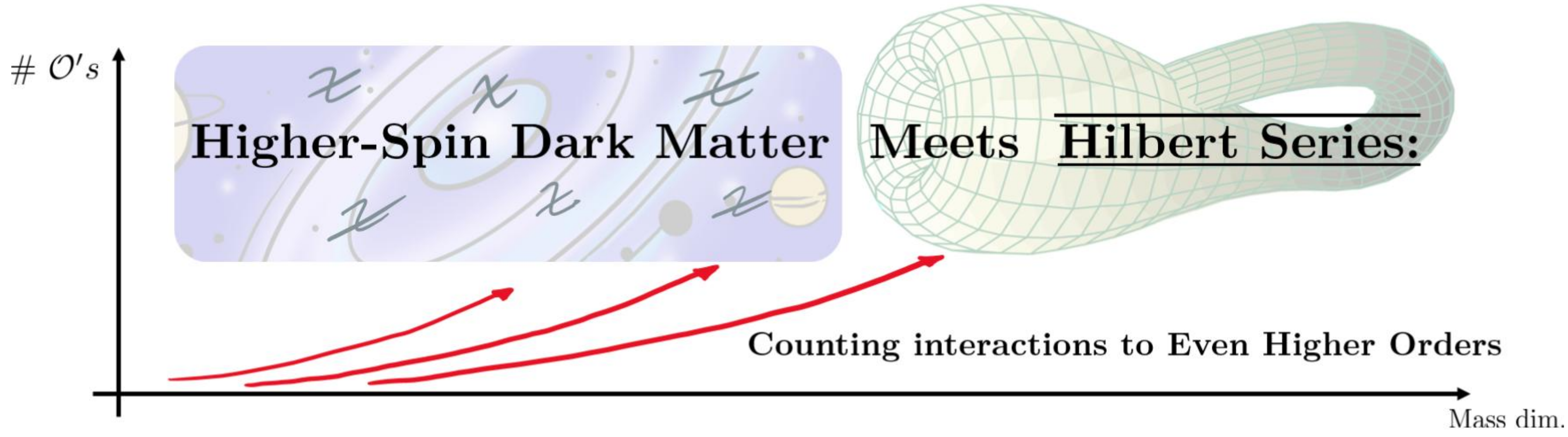
Higher-Spin Dark Matter Meets Hilbert Series:

Counting interactions to Even Higher Orders



Bruno Siqueira Eduardo
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work in progress in collaboration with:
Oscar Eboli (Universidade de São Paulo)
Enrico Bertuzzo (Università degli Studi di Modena e Reggio Emilia)

CLUSTER OF EXCELLENCE QUANTUM UNIVERSE
HELMHOLTZ
DESY THEORY WORKSHOP
23 – 26 September 2025 DESY Hamburg, Germany

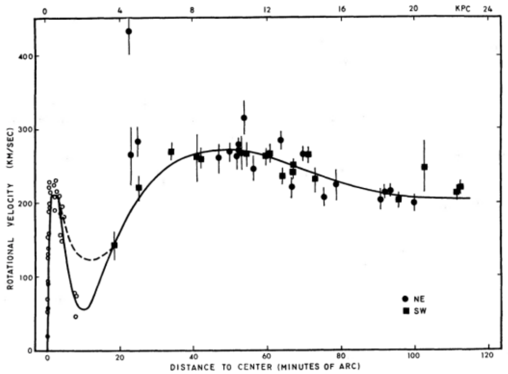
Evidences for Dark Matter

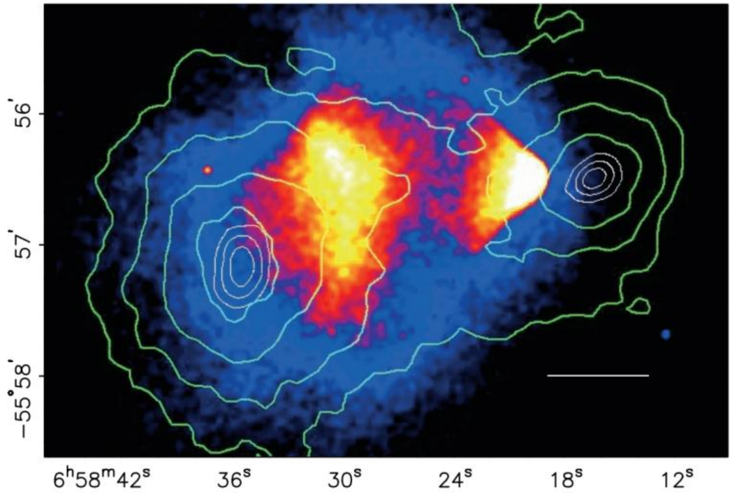
Length Scale

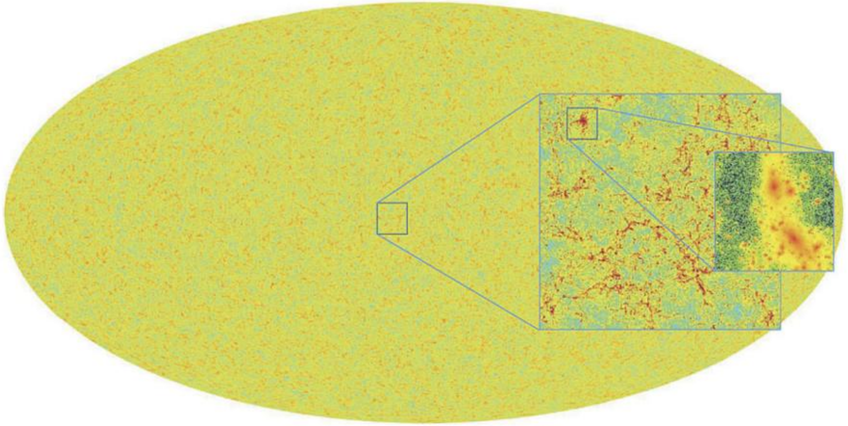
~10^2 kpc

~1 Mpc

~1 Gpc







Vera C. Rubin and W. Kent Ford, Jr., “Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions”, 1970.
D. Clowe et al., “A direct empirical proof of the existence of dark matter”, 2006.
D. Potter et al., “PKDGRAV3: Beyond Trillion Particle Cosmological Simulations for the Next Era of Galaxy Surveys”, 2017.

