Bubble wall dynamics from nonequilibrium QFT

Based on 2504.13725

with W.Ai, B.Garbrecht, C.Tamarit, M.Vanvlasselaer

Matthias Carosi

Theoretical Physics of the Early Universe
TUM School of Natural Sciences
Technical University of Munich

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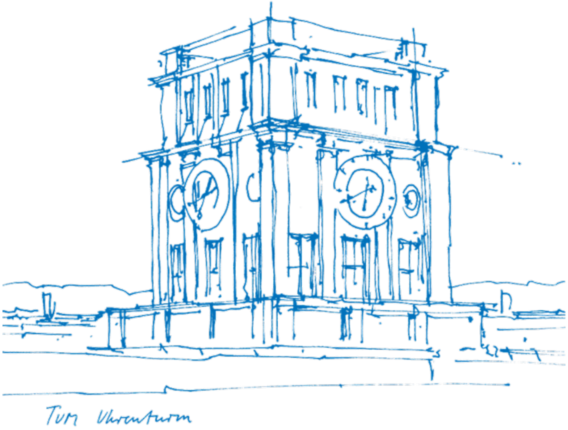

Outline

1. The dynamics of a single bubble
   - Evolution of the single bubble
   - Kinetic vs. kick pictures
2. The language of nonequilibrium QFT: CTP and 2PI
3. Friction from pair production
4. Conclusions and outlook

Evolution of the single bubble

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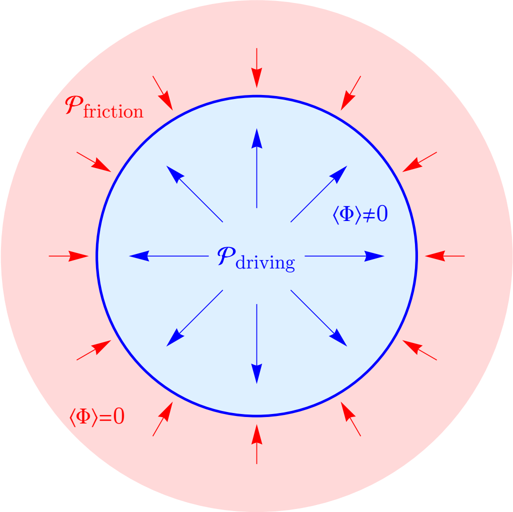

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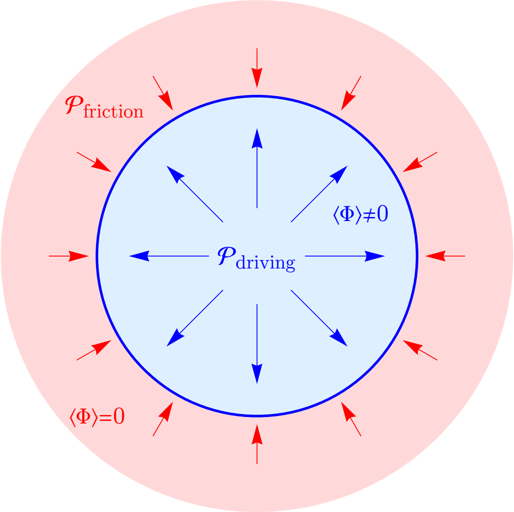

When the two pressures balance

$$\mathcal{P}_{\text{friction}} = \mathcal{P}_{\text{driving}}$$

the system reaches a *steady state*

$\Longrightarrow$terminal wall velocity

$\equiv \; v_w$

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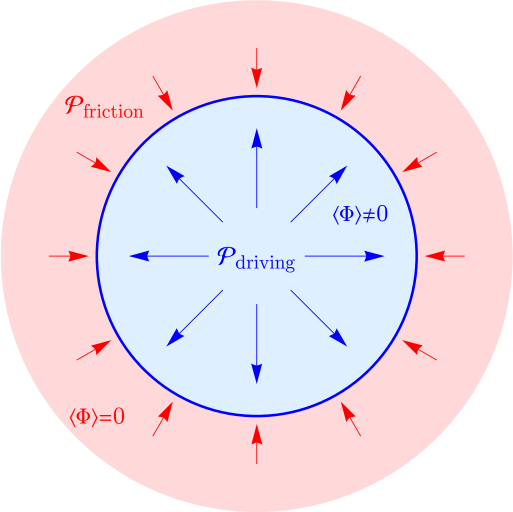

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Goal: identify v_w from the steady state condition.

