b} + \frac{g_W}{c_W}t^3_{L,ab}\overline{q}_I^{i,a}\overline{\not{Z}}\mathbb{P}_{\text{L}}q_I^{i,b} + (1-c_{\text{QED}})\frac{s_W}{c_W}eQ_{ab}\overline{q}_I^{i,a}\widehat{\not{Z}}q_I^{i,b}$$

- $W_\mu^\pm$ interaction current

$$\frac{g_W}{\sqrt{2}}\tau^+_{ab}\,\overline{q}_I^{i,a}\overline{\not{W}^+}\mathbb{P}_{\text{L}}q_I^{i,b} + \frac{g_W}{\sqrt{2}}\tau^-_{ab}\,\overline{q}_I^{i,a}\overline{\not{W}^-}\mathbb{P}_{\text{L}}q_I^{i,b}$$

- gluon interaction current

$$g_s t^A_{s,ij}\overline{q}_I^{i,a}\not{G}^A q_I^{j,a} - (1-c_{\text{QCD}})g_s t^A_{s,ij}\overline{q}_I^{i,a}\widehat{\not{G}^A}q_I^{j,a}$$

Thank You

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BackUp Slides

Chiral Gauge Theories and the γ_5-Problem

- Electroweak interactions act on chiral fermions
  - Left-handed and right-handed fermions interact differently with gauge bosons
  - SM and all its extensions for potential new physics are chiral gauge theories
- γ_5 is manifestly 4-dimensional
- Cannot simultaneously retain the following properties in D dimensions

$$\{\gamma_5, \gamma^\mu\} = 0 \tag{4}$$

$$\mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1) \tag{5}$$

$$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4i\varepsilon^{\mu\nu\rho\sigma} \tag{6}$$

- Inconsistent in $D \neq 4 \implies \gamma_5$-problem
- Give up at least one property
  - Breitenlohner-Maison/'t Hooft-Veltman (BMHV) scheme [HV'72, BM'77]
  - Alternative γ_5-schemes (Naive anticommuting? Reading point? ...)

Technical Aspects of the γ-Algebra in the BMHV Scheme

1. Orthogonality of $\mathbb{M}_4$ and $\mathbb{M}_{-2\epsilon} \implies 4D$ – evanescent contractions vanish

$$\overline{\eta}_{\mu\nu}\mathrm{Tr}(\ldots\widehat{\gamma}^{\nu}\ldots) = 0$$

2. Normal form of γ-traces:

$$\mathrm{Tr}(\widehat{\gamma}^{\nu_1}\ldots\widehat{\gamma}^{\nu_m}\overline{\gamma}^{\mu_1}\ldots\overline{\gamma}^{\mu_n}\mathbb{\Lambda}), \qquad \mathbb{\Lambda} \in \{\mathbb{1}, \gamma_5, \mathbb{P}_{\mathrm{L}}, \mathbb{P}_{\mathrm{R}}\}$$

3. γ-trace factorisation:

$$\mathrm{Tr}(\widehat{\gamma}^{\nu_1}\ldots\widehat{\gamma}^{\nu_m}\overline{\gamma}^{\mu_1}\ldots\overline{\gamma}^{\mu_n}\mathbb{\Lambda}) = \frac{1}{4}\mathrm{Tr}(\widehat{\gamma}^{\nu_1}\ldots\widehat{\gamma}^{\nu_m})\mathrm{Tr}(\overline{\gamma}^{\mu_1}\ldots\overline{\gamma}^{\mu_n}\mathbb{\Lambda})$$

Symmetries — Slavnov-Taylor Identity

Consistent theory: $\mathcal{S}(\Gamma_{\text{ren}}) \stackrel{!}{=} 0$

- physical Hilbert space with positive norm states
- unitary and gauge independent physical S-matrix

Distinguish two cases: [OP,SS'95, HB et al.'23]

1. Regularization preserves symmetries: $\mathcal{S}(\Gamma_{\text{reg}}) = 0$
   $\Longrightarrow$ standard multiplicative renormalization
2. Regularization breaks symmetries: $\mathcal{S}(\Gamma_{\text{reg}}) \neq 0$
   $\Longrightarrow$ include symmetry-restoring counterterms, satisfying $\mathcal{S}(\Gamma^{(n)}_{\text{subren}}) = -bS^n_{\text{ct}}$

Regularization induced symmetry breakings need to be restored

- Symmetries usually valid in DReg
- However, no gauge-invariant regularization that preserves chiral symmetry $\Longrightarrow$ γ_5-problem

