Dirac Leptogenesis Via Scattering

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Outline

Leptogenesis

Leptogenesis: the dynamical generation of a matter–antimatter asymmetry generating lepton number violation.

$\mathbf{L} = N_L - N_{\bar{L}}$

(number of leptons minus anti-leptons)

Motivation

Rise of the Big Bang Theory:

- Theoretical predictions
- and observational discoveries i.e Cosmic expansion and cosmic microwave background (CMB)
- Indication of a hot universe in early stages and presence of antimatter
- Lack of observational evidence for a large amount of antimatter in the observable universe

Motivation

The Asymmetry of the Universe can be estimated by:

- The baryon-to-photon ratio: $\eta = \frac{n_B}{n_\gamma}$

  Where $\eta_B \approx (\frac{s}{n_\gamma}) \cdot \frac{n_B}{s} = (\frac{s}{n_\gamma}) \cdot Y_\eta = 7.04\ Y_{\Delta B}$
- η can be determined independently in two different ways:
  - From the abundances of light elements in the intergalactic medium (IGM)
  - From the power spectrum of temperature fluctuations in the CMB
- Both consistently give values of order 10^{-10}
- Discovery of small violations of P and CP invariance (and thus also C invariance) hints at the possibility of baryogenesis/leptogenesis in the early Universe.
- Progress in non-equilibrium field theories

Sakharov Conditions (Conditions for Baryogenesis)(1967)

- B number violation (the presence of baryon-number violating interactions)
- C and CP violation.
- Departure from thermal equilibrium.

. Note: Smallness of the CP violation in the SM requires us to consider BSM Theories.

Need for BSM Theories

- Many models have been proposed in accordance with Sakharov's conditions since his original suggestion.
- These models are often grouped into two main categories:

  (a) Models that directly generate a matter–antimatter asymmetry directly in the Baryonic sector.(**Baryogenesis**).

  (b) Lepton number violation is transferred to the baryonic sector, either by SM sphalerons or through processes involving physics beyond the Standard Model (BSM).**Baryogenesis via Leptogenesis**

Extensions to the Standard Model for successful Leptogenesis

- **Standard Leptogenesis (Simplest Extension to explain Leptogenesis):**
  - Includes 3 right-handed neutrinos (Majorana Particles) in addition to the 3 left-handed ones in the SM plus a heavy scalar.
  - A lepton asymmetry is generated in the early universe via the decay of heavy right-handed Majorana neutrinos ($F \to f\phi^*$).
  - Heavy right-handed Majorana neutrinos go out of the equilibrium producing lepton asymmetry and Seesaw Mehanism is used to explain the smallness of the mass of the Standard Model neutrinos.
  - This asymmetry is then converted into the observed baryon asymmetry through sphaleron processes.

Dirac Leptogenesis via Scattering

- In Dirac Leptogenesis via Scattering:
  - Scatterings (2→2 processes) between light degrees of freedom:
    - Standard Model (SM) particles
    - Right handed neutrinos (two generations are enough for our scenario).
    - Heavy Scalar with two flavors
  - These interactions are sufficient to generate the asymmetry.
- Sakharov's Conditions:
  - The right-handed neutrino partners are out of the equilibrium. (This satisfies Sakharov's conditions for baryogenesis)
- Off-Shell Heavy Degrees of Freedom:
  - Heavy degrees of freedom are not needed to be produced in the early universe.
  - The reheating temperature can be below the mass scale of these heavy particles.
  - $T_{reh} = 10^8 GeV$

Closed Time Path (CTP) Formalism

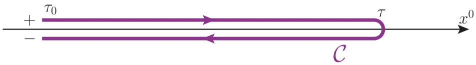


Figure: The Closed-Time Path (CTP)

For a two-point function G^{ab}, we write:

$$\text{Wightman Functions:} \quad G^{>}(x,y) = G^{-+}(x,y), \quad G^{<}(x,y) = G^{+-}(x,y), \quad (1)$$

$$\text{(Anti-)Time ordered two pt. fns. :} \quad G^{T}(x,y) = G^{++}(x,y), \quad G^{\bar{T}}(x,y) = G^{--}(x,y), \quad (2)$$

$$\text{Spectral function:} \quad G^{\mathcal{A}} = \frac{i}{2}\left(G^{>} - G^{<}\right) \quad (3)$$