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Two main approaches exist for studying the dynamics of a single bubble

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Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + V'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi} \int_{\mathbf{p}} f_i(\mathbf{p}, x) = \ 0 \\ \frac{\mathrm{d}f_i}{\mathrm{d}t} = \ -\mathcal{C}[f, \varphi] \end{cases}$$

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[Dine et al. '92, Bodeker and Moore '09, '17]

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$$\mathcal{P}_{\mathrm{kick}} = \sum_{i,X} \int_{\mathbf{p}} 2p^z \, \mathrm{d}\mathbb{P}_{i\to X}(\mathbf{p}) \, f_i(\mathbf{p}) \, \Delta p^z_{i\to X}$$

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Why would we care about processes which are higher order in the couplings?

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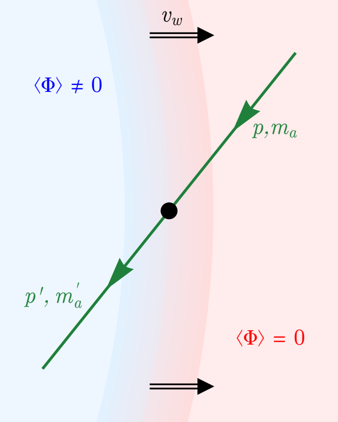

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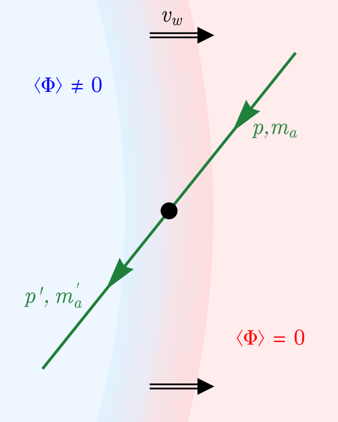

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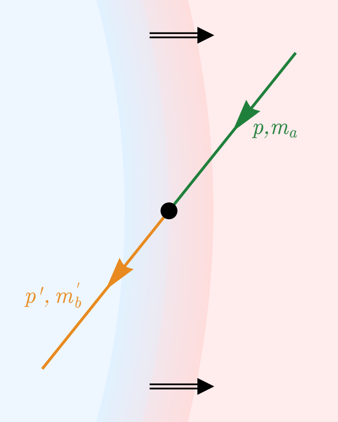

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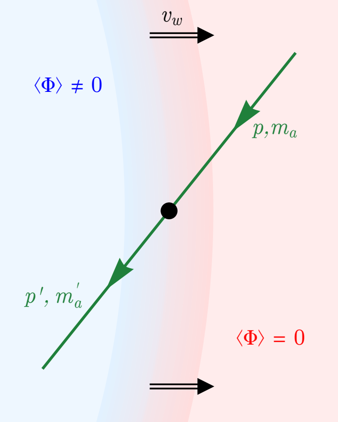

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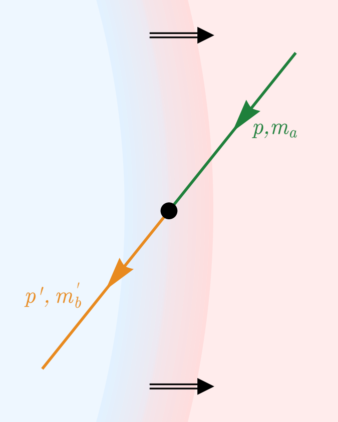

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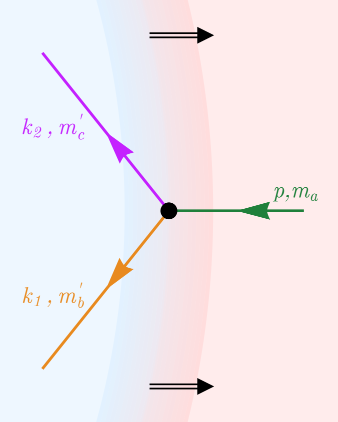

Particle production

$\sim \log \gamma_w$ for scalars

$\sim \gamma_w$ for gauge bosons

Outline

The tools of nonequilibrium QFT

real time correlators	$\Longrightarrow$	CTP formalism
dynamical equations	$\Longrightarrow$	2PI effective action

