Renormalization and UV behaviour of 5D gauge theories

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~ DESY Theory Workshop ~

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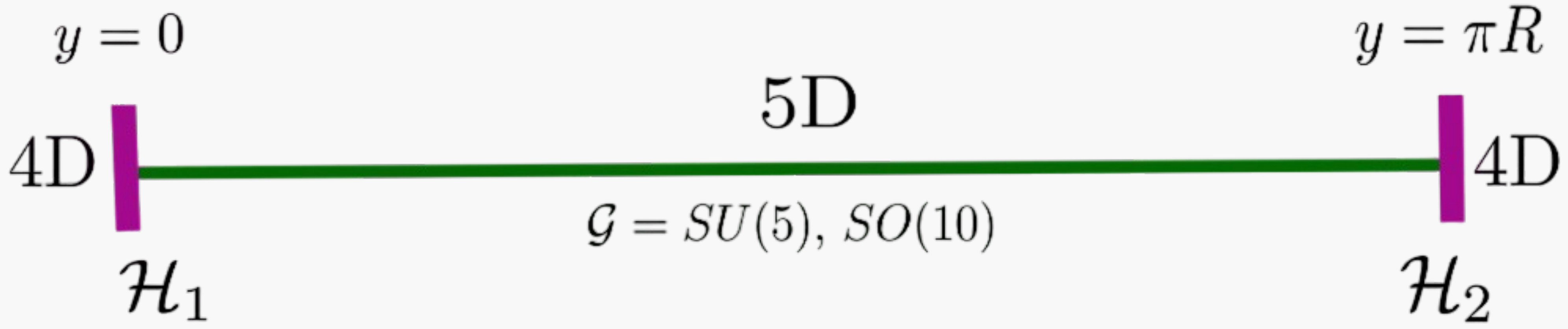
Outline

- Some comments on asymptotic GUTs (see Wanda's slides)
- RGEs: from 4D to 5D
- Renormalization of 5D gauge theories (one loop)
- Conclusions

Asymptotic GUTs

- **5D models**: one extra dimension [1] compactified on $K = S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2$



- Symmetry is broken via orbifolding ($\mathcal{G} \to \mathcal{H}_i$ on each boundary [2])

$$\mathcal{G}_{\rm 4D} \equiv \mathcal{H}_i \cap \mathcal{H}_j \supset \mathcal{G}_{\rm SM}$$

remnant 4D theory

[1] A. Hebecker, J. March-Russell, Nuclear Phys. B 625 (2002)
[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

Kaluza Klein decomposition

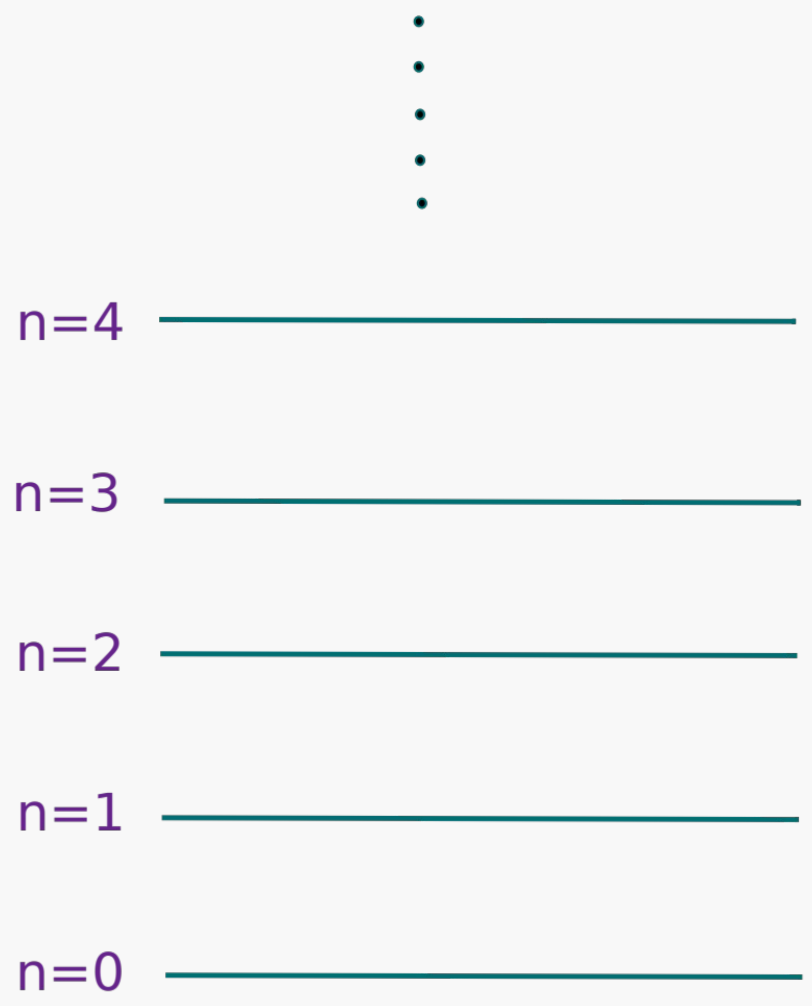
$$\Phi(x^\mu, y) = \sum_{n=0}^{\infty} \phi_+^{(n)}(x^\mu)\cos(ny/R) + \sum_{n=1}^{\infty} \phi_-^{(n)}(x^\mu)\sin(ny/R)$$

