Electroweak phase transition seeded by DFSZ axion string

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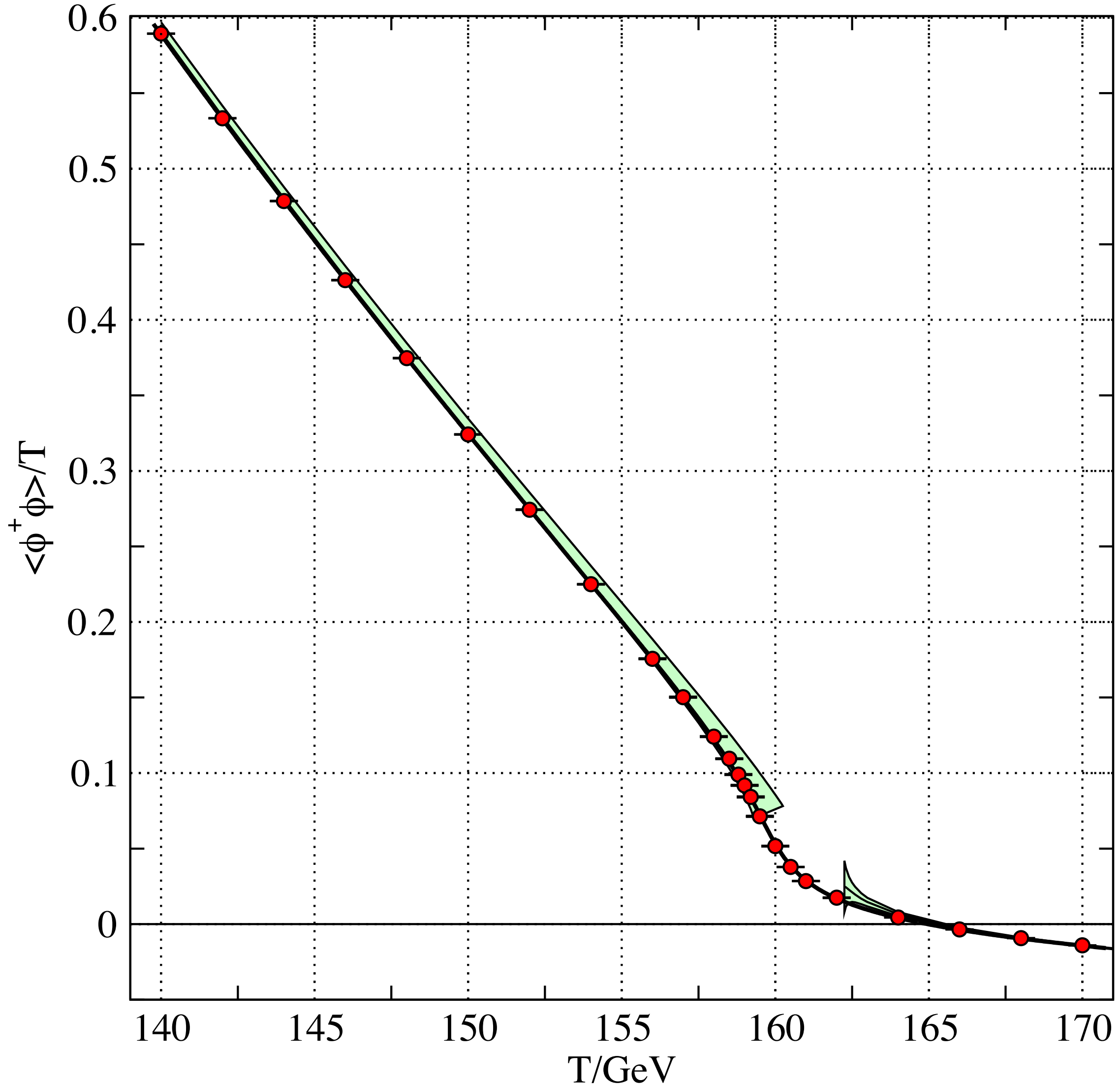
Work in progress w/ Simone Blasi (DESY)

DESY Theory Workshop, 24th Sep 2025

Introduction

Electroweak Phase Transition

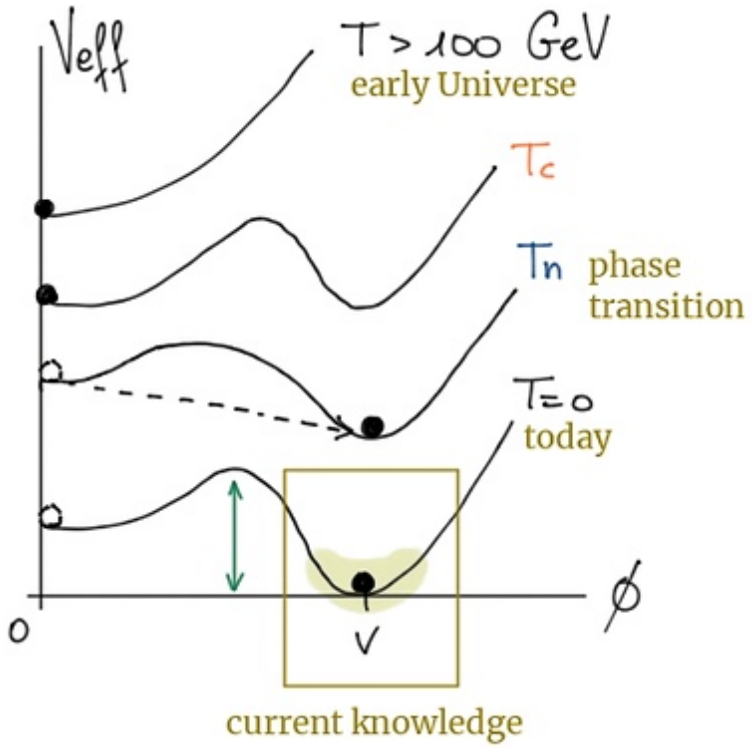
- SM predicts EW crossover, but we don't really know what happened at $T \sim v_{\mathrm{EW}}$

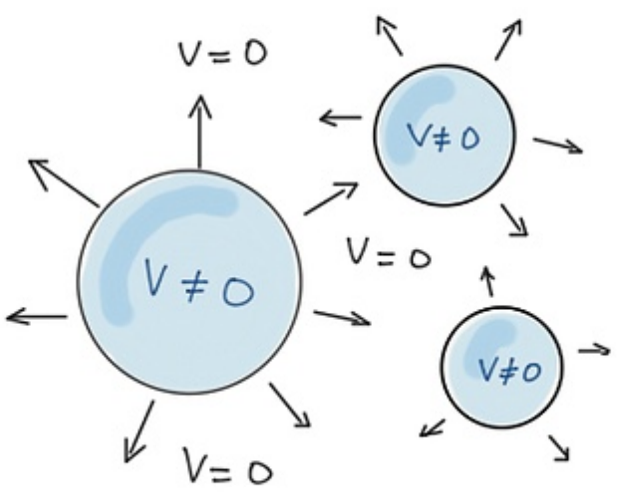


[D'Onofrio-Rummukainen, 1508.07161]

1st-order EWPT in BSM

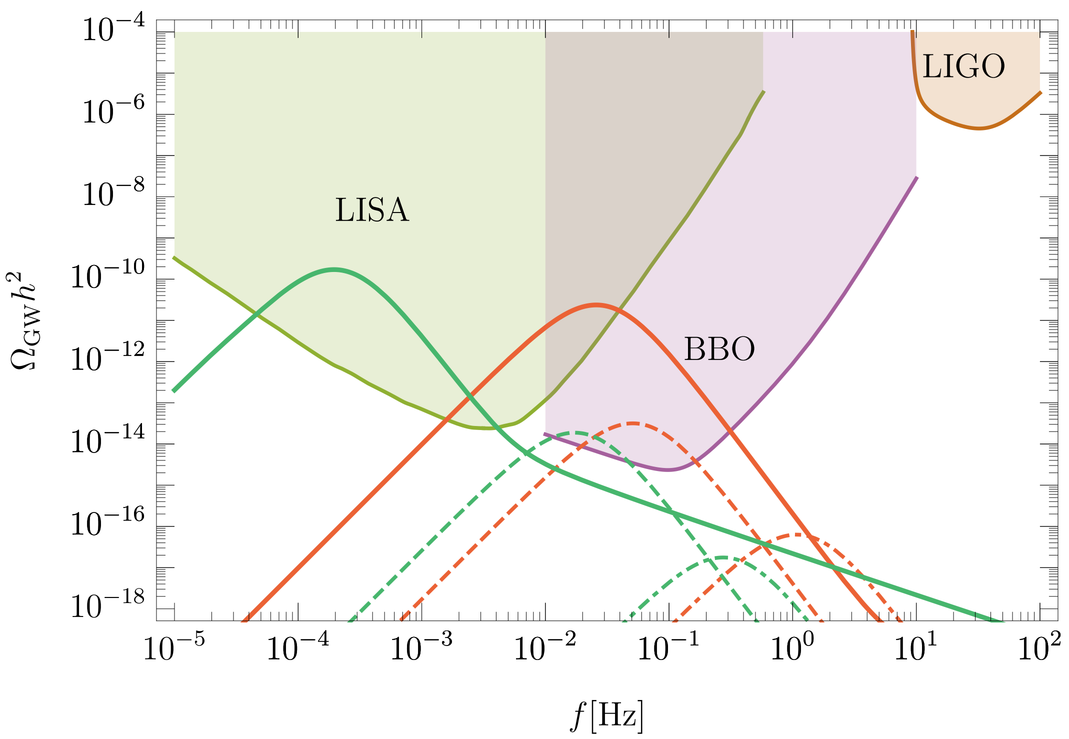
- A lot of motivation: EW Baryogenesis, Gravitational wave, etc





(Fig credit: Kateryna Radchenko)

[Beniwal+, 1702.06124]



- **Bubble nucleation seeded by impurities** **Cf) Simone's talk**

ubiquitous in daily life, but still to be explored in cosmology

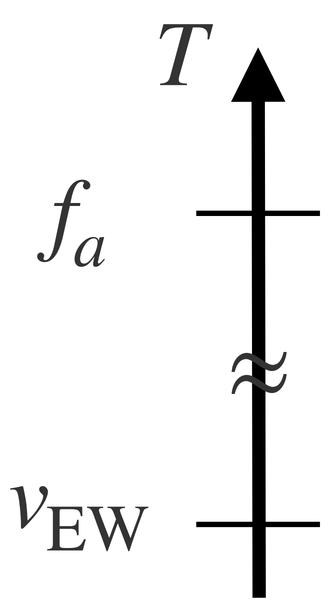
DFSZ axion model

[Zhitnitsky, '80] [Dine-Fischler-Srednicki, '83]

- 2HDM $H_{1,2}$ + Singlet PQ scalar S

$$\mathscr{L} = \mathscr{L}_{\text{kin.}} - V_S(|S|) - V_{\text{EW}}(H_1, H_2) - \left(\underline{\kappa S^2 H_1^\dagger H_2} + \text{h.c.}\right)$$

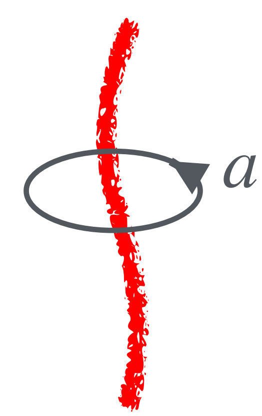
$H_{1,2}$ charged under $U(1)_{PQ}$



$\langle H_{1,2} \rangle = 0 \qquad \langle S \rangle = f_a/\sqrt{2}$ → $\mathscr{L} \supset \# \frac{a}{f_a} G\tilde{G}$

$\langle H_{1,2} \rangle \simeq v_{\text{EW}} \qquad \langle S \rangle = f_a/\sqrt{2}$

axion string



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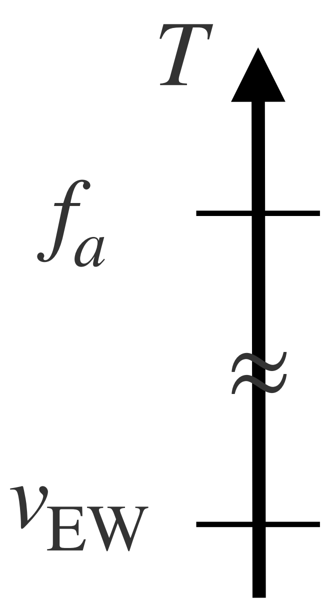

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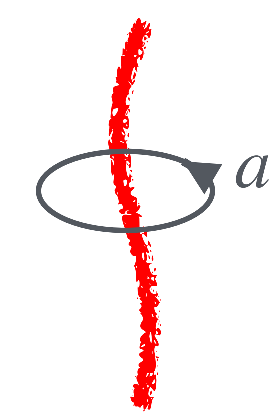


- **Nice setup to consider seeded EWPT**
  - **SSB of $U(1)_{PQ}$ → impurity (axion string)**
  - **Two doublets $H_{1,2}$ → can have 1st-order EWPT**

Toy model inspired by DFSZ

DFSZ-like toy model

- Two complex scalar fields: φ (SM Higgs like), Φ (PQ-like)

$$\mathscr{L} = |\partial_\mu \varphi|^2 + |\partial_\mu \Phi|^2 - V_{PQ}(|\Phi|) - V(|\varphi|) + \left(\kappa \varphi^2 \Phi^2 + \mathrm{h.c.}\right)$$

$|\varphi|^2 |\Phi|^2$ in KSVZ

- κ-term is DFSZ-ish operator

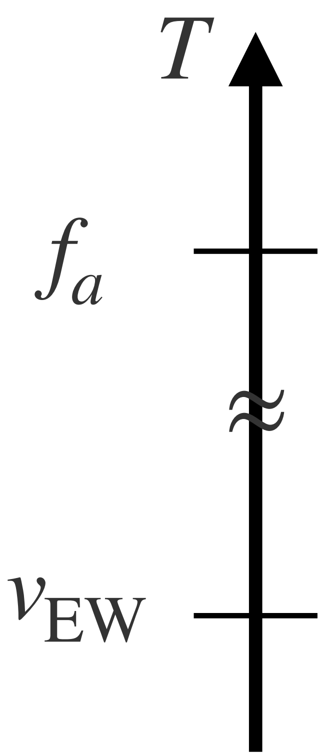
- The model has global $U(1)_{PQ} \times \mathbb{Z}_2$ sym.

$$\begin{cases} \Phi \to e^{i\xi}\Phi \\ \varphi \to e^{-i\xi}\varphi \end{cases} \qquad \begin{cases} \Phi \to i\Phi \\ \varphi \to i\varphi \end{cases}$$

φ is involved for $U(1)_{PQ}$ (feature of DFSZ)

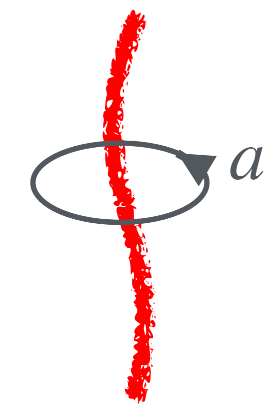
DFSZ-like toy model

- Heavier scalar Φ takes VEV earlier → breaking of $U(1)_{PQ}$ → axion!