Dark Matter Frameworks

DARK MATTER

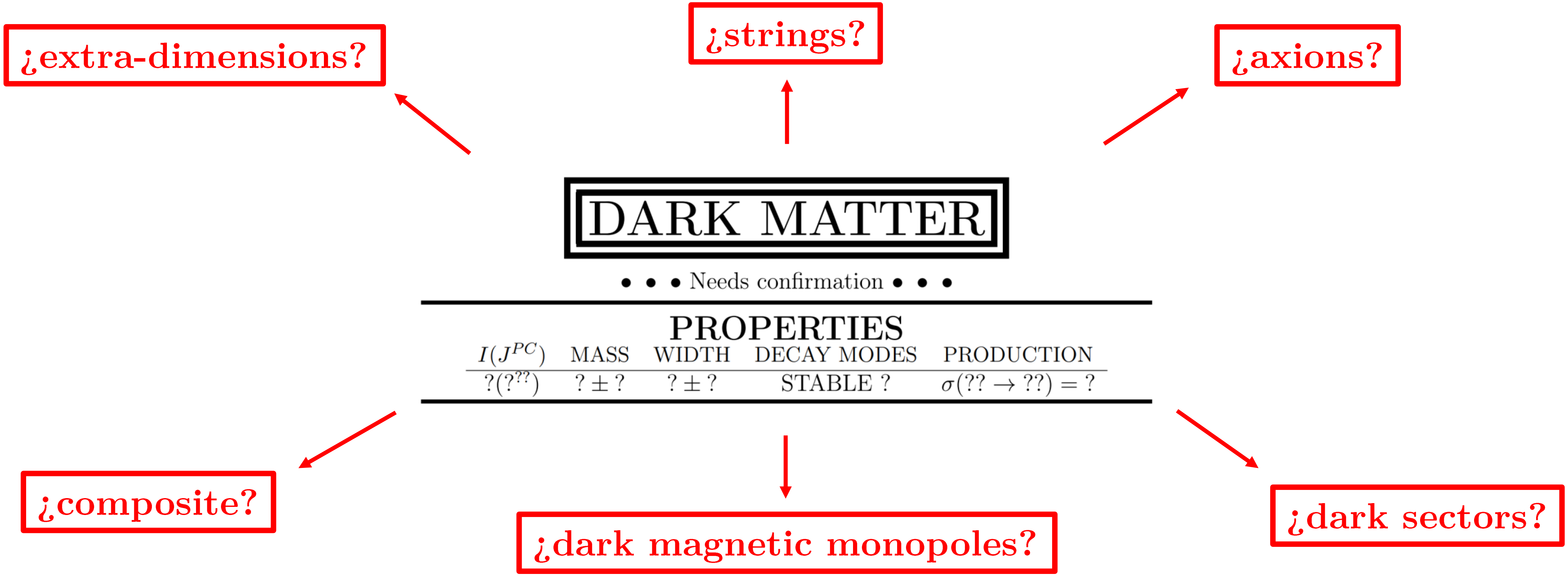
• • • Needs confirmation • • •

PROPERTIES

$I(J^{PC})$	MASS	WIDTH	DECAY MODES	PRODUCTION
$?(?^{??})$	$? \pm ?$	$? \pm ?$	STABLE ?	$\sigma(?? \to ??) = ?$

M. Cirelli et al., “Dark Matter”, 2024.

Dark Matter Frameworks



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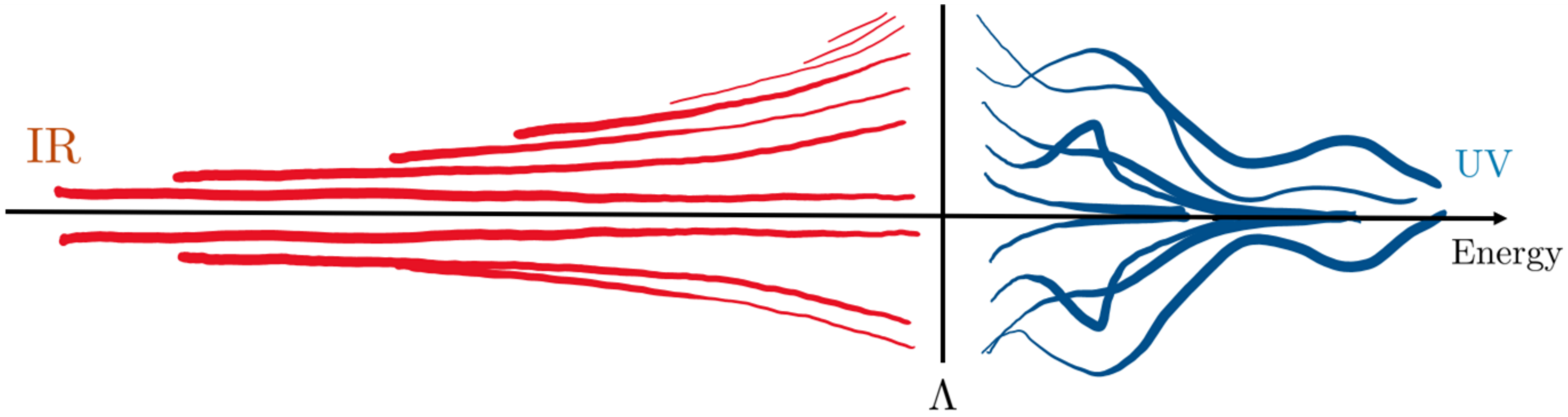
Effective field theories

"All physical theories are effective theories"

Riccardo Penco, "An Introduction to Effective Field Theories", 2020.



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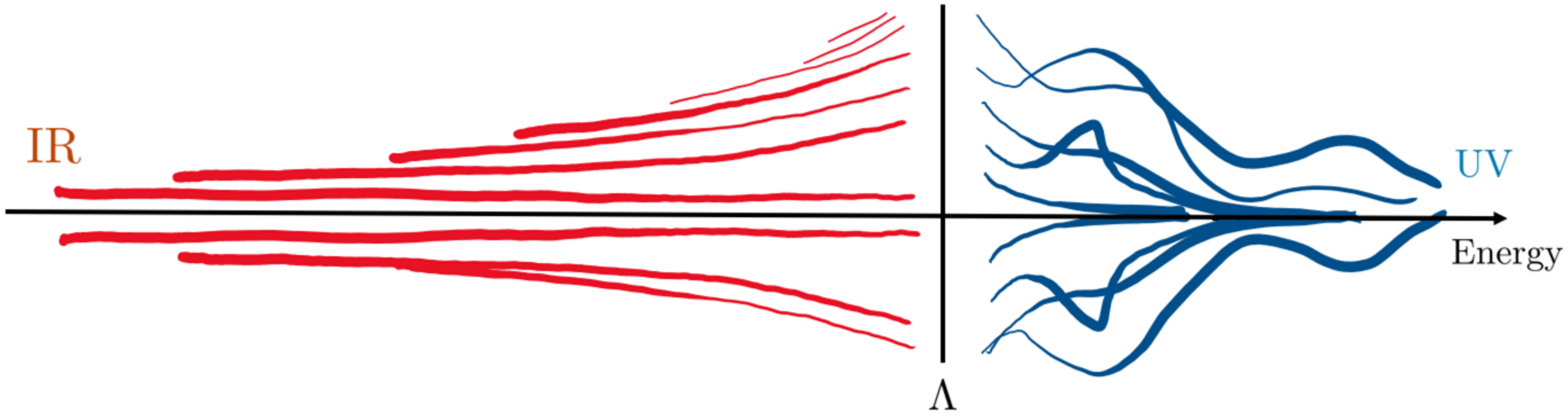

Parametrizes our ignorance by including all allowed terms in

$$\mathcal{L} = \mathcal{L}_0 + \sum_j \frac{c_j}{\Lambda^{\Delta_j - d}} \mathcal{O}_j$$

Which EFT?

