Particle Accelerators - Part 3

DESY Summer Student Lectures 2025

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HELMHOLTZ

DESY.

4 Lectures

Yesterday

Part 1: General introduction:

- What are particle accelerators?
- Why and where do we need them?
- What types do exist?

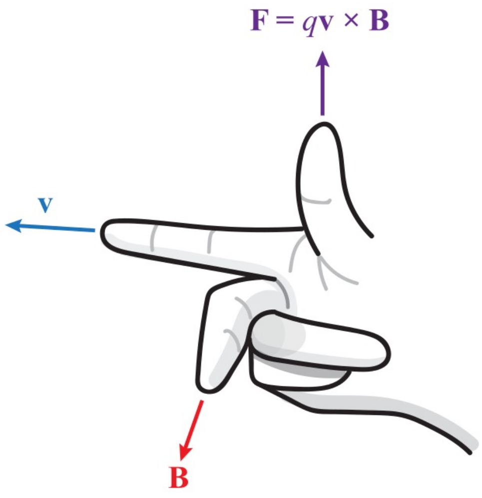
Part 2: Accelerator Technology (Gregor Loisch)

Today

Part 3: How to build a particle accelerator

Part 4: What Users need and how to deliver

A *charged* particles that travels through an electro-magnetic field feel the Lorentz force



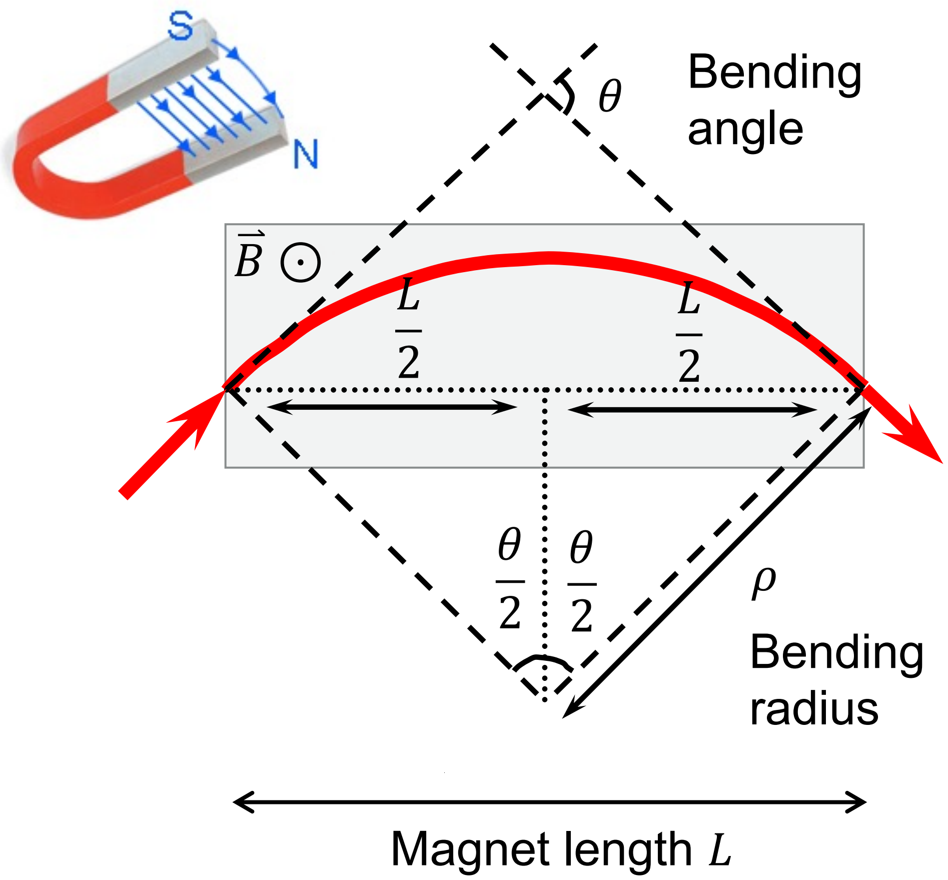
$$\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$$

Magnetic field *B*:
Force acts perpendicular to path.
→ **Can change direction of particle**
→ cannot accelerate

Electric field E:
Force acts parallel to path.
→ **Can accelerate**
→ not optimal for deflection

Charged Particles are Deflected in a Magnetic Field

With a bending radius proportional to its momentum



The ideal circular orbit

Lorentz Force $\quad F_L = q\, v\, B$

Centrifugal Force $\quad F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$

$$F_L = F_{centr}$$

$$\frac{p}{q} = B\,\rho$$

$B\,\rho =$ **Beam rigidity**

Source gif: http://www.lhc-facts.ch

q charge

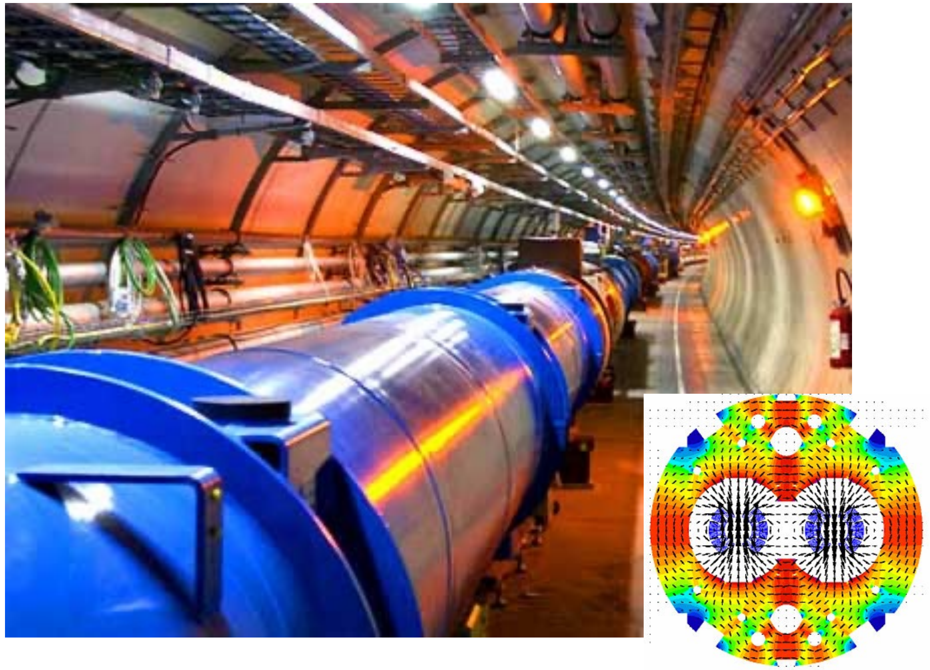
$p = \gamma\, m_0 v$ momentum

B magn. field strength

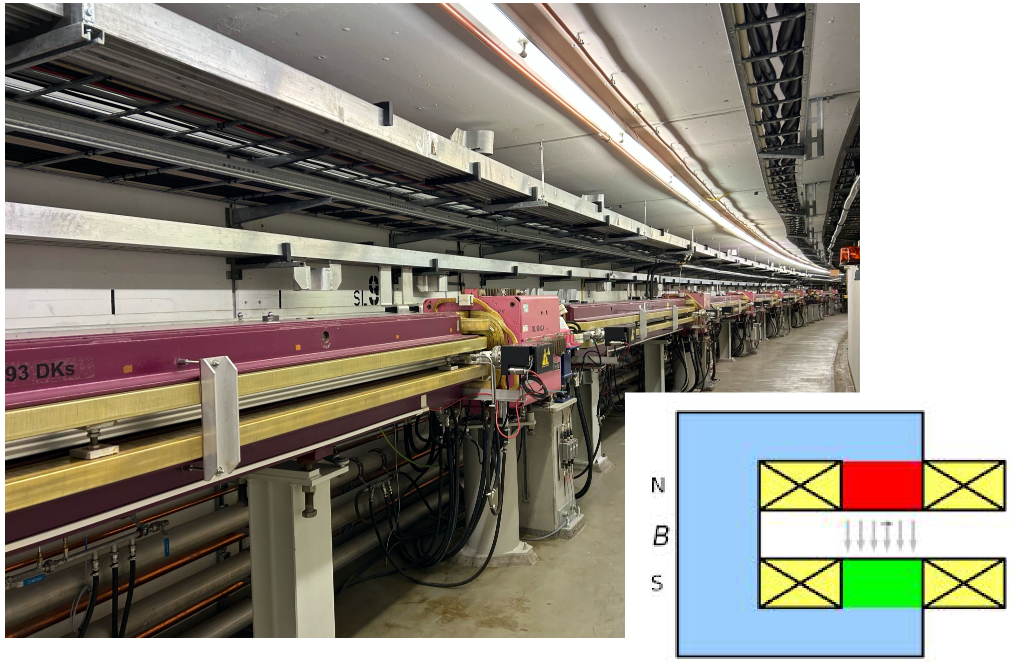
ρ bending radius

Bending (Dipole) Magnets – Keep Particles on Circular Orbit

Vertical magnetic field to bend in horizontal plane.



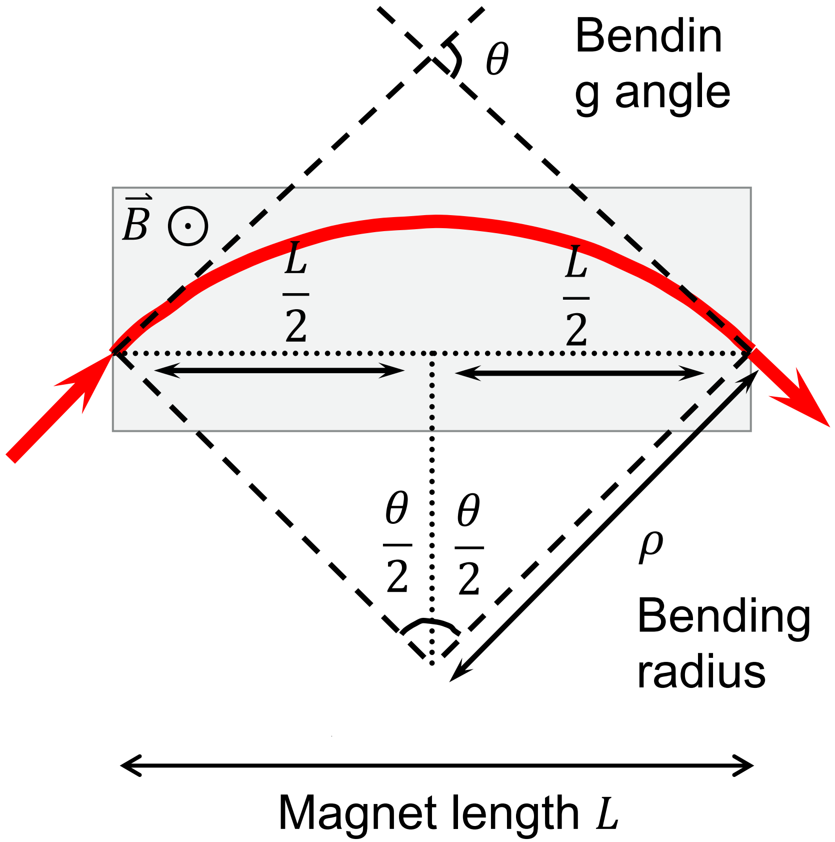
LHC has 1232 superconducting dipole magnets, each 15m long and able to deflect the beam by 0.29^{o}.

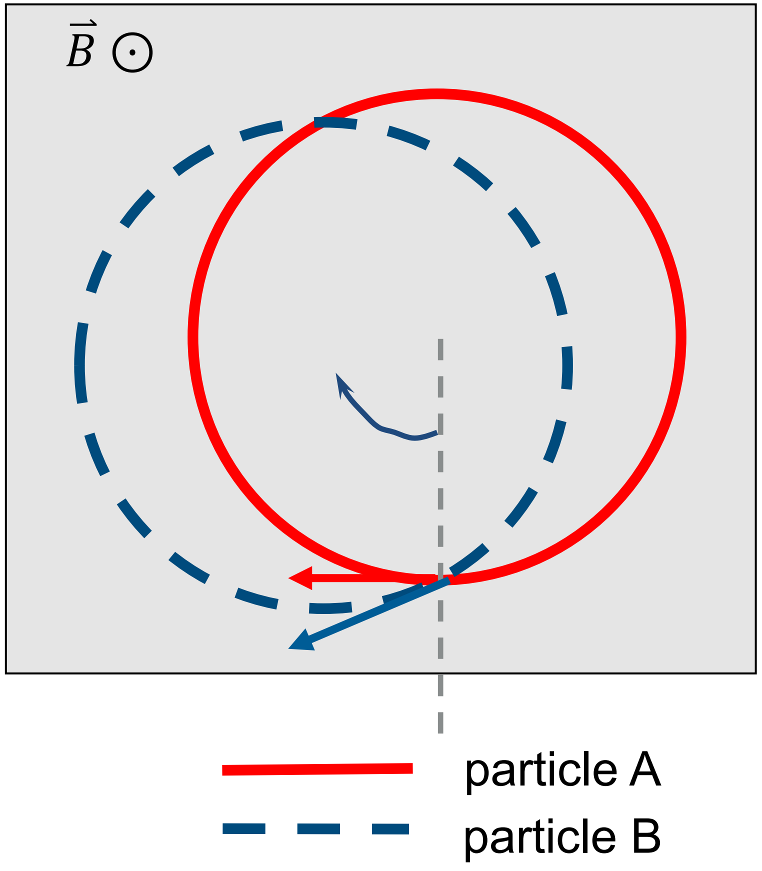


PETRA III has about 200 5.6m-long arc dipole magnets, each deflecting the beam by 1.6^{o} *(experimental halls have different lattice design)*

Two equal charged particles draw the same circle in an homogeneous magnetic field

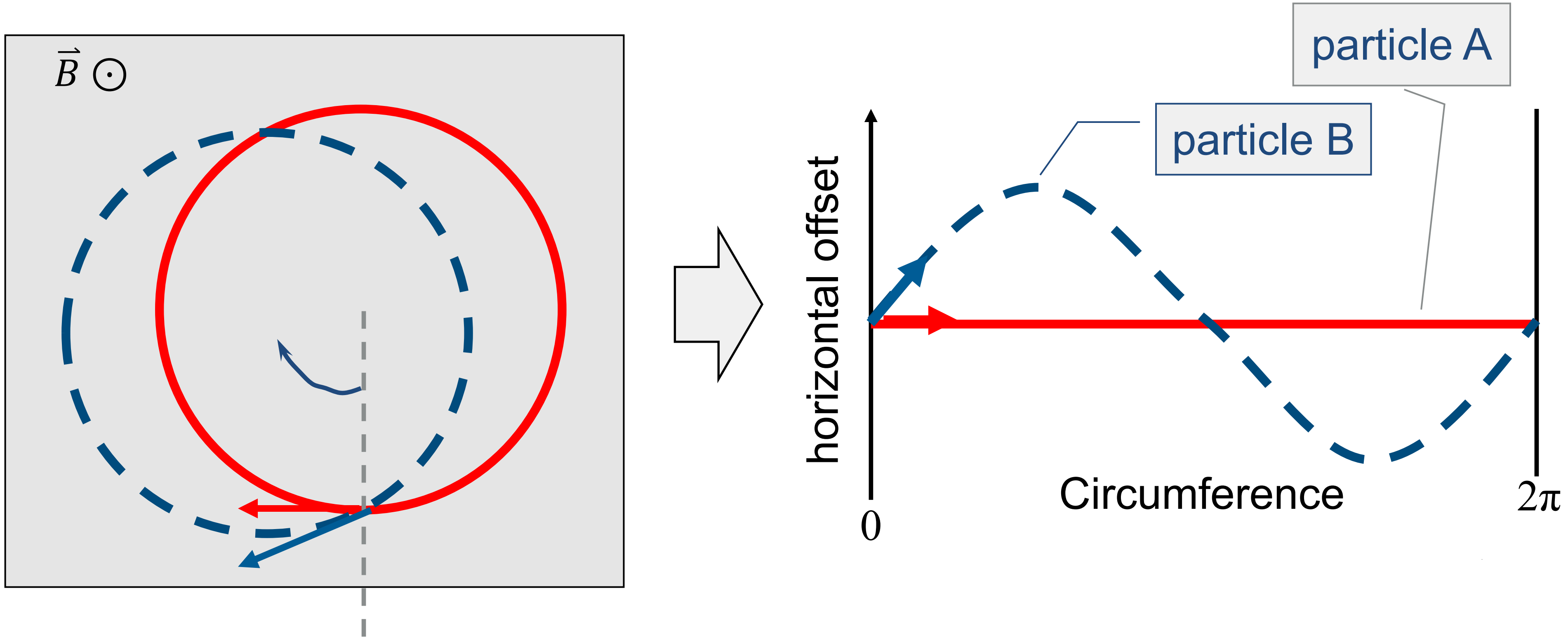
BUT: Variation of the initial angle & position change the position of the circle in the plane





Particles Oscillate Around the Design Orbit

horizontal movement

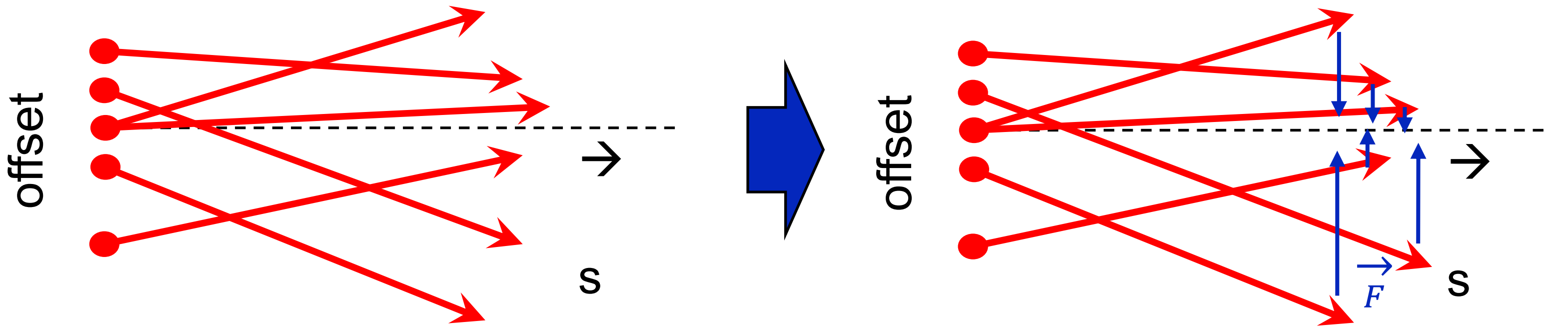


In an homogeneous magnetic field, particles with varying initial conditions fulfil oscillations around the design orbit → **Betatron-Oscillation**

design orbit = trajectory of ideal particle → defined by dipole magnets

A bunch contains many particles with different initial conditions

Focusing is needed to keep the particles close to the design orbit



Many different positions, angles and energy offsets

We need a focusing force that keeps the particles close to the design orbit.

Focusing force should rise as a function of the distance to the design orbit.