Proposed General Scenario

Consider a scenario given by the following Lagrangian:

$$\mathcal{L} = \overline{e}_L^c F_i \nu_L \overline{\chi}_i + \overline{e}_R^c G_i \nu_R \overline{\chi}_i + \text{h.c.}, \tag{4}$$

Originally proposed by [Julian Heeck, Jan Heisig, and Anil Thapa, (June 2023)]

Quantum numbers for particle χ_i:

$SU(3) \times SU(2) \times U(1) = (\mathbf{1}, \mathbf{1}, -1)$

Color singlet, Weak singlet with hypercharge -1 and electric charge $\frac{-1}{2}$

Asymmetry

For calculating the equilibration rate, we add yields for particle and anti-particles and for calculating the washout rate and source term subtract the yields:

$$Y_{\Sigma_{\nu_R}} = Y_{\nu_R} + Y_{\bar{\nu}_R} \tag{5}$$

$$Y_{\Delta_{\nu_R}} = Y_{\nu_R} - Y_{\bar{\nu}_R} \tag{6}$$

Where Baryon Asymmetry is given by:

$$Y_{\Delta B} = \frac{28}{79} Y_{\Delta(B-L_{SM})} = \frac{28}{79} Y_{\Delta\nu_R}, \qquad c_s = \frac{28}{79} \tag{7}$$

Calculation for the Source Term

$\nu_R(q) =$

$=$

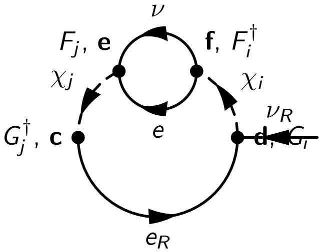
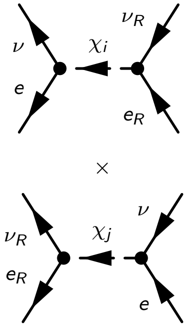


Figure: One loop Wavefunction correction to the Source term

Representation as Interference of the Two Tree-Level Diagrams

This could be represented as interference of the two tree-level diagrams after applying the cut on the fermionic loop inside the scalar propagator that goes all the way to ν_R:



Collision Term for ν_R for One-Loop wavefunction Correction to the Scalar Propagator

The collision term is given in the form of a kinetic equation:

$$C_{\nu_R}(\mathbf{q}) = \frac{d}{dt}\left(f_{\nu_R}(\mathbf{q}) - \bar{f}_{\nu_R}(\mathbf{q})\right) = \int \frac{dq^o \, sign(q^o)}{2\pi} \operatorname{tr}\left[i\Sigma^{\mathbf{wf1},>}_{\nu_R}(q) iS^{<}_{\nu_R}(q) - i\Sigma^{\mathbf{wf1},<}_{\nu_R}(q) iS^{>}_{\nu_R}(q)\right] \tag{8}$$

And the number density evolution is given by:

$$\dot{n}_{\nu_R}(\mathbf{q}) = \int \frac{d^3 q}{(2\pi)^3} \mathcal{C}_{\nu_R}(\mathbf{q}) = \int \frac{d^4 q}{(2\pi)^4} \operatorname{tr}\left[i\Sigma^{\mathbf{wf1},>}_{\nu_R}(q) iS^{<}_{\nu_R}(q) - i\Sigma^{\mathbf{wf1},<}_{\nu_R}(q) iS^{>}_{\nu_R}(q)\right] \tag{9}$$

For out-of-equilibrium ν_R, replace:

$$iS^{<,>,T,\bar{T}}_{\nu_R} = i\delta S_{\nu_R}$$

Number density evolution for one loop Source term

ν_R going out of the equilibrium in the internal loop.

$$\dot{n}_{\nu_R} = 4\sum_{ij}\mathbf{tr}[F_iF_j^\dagger]\mathbf{tr}[G_i^\dagger G_j] i\Delta\chi_j^{\bar{T}}(p) i\Delta\chi_i^{T}(p)\int\frac{d^4p}{(2\pi)^4}$$

$$\int_{\vec{p_1}\vec{p_2}\vec{q}\vec{q}'}\delta_{(p-q-q')}\delta_{(p_1+p_2-p)}(p_1\cdot p_2)(q\cdot q')\Bigg[\Bigg(\delta f_{\nu_R}^{\beta}(|\mathbf{p_2}|) - \delta\bar{f}_{\nu_R}^{\beta}(|\mathbf{p_2}|)\Bigg)$$