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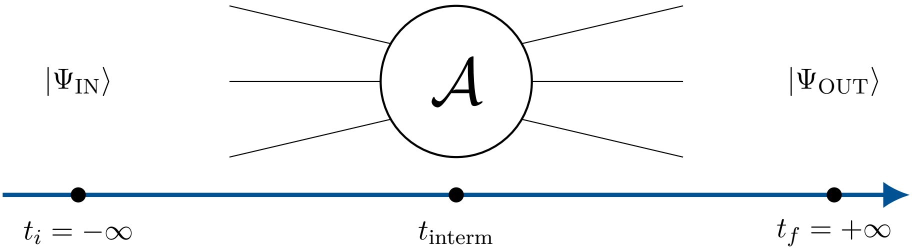

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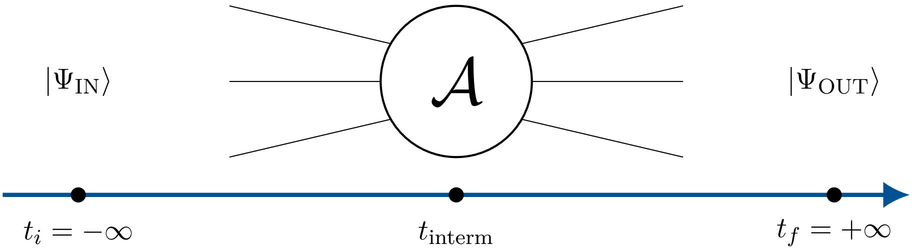

Using it, we compute transition amplitudes between **asymptotic states**

$$\mathcal{A} = \langle \Psi_{\rm OUT}|\,\mathcal{O}(\hat{\phi})\,|\Psi_{\rm IN}\rangle = \mathcal{N}\int [\mathcal{D}\phi]\,\Psi^*_{\rm OUT}(\phi)\mathcal{O}(\phi)\Psi_{\rm IN}(\phi)e^{iS[\phi]}$$

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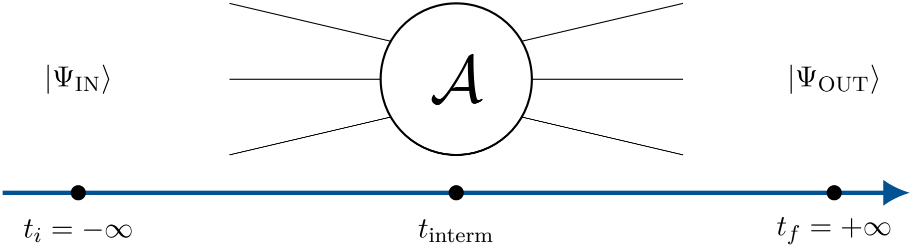

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But how can we compute time (and space) dependent correlators?

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Setting: we know the state at some initial time t_i and want to know what it will be at time t_f.

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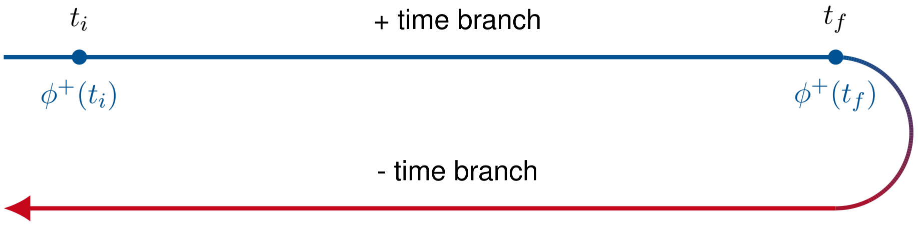

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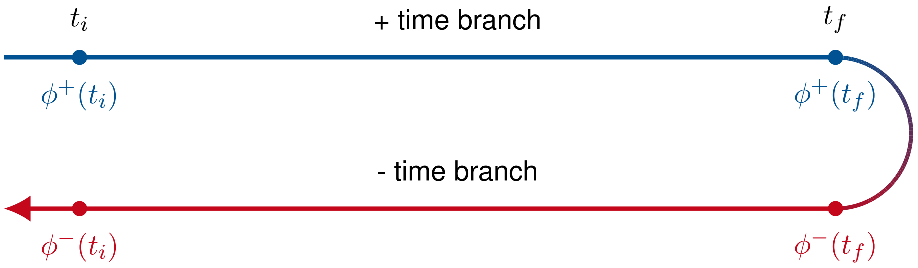