Linearly increasing magnetic field with distance from design orbit

Focusing of particles with quadrupoles

$$F(x) = q \cdot v \cdot B(x)$$

with the vertical (y) and horizontal (x) quadrupole fields

$$B_y = g \cdot x$$
$$B_x = g \cdot y$$

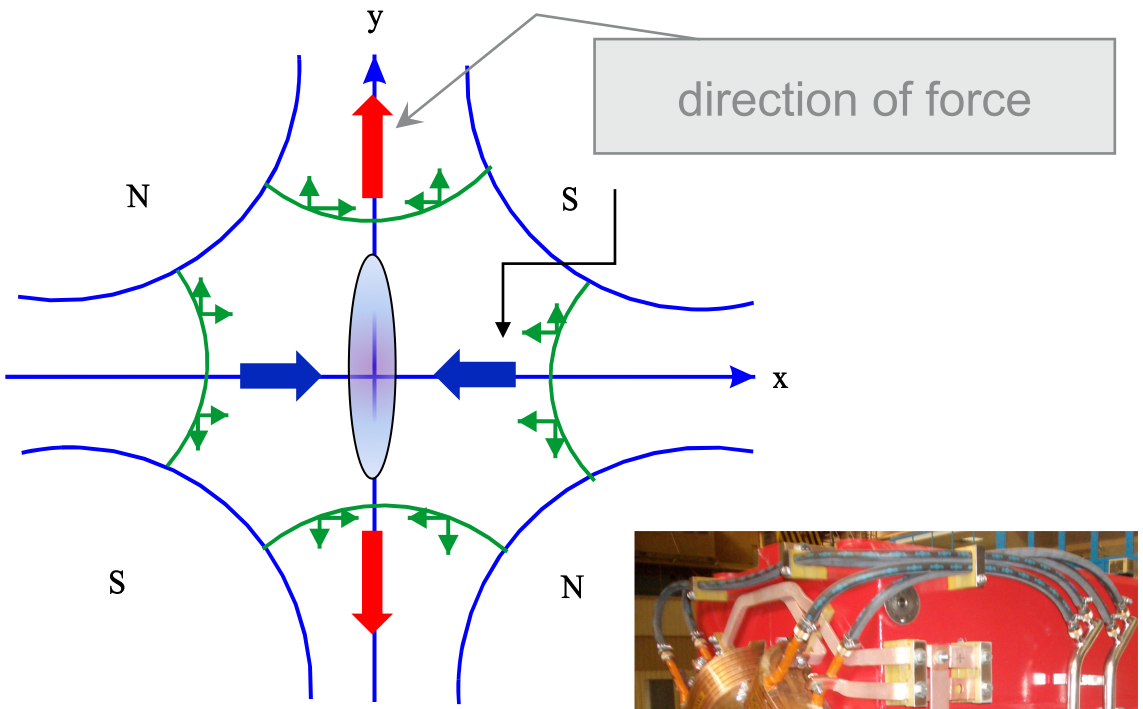
where g is the gradient

$$g = \frac{2\mu_0 n I}{r^2}\left[\frac{T}{m}\right]$$

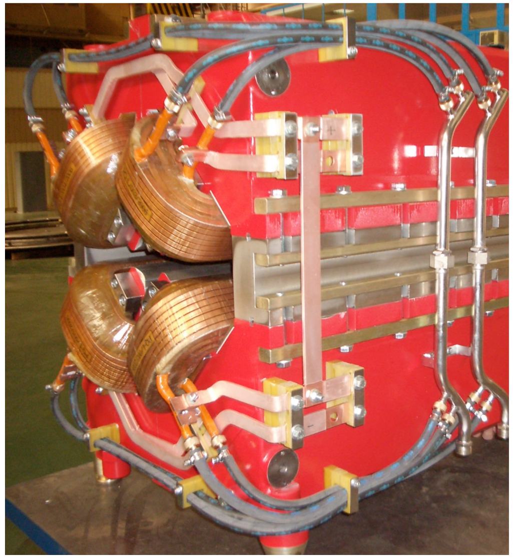
Normalized gradient = focusing strength

$$k = \frac{g}{p/q}\left[m^{-2}\right]$$

I coil current
n number of windings
r distance magnet center to pole
μ_0 permeability of free space



Quadrupoles focus in one plane, but defocus in the other!



quadrupole magnet

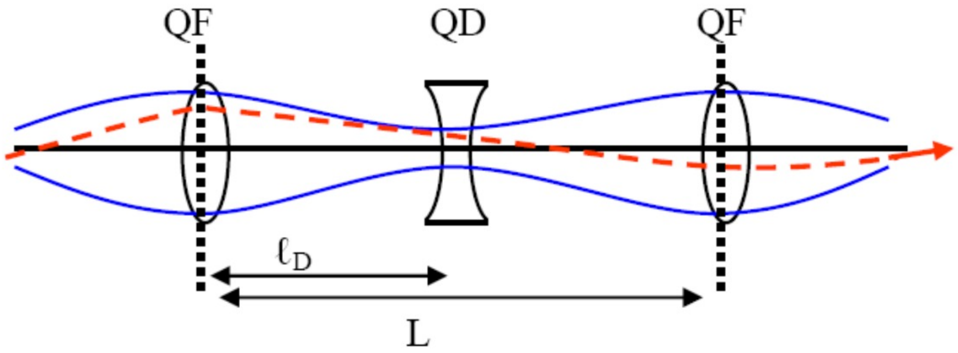
Focusing of particles with quadrupoles is similar to focusing of light with lenses.

A series of alternating focusing and defocusing lenses will focus:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

In a synchrotron quadrupoles are lenses with the focal length:

$$f = \frac{1}{k \cdot l_Q}$$



Consider:

$$f_1 = f$$
$$f_2 = -f$$

Then:

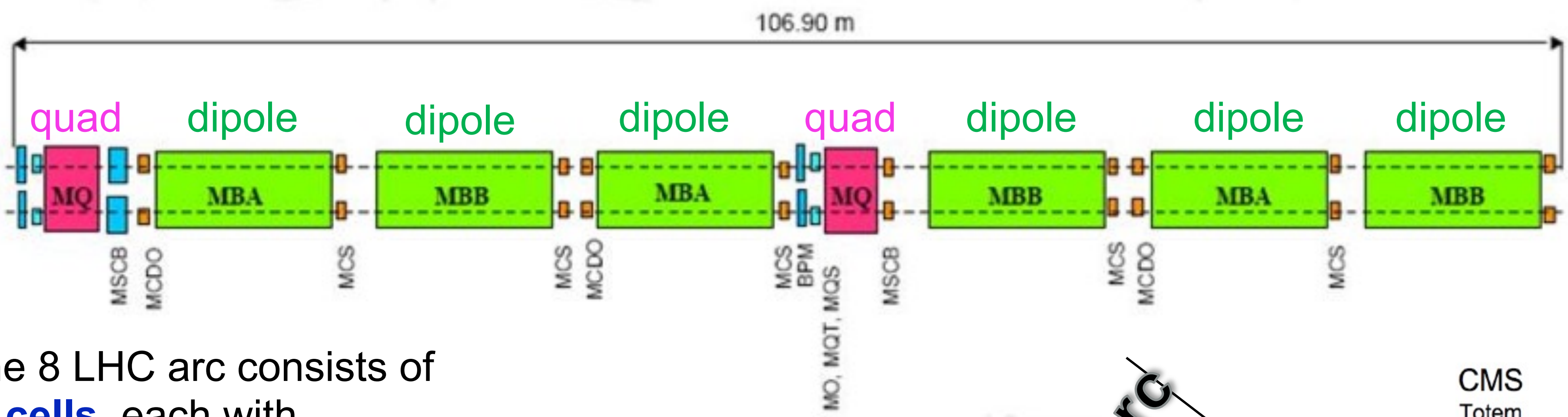
$$F = \frac{f^2}{d} > 0$$

Typical alternating

F = focusing
0 = nothing (dipole, RF, …)
D = defocusing
0

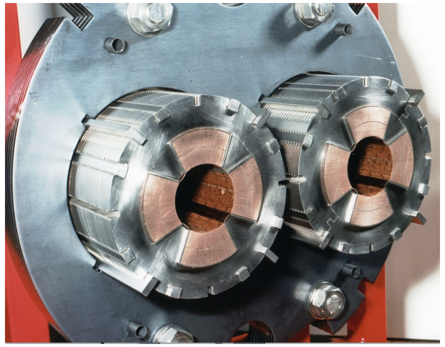
lattice of quadrupoles in an accelerator

The LHC FODO cell



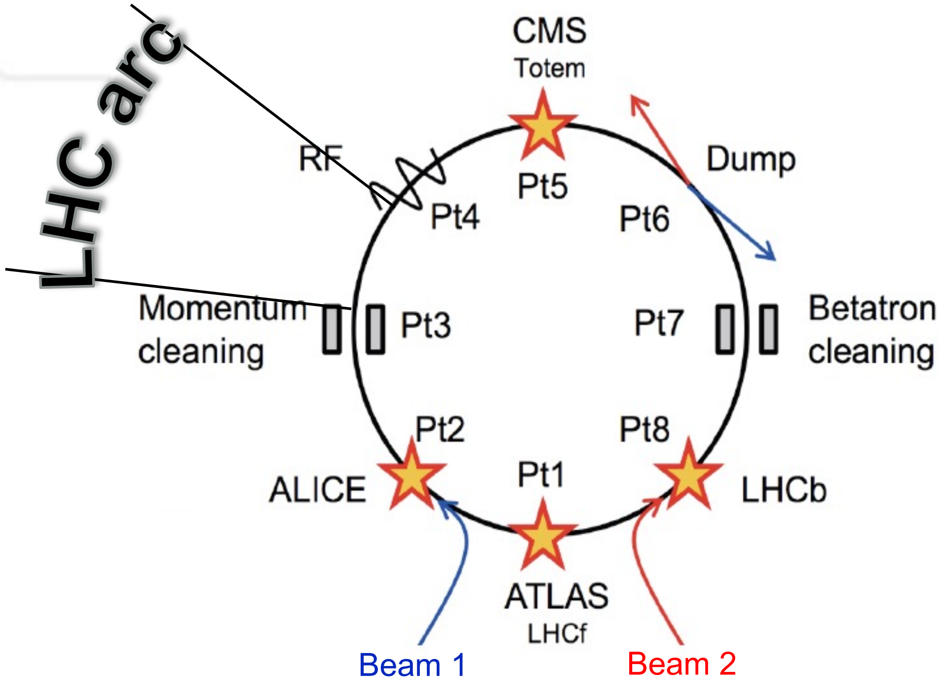
Each of the 8 LHC arc consists of
23 FODO cells, each with

- **2 Quadrupoles**
- **6 Dipoles**
- Additional instrumentation and corrector magnets are installed in between for beam control.



LHC quadrupole

A focusing magnet for Beam 1 is a defocusing for Beam 2 in the same plane.

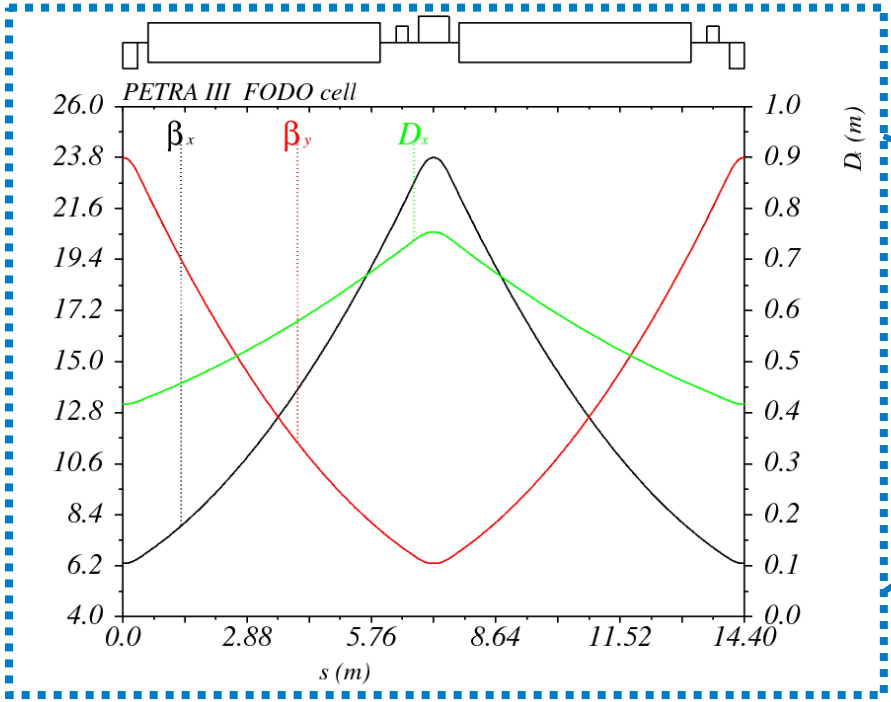


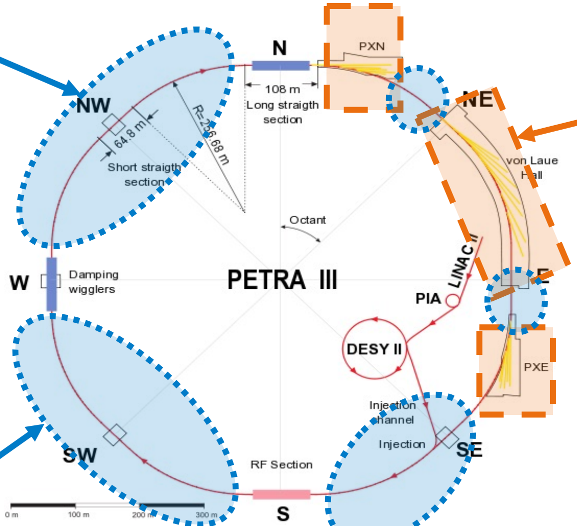
Hybrid Lattice in PETRA III

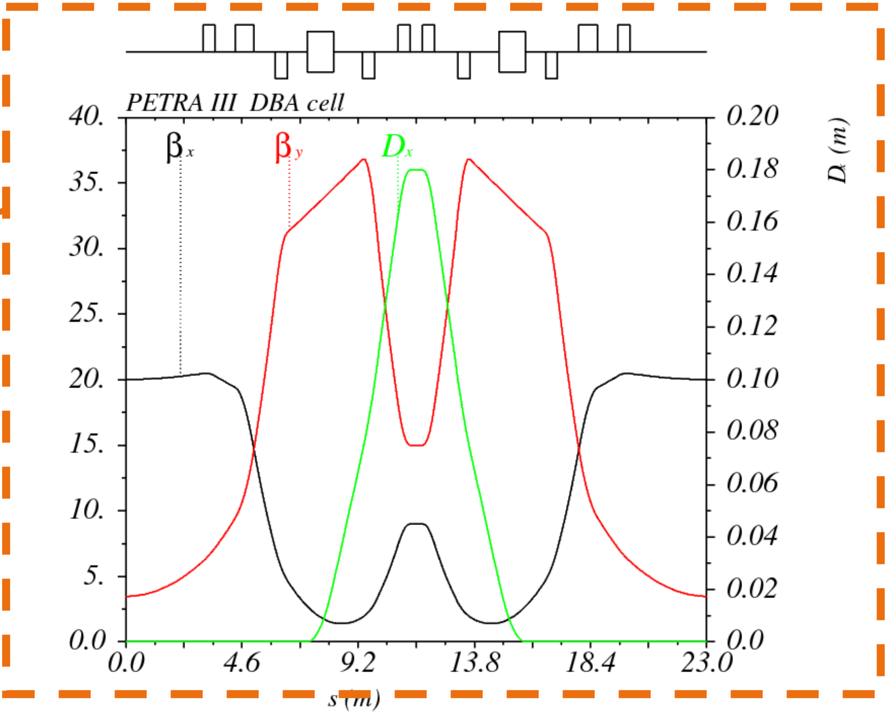
FODO in standard arcs, Double Bend Achromat (DBA) lattice in experimental halls

FODO Arc:
- 14 cells (14.4 m)
- 28 dipoles (28 mrad, L = 5.35 m)

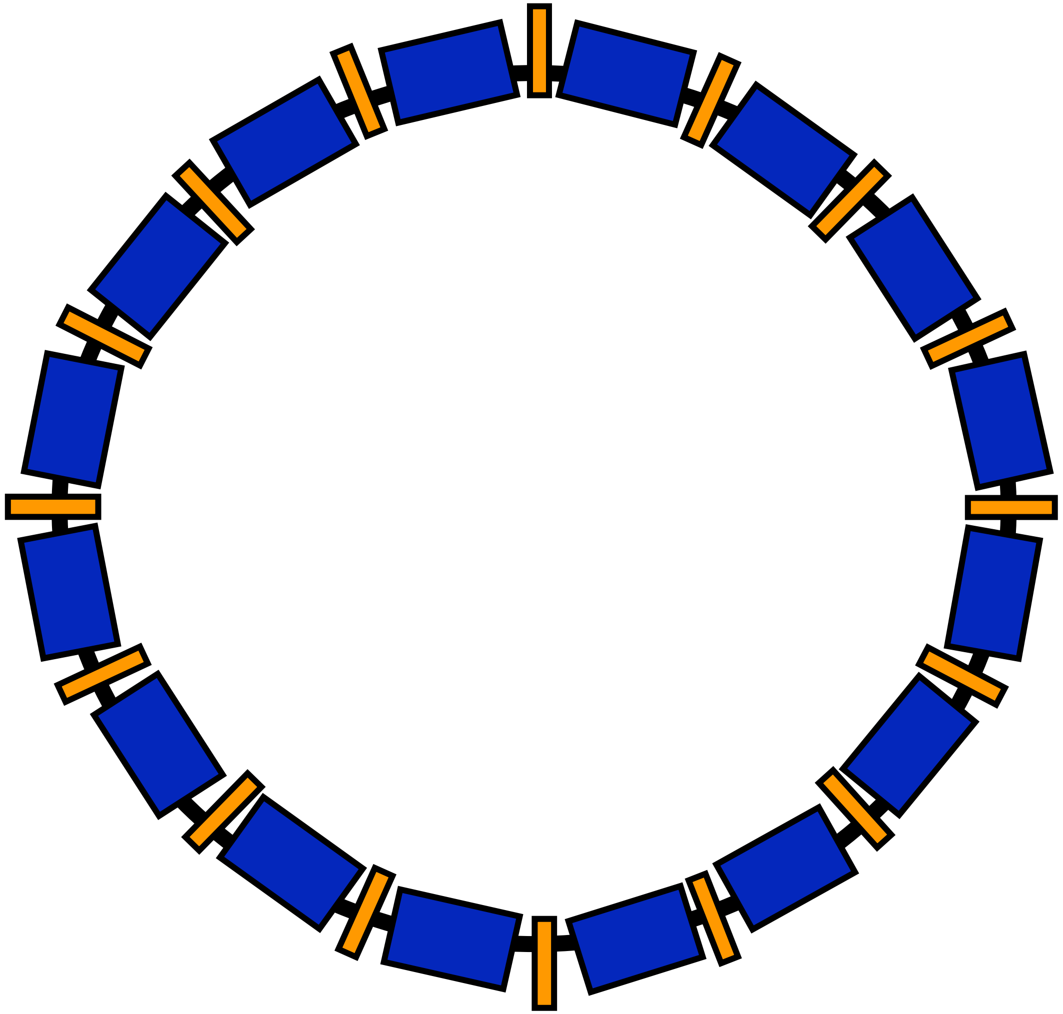
DBA octant (Max von Laue Hall):
- 8 cells (23 m)
- 18 dipoles (43.3 mrad, L = 1m)
- \+ 5 canting dipoles (5 mrad, L = 0.3 m)







How does a particle move in an accelerator?