Any 5D field

$$\Phi_{5\mathrm{D}}$$

(scalar, fermion, gauge boson)

Infinite tower of 4D fields



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Any 5D field

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(scalar, fermion, gauge boson)

Infinite tower of 4D fields

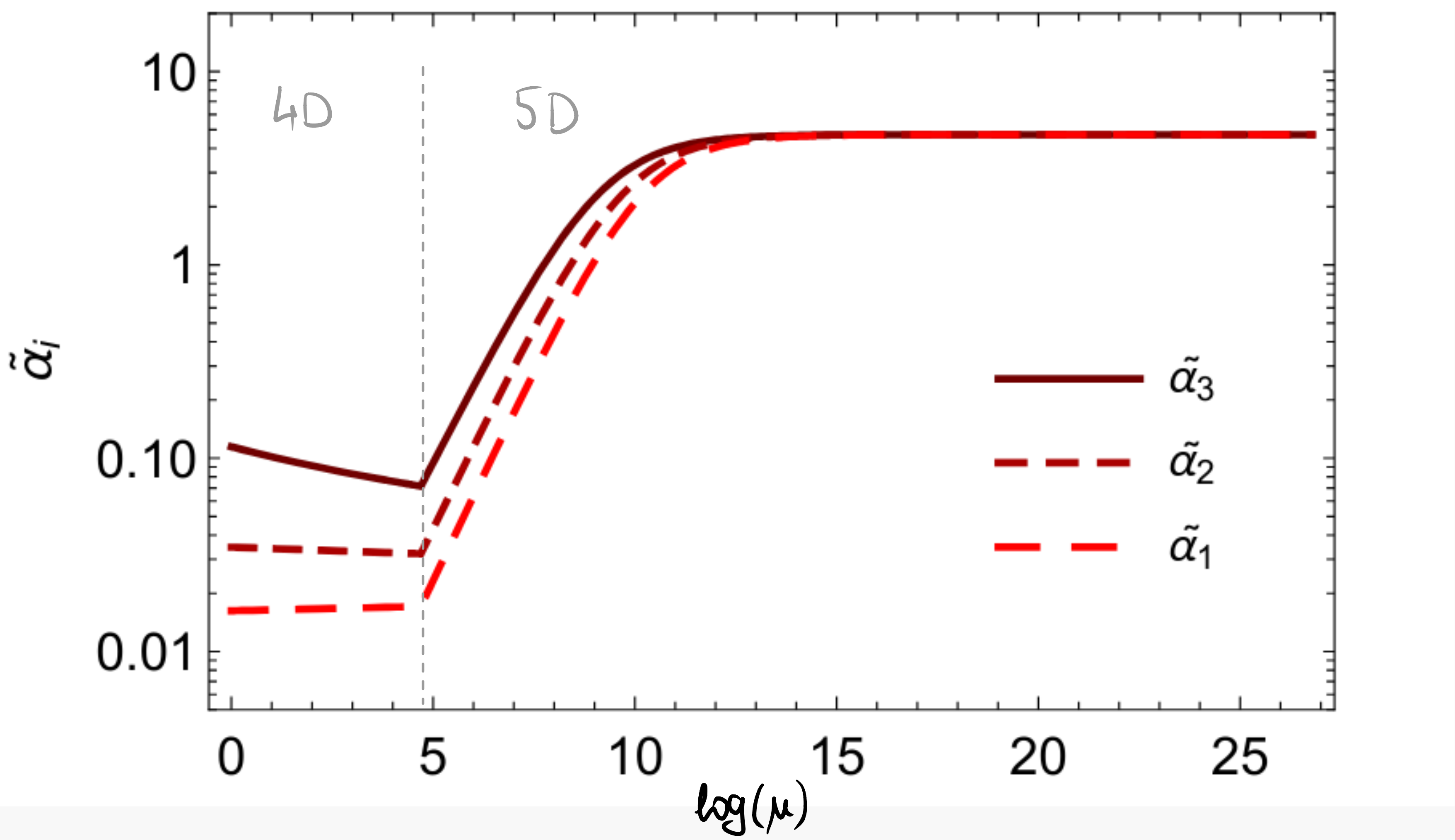
RGEs: from 4D to 5D

- 4D: logarithmic running $$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3$$

- 5D: power-law running [2]