SM + DM candidate

- ✅ massive
- 🟢 SM gauge singlets
- 🔴 higher spin (3/2 and 2)

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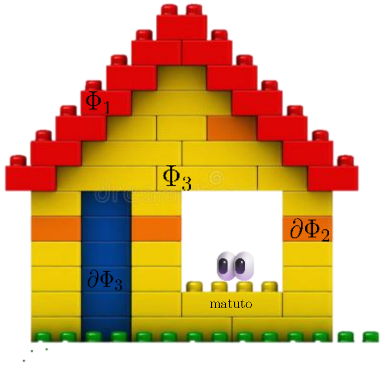
Problem:

EOM, IBP, Fierz...

... SYSTEMATICALLY!
... to ALL ORDERS!



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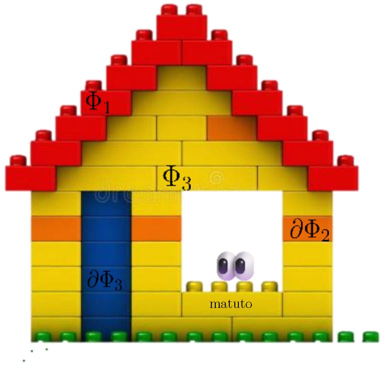

Remedy:

HILBERT SERIES

Hilbert Series 101

Field content: $\{\Phi_1, ..., \Phi_N\}$

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$$H(\mathcal{D}, \{\phi_i\}) \equiv \sum_{n=0}^{\infty} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} \cdots \sum_{r_N=0}^{\infty} d_{n,r_1,r_2,\ldots,r_N} \mathcal{D}^n \phi_1^{r_1} \phi_2^{r_2} \cdots \phi_N^{r_N}$$

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complex variables
("spurions")

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$\mathcal{D}, \{\phi_i\}$: complex variables ("spurions")

$d_{n,r_1,r_2,\ldots,r_N}$: number of $\mathcal{O}$'s with

- r_1 fields Φ_1;
- ...
- r_N fields Φ_N;
- n derivatives.

Hilbert Series 101

Example: SMEFT (1 generation)

$$H(\mathcal{D}, \{\phi_i\}) = H(\mathcal{D}, Q, Q^\dagger, L, L^\dagger, H, H^\dagger, u, u^\dagger, d, d^\dagger, e, e^\dagger, B_L, B_R, W_L, W_R, G_L, G_R)$$

B. Henning, X. Lu, T. Melia, H. Murayama. "Higher dimension operators in the SM EFT", 2019.

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Dim-5: $H^2L^2 + H^{\dagger\,2}L^{\dagger\,2}$

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Dim-6:

$$\begin{aligned}
&H^3H^{\dagger\,3} + u^\dagger Q^\dagger H H^{\dagger\,2} + 2Q^2Q^{\dagger\,2} + Q^{\dagger\,3}L^\dagger + Q^3L + 2QQ^\dagger LL^\dagger + L^2L^{\dagger\,2} + uQH^2H^\dagger \\
&+2uu^\dagger QQ^\dagger + uu^\dagger LL^\dagger + u^2u^{\dagger\,2} + e^\dagger u^\dagger Q^2 + e^\dagger L^\dagger H^2H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + eLHH^{\dagger\,2} + euQ^{\dagger\,2} \\
&+2euQL + ee^\dagger QQ^\dagger + ee^\dagger LL^\dagger + ee^\dagger uu^\dagger + e^2e^{\dagger\,2} + d^\dagger Q^\dagger H^2H^\dagger + 2d^\dagger u^\dagger Q^{\dagger\,2} + d^\dagger u^\dagger QL \\
&+d^\dagger e^\dagger u^{\dagger\,2} + d^\dagger eQ^\dagger L + dQHH^{\dagger\,2} + 2duQ^2 + duQ^\dagger L^\dagger + de^\dagger QL^\dagger + deu^2 + 2dd^\dagger QQ^\dagger + dd^\dagger LL^\dagger \\
&+2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + d^2d^{\dagger\,2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger HG_R + HH^\dagger G_R^2 + G_R^3 + uQHG_L \\
&+dQH^\dagger G_L + HH^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger HW_R + d^\dagger Q^\dagger HW_R + HH^\dagger W_R^2 + W_R^3 \\
&+uQHW_L + eLH^\dagger W_L + dQH^\dagger W_L + HH^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger HB_R \\
&+d^\dagger Q^\dagger HB_R + HH^\dagger B_RW_R + HH^\dagger B_R^2 + uQHB_L + eLH^\dagger B_L + dQH^\dagger B_L + HH^\dagger B_LW_L \\
&+HH^\dagger B_L^2 + 2QQ^\dagger HH^\dagger\mathcal{D} + 2LL^\dagger HH^\dagger\mathcal{D} + uu^\dagger HH^\dagger\mathcal{D} + ee^\dagger HH^\dagger\mathcal{D} + d^\dagger uH^2\mathcal{D} + du^\dagger H^{\dagger\,2}\mathcal{D} \\
&+dd^\dagger HH^\dagger\mathcal{D} + 2H^2H^{\dagger\,2}\mathcal{D}^2
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Computing the Hilbert Series

Fields Φ_i transform under (Lorentz/gauge) group irreps V_i

$$V_i \otimes V_j = \bigoplus_k V_k$$

F. Peter and H. Weyl, “Die Vollst¨andigkeit der primitiven Darstellungen einer geschlossenen kontinuierlichen Gruppe”, 1927.