Particle Motion

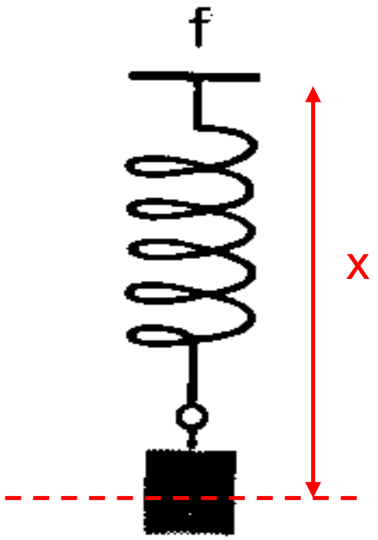
Focusing force that keeps the particles close to the design orbit, which rises as a function of the distance.

$$F(x) = q \cdot v \cdot B(x)$$

Classical free harmonic oscillator

→ restoring force proportional to the displacement x
when displaced from equilibrium position

Second law of motion: $\vec{F} = m\vec{a} \Rightarrow m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = m\ddot{x} = -kx$



Solution of equation of motion is sinusoidal oscillation: $x(t) = A\cos(\omega t + \varphi)$

Coordinate System follows the particles trajectory

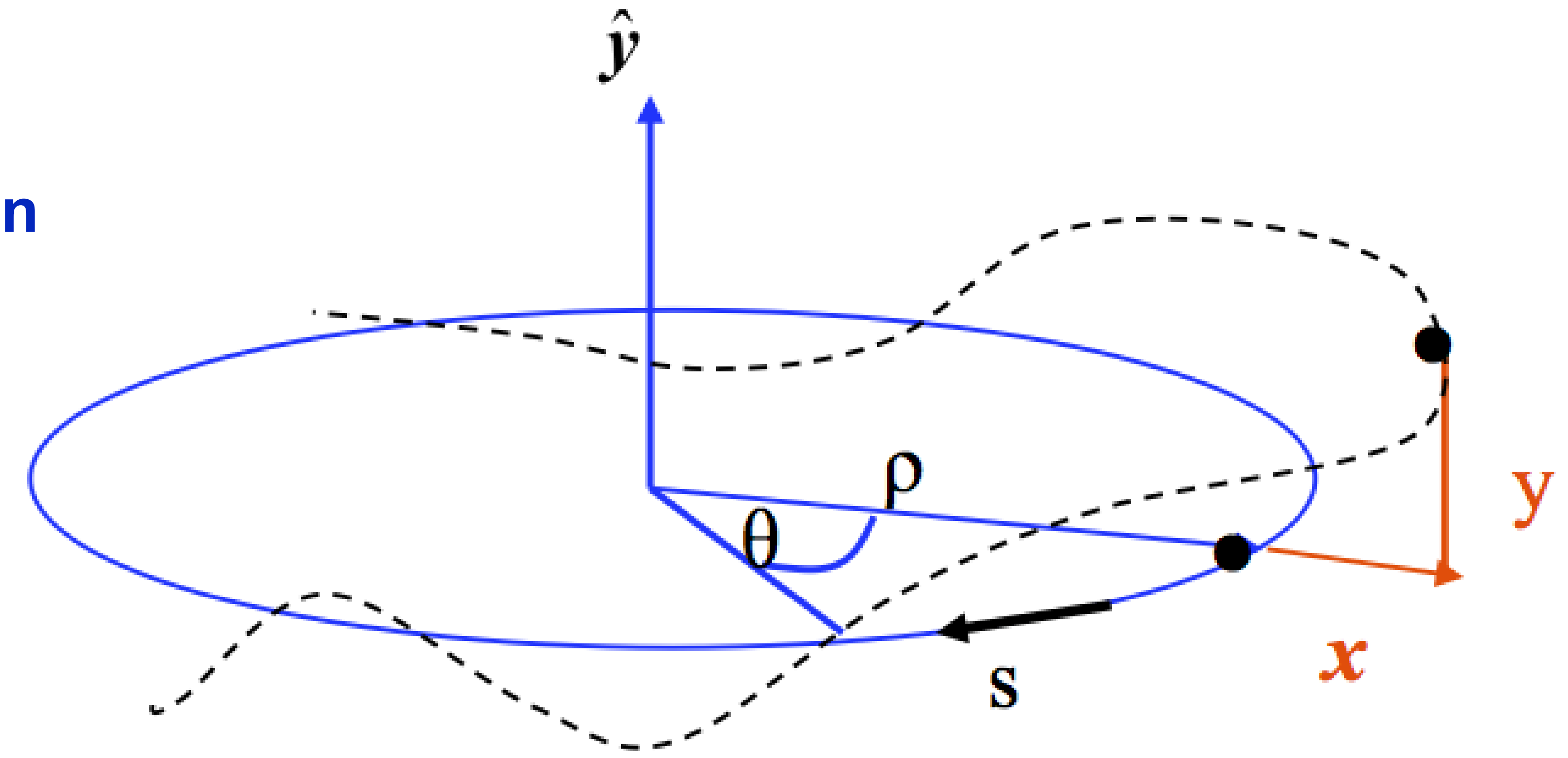
Frenet-Serret rotating frame

Assumptions:

- The ideal particle defines the “design” trajectory: ***x=0, y=0***
  → *travels through the center of all magnets.*

- *x, y* << ρ

Look at the particle motion along the path length *s*.



x = particle amplitude
x’ = angle of particle trajectory (wrt ideal path line)

Equation of Motion

Assuming uncoupled horizontal and vertical motion

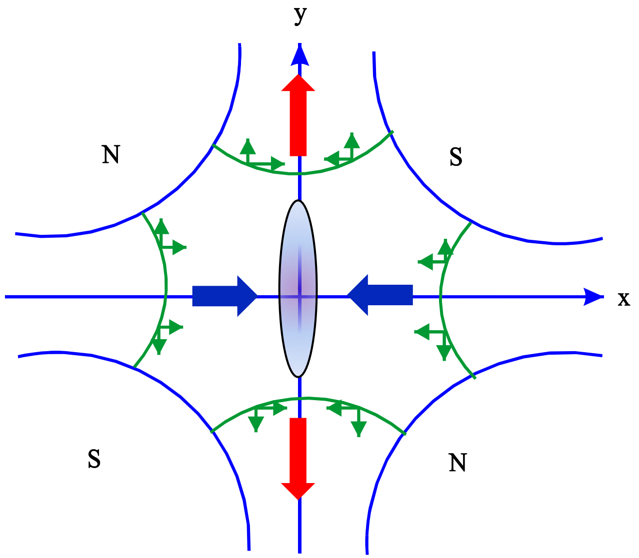
For how to get here, see some hints in the back-up slides.

Horizontal motion: $$x'' + Kx = 0$$

Vertical motion: $$y'' - ky = 0$$

Where $K = \frac{1}{\rho^2} + k$

with k as the quadrupole focusing strength and ρ the bending radius.



In vertical:

→ In general, no dipoles: $\frac{1}{\rho^2} = 0$

→ Sign change of force direction: $k \Longleftrightarrow -k$

Assuming the motion in the horizontal and vertical plane are independent
→ Particle motion in x & y is uncoupled

Solving the Equation of Motion results in a Transfer Matrix

Each element type has its own transfer matrix describing the change of a particle's coordinates after a passage through

Equation of motion in horizontal plane

$$x'' + Kx = 0$$

Equation of the **harmonic oscillator** with spring constant K.

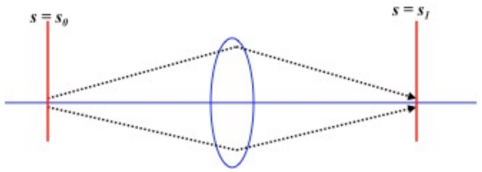
Examples of transfer matrices for other elements are in the back-up slides

Ansatz

For **K > 0 (focusing)** the solution can be found with this ansatz and boundary conditions:

$$x(s) = a_1 \cos(\omega s) + a_2 \sin(\omega s)$$

$$s = 0 \rightarrow \begin{cases} x(0) = x_0, \\ x'(0) = x'_0 \end{cases}$$



Solution

Inserting these into the equation of motion yields:

$$x(s) = x_0 \cos(\sqrt{K} s) + x'_0 \frac{1}{\sqrt{K}} \sin(\sqrt{K} s)$$

$$x'(s) = -x_0 \sqrt{K} \sin(\sqrt{K} s) + x'_0 \cos(\sqrt{K} s)$$

Transfer Matrix

Use matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix} = M_{foc} \cdot \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

Focusing Quadrupole

$$M_{foc} = \begin{pmatrix} \cos(\sqrt{K} s) & \frac{1}{\sqrt{K}} \sin(\sqrt{K} s) \\ -\sqrt{K} \sin(\sqrt{K} s) & \cos(\sqrt{K} s) \end{pmatrix}$$

Particle Tracking

Predicting the particles path through the accelerator elements

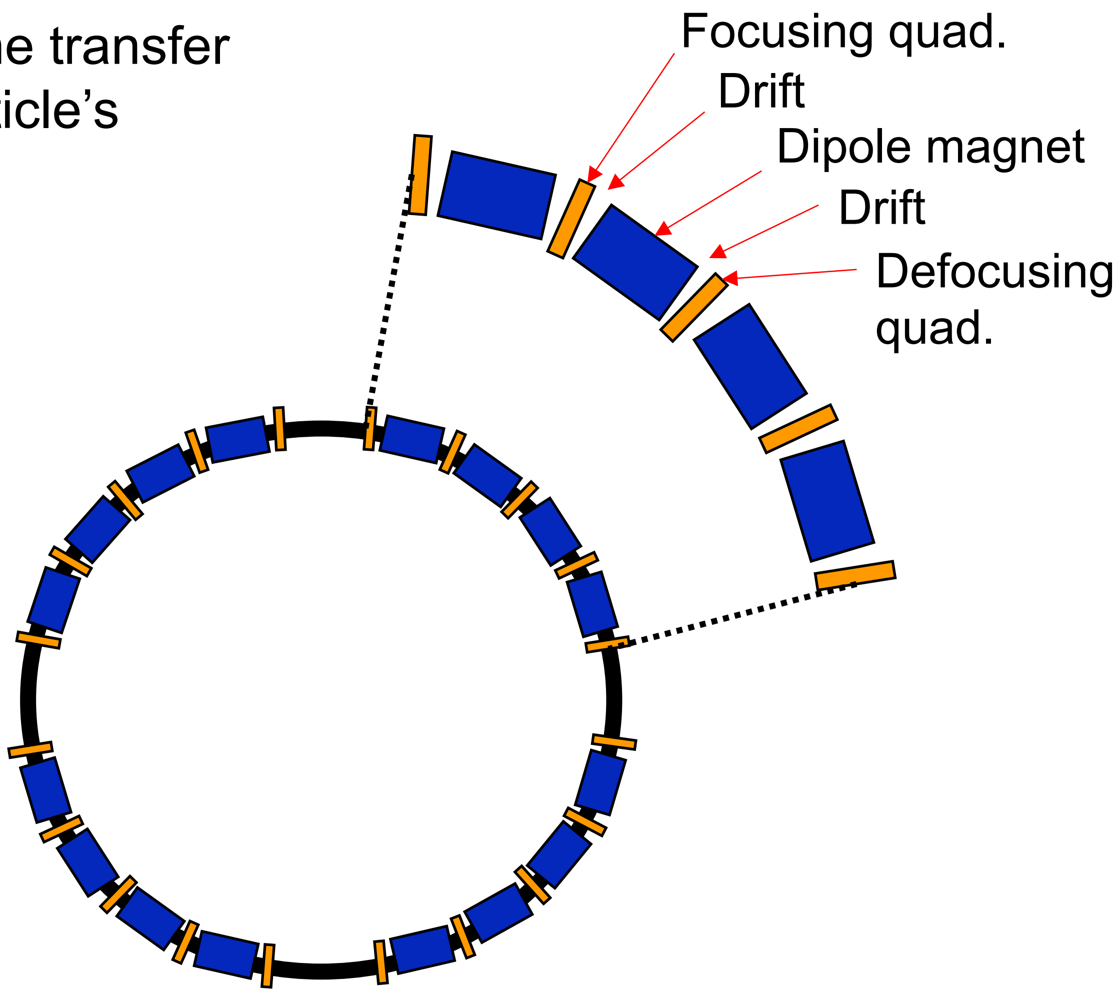
Knowing the initial coordinates at $s=s_0$, we can use the transfer matrix to calculate the effect of an element to the particle's trajectory and get it's new coordinates at $s=s_1$.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$

For a sequence of elements:

$$M_{total} = M_{QF} \cdot M_D \cdot M_{Bend} \cdot M_D \cdot M_{QD} \cdot \ldots$$

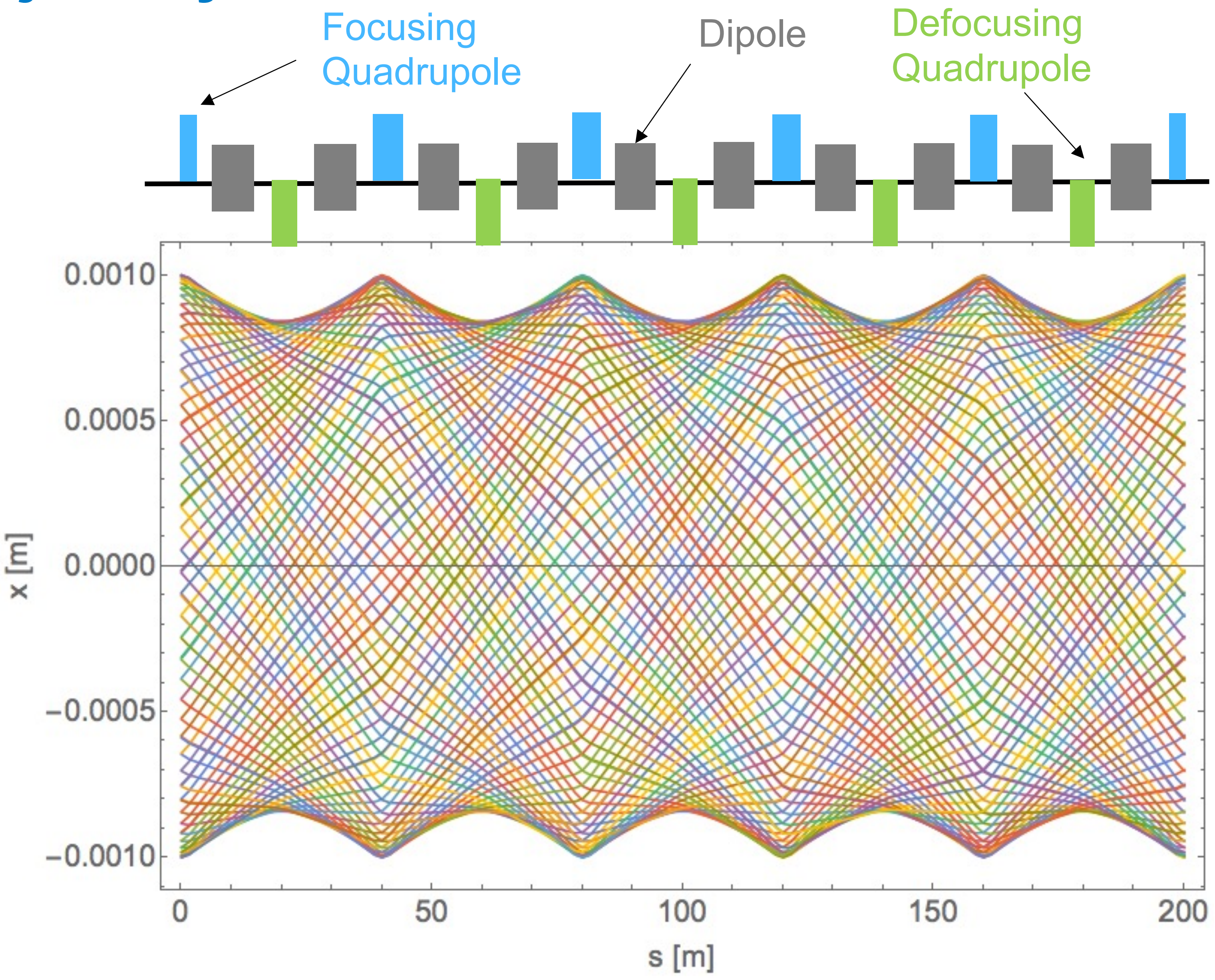
Building up the particles path through the accelerator …



How does a particle trajectory look like?

Initial coordinates
x_0 = 0.001m
x_0' = 0

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_1} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{s_0}$$



Hill's Equation

We had ...

$$x'' + Kx = 0$$

But, around the accelerator *K* is not constant and does depend on *s*!

$$x''(s) + K(s)x(s) = 0$$

Hill's equation

For

- restoring force ≠ const.,
- $K(s)$ depends on the position *s*
- $K(s+L) = K(s)$ periodic function, where L is the "lattice period"

Describing a **quasi harmonic oscillation**, where **amplitude and phase depend on the position s** in the ring.

The Beta Function determines the Focusing Properties

General solution of Hill's equation

$$x(s) = \sqrt{2J_x\beta_x(s)}\cos(\psi(s) + \phi)$$

Integration constants: determined by initial conditions

The **beta function** is a periodic function determined by the focusing properties of the lattice, i.e. quadrupoles

$$\beta(s+L) = \beta(s)$$

The **"phase advance"** of the oscillation between the point s_0 and point s in the lattice.

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)}$$

The number of oscillations per turn is called "tune"

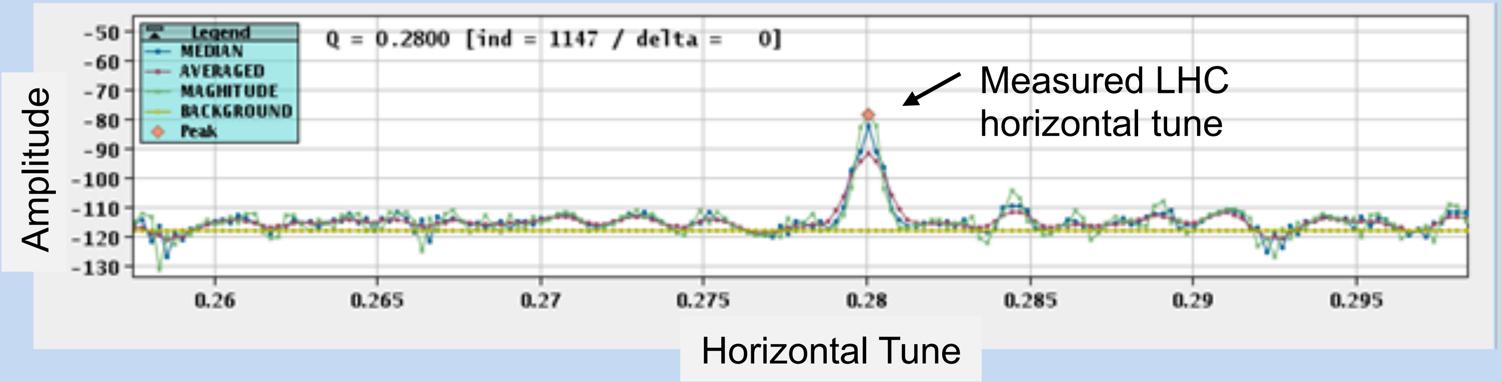
The tune is an important parameter for the stability of motion over many turns.

$$\psi(s) = \int_0^s \frac{ds}{\beta(s)} \xrightarrow{\text{full turn}} Q = \frac{1}{2\pi} \int \frac{ds}{\beta(s)}$$

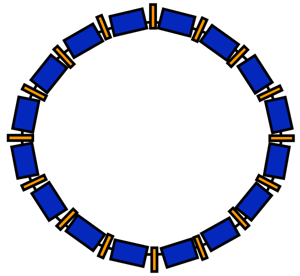
The tune has to be **chosen appropriately, measured and corrected**.

Tune Measurement

1) Measure beam position at one location turn by turn.
2) Beam position will change $\propto \cos(2\pi Q i)$.
3) Perform FFT to get frequency of oscillation → tune

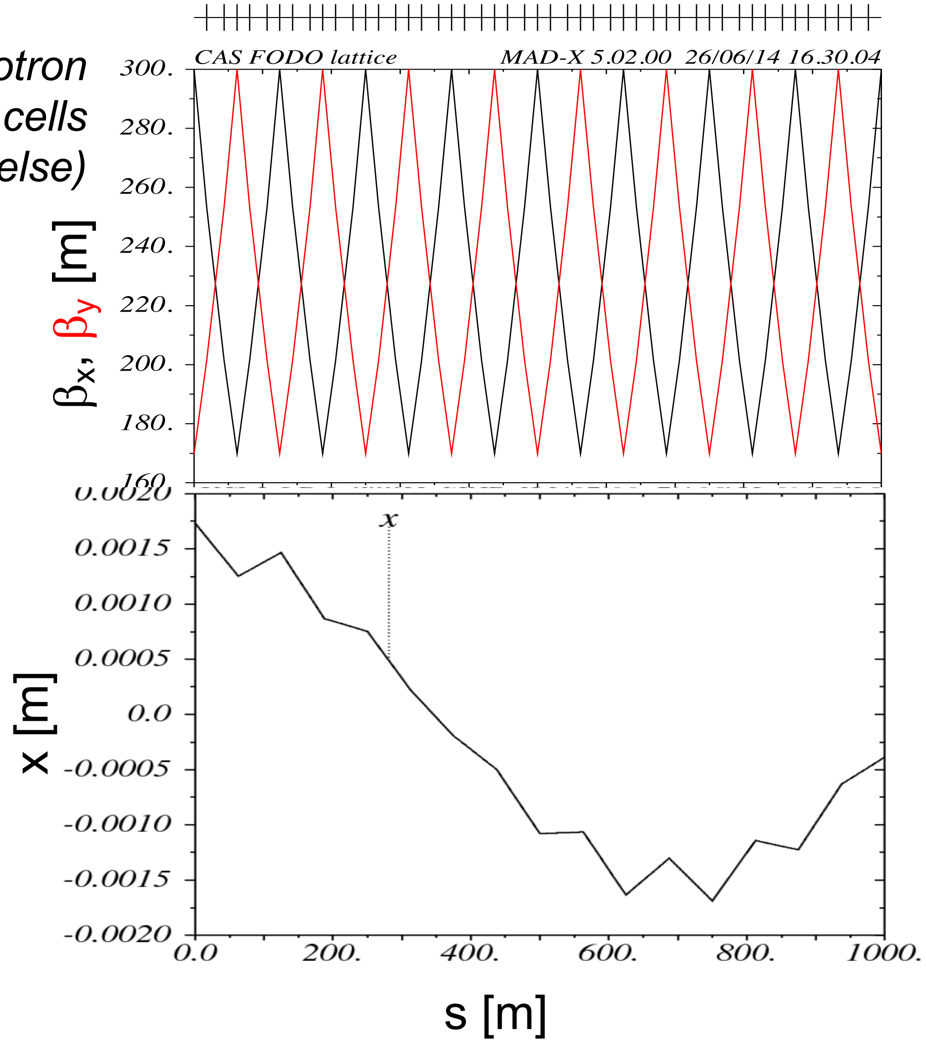


Toy Lattice – a Simple Example



Maximum beta: 300 m
Maximum trajectory offset: 1.5 mm

Example synchrotron with 8 FODO cells (and nothing else)



The ***β*-function** reflects the periodicity of the magnet structure:
→ 8 FODO cells
→ 8 oscillations of the β-function

In this example, the number of trajectory (betatron) oscillations around the ring is **Q<1**.

