IMPERIAL

Causality Bounds in the Primordial Power Spectrum

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Based on 2502.19477 with M. Carrillo Gonzalez

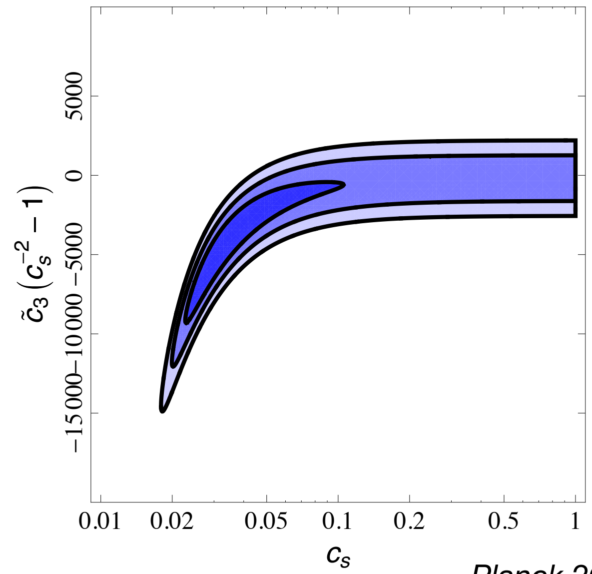
Also 2109.00567 with G. Ballesteros and L. Santoni

How to constraint models of inflation

- The use of effective field theories in cosmology is very extensive since it is a useful guide to parametrise many models

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2|\dot{H}|c_s^{-2}}{H^2}\left(\dot{\zeta}^2 - c_s^2\frac{(\partial_i\zeta)^2}{a^2}\right) - \frac{M_{\rm Pl}^2|\dot{H}|}{H^3}(1-c_s^{-2})\left(\left(1-\frac{4}{3}\frac{\tilde{c}_3}{c_s^2}\right)\dot{\zeta}^3 - \frac{(\partial_i\zeta)^2}{a^2}\dot{\zeta}\right)\right]$$

Cheung et al '05



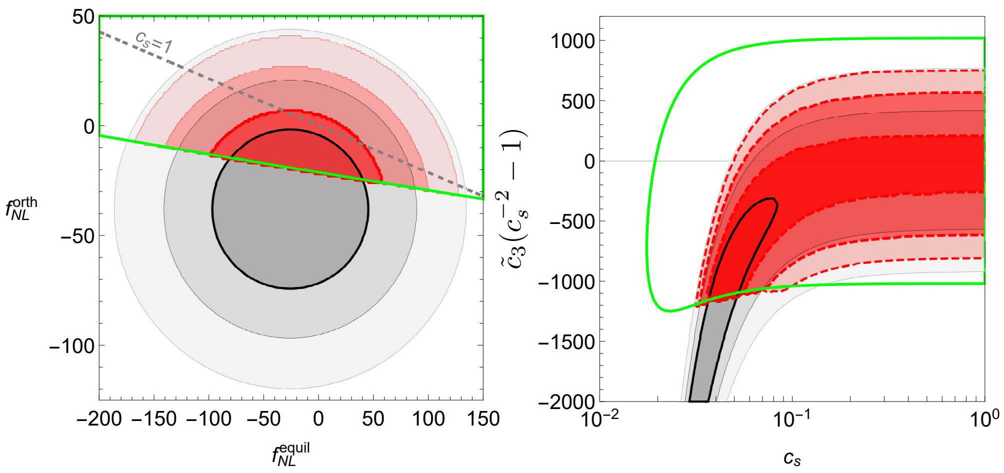
Planck 2018

- Theoretical priors can drastically change estimation of cosmological parameters

Constraining the EFT of Inflation

- Is it possible to constrain the EFT of inflation using positivity, analyticity and causality?

