Universität Münster
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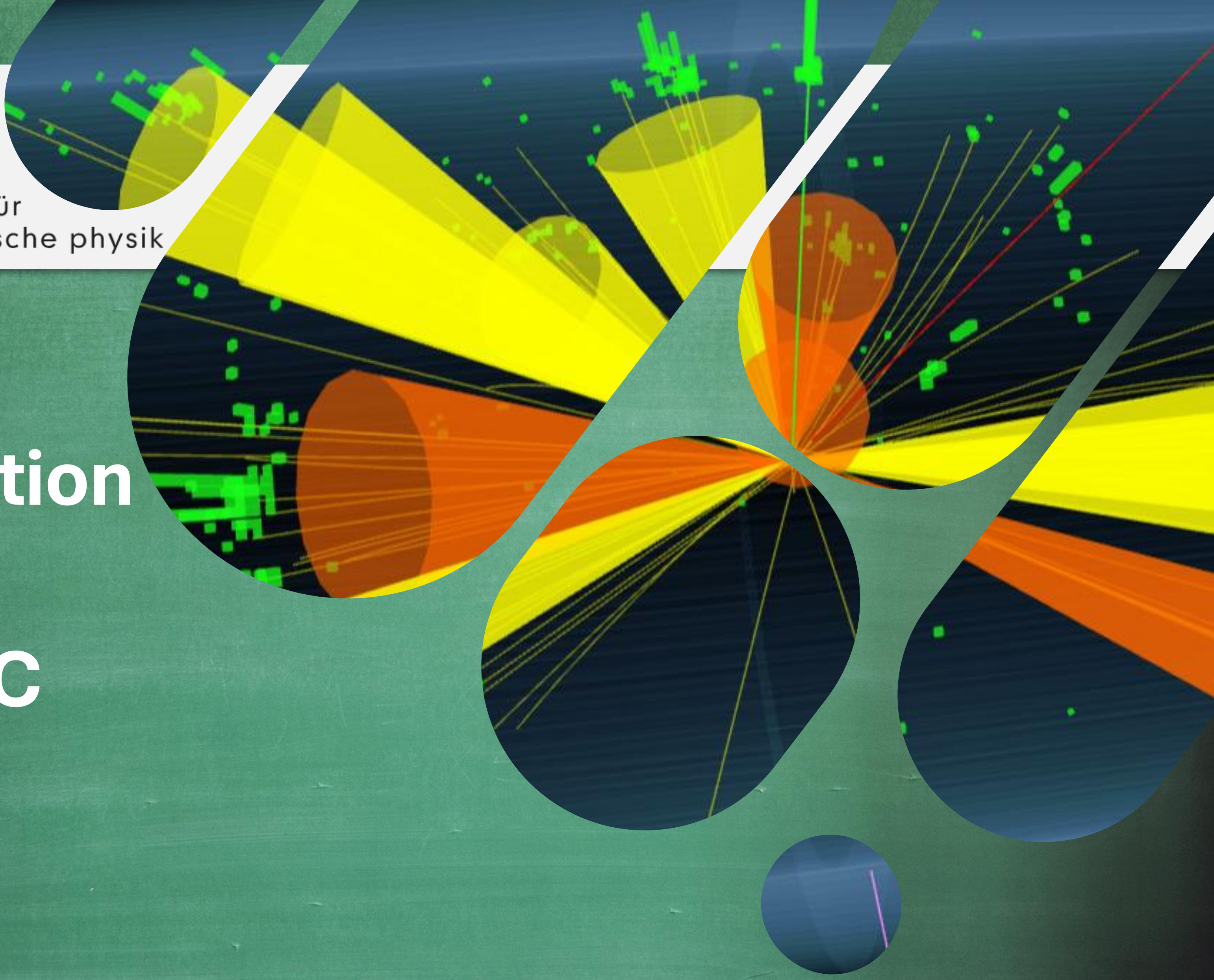


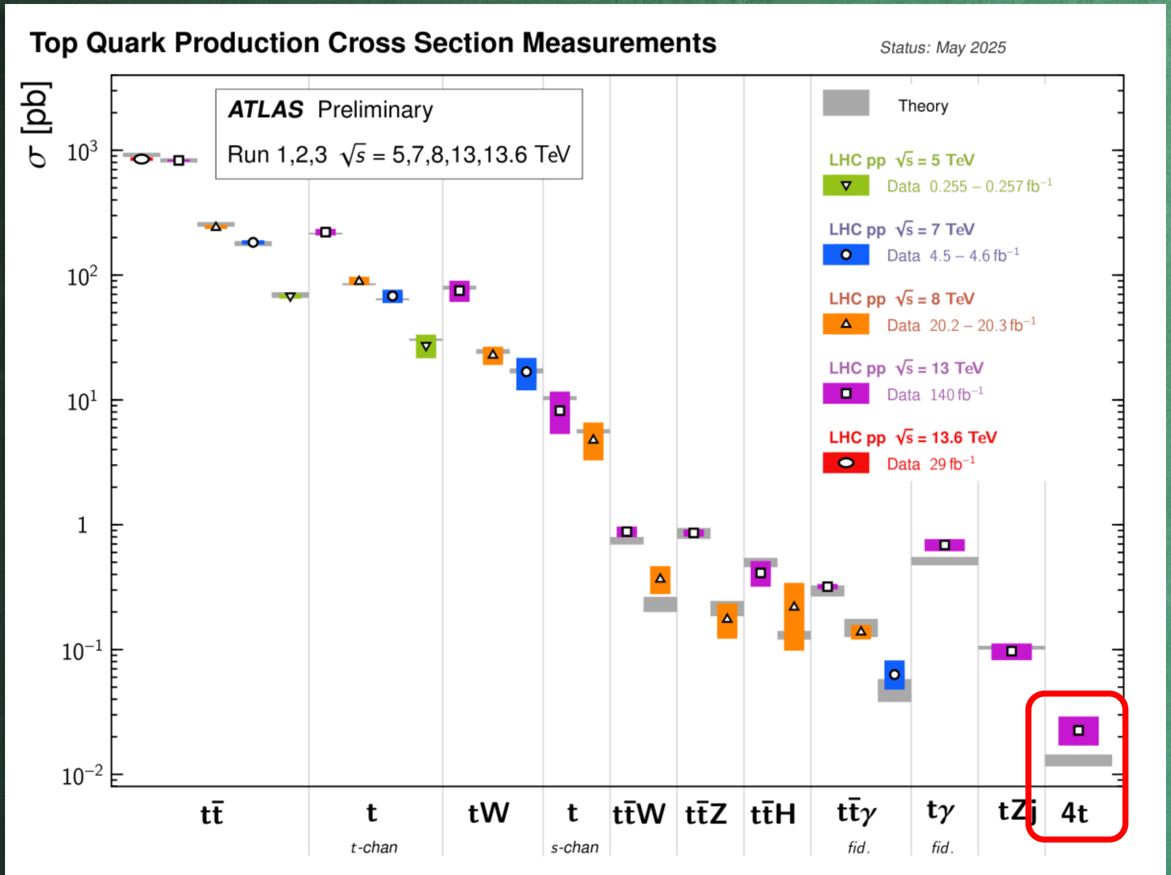
Invariant-Mass Threshold Resummation for Four Top-Quark Production at the LHC

Based on: **2505.10381** (submitted to JHEP)