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Haar measure

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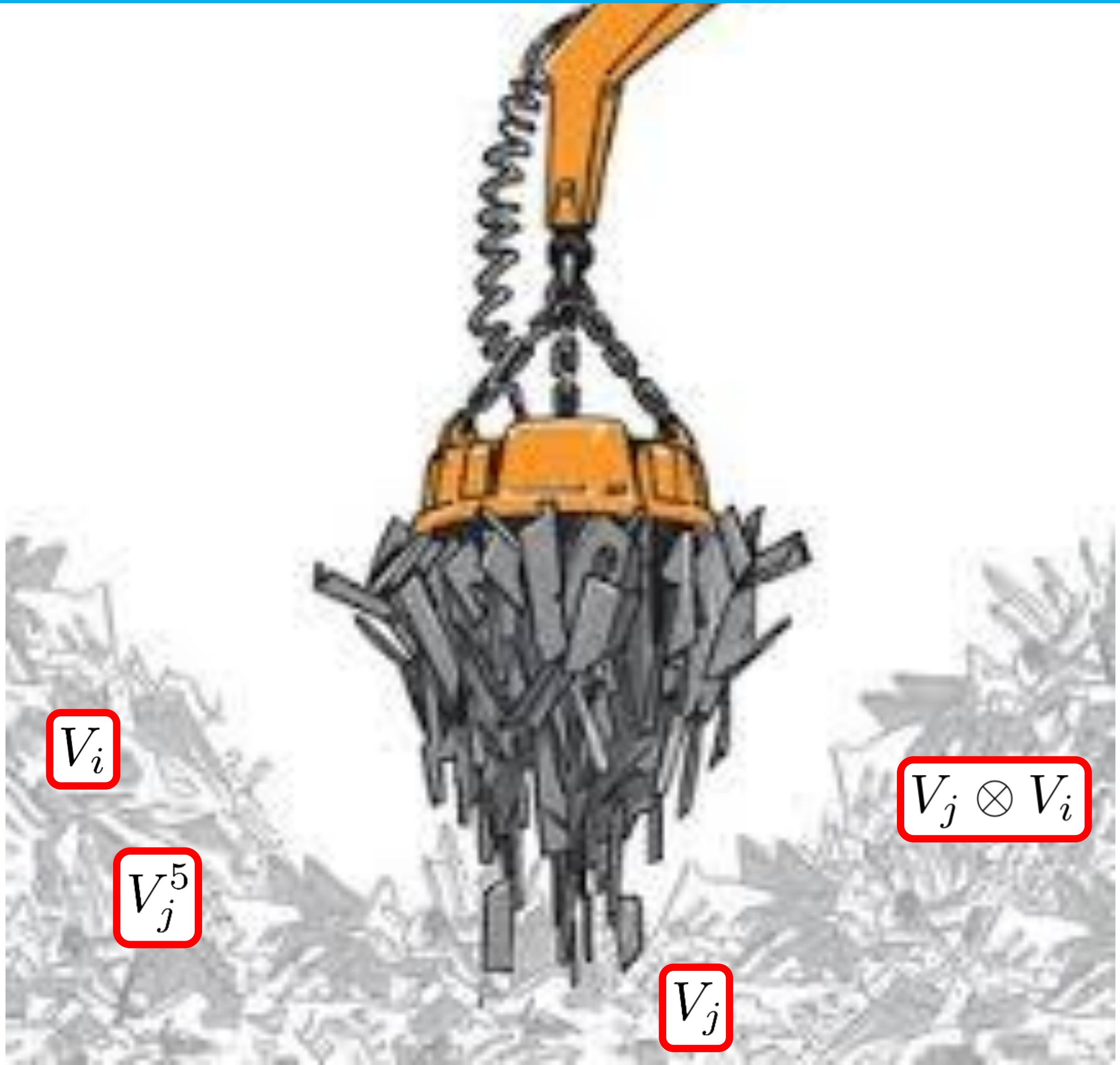
Character orthogonality



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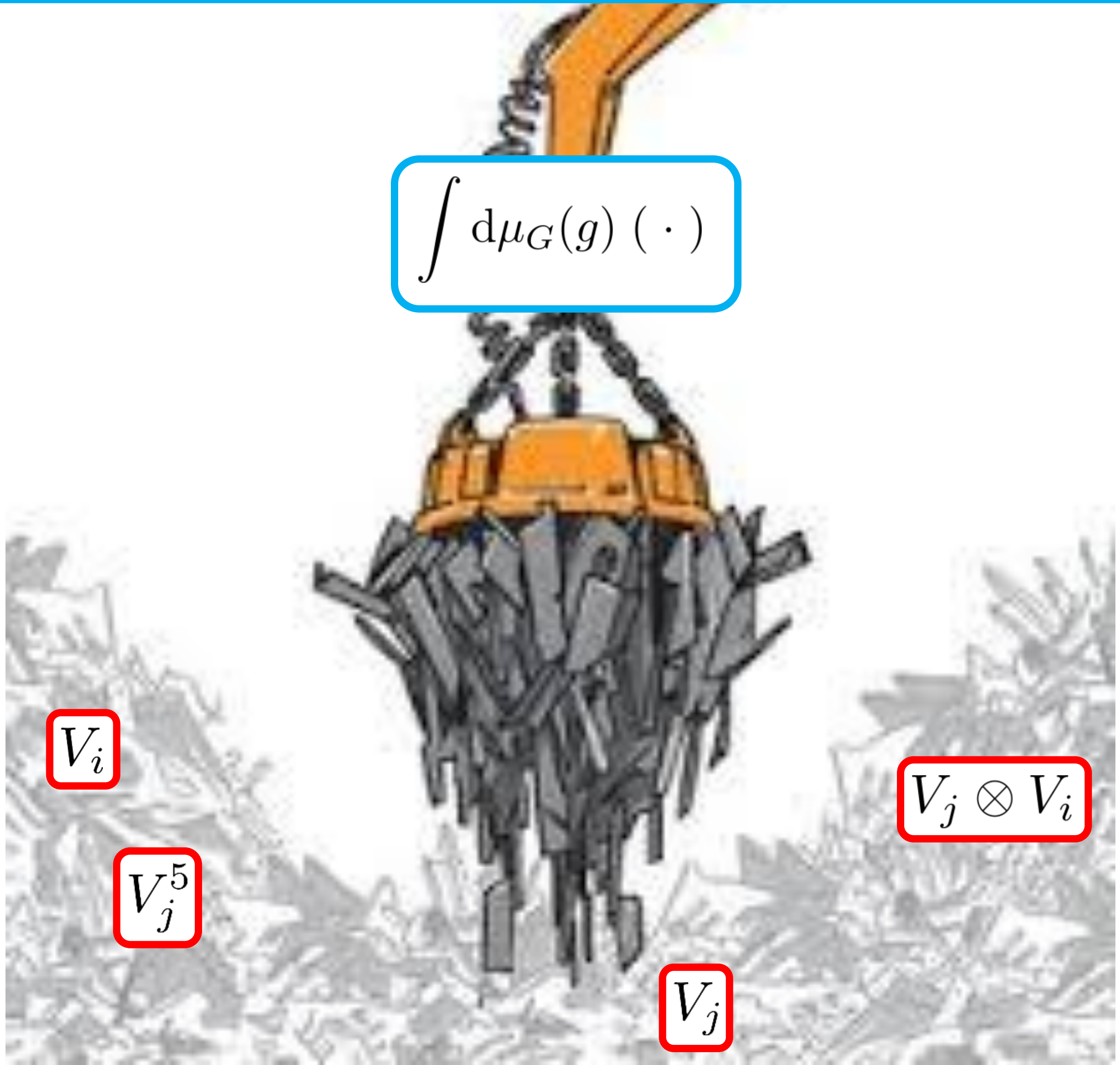