The periodicity of the betatron oscillation is NOT the same as the periodicity of the magnetic structure.

Particles describe an Ellipse in x, x' Phase-Space

Shape is defined by the Courant-Snyder parameters

General solution of Hill's equation: $x(s) = \sqrt{2J_x\beta_x(s)}\cos(\psi(s) + \phi)$

J_x is called **action** and can be written as:

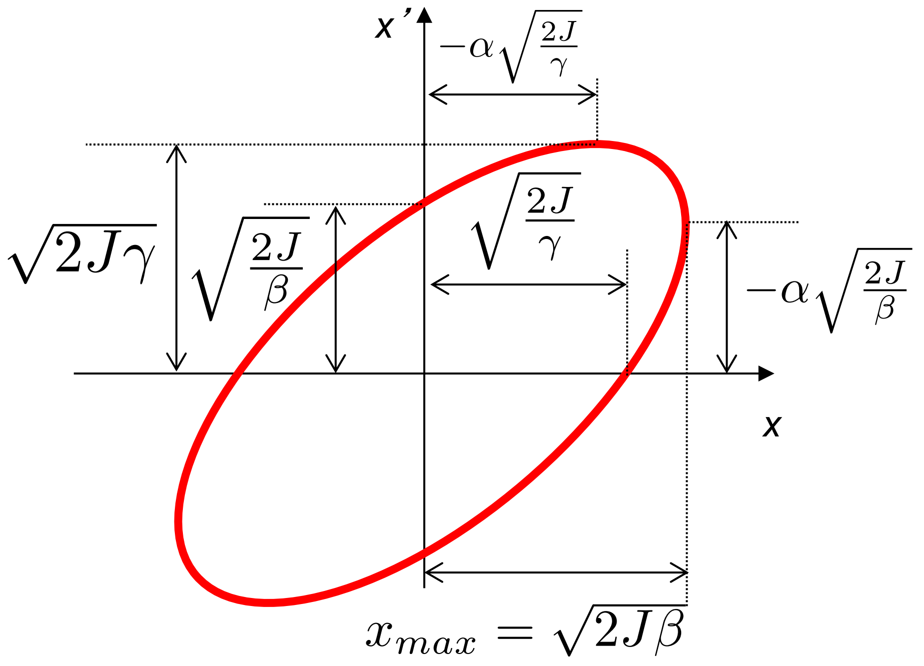
$$J_x = \frac{1}{2}\left(\gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2\right)$$

which is the equation of an **ellipse** in the **phase-space *x, x'*.**

The shape and orientation of ellipse are defined by the Courant-Snyder parameters.

The area of the ellipse is

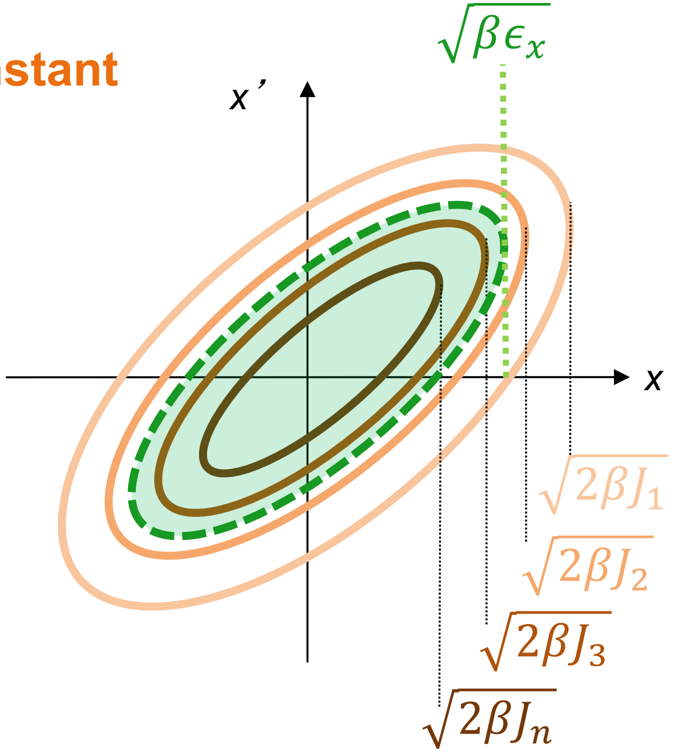
$$A = 2 \cdot \pi \cdot J_x$$



x-x' phase space
(trajectory offset vs. angle)

Emittance and Beam Size

The beam size varies along the lattice, while the emittance is constant



At a given location: $x = \sqrt{2\beta_x J_x}\cos\psi_x$

The mean square value of this is:

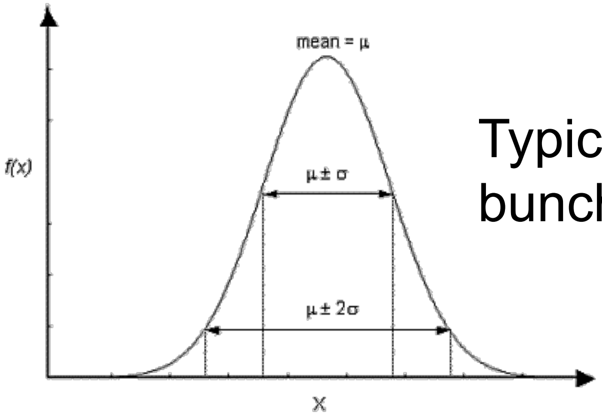
$$\langle x^2\rangle = 2\beta_x\langle J_x\cos^2\psi_x\rangle = \beta_x\langle J_x\rangle = \beta_x\epsilon_x$$

assumes action and phase uncorrelated, and uniform distribution in phase from 0 to 2π.

Defines *emittance* of particle distribution:

$$\langle J_x\rangle = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2} := \epsilon_x$$

ϵ_x is an **intrinsic beam property** that is defined at it's creation.

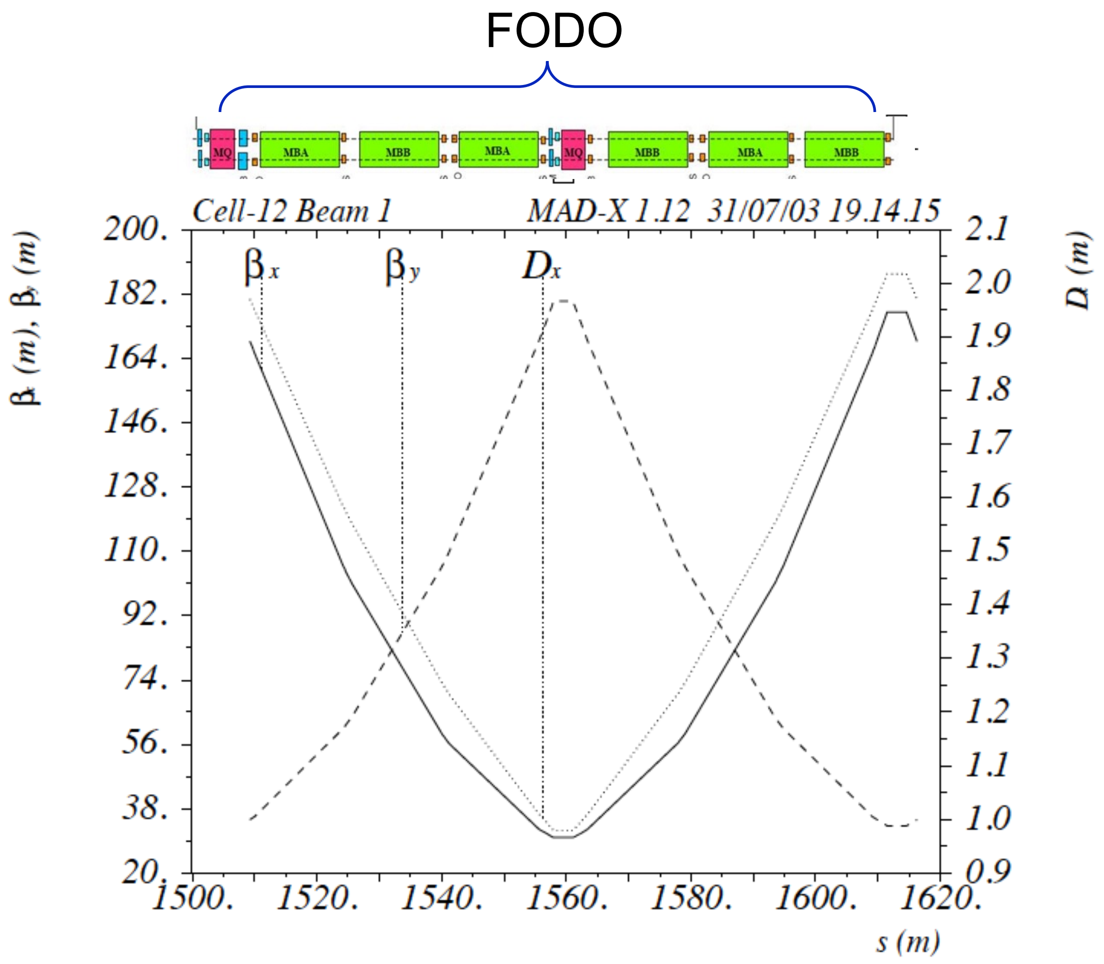


Typically the distribution of particles in a bunch follows a Gaussian shape:

$$\rho(x) = \frac{N}{\sqrt{2\pi}\sigma_x}\cdot e^{-\frac{x^2}{2\sigma_x^2}}$$

Therefore, $\sigma_x = \sqrt{\langle x^2\rangle} = \sqrt{\epsilon_x\beta_x}$ describes the one sigma **beam size**.

The Beam Size changes around the Accelerator



The β-function is periodic

→ It changes along the cell.

→ The beam size changes along the cell!

$$\sigma = \sqrt{\varepsilon \beta}$$

Max. horizontal beam size in the **focusing quadrupoles**

Max. vertical beam size in the **defocusing quadrupoles**

The regular LHC FODO cell:
- *Phase advance: 90°*
- *Maximum beta: 180 m*

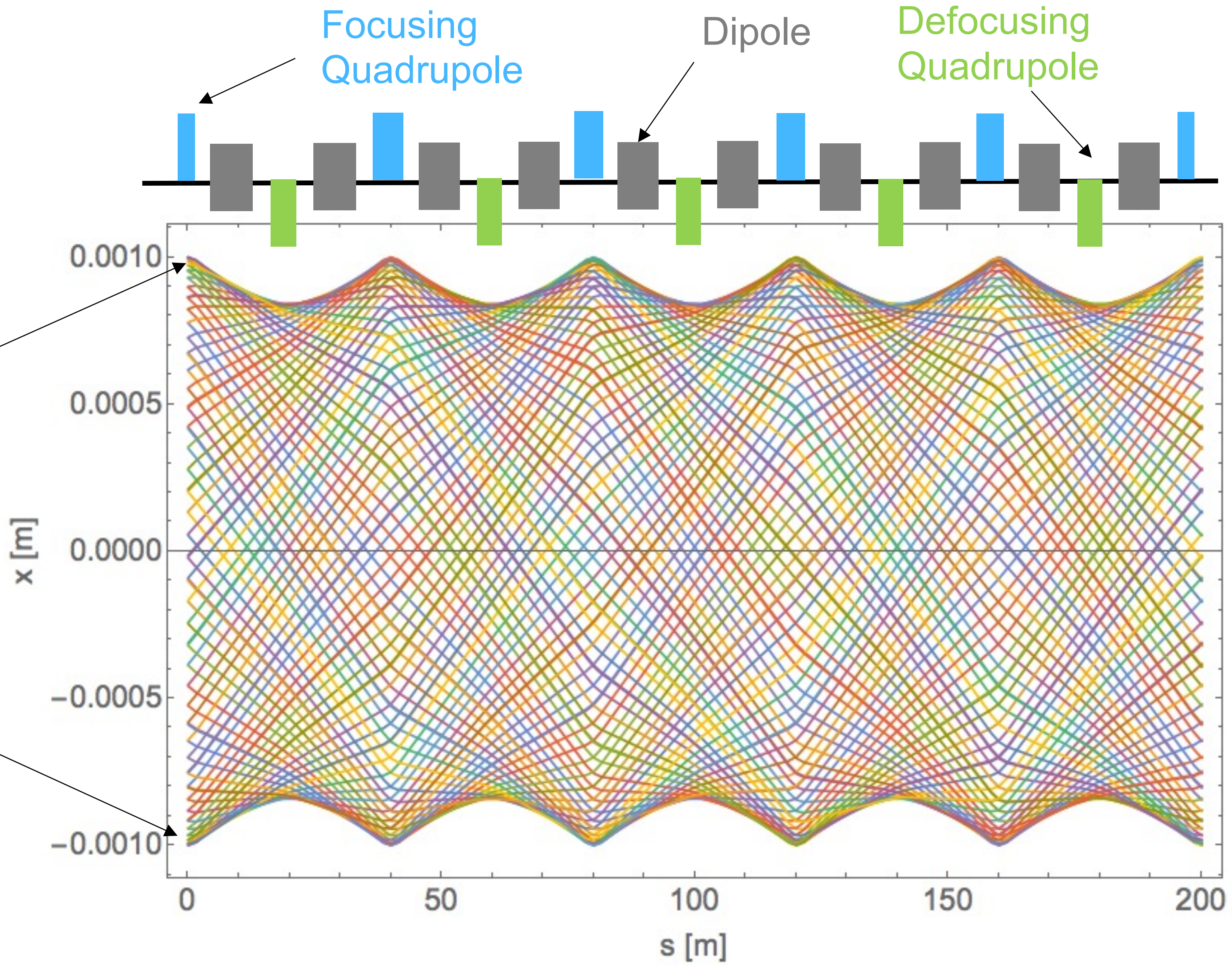
The Beam Envelope

Initial coordinates
$x_0 = 0.001\text{m}$
$x_0' = 0$

1-σ beam envelope

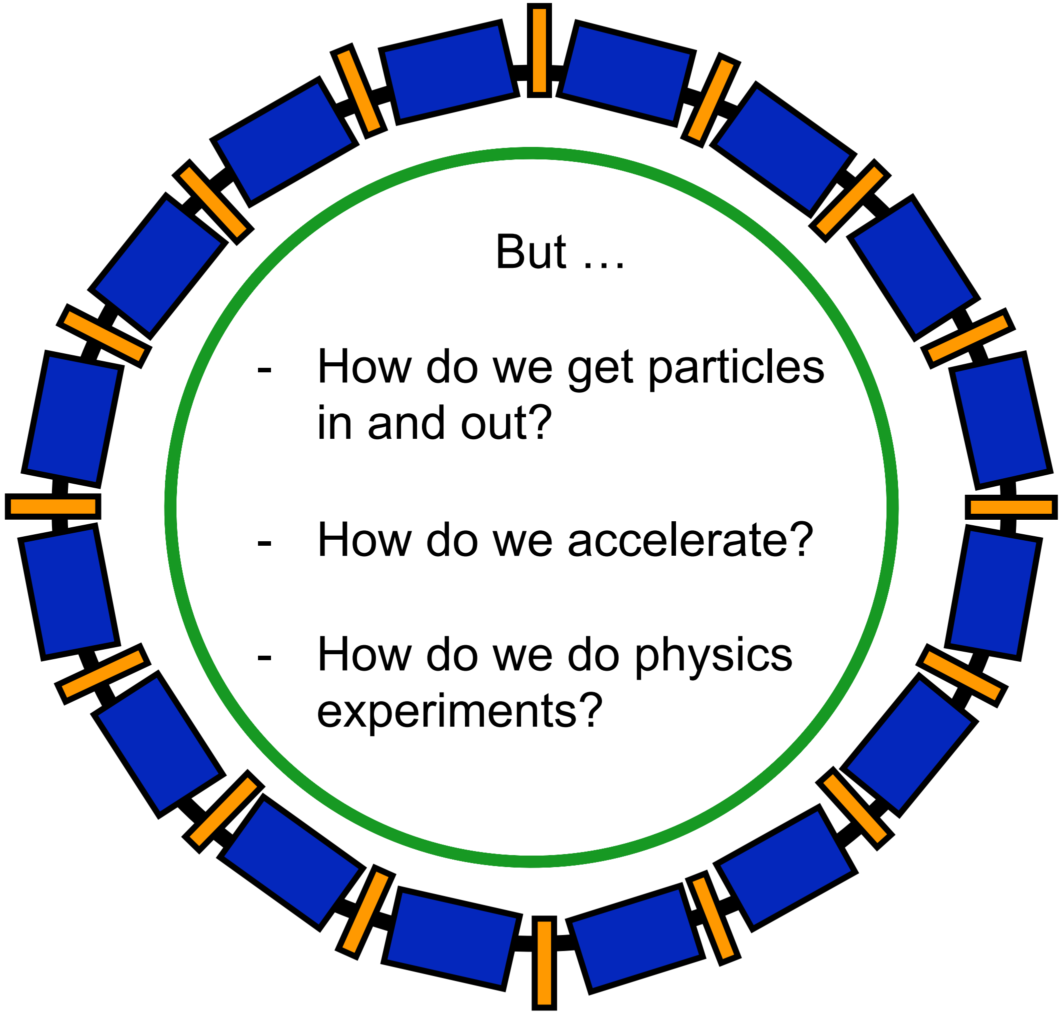
for beam with σ = 1mm
in focusing quadrupole

$$\sigma_x = \sqrt{\varepsilon \beta_x}$$



What have we learned so far?

We know, how particles behave along the magnetic lattice of an accelerator.

