Two-loop renormalisation of the 2HDM and phenomenological applications

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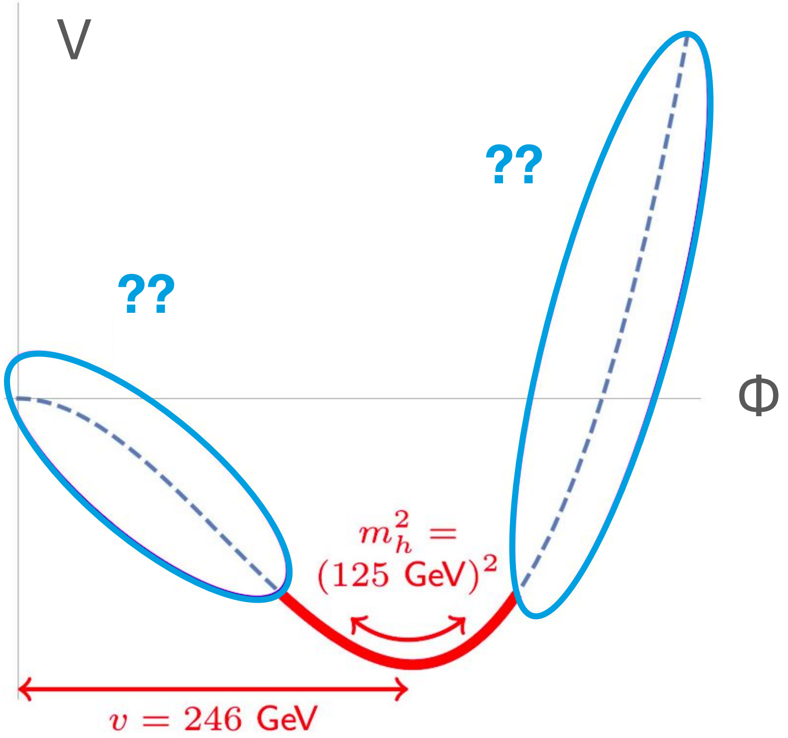
Motivation

- Why trilinear Higgs couplings?
  - Not well constrained: good portal for new physics
  - Large impact in hh production
  - Related to evolution of early universe: SFOEWPT and BAU

- Why loop corrections?
  - Radiative corrections from extended scalar sectors can be very significant

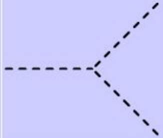
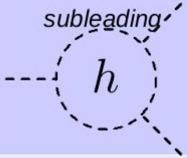
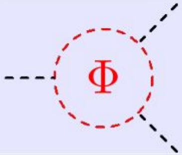
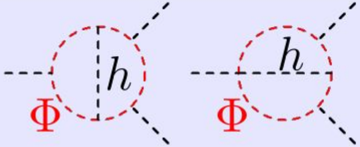
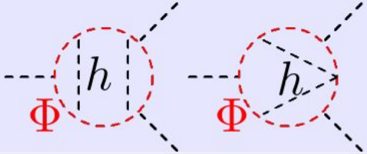
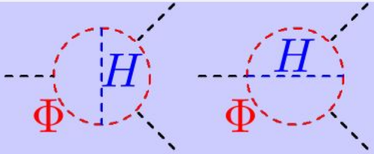
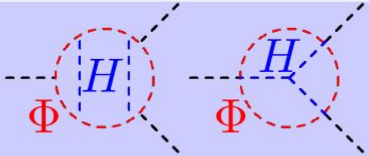
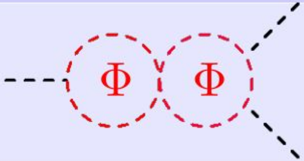
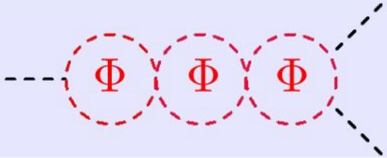


Taken from J. Braathen

Why two loops?

New types of contributions can enter from the loop level, and can be much larger than tree-level contributions

One loop corrections are LO in BSM and two loop are NLO

Coupling/Order	0L	1L	2L	3L
g_{hhhh}			*subleading*	*subleading*
$g_{(h)h\Phi\Phi}$ $\left[g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}\right]$	-			
$g_{(h)H\Phi\Phi'}$ [$g_{(h)G\Phi\Phi'}$ case similar]	-	-		
$g_{\Phi\Phi\Phi'\Phi'}$ [2 BSM scalars of species Φ, 2 of species Φ']	-	-		

Set-up

Previous set-up: [J. Braathen, S, Kanemura, '19] Loop-induced deviations from alignment were neglected (as subleading)

MODEL → Alignment limit $\Lambda_6=0$ → Renormalization → Loop corrections → **No Λ_6 counter term**

Our set-up: Following: [G. Degrassi, P. Slavich, '23]

MODEL → Renormalization → Loop corrections → Alignment limit $\Lambda_6=0$ → **Λ_6 counter term included**

The Λ_6 counter term ensures the alignment limit after loop corrections

THDM

General model

Doublets:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + \sigma_1 i) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + \sigma_2 i) \end{pmatrix}$$

$$\boxed{\begin{aligned} V(\Phi_1, \Phi_2) = {} & \tilde{M}_{11}^2|\Phi_1|^2 + \tilde{M}_{22}^2|\Phi_2|^2 - \tilde{M}_{12}^2(\Phi_2^\dagger\Phi_1 + \mathrm{h.c.}) + \frac{\Lambda_1}{2}|\Phi_1|^4 + \frac{\Lambda_2}{2}|\Phi_2|^4 + \\ & + \tilde{\Lambda}_3|\Phi_1|^2|\Phi_2|^2 + \tilde{\Lambda}_4|\Phi_2^\dagger\Phi_1|^2 + \left[\frac{\tilde{\Lambda}_5}{2}(\Phi_2^\dagger\Phi_1)^2 + (\tilde{\Lambda}_6|\Phi_1|^2 + \tilde{\Lambda}_7|\Phi_2|^2)\Phi_1^\dagger\Phi_2 + \mathrm{h.c}\right] \end{aligned}}$$