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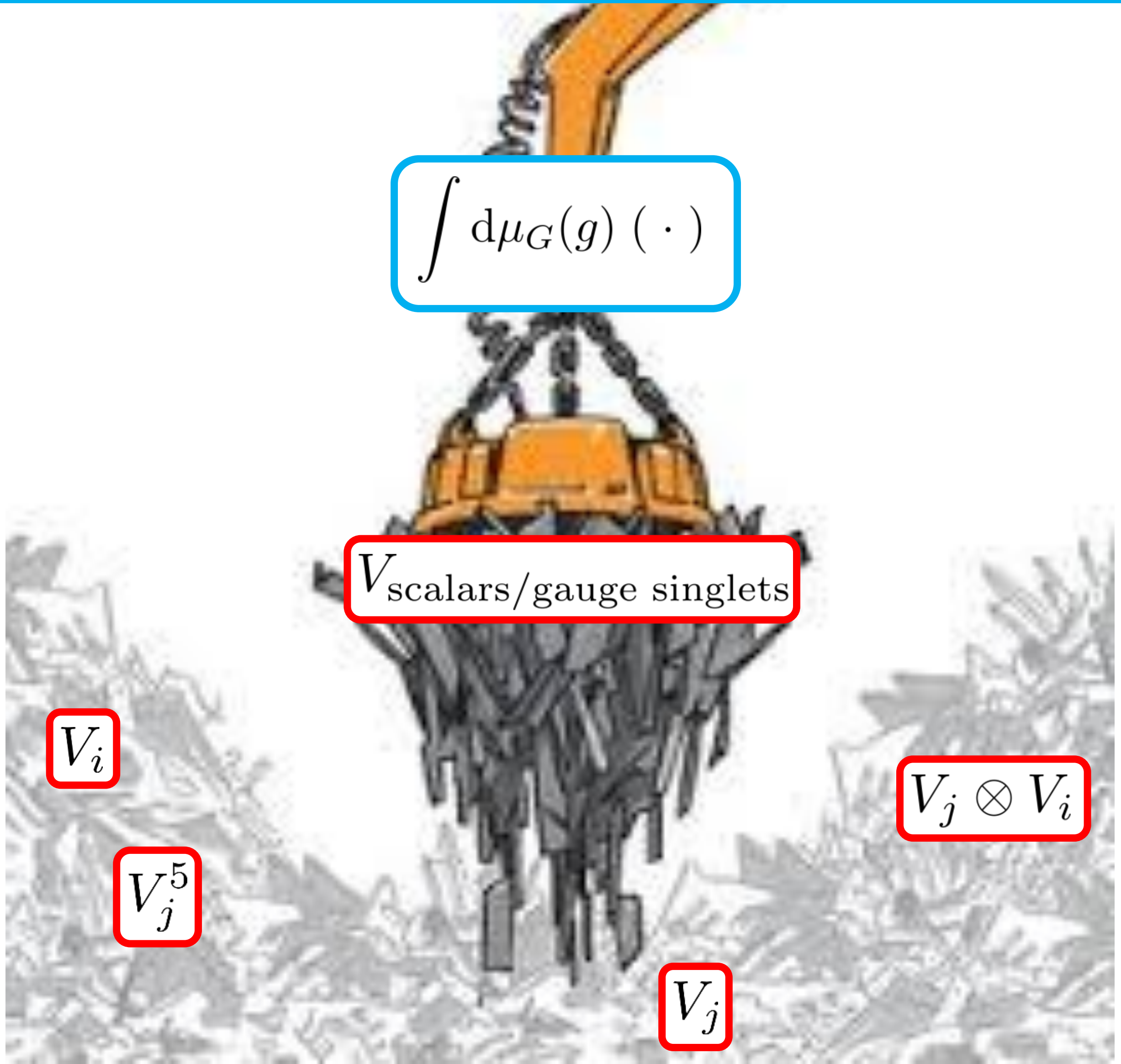

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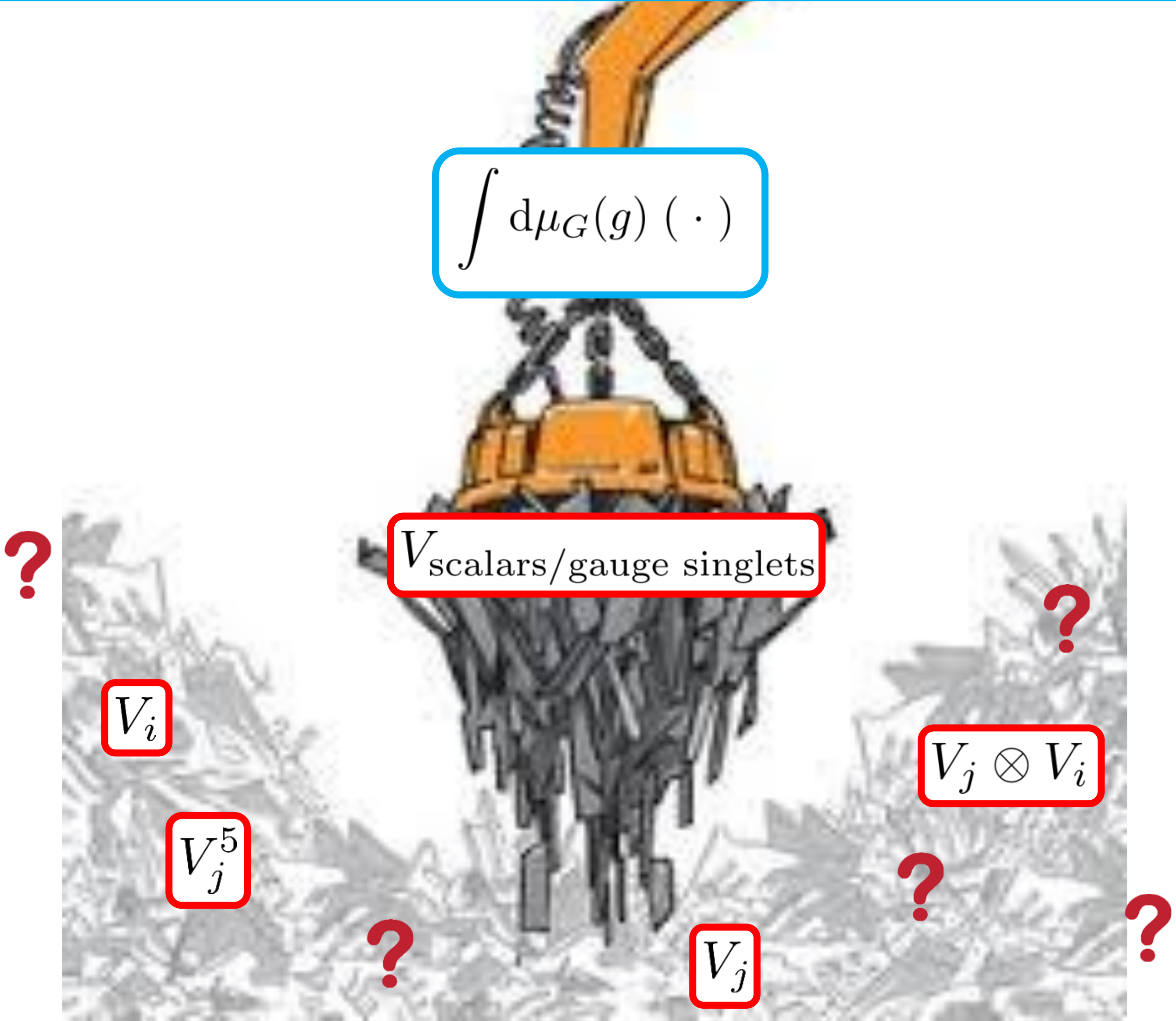

The plethystic exponential

complete operator basis

$\Leftrightarrow$

selecting $V_{\text{scalars/gauge singlets}}$ from a representation that contemplates **ALL** possible **(anti)symmetric tensor powers** of the (fermionic) bosonic field irreps.