Michele Lupattelli
in collaboration with
M. van Beekveld, A. Kulesza, T. Saracco

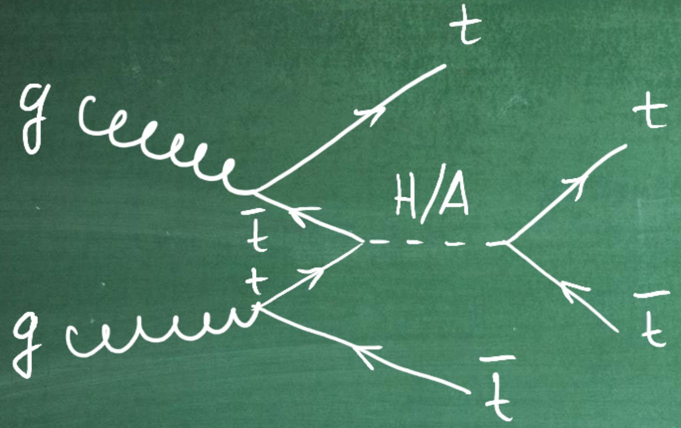
DESY Theory Workshop, 24 September 2025

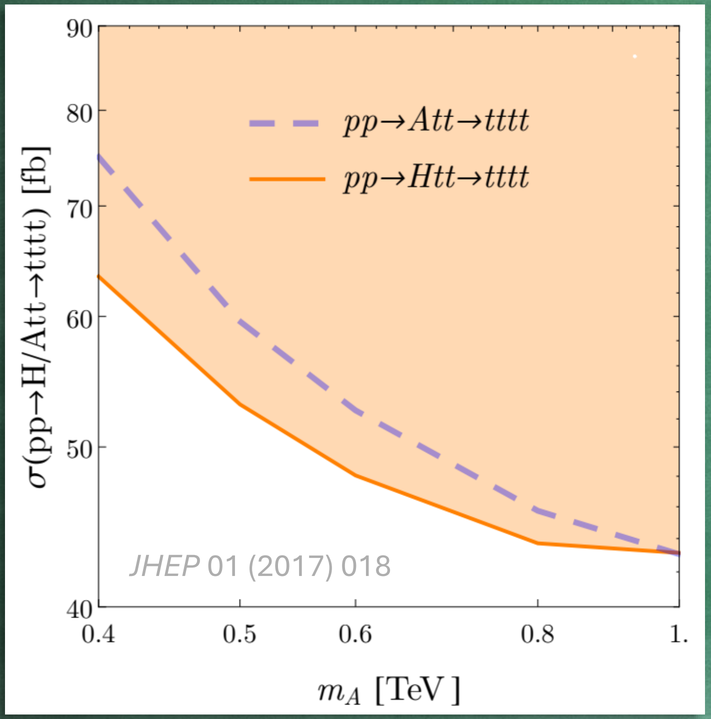
Motivations



$t\bar{t}t\bar{t}$ is a very rare process

Motivations

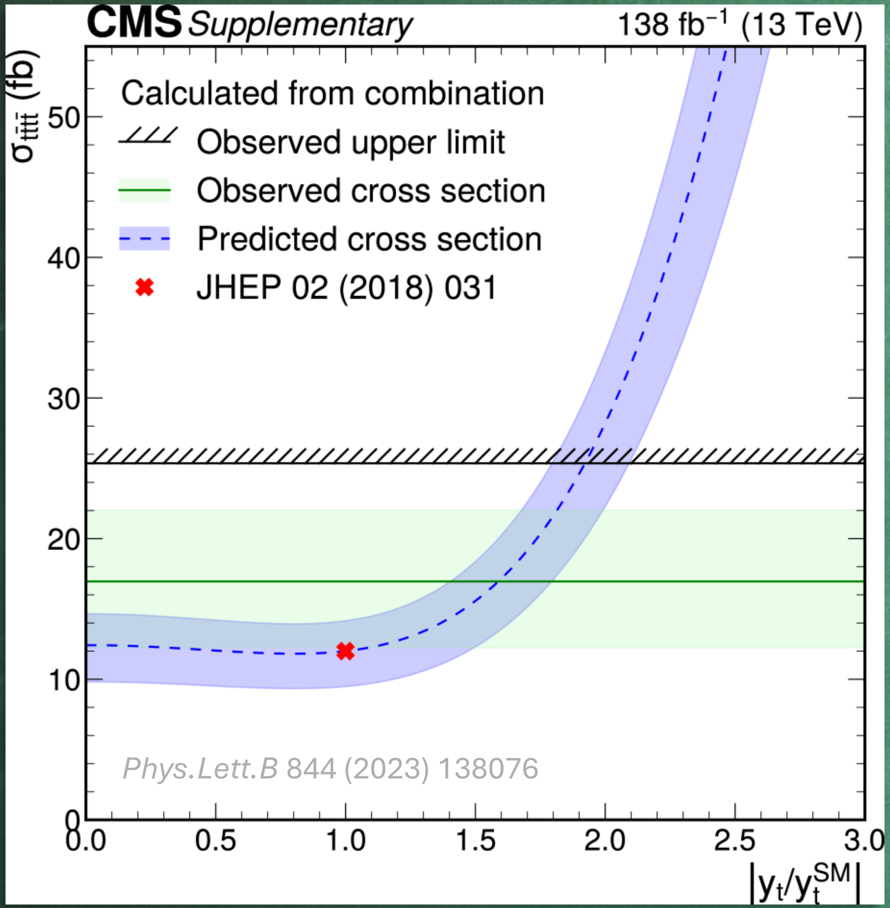




$t\bar{t}t\bar{t}$ is a very rare process:

- can hide new physics BSM

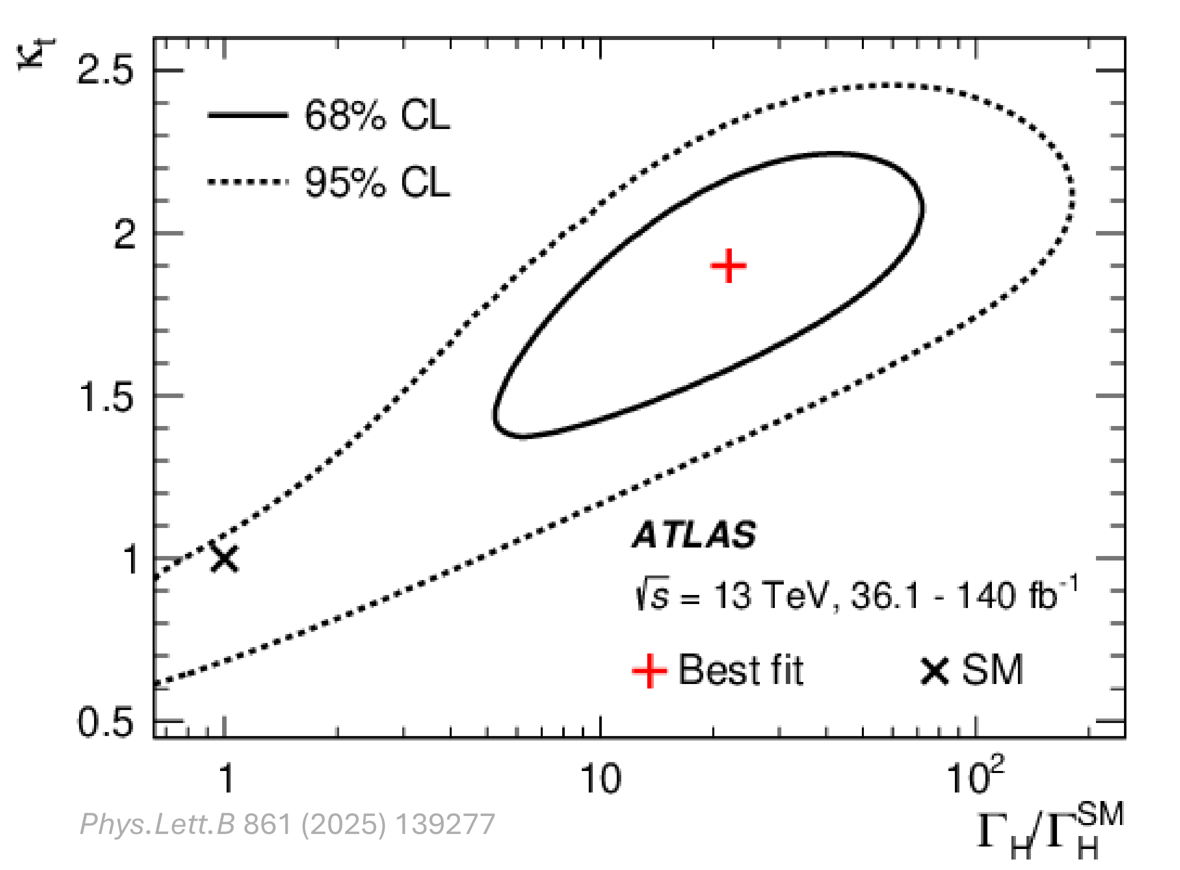
Motivations



$t\bar{t}t\bar{t}$ is a very rare process:

- can hide new physics BSM
- sensitive to top-Yukawa coupling

Motivations

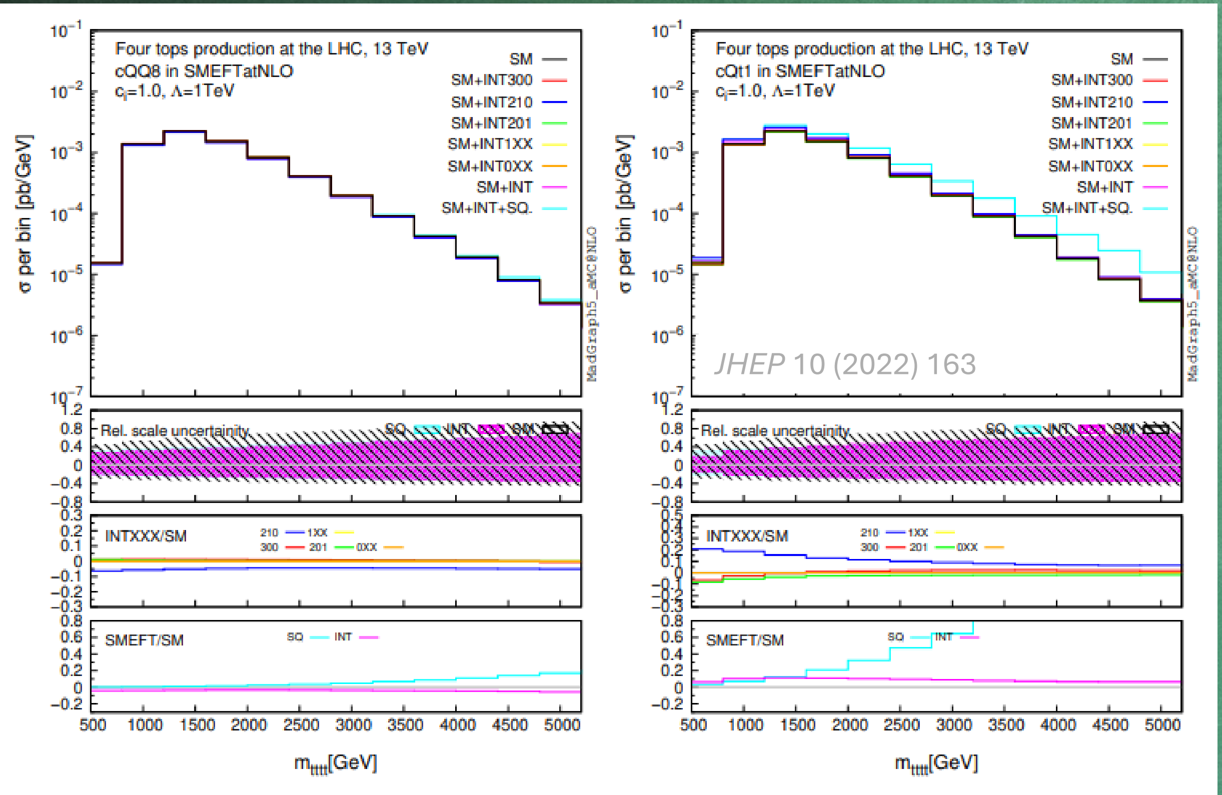


Phys.Lett.B 861 (2025) 139277

$t\bar{t}t\bar{t}$ is a very rare process:

- can hide new physics BSM
- sensitive to top-Yukawa coupling
- constrain width of Higgs boson

Motivations



$t\bar{t}t\bar{t}$ is a very rare process:

- can hide new physics BSM
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- constrains operators in EFT

Motivations

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Regular Article - Experimental Physics

Observation of four-top-quark production in the multilepton final state with the ATLAS detector

ATLAS Collaboration*
CERN, 1211 Geneva 23, Switzerland

Received: 29 March 2023 / Accepted: 2 May 2023 / Published online: 12 June 2023

Phys. Lett. B 847 (2023) 138290
Contents lists available at ScienceDirect
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journal homepage: www.elsevier.com/locate/physletb

Letter

Observation of four top quark production in proton-proton collisions at $\sqrt{s} = 13\,\text{TeV}$

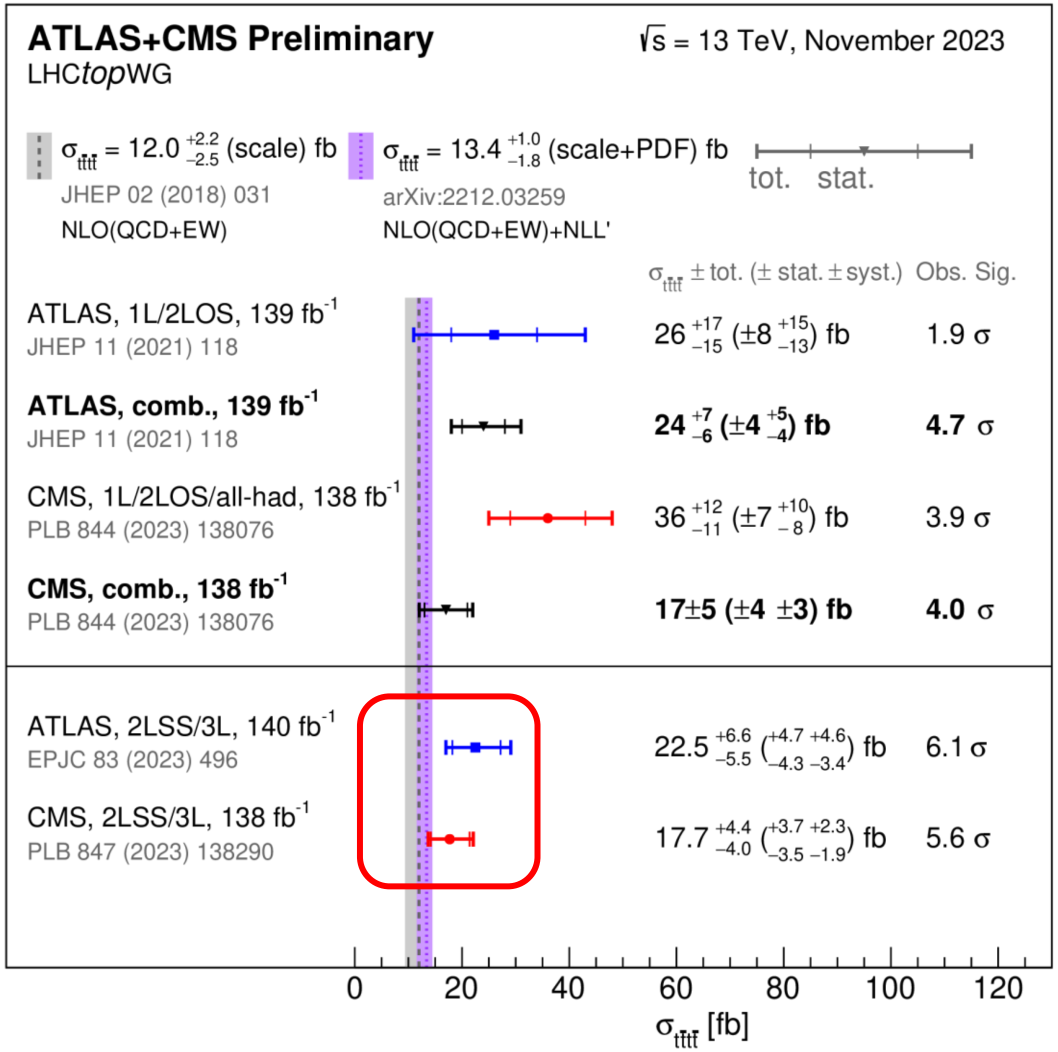
The CMS Collaboration *
CERN, Geneva, Switzerland

$t\bar{t}t\bar{t}$ is a very rare process:

- can hide new physics BSM
- sensitive to top-Yukawa coupling
- constrain width of Higgs boson
- constrain operators in EFT

Eureka!

Motivations



Consistent with Standard Model predictions:

- ATLAS: 1.8, 1.7 standard deviations;
- CMS: 1.3, 1.1 standard deviations.

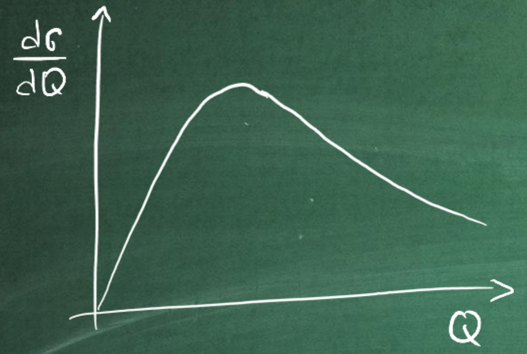
HL-LHC → reduction of experimental uncertainties.

Accuracy of theoretical predictions must improve as well!

State-of-the-art $t\bar{t}t\bar{t}$ theory

- First calculations of NLO QCD corrections in [Bevilacqua, Worek '12]
- Matched with parton shower and studied in aMC@NLO [Alwall et al. '14][Maltoni, Pagani, Tsinikos '15]
- Full set of EW corrections added in [Frederix, Pagani, Zaro '17]
- Spin correlations in LO top quark decays within the framework of Powheg Box [Jezo, Krauss '21]
- Effect of soft-gluon corrections at NLO+NLL' in the absolute-mass threshold formalism studied for the first time in [van Beekveld, Kulesza, Moreno Valero '22]
- Spin correlations in NLO top quark decays using NWA [Stremmer, Worek '24]
- Effect of soft-gluon corrections at NLO+NLL' in the invariant-mass threshold formalism [presented today]

Invariant-Mass Threshold Resummation



- Observable: invariant mass $d\sigma/dQ$

Invariant-Mass Threshold Resummation

from KLN theorem:

$$\alpha_s^n \left[\frac{\log^m(1-\hat{\rho})}{1-\hat{\rho}} \right]_+ , \quad m \leq 2n-1$$

- Observable: invariant mass $d\sigma/dQ$
- Threshold variable: $\hat{\rho} = Q^2/s$

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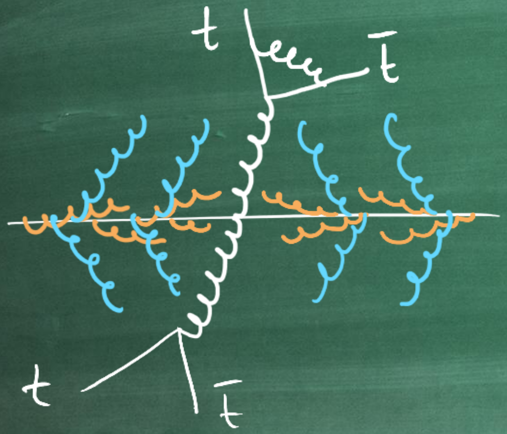
- Threshold limit: $\hat{\rho} \to 1$
  - Enhancement of the cross section



- hard process
- soft emissions
- collinear emissions

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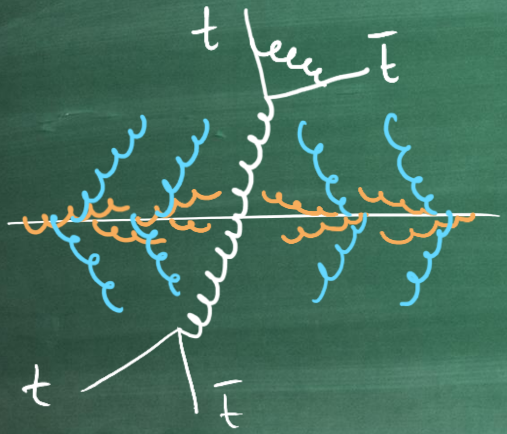
Mellin transform

$$\frac{d\tilde{\sigma}_{pp}(N)}{dQ} = \int_0^1 d\rho\, \rho^{N-1} \frac{d\sigma_{pp}(\rho)}{dQ}, \quad \alpha_s^n \log^m N$$



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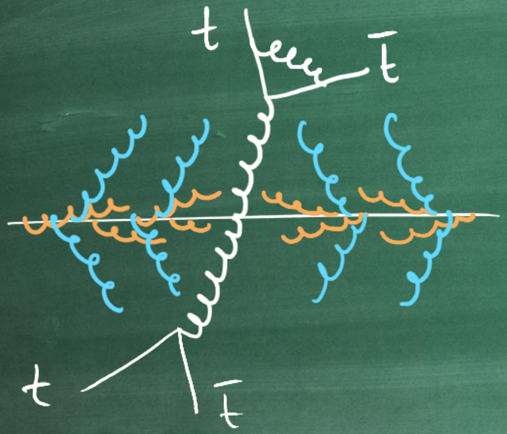