$$\Bigg(-f_{\nu_R}^{\alpha}(|\mathbf{q}'|)f_{e_R}(|\mathbf{q}|) - f_{e_R}(|\mathbf{q}|)f_{e_R}(|\mathbf{p_1}|) - f_{\nu_R}^{\alpha}(|\mathbf{q}'|)f_{e_R}(|\mathbf{p_1}|) - f_{e_R}(|\mathbf{p_1}|) - f_{\nu_R}^{\alpha}(|\mathbf{q}'|) - f_{e_R}(|\mathbf{q}|)\Bigg)\Bigg] \tag{10}$$

$$\delta f_{\nu_R}^{\beta}(\mathbf{p_2}) \neq \delta\bar{f}_{\nu_R}^{\beta}(\mathbf{p_2}), \quad \Rightarrow \tag{11}$$

But Asymmetry only transfers between different flavors of ν_R.

Calculation for the $\dot{n}_{\nu_R}$ for the Washout and the Equilibration rate

(ν_R Going Out of Equilibrium) for the One-Loop correction to the scalar propagator in the external loop.
The number density evolution is given by:

$$\dot{n}_{\nu_R}(\mathbf{q}) = \int \frac{d^4q}{(2\pi)^4}\,\mathrm{tr}\left[i\not{\Sigma}^{\mathbf{wf1},>}_{\nu_R}(\mathbf{q})\, i\delta S_{\nu_R}(\mathbf{q}) - i\not{\Sigma}^{\mathbf{wf1},<}_{\nu_R}(\mathbf{q})\, i\delta S_{\nu_R}(\mathbf{q})\right] \tag{12}$$

$$= \int \frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\left[i\delta\Pi^{<}_{R}(p)\, i\Pi^{>}(p) - i\delta\Pi^{>}_{R}(p)\, i\Pi^{<}(p)\right] \tag{13}$$

Self-Energy Contribution

$$i\Pi^{>}(p) = -\mathbf{tr}[G_i G_j^{\dagger}]\frac{p^2}{8\pi|\mathbf{p}|}\Big[.....\Big] \qquad (14)$$

- **For** $p^2 > 0, \quad p^0 > 0$:

$$\int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{q}|\big(-1 + f_{\nu_L}(|p^o| - |\mathbf{q}|)\big)\big(-1 + f_{e_L}(|\mathbf{q}|)\big)$$

- **For** $p^2 > 0, \quad p^0 < 0$:

$$\int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{q}|\Big(f_{\nu_L}(|p^o| - |\mathbf{q}|)f_{e_L}(|\mathbf{q}|)\Big)$$

- **For** $p^2 < 0$:

$$\int_{\frac{1}{2}(|\mathbf{p}|-|p^o|)} d|\mathbf{q}|f_{\nu_L}(p^o + |\mathbf{q}|)\big(-1 + f_{e_L}(|\mathbf{q}|)\big) + \int_{\frac{1}{2}(|\mathbf{p}|+|p^o|)} d|\mathbf{q}|\big(-1 + f_{\nu_L}(|\mathbf{q}| - p^o)\big)f_{e_L}(|\mathbf{q}|)$$

$\dot{n}_\nu$ contribution for the Equilibration Rate and the Washout Rate

$$\dot{n}_{\nu_R} = \frac{\mathbf{tr}[G_i G_j^\dagger]\,\mathbf{tr}[F_i^\dagger F_j]}{256\pi^5 M_i^2 M_j^2} \int_0^\infty dp^0 \int_0^{|p^o|} d|\mathbf{p}| \left((p^0 + |\mathbf{p}|)(p^0 - |\mathbf{p}|)\right)^2 \times \left[\int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{p_1}| d|\mathbf{q}| \delta f_{\nu_R}^\alpha(|p^o| - |\mathbf{q}|) \Big(f_{e_R}(|\mathbf{q}|) + f_{\nu_L}(|p^o| - |\mathbf{p_1}|) f_{e_L}(|\mathbf{p_1}|) \Big) \right] \quad (15)$$