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$$\prod_{i=1}^{N} \mathrm{PE}_i[\phi_i \chi_i] \equiv \prod_{i=1}^{N} \sum_{n_i=0}^{\infty} \phi_i^{n_i} \cdot \begin{cases} S^{n_i}(\chi_i), & \text{if } \Phi_i \text{ is a boson} \\ \Lambda^{n_i}(\chi_i), & \text{if } \Phi_i \text{ is a fermion} \end{cases}$$

Projecting out scalars (+ removing IBP)

IBP relations:

total divergences ~ 0

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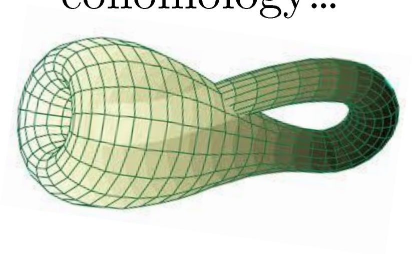
cohomology...



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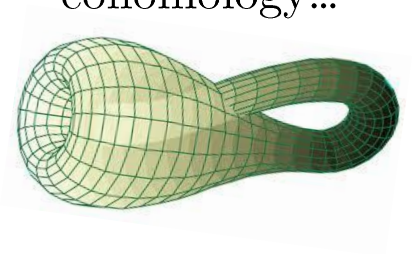

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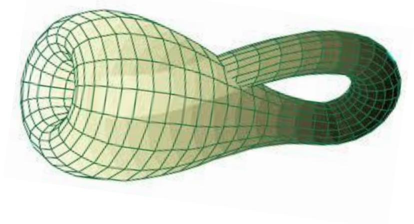

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$$\#(\text{not co-exact 0-forms}) = \sum_{k=0}^{d} (-\mathcal{D})^k \#(k\text{-forms}) - \sum_{k=1}^{d} (-\mathcal{D})^k \#(\text{co-closed not co-exact } k\text{-forms})$$

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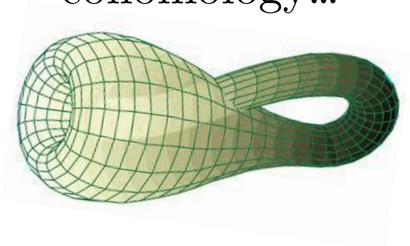

$\Rightarrow$ operator basis consists of **not co-exact 0-forms**

$$\#(\text{not co-exact 0-forms}) \qquad \text{mass dim.} \leq d$$

$$= \sum_{k=0}^{d} (-\mathcal{D})^k \#(k\text{-forms}) - \sum_{k=1}^{d} (-\mathcal{D})^k \#(\text{co-closed not co-exact } k\text{-forms})$$

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Further projecting out scalars (+removing IBP)

IBP relations:
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with cohomology...
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operator basis consists of
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$$\#(\text{not co-exact 0-forms}) \approx \sum_{k=0}^{d} (-\mathcal{D})^k \#(k\text{-forms})$$

A $k-$form transforms as $\Lambda^k \Box$ under $SO(d)$:

$$H(\mathcal{D}, \{\phi_i\}) = \int d\mu_{G_{\text{gauge}}} \int d\mu_{SO(d)} \underbrace{\sum_{k=0}^{d} (-\mathcal{D})^k \overline{\chi_{\Lambda^k \Box}}}_{\equiv \frac{1}{P}} \prod_{i=1}^{N} \text{PE}_i[\phi_i \chi_i]$$

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Removing EOM redundancies

Single Particle Module (SPM)

$$R_\Phi = \begin{pmatrix} \Phi \\ \partial_\mu \Phi \\ \partial_{\{\mu_1}\partial_{\mu_2\}}\Phi \\ \vdots \end{pmatrix}$$

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distinct interpolating fields for an asymptotic one-particle state

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$$\chi_{R_\Phi} ???$$

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Single Particle Module (SPM)

$$R_\Phi = \mathrm{Ind}_{SO(3)}^{SO(4)}$$

Ind_H^G: generates G reps. from $H \subset G$ irreps.

L. Gráf, B. Henning, X. Lu, T. Melia, H. Murayama. "Hilbert Series, the Higgs Mechanism, and HEFT", 2022.

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$$\Rightarrow \chi_{R_\Phi} = \sum_{n=0}^{\infty} \sum_{m=-k}^{k} \mathcal{D}^{\Delta_{m,n}} \chi_{(k+n,m)}$$

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$\Delta_{m,n}$ → smallest j such that $\partial^j \Phi \supset (k+n,m)$

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Higher-spin characters

spin	$s = 3/2$
field	ψ_μ (Rarita-Schwinger)
Lorentz rep. - $SU(2) \times SU(2)$	$\left(1, \frac{1}{2}\right) \oplus \left(\frac{1}{2}, 1\right)$
Lorentz rep. - $SO(4)$	$\left(\frac{3}{2}, \frac{1}{2}\right) \oplus \left(\frac{3}{2}, -\frac{1}{2}\right)$
SPM character	$P(1-\mathcal{D})\left[\chi_{(3/2,1/2)} + \chi_{(3/2,-1/2)} - \mathcal{D}\left(\chi_{(1/2,1/2)} + \chi_{(1/2,-1/2)}\right)\right]$

Higher-spin characters

spin	$s = 2$
field	$h_{\mu\nu}$
Lorentz rep. - $SU(2) \times SU(2)$	$(1, 1)$
Lorentz rep. - $SO(4)$	$(2, 0)$
SPM character	$P(1 - \mathcal{D}^2)(\chi_{(2,0)} - \mathcal{D}\chi_{(1,0)})$

Counting the effective operators

$$3H^\dagger L^\dagger \psi_\mu \mathcal{D} + 3HL\psi_\mu \mathcal{D} + 2H^\dagger H \psi_\mu^2$$