$$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3 + (S(t) - 1) b^i_5 g_i^3$$

where $S(t) = \begin{cases} \mu\ \mathrm{R} = M_Z\ \mathrm{R}\ e^t, & \text{for } \mu \geq 1/R \\ 1, & \text{otherwise} \end{cases}$



G. Cacciapaglia ,et.al., 104, PRD 2021

[2] G. Cacciapaglia, arXiv:2309.10098 (2023)

RGEs: from 4D to 5D

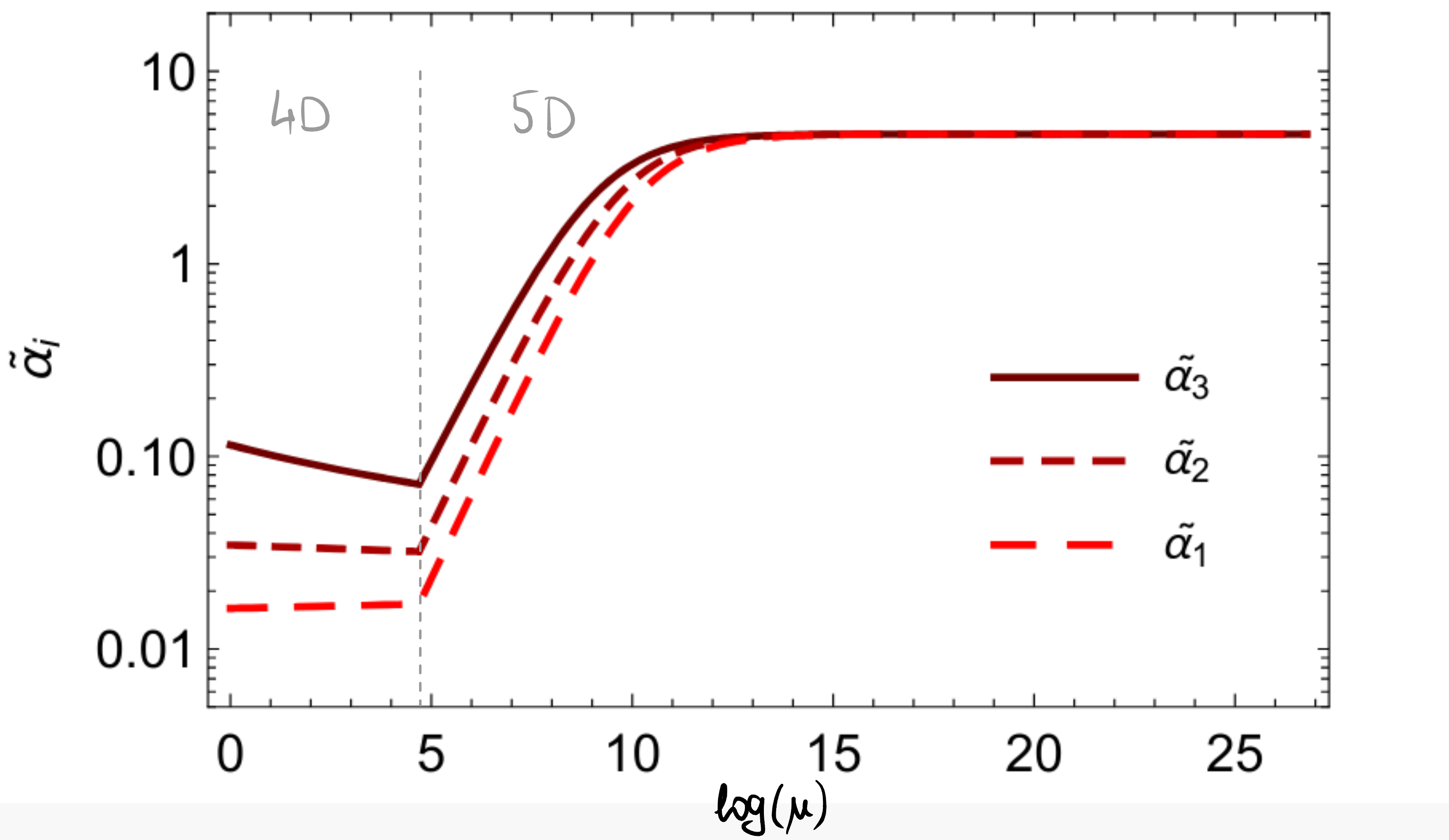
- 5D: couplings flow asymptotically towards a UV fixed point