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- JET FUNCTIONS

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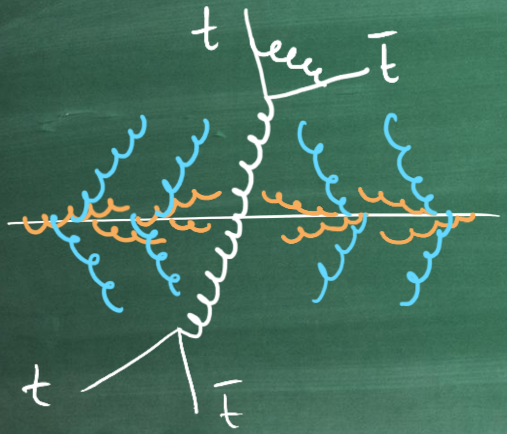

$$\mu \frac{d}{d\mu} \tilde{\sigma}_{ij}(N) = 0$$

- hard FUNCTION
- soft FUNCTION
- JET FUNCTIONS

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- RGEs ⇒ inclusion of all orders

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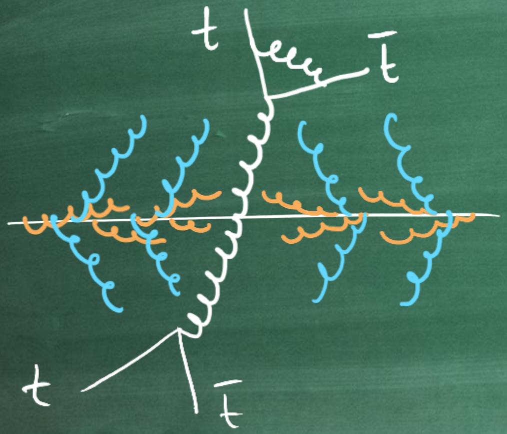

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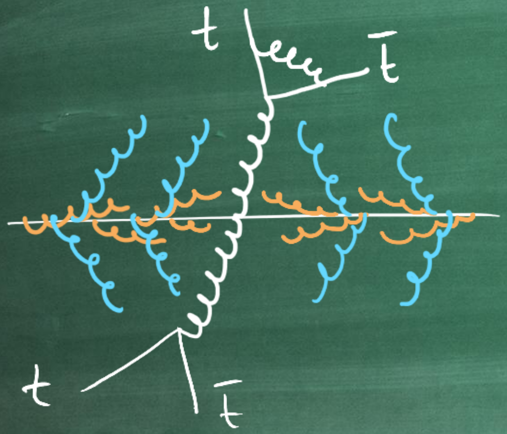
  - Factorization in Mellin space
- RGEs ⇒ inclusion of all orders
- Transform back to momentum space

NLL' accuracy

$$\frac{d\tilde{G}^{res}_{ij}(N)}{dQ} = \mathrm{Tr}\{\bar{S}_{ij}(N+1)H_{ij}\}\Delta_i(N+1)\Delta_j(N+1)$$



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- hard FUNCTION
- soft FUNCTION
- JET FUNCTIONS

NLL' accuracy

$$H_{ij} = \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^k H_{ij}^{(k)}$$

$$S_{ij} = \bar{U}_{ij}\, \tilde{S}_{ij}\, U_{ij} \quad , \quad \tilde{S}_{ij} = \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{4\pi}\right)^k \tilde{S}_{ij}^{(k)}$$

$$U_{ij} = P \exp\left[\frac{1}{2} \int_{\mu_R^2}^{Q^2/\bar{N}^2} \frac{d\mu^2}{\mu^2}\, \Gamma_{ij}\left(\mu^2, \alpha_s(\mu^2)\right)\right]$$

$$\Delta_i = \exp\left[\sum_{k=1}^{\infty} \alpha_s^{k-2}\, g_k(\lambda)\right] \quad , \quad \lambda = \alpha_s b_0 \log \bar{N}$$

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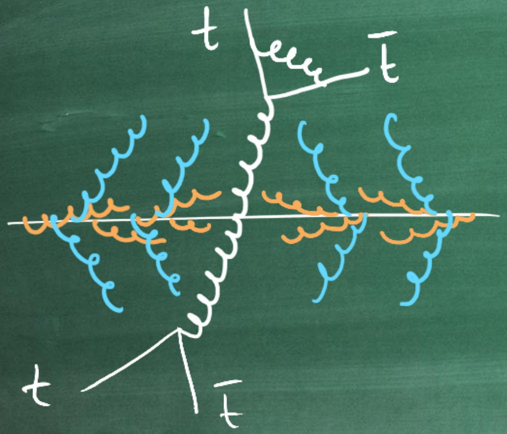

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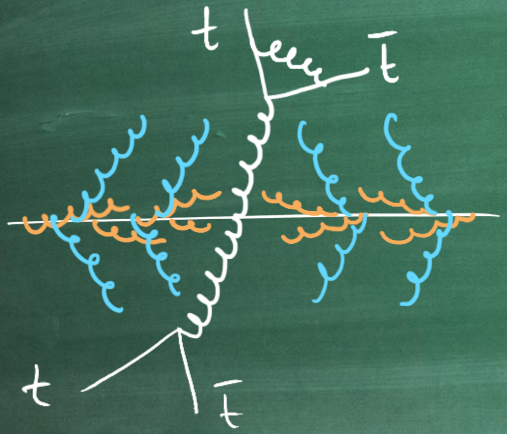

- hard FUNCTION
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$$\mathbf{H} = \mathbf{H}^{(0)} + \frac{\alpha_s}{4\pi} \mathbf{H}^{(1)}$$

$$\tilde{\mathbf{S}} = \tilde{\mathbf{S}}^{(0)} + \frac{\alpha_s}{4\pi} \tilde{\mathbf{S}}^{(1)}$$

$$\mathbf{\Gamma} = \frac{\alpha_s}{4\pi} \mathbf{\Gamma}^{(1)}$$

$$\Delta_i = \exp\{g_1 \log N + g_2\}$$

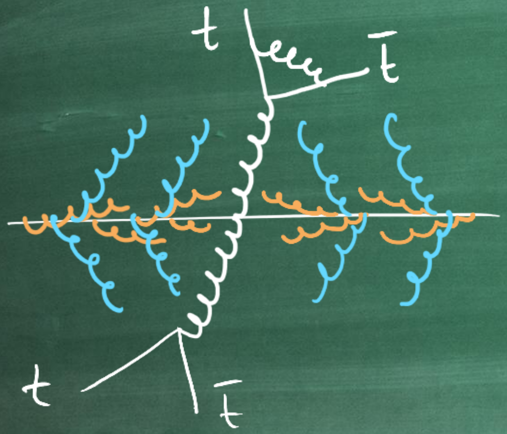
NLL' accuracy

Matching to NLO: **NLO+NLL'**

$$d\sigma^{\text{f.o.+res}} = d\sigma^{\text{f.o.}} + \left[d\sigma^{\text{res}} - d\sigma^{\text{res}}\big|_{\mathcal{O}(\alpha_s^n)} \right]$$

NLO obtained with MG5_aMC@NLO. *(JHEP 07 (2014) 079 - JHEP 07 (2018) 185)*
It includes also EW corrections.

$$\frac{d\tilde{G}^{res}_{ij}(N)}{dQ} = \mathrm{Tr}\left\{ \bar{S}_{ij}(N+1) H_{ij} \right\} \Delta_i(N+1)\Delta_j(N+1)$$