General basis:

$$v_1, v_2, \tilde{M}_{11}^2, \tilde{M}_{22}^2, \tilde{M}_{12}^2, \tilde{\Lambda}_1, \tilde{\Lambda}_2, \tilde{\Lambda}_3, \tilde{\Lambda}_4, \tilde{\Lambda}_5, \tilde{\Lambda}_6, \tilde{\Lambda}_7$$

THDM

softly broken $\mathbb{Z}_2$ basis

Doublets:
$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + \sigma_1 i) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + \sigma_2 i) \end{pmatrix}$$

$$\boxed{\begin{aligned} V(\Phi_1,\Phi_2) = {} & \tilde{M}_{11}^2|\Phi_1|^2 + \tilde{M}_{22}^2|\Phi_2|^2 - \tilde{M}_{12}^2(\Phi_2^\dagger\Phi_1 + \text{h.c.}) + \frac{\Lambda_1}{2}|\Phi_1|^4 + \frac{\Lambda_2}{2}|\Phi_2|^4 + \\ & + \tilde{\Lambda}_3|\Phi_1|^2|\Phi_2|^2 + \tilde{\Lambda}_4|\Phi_2^\dagger\Phi_1|^2 + \left[\frac{\tilde{\Lambda}_5}{2}(\Phi_2^\dagger\Phi_1)^2 + (\cancel{\tilde{\Lambda}_6}|\Phi_1|^2 + \tilde{\Lambda}_7\cancel{|\Phi_2}|^2)\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \end{aligned}}$$

General basis: $v_1, v_2, \tilde{M}_{11}^2, \tilde{M}_{22}^2, \tilde{M}_{12}^2, \tilde{\Lambda}_1, \tilde{\Lambda}_2, \tilde{\Lambda}_3, \tilde{\Lambda}_4, \tilde{\Lambda}_5, \cancel{\tilde{\Lambda}_6}, \cancel{\tilde{\Lambda}_7}$

$\mathbb{Z}_2$-symmetric mass basis: $v, \alpha, \beta, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, M^2$

Tad. eqs. solved for: $\tilde{M}_{11}^2, \tilde{M}_{22}^2$

THDM

Higgs basis

Doublets: $\Phi_{\rm SM} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \phi_{\rm SM} + iG^0) \end{pmatrix}, \quad \Phi_{\rm BSM} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\cancel{v_{\rm BSM}} + \phi_{\rm BSM} + iA) \end{pmatrix}$

$$\boxed{\begin{aligned} V(\Phi_1,\Phi_2) = {}& \tilde{M}_{11}^2|\Phi_1|^2 + \tilde{M}_{22}^2|\Phi_2|^2 - \tilde{M}_{12}^2(\Phi_2^\dagger\Phi_1 + \text{h.c.}) + \frac{\Lambda_1}{2}|\Phi_1|^4 + \frac{\Lambda_2}{2}|\Phi_2|^4 + \\ & + \tilde{\Lambda}_3|\Phi_1|^2|\Phi_2|^2 + \tilde{\Lambda}_4|\Phi_2^\dagger\Phi_1|^2 + \left[\frac{\tilde{\Lambda}_5}{2}(\Phi_2^\dagger\Phi_1)^2 + (\tilde{\Lambda}_6|\Phi_1|^2 + \tilde{\Lambda}_7|\Phi_2|^2)\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] \end{aligned}}$$

General basis: $v_1, \cancel{v_2}, \tilde{M}_{11}^2, \tilde{M}_{22}^2, \tilde{M}_{12}^2, \tilde{\Lambda}_1, \cancel{\tilde{\Lambda}_2}, \tilde{\Lambda}_3, \tilde{\Lambda}_4, \tilde{\Lambda}_5, \tilde{\Lambda}_6, \tilde{\Lambda}_7$

FC~~N~~C

$\tilde{\Lambda}_2 = \tilde{\Lambda}_2(\tilde{\Lambda}_7)$

Higgs basis: $v, \cancel{v_{BSM}}, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, M_{22}^2, \Lambda_6, \Lambda_7$

Tad. eqs. solved for: $\tilde{M}_{11}^2, \tilde{M}_{12}^2$

Effective potential approach

$$\boxed{V_{eff} \equiv V^{(0)} + \kappa V^{(1)} + \kappa^2 V^{(2)}}$$

Loop factor:

$$\kappa = \frac{1}{16\pi^2}$$

Leading contributions from top quark and BSM scalars:

-One loop:

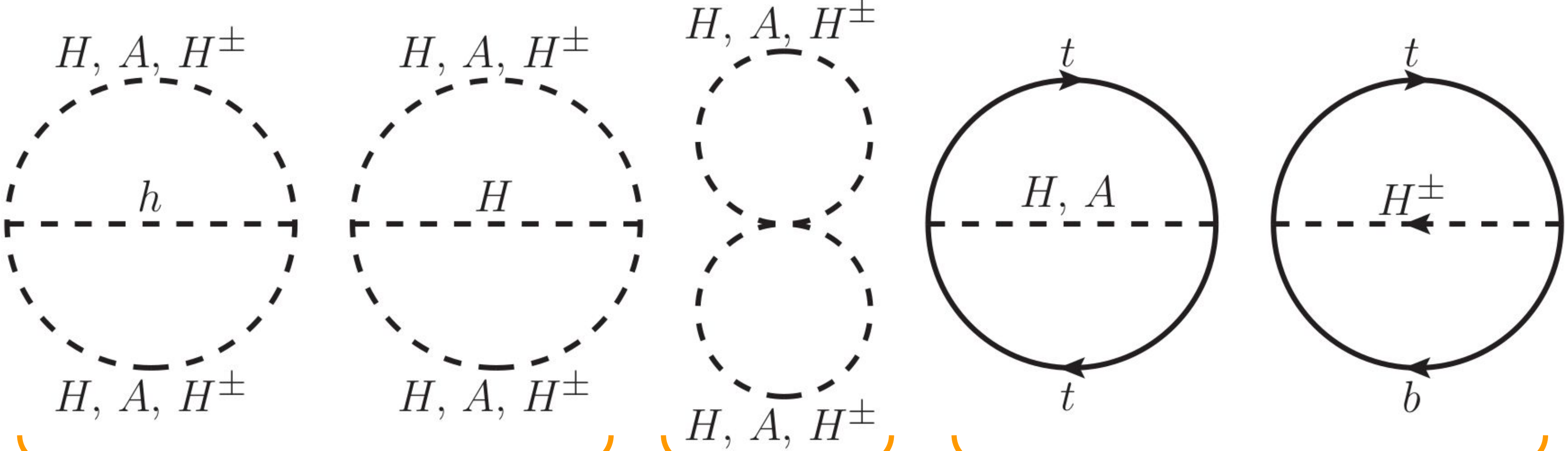
$$V^{(1)}(\phi) = -3m_t^2(\phi)\left(\overline{\log} m_t^2(\phi) - \frac{3}{2}\right) + \sum_{\phi_i} \frac{n_{\phi_i} m_{\phi_i}^4(\phi)}{4}\left(\overline{\log} m_{\phi_i}^2(\phi) - \frac{3}{2}\right)$$

-Two loops:

$$V^{(2)}(\phi) = V^{(2)}_{SSS}(\phi) + V^{(2)}_{SS}(\phi) + V^{(2)}_{FFS}(\phi)$$

Effective potential approach

Figure from: [J. Braathen, S, Kanemura, '19]

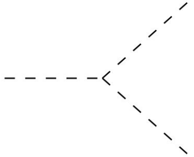


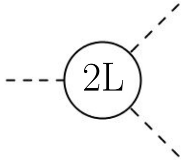
$V^{(2)}_{SSS}(\phi)$ $V^{(2)}_{SS}(\phi)$ $V^{(2)}_{FFS}(\phi)$

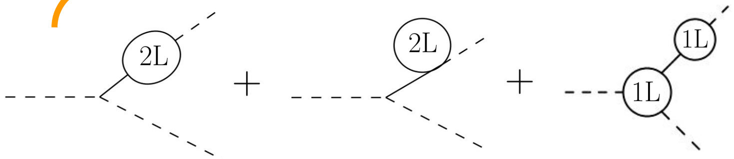
$\phi = h, H$

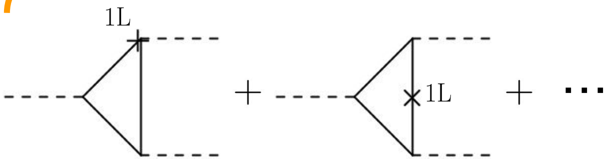
Diagrammatic approach

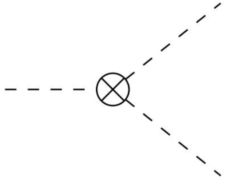
$$\hat{\lambda}_{ijk} = \lambda^{(0)}_{ijk} + \delta^{\text{gen}}\lambda_{ijk} + \delta^{\text{WFR}}\lambda_{ijk} + \delta^{\text{sub}}\lambda_{ijk} + \delta^{\text{CT}}\lambda_{ijk}$$











Definition of the countertems

Parameters for which we need one-loop counterterms:

$$v, v_{BSM}, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, m_t^2, M_{22}^2, \Lambda_6, \Lambda_7$$

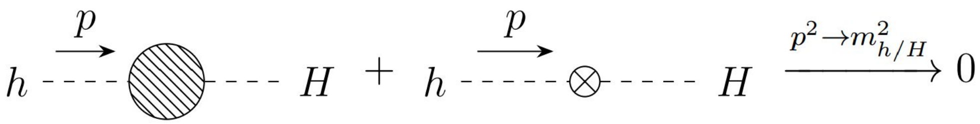
Parameters for which we need one- and two-loop counterterms:

$$v_{BSM}, m_h^2, \Lambda_6$$

Λ_6 counterterm

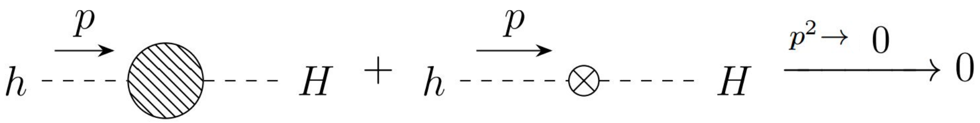
We start with the **on-shell condition** for the two point function

$$\mathrm{Re}[\hat{\Sigma}_{hH}(\ 0\ )] = \mathrm{Re}[\hat{\Sigma}_{Hh}(\ 0\ )] = 0$$



Then **modifying it**, expanding it and imposing the condition at every loop order we can obtain the one and two loop **Λ_6 counterterms**:

$$\mathrm{Re}[\hat{\Sigma}_{hH}(\ 0\ )] = \mathrm{Re}[\hat{\Sigma}_{Hh}(\ 0\ )] = 0$$