It's existence ⟺ good behavior in the UV

- gauge couplings

$$16\pi^2 \frac{dg_i}{dt} = b^i_{SM}\, g_i^3 + (S(t)-1) b^i_5 g_i^3$$

Fixed point exists for $b^i_5 \leq 0$



G. Cacciapaglia ,et.al., 104, PRD 2021

RGEs: from 4D to 5D

- Yukawa couplings

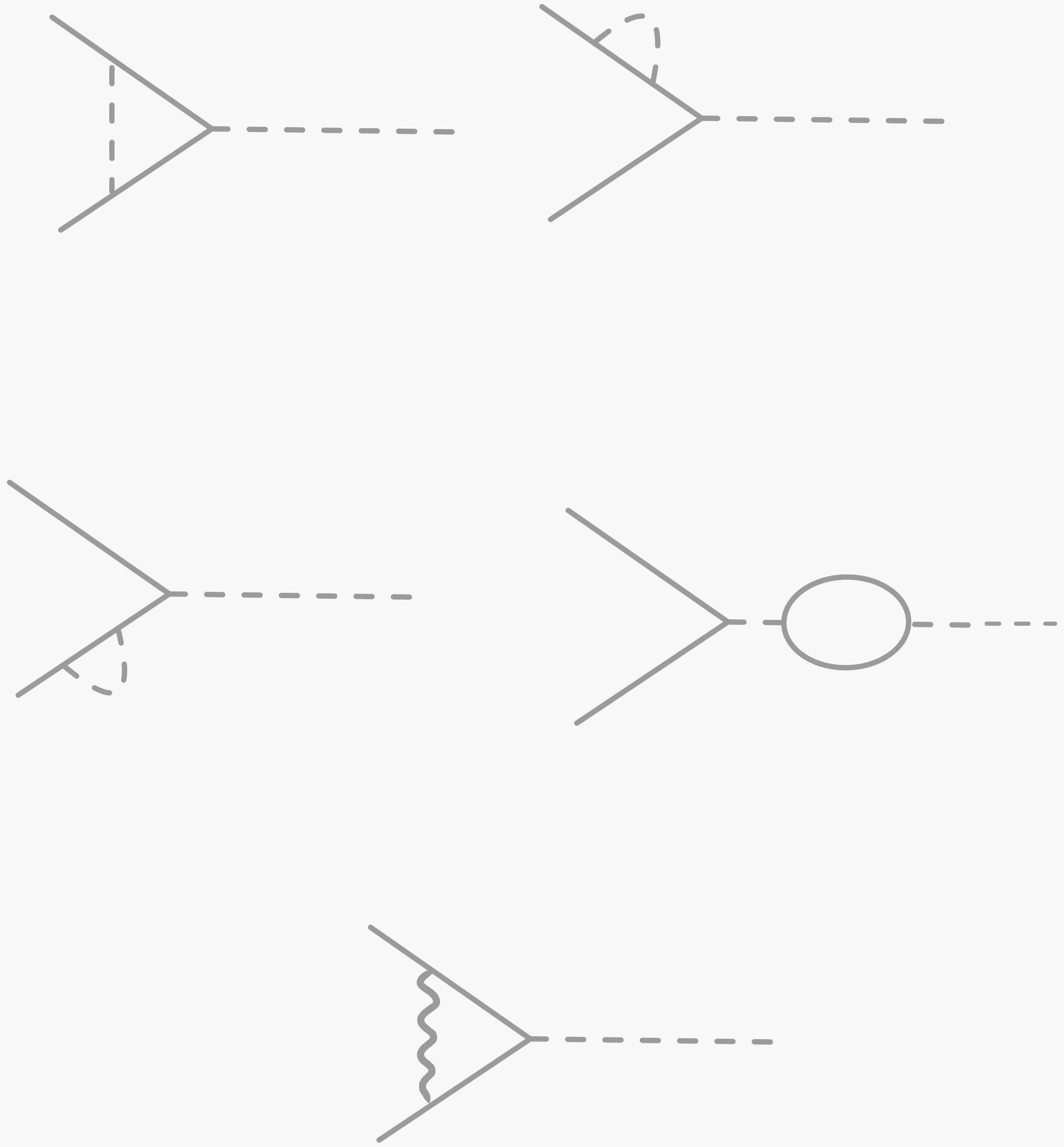
$$16\pi^2 \frac{dy}{dt} = \beta^y_{SM} + (S(t) - 1)\beta^y_5$$

