QCD

Part 1

M. Diehl

Deutsches Elektronen-Synchroton DESY

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Plan of lectures

- Brief introduction
- Renormalisation, running coupling, running masses
  scale dependence of observables
- $e^+e^- \to$ hadrons
  some basics of applied perturbation theory
- Factorisation and parton densities
  using perturbation theory in ep and pp collisions

these lectures: present theory concepts
for measurements see Lydia Beresford's lectures on LHC physics

Quantum chromodynamics (QCD)

- theory of interactions between quarks and gluons
- different from weak and electromagnetic interactions because coupling α_s is large at small momentum scales
  - quarks and gluons are confined inside bound states: hadrons (proton, neutron, pion, ...)
  - weak-coupling expansion in α_s at high momentum scales, "asymptotic freedom"
- symmetries
  - gauge invariance: group SU(3) $\leftrightarrow$ colour charge
    electromagnetism: U(1) $\leftrightarrow$ electric charge
  - Lorentz invariance and discrete symmetries:
    P (parity = space inversion) T (time reversal)
    C (charge conjugation)
  - chiral symmetry for zero masses of u, d and s
- embedded in Standard Model: quarks couple to γ, W, Z and H

Why care about QCD?

- without quantitative understanding of QCD would have very few physics results from LHC, Belle, ...
- α_s and quark masses are fundamental parameters of nature need e.g.
  - m_t to compute many electroweak effects $\rightarrow$ Higgs physics
  - α_s to discuss possible unification of forces
- QCD is the one strongly interacting quantum field theory we can study in experiment.
  many interesting phenomena:
  - structure of proton
  - confinement
  - chiral symmetry and its breaking
    (blueprint for many composite Higgs models)
  - mathematics: (non)-convergence of perturbative series

Basics of QCD perturbation theory

- split Lagrangian into free and interacting parts:

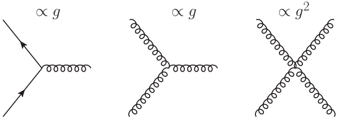
$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}}$$

  - $\mathcal{L}_{\text{int}}$: interaction terms $\propto g$ or g^2 $\qquad \alpha_s = g^2/(4\pi)$
  - expand process amplitudes, cross sections, etc. in powers of g
  - Feynman graphs visualise individual terms in expansion

- from $\mathcal{L}_{\text{free}}$: free quark and gluon propagators

  - in position space: propagation from x^μ to y^μ
  - in momentum space: propagation with four-momentum k^μ

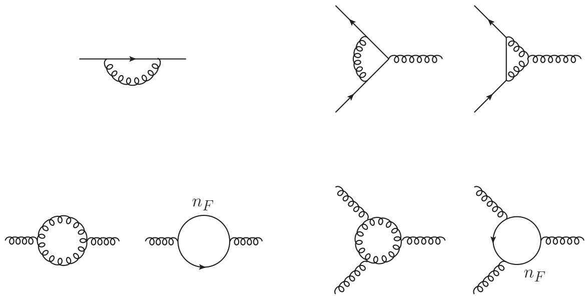
- from $\mathcal{L}_{\text{int}}$: elementary vertices



Loop corrections

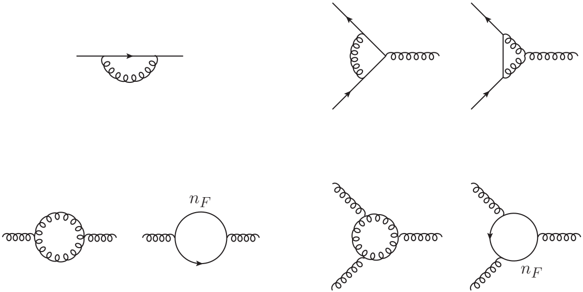
- in loop corrections find ultraviolet (UV) divergences
- only appear in corrections to

propagators | elementary vertices



Exercise: Draw the remaining one-loop graphs for all propagators and elementary vertices

- origin of UV divergences: region of ∞ ly large loop momenta
  $\leftrightarrow$ quantum fluctuations at ∞ ly small space-time distances
- idea: encapsulate UV effects in (a few) parameters
  when describing physics at a scale μ $\rightsquigarrow$ renormalisation



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- idea: encapsulate UV effects in (a few) parameters when describing physics at a scale μ $\rightsquigarrow$ renormalisation
- technically:
  1. regulate: artificial change of theory making div. terms finite
     - physically intuitive: momentum cutoff
     - in practice: dimensional regularisation (dim. reg.)
  2. renormalise: absorb UV effects into
     - coupling constant $\alpha_s(\mu)$
     - quark masses $m_q(\mu)$
     - quark and gluon fields (wave function renormalisation)
  3. remove regulator: quantities are finite when expressed in terms of renormalised parameters and fields
- renormalisation scheme: choice of which terms to absorb
  "∞" is as good as "$\infty + \log(4\pi)$"