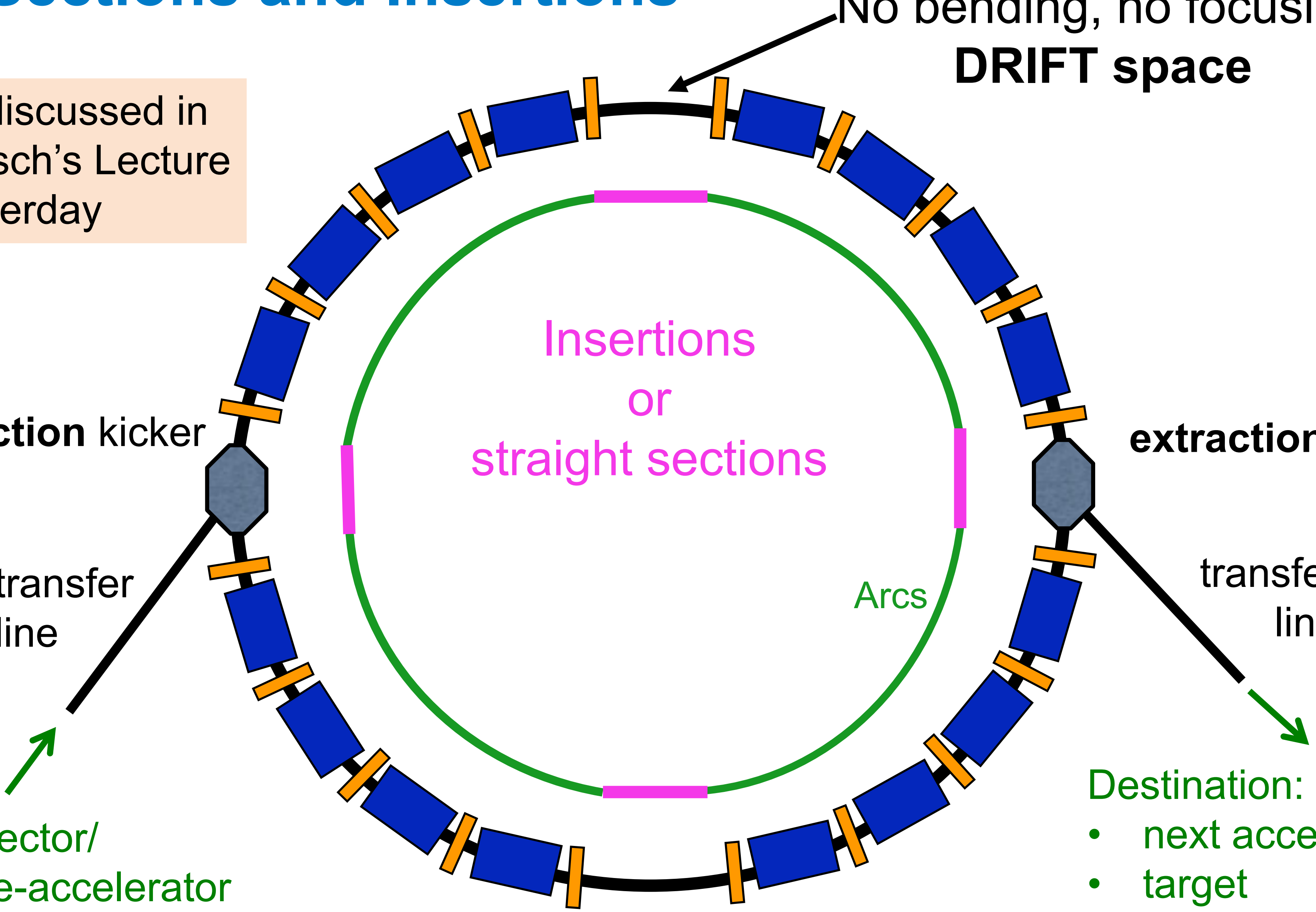


But …

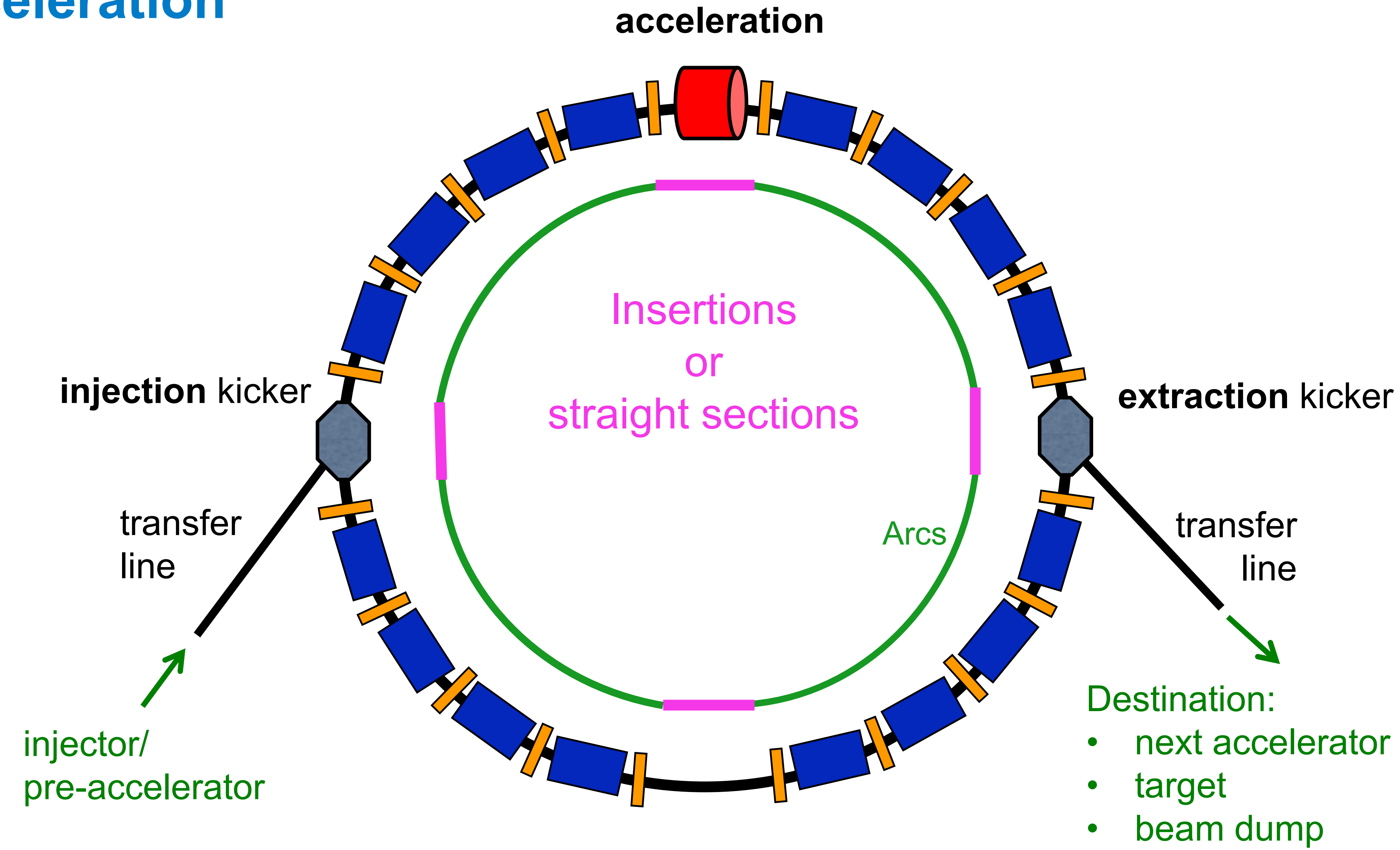
- How do we get particles in and out?
- How do we accelerate?
- How do we do physics experiments?

Straight Sections and Insertions

Injection discussed in Gregor Loisch’s Lecture yesterday

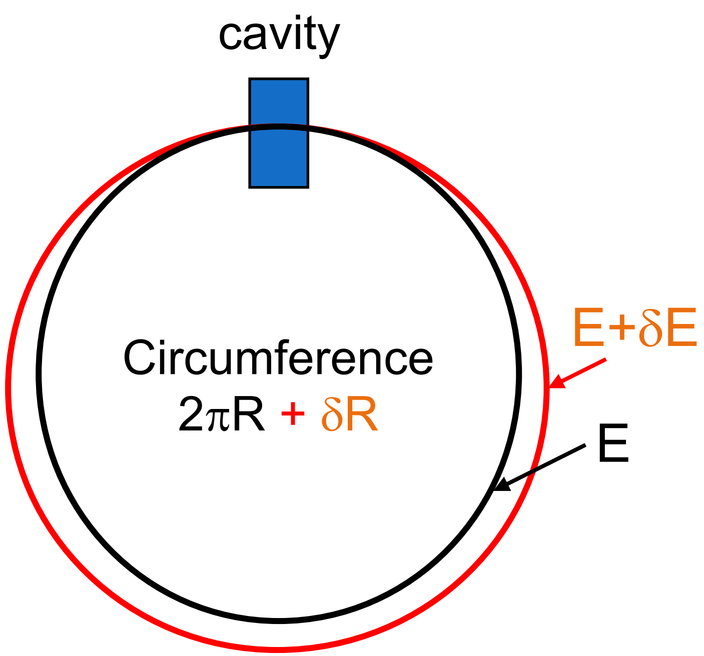


Acceleration



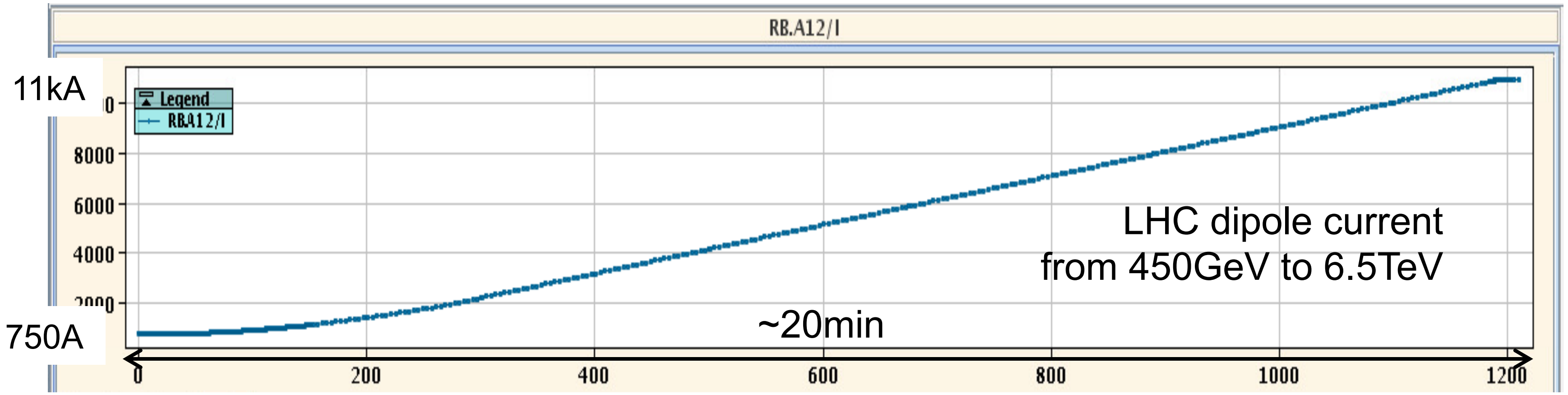
RF Acceleration and Magnetic Field Increase

What about the magnetic field during acceleration?

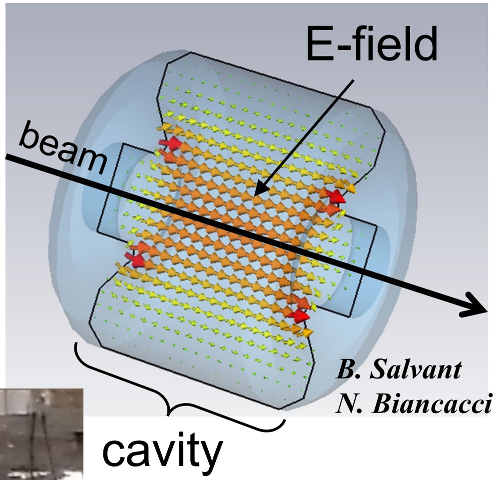


Beam rigidity needs to be increased proportionally to increasing energy.
→ Machine radius is constant.
→ Need to increase dipole field accordingly!

$$\frac{p}{q} = B\,\rho$$



Where do we accelerate?



Use RF cavities to apply the same accelerating voltage on each passage.

→ Gradually increase total energy by **gaining a small amount each turn**.

LHC has
- 8 superconducting cavities per beam
- Accelerating field 5MV/m
- Can deliver 2MV/cavity
- Operating at 400MHz

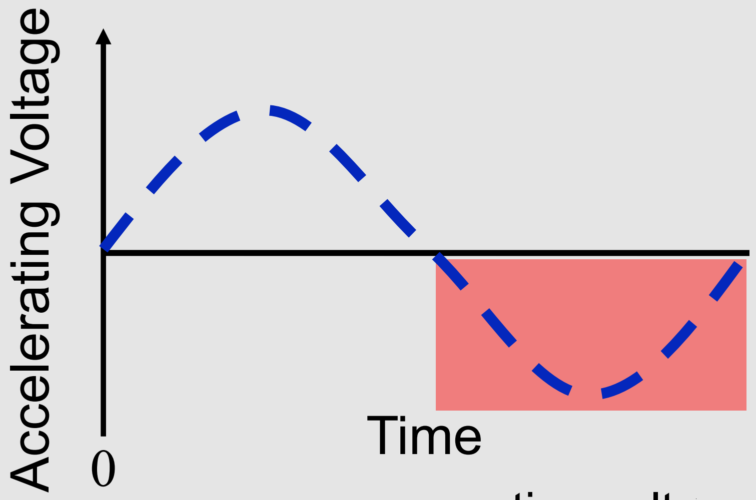
PETRA III has
- 12 seven-cell copper cavities
- Accelerating field ~2.8MV/m
- Can deliver 1.6MV/cavity
- Operating at 500MHz

Accelerating voltage is changing with time

That has two consequences

Need ***synchronization*** between beam and RF phase to gain energy.

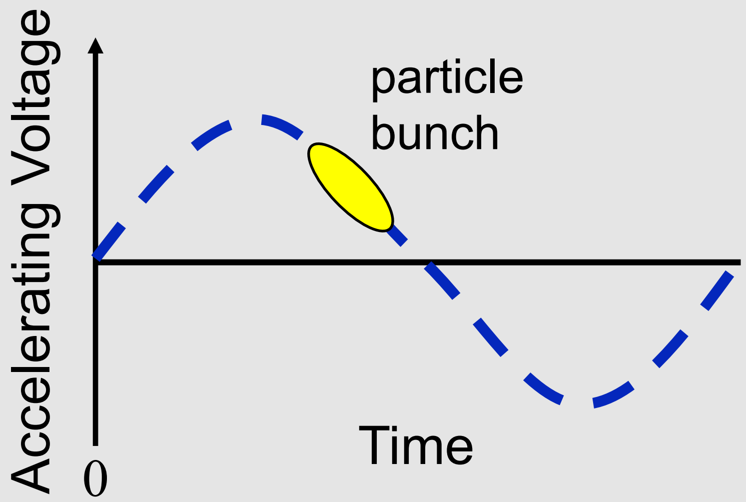
There is a ***synchronous RF phase*** for which the energy gain fits the increase of the magnetic field.



negative voltage
→ deceleration

Not all particles see the same voltage, because they arrive at different times.

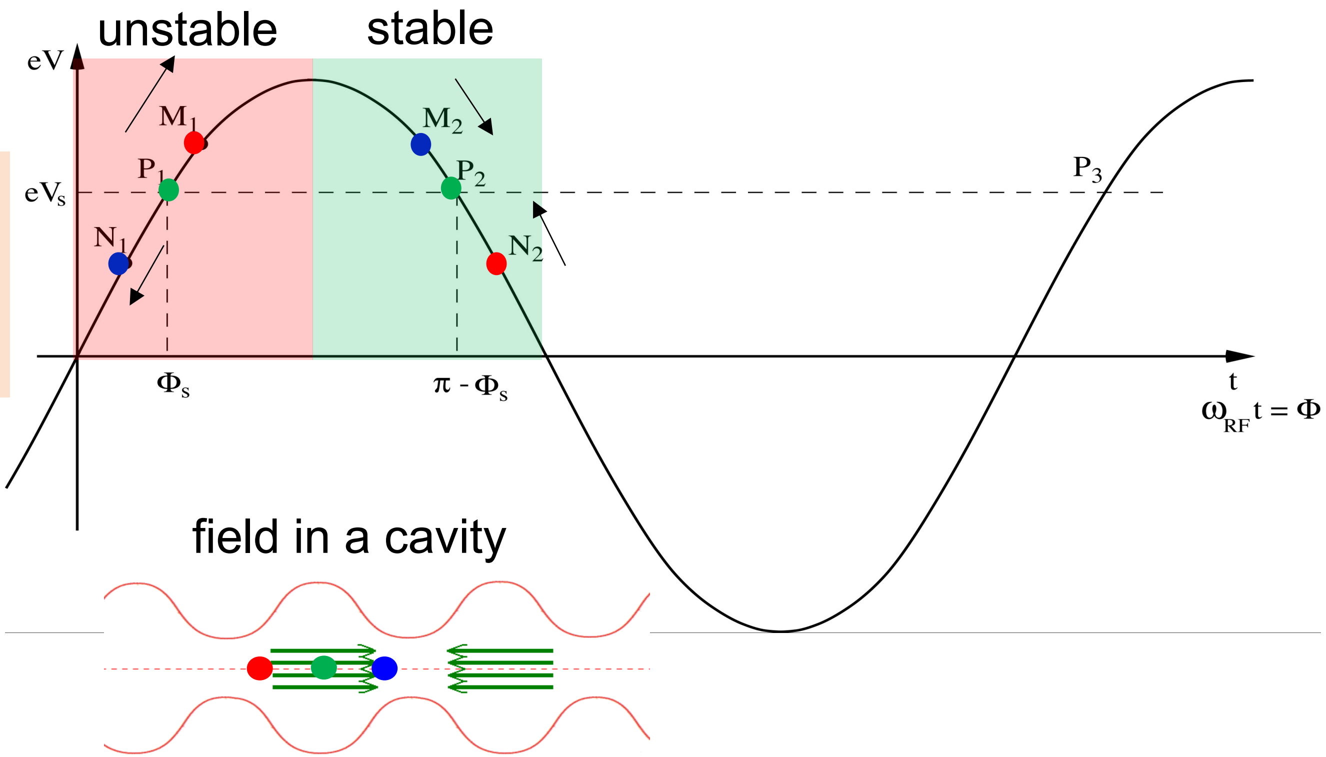
Not all particles gain the same energy.



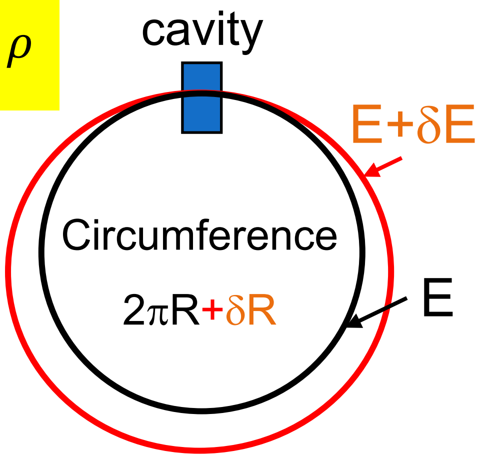
Phase Stability (relativistic regime → $v \approx c$)

An increase in momentum transforms into a longer orbit and thus a longer revolution time.

M_1 & N_1 will go away from P_1 → unstable (and finally be lost)



$$\frac{p}{q} = B\,\rho$$

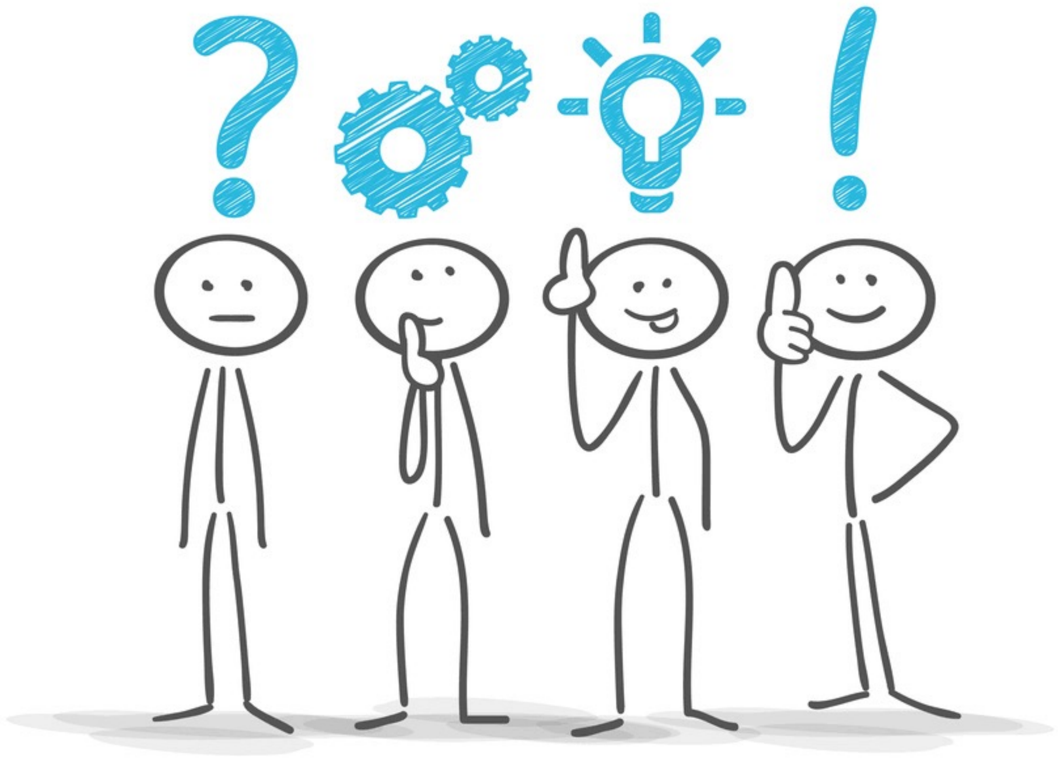


- Ideal particle
- Particle with Δt < 0 → higher energy gain → gets longer orbit
- Particle with Δt > 0 → lower energy gain → gets shorter orbit

M_2 & N_2 will move towards P_2 → stable

Longitudinal (phase) focusing keeps particles close to each other ... forming a „bunch“

Courtesy F. Tecker for drawings



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Everything clear! Hmm ….