``axion string''

$\langle \varphi \rangle = 0, \quad \langle \Phi \rangle = f_a/\sqrt{2} \quad \rightarrow \Phi \sim \dfrac{f_a}{\sqrt{2}} e^{i\theta}$



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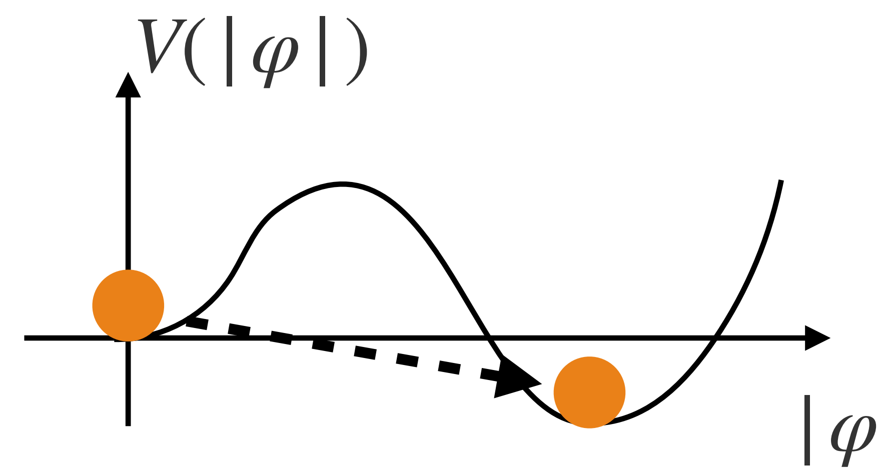

How is this nucleation influenced by axion string?

Critical bubble in toy model

- The critical bubble in the background of axion string

$$\Phi \sim \frac{f_a}{\sqrt{2}} e^{i\theta} \quad \leftarrow \text{axion string}$$

$$\varphi = \frac{1}{\sqrt{2}} h(x)\, e^{i\phi(x)} \quad \leftarrow \text{polar decomp. of Higgs}$$

$$\mathscr{L} = \frac{1}{2}(\partial_\mu h)^2 + \frac{h^2}{2}(\partial_\mu \phi)^2 - V(h) + 2\kappa f_a^2 h^2 \cos 2(\theta + \phi)$$

position-dependent cos potential

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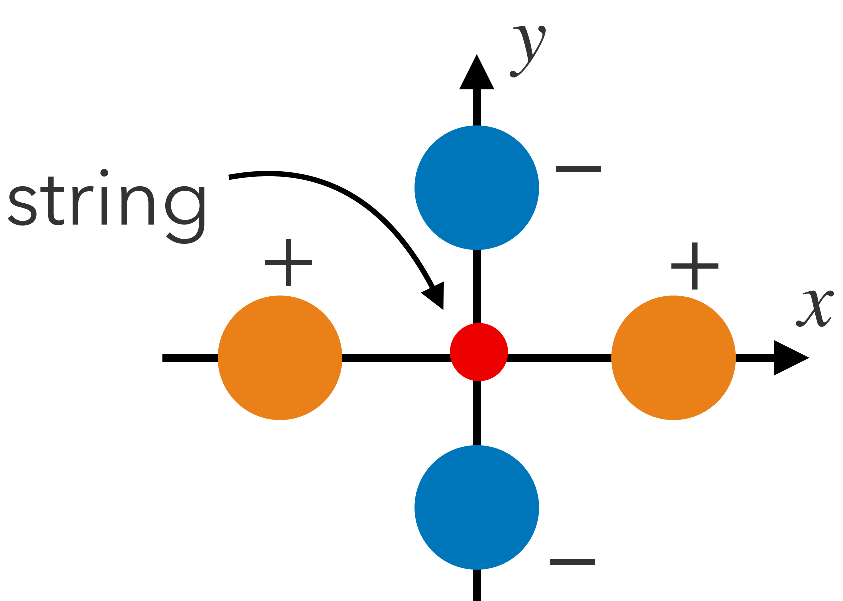
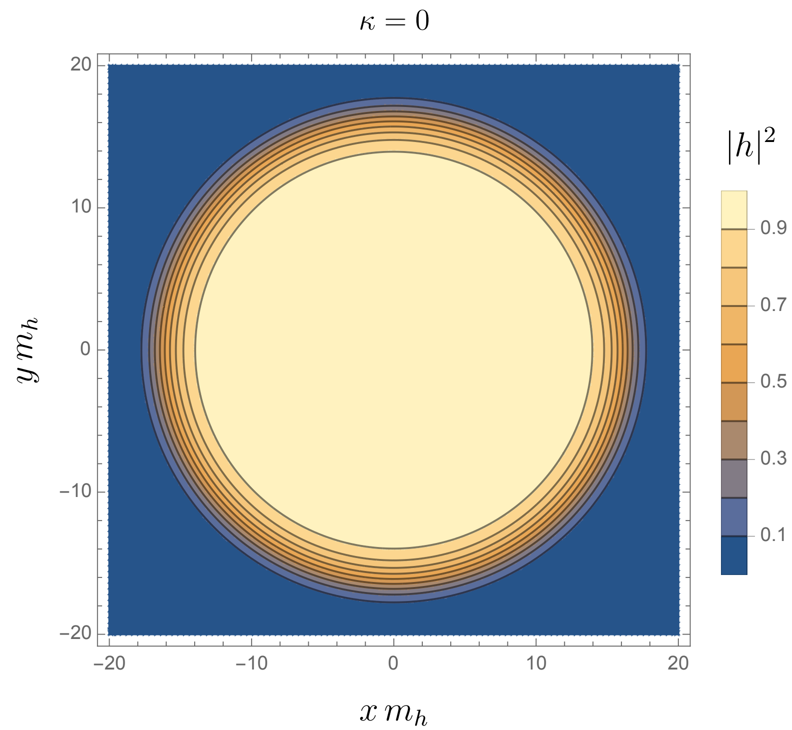


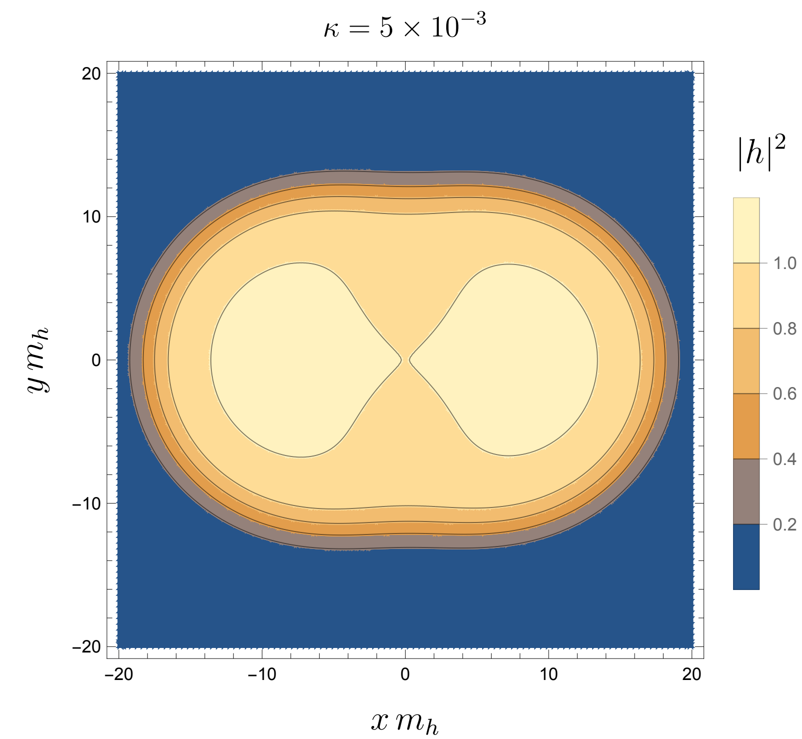
position-dependent cos potential

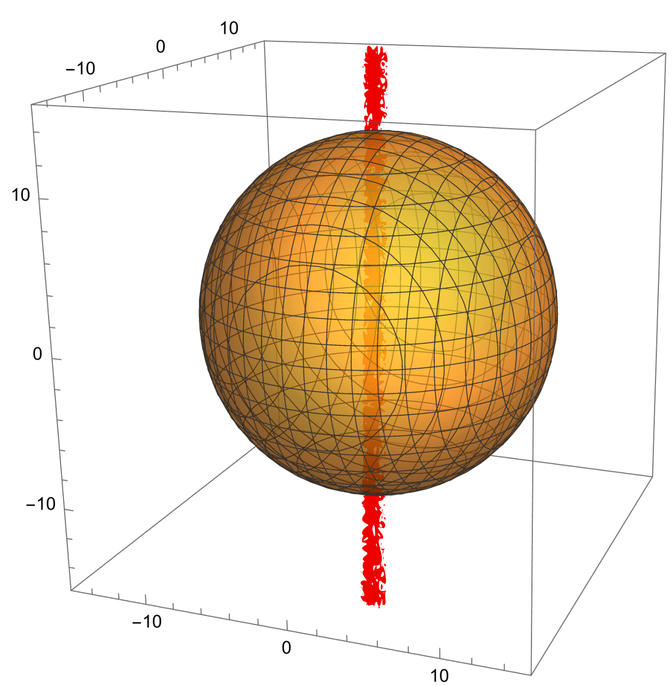
$$\cos 2\phi \;@\; \theta = 0, \pi$$

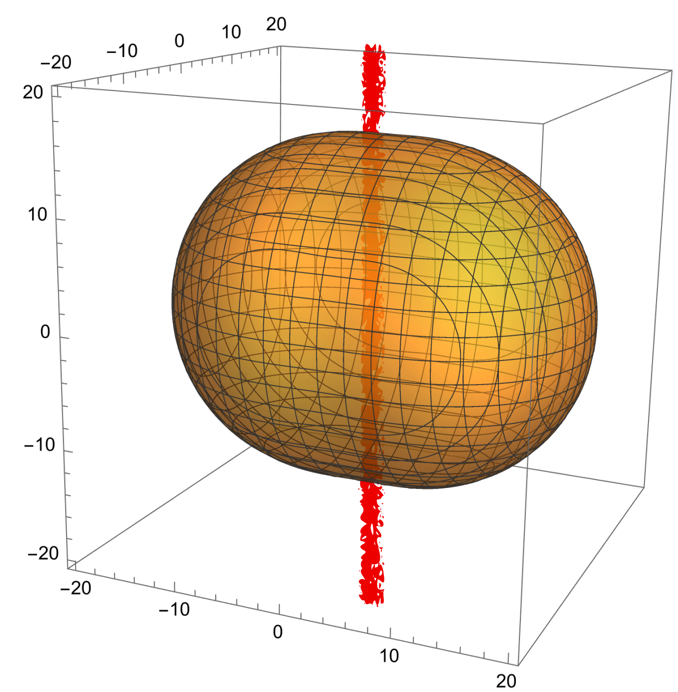
$$-\cos 2\phi \;@\; \theta = \pi/2, 3\pi/2$$