where the β-function is given by [3]

$$\beta_y = y(c_y y^2 - d_y g^2)$$

Fixed point $\tilde{\alpha}^*_y = \dfrac{d_y \tilde{\alpha}^*_g - 2\pi}{c_y}$ exists for $\mathrm{d}_y \tilde{\alpha}^*_g \geq 2\pi$

$$\tilde{\alpha}_{y(g)} = \alpha_{y(g)} \mu R$$



[3] G. Cacciapaglia, W. Isnard, R. Pasechnik, **AP**, (in preparation)

RGEs: from 4D to 5D

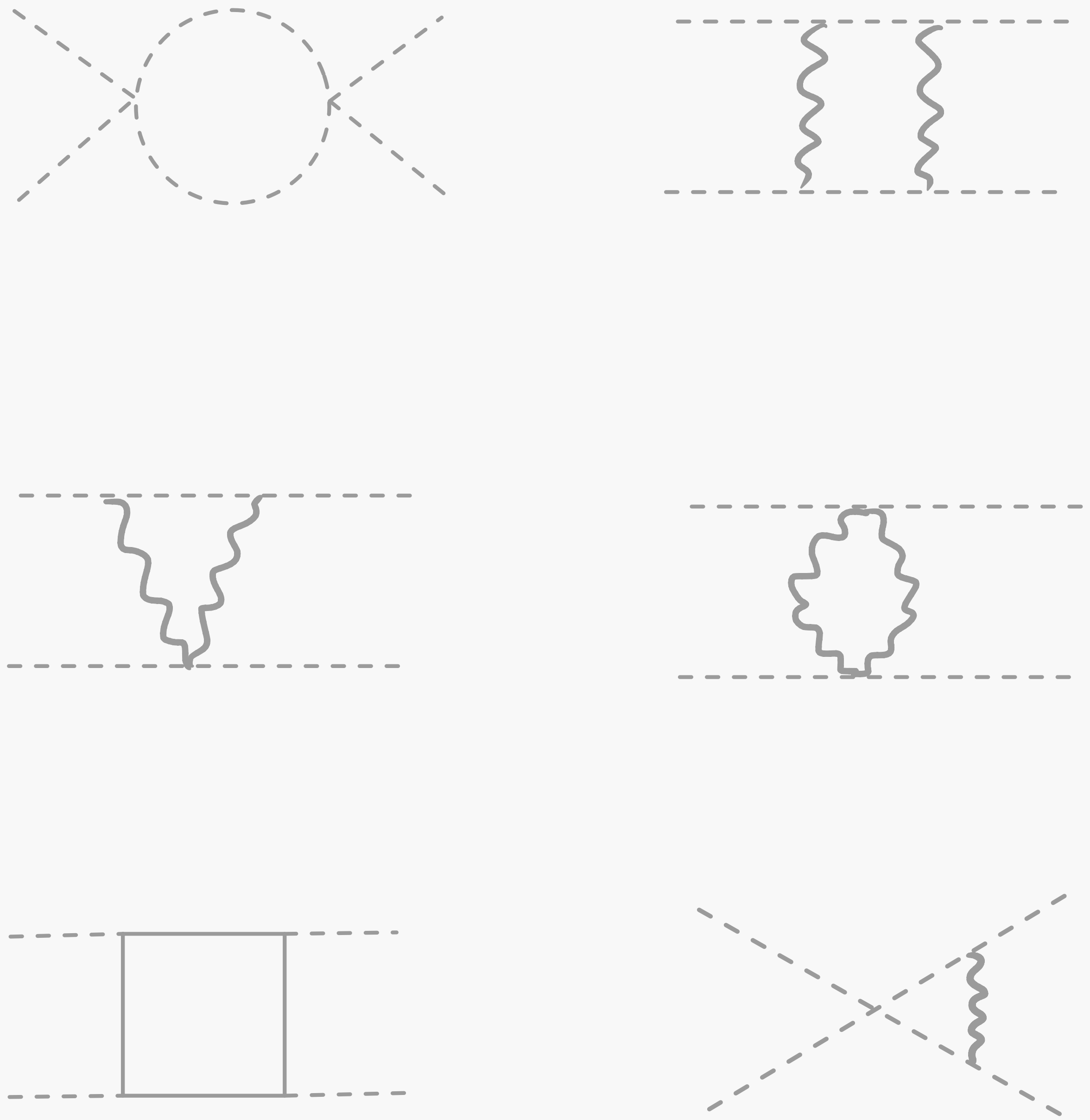
- Scalar couplings $\frac{1}{4!}\lambda\phi^4$

$$16\pi^2\frac{d\lambda}{dt} = \beta^{\lambda}_{SM} + (S(t)-1)\beta^{\lambda}_{5}$$