- Deep in the horizon $\frac{k}{aH} \gg 1$ the dynamics is the same as in flat space

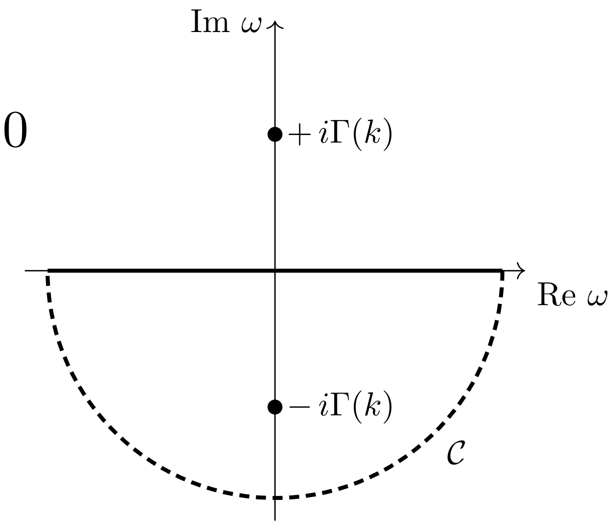


Grall, Melville '21

- Crucially there is no analyticity, hence the bounds are not necessarily true

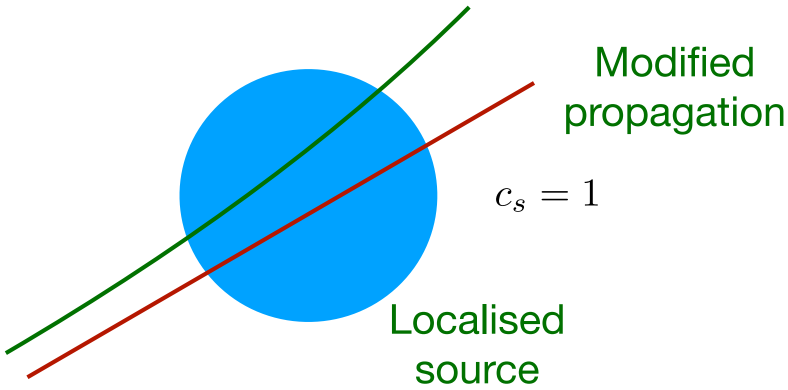
Causality

- Microcausality $G_R(x-y) = 0 \text{ for } (x-y)^2 > 0$



Usual argument requires Lorentz invariance

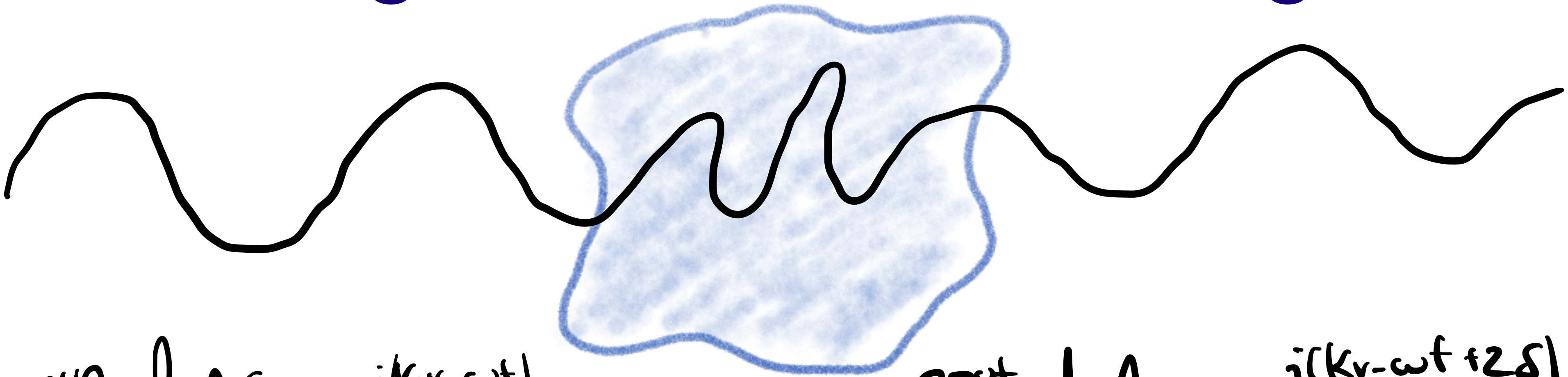
- EFT description (low frequency). Infrared causality



- Diagnose by looking at time delay

$$\Delta T = -i\langle \text{in}|\hat{S}^\dagger \frac{\partial}{\partial \omega}\hat{S}|\text{out}\rangle$$

Scattering off non-trivial background



$$\varphi^{in} = \int_k \frac{A(k)}{r} e^{i(k\cdot r-\omega t)} \qquad \varphi^{out} = \int \frac{A(k)}{r} e^{i(kr-\omega t+2\delta)}$$