EPACE

European Compact accelerators, Applications, Entrepreneurship

- 15 PhD in plasma acceleration in Europe, combined with training in innovation & entrepreneurship
- Funded as an MSCA Doctoral Network, Start in Sept. 2025
- Contact: lisa.crinon@desy.de
- **visit our website for current opportunities!** www.epace.eu

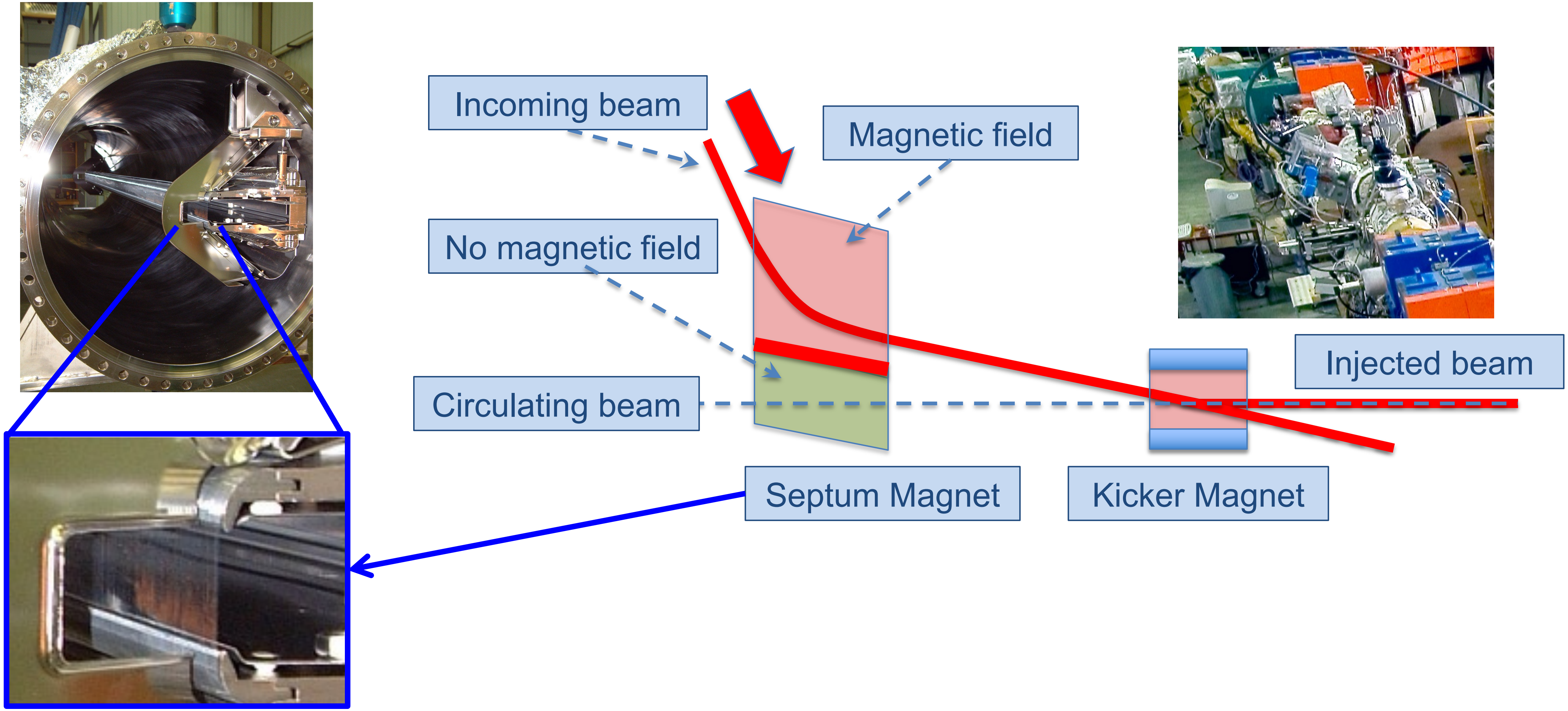
Research projects:

- kHz laser-wakefield acceleration
- Snapshot tomography of laser-plasma acceleration
- Machine-Learning-Enhanced Laser Plasma Accelerators
- Tailored plasma targets for Laser Wakefield Acceleration
- Production of high-density spin-polarized hydrogen-atom target
- Spin polarisation in plasma accelerators
- Very high energy electrons (VHEE) radiotherapy with beams from a wakefield accelerator
- Compact muon and electron source combined with the GScan detector system: the radiological system for medical applications
- Advancing radiotherapy with laser-plasma accelerators
- ICS soft x-ray source for semiconductor wafer metrology
- Inverse Compton Scattering (ICS) x-ray source from a high repetition rate laser wakefield accelerator
- Controlling plasma sources on hydrodynamic time scales to better plasma accelerators
- Theoretical study of superluminal laser-plasma acceleration
- Plasma Mirrors: towards extreme intensity light sources and high-quality compact electron accelerators
- Better beam quality in plasma accelerators through high-performance computing

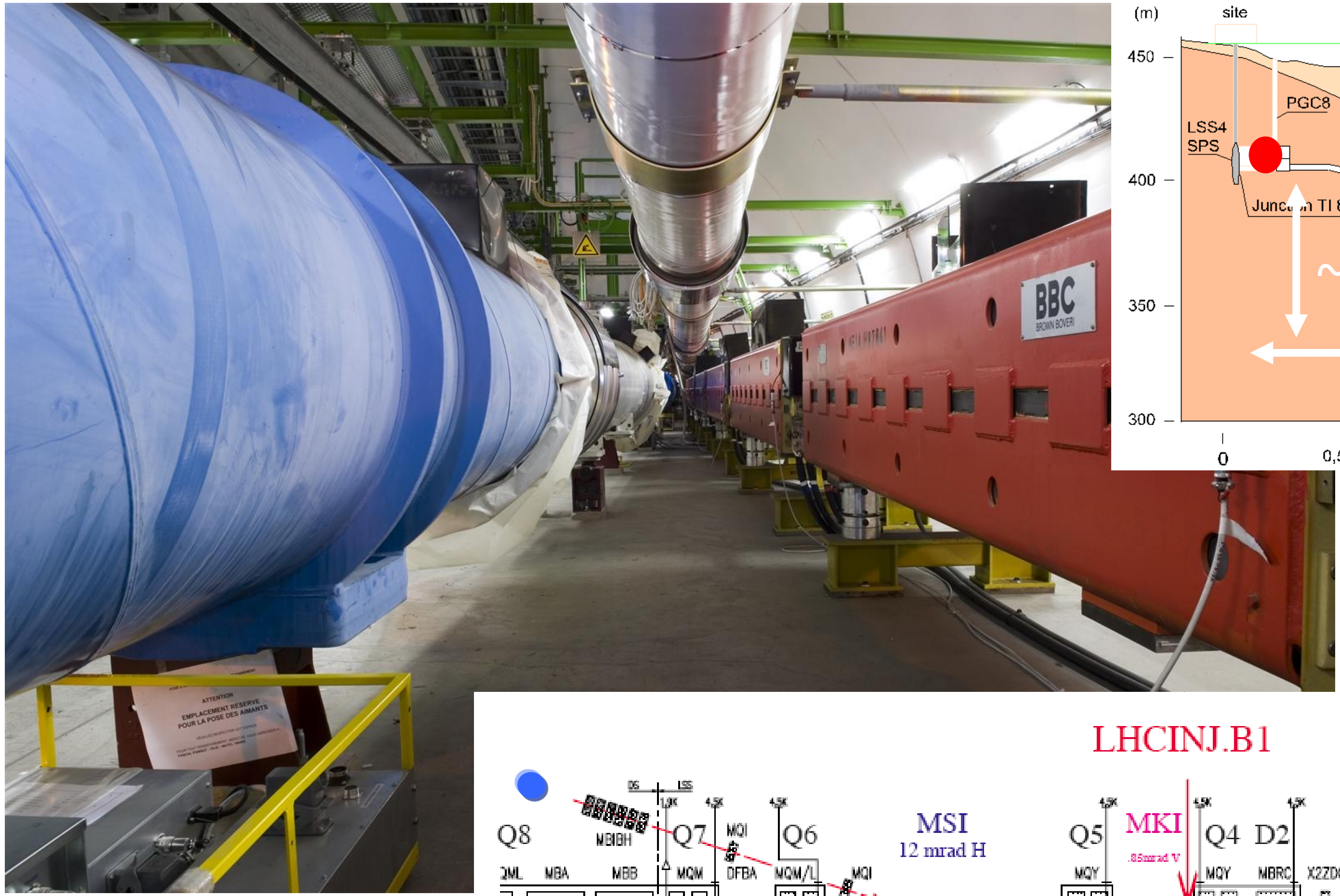
Back-up Slides

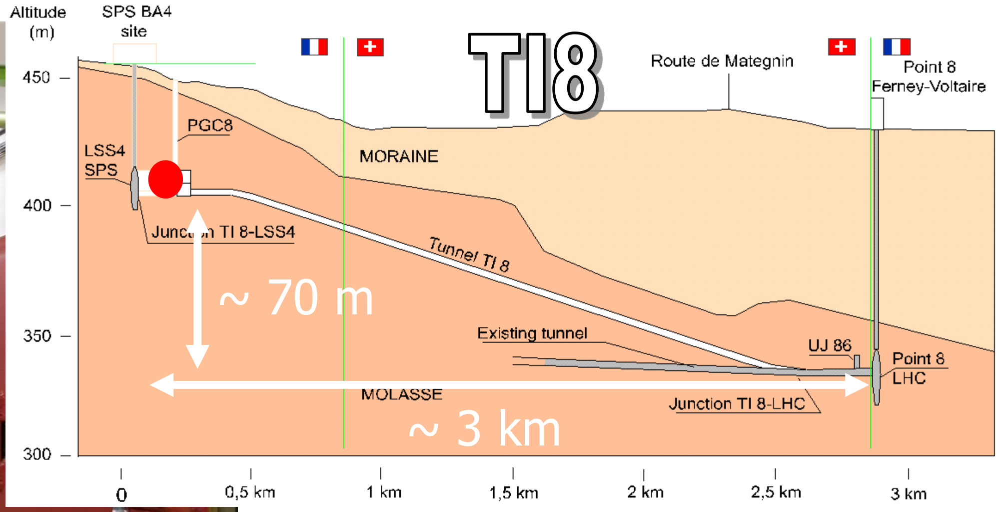
On-Axis (swap out) Injection

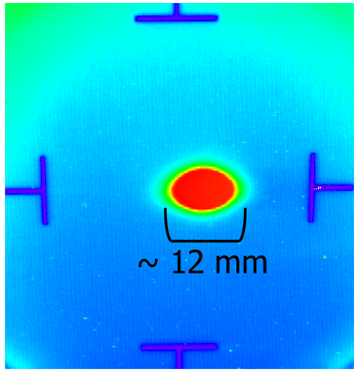
Extraction follows the same principle, but the beam travels in the opposite direction.

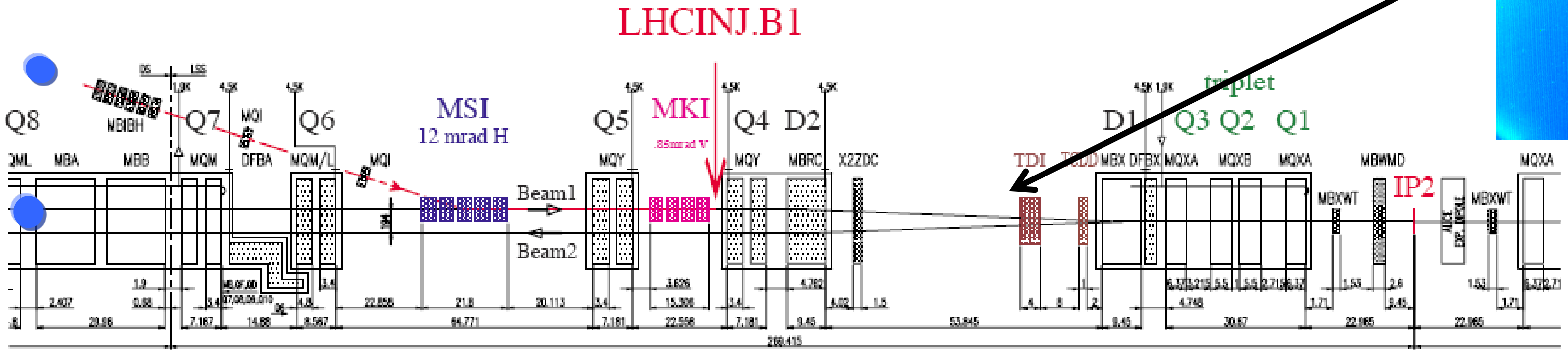


Injection of Beam 2 into LHC







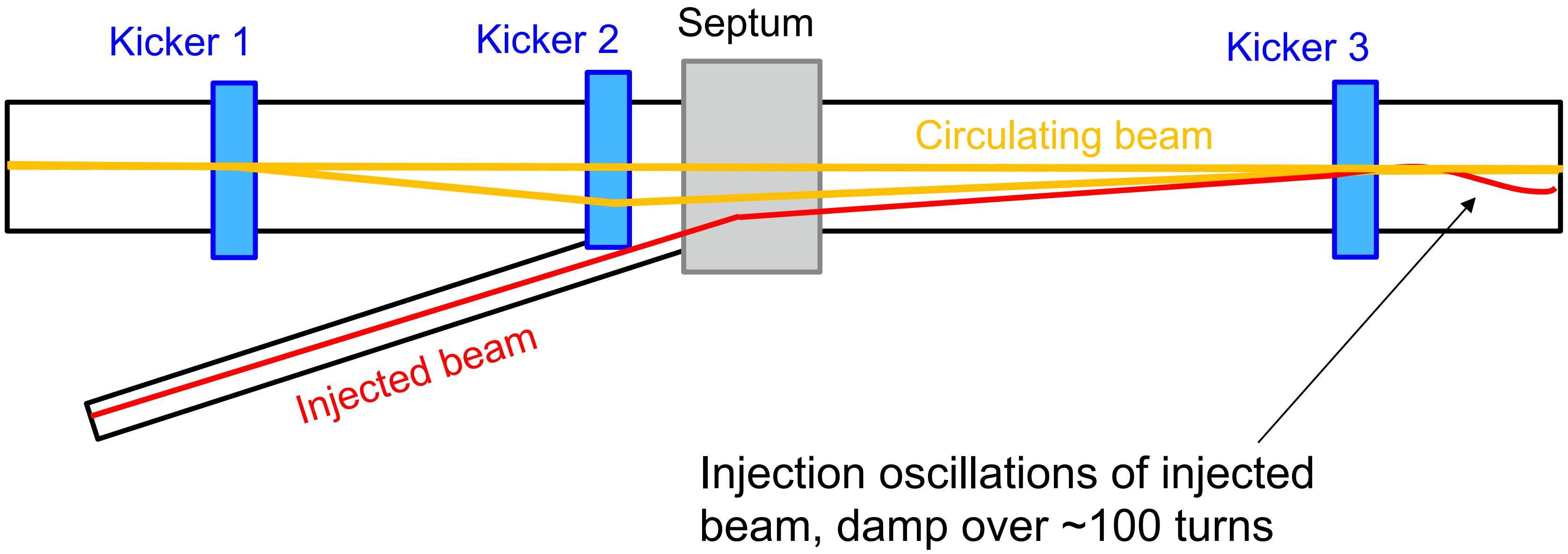


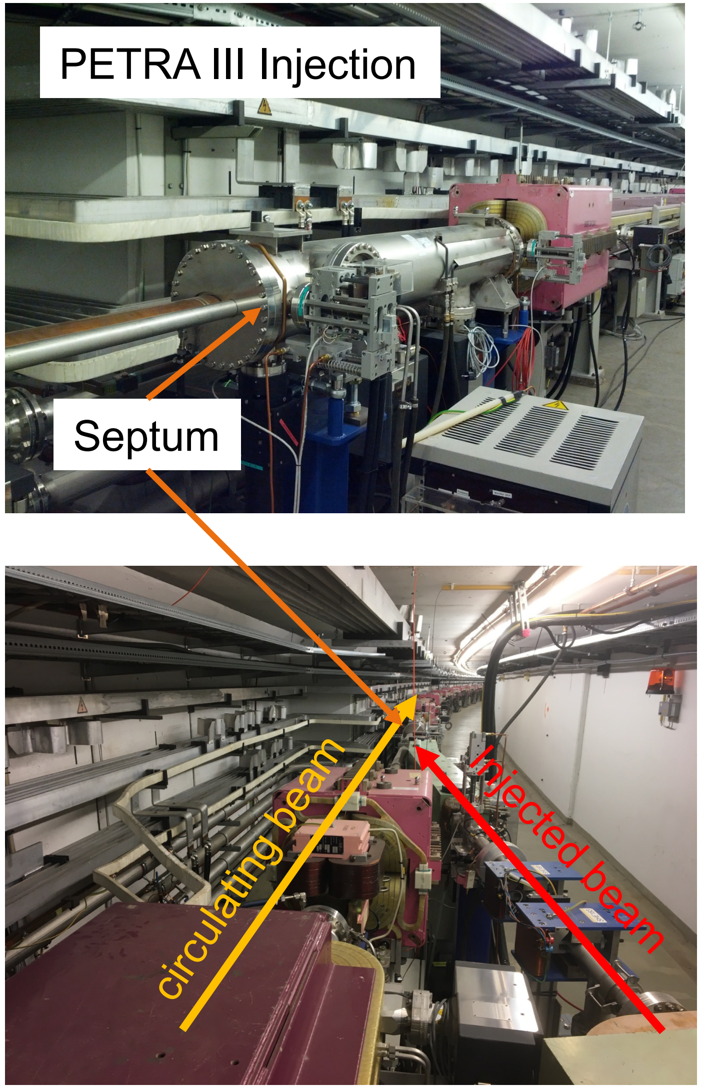
court.
R. Alemany

Off-Axis Top-up Injection

3-Kicker orbit bump to bring circulating beam close to injected beam

TopUp Injection only practical for electrons.





Lorentz force linearly increasing as function of distance from design orbit

→ Linearly increasing magnetic field

$$F(x) = q \cdot v \cdot B(x)$$

Taylor series as function of distance from magnet center

$$B_y(x) = B_{y0} + \frac{\partial B_y}{\partial x} x + \frac{1}{2} \frac{\partial^2 B_y}{\partial x^2} x^2 + \frac{1}{3!} \frac{\partial^3 B_y}{\partial x^3} x^3 + \ldots$$

dipole — *quadrupole* — *sextupole* — *octupole* — *higher orders*

Normalize to p/q: $\frac{p}{q} = B\,\rho$

$$\frac{B_y(x)}{p/q} = \frac{1}{\rho} + kx + \frac{1}{2} m x^2 + \frac{1}{3!} n x^3 + \ldots$$

quadrupole

Towards the Equation of Motion

$$F_x = m \cdot \ddot{x}$$ Describes motion as a function of time.

But what we need is something like $F_x = Mx''$

$$\dot{x} = \frac{dx}{dt}$$

$$x' = \frac{dx}{ds}$$

→ Replace free parameter time ***t*** by path length ***s.***

→ Compare to Lorentz force $F(x) = q \cdot v \cdot B(x)$

Taylor expansion of normalize magnetic field:

$$\frac{B_y(x)}{p/q} = \frac{1}{\rho} + kx + \frac{1}{2}mx^2 + \frac{1}{3!}nx^3 + \ldots$$

dipole *quadrupole* *sextupole* *octupole* *higher orders*

Only consider **linear** terms: **dipole & quadrupole** fields!

$$\frac{B_y(x)}{p/q} \approx \frac{1}{\rho} + kx$$

Defocusing Quadrupole, Drift Space & Dipole

Equivalent strategy using individual Ansatz

Equation of motion
in horizontal plane

$$x'' + Kx = 0$$

Equation of the
harmonic oscillator
with spring constant K.

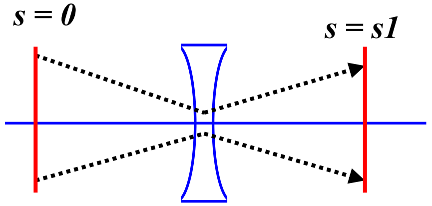
Defocusing Quadrupole

For **K < 0 (defocusing)**
the new ansatz is:

$$x(s) = a_1 \cosh(\omega s) + a_2 \sinh(\omega s)$$

Defocusing Quadrupole

$$M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|}s) & \frac{1}{\sqrt{|K|}} \sinh(\sqrt{|K|}s) \\ \sqrt{|K|} \sinh(\sqrt{|K|}s) & \cosh(\sqrt{|K|}s) \end{pmatrix}$$

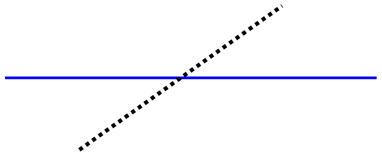


Drift Space

For **K = 0 (drift)** the ansatz is: $x(s) = x'_0 s$

Drift Space

$$M_{drift} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$



Dipole

For **K = 1/ρ² (dipole)** use the result for a focusing dipole and insert K.