Numerical results for critical bubble



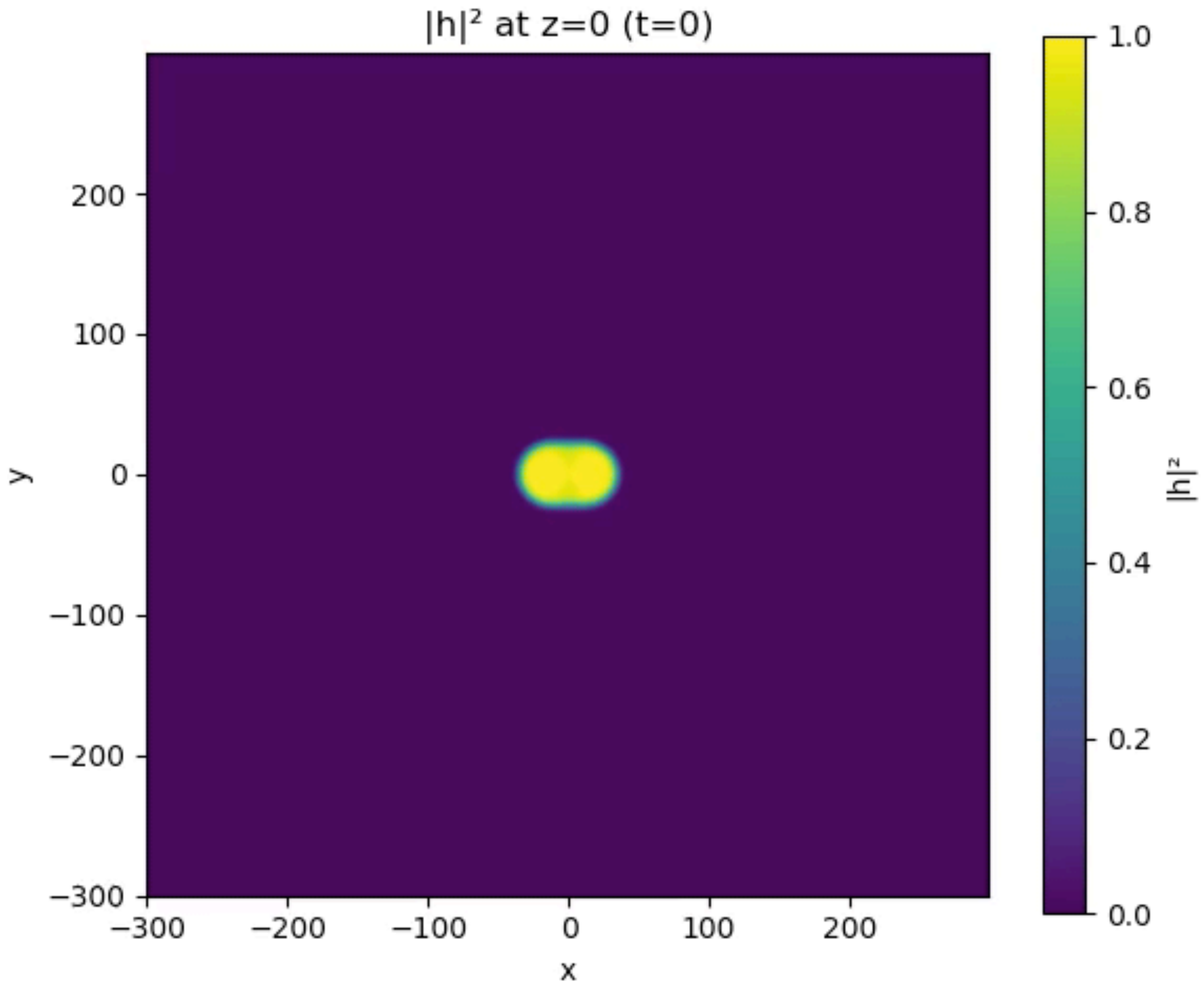


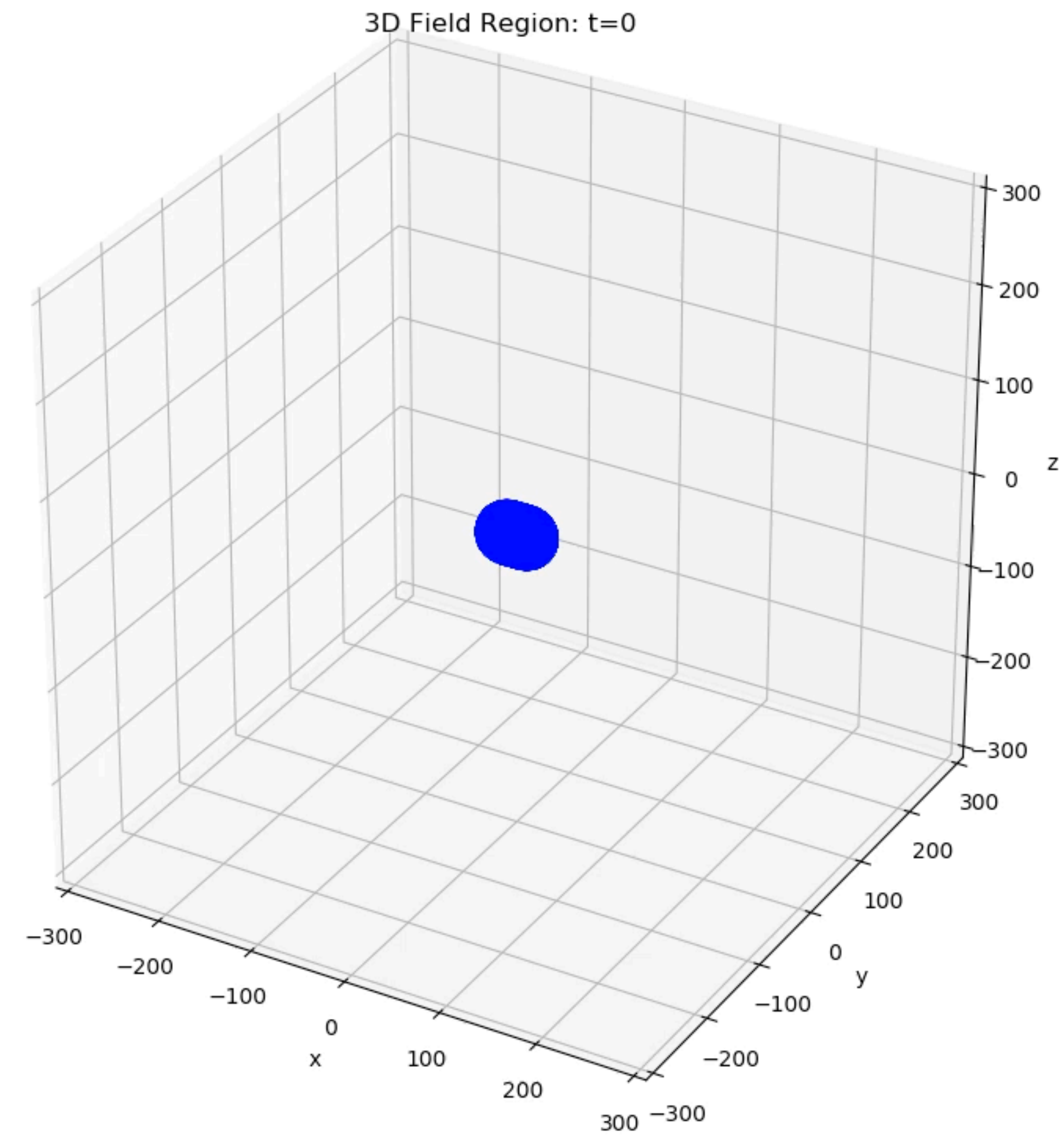




no O(3) or O(2) sym.

Expansion after nucleation





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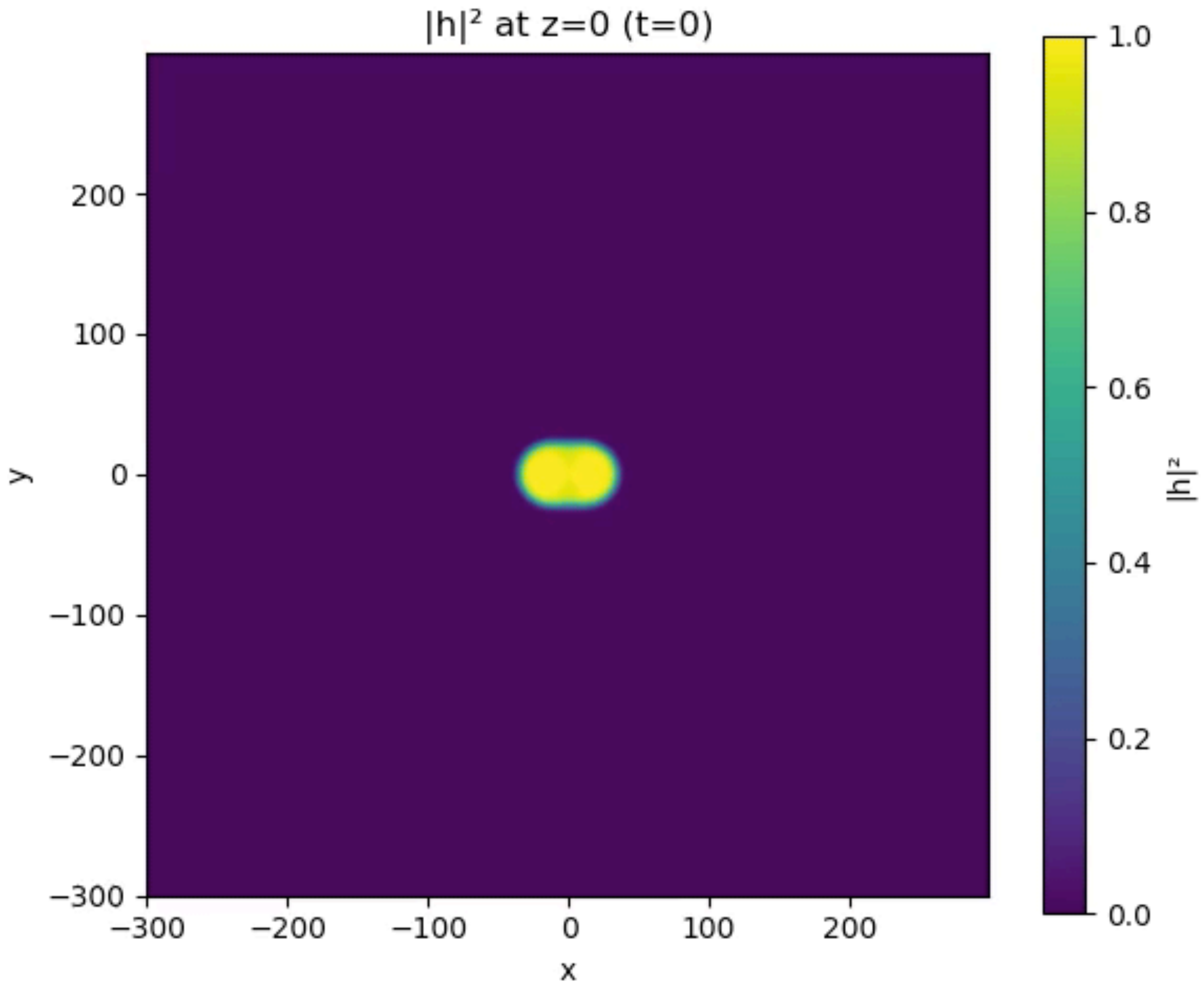

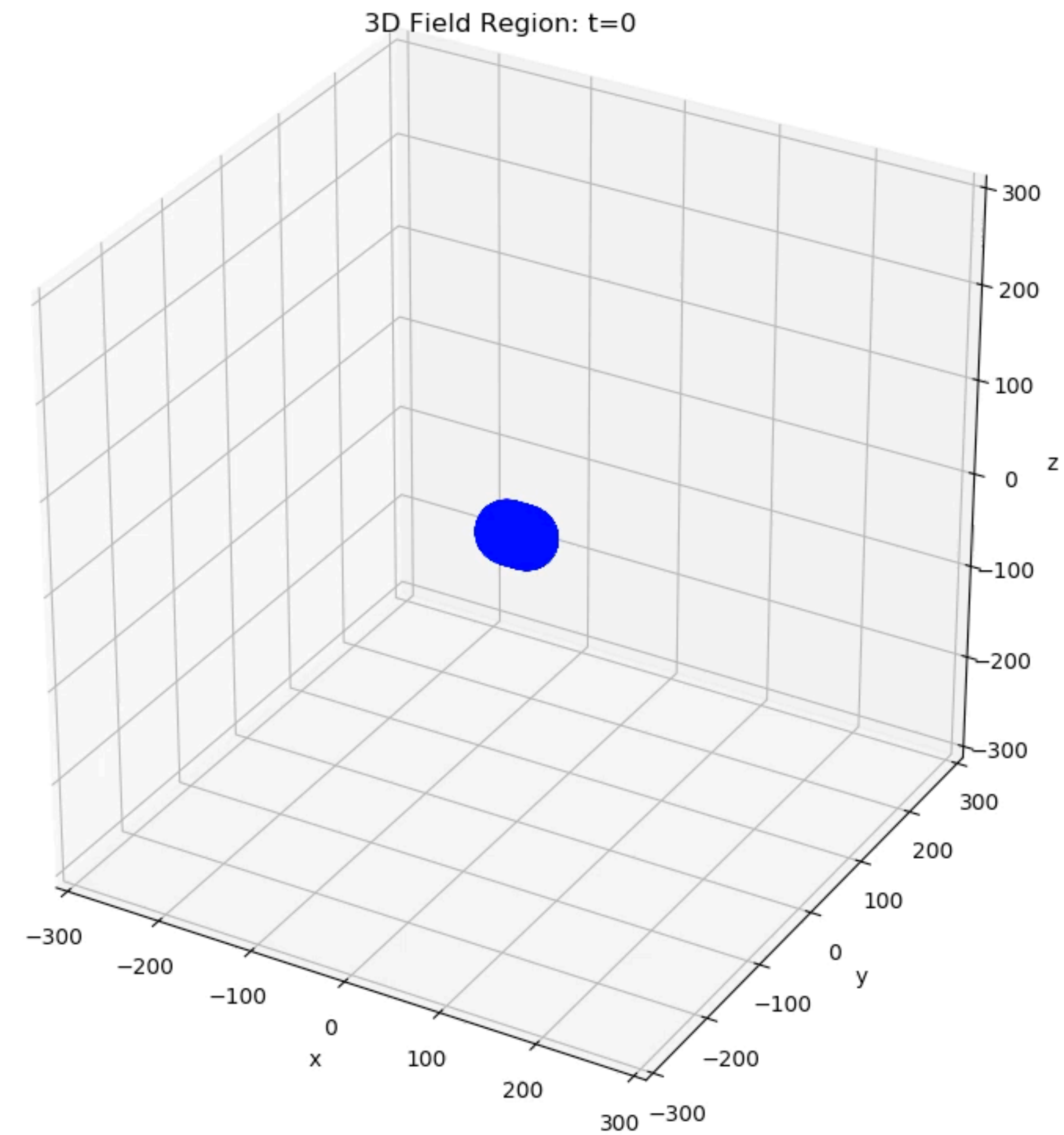

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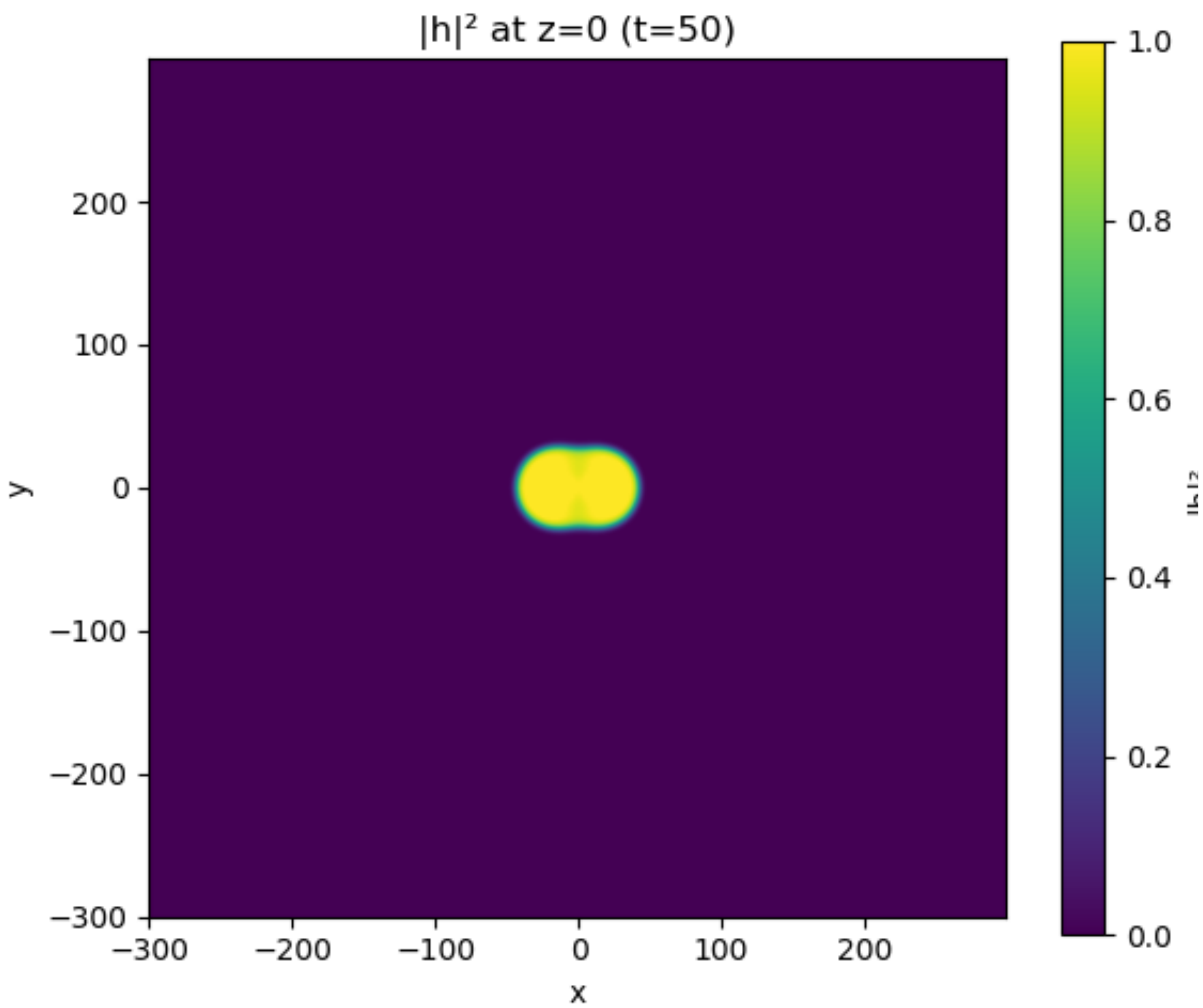