$$\dot{\bar{n}}_{\nu_R} = \frac{\mathbf{tr}[G_i G_j^\dagger]\,\mathbf{tr}[F_i^\dagger F_j]}{256\pi^5 M_i^2 M_j^2} \int_{-\infty}^0 dp^0 \int_0^{|p^o|} d|\mathbf{p}| \Big((|p^o| + |\mathbf{p}|)(|p^o| - |\mathbf{p}|) \Big)^2 \times \int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{p_1}| d|\mathbf{q}| \left[\Big(\delta \bar{f}_{\nu_R}^\alpha(|p^o| - |\mathbf{q}|)\Big) \Big(- f_{\nu_L}(|p^o| - |\mathbf{p_1}|) f_{e_L}(|\mathbf{p_1}|) - f_{e_R}(|\mathbf{q}|) \Big) \right] \quad (16)$$

$\dot{n}_{\nu_{R(Total)}}$ contribution for the Equilibration Rate

$$\dot{n}_{\nu_{R(Total)}} = \dot{n}_{\nu_R} + \dot{n}_{\bar{\nu}_R} = \dot{n}_{\Sigma\nu_R} \tag{17}$$

Using Maxwell-Boltzmann Statistics and solving on Mathematica, we get;

$$\dot{n}_{\Sigma\nu_R} = \frac{51}{32\pi^5} \times \frac{\mathbf{tr}[G_i G_j^\dagger]\,\mathbf{tr}[F_i^\dagger F_j]}{M_j^2 M_i^2} \mu_{\Sigma\nu_R} T^7 \tag{18}$$

$$= \frac{a}{\pi^5} \times \frac{\mathbf{tr}[G_i G_j^\dagger]\,\mathbf{tr}[F_i^\dagger F_j]}{M_j^2 M_i^2} \mu_{\Sigma\nu_R} T^7 \tag{19}$$

$\dot{n}_{\nu_{R(Total)}}$ contribution for the Washout Rate

$$\dot{n}_{\nu_{R(Total)}} = \dot{n}_{\nu_R} - \dot{n}_{\bar{\nu}_R} = \dot{n}_{\Delta\nu_R} \tag{20}$$

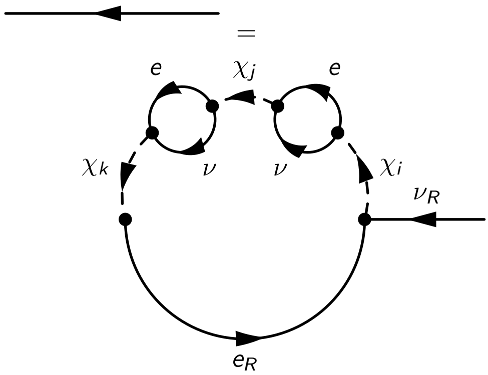
Using Maxwell-Boltzmann Statistics and solving on Mathematica, we get;

$$\dot{n}_{\Delta\nu_R} = \frac{51}{32\pi^5} \times \frac{\mathbf{tr}[G_i G_j^\dagger]\,\mathbf{tr}[F_i^\dagger F_j]}{M_j^2 M_i^2} \mu_{\Delta\nu_R} T^7 \tag{21}$$

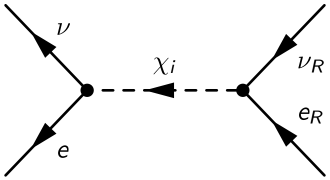
$$= \frac{a}{\pi^5} \times \frac{\mathbf{tr}[G_i G_j^\dagger]\,\mathbf{tr}[F_i^\dagger F_j]}{M_j^2 M_i^2} \mu_{\Delta\nu_R} T^7 \tag{22}$$

$\dot{n}_{\nu_R}$ for two-Loop wavefunction correction to the scalar propagator to get the Source Term

$\nu_R(q) =$



Two-loop diagram as Interference of Tree-Level and one-loop level diagrams and Real Intermediate States (RIS) Subtraction



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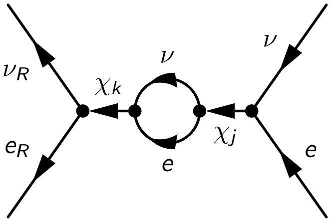

Number density evolution for ν_R for two loop correction to the scalar propagator

$$\dot{n}_{\nu_R} = \int \frac{d^4 p}{(2\pi)^4} \operatorname{tr}\left[i\Sigma_{\nu_R}^{\mathbf{wf2},>}(\mathbf{q}) iS_{\nu_R}^{<}(\mathbf{q}) - i\Sigma_{\nu_R}^{\mathbf{wf2},<}(\mathbf{q}) iS_{\nu_R}^{>}(\mathbf{q}) \right] \tag{23}$$