At fixed r $\quad \Delta T = 2\,\partial_\omega \delta \quad$ for ω conserved

At fixed t $\quad \Delta r = -2\,\partial_k \delta \quad$ for k conserved

De Rham, Tolley '19
Carrillo-Gonzalez '23

Applications to Cosmology

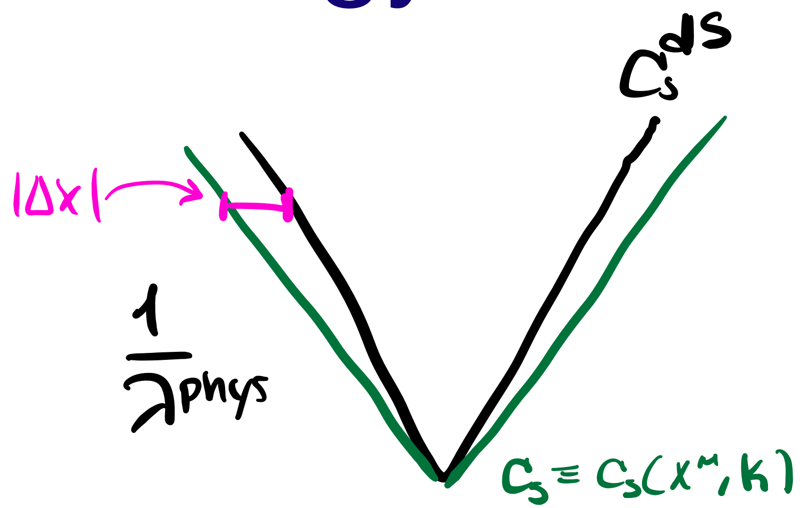
Consider fixed FLRW background

$$ds^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2)$$

Field with background $\bar{\Phi}(t)$

experiences spatial shift

$$\frac{k}{a(t_f)}\left(a(t_f)\Delta r\right) \sim k \int_{t_i}^{t_f} \frac{dt}{a(t)} \left(c_s^{EFT}(k,t) - c_s^{FRW}(k,t) \right) < 1$$



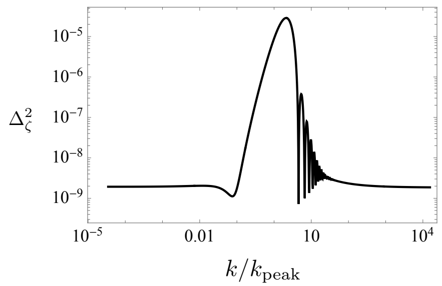
Similar ideas (same at LO) Bittermann, Mc. Loughlin, Rosen

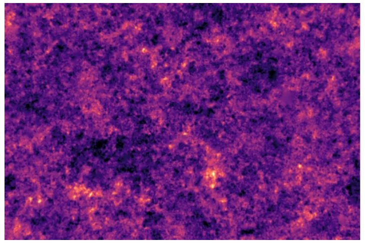
Causality in Inflation

- Let's consider the following EFT

$$S_\zeta^{(2)} = \int d^4x\, a^3 H^{-2}\left(2M_2^4 - M_{\rm Pl}^2\dot{H}\right)\left[\dot{\zeta}^2 - c_s^2\frac{(\partial_i\zeta)^2}{a^2} - \alpha\frac{(\partial_i^2\zeta)^2}{H^2a^4}\right]$$

- This model can produce PBH's within the regime of the EFT





- In general a only c_s^2 can only lead to $\Delta_\zeta^2 \sim 10^{-2}$

$$\Lambda_\star \gg H \rightarrow c_s^4 \gg \Delta_\zeta^2$$

- Using more parameters in the dispersion relation allows to increase

$$\omega^2(k,N) = c_s^2(N)\frac{k^2}{a^2H^2} + \alpha(N)\frac{k^4}{a^4H^4} \equiv (c_s^{\rm eff}(k,N))^2\frac{k^2}{a^2H^2}$$