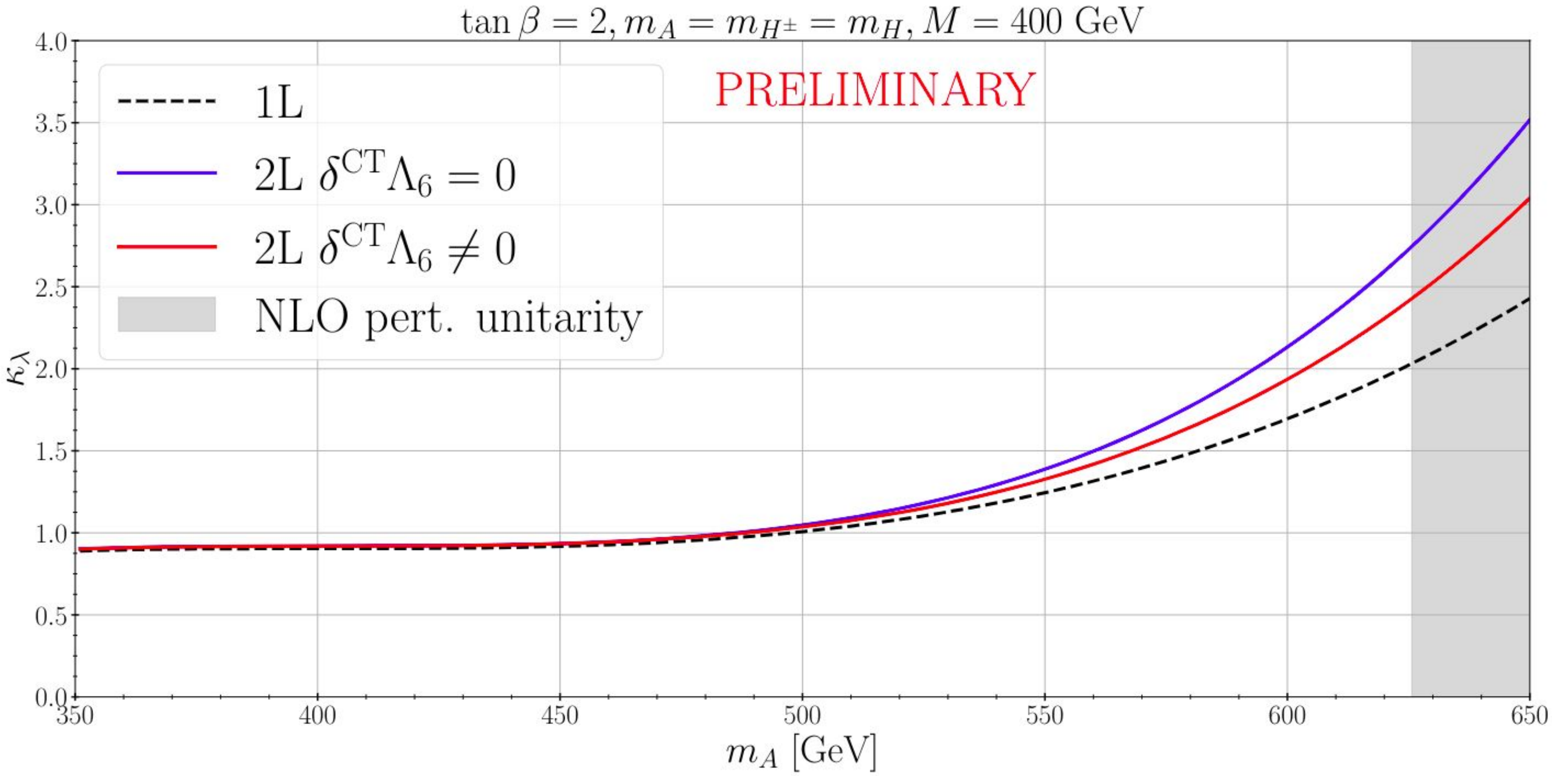


This condition is equivalent to impose that the mass matrix of the Higgs even sector is diagonal at tree level and to every loop order for 0 external momenta:

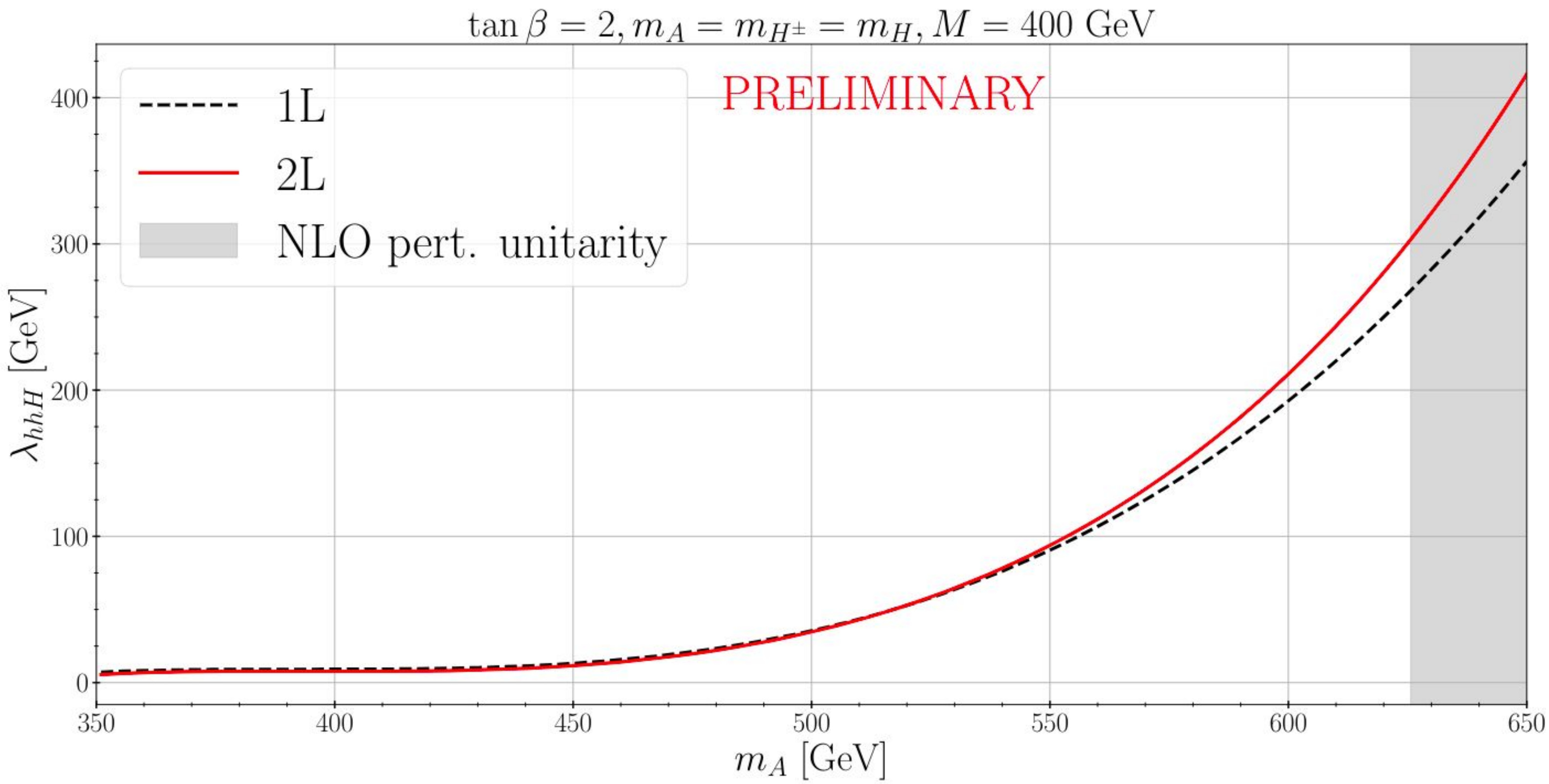
$$\hat{\mathcal{M}}^2 = \hat{M}^2 - \hat{\Sigma}_\phi(0) = \begin{pmatrix} M^2_{SM} & \Lambda_6 v^2 \\ \Lambda_6 v^2 & M^2_{BSM} \end{pmatrix} - \begin{pmatrix} \hat{\Sigma}_{hh}(0) & \hat{\Sigma}_{hH}(0) \\ \hat{\Sigma}_{Hh}(0) & \hat{\Sigma}_{HH}(0) \end{pmatrix}$$