Simplifying leads to:

$$\dot{n}_{e_L} = \int \frac{d^3 q}{(2\pi)^3} C_{e_L}(\mathbf{q}) = \frac{-i}{M_k^2 M_j^2 M_i^2} \int \frac{d^4 p}{(2\pi)^4} \left[\Pi_R^A \left(i\delta\Pi_{R,kj}^{>} i\Pi_{L,ji}^{<} - i\delta\Pi_{R,kj}^{<} i\Pi_{L,ji}^{>} + i\Pi_{L,kj}^{>} i\delta\Pi_{R,ji}^{<} - i\Pi_{L,kj}^{<} i\delta\Pi_{R,ji}^{>} \right) \right] \tag{24}$$

$$\text{where} \quad \left(i\Delta_{\chi_k}^{T} i\Delta_{\chi_j}^{\bar{T}} i\Delta_{\chi_i}^{T} \right) = \frac{i^3}{(-M_k^2)(M_j^2)(-M_i^2)} = \frac{-i}{M_i^2 M_j^2 M_k^2}, \quad \text{for } M^2 >> p^2 \tag{25}$$

Number Density Evolution for the Two Loop Source Term from positive energy

$$\dot{n}_{\nu_R}(\mathbf{q}) = \frac{i\mathbf{tr}[G_i G_k^\dagger]}{4096\pi^6 M_i^2 M_j^2 M_k^2} \int_0^\infty dp^0 \int_0^{|p^o|} d|\mathbf{p}| \frac{\Big((p^0+|\mathbf{p}|)(p^0-|\mathbf{p}|)\Big)^3}{|\mathbf{p}|} \int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{q}|$$
$$\times \Bigg[\mathbf{tr}[F_i^\dagger F_j]\mathbf{tr}[G_j^\dagger G_k] \int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{p_1}| d|\mathbf{k_1}| \delta f_{\nu_R}^{\beta}(|p^o|-|\mathbf{k_1}|)$$
$$\times \Big(- f_{\nu_L}(|p^o|-|\mathbf{p_1}|) f_{e_L}(|\mathbf{p_1}|) - f_{e_R}(|\mathbf{k_1}|)\Big)$$
$$+ \mathbf{tr}[F_j^\dagger F_k]\mathbf{tr}[G_i^\dagger G_j] \int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{p_1}| d|\mathbf{k_1}| \delta f_{\nu_R}^{\beta}(|p^o|-|\mathbf{p_1}|)$$
$$\times \Big(f_{e_R}(|\mathbf{p_1}|) + f_{e_L}(|\mathbf{k_1}|) f_{\nu_L}(|p^o|-|\mathbf{k_1}|)\Big)\Bigg] \tag{26}$$

Number Density Evolution for the Two Loop Source Term from negative energy

$$\dot{n}_{\bar{\nu}_R} = \frac{i\mathbf{tr}[G_i G_k^\dagger]}{4096\pi^6 M_i^2 M_j^2 M_k^2} \int_{-\infty}^{0} dp^0 \int_0^{|p^o|} d|\mathbf{p}| \frac{\Big((p^0+|\mathbf{p}|)(p^0-|\mathbf{p}|)\Big)^3}{|\mathbf{p}|} \int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{q}|$$

$$\times \Bigg[\mathbf{tr}[G_j^\dagger G_k]\mathbf{tr}[F_i^\dagger F_j] \int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{k_1}|d|\mathbf{p_1}|\delta\bar{f}^{\beta}_{\nu_R}(|p^o|-|\mathbf{k_1}|)$$

$$\times \Big(f_{e_R}(|\mathbf{k_1}|) + f_{\nu_L}(|p^o|-|\mathbf{p_1}|)f_{e_L}(|\mathbf{p_1}|)\Big)$$

$$+\mathbf{tr}[F_j^\dagger F_k]\mathbf{tr}[G_i^\dagger G_j] \int_{\frac{1}{2}(|p^o|-|\mathbf{p}|)}^{\frac{1}{2}(|p^o|+|\mathbf{p}|)} d|\mathbf{k_1}|d|\mathbf{p_1}|\delta\bar{f}^{\beta}_{\nu_R}(|p^o|-|\mathbf{p_1}|)$$

$$\times \Big(-f_{\nu_L}(|p^o|-|\mathbf{k_1}|)f_{e_L}(|\mathbf{k_1}|) - f_{e_R}(|\mathbf{p_1}|)\Big)\Bigg] \quad (27)$$