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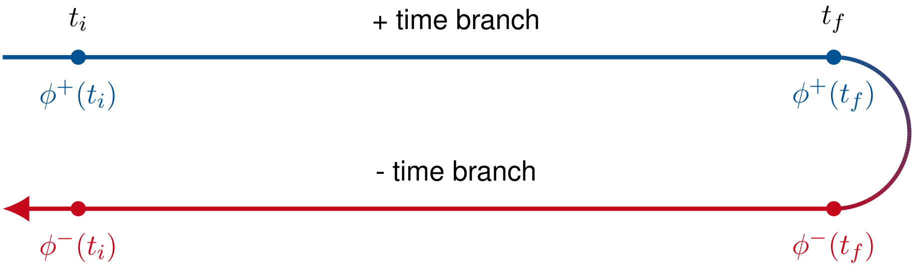

We introduce the label $\pm$ for the time branch, double our degrees of freedom, and can now use all the tools from the path integral formalism.

The tools of nonequilibrium QFT

real time correlators $\Longrightarrow$ CTP formalism ✓

dynamical equations $\Longrightarrow$ 2PI effective action

Introducing the 2PI effective action

We introduce the generator of connected one-   point functions

$$e^{W[J\quad]} = Z[J\quad] = \int [\mathcal{D}\phi]\, e^{iS[\phi]+\int_x J(x)\phi(x)}$$

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Equations for the one- and two-point functions are then easily generated

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi(x)} = 0 \qquad\qquad \frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta(x,y)} = 0}$$

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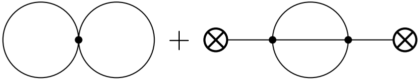
For a scalar theory with quartic self-interaction, we have

$$\mathcal{L}_{\mathrm{int}} = -\frac{\lambda}{4!}\phi^4 \longrightarrow i\Gamma_2 =$$



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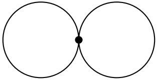
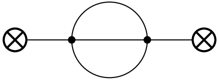
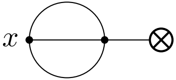
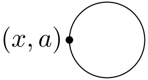
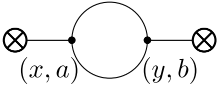
$$\mathcal{L}_{\mathrm{int}} = -\frac{\lambda}{4!}\phi^4 \longrightarrow i\Gamma_2 =$$  $+$  $+ \dots$

$$i\frac{\delta \Gamma_2}{\delta \varphi(x)} \supset$$ 

$$2i\frac{\delta \Gamma_2}{\delta \Delta^{ab}(x,y)} \supset$$ 

$+$ 

The Wigner transform

To put the equations in a useful form, we go to Wigner space

$$\overline{\Delta}^{ab}(k,x) = \int \mathrm{d}^4 r \, e^{ik\cdot r} \Delta^{ab}\left(x + \frac{r}{2}, x - \frac{r}{2}\right)$$

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$$\overline{\Delta}^{T}(k,x) = \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\varepsilon} + 2\pi\delta(k^2 - m^2)\left[\vartheta(k^0) f(\mathbf{k}, x) + \vartheta(-k^0) f(-\mathbf{k}, x)\right]$$

$$\overline{\Delta}^{<}(k,x) = 2\pi\delta(k^2 - m^2)\left[\vartheta(k^0) f(\mathbf{k}, x) + \vartheta(-k^0)(1 + f(-\mathbf{k}, x))\right]$$

The bubble wall equation of motion

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Having solved for the two-point function at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + V_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int \mathrm{d}^4y\,\Pi^R(x,y)\varphi(y) = 0$$

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$$\frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \frac{\lambda}{2}\varphi(x)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon} + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}}f(\mathbf{k},x)$$