where the β-function is given by [3]

$$\beta_\lambda = a_\lambda\lambda^2 - b_\lambda y^4 - c_\lambda\lambda g^2 + d_\lambda g^4 + e_\lambda\lambda y^2$$

Fixed point condition is more complicated...



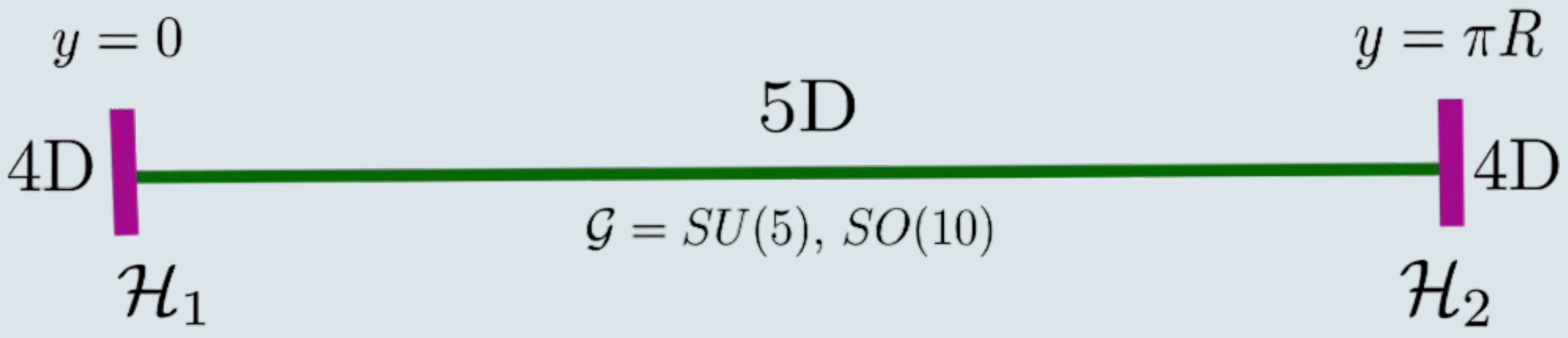
[3] G. Cacciapaglia, W. Isnard, R. Pasechnik, **AP**, (in preparation)

Assume a **theory** with couplings that flow asymptotically towards **fixed points**

+ the fixed points are perturbative

Can we **renormalize** the theory in the bulk and on the boundary?