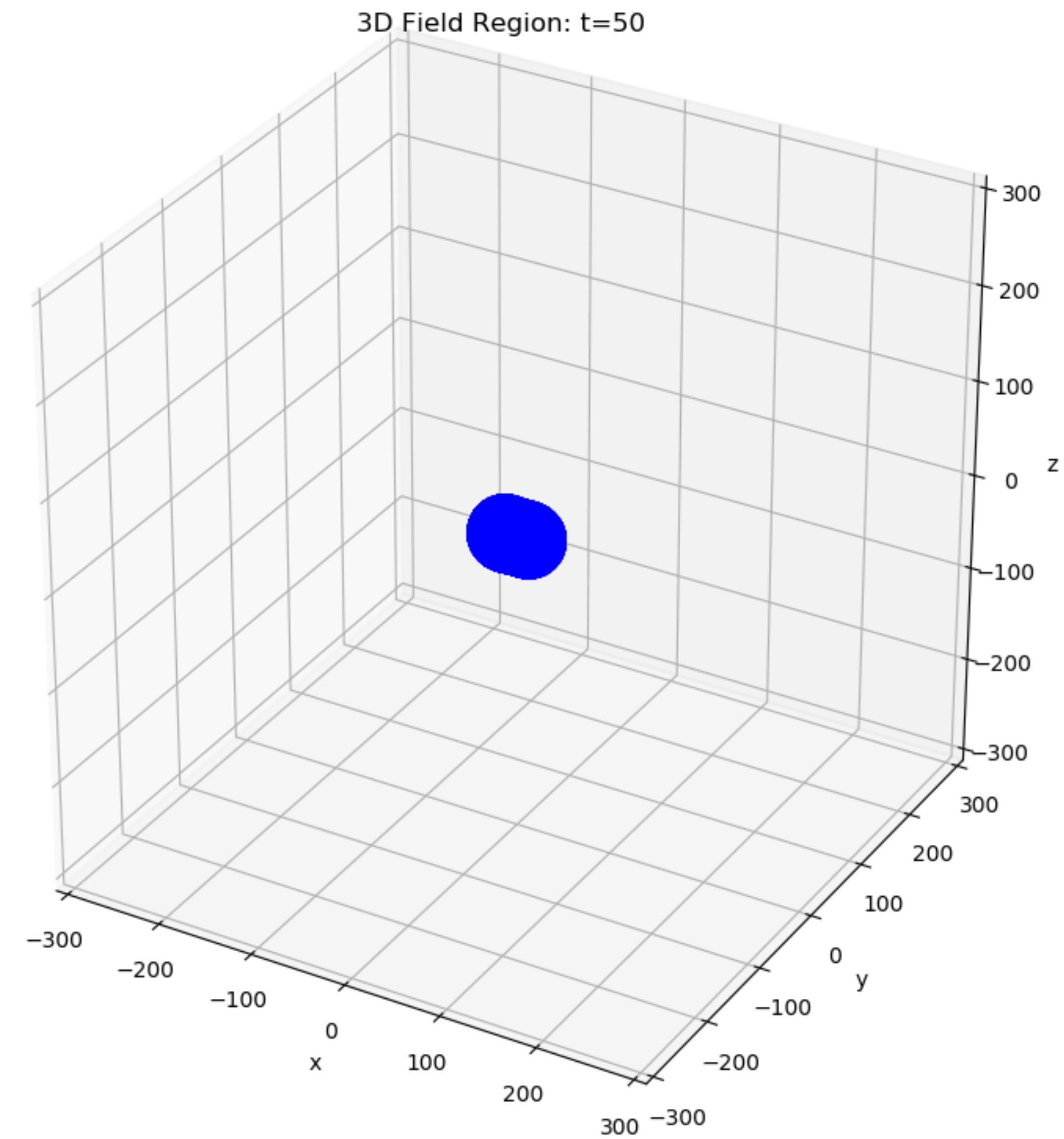

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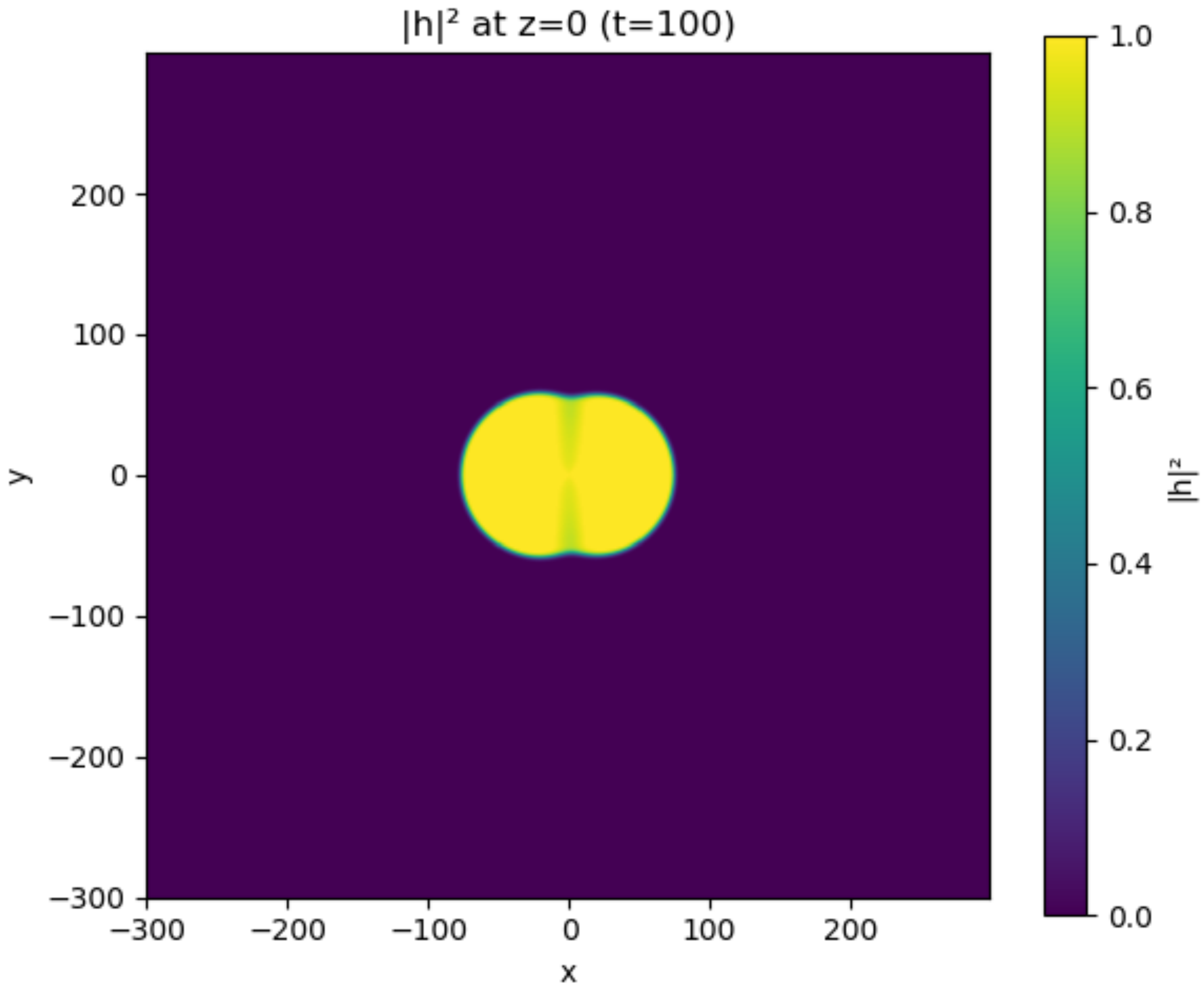

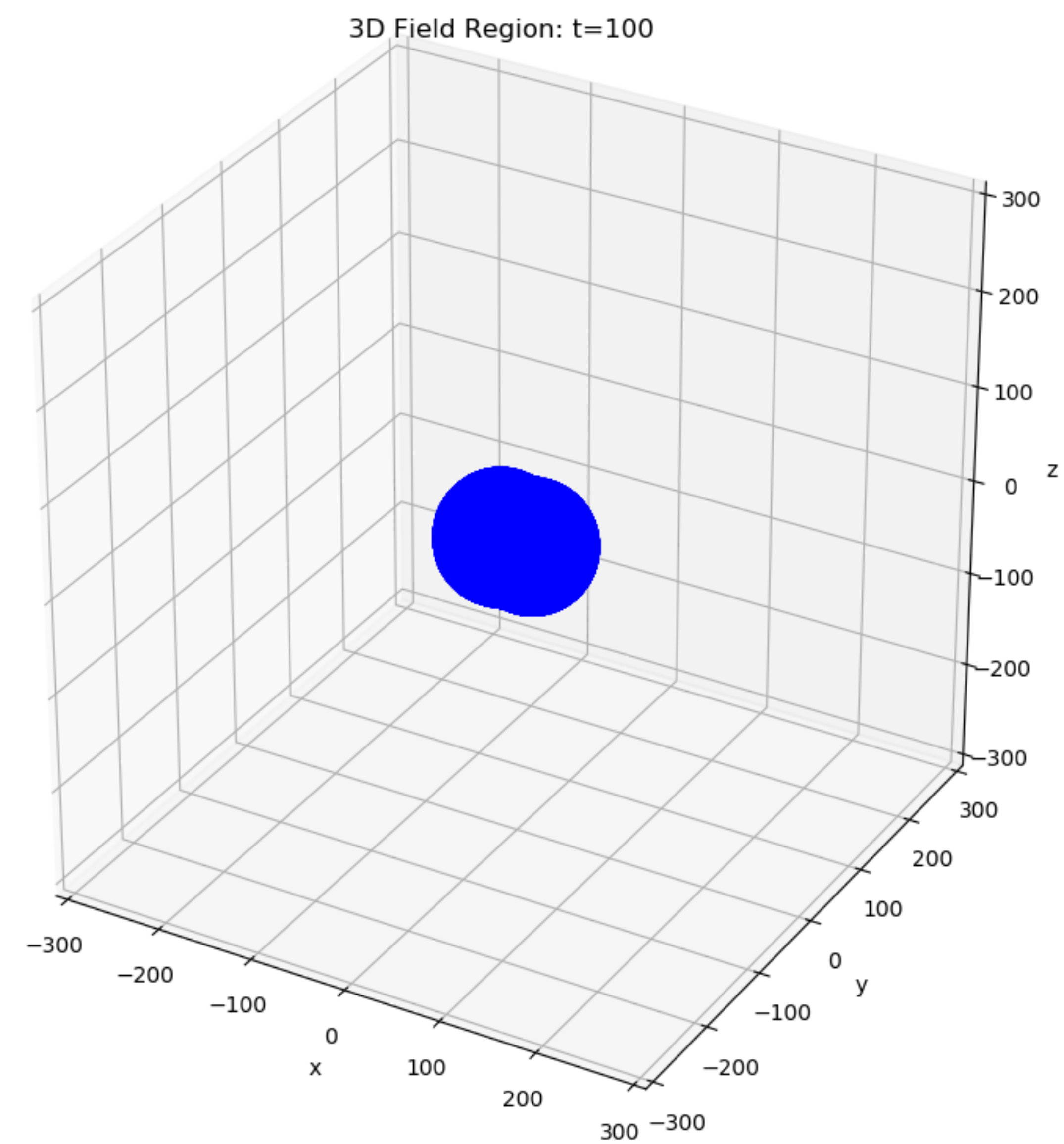

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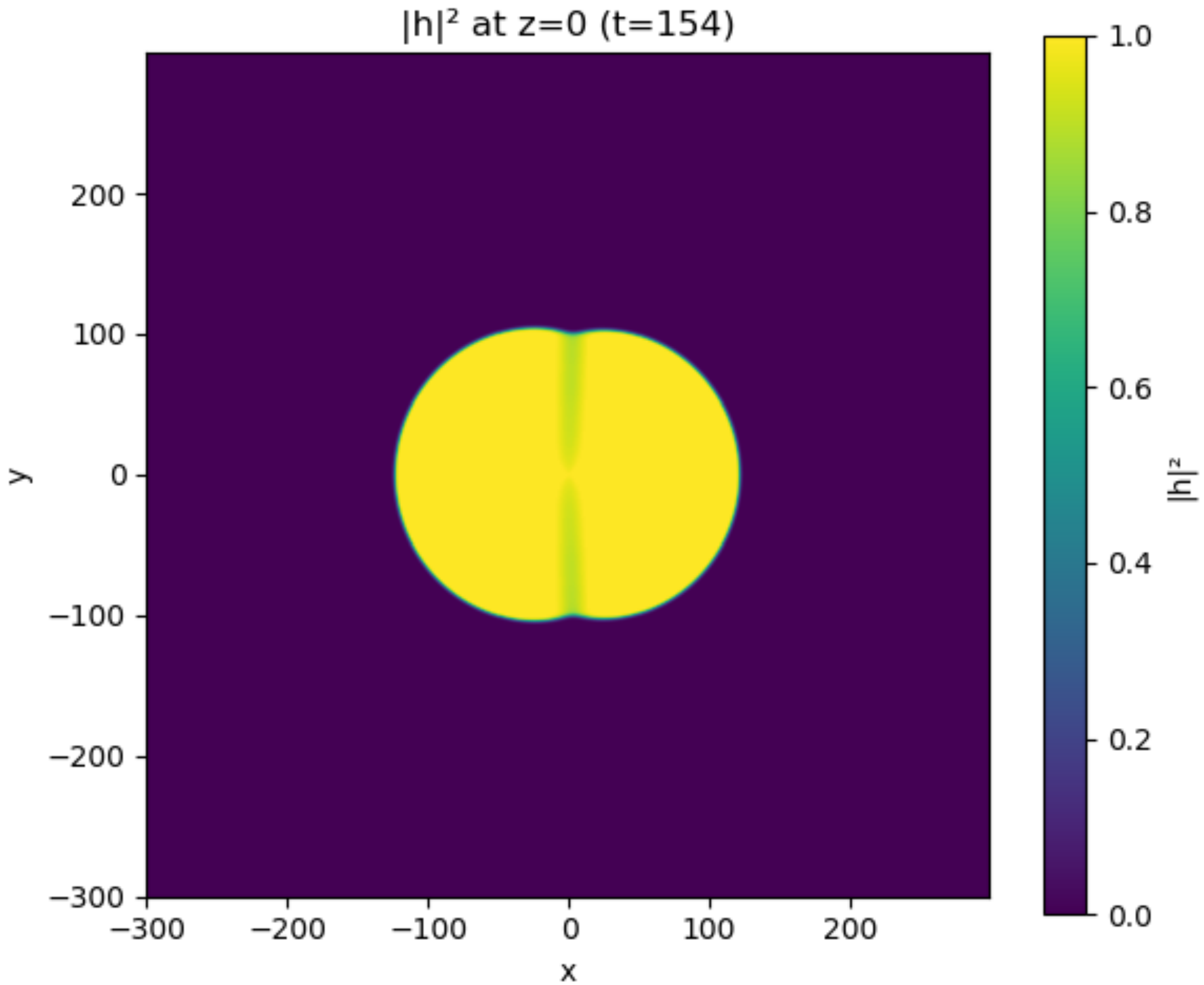