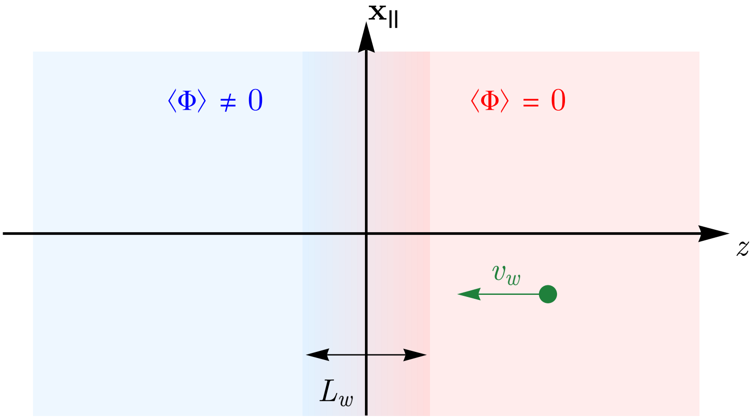
- Two-loop term: retarded self-energy

$$\Pi^R(x,y) = \Pi^{++}(x,y) - \Pi^{-+}(x,y)$$

Identifying sources of friction

Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**



Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + V'_{\mathrm{eff}}(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \underbrace{\pi^R(z, z')}_{\int \mathrm{d}\mathbf{x}_{\parallel} \mathrm{d}\mathbf{x'}_{\parallel} \Pi^R(\mathbf{x},\mathbf{x'})} \varphi(z') \quad = 0$$

Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + V'_{\mathrm{eff}}(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

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Identifying sources of friction

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Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

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$$- \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) \qquad \equiv \mathcal{P}_{\text{dissipative}}$$

$$\left. - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

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$$\begin{aligned}
\Rightarrow \underbrace{\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\cancel{\frac{1}{2} \left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \right)^2} + V_{\text{eff}}(\varphi(z), T) \right]}_{\Delta V_{\text{eff}} \equiv \mathcal{P}_{\text{driving}}} = \int_{-\delta}^{\delta} \mathrm{d}z \, \Bigg[& \frac{\partial V_{\text{eff}}}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z} && \equiv \mathcal{P}_{\text{LTE}} \\
& - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) && \equiv \mathcal{P}_{\text{dissipative}} \\
& - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \Bigg] && \equiv \mathcal{P}_{\text{vertex}}
\end{aligned}$$

Outline

The self-energy

The self-energy

At leading order in the gradient expansion

$$\mathcal{P}_{\text{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} iq^z \, |\tilde{\varphi}(q^z)|^2 \, \tilde{\pi}^R(-q^z)$$

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Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

The self-energy

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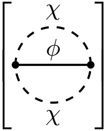
$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}$ []

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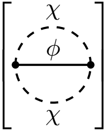
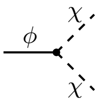
$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}$ [] = |  |^2

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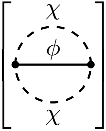
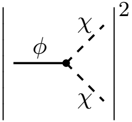
$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}$ 

$=$ 

$\implies$ pair production!

The self-energy

The imaginary part of the self-energy is computed via CTP cutting rules

$$\mathrm{Im}\tilde{\pi}^R(q^z) = -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right)$$

The self-energy

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\begin{aligned}
\mathrm{Im}\tilde{\pi}^R(q^z) &= -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right) \\
&\simeq \frac{g^2}{4}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2}(2\pi)^3\delta^{(3)}(\mathbf{q}-\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2)(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\left[f_\phi(\mathbf{p}) - f_\phi(-\mathbf{p})\right]
\end{aligned}
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$$\begin{aligned}\mathrm{Im}\tilde{\pi}^R(q^z) &= -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right)\\ &\simeq \frac{g^2}{4}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2}(2\pi)^3\delta^{(3)}(\underset{(0,0,q^z)}{\mathbf{q}} - \mathbf{p} + \mathbf{k}_1 + \mathbf{k}_2)(2\pi)\delta(E_{\mathbf{p}}^{(\phi)} - E_{\mathbf{k}_1}^{(\chi)} - E_{\mathbf{k}_2}^{(\chi)})\left[f_\phi(\mathbf{p}) - f_\phi(-\mathbf{p})\right]\end{aligned}$$

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\end{aligned}
$$

and the pressure due to pair production reads

$$
\begin{aligned}
\mathcal{P}_{\phi\to\chi\chi} = \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2}&(2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2}) \\
&\times f_\phi(\mathbf{p}) \quad \Delta p^z \quad |\tilde{\varphi}(\Delta p^z)|^2
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&\times f_\phi(\mathbf{p}) \quad \Delta p^z \quad |\tilde{\varphi}(\Delta p^z)|^2
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density of incoming particles