Ward Identities in abelian chiral Gauge Theories

Abelian gauge theories: Slavnov-Taylor identity $\mathcal{S}(\Gamma) = 0 \implies$ Ward identities

$$
\begin{aligned}
p_\nu \, \widetilde{\Gamma}^{\nu\mu}_{AA} &= 0, \\
p_\sigma \, \widetilde{\Gamma}^{\sigma\rho\nu\mu}_{AAAA} &= 0, \\
\widetilde{\Gamma}^{ji,\mu}_{\psi\overline{\psi}A}(q=0) &= -e\,\mathcal{Y}_{Rjk}\,\frac{\partial}{\partial p_\mu}\widetilde{\Gamma}^{ki}_{\psi\overline{\psi}}
\end{aligned}
$$

Distinguish two cases

1. Regularization preserves symmetries
   $\implies$ standard multiplicative renormalization
2. Regularization breaks symmetries
   $\implies$ include symmetry-restoring counterterms, satisfying $\mathcal{S}(\Gamma^{(n)}_{\text{subren}}) = -bS^n_{\text{ct}}$

[OP,SS'95, HB et al.'23]

Algebraic Renormalization

- Symmetry restoration $\longrightarrow$ inductive procedure
- Symmetry breaking is restricted in two ways
  - $\Delta =$ local polynomial restricted by power counting
  - Wess-Zumino consistency condition: $b\,\Delta = 0$
- Trivial elements of the BRST Cohomology, i.e. $\exists\,\Delta'$ with $\Delta = b\,\Delta'$
  - Symmetry can be established to all orders
  - Original action can be equipped with a new counterterm

$$S_{\text{fct}} = -\Delta'$$

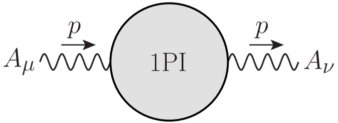
- Non-trivial elements of the BRST Cohomology
  - Anomaly; the breaking cannot be restored
  - Anomaly cancellation conditions can be derived, e.g.

$$\mathrm{Tr}(\mathcal{Y}_R^3) = 0$$

Symmetry Restoration — Explicit Evaluation

Explicit evaluation of all Green functions in $\mathcal{S}_D(\Gamma^{(n)}_{\text{subren}} + S^n_{\text{sct}})$ [HB et al. '21,'23]

$$i\,\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^1 =$$



$$\Big|^1$$

$$= \frac{ie^2}{16\pi^2}\frac{\mathrm{Tr}(\mathcal{Y}_R^2)}{3}\Bigg\{\frac{2}{\epsilon}\left[\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \frac{1}{2}\widehat{p}^2\overline{\eta}^{\mu\nu}\right]$$

$$+\left[\left(\frac{10}{3} - 2\ln(-p^2)\right)\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \left(\overline{p}^2 + \widehat{p}^2\left(\frac{8}{3} - \ln(-p^2)\right)\right)\overline{\eta}^{\mu\nu}\right]\Bigg\}$$

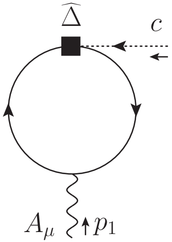
Exhibit the breaking explicitly — probe the gauge boson Ward identity

$$[\mathcal{S}(\Gamma)]^1_{A^\mu c} = i\,p_\nu\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^1 = \frac{ie^2}{16\pi^2}\frac{\mathrm{Tr}(\mathcal{Y}_R^2)}{3}\left[-\frac{1}{\epsilon}\widehat{p}^2\overline{p}^\mu - \overline{p}^2\overline{p}^\mu\right] \neq 0$$

Symmetry Restoration — Quantum Action Principle

Breaking of the gauge boson self energy at 1-loop [HB et al. '21,'23]

$$i\left(\widehat{\Delta}\cdot\Gamma^1\right)_{A_\mu c} =$$



$$= \frac{e^2}{16\pi^2}\frac{\text{Tr}(\mathcal{Y}_R^2)}{3}\left[-\frac{1}{\epsilon}\widehat{p}^2\overline{p}^\mu - \overline{p}^2\overline{p}^\mu\right]$$

$$= -i\left[\mathcal{S}(\Gamma)\right]^1_{A^\mu c} = p_\nu \widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^1$$

1-loop breaking of the Slavnov-Taylor identity

$$\left(\widehat{\Delta}\cdot\Gamma\right)^1 = -\frac{1}{16\pi^2}\int d^Dx\,\frac{e^2\,\text{Tr}\left(\mathcal{Y}_R^2\right)}{3}\left[\frac{1}{\epsilon}\,c\,\overline{\partial}_\mu\,\widehat{\partial}^2\,\overline{A}^\mu + c\,\overline{\partial}_\mu\,\overline{\partial}^2\,\overline{A}^\mu\right] + \ldots$$

Quantum Action Principle vs. Explicit Evaluation

Simplification is threefold

+ Only UV-divergent part of Green functions contributes
+ Only power-counting divergent Green functions required
+ In general fewer diagrams with Δ insertion

Cons

– Requires special Δ-operator inserted Green functions

$\Longrightarrow$ Procedure with the Quantum Action Principle is much more efficient!