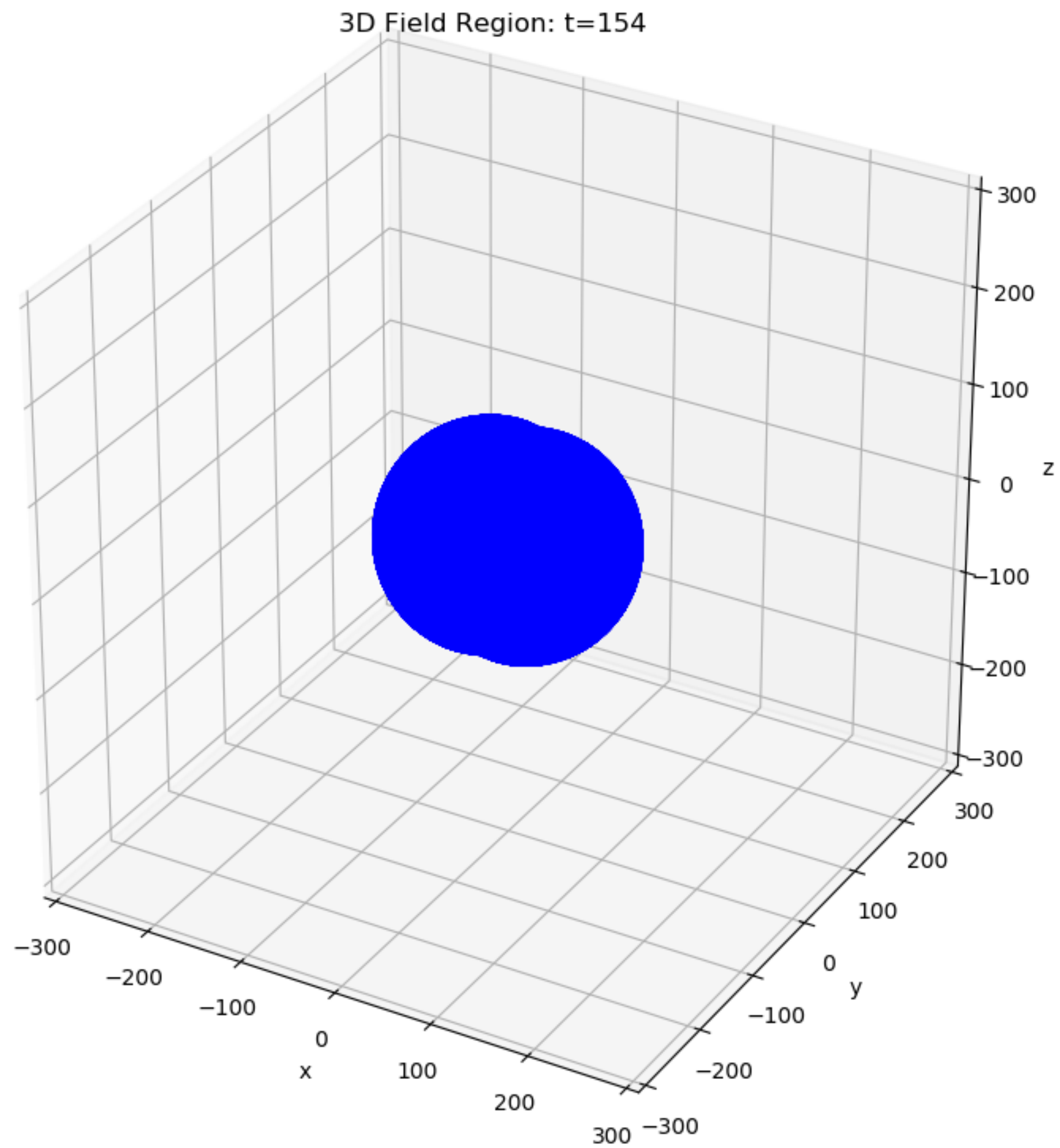

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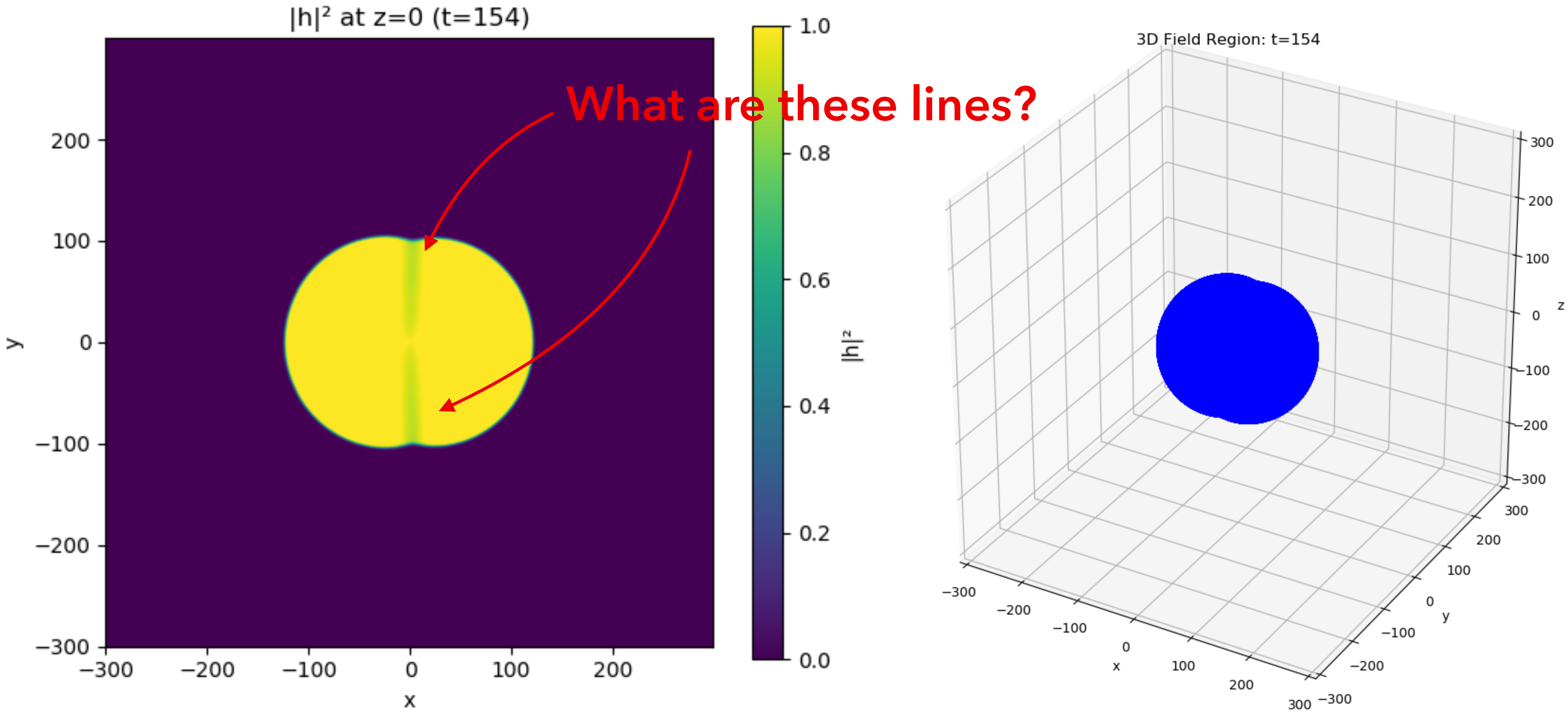
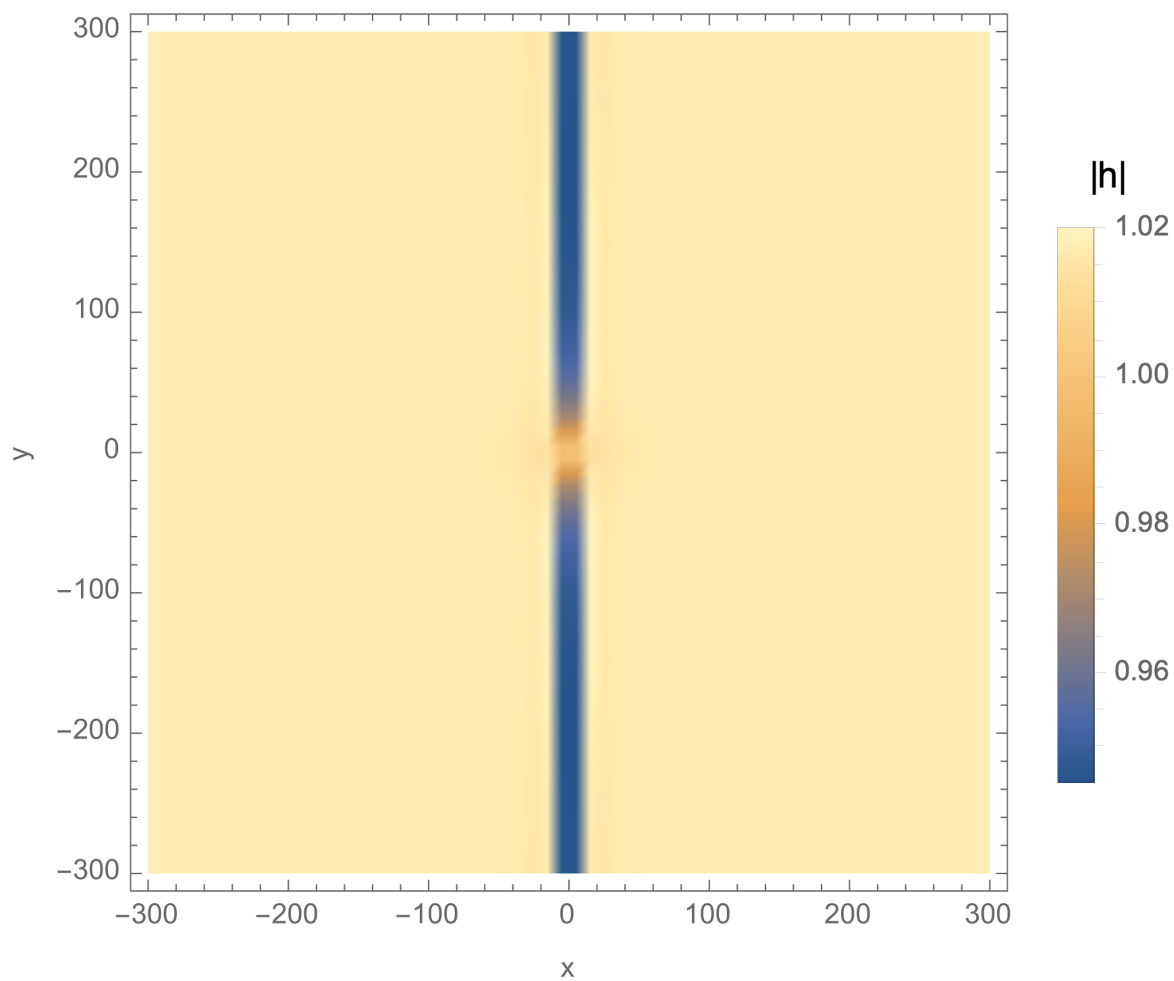


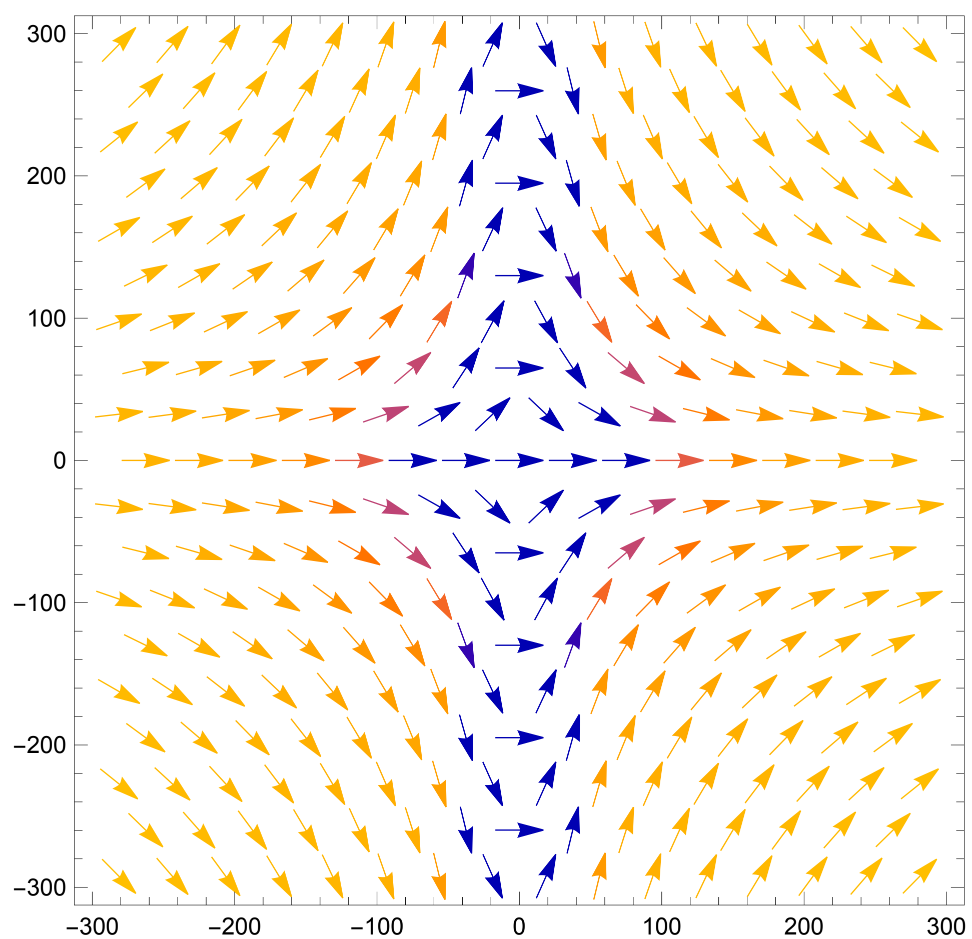
Domain wall

- DW exists in this model! → **New mechanism of DW production**

$$\mathscr{L} = |\partial_\mu \varphi|^2 + |\partial_\mu \Phi|^2 - V(|\varphi|) - V_{PQ}(|\Phi|) + \left(\kappa \varphi^2 \Phi^2 + \mathrm{h.c.}\right) \qquad U(1)_{PQ} \times \mathbb{Z}_2 \text{ sym.}$$



$(\mathrm{Re}\,\varphi, \mathrm{Im}\,\varphi)$



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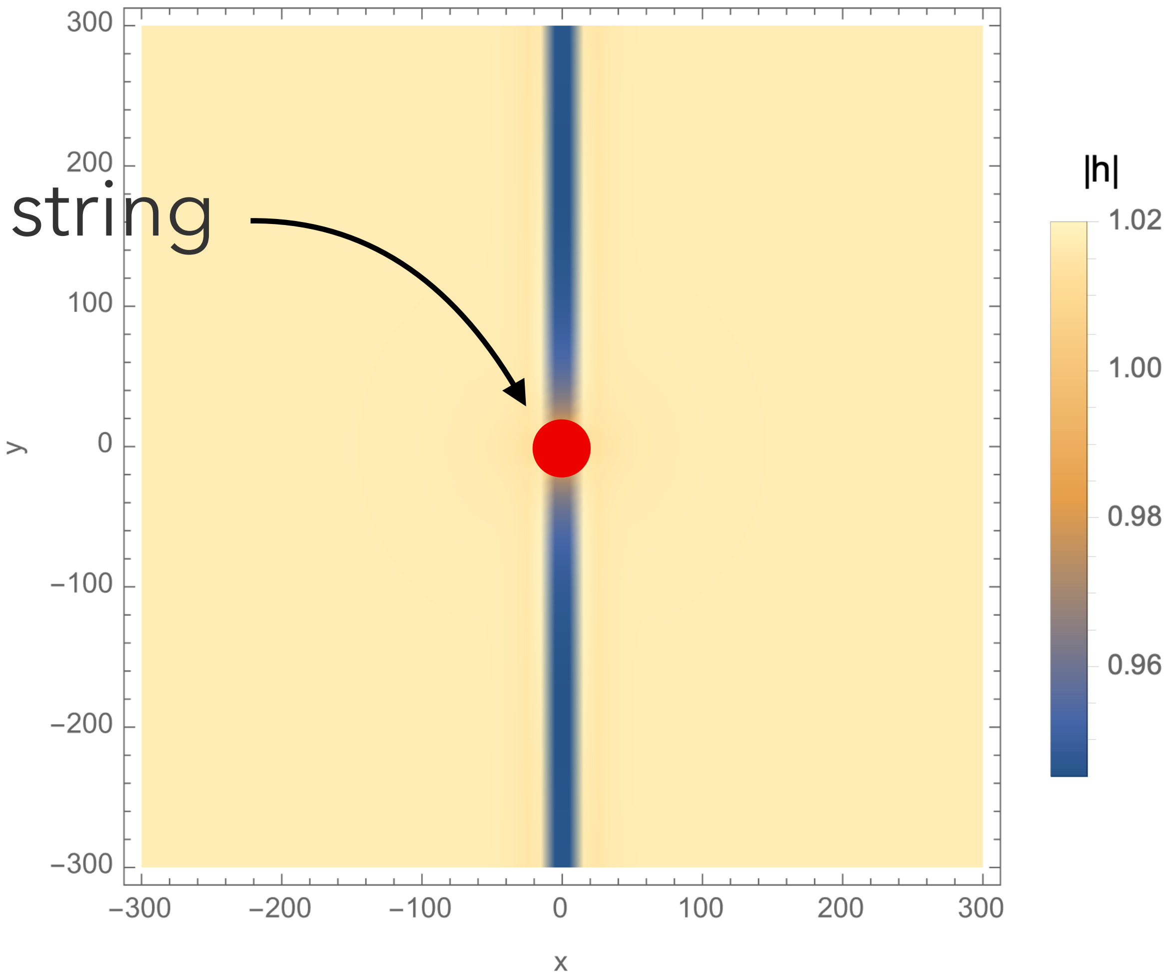