Ballesteros, SC and Santoni '21

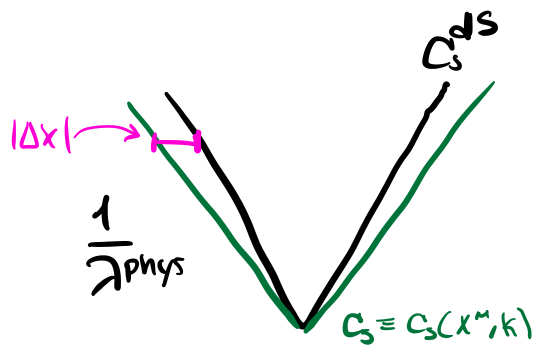
Causality in Inflation

- Let's consider the following EFT

$$S_\zeta^{(2)} = \int d^4x\, a^3 H^{-2} \left(2M_2^4 - M_{\rm Pl}^2 \dot{H}\right) \left[\dot{\zeta}^2 - c_s^2 \frac{(\partial_i \zeta)^2}{a^2} - \alpha \frac{(\partial_i^2 \zeta)^2}{H^2 a^4}\right]$$

- Solve the modes in WKB $\phi_k \sim \dfrac{e^{ik\tau}}{\sqrt{\omega(k,N)}} e^{-ik\left(\int_{-\infty}^{\tau}\left((c_s^{\rm eff})^2 - 1\right) d\tilde{\tau}\right)}$

Causality if $k\left(\int_{N_{\rm ini}}^{N_{\rm final}} \left(\partial_k \omega(k,N) - \frac{1}{aH}\right) dN\right) \leq 1$



$$\omega^2(k,N) = c_s^2(N) \frac{k^2}{a^2 H^2} + \alpha(N) \frac{k^4}{a^4 H^4} \equiv \left(c_s^{\rm eff}(k,N)\right)^2 \frac{k^2}{a^2 H^2}$$