Total Number Density Evolution for the Two Loop Source Term

Solving on Mathematica gives;

$$\dot{n}_{\nu_R(Total)} = \dot{n}_{\nu_R} - \dot{n}_{\bar{\nu}_R} = \dot{n}_{\Delta\nu_R} \tag{28}$$

$$\dot{n}_{\Delta\nu_R}(\mathbf{q}) = \frac{297}{64\pi^6} \times \frac{i\,\mathrm{tr}[G_i G_k^\dagger]}{M_k^2 M_j^2 M_i^2}\left[\mathbf{tr}[F_j^\dagger F_k]\mathbf{tr}[G_i^\dagger G_j] - \mathbf{tr}[F_i^\dagger F_j]\mathbf{tr}[G_j^\dagger G_k]\right] \times \mu_{\Sigma\nu_R} T^9 \tag{29}$$

$$\dot{n}_{\Delta\nu_R}(\mathbf{q}) = \frac{b}{\pi^6} \times \frac{i\,\mathrm{tr}[G_i G_k^\dagger]}{M_k^2 M_j^2 M_i^2}\left[\mathbf{tr}[F_j^\dagger F_k]\mathbf{tr}[G_i^\dagger G_j] - \mathbf{tr}[F_i^\dagger F_j]\mathbf{tr}[G_j^\dagger G_k]\right] \times \mu_{\Sigma\nu_R} T^9 \tag{30}$$

Fluid Equations and Yield

$$\dot{n}_{\Sigma\nu_R} = \delta n_{\Sigma\nu_R} \cdot \frac{51 \times \mathbf{tr}[G_i^\dagger G_j]\,\mathbf{tr}[F_i F_j^\dagger]}{32\pi^3 M_j^2 M_i^2} T^5 = \delta n_{\Sigma\nu_R} \cdot \mathcal{C}_{\Sigma\nu_R} \quad (31)$$

We use,

$$\dot{Y} = \frac{1}{s}(\dot{n}_{diluted} + 3Hn) \quad (32)$$

$$\dot{Y} = \frac{1}{s}(\dot{n} - 3Hn + 3Hn) \quad (33)$$

$$\dot{Y} = \frac{\dot{n}}{s} \quad (34)$$

So, we get:

$$\delta\dot{Y}_{\Sigma\nu_R} = \delta Y_{\Sigma\nu_R} \cdot \mathcal{C}_{\Sigma\nu_R} \quad (35)$$

where $\mathcal{C}_{\Sigma\nu_R}$ is the equilibration rate.

Fluid Equations

In conformal spacetime:

$$\frac{d\,\delta Y_{\Sigma\nu_R}}{dz} = \mathcal{C}_{\Sigma\nu_R}\left(Y_{\Sigma\nu_R} - Y^{eq}_{\Sigma\nu_R}\right) \tag{36}$$

$$\frac{d\,\delta Y_{\Delta\nu_R}}{dz} = \left(\mathcal{S}_{\Sigma\nu_R}\,\delta Y_{\Sigma\nu_R} + \mathcal{W}_{\Delta\nu_R}\,\delta Y_{\Delta\nu_R}\right) \tag{37}$$

Now, we have the expression for $Y_{\nu_R}(z')$ in the integral form as:

$$Y_{\Delta\nu_R}(z') = \int_{z_i}^{z} \frac{\mathcal{S}_{\Sigma\nu_R}(z')}{\mathcal{C}_{\Sigma\nu_R}(z'')}\frac{dY_{\Sigma\nu_R}}{dz''}\, e^{\int_{z'}^{z} \mathcal{W}_{\Delta\nu_R}(z'')dz''}\, dz' \tag{38}$$

$$Y_{\Delta\nu_R}(z') = \int_{z_i}^{z} \mathcal{S}_{\Sigma\nu_R}(z')\delta Y_{\Sigma\nu_R}(z'')e^{\int_{z'}^{z} \mathcal{W}_{\Delta\nu_R}(z'')dz''}\, dz' \tag{39}$$

$$\delta Y_{\Sigma\nu_R} = \delta Y_{\Sigma\nu_R}(z_i)\exp\int_{z'}^{z} dz''\mathcal{C}_{\nu_R}(z'') \tag{40}$$