The self-energy

The imaginary part of the self-energy is computed via CTP cutting rules

$$\mathrm{Im}\tilde{\pi}^R(q^z) = -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right)$$
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$$\times f_\phi(\mathbf{p}) \quad \Delta p^z \quad |\tilde{\varphi}(\Delta p^z)|^2$$

density of incoming particles

momentum exchange

The self-energy

The imaginary part of the self-energy is computed via CTP cutting rules

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$$\times\, f_\phi(\mathbf{p}) \quad \Delta p^z \quad |\tilde{\varphi}(\Delta p^z)|^2$$

density of incoming particles

Fourier tf. of the wall

momentum exchange

The self-energy

The imaginary part of the self-energy is computed via CTP cutting rules

$$\mathrm{Im}\tilde{\pi}^R(q^z) = -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right)$$

$$\simeq \frac{g^2}{4}\int_{\mathbf{p},\mathbf{k}_1,\ldots} \ldots \, E^{(\chi)}_{\mathbf{k}_2})\left[f_\phi(\mathbf{p}) - f_\phi(-\mathbf{p})\right]$$

recall: $\mathcal{P}_{\text{kick}} = \sum_{i,X}\int_{\mathbf{p}} 2p^z\,\mathrm{d}\mathbb{P}_{i\to X}(\mathbf{p})\,f_i(\mathbf{p})\,\Delta p^z_{i\to X}$

and the pressure ...

$$\mathcal{P}_{\phi\to\chi\chi} = \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2}(2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})$$

density of incoming particles $\quad \times\, f_\phi(\mathbf{p}) \quad \Delta p^z \quad |\tilde{\varphi}(\Delta p^z)|^2$ Fourier tf. of the wall

momentum exchange

Outline

Conclusions and outlook [2504.13725]

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- The full bubble wall dynamics can be described using the language of nonequilibrium QFT (CTP) and the 2PI effective action.

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Conclusions and outlook [2504.13725]

- The full bubble wall dynamics can be described using the language of nonequilibrium QFT (CTP) and the 2PI effective action.
- Within this framework, we provide a *first-principle derivation* of the kick picture.
- In the gradient expansion, we find dynamical equations amenable to numerical implementation, e.g. in *WallGo* [Eckstedt et al. '24]. These include, order by order, all **thermal** and **quantum** effects.

Conclusions and outlook [2504.13725]

- The full bubble wall dynamics can be described using the language of nonequilibrium QFT (CTP) and the 2PI effective action.
- Within this framework, we provide a *first-principle derivation* of the kick picture.
- In the gradient expansion, we find dynamical equations amenable to numerical implementation, e.g. in *WallGo* [Eckstedt et al. '24]. These include, order by order, all **thermal** and **quantum** effects.

Future directions

- investigate out-of-equilibrium effects, such as gauge boson saturation,
- find general bounds for friction strength,
- study numerically the effect of quantum effects for intermediate wall velocities.

BACK-UP SLIDES

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

$$\nabla\varphi \sim \frac{1}{L_w}\,, \qquad L_w \equiv \text{wall width}$$

$$k \sim \gamma_w T \;\equiv\; \text{typical momentum of a particle } \textbf{in the wall frame}$$

$$\implies \gamma_w T L_w \gg 1$$

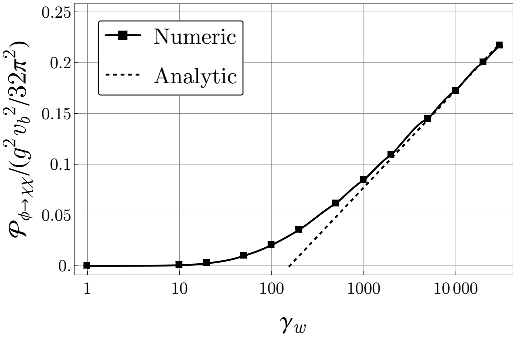
The gradient expansion is valid if the wall is either **fast** or **thick**. For the numerical and analytical results, we assumed the plasma outside the bubble to be **in equilibrium**, which is once again only valid if the wall is very fast.

Pair production in the ultrarelativistic limit

Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24\times 32\pi^2}\log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

which approach the result from the kick picture. Similarly, we show in our work that particle mixing and transition radiation are also captured within this framework.



Mixing

Assume two mixing scalar species χ and s interacting through the background

$$\mathcal{L}_{\text{int}} \supset -\kappa\varphi\chi s\,, \qquad \text{and} \quad m_\chi \gg m_s$$

Particles χ are absent in the plasma but are generated via mixing as s-particles go through the wall. In the ultrarelativistic limit

$$\mathcal{P}^{\gamma_w\to\infty}_{s\to\chi} = \frac{2\kappa^2 v_b^2}{m_\chi^2}\frac{T^2}{24}$$