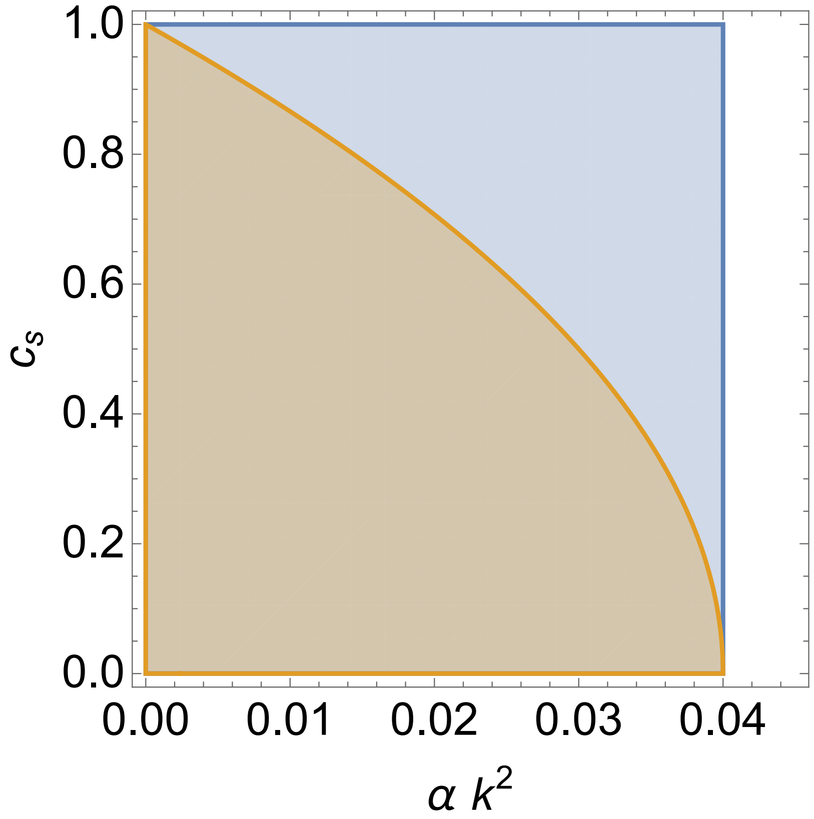
Bounds on constant parameters

- Let's keep constant $c_s, \ \alpha$

$$\omega^2 = c_s^2 \frac{k^2}{a^2 H^2} + \alpha \frac{k^4}{a^4 H^4}$$

- Not all parameters of the theory are causal

$$c_s = 1, \ \alpha = 0$$

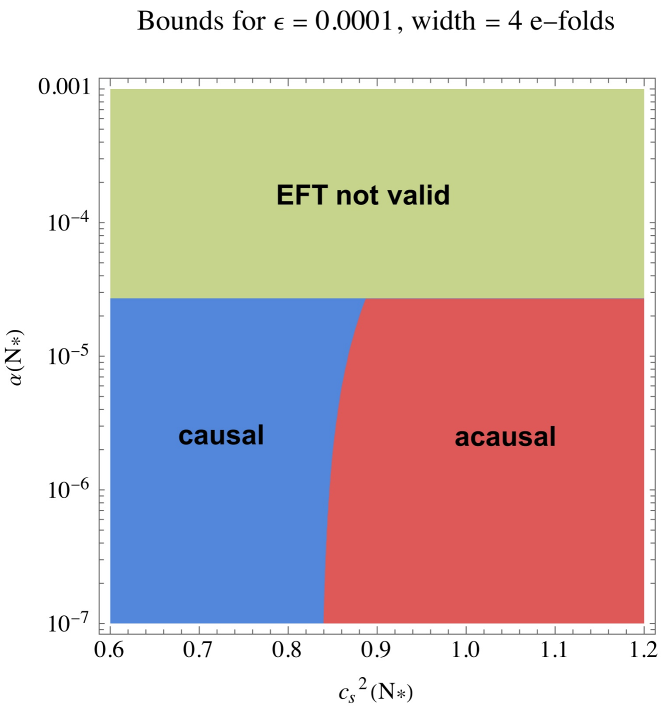


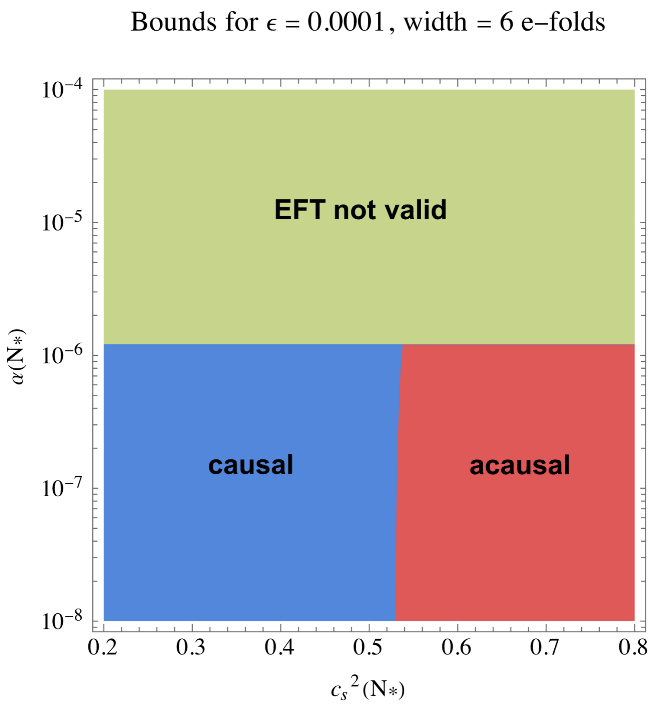
Bounds on c_s and α

$$\omega^2 = c_s^2 \frac{k^2}{a^2 H^2} + \alpha \frac{k^4}{a^4 H^4}$$