$$\kappa_\lambda = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\mathrm{SM},(0)}}$$

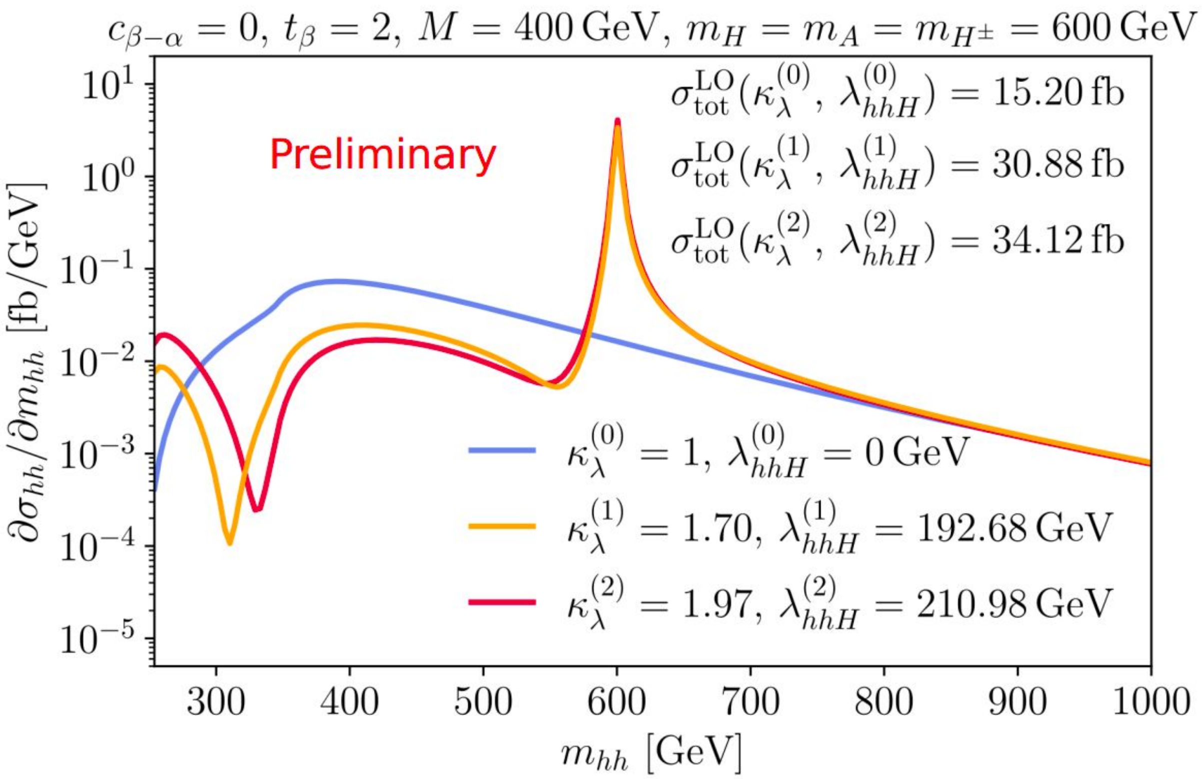
Results for λ_{hhh}



Results for λ_{hhH}



Results for di-Higgs production



Di-Higgs cross-section and m_{hh} distributions computed with anyHH [Bahl et al. WIP], using our 2L predictions for λ_{hhh} and λ_{hhH} (thanks to Kateryna Radchenko Serdula for running the distributions)

Summary

We use the **Higgs Basis** to renormalise the alignment condition more conveniently

We compute the **two-loop corrections** to λ_{ijk} following **two different approaches.**