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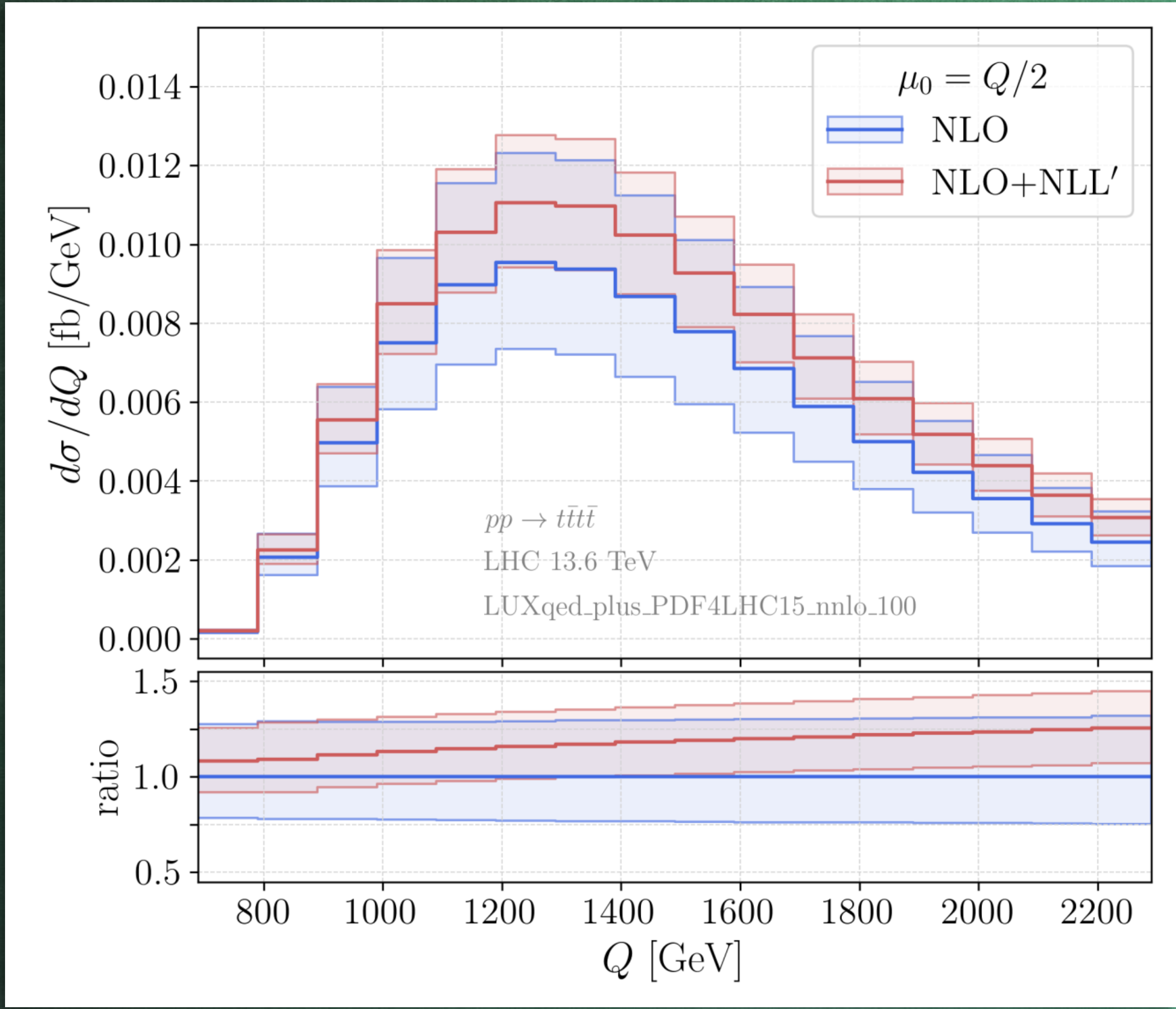
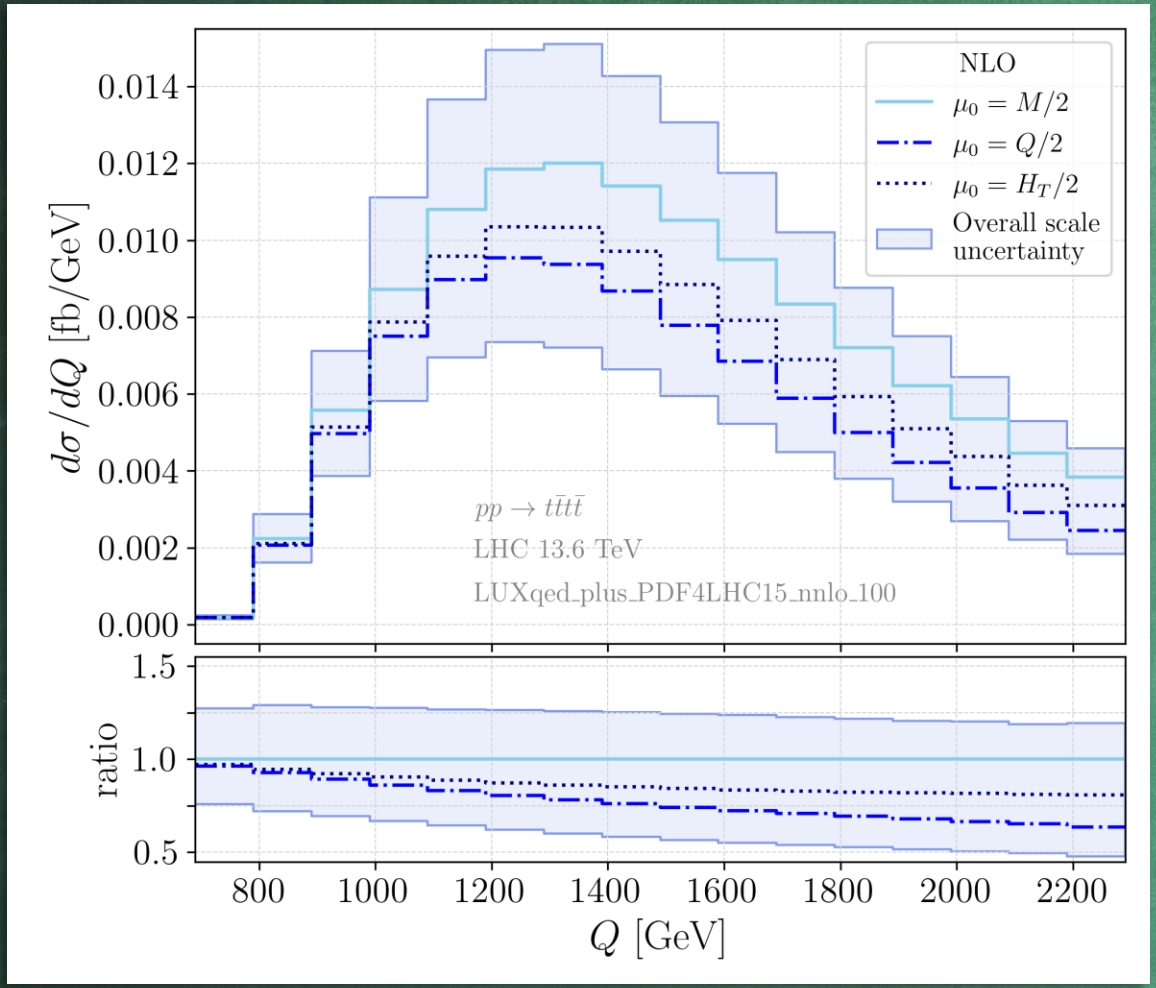
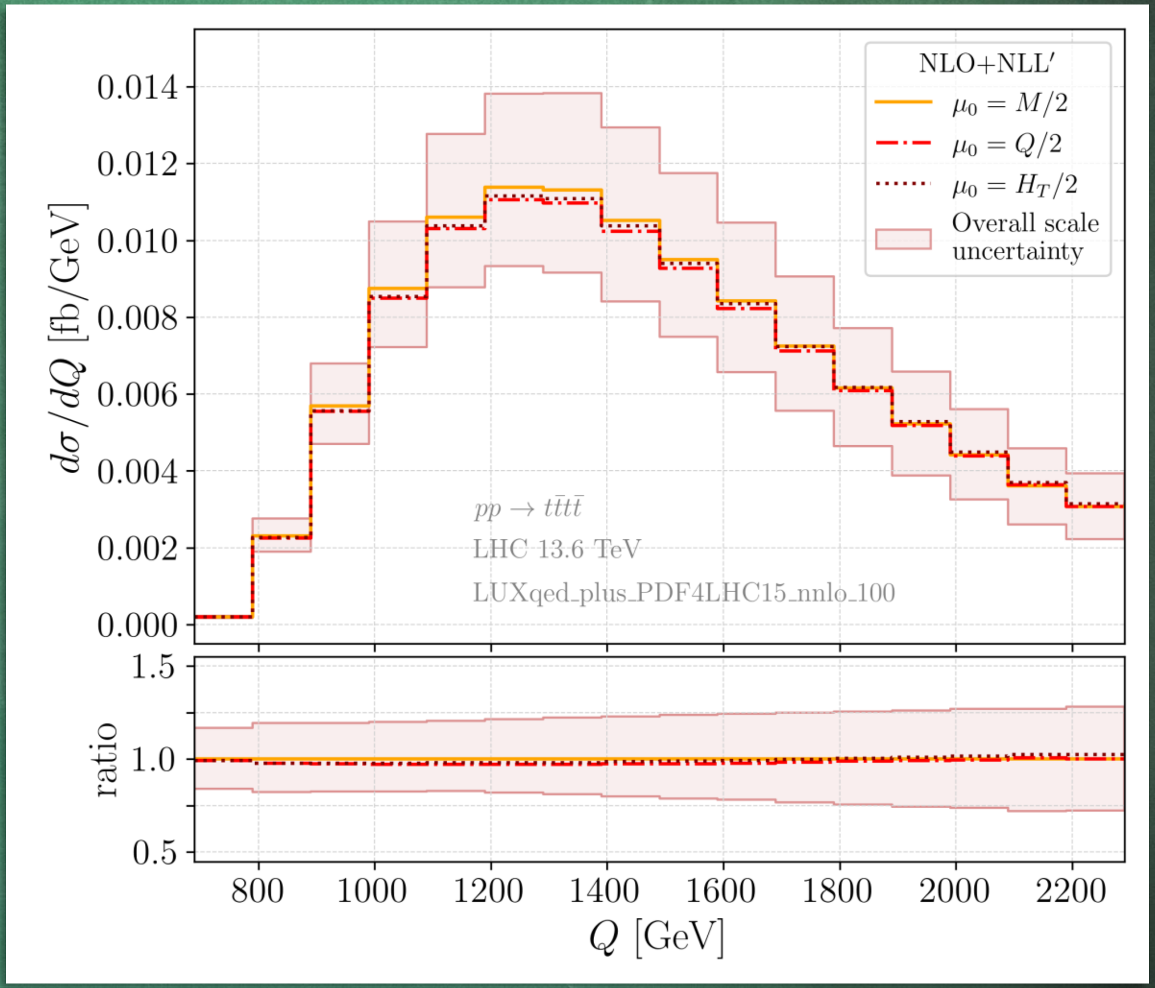
Results – Invariant-Mass distribution



- Change in shape substantial
- NLL’ corrections vary in range [8%,25%]
- NLL’ corrections increasingly positive with Q
- Scale uncertainty substantially reduced