$$c_s = 1$$

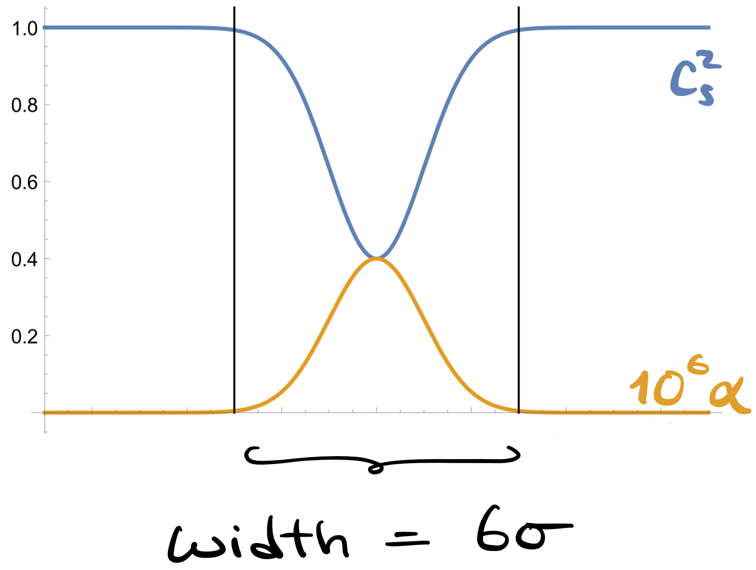
only consistent on a free theory

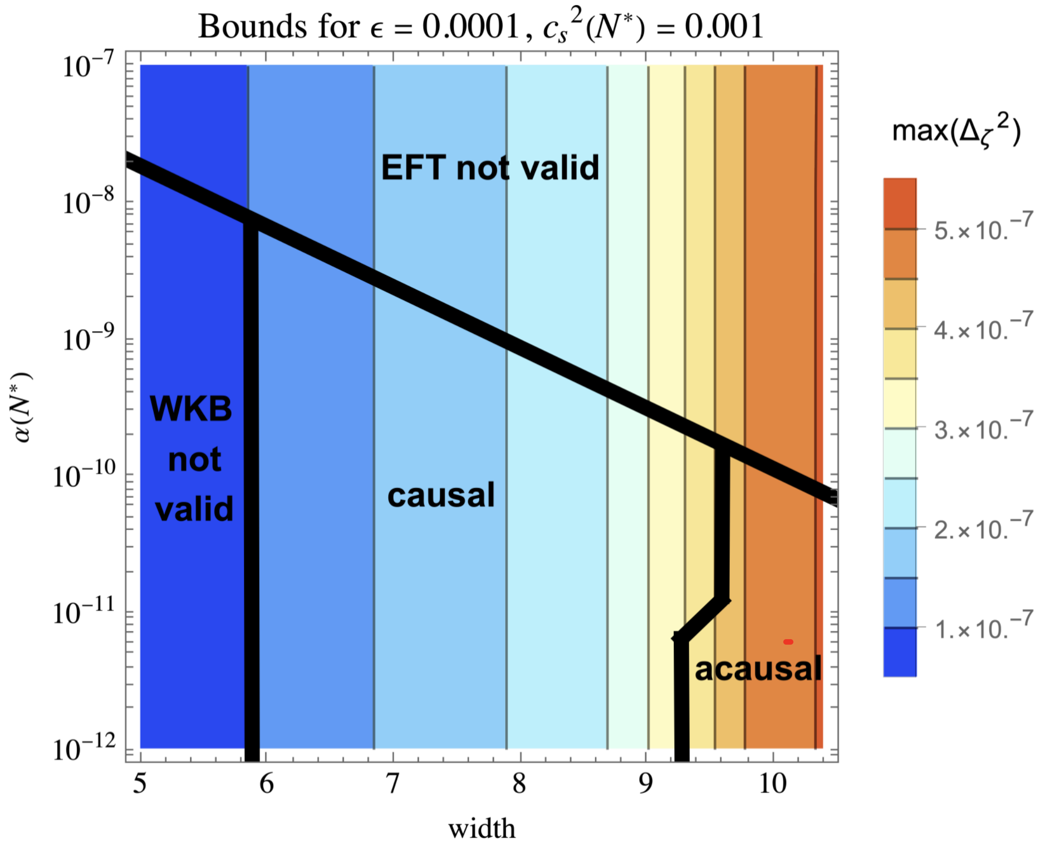




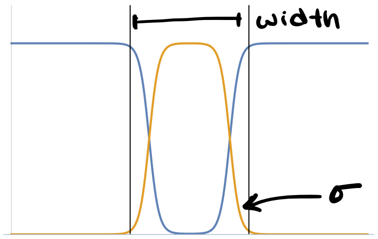
Bounds on primordial power spectrum

Gaussian transition $e^{-\frac{(N-N^*)^2}{2\delta^2}}$

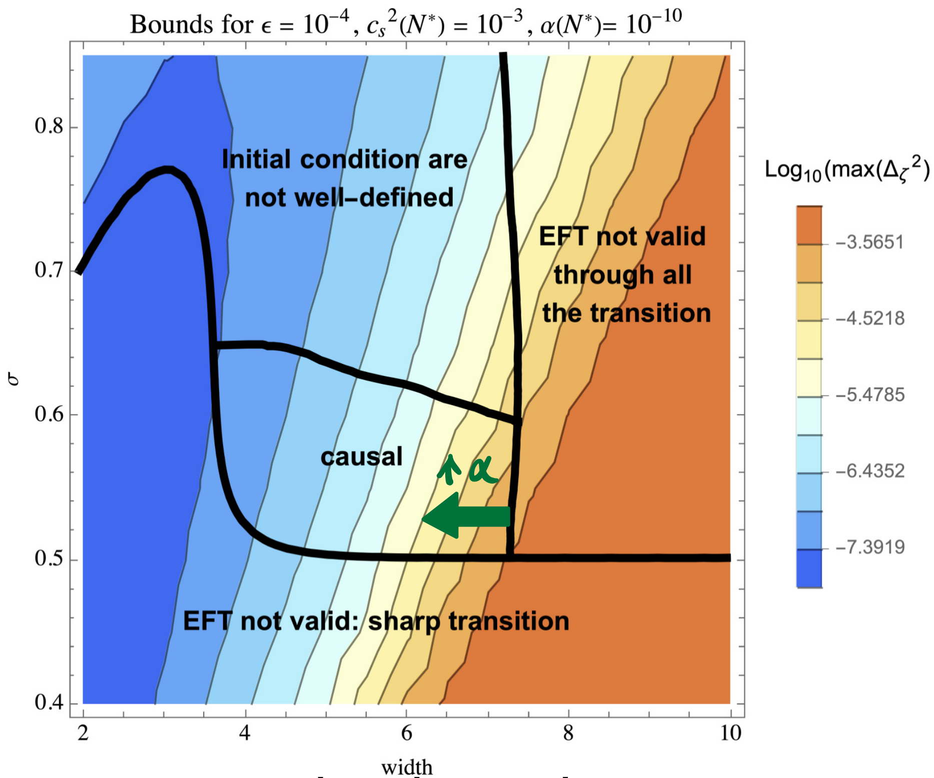




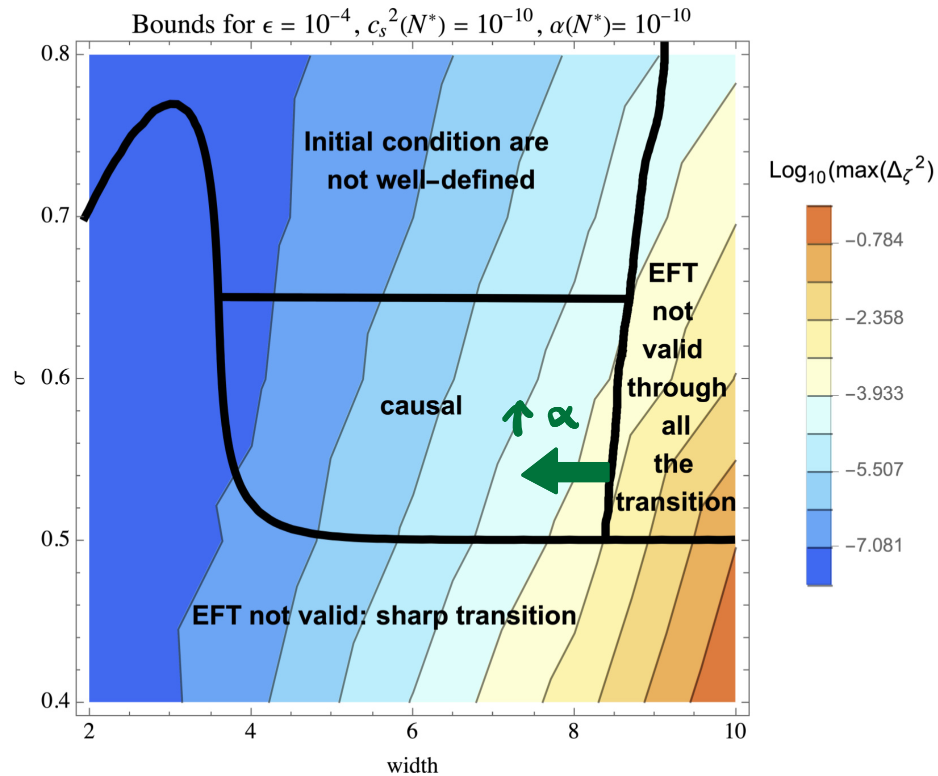
Bounds on primordial power spectrum



Tanh transition



c_s dominated



α dominated

Conclussions

- Using causality we can constrain some of the growth of the power spectrum
- This rules out most of the sensitive cases when PBH can be formed in this model
- Can we use this to constrain other interesting cases (for example leading to GWs)
- Future work - Constrains on non Gaussianity using causality