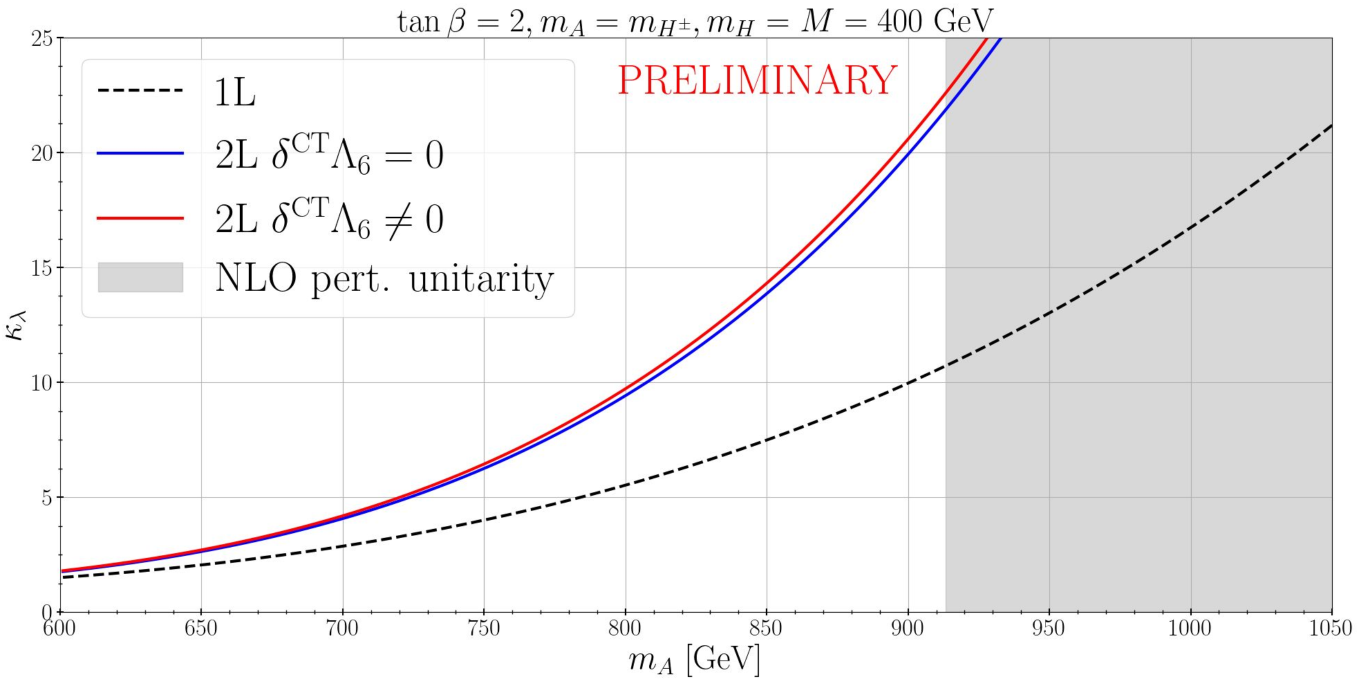
We impose the **alignment limit** not only at tree level but also at **every loop level** and we take into account the renormalisation of Λ_6 at one- and two-loop orders.

We investigate the **impact** of the two-loop correction **on di-Higgs production**