Results – Invariant-Mass distribution

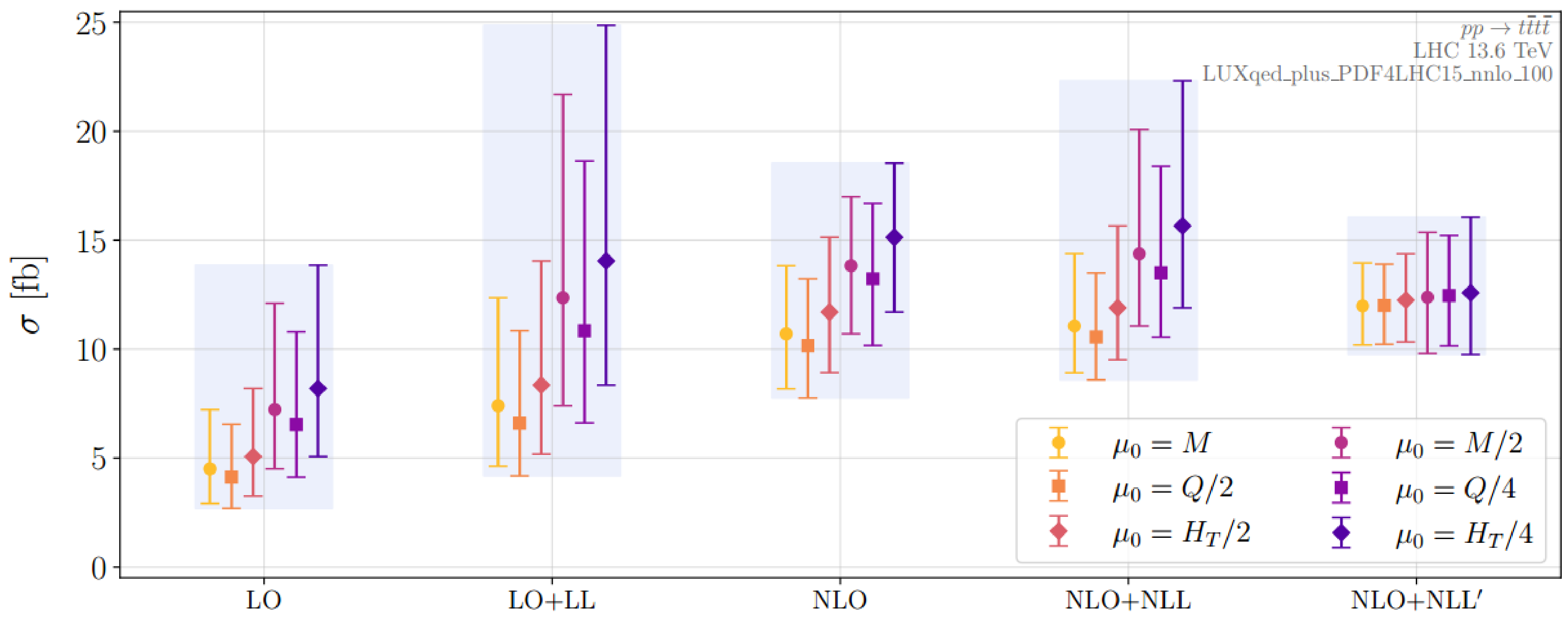




- Central values differ up to 36%

- Better convergence (3% differences at most)
- Lower overall scale uncertainty

Results – Integrated cross section



$\sqrt{S}$ [TeV]	μ_0	NLO [fb]	NLO+NLL [fb]	$\mathcal{K}^{\mathrm{NLL}}$	NLO+NLL′ [fb]	$\mathcal{K}^{\mathrm{NLL}'}$
13.6	$M/2$	$13.83^{+23.0\%}_{-22.6\%}$	$14.38^{+39.6\%}_{-23.1\%}$	1.04	$12.38^{+24.1\%}_{-20.8\%}$	0.90
	$Q/2$	$10.16^{+30.1\%}_{-23.6\%}$	$10.55^{+27.9\%}_{-18.6\%}$	1.04	$12.00^{+15.8\%}_{-14.9\%}$	1.18
	$H_T/2$	$11.70^{+29.3\%}_{-23.8\%}$	$11.89^{+31.7\%}_{-20.1\%}$	1.02	$12.25^{+17.3\%}_{-15.7\%}$	1.05

Results – Comparing with experimental data

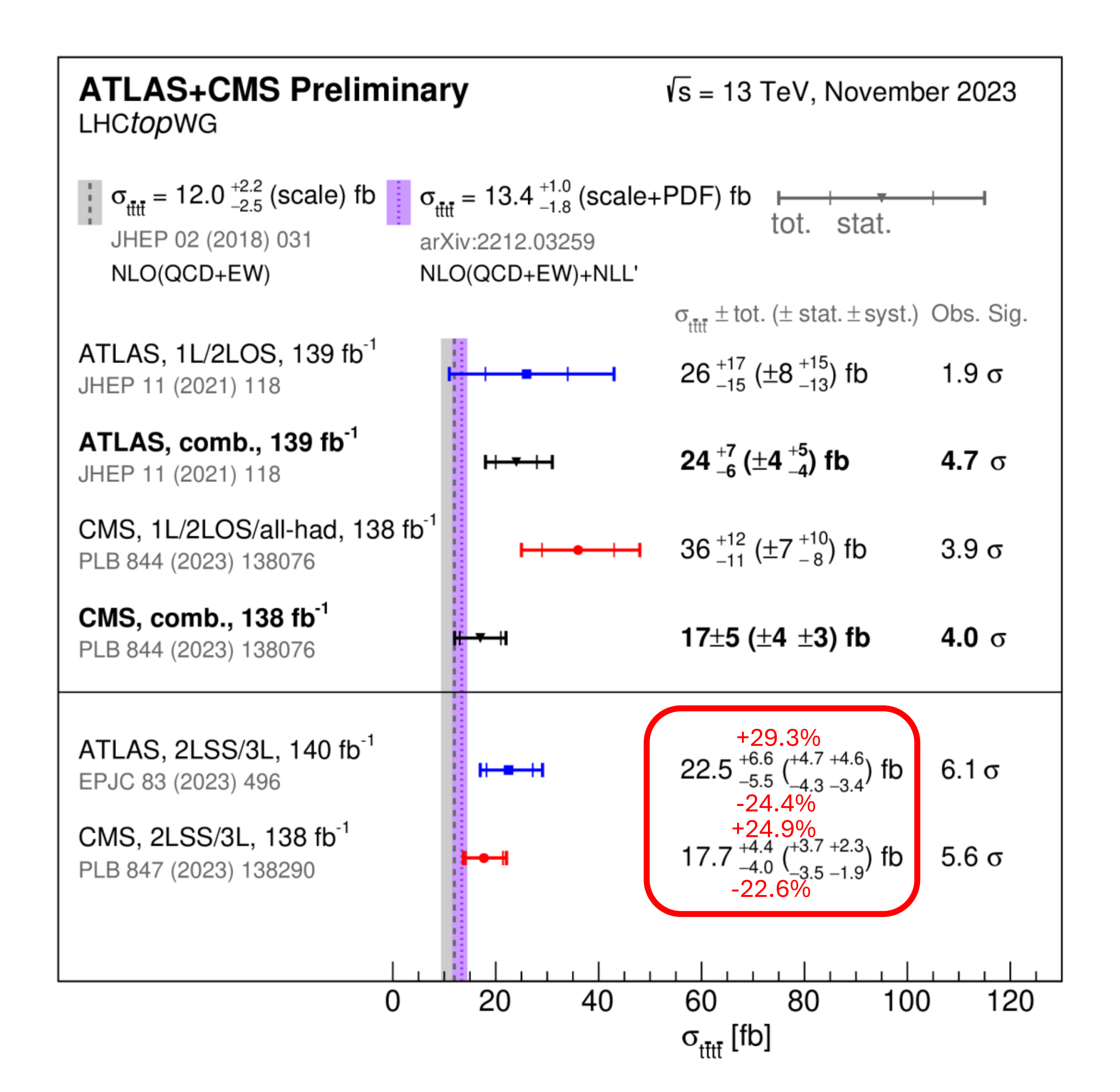
$\sqrt{S}$ [TeV]	μ_0	NLO+NLL′ [fb]
13	$M/2$	$10.43^{+23.6\%}_{-20.8\%}$
	$Q/2$	$10.16^{+15.7\%}_{-14.8\%}$
	$H_T/2$	$10.35^{+17.1\%}_{-15.7\%}$

$\mu_0 = Q/2$:

- 1.8 σ from CMS
- 2.2 σ from ATLAS

$\mu_0 = M/2$:

- 1.5 σ from CMS
- 2.0 σ from ATLAS



Conclusions

- I presented the most accurate QCD predictions for $t\bar{t}t\bar{t}$ to date. The NLO results have been combined with NLL' (NLO+NLL'), and thus include all-order corrections in the soft gluon emission limit.
- The NLL' corrections reduce the theoretical uncertainty and improve the convergence of the predictions.
- For the first time, soft-gluon corrections to the invariant mass distribution Q of the $t\bar{t}t\bar{t}$ system have been obtained.

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Outlook

- The new theoretical predictions are in agreement with the experimental results. However, both the theoretical uncertainty and the experimental error are still quite large. With HL-LHC, further effort from theory side is needed.
- Next step: performing the calculation at NLO+NNLL accuracy.


Thank you!