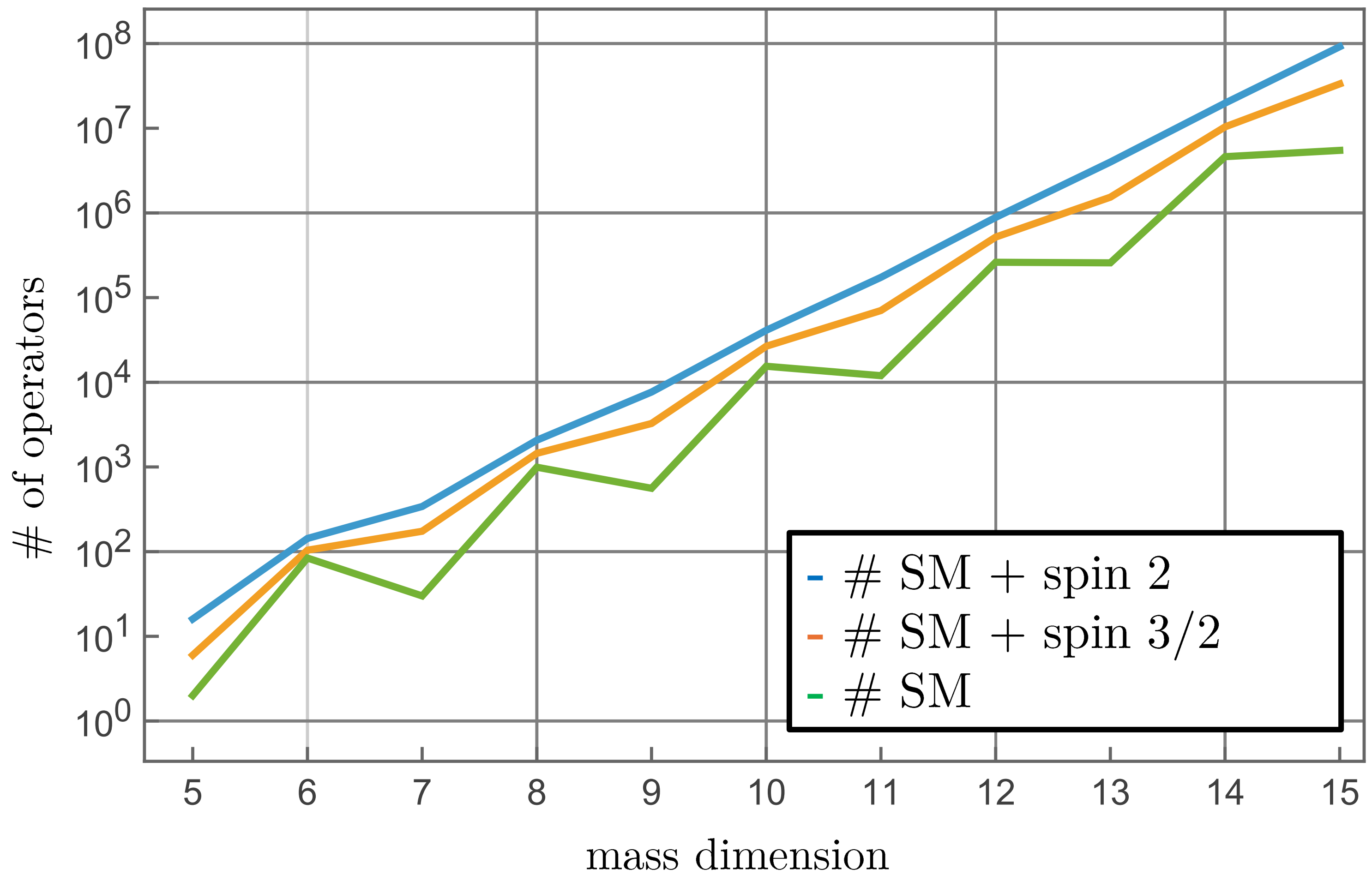
dim 5, spin 3/2
Total: 8

$$3BH^\dagger \psi_\mu L^\dagger + 3BHL\psi_\mu + 27eud^\dagger \psi_\mu + 9dd^\dagger \psi_\mu^2 + 9d^\dagger \psi_\mu \left(Q^\dagger\right)^2 + 27de^\dagger \psi_\mu u^\dagger + 9dQ^2\psi_\mu + 3\mathcal{D}e^\dagger \left(H^\dagger\right)^2 \psi_\mu + 9ee^\dagger \psi_\mu^2 + 3eH^2\mathcal{D}\psi_\mu + 3WH^\dagger \psi_\mu L^\dagger + H\mathcal{D}H^\dagger \psi_\mu^2 + 3HLW\psi_\mu + 9L\psi_\mu^2 L^\dagger + 27Q\psi_\mu L^\dagger u^\dagger + 27Lu\psi_\mu Q^\dagger + 9Q\psi_\mu^2 Q^\dagger + 9u\psi_\mu^2 u^\dagger + 4\psi_\mu^4$$

dim 6, spin 3/2
Total: 194

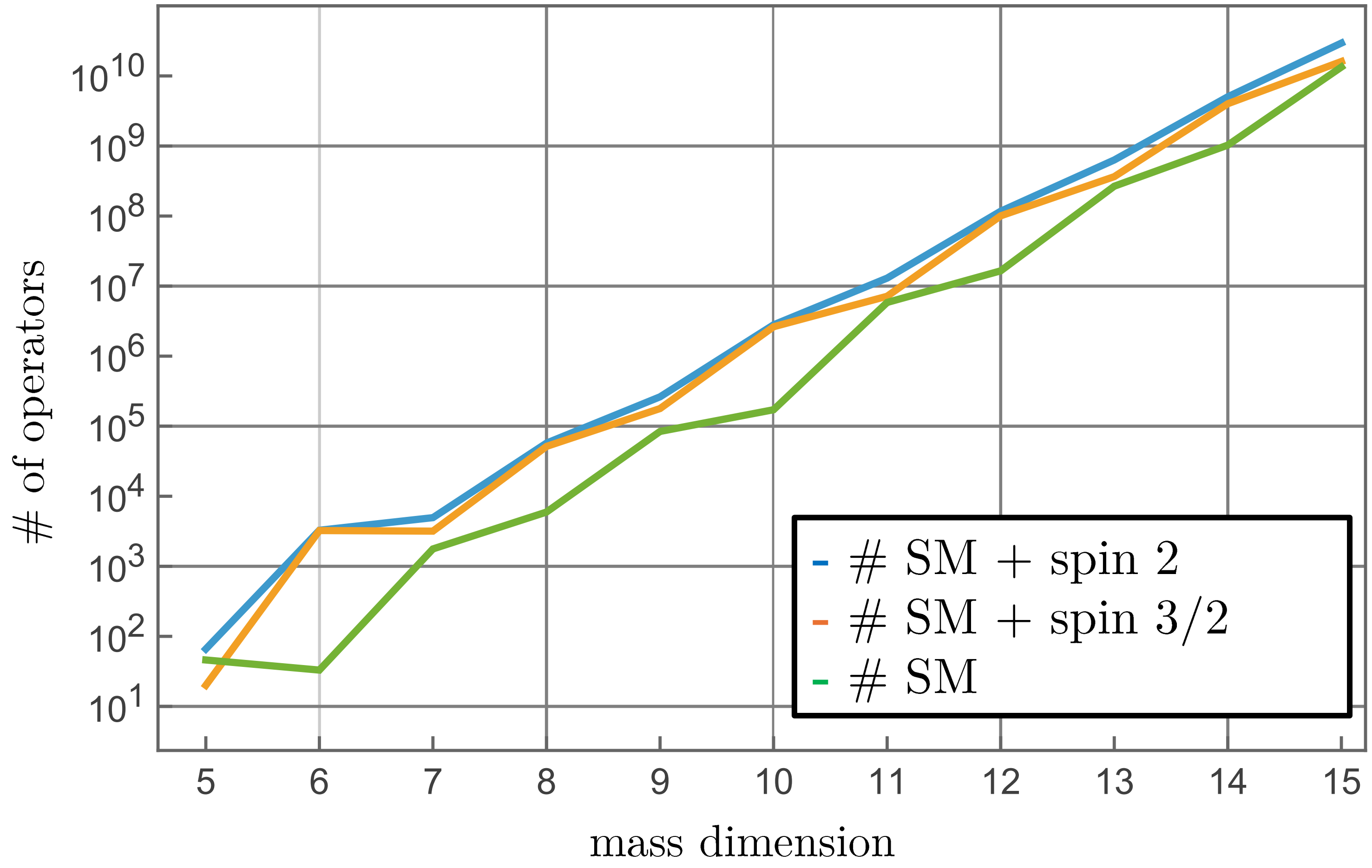
C. B. Marinissen, R. Rahn, W. J. Waalewijn. "Efficient Hilbert Series for Effective Theories", 2020.