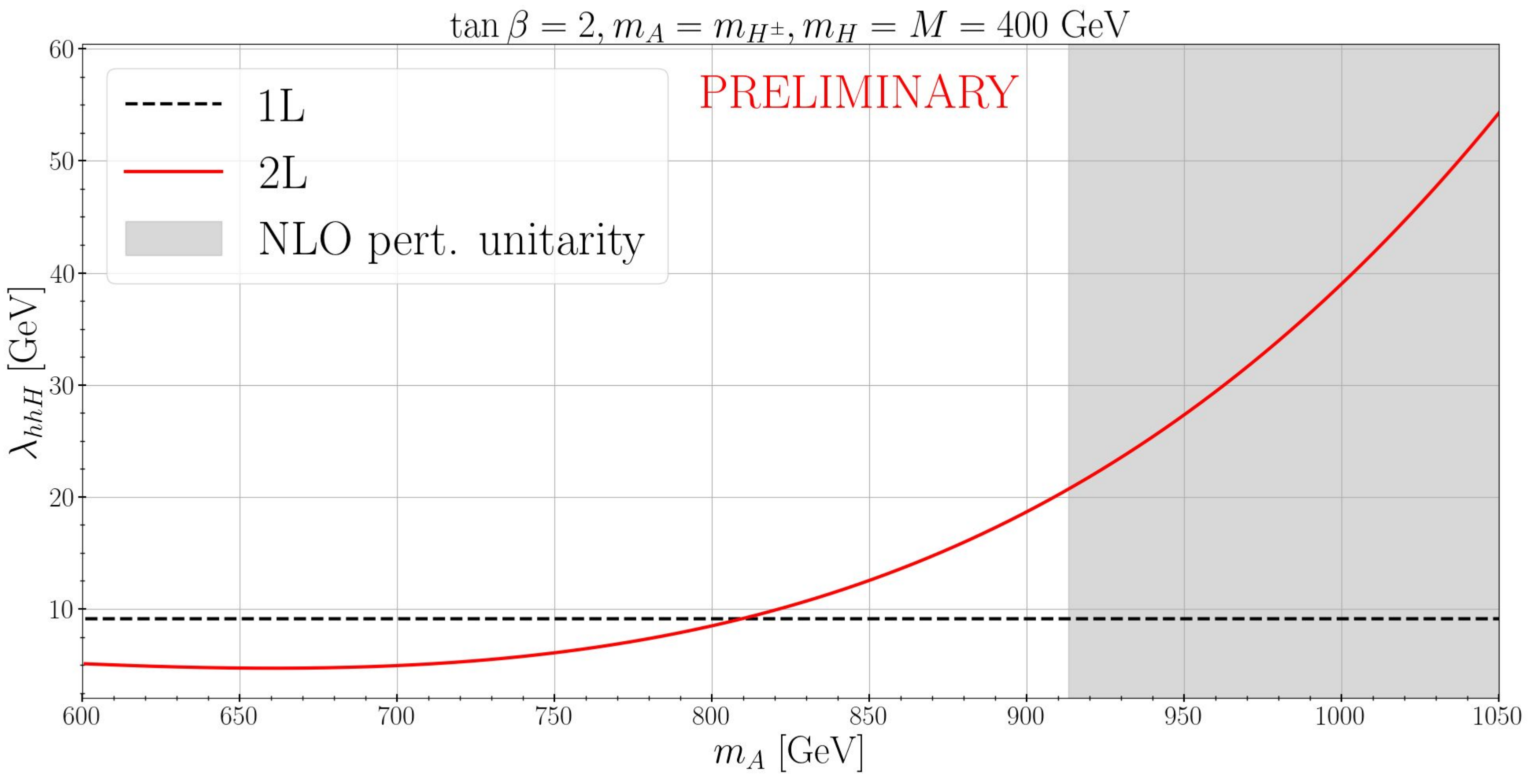
Thank you for your attention

$$\boxed{\kappa_\lambda = \frac{\lambda_{hhh}}{\lambda_{hhh}^{\text{SM},(0)}}}$$

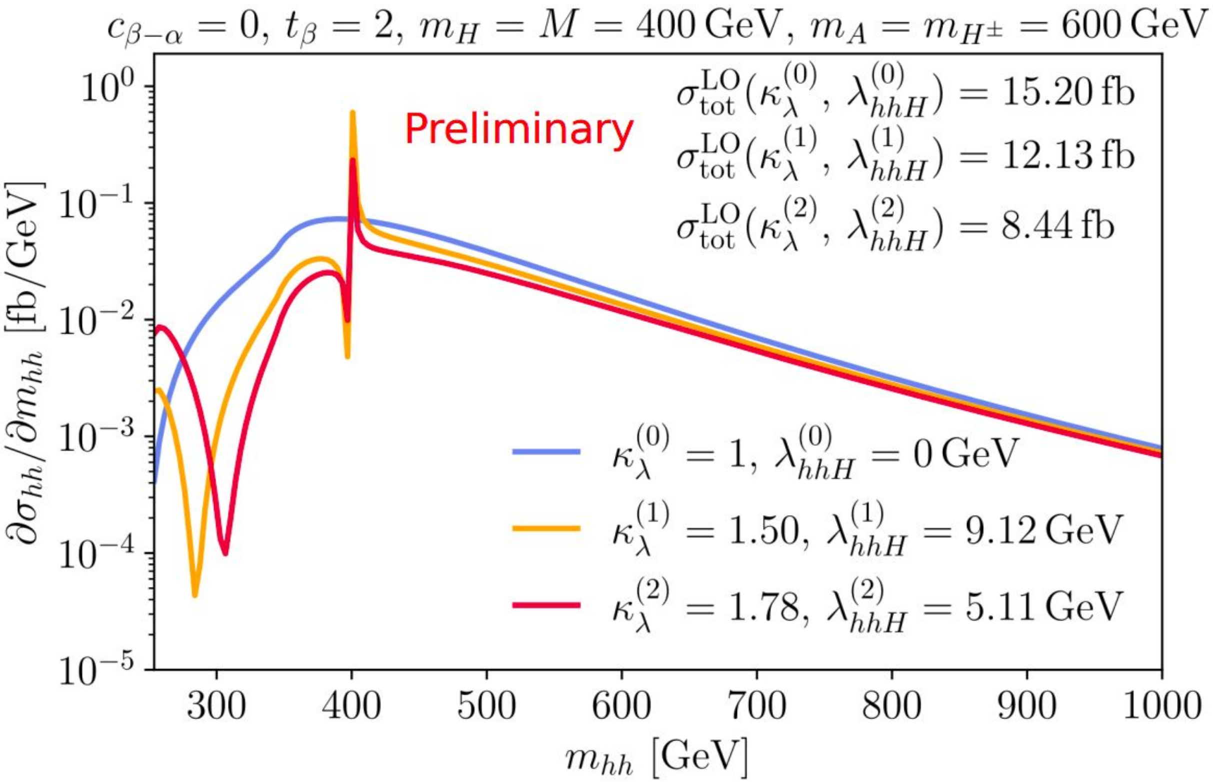
Results for λ_{hhh}



Results for λ_{hhH}



Results for di-Higgs production

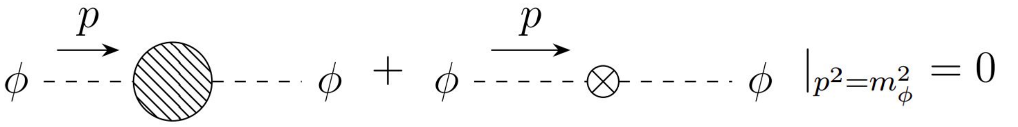


Field and mass renormalisation

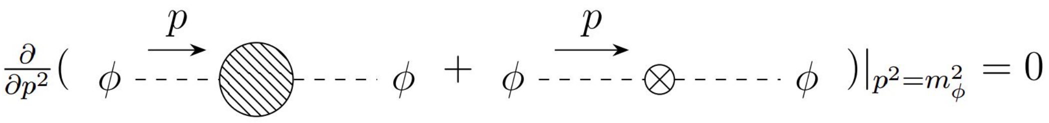
In the renormalisation procedure of the 2-point functions we fix two renormalization conditions following the OS scheme. To determine the $\Lambda_6$counterterm we will use a modified version of the third OS condition.