Final Yield of ν_R

Solving on Mathematica gives:

$$Y_{\Delta\nu_R}(z) = k_S \cdot \exp\left(-kz^{-8}\right) \cdot \frac{2\sqrt{k}\left(-\frac{e^{k/z^8}}{z^4} + \frac{e^{k/z_i^8}}{z_i^4}\right) + \sqrt{\pi}\left(\text{Erfi}\left(\frac{\sqrt{k}}{z^4}\right) - \text{Erfi}\left(\frac{\sqrt{k}}{z_i^4}\right)\right)}{16\,k^{3/2}} \quad (41)$$

Setting z_i equals to '1' and z equals to Infinity as we set same final limit, we get:

$$Y_{\Delta\nu_R}(z=\infty) = k_S \cdot \frac{e^k\left(\sqrt{k} - \text{DawsonF}(\sqrt{k})\right)}{8\,k^{3/2}} \quad (42)$$

where,

$$k = \frac{k_{\mathcal{C}_{\Sigma\nu_R}} + k_{\mathcal{W}_{\Delta\nu_R}}}{8} = \frac{k_{\mathcal{W}_{\Delta\nu_R}} + k_{\mathcal{W}_{\Delta\nu_R}}}{8} = \frac{k_{\mathcal{W}_{\Delta\nu_R}}}{4} \quad (43)$$

Plot for Yield as a function of K

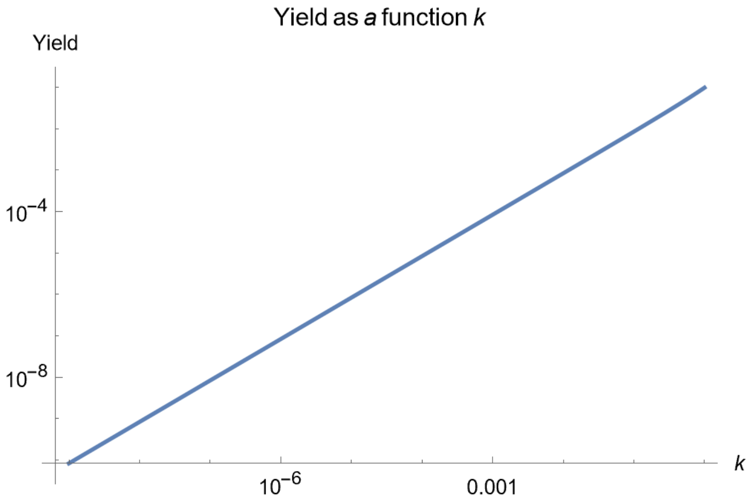

Figure: $k > 10^{-9}$

Plot for Yield as a function of z

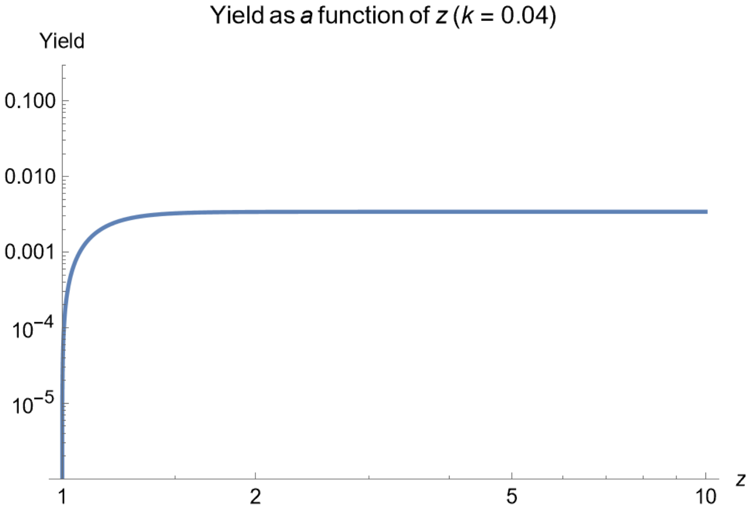

Figure: Production of ν_R in the early universe and then the sudden Freeze-In

Conclusion

- We can say that introducing flavors to the scenario successfully generates the asymmetry.
- CTP Formalism has the advantage of Real Intermediate Subtraction as it naturally incorporates it within its formalism.