Operator growth (Nf = 1)



C. B. Marinissen, R. Rahn, W. J. Waalewijn. “Efficient Hilbert Series for Effective Theories”, 2020.

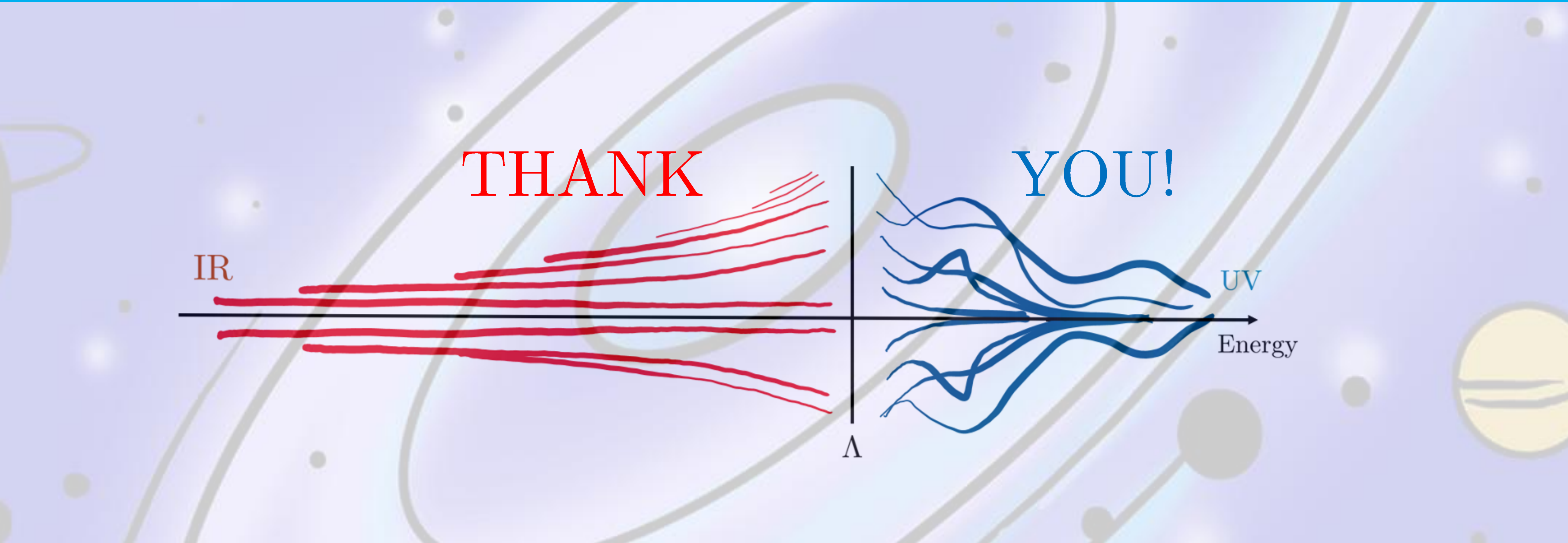
Operator growth (Nf = 3)



C. B. Marinissen, R. Rahn, W. J. Waalewijn. "Efficient Hilbert Series for Effective Theories", 2020.

Conclusions

- Compelling evidence for DM and a plethora of models to describe it;
- EFTs is a powerful model independent tool;
- Hilbert Series can address IBP and EOM redundancies;
- Higher-spin fields can be embedded into the Hilbert Series formalism;
- Higher-spin fields => more operators allowed.


THANK
YOU!
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EXTRA

Counting the effective operators

$$
\begin{aligned}
&3H^\dagger L^\dagger \psi_\mu \mathcal{D} \\
&+3HL\psi_\mu \mathcal{D} \\
&+2H^\dagger H \psi_\mu^2
\end{aligned}
$$

dim 5, spin 3/2

$$
\begin{aligned}
&3h_{\mu\nu}^3 \mathcal{D}^2 + h_{\mu\nu}^5 \\
&+ h_{\mu\nu}(G^2 + W^2 + B^2) \\
&+ 9h_{\mu\nu}\mathcal{D}(e^\dagger e + L^\dagger L + d^\dagger d + u^\dagger u + q^\dagger q) \\
&+ h_{\mu\nu}\mathcal{D}^2 H^\dagger H \\
&+ h_{\mu\nu}^3 H^\dagger H
\end{aligned}
$$

dim 5, spin 2

C. B. Marinissen, R. Rahn, W. J. Waalewijn. "Efficient Hilbert Series for Effective Theories", 2020.

Counting the effective operators

$$3BH^\dagger\psi_\mu L^\dagger + 3BHL\psi_\mu + 27eud^\dagger\psi_\mu$$
$$+9dd^\dagger\psi_\mu^2 + 9d^\dagger\psi_\mu\left(Q^\dagger\right)^2 + 27de^\dagger\psi_\mu u^\dagger$$
$$+9dQ^2\psi_\mu + 3\mathcal{D}e^\dagger\left(H^\dagger\right)^2\psi_\mu + 9ee^\dagger\psi_\mu^2$$
$$+3eH^2\mathcal{D}\psi_\mu + 3WH^\dagger\psi_\mu L^\dagger + H\mathcal{D}H^\dagger\psi_\mu^2$$
$$+3HLW\psi_\mu + 9L\psi_\mu^2 L^\dagger + 27Q\psi_\mu L^\dagger u^\dagger$$
$$+27Lu\psi_\mu Q^\dagger + 9Q\psi_\mu^2 Q^\dagger + 9u\psi_\mu^2 u^\dagger + 4\psi_\mu^4$$

dim 6, spin 3/2

$$5B^2h_{\mu\nu}^2 + 9Qd^\dagger h_{\mu\nu}^2 H^\dagger + 27d\mathcal{D}d^\dagger h_{\mu\nu}^2$$
$$+9dHh_{\mu\nu}^2 Q^\dagger + 9Le^\dagger h_{\mu\nu}^2 H^\dagger + 27e\mathcal{D}e^\dagger h_{\mu\nu}^2$$
$$+9eHh_{\mu\nu}^2 L^\dagger + 5G^2h_{\mu\nu}^2 + 2Hh_{\mu\nu}^4 H^\dagger$$
$$+6H\mathcal{D}^2h_{\mu\nu}^2 H^\dagger + 9uh_{\mu\nu}^2 H^\dagger Q^\dagger$$
$$+9HQh_{\mu\nu}^2 u^\dagger + 27L\mathcal{D}h_{\mu\nu}^2 L^\dagger + 3h_{\mu\nu}^6$$
$$+11\mathcal{D}^2h_{\mu\nu}^4 + 27\mathcal{D}Qh_{\mu\nu}^2 Q^\dagger + 27\mathcal{D}uh_{\mu\nu}^2 u^\dagger$$
$$+5W^2h_{\mu\nu}^2 + H^2h_{\mu\nu}^2\left(H^\dagger\right)^2$$

dim 6, spin 2

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