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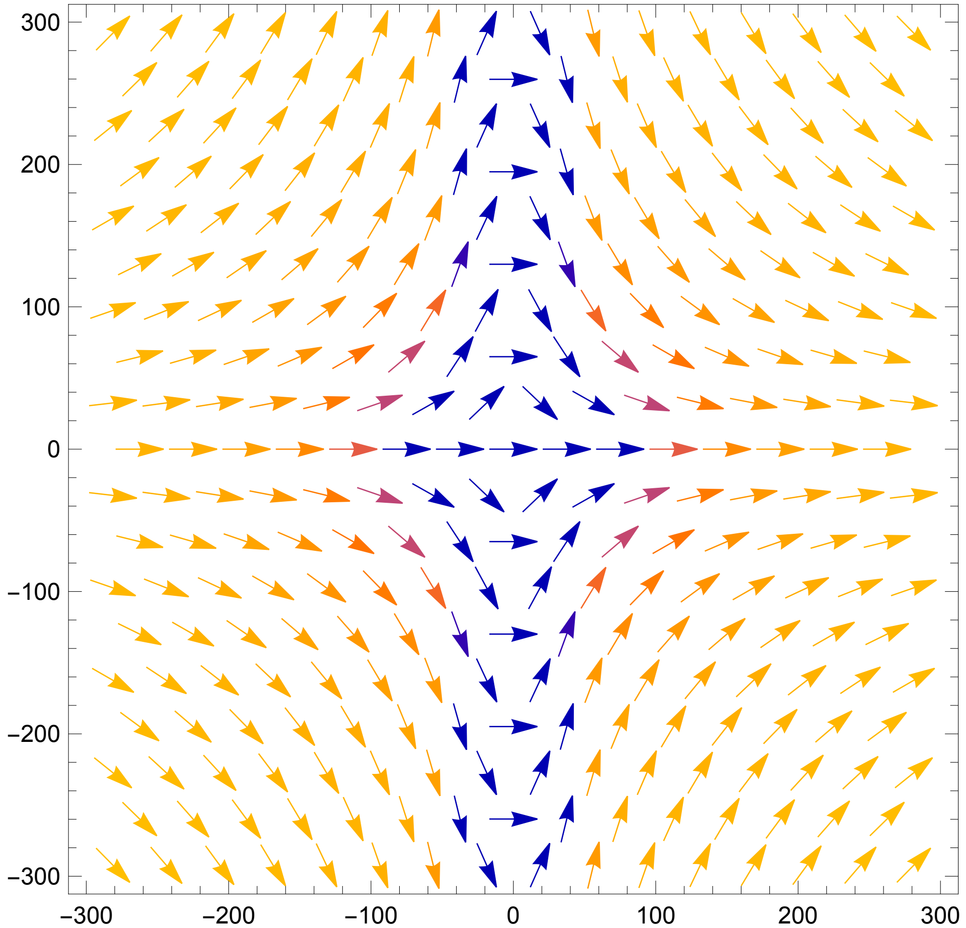
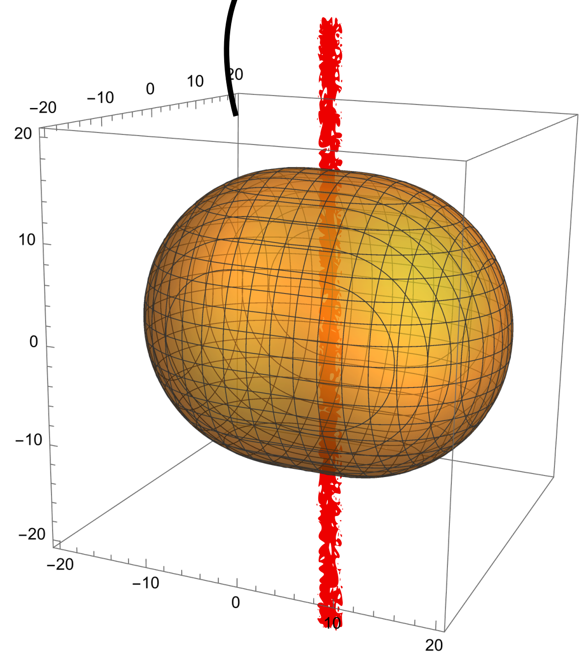


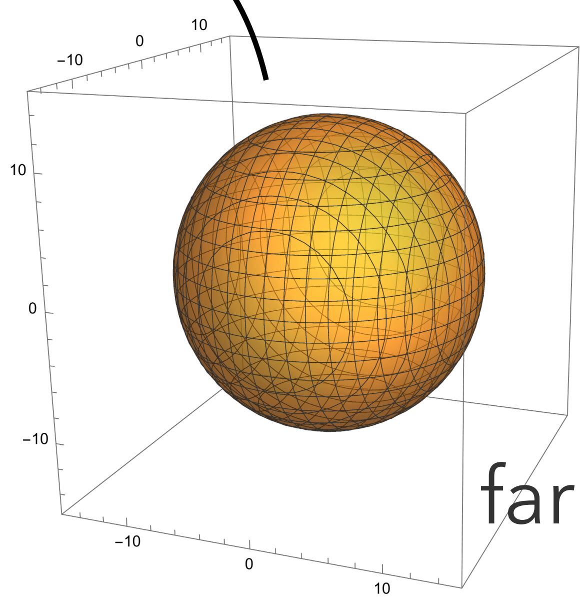
Tunneling Probability

- This κ-term makes phenomenology much richer, but **doesn't enhance the seeded tunneling itself.** 😢

$$\because\ \Delta E_c \equiv E_c|_{\mathrm{Seeded}} - E_c|_{\mathrm{Hom}} \sim \kappa f_a^2 v_{\mathrm{EW}}^2 R^3$$

on the string





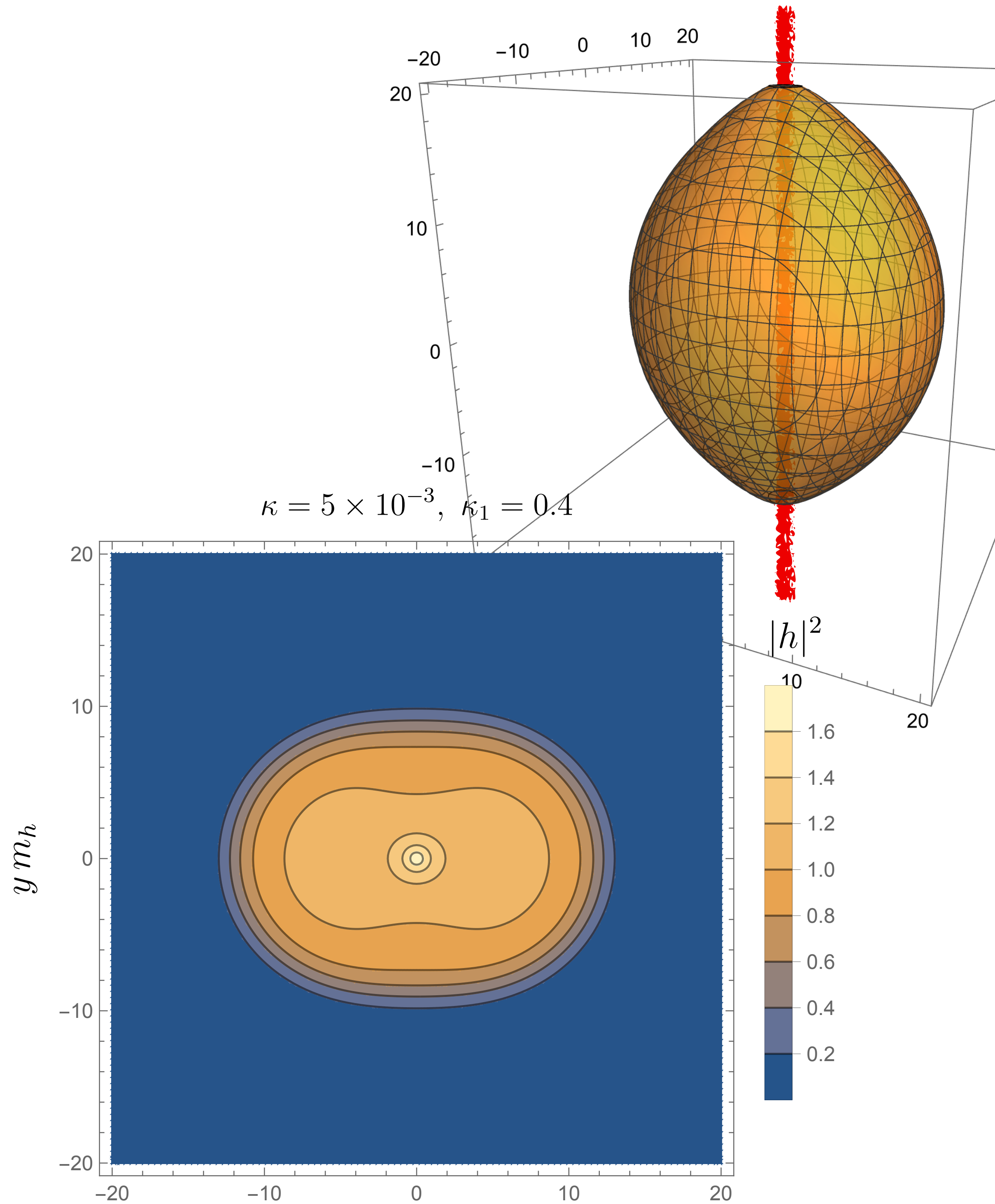
far from the string

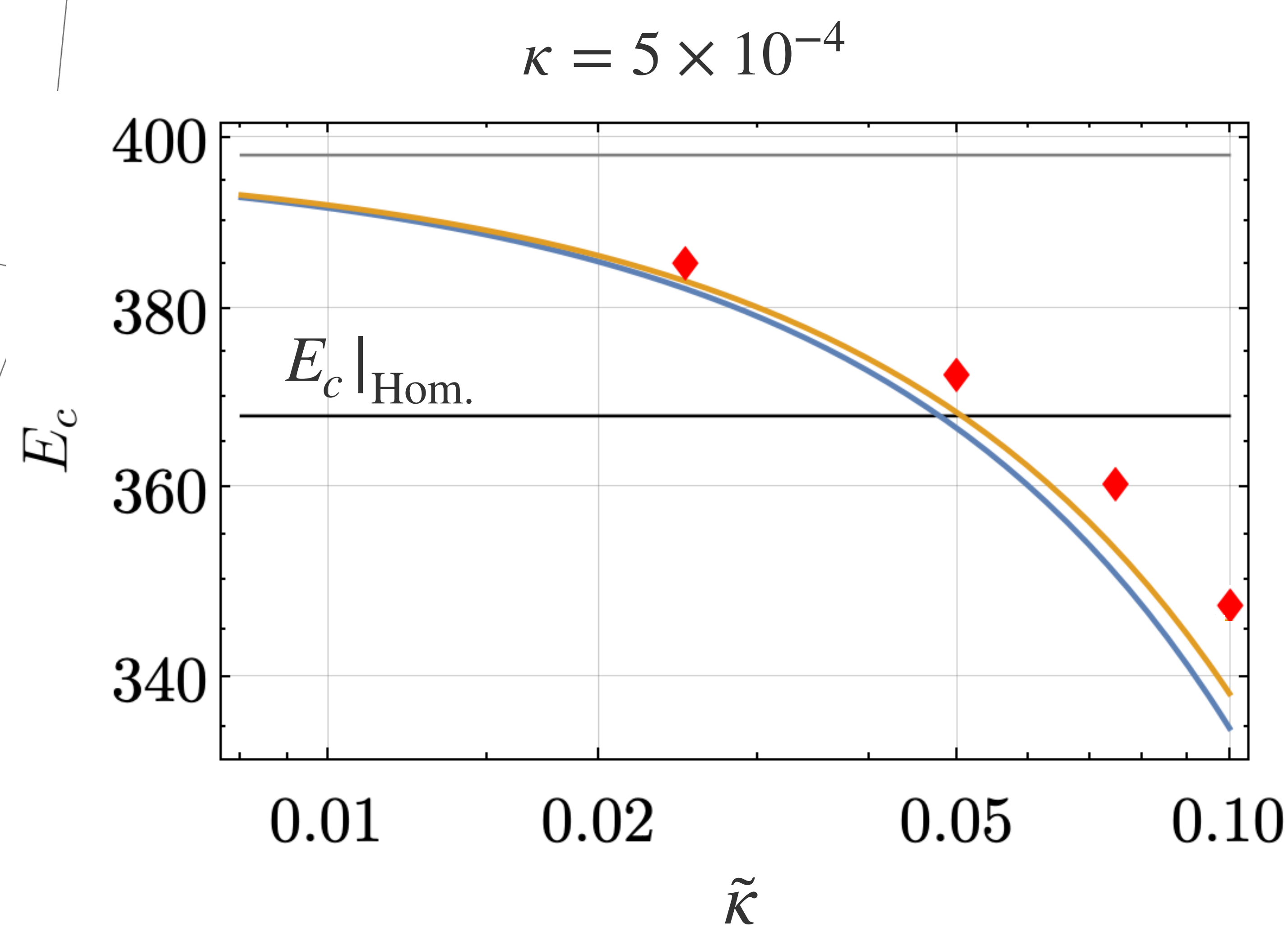
→ **add** $\tilde{\kappa}|\varphi|^2|\Phi|^2$ **like KSVZ setup [cf. Simone's talk]**

[Blasi-Mariotti, 2405.08060]

Critical bubble with KSVZ-like op.