Thank you for your Patience

Questions?

CTP Propagators for left-handed fermions are given by:

$$iS^{<}(p) = -2\pi\delta(p^2)P_L \not{p} P_R \left[\vartheta(p_0) f_{_L}(\mathbf{p}) - \vartheta(-p_0)\left(1 - \bar{f}_{_L}(-\mathbf{p})\right)\right], \tag{44}$$

$$iS^{>}(p) = -2\pi\delta(p^2)P_L \not{p} P_R \left[-\vartheta(p_0)\left(1 - f_{_L}(\mathbf{p})\right) + \vartheta(-p_0)\bar{f}_{_L}(-\mathbf{p})\right], \tag{45}$$

$$iS^{T}(p) = P_L \frac{i\not{p}}{p^2 + i\epsilon} P_R - 2\pi\delta(p^2)P_L \not{p} P_R \left[\vartheta(p_0) f_{_L}(\mathbf{p}) + \vartheta(-p_0)\bar{f}_{_L}(-\mathbf{p})\right], \tag{46}$$

$$iS^{\bar{T}}(p) = -P_L \frac{i\not{p}}{p^2 - i\epsilon} P_R - 2\pi\delta(p^2)P_L \not{p} P_R \left[\vartheta(p_0) f_{_L}(\mathbf{p}) + \vartheta(-p_0)\bar{f}_{_L}(-\mathbf{p})\right]. \tag{47}$$

CTP Propagators for right-handed fermions are given by:

$$iS^{<}(p) = -2\pi\delta(p^2)P_R \not{p} P_L \left[\vartheta(p_0)f_{\ell}(\mathbf{p}) - \vartheta(-p_0)\left(1-\bar{f}_{\ell}(-\mathbf{p})\right)\right], \tag{48}$$

$$iS^{>}(p) = -2\pi\delta(p^2)P_R \not{p} P_L \left[-\vartheta(p_0)\left(1-f_{\ell}(\mathbf{p})\right) + \vartheta(-p_0)\bar{f}_{\ell}(-\mathbf{p})\right], \tag{49}$$

$$iS^{T}(p) = P_R \frac{i\not{p}}{p^2+i\epsilon} P_L - 2\pi\delta(p^2)P_R \not{p} P_L \left[\vartheta(p_0)f_{\ell}(\mathbf{p}) + \vartheta(-p_0)\bar{f}_{\ell}(-\mathbf{p})\right], \tag{50}$$

$$iS^{\bar{T}}(p) = -P_R \frac{i\not{p}}{p^2-i\epsilon} P_L - 2\pi\delta(p^2)P_R \not{p} P_L \left[\vartheta(p_0)f_{\ell}(\mathbf{p}) + \vartheta(-p_0)\bar{f}_{\ell}(-\mathbf{p})\right]. \tag{51}$$

CTP propagators for massive scalars are given by:

$$i\Delta_\phi^<(p) = 2\pi\delta(p^2 - M^2)\left[\vartheta(p_0)f_\phi(\mathbf{p}) + \vartheta(-p_0)\left(1 + \bar{f}_\phi(-\mathbf{p})\right)\right], \tag{52}$$

$$i\Delta_\phi^>(p) = 2\pi\delta(p^2 - M^2)\left[\vartheta(p_0)\left(1 + f_\phi(\mathbf{p})\right) + \vartheta(-p_0)\bar{f}_\phi(-\mathbf{p})\right], \tag{53}$$

$$i\Delta_\phi^T(p) = \frac{i}{p^2 - M^2 + i\epsilon} + 2\pi\delta(p^2 - M^2)\left[\vartheta(p_0)f_\phi(\mathbf{p}) + \vartheta(-p_0)\bar{f}_\phi(-\mathbf{p})\right], \tag{54}$$

$$i\Delta_\phi^{\bar{T}}(p) = \frac{-i}{p^2 - M^2 - i\epsilon} + 2\pi\delta(p^2 - M^2)\left[\vartheta(p_0)f_\phi(\mathbf{p}) + \vartheta(-p_0)\bar{f}_\phi(-\mathbf{p})\right]. \tag{55}$$