Backup Slides

Absolute- vs Invariant-Mass Threshold Resummation

NLO	$\sqrt{S} = 13.6$ TeV	–	$\mu_0 = M/2$	–	Cross sections in [fb]
$13.83^{+23.0\%}_{-22.6\%}$					

NLO+NLL$'_{\text{IMT-res}}$	$\mathcal{K}_{\text{IMT-res}}$	NLO+NLL$'^{\text{test}}_{\text{IMT-res}}$	$\mathcal{K}^{\text{test}}_{\text{IMT-res}}$	NLO+NLL$'_{\text{AMT-res}}$	$\mathcal{K}_{\text{AMT-res}}$
$12.38^{+24.1\%}_{-20.8\%}$	0.90	$17.91^{+13.8\%}_{-20.8\%}$	1.30	$17.36^{+8.3\%}_{-17.5\%}$	1.26

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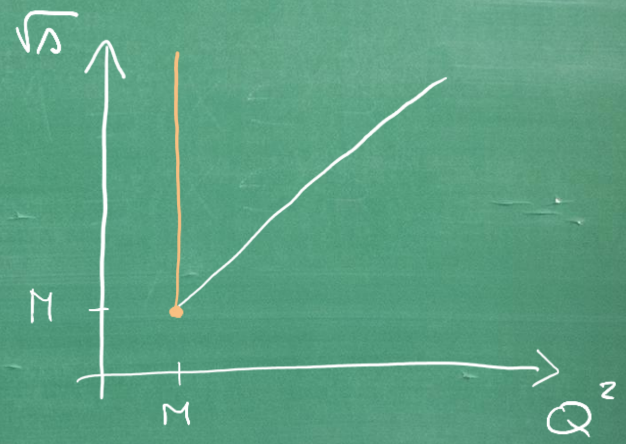
- AMT-res: soft-gluon corrections originating from phase-space region close to the four-top production threshold $M = 4m_t$



treshold: $\hat{\rho} = 1$

$\hat{\rho} = \frac{M^2}{s}$

Absolute- vs Invariant-Mass Threshold Resummation

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NLO+NLL$'_{\text{IMT-res}}$	$\mathcal{K}_{\text{IMT-res}}$	NLO+NLL$'^{\,\text{test}}_{\text{IMT-res}}$	$\mathcal{K}^{\text{test}}_{\text{IMT-res}}$	NLO+NLL$'_{\text{AMT-res}}$	$\mathcal{K}_{\text{AMT-res}}$
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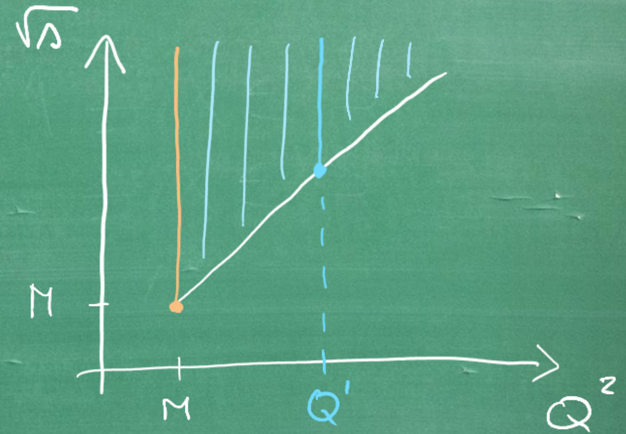
- IMT-res: soft-gluon corrections originating from phase-space region close to the four-top invariant mass Q



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Absolute- vs Invariant-Mass Threshold Resummation

NLO	$\sqrt{S} = 13.6$ TeV – $\mu_0 = M/2$ – Cross sections in [fb]				
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NLO+NLL$'_{\text{IMT-res}}$	$\mathcal{K}_{\text{IMT-res}}$	NLO+NLL$'^{\,\text{test}}_{\text{IMT-res}}$	$\mathcal{K}^{\text{test}}_{\text{IMT-res}}$	NLO+NLL$'_{\text{AMT-res}}$	$\mathcal{K}_{\text{AMT-res}}$
$12.38^{+24.1\%}_{-20.8\%}$	0.90	$17.91^{+13.8\%}_{-20.8\%}$	1.30	$17.36^{+8.3\%}_{-17.5\%}$	1.26

- AMT-res: soft-gluon corrections originating from phase-space region close to the four-top production threshold $M = 4m_t$
- IMT-res: soft-gluon corrections originating from phase-space region close to the four-top invariant mass Q

$$g_2(\lambda) = \frac{A^{(1)} b_1}{2\pi b_0^3}\left[2\lambda + \log(1-2\lambda) + \frac{1}{2}\log^2(1-2\lambda)\right] - \frac{A^{(2)}}{2\pi^2 b_0^2}\left[2\lambda + \log(1-2\lambda)\right] + \frac{A^{(1)}}{2\pi b_0}\left[\log(1-2\lambda)\log\left(\frac{M^2}{\mu_R^2}\right) + 2\lambda\log\left(\frac{\mu_F^2}{\mu_R^2}\right)\right]$$

Absolute- vs Invariant-Mass Threshold Resummation

NLO	$\sqrt{S} = 13.6$ TeV	–	$\mu_0 = M/2$	–	Cross sections in [fb]
$13.83^{+23.0\%}_{-22.6\%}$					
NLO+NLL$'_{\text{IMT-res}}$	$\mathcal{K}_{\text{IMT-res}}$	NLO+NLL$'^{\,\text{test}}_{\text{IMT-res}}$	$\mathcal{K}^{\text{test}}_{\text{IMT-res}}$	NLO+NLL$'_{\text{AMT-res}}$	$\mathcal{K}_{\text{AMT-res}}$
$12.38^{+24.1\%}_{-20.8\%}$	0.90	$17.91^{+13.8\%}_{-20.8\%}$	1.30	$17.36^{+8.3\%}_{-17.5\%}$	1.26

- AMT-res: soft-gluon corrections originating from phase-space region close to the four-top production threshold $M = 4m_t$
- IMT-res: soft-gluon corrections originating from phase-space region close to the four-top invariant mass Q

$$g_2(\lambda) = \frac{A^{(1)} b_1}{2\pi b_0^3}\left[2\lambda + \log(1-2\lambda) + \frac{1}{2}\log^2(1-2\lambda)\right] - \frac{A^{(2)}}{2\pi^2 b_0^2}\left[2\lambda + \log(1-2\lambda)\right] + \frac{A^{(1)}}{2\pi b_0}\left[\log(1-2\lambda)\log\left(\frac{Q^2}{\mu_R^2}\right) + 2\lambda\log\left(\frac{\mu_F^2}{\mu_R^2}\right)\right]$$

A complicated calculation

Color decomposition of the amplitude

Soft radiation sensitive to overall color structure of hard process

$\Longrightarrow$ H and S are matrices in colour space:

- $q\bar{q}$ channel: 6-dimensional colour space
- gg channel: 14-dimensional colour space

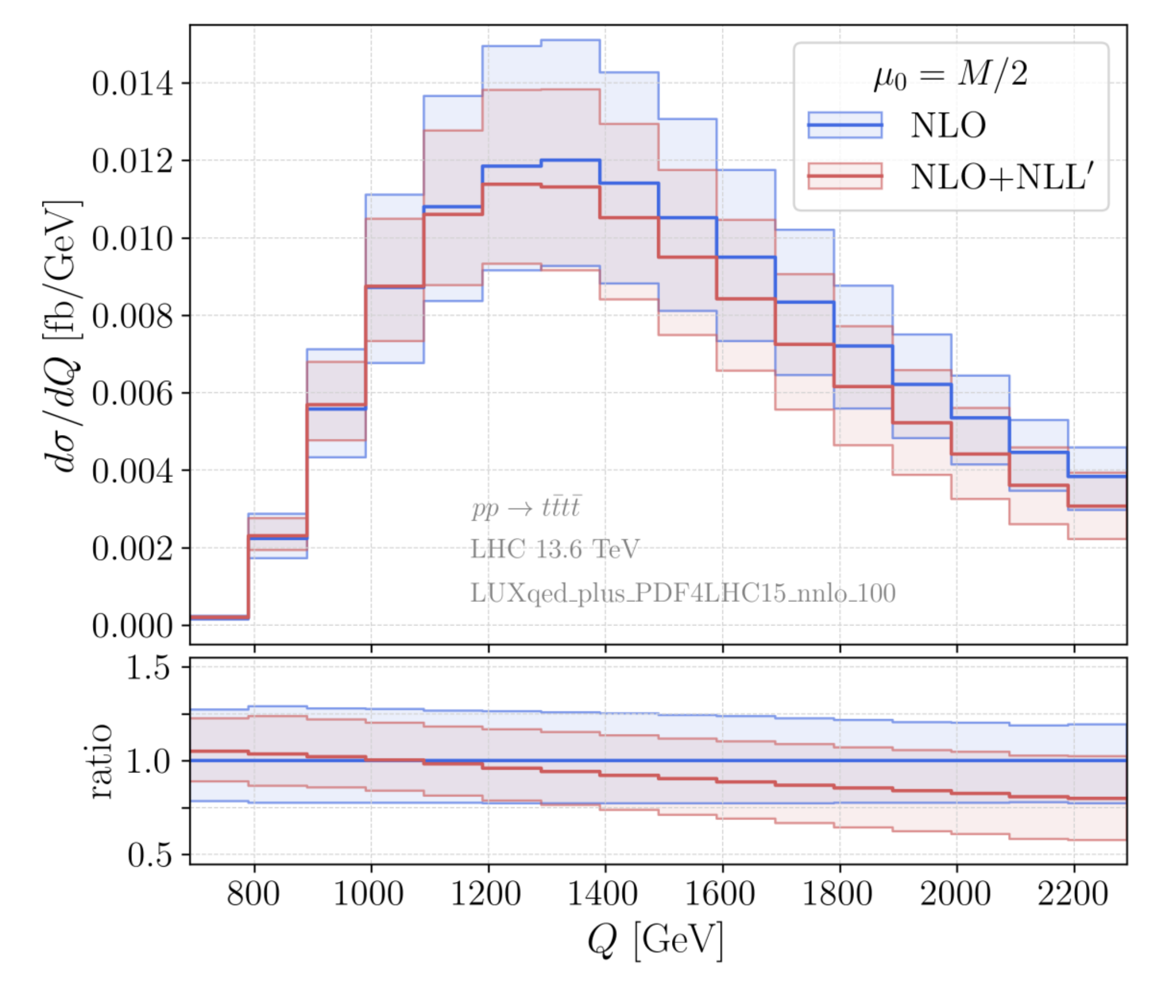
Color decomposed amplitudes extracted from custom version of OpenLoops (Eur. Phys. J. C 79 (2019) 866)