OS conditions:

$$\mathrm{Re}[\hat{\Sigma}_{hh}(m_h^2)] = \mathrm{Re}[\hat{\Sigma}_{HH}(m_H^2)] = 0$$



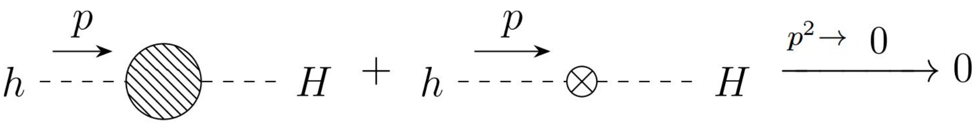
$$\mathrm{Re}\left[\frac{\partial\hat{\Sigma}_{hh}(p^2)}{\partial p^2}\bigg|_{p^2=m_h^2}\right] = \mathrm{Re}\left[\frac{\partial\hat{\Sigma}_{HH}(p^2)}{\partial p^2}\bigg|_{p^2=m_H^2}\right]$$



$\phi = h, H$

Modified OS condition:

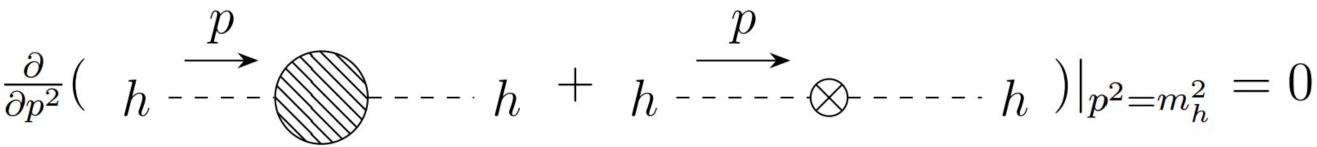
$$\mathrm{Re}[\hat{\Sigma}_{hH}(\ 0\ )] = \mathrm{Re}[\hat{\Sigma}_{Hh}(\ 0\ )] = 0$$



External leg corrections

Taking the **second condition**, expanding it and imposing the condition at every loop order we can obtain the one and two loop **diagonal Z factor counterterms:**

$$\mathrm{Re}\left[\frac{\partial\hat{\Sigma}_{hh}(p^2)}{\partial p^2}\bigg|_{p^2=m_h^2}\right] = 0$$



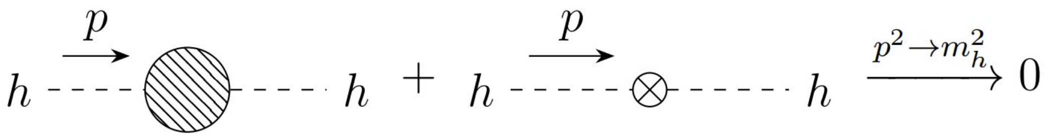
We can follow the same procedure for all the particles involved in the computation:

$$h, H, A, H^{\pm}$$

Mass counterterms

Taking the **first condition**, expanding it and imposing the condition at every loop order we can obtain the one and two loop **mass counterterms:**

Condition:



Counterterms:

$$\left\{ \begin{aligned} (\delta m_h^2)^{(1)} &= \Sigma_{hh}^{(1)}(m_h^2) - \delta T_{hh}^{(1)} \,, \\ (\delta m_h^2)^{(2)} &= \Sigma_{hh}^{(2)}(m_h^2) - \delta T_{hh}^{(2)} - (\delta D_{hh}^2)^{(1)} \delta Z_{hh}^{(1)} \end{aligned} \right\}$$

We can follow the same procedure for all the particles which masses need a counterterm:

$$h, H, A, H^{\pm}, t$$

Vev counterterm

We express the EW vev in terms of the electroweak parameters and we follow the same procedure that for the SM:

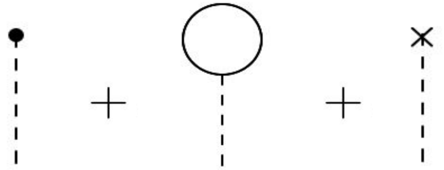
Definition of the EW vev:

$$v^2 = \frac{m_W^2}{\pi \alpha_{EM}} \left(1 - \frac{m_W^2}{m_Z^2} \right)$$

$$\boxed{\frac{\delta v}{v} = \frac{1}{2} \left(\frac{s_W^2 - c_W^2}{s_W^2} \frac{\mathrm{Re}[\Sigma_{WW}(m_W^2)]}{m_W^2} + \frac{c_W^2}{s_W^2} \frac{\mathrm{Re}[\Sigma_{ZZ}(m_Z^2)]}{m_Z^2} - \frac{d}{dp^2} \Sigma_{\gamma\gamma}(p^2) \Big|_{p^2=0} - \frac{2 s_W}{c_W} \frac{\Sigma_{\gamma Z}(0)}{m_Z^2} \right)}$$

Tadpole counterterms

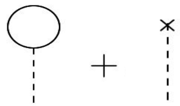
$$\boxed{\hat{t}_i = t_i^{(0)} + \delta t_i + \delta t_i^{\mathrm{CT}} = 0}$$



OS/standard scheme

$$t_i^{(0)} = 0$$

$$\left\{ \delta t_i + \delta t_i^{\mathrm{CT}} = 0 \right\}$$



For each loop order

Now we do not have to include tadpole diagrams in the calculation

Conditions to avoid FCNCs

The $\mathbb{Z}_2$ symmetry is imposed to avoid the flavor changing neutral currents (FCNC) but in the Higgs basis we do not impose this symmetry.

Comparing the Higgs basis with the $\mathbb{Z}_2$ symmetric case

$$\left\{ \begin{array}{c} \Lambda_2 = \Lambda_1 + 2(\Lambda_6 + \Lambda_7)\cot 2\beta, \\ \Lambda_3 + \Lambda_4 + \Lambda_5 = \Lambda_1 + 2\Lambda_6 \cot 2\beta - (\Lambda_6 - \Lambda_7)\tan 2\beta \end{array} \right\}$$

Conditions from: [J. Bernon, J.F. Guinon, H.E. Haber, Y. Jiang, S. Kraml, '15]

$$\boxed{\Lambda_2 \overset{\text{alignment}}{=} \frac{m_h^2}{v^2} - \frac{2v^2\Lambda_7^2}{2M_{22}^2 + m_h^2 - 2m_H^2}}$$

Checks of the results

Compare **our counterterms** with the **RGEs of SARAH**

Check that the diagrammatic result is **UV finite**

Cross check the results between the **diagrammatic and the effective potential approach**

Cross check results with the **literature**