Dipole

$$M_{dipole} = \begin{pmatrix} \cos(\frac{s}{\rho}) & \rho \sin(\frac{s}{\rho}) \\ -\frac{1}{\rho} \sin(\frac{s}{\rho}) & \cos(\frac{s}{\rho}) \end{pmatrix}$$

Courant-Snyder Parameters: $\alpha(s), \beta(s), \gamma(s)$

Provide an alternative description of the single particle trajectory through the lattice

General solution of Hill's equation: $x(s) = \sqrt{2J_x\beta_x(s)}\cos(\psi(s) + \phi)$

Define: $\alpha(s) = -\frac{1}{2}\beta'(s) \qquad \gamma(s) = \frac{1+\alpha(s)^2}{\beta(s)}$

$\alpha(s), \beta(s), \gamma(s)$ are called
Courant-Snyder or Optics parameters

Let's assume for $s(0) = s_0$, $\psi(0) = 0$, $\beta(0) = \beta_0$ and $\alpha(0) = \alpha_0$
Defines ϕ from initial conditions: x_0 and x'_0, β_0 and α_0.

Re-write transfer matrix with optics parameters:

$$M = \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}(\cos\psi + \alpha_0 \sin\psi) & \sqrt{\beta\beta_0}\sin\psi \\ \frac{(\alpha_0-\alpha)\cos\psi-(1+\alpha\alpha_0)\sin\psi}{\sqrt{\beta\beta_0}} & \sqrt{\frac{\beta_0}{\beta}}(\cos\psi - \alpha\sin\psi)) \end{pmatrix}$$

Once we know α, β, we can compute the single particle trajectories between two locations without remembering the exact lattice structure and strength of each element!

How does the bending radius changes, when accelerating without adjusting the magnetic field?

LHC magnetic dipole field at 450 GeV:

$$B = \frac{p}{q\rho} = \frac{450\,\mathrm{GeV}/c}{e \times 2803\,\mathrm{m}} = 0.535\,\mathrm{T}$$

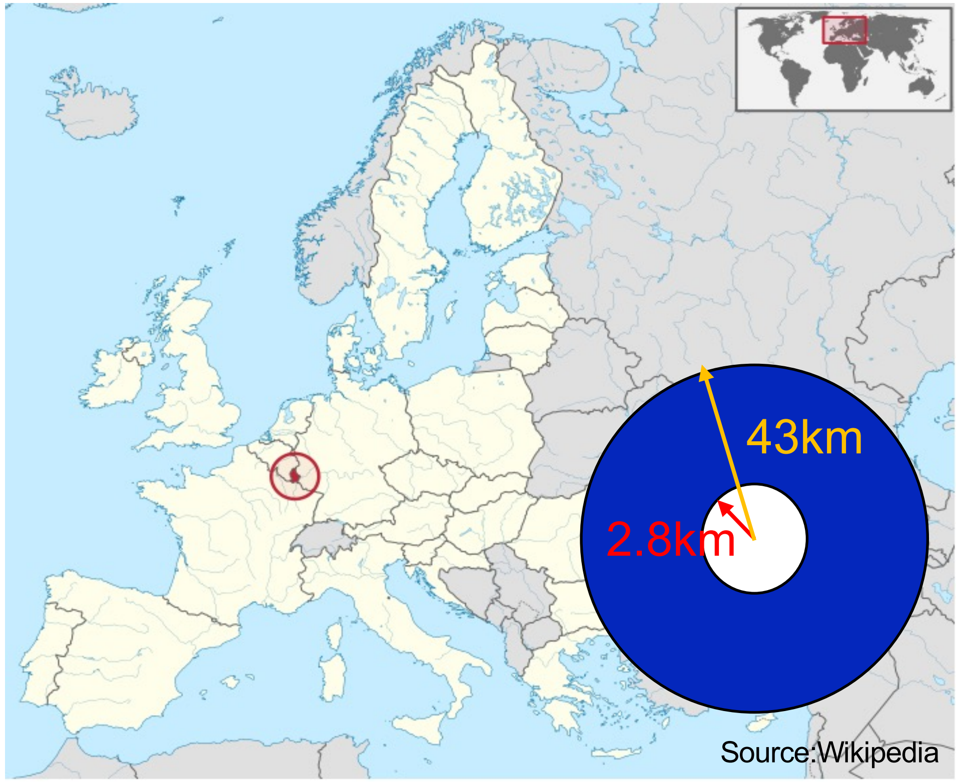
Required bending radius at 7 TeV with B_{inj}=0.5T:

$$\rho = \frac{p}{qB} = \frac{7\,\mathrm{TeV}/c}{e \times 0.535\,\mathrm{T}} = 43.6\,\mathrm{km}$$

$$\frac{p}{q} = B\,\rho$$

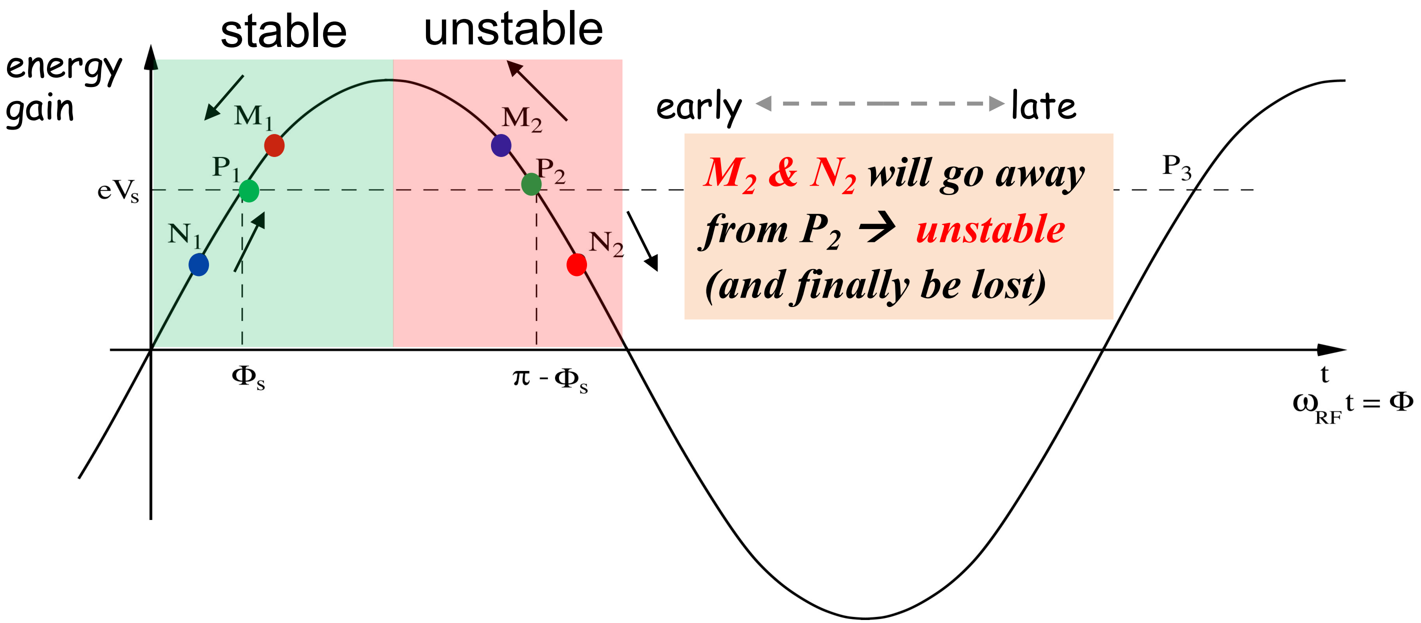
Equivalent to 270km circumference
(pure dipole field! without any insertions or quadrupoles)

Magnet surface = 5800km^2
→Area of Brunei (South-Eastern Asia)
→ Area of 2x Luxemburg



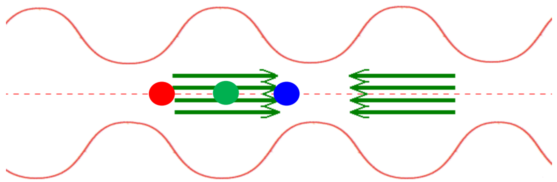
Phase Stability (non-relativistic regime)

Energy increase is transferred into a velocity increase



Longitudinal (phase) focusing keeps particles close to each other ... forming a „bunch“

field in a cavity



Particles P_1, P_2 have the synchronous phase.

- Ideal particle
- Particle with $\Delta t < 0$ (early) →lower energy gain → gets slower
- Particle with $\Delta t > 0$ (late) →higher energy gain → gets faster

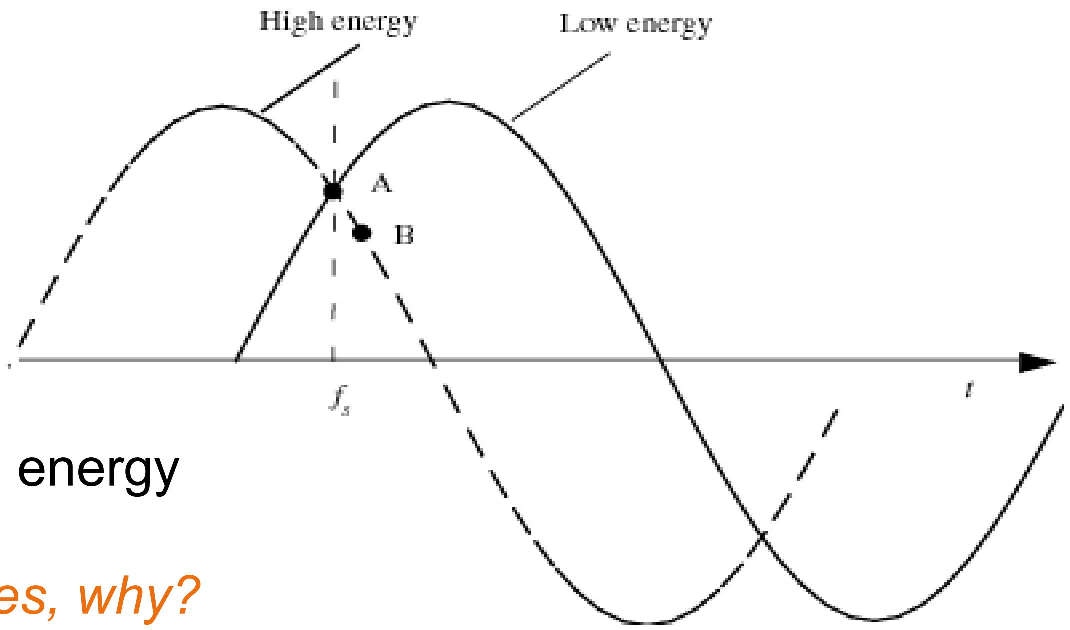
→ M_1 & N_1 will move towards P_1 → stable

Courtesy F. Tecker for drawings

Crossing Transition

The previously stable synchronous phase becomes unstable when v => c and the gain in path length overtakes the gain in velocity → ***Transition***

Transition from one slope to the other during acceleration → ***Crossing Transition.***
The RF system needs to make a rapid change of the RF phase, a ‘phase jump’.



In the PS: γ_t is at ~6 GeV, injection at 1.4GeV
In the SPS: γ_t= 22.8, injection at γ=27.7
=> no transition crossing!
In the LHC: γ_t is at ~55 GeV, also far below injection energy

Transition crossing is not needed in leptons machines, why?

Synchrotron Oscillation

Like in the transverse plane the particles are oscillating in longitudinal space

Simple case (no accel.): B = const.
- Stable phase: $\phi_0 = 0$
- Particle B oscillates around ϕ_0.

Particles keep *oscillating around the stable synchronous particle* varying phase and dp/p.

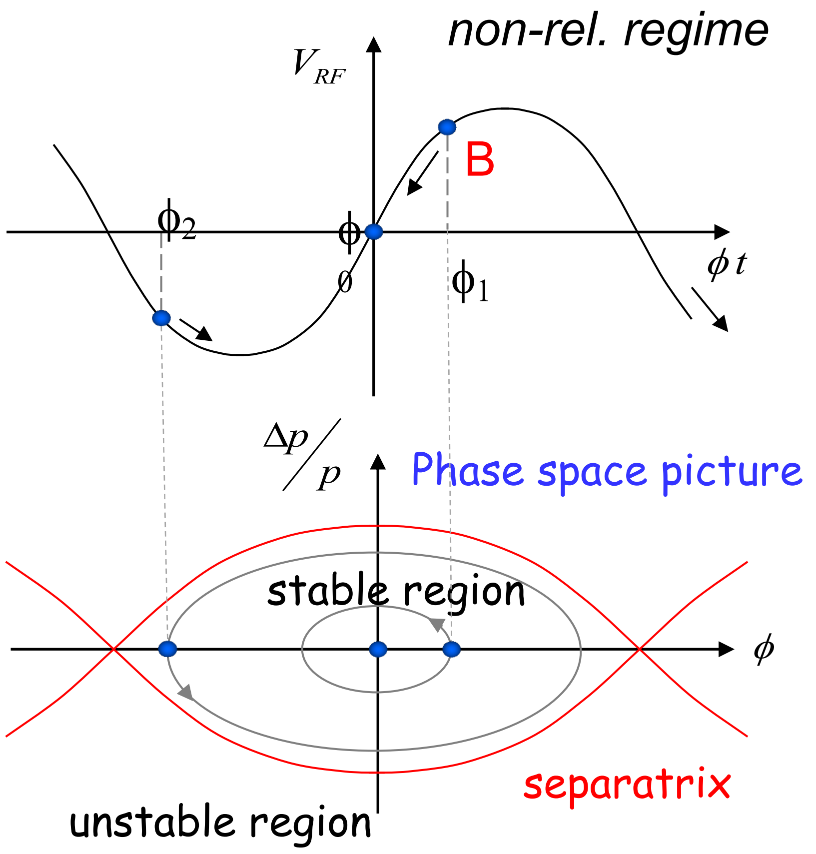
Typically one synchrotron oscillation takes many turns (much slower than betatron oscillation)

Phase-space ellipse defines ***longitudinal emittance***.

Separatrix is the trajectory separating stable and unstable motion.

Stable region is also called ***bucket***.
→ Harmonic number *h* = number of buckets:

$$f_{RF} = h\, f_{rev}$$



Courtesy F. Tecker for drawings

Emittance during Acceleration

What happens to the emittance if the reference momentum P0 changes?

We can write down the transfer matrix for reference momentum change:

$$M_x = \begin{pmatrix} 1 & 0 \\ 0 & P_0/P_1 \end{pmatrix} \Rightarrow \epsilon_{x1} = \frac{P_0}{P_1}\epsilon_{x0}$$

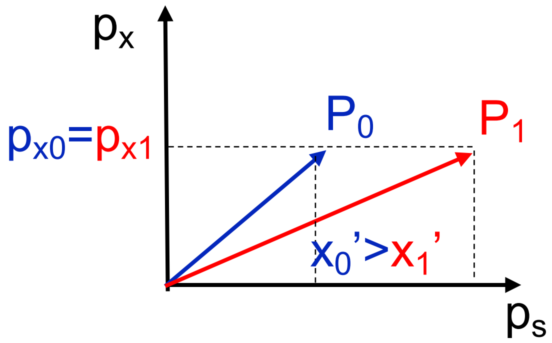
The emittance shrinks with acceleration!

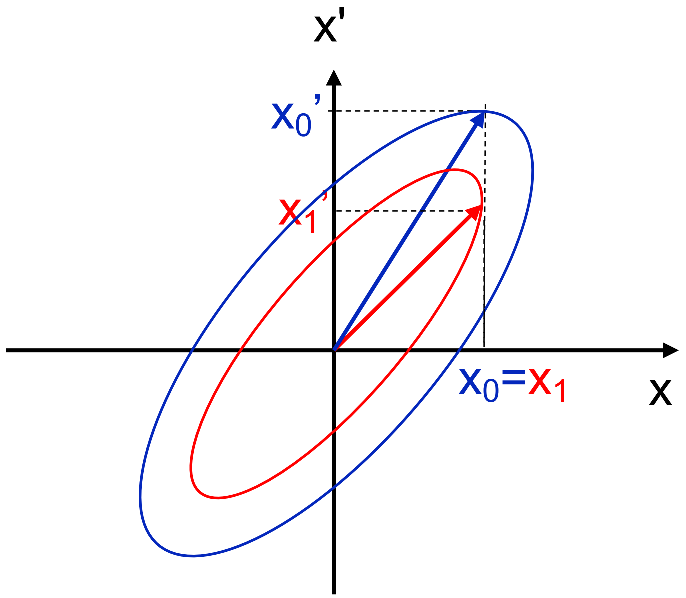
With $P = \beta\gamma mc$ where γ, β are the relativistic parameters.

The conserved quantity is

$$\beta_1\gamma_1\epsilon_{x1} = \beta_0\gamma_0\epsilon_{x0}$$

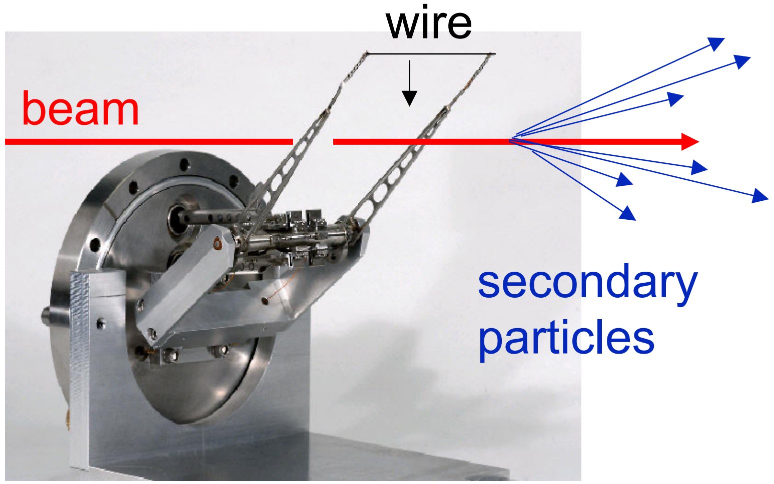
It is called ***normalized emittance***.

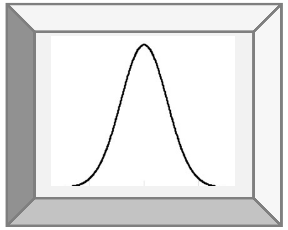




Measuring Beam Size and Emittance

Principle of a wire-scanner beam size measurement

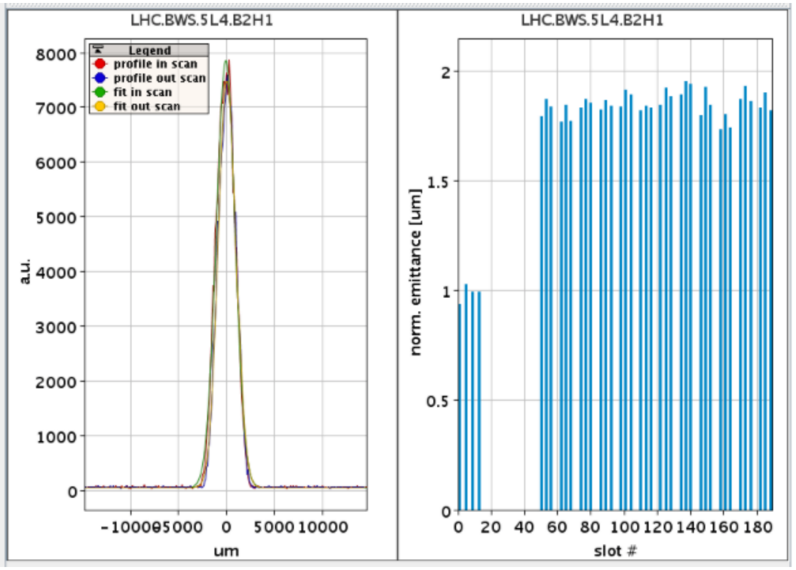




$$\sigma_x = \sqrt{\varepsilon \beta_x}$$

Gaussian fit to profile → beam size σ
Knowledge of β-function → emittance ε

Single horizontal bunch profile



LHC measurement

Emittance calculated from profile measurement.
All circulating bunches.

How big are the beams in the LHC?

Normalized emittance at LHC : ε_n = 3.5 μm
→ ε_n preserved during acceleration.