Dimensional regularisation in a nutshell

- choice of regulator ≈ choice between evils
- dim. reg.: no physics intuition, but keeps intact essential symmetries (gauge and Lorentz invariance)
- idea: integrals for Feynman graphs become UV finite in lower space-time dimension, e.g.

$$\int \frac{d^D k}{(2\pi)^D} \, \frac{1}{k^2 - m^2} \, \frac{1}{(k-p)^2 - m^2}$$

log. div. for $D = 4$
converg. for $D = 3, 2, 1$

- more detail ⇝ blackboard

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- procedure:
  1. formulate theory in D dimensions (with D small enough)
  2. analytically continue results from integer to complex D
     original divergences appear as poles in $1/\epsilon$ ($D = 4 - 2\epsilon$)
  3. renormalise ($\overline{\text{MS}}$ scheme: subtract poles and a const.)
  4. take $\epsilon \to 0$

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- enter: a mass scale μ
  - coupling in $4 - 2\epsilon$ dimensions is $\mu^\epsilon g$ with g dimensionless
    necessary to get dimensionless action $\int d^D x \, \mathcal{L}$
  - any other regularisation introduces a mass parameter as well

$\rightsquigarrow$ renormalised quantities depend on μ

Renormalisation group equations (RGE)

- scale dependence of renormalised quantities described by differential equations:

$$\frac{d}{d\log\mu^2}\,\alpha_s(\mu) = \beta\big(\alpha_s(\mu)\big) \qquad\qquad \alpha_s = \frac{g^2}{4\pi}$$

$$\frac{d}{d\log\mu^2}\,m_q(\mu) = m_q(\mu)\,\gamma_m\big(\alpha_s(\mu)\big)$$

- β, γ_m = perturbatively calculable functions in region where $\alpha_s(\mu)$ is small enough

$$\beta = -b_0\,\alpha_s^2\,\big[1 + b_1\alpha_s + b_2\,\alpha_s^2 + b_3\,\alpha_s^3 + \ldots\big]$$

$$\gamma_m = -c_0\,\alpha_s\,\big[1 + c_1\alpha_s + c_2\,\alpha_s^2 + c_3\,\alpha_s^3 + \ldots\big]$$

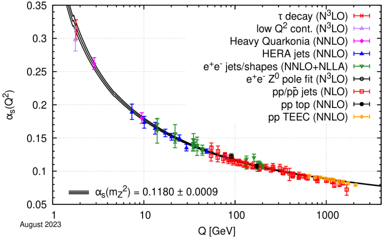
coefficients known including b_4, c_4 (five loops) (b_4 since 2016)

$$b_0 = \tfrac{1}{4\pi}\big(11 - \tfrac{2}{3}n_F\big) \qquad\qquad c_0 = \tfrac{1}{\pi}$$

The running of α_s

- $\beta_{\text{QCD}} < 0$
  $\Rightarrow \alpha_s(\mu)$ decreases with μ

Nobel prize  2004 for
Gross, Politzer and Wilczek



plot: Review of Particle Properties 2024

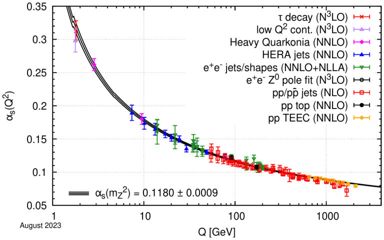
- asymptotic freedom at large μ
- perturbative expansion becomes invalid at low μ
  quarks and gluons are strongly bound inside hadrons: confinement
  momenta below $1\,\text{GeV}$ $\leftrightarrow$ distances above $0.2\,\text{fm}$

The running of α_s

- truncating $\beta = -b_0\, \alpha_s^2\, (1 + b_1 \alpha_s)$ get

$$\alpha_s(\mu) = \frac{1}{b_0 L} - \frac{b_1 \log L}{(b_0 L)^2} + \mathcal{O}\Big(\frac{1}{L^3}\Big)$$

with $L = \log \dfrac{\mu^2}{\Lambda_{\text{QCD}}^2}$



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- dimensional transmutation:
  mass scale Λ_{QCD} not in Lagrangian, reflects quantum effects
- more detail $\rightsquigarrow$ blackboard