(one loop)

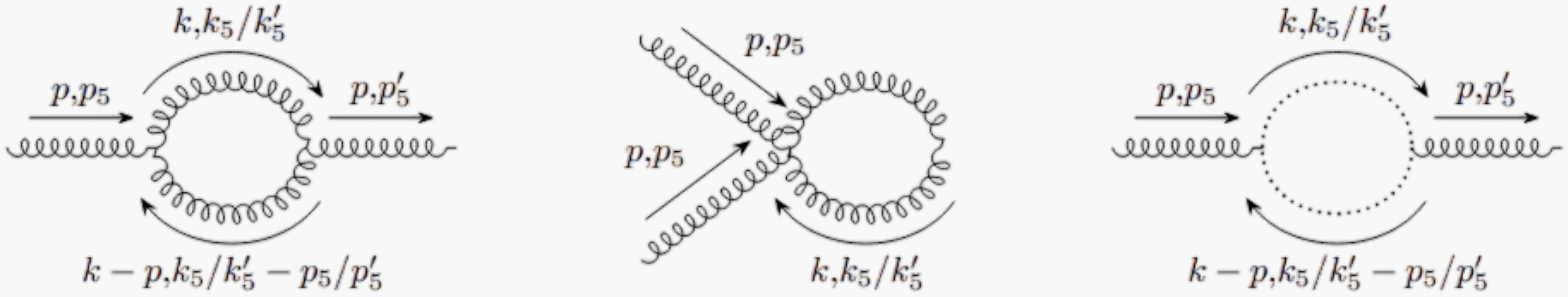


Renormalization of 5D gauge theories

Common lore: **5D** gauge theories are **non-renormalizable** [4]

... but under certain conditions a fixed point exists → theories valid up to arbitrarily high energy scales [5]

Pure Yang Mills theory $$\mathcal{L}_{\rm YM} = -\frac{1}{4} F^{MN} F_{MN}$$

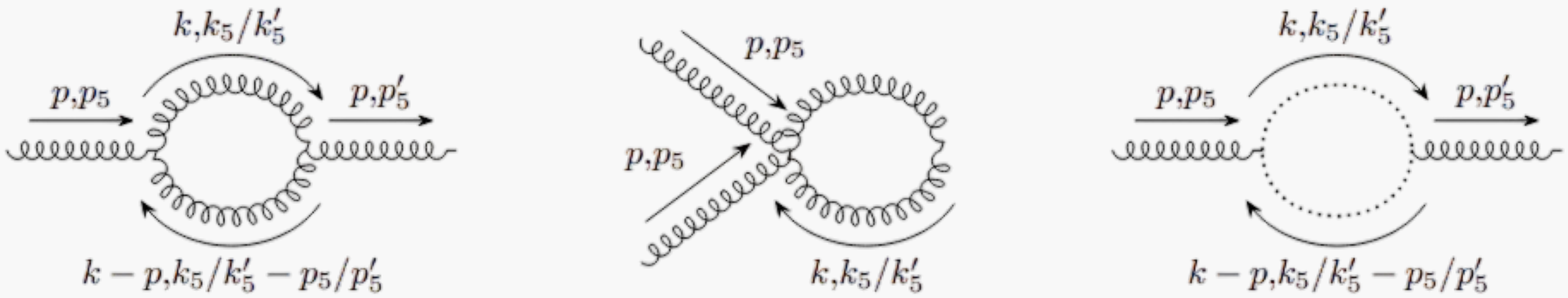


[4] H. Gies, Phys. Rev. D 68 (2003)

[5] T. Morris, JHEP 2005 (2005)

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\text{YM}} = -\frac{1}{4} F^{MN} F_{MN}$



$$i\Sigma = [p_5 - \text{conserving terms}] + [p_5 - \text{non-conserving terms}]$$

bulk boundary

Bulk: structure of divergencies same as in 4D (at one loop)