- Comparison between seeded tunneling and normal one





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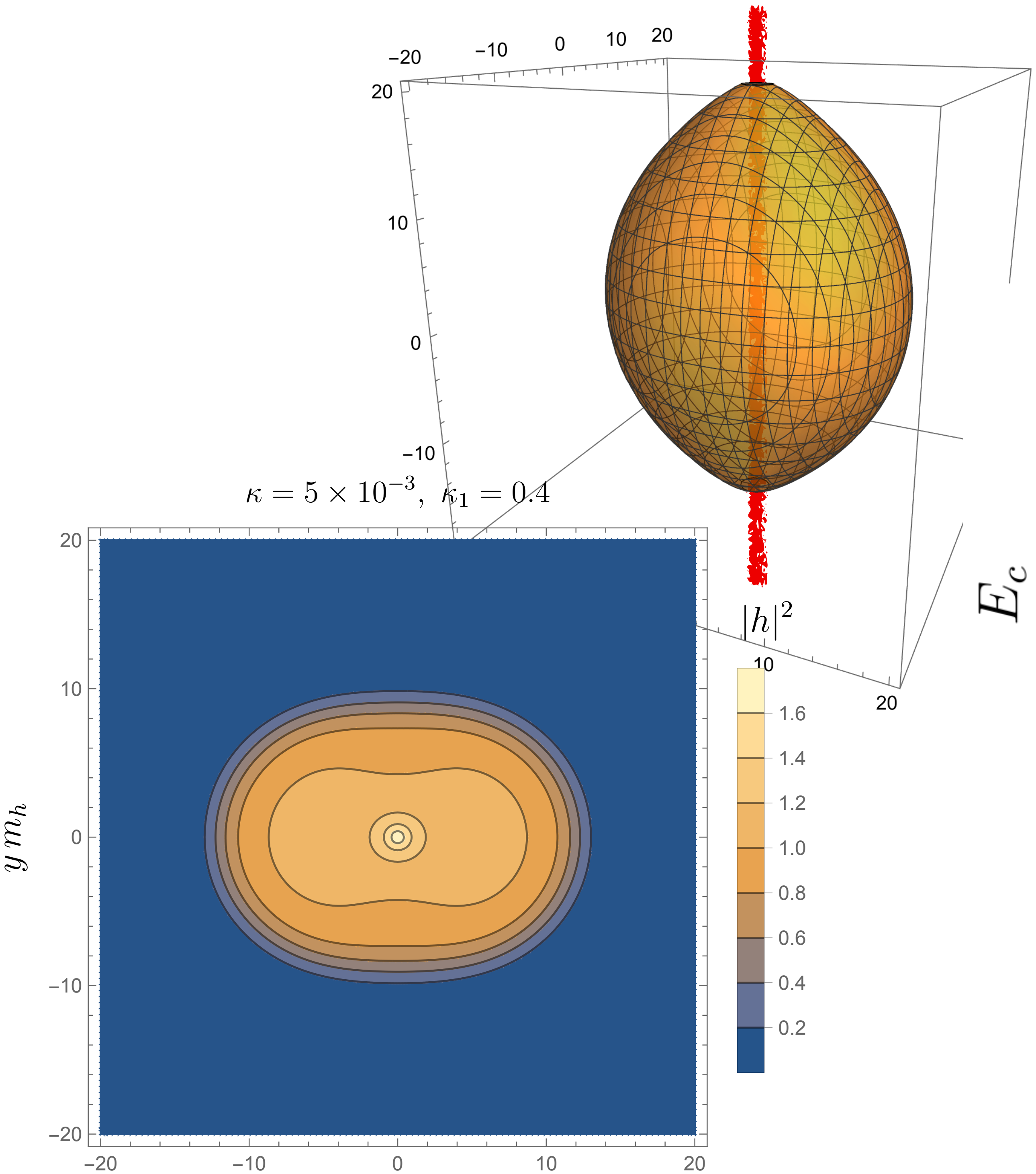
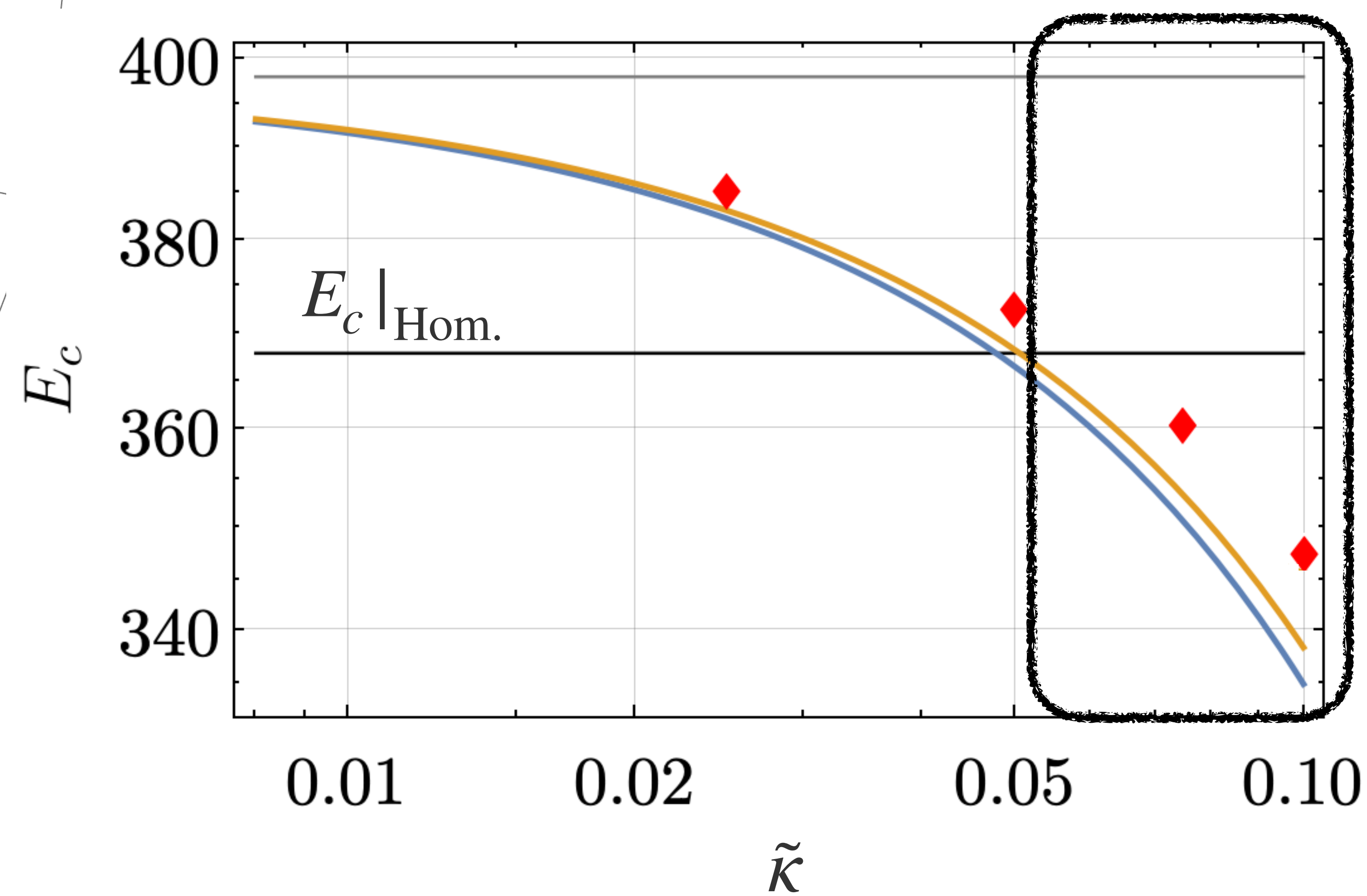
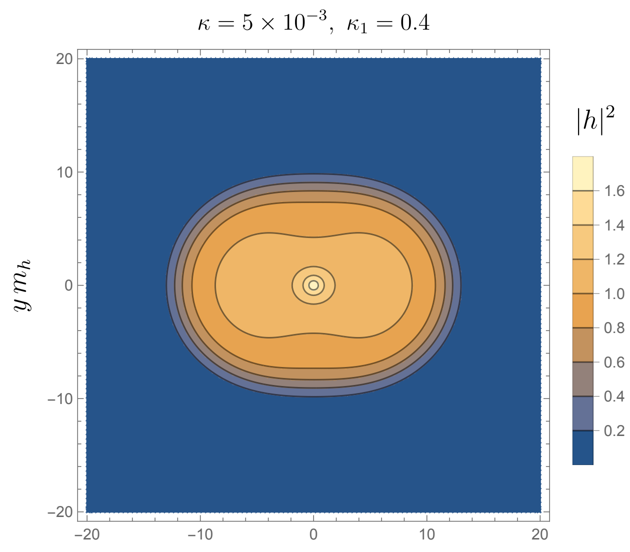


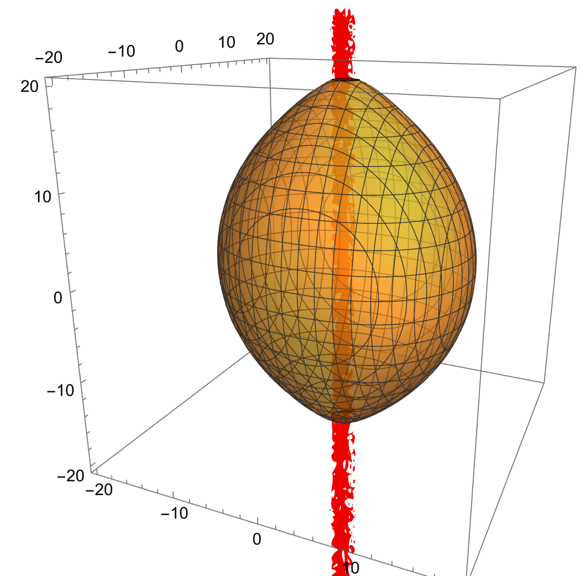
funny bubble is favored



Summary

- PT seeded by impurity is very interesting.
- A lot of future directions:
    - real DFSZ
    - funny shape of bubble → enhancement of GW signal?
    - how affects EW baryogenesis?
    - What is consequence of DW production?





Backup

Perturbation theory

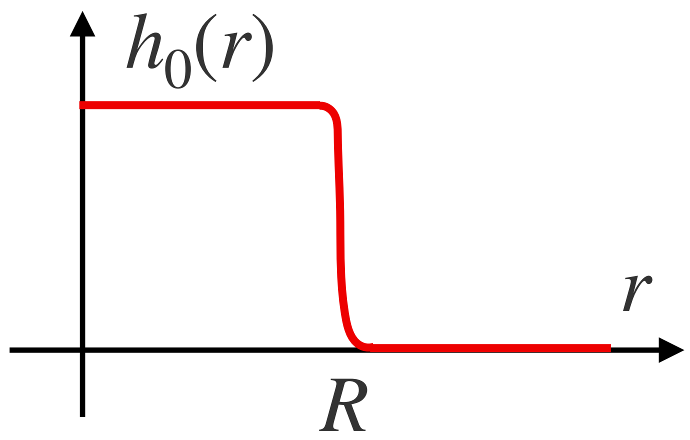
- It makes sense to consider perturbation w.r.t. κ :

higgs mode:

$$h(x) = h_0(x) + \delta h(x)$$

angular mode:

$$\phi(x) = \phi_0 + \delta\phi(x)$$



thin-wall as $\mathcal{O}(\kappa^0)$

Perturbation theory

- It makes sense to consider perturbation w.r.t. κ :

bounce solution w/ $\kappa = 0$

const (=0)

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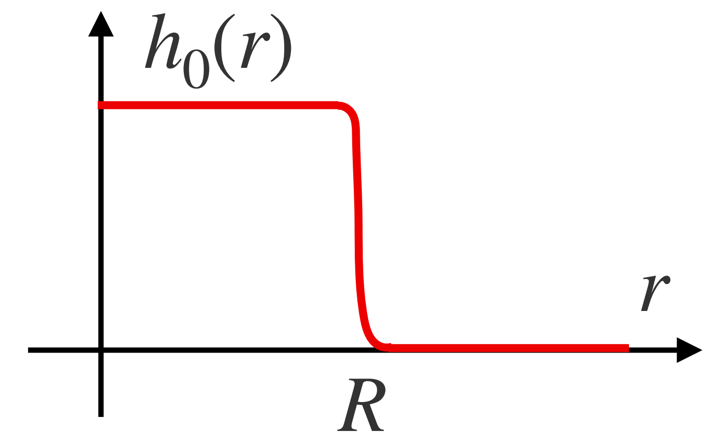

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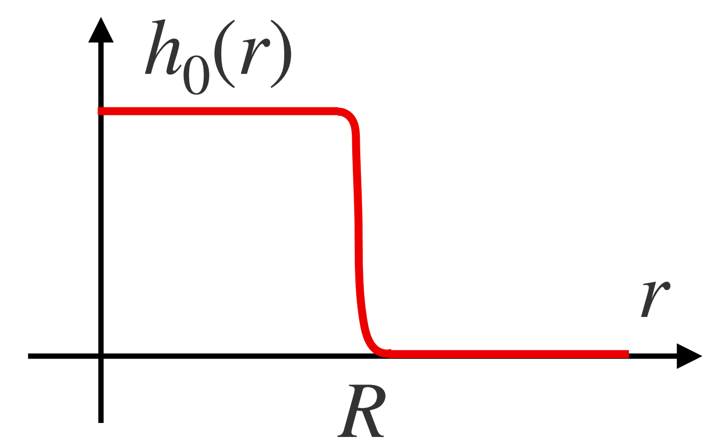
$\mathcal{O}(\kappa^1)$ correction



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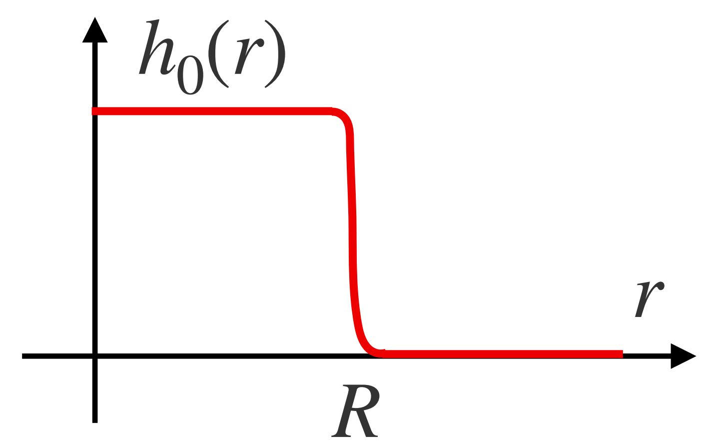


thin-wall as $\mathcal{O}(\kappa^0)$

- EOM for $\mathcal{O}(\kappa^1)$ fluctuations:

higgs mode: $$-\partial_i^2 \delta h + V''_{\mathrm{EW}}(h_0)\delta h = \kappa h_0(r) f_a^2 \cos 2\theta$$

angular mode: $$-\partial_i \left[h_0^2(r)\partial_i \delta\phi\right] = -\,\kappa f_a^2\, h_0^2(r)\ \sin 2\theta$$

Linearized EOM for δh

$$\underbrace{\left(-\partial_i^2 + V''_{\rm EW}(h_0)\right)\delta h}_{\equiv \hat{\mathcal{O}}\delta h} = \kappa h_0(r) f_a^2 \cos 2\theta \equiv S(x) \quad \text{source}$$

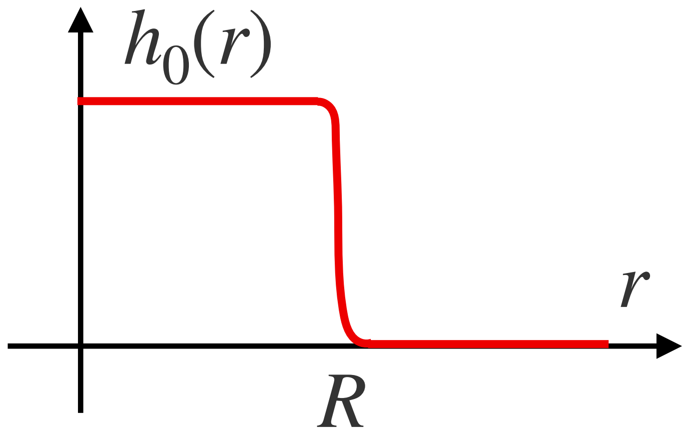
eigenvalue problem: $\hat{\mathcal{O}} f_\omega(x) = \omega^2 f_\omega(x)$