The **geometric emittance:**

- Injection energy of 450 GeV: ε = 7.3 nm
- Top energy of 7 TeV: ε = 0.5 nm

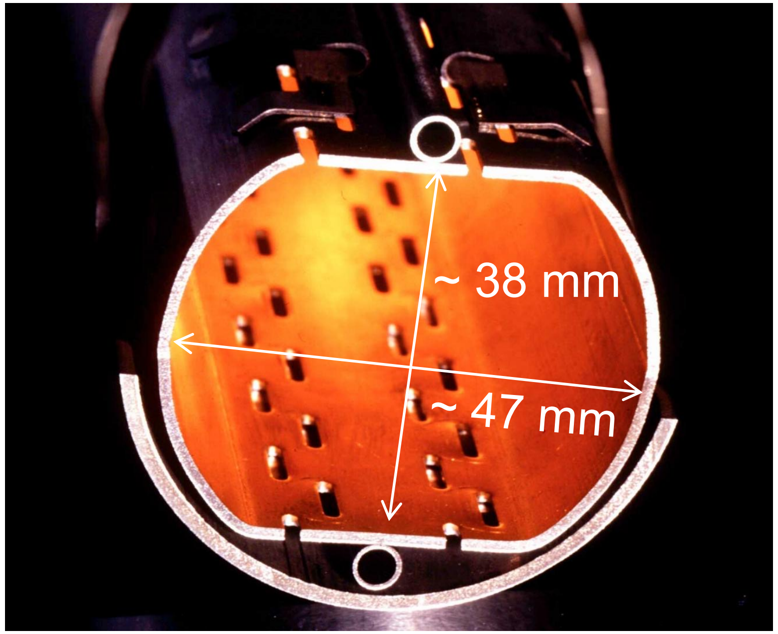
$$\varepsilon_{7TeV} = \varepsilon_{450GeV} \frac{\gamma_{450GeV}}{\gamma_{7TeV}}$$

The corresponding max. **beam sizes** in the arc,
at the location with the maximum beta function (β_{max} = 180 m):

– σ_{450GeV} = 1.1 mm
– σ_{7TeV} = 300 μm

Aperture requirement: a > 10 σ
LHC beam pipe radius:

- Vertical plane: 19 mm ~ 17 σ @ 450 GeV
- Horizontal plane: 23 mm ~ 20 σ @ 450 GeV

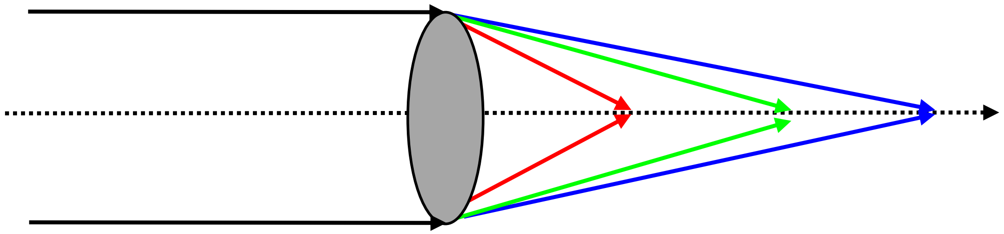


Chromaticity

Dipole magnets generate dispersion, which is then focused by quadrupoles.

focusing strength

$$k = \frac{g}{p/q} [m^{-2}]$$



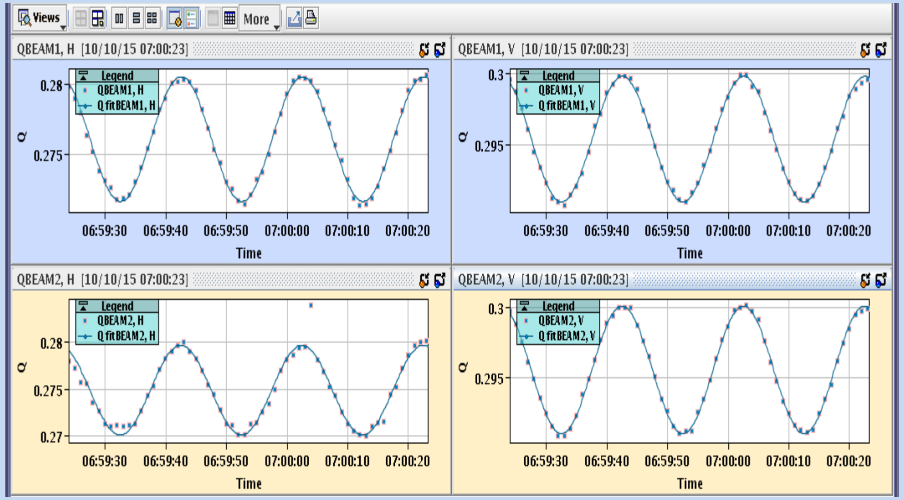
particle having ...
to high energy
to low energy
ideal energy

Chromaticity Q' acts like a quadrupole error and leads to a tune spread.
Definition of Chromaticity:

$$\Delta Q = Q' \frac{\Delta p}{p} \qquad Q' = -\frac{1}{4\pi} \oint k(s)\beta(s)ds$$

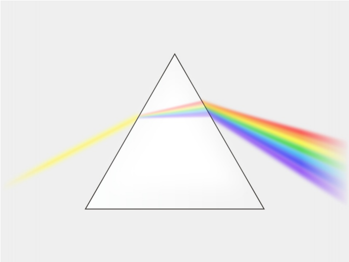
Q' measurement at LHC

The chromaticity is measured by changing / **modulating the energy** offset dp/p **through the RF frequency** while recording the tune change ΔQ.



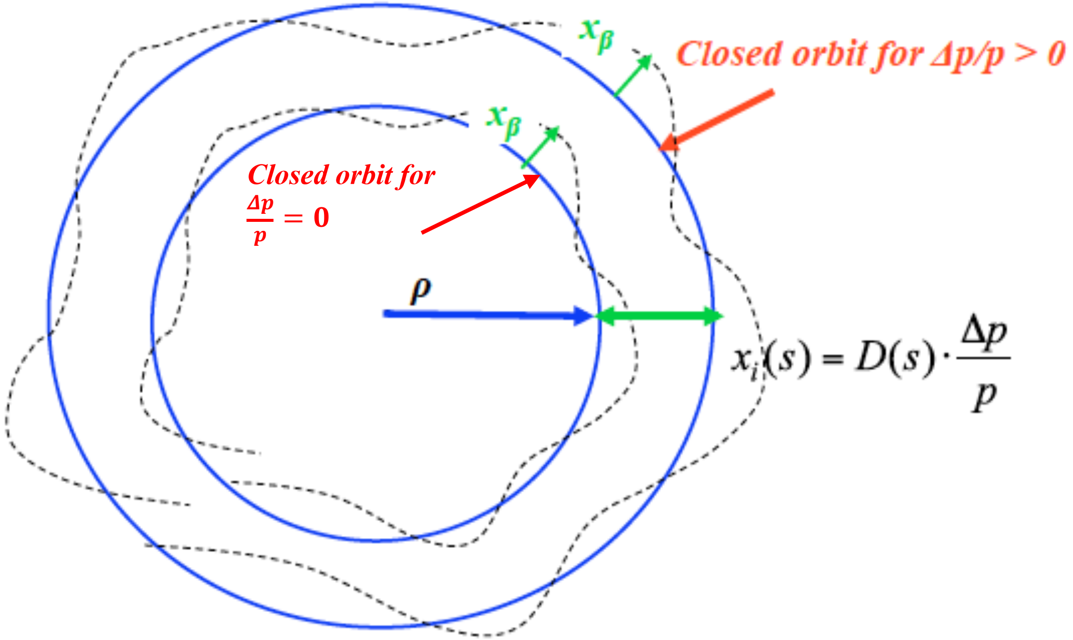
Transverse-Longitudinal Coupling: Dispersion

Dipole magnets generate dispersion → Particles with different momentum are bent differently



Due the momentum spread in the beam $\frac{\Delta p}{p}$, this has to be taken into account for the particle trajectory.

Dispersion function *D(s)*
corresponds to the trajectory of a particle with momentum offset $\frac{\Delta p}{p} = 1$.

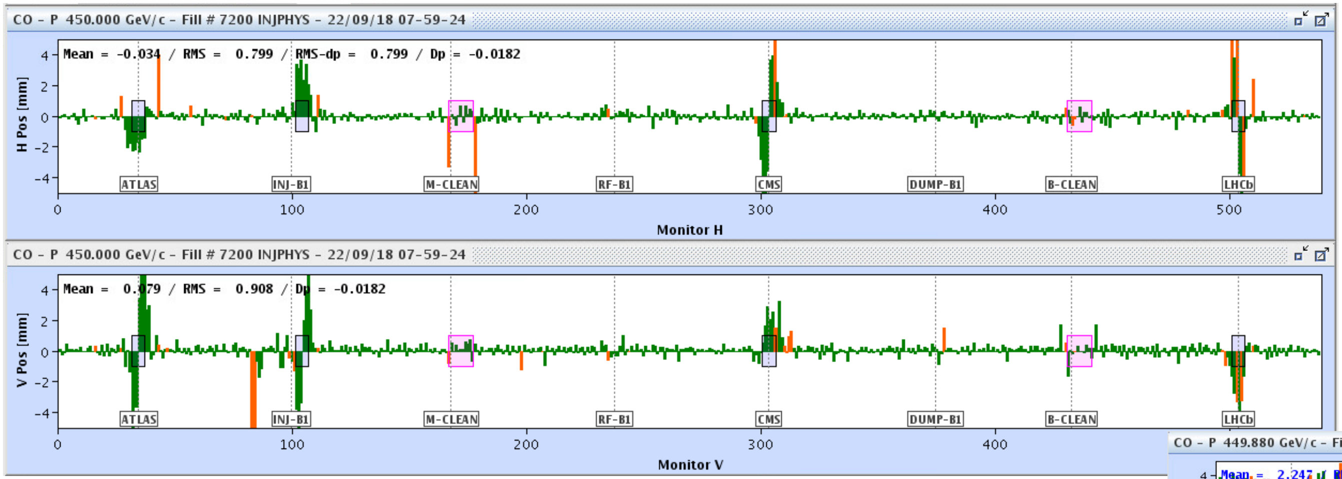


$$x(s) = x_\beta(s) + D(s)\frac{\Delta p}{p}$$

This also has an effect on the beam size:

$$\sigma = \sqrt{\beta\varepsilon} \quad \rightarrow \quad \sigma = \sqrt{\beta\varepsilon + D^2\left(\frac{\Delta p}{p}\right)^2}$$

Dispersive Orbit



On-momentum orbit in LHC

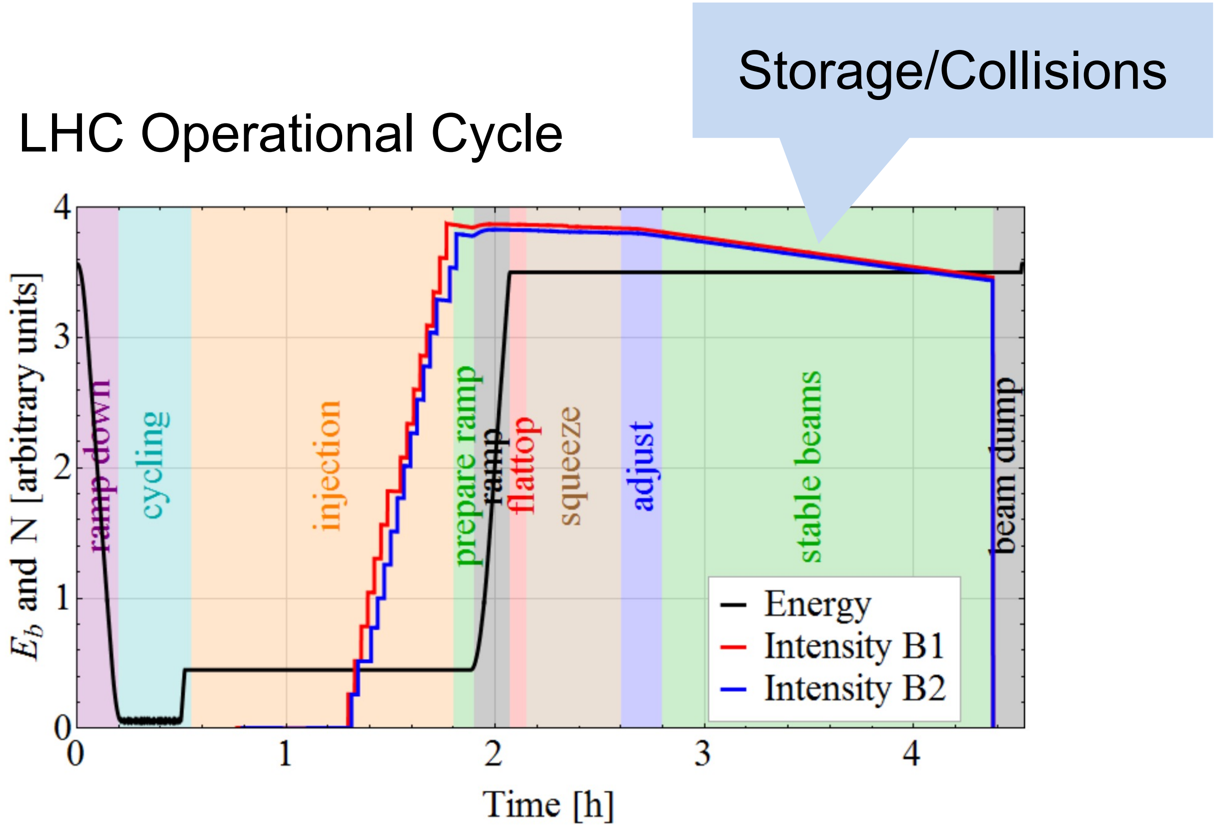
Off-momentum orbit

Dedicated energy (i.e. f_{RF}) change of the stored beam.

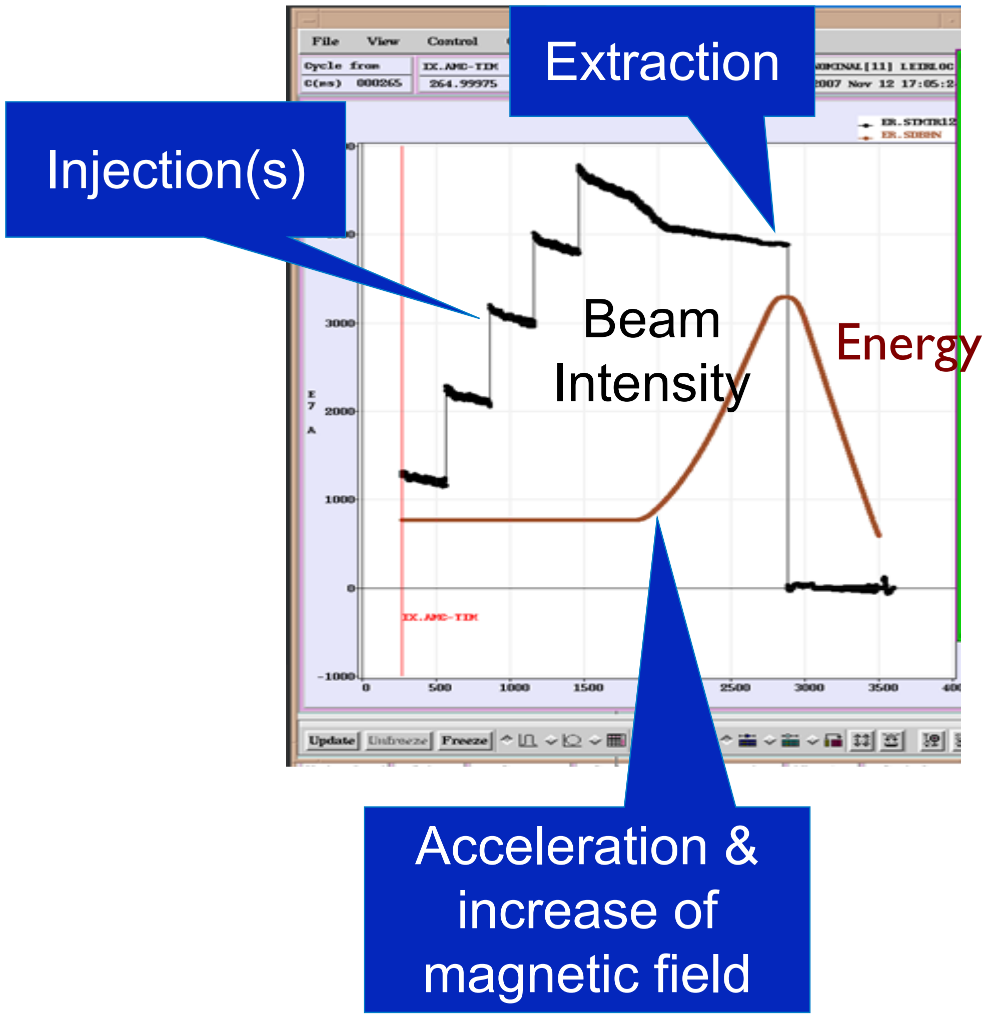
- Horizontal orbit is moved to a dispersions trajectory.
- Vertical orbit unchanged (no vertical dispersion)

Accelerator Cycle

LHC Operational Cycle



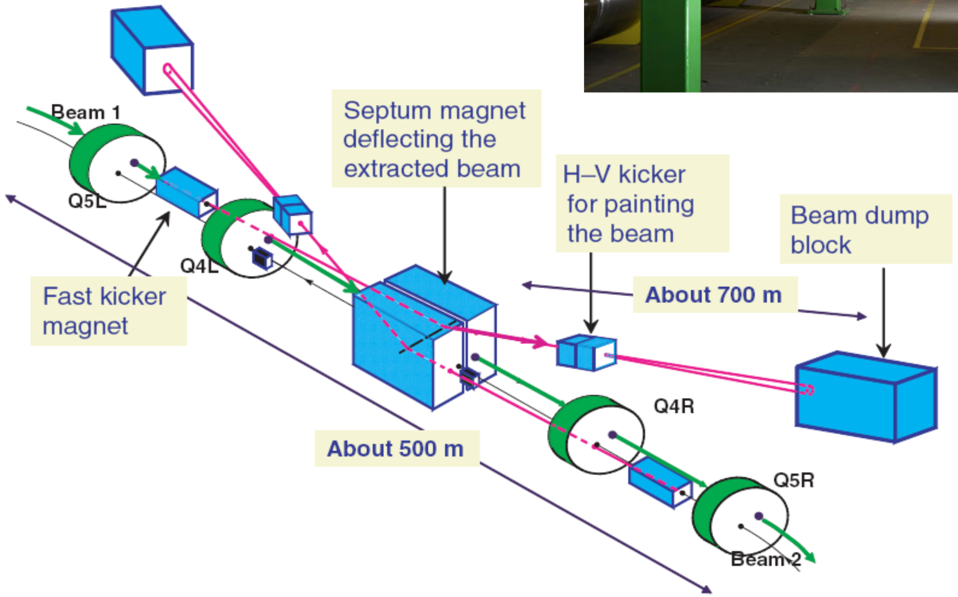
LEIR Operational Cycle

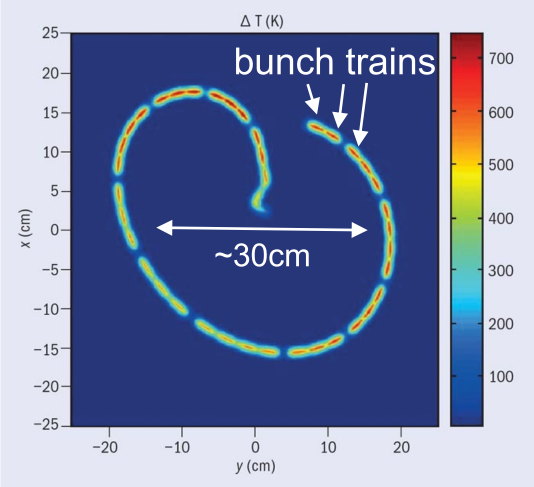


Beam Dump – How to safely kill the LHC beam

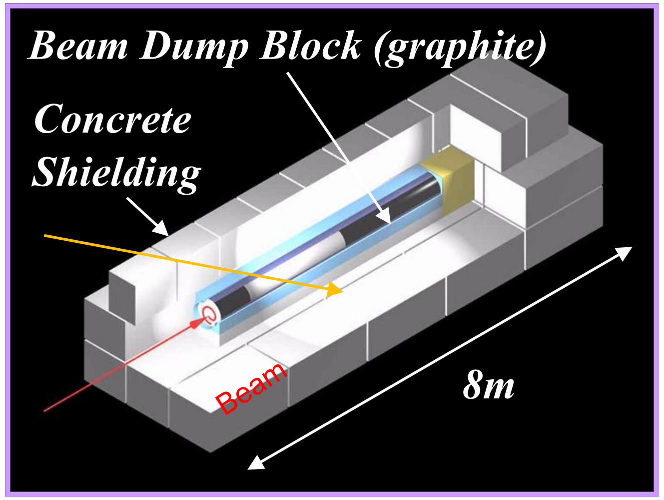
LHC beam stores ~360MJ energy.

Schematic of LHC beam dump system





Sweep of beam on beam dump window



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