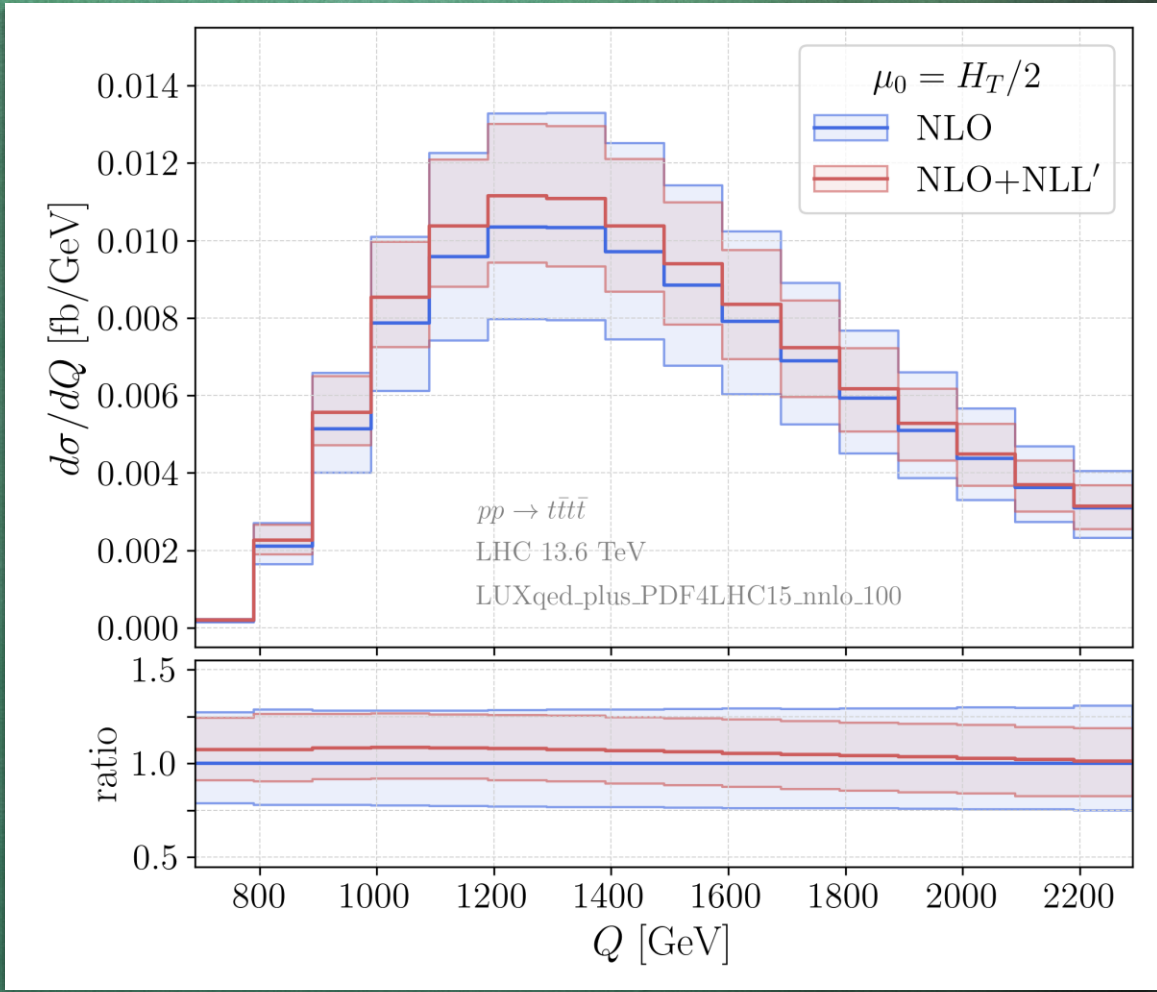
Diagonalization of soft anomalous dimension

Diagonalization necessary to get rid of path-ordering operator. Performed for every phase-space point.

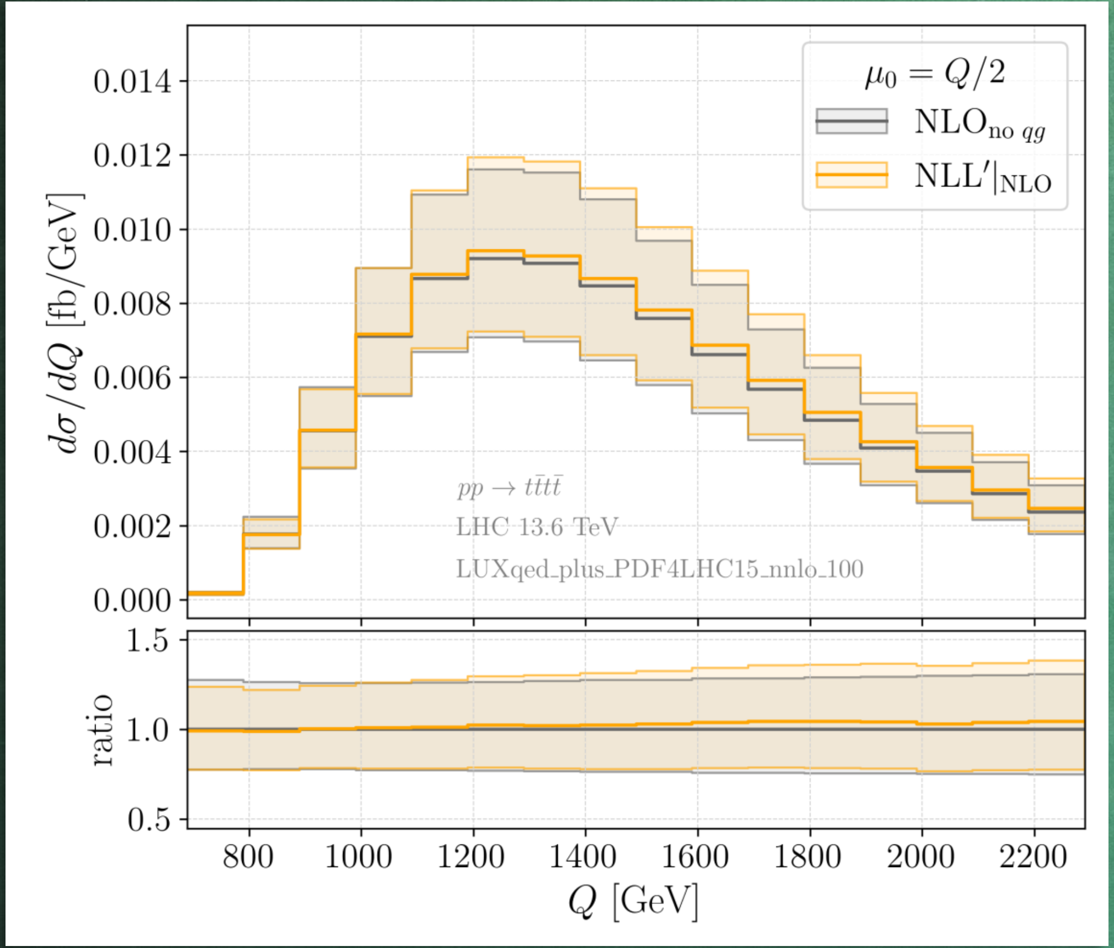
$$\mathbf{U} = \mathcal{P}\exp\left[\frac{1}{2}\int_{\mu^2}^{Q^2/\bar{N}^2}\frac{\mathrm{d}\mu^2}{\mu^2}\mathbf{\Gamma}\left(\mu^2, \alpha_s(\mu^2)\right)\right]$$

Invariant-Mass distribution (additional scales)





Approximate NLO



μ_0	$\mathrm{NLO}_{\mathrm{QCD}}$ [fb]	$\mathrm{NLO}_{\mathrm{no}\ qg}$ [fb]	$\mathrm{NLL}'\|_{\mathrm{NLO}}$ [fb]
$M/2$	$13.13^{+25.2\%}_{-24.5\%}$	$13.05^{+20.2\%}_{-21.1\%}$	$13.45^{+21.6\%}_{-21.9\%}$
$Q/2$	$9.38^{+33.3\%}_{-25.8\%}$	$9.77^{+28.1\%}_{-23.9\%}$	$9.92^{+28.7\%}_{-24.1\%}$
$H_T/2$	$10.88^{+32.3\%}_{-25.8\%}$	$11.22^{+26.0\%}_{-23.7\%}$	$11.44^{+27.0\%}_{-24.0\%}$

- NLL' expanded reproduces $NLO_{no\ qg}$ reliably, both at the differential and integrated level.
- qg contribution to the cross section is very small.
- Differences between $NLL'|_{NLO}$ and $NLO_{no\ qg}$ do not exceed 3%.
- Differences between $NLL'|_{NLO}$ and NLO are at most 6%.