- Spectral decomposition works well

$$\delta h = \sum_\omega \sum_{l,m} \frac{c_{\omega,l,m}}{\omega^2} Y_{lm}(\Omega) f_\omega(\rho, z)$$

$$\to c_{\omega,l,m} = \int d^3x\, S(x) Y_{l,m} f_\omega \propto \delta_{l2}\delta_{m2}$$

$Y_{l,m}$: harmonic func



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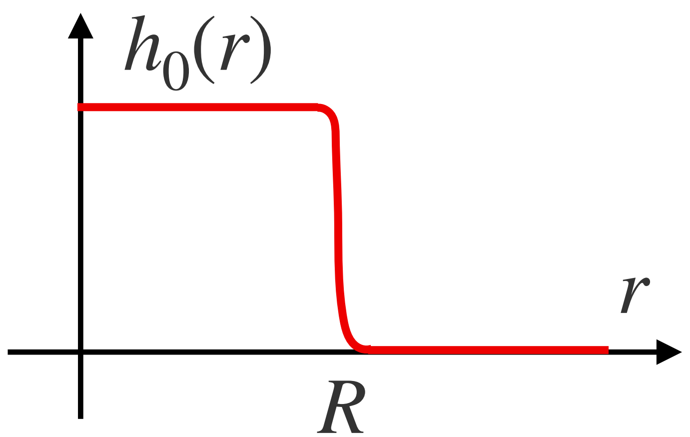

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$Y_{l,m}$: harmonic func

- only $Y_{2,2} \propto \cos 2\theta$ survives → $\delta h \propto \cos 2\theta$
- leading is quasi-zeromodes: translation of wall → $\omega \sim 1/R$

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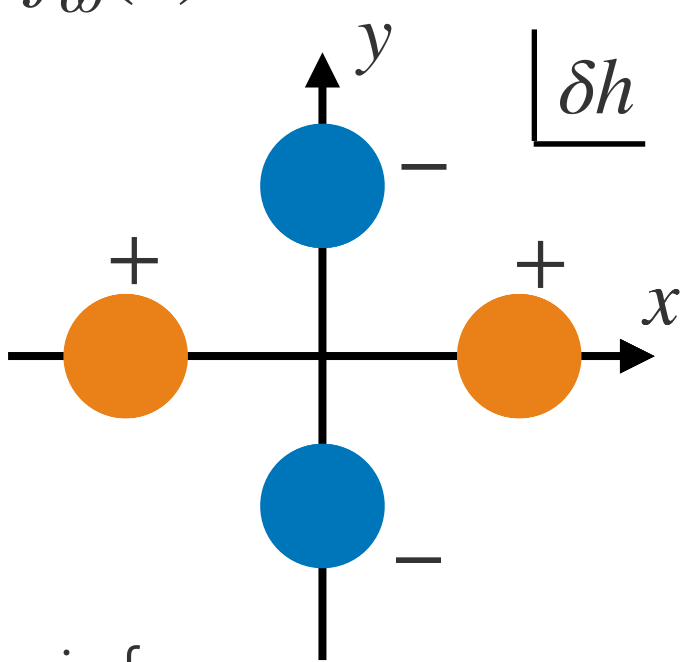

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Linearized EOM for δh

$$\underbrace{\left(-\partial_i^2 + V''_{\rm EW}(h_0)\right)\delta h}_{\equiv\, \hat{\mathcal{O}}\delta h} = \kappa h_0(r) f_a^2 \cos 2\theta \equiv S(x) \quad \text{source}$$

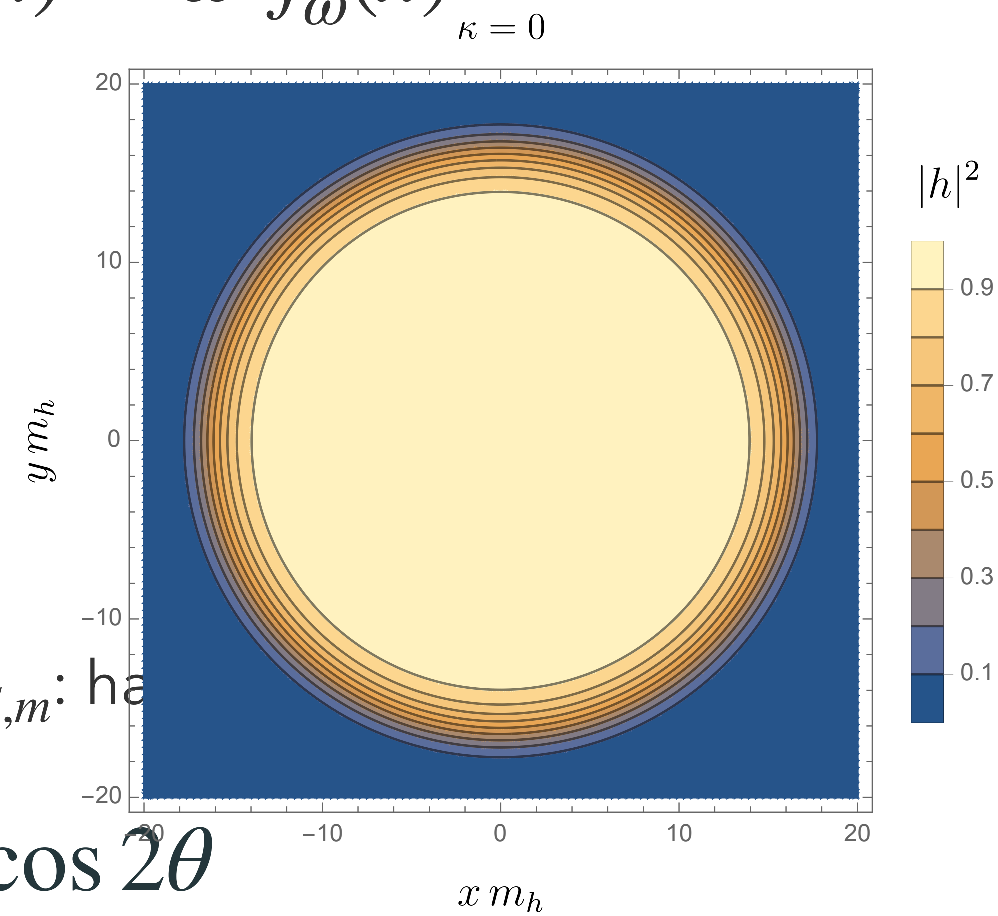
eigenvalue problem: $\hat{\mathcal{O}} f_\omega(x) = \omega^2 f_\omega(x)$

- Spectral decomposition works well

$$\delta h = \sum_\omega \sum_{l,m} \frac{c_{\omega,l,m}}{\omega^2} Y_{lm}(\Omega) f_\omega(\rho, z)$$

$$\to c_{\omega,l,m} = \int d^3x\, S(x) Y_{l,m} f_\omega \propto \delta_{l2}\delta_{m2}$$

$Y_{l,m}$: ha



- only $Y_{2,2} \propto \cos 2\theta$ survives → $\delta h \propto \cos 2\theta$
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Linearized EOM for δh

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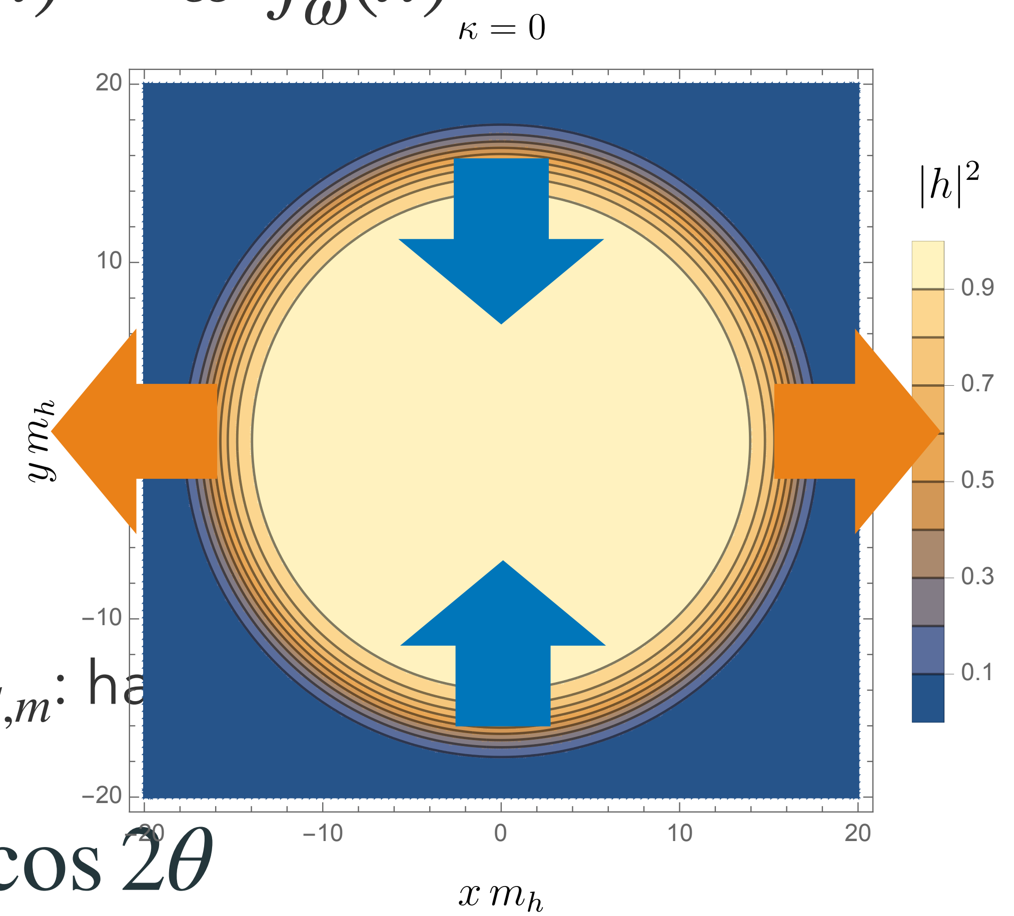

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  - only $Y_{2,2} \propto \cos 2\theta$ survives → $\delta h \propto \cos 2\theta$
  - leading is quasi-zeromodes: translation of wall → $\omega \sim 1/R$

Linearized EOM for δh

$$\left(-\partial_i^2 + V''_{\rm EW}(h_0)\right)\delta h = \kappa h_0(r) f_a^2 \cos 2\theta$$

$$\equiv \hat{\mathcal{O}}\delta h \qquad\qquad \equiv S(x)$$ source

eigenvalue problem: $\hat{\mathcal{O}} f_\omega(x) = \omega^2 f_\omega(x)$

- Green function method works well

$Y_{l,m}(\Omega)$: harmonic func

$$\delta h = \sum_\omega \sum_{l,m} \frac{c_{\omega,l,m}}{\omega^2} Y_{lm}(\Omega) f_\omega(\rho, z) \quad \to c_{\omega,l,m} = \int d^3x\, S(x) Y_{l,m} f_\omega$$

$$= \int d^3x' \sum_\omega \sum_{l,m} \frac{1}{\omega^2} Y_{lm}(\theta) f_\omega(x) Y^*_{l,m}(\theta') f^*_\omega(x')\, S(x')$$

Green function

Perturbation theory

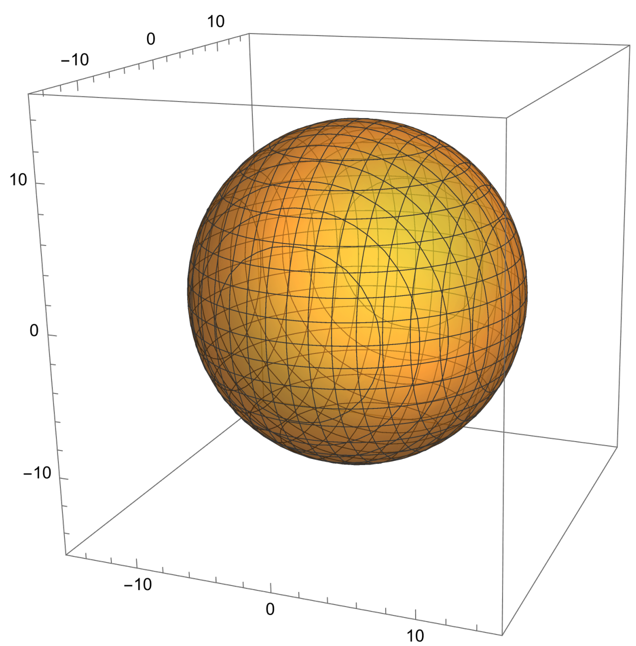
- It makes sense to consider perturbation w.r.t. κ :

bounce solution w/ $\kappa = 0$

const (=0)

$$h(x) = h_0(x) + \delta h(x) \qquad \phi(x) = \phi_0 + \delta\phi(x)$$

$\mathcal{O}(\kappa^1)$ correction



bounce solution w/ $\kappa = 0$

bounce up to $\mathcal{O}(\kappa^1)$

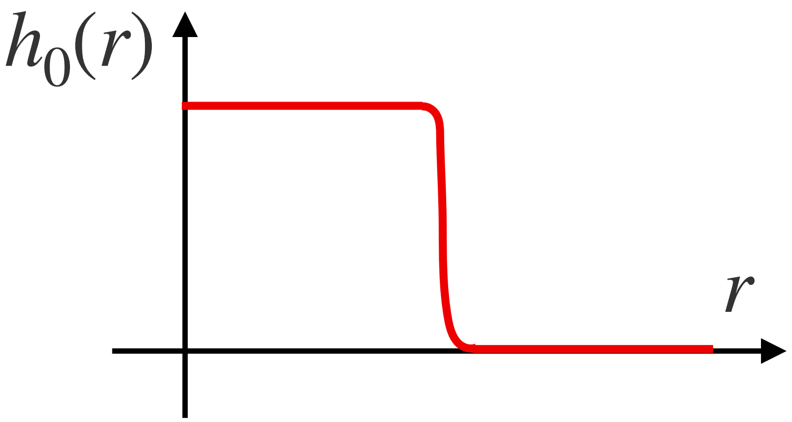
Linearized EOM

- EOM for $\mathcal{O}(\kappa^1)$ fluctuations:

$$-\partial_i^2\delta h + V''_{\rm EW}(h_0)\delta h = \kappa h_0(r) f_a^2 \cos 2\theta$$

$$\equiv \hat{\mathcal{O}}\delta h$$

$$-\partial_i\left[h_0^2(r)\partial_i\delta\phi\right] = -\kappa f_a^2\, h_0^2(r)\ \sin 2\theta$$



thin-wall as BG

- Eigenmode decomposition:

coefficient of expansion

harmonic funcs

$$\delta h = \sum_\omega \sum_{l,m} \frac{c_{\omega,l,m}}{\omega^2} Y_{lm}(\theta) f_\omega(x)$$

eigenvalue of "kernel": $\hat{\mathcal{O}} f_\omega(x) = \omega^2 f_\omega(x)$

solving Eq ⇔ determining $c_{\omega,l,m}$