→ Scaling changes from logarithmic 4D ($\sim \log \Lambda$) to 5D linear ($\sim \Lambda$)

→ Same renormalization procedure as in 4D; **finite number of counterterms** are needed

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\rm YM} = -\frac{1}{4} F^{MN} F_{MN}$

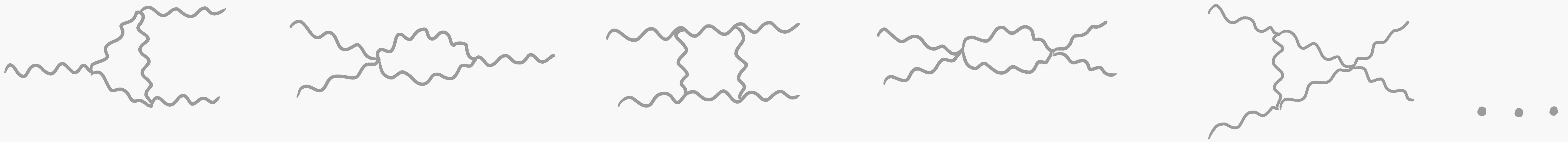
Boundary: renormalizability of a pure Yang Mills at the fixed points

→ less straightforward than bulk

Two-point function: [3,6]

$$i\Sigma = \frac{g^2}{64\pi^2}\frac{1}{\epsilon}\left[(g_{\mu\nu} - p_\mu p_\nu)\left(\frac{11}{3} - (\xi - 1)\right) C(G) + g_{\mu\nu}\frac{p_5^2 + {p'_5}^2}{2}(4 + (\xi - 1))C(G)\right]$$

To reconstruct the full counterterm on the boundary we need **higher vertex corrections** as well (3-pt and 4-pt functions)



(work in progress)

[3] G. Cacciapaglia, W. Isnard, R. Pasechnik, **AP**, (in preparation)

[6] HC Cheng, et al., Phys. Rev. D, 66 (2002)

Renormalization of 5D gauge theories

Pure Yang Mills theory $\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4} F^{MN} F_{MN}$

Boundary: renormalizability of a pure Yang Mills at the fixed points

- → less straightforward than bulk
- → divergencies can be absorbed by a **finite number of counterterms**
- → **"magic gauge"** ($\xi = -3$): coefficients of the 2-,3- and 4-pt functions are the same

  localized counterterm can be unified into a single term $-\frac{1}{4} K \delta \left(F_{\mu\nu} F^{\mu\nu} \right)$

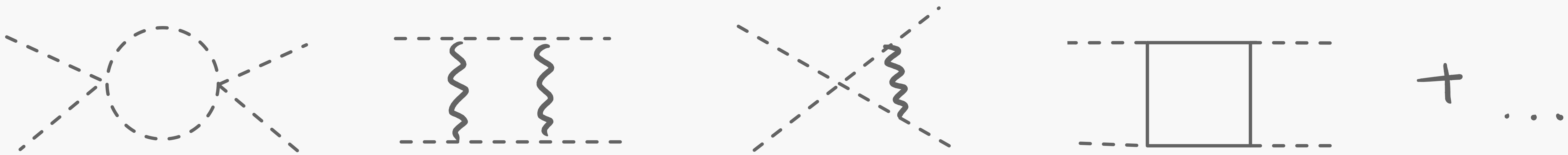
! Theory is renormalizable both in the bulk and on the boundary (at one loop)

(finite number of operators to cancel divergencies)

Renormalization of 5D gauge theories

Adding scalars and fermions

- realistic models: go beyond pure Yang Mills
- Yukawa interactions: finite on the boundary [7]
- fermion and scalar 2-pt functions introduce finite number of counterterms [6]
- in progress: renormalizing scalar quartic interactions



[6] HC Cheng, et al., Phys. Rev. D, 66 (2002)

[7] H. Georgi, et al, Phys. Let. B 506 (2001)

Summary and conclusions

- RGEs of couplings change in 5D: **power law running** (compared to 4D logarithmic)
- Consistency of models requires the existence of **UV fixed points**
- Fixed points $\longleftrightarrow$ renormalizability
- Pure **Yang Mills renormalizable** at one loop (both bulk and boundary)
- Going beyond Yang Mills (in progress): so far also renormalizable