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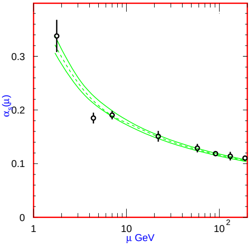

plot: Review of Particle Properties 2003

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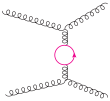
Scale dependence of observables

- observables computed in perturbation theory depend on renormalisation scale μ
  - implicitly through $\alpha_s(\mu)$
  - explicitly through terms $\propto \log(\mu^2/Q^2)$ where $Q =$ typical scale of process



e.g. $Q = p_T$ for production of particles with high p_T
$Q = M_H$ for decay Higgs $\to$ hadrons
$Q =$ c.m. energy for $e^+e^- \to$ hadrons

- μ dependence of observables must cancel at accuracy of the computation

  see how this works $\rightsquigarrow$ blackboard

Scale dependence of observables

- for generic observable C have expansion

$$C(Q) = \alpha_s^n(\mu)\left[C_0 + \alpha_s(\mu)\left\{C_1 + nb_0C_0\log\frac{\mu^2}{Q^2}\right\} + \mathcal{O}\big(\alpha_s^2\big)\right]$$

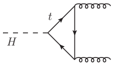
- Exercise: check that this satisfies

$$\frac{d}{d\log\mu^2}\,C = \mathcal{O}\big(\alpha_s^{n+2}\big)$$

$\Rightarrow$ residual scale dependence when truncate perturbative series

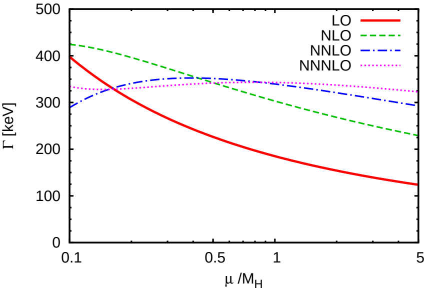
- at higher orders:
  $\alpha_s^{n+k}(\mu)$ comes with up to k powers of $\log(\mu^2/Q^2)$
  - choose $\mu \sim Q$ so that $|\alpha_s \log(\mu/Q)| \ll 1$
    otherwise higher-order terms spoil series expansion

Example



- inclusive hadronic decay of Higgs boson
  via top quark loop (i.e. without direct coupling to b quark)

- in perturbation theory: $H \to 2g$, $H \to 3g$, $\ldots$
  calculated to N^{3}LO — Baikov, Chetyrkin 2006
  and to N^{4}LO — Herzog et al 2017



- scale dependence decreases at higher orders
- choice $\mu < M_H$ more appropriate than $\mu = M_H$
- scale variation by factor 2 up and down often taken as estimate of higher-order corrections
  simple and easy to do
  but must not be over-interpreted

Quark masses

- recall: α_s and m_q depend on renormalisation scheme
  - standard in QCD: $\overline{\text{MS}}$ scheme $\rightsquigarrow$ running $\alpha_s(\mu)$ and $m_q(\mu)$
  - for heavy quarks c, b, t can also use pole mass/on-shell scheme

    standard in QED for electron, muon, etc.

    scheme transformation:

$$m_{\text{pole}} = m(\mu) \left[1 + \frac{\alpha_s(\mu)}{\pi} \left(\frac{4}{3} - \log \frac{m^2(\mu)}{\mu^2} \right) + \mathcal{O}(\alpha_s^2) \right]$$

- $\overline{\text{MS}}$ masses from Review of Particle Properties 2024

$$m_u = 2.16(7)\ \text{MeV} \qquad m_d = 4.70(7)\ \text{MeV} \qquad m_s = 93.5(0.8)\ \text{MeV}$$

at $\mu = 2\,\text{GeV}$

$$\overline{m}_c = 1.2730(46)\ \text{GeV} \quad \overline{m}_b = 4.183(7)\ \text{GeV} \quad \overline{m}_t = 162.5^{+2.1}_{-1.5}\ \text{GeV}$$

with $m_q(\mu = \overline{m}_q) = \overline{m}_q$

Summary of Part 1

- beyond all technicalities reflects physical idea:
  eliminate details of physics at scales $\gg$ scale Q of an observable
- running of α_s $\rightsquigarrow$ characteristic features of QCD:
  - asymptotic freedom at high scales $\rightsquigarrow$ use perturbation theory
  - strong interactions at low scales $\rightsquigarrow$ need other methods
  - introduces mass scale Λ_{QCD} into theory
- dependence of observable on μ governed by RGE
  reflects (and estimates) particular higher-order corrections

  . . . but not all