QCD

Part 2

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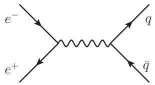
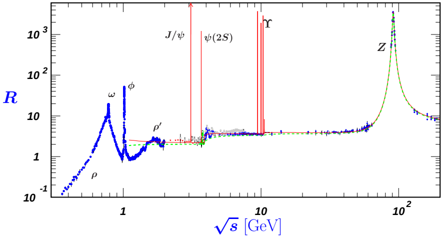
$e^+e^- \to$ hadrons

$$R = \frac{\sigma(e^+e^- \to X)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

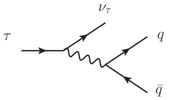
for $\sqrt{s} \gg$ resonance masses



plot: Review of Particle Properties 2021



- removing electroweak part $\rightsquigarrow \sum_X |\mathcal{A}(\gamma^* \text{ or } Z^* \to X)|^2$
- among simplest applications of perturbative QCD
  - fully inclusive final state
  - no hadrons in initial state
- closely related theory description for

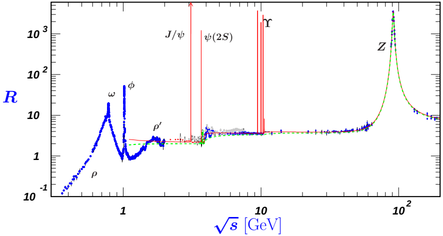


$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau + X)}{\Gamma(\tau \to \nu_\tau + e\nu_e)} \quad \rightsquigarrow \quad \sum_X |\mathcal{A}(W^* \to X)|^2$$

at lowest order in α_s:

$$R_0 = N_c \sum_q e_q^2$$

from $\gamma^* \to q\bar{q}$ with $m_q = 0$



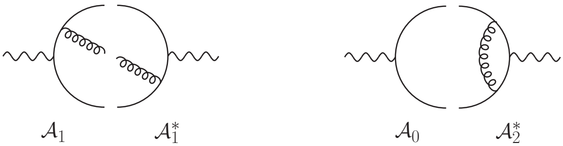
- expansion known up to $R = R_0 \left[1 + \frac{1}{\pi}\alpha_s + C_2\,\alpha_s^2 + C_3\,\alpha_s^3 + C_4\,\alpha_s^4 \right]$
  - quark mass corrections also partly known
  - same for τ decays
  - suitable observables for α_s determination
- underlying concept: parton-hadron duality:

$$\sum_{X \in \text{partons}} |\mathcal{A}(\gamma^* \to X)|^2 = \sum_{X \in \text{hadrons}} |\mathcal{A}(\gamma^* \to X)|^2$$

  - $\gamma^* \to$ partons valid description for short space-time $\sim 1/\sqrt{s}$
  - subsequent dynamics changes final state, but not inclusive rate

A closer look at the $\mathcal{O}(\alpha_s)$ corrections

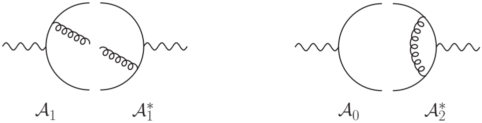
- expand $\mathcal{A}(q\bar{q}g) = g\mathcal{A}_1 + \ldots$ and $\mathcal{A}(q\bar{q}) = \mathcal{A}_0 + g^2\mathcal{A}_2 + \ldots$



real corrections: extra partons in final state
virtual corrections: loops in $\mathcal{A}$ or $\mathcal{A}^*$

- virtual corrections have UV divergences
  $\to$ standard renormalisation procedure
- real and virtual corrections: soft and collinear divergences
  - regions where gluon momentum $\to 0$ or $\propto$ momentum of q or $\bar{q}$
  - cancel in sum over all graphs

A closer look at soft and collinear divergences



- more detail $\rightsquigarrow$ blackboard

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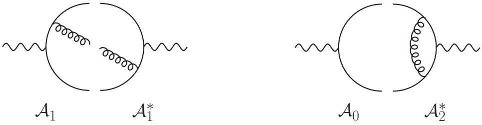
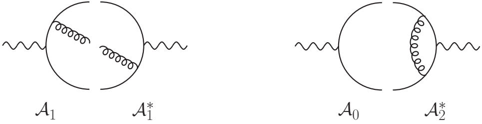


- have soft (= IR) div. because of massless gluons
  same phenomenon in QED: soft photons $\to$ "IR catastrophe"
- have collinear (= mass) div. if set quark masses to zero

  could formally keep $m_q \neq 0$, but perturbation theory
  not trustworthy at scales $\sim m_u, m_d, m_s$
- divergences cancel, result dominated by large virtualities

  otherwise could not use parton-hadron duality
- technical difficulty: cancellations take place in $D \neq 4$ dimensions
  for differential cross sections and numerics want $D = 4$

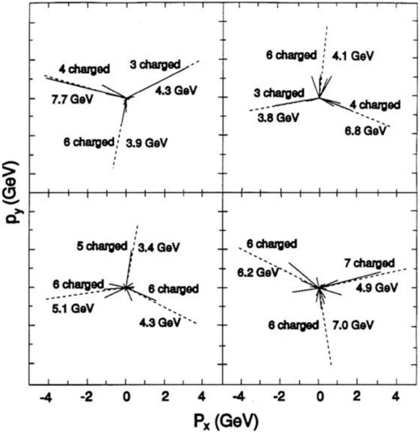
A footprint of divergences: large logarithms



- both soft and collinear divergences are logarithmic: $\int dE/E \int d\theta/\theta$
- fixing final-state momenta restricts integration region in real corrections, but not in virtual ones
  - for each emission get double logarithm $\propto \alpha_s \log^2(\ldots)$ "Sudakov logarithms"
  - if logarithms are large must sum them to all orders in α_s "resummation"
  - can be done analytically for certain cases
  - done by "parton showers" in Monte Carlo generators

Beyond inclusive final states: hadronic jets

- jet = "bunch of hadrons moving approx. in same direction"
- perhaps the most direct manifestation of quarks or gluons



three-jet events in e^+e^- annihilation at $\sqrt{s} = 27.4\,\mathrm{GeV}$
TASSO (DESY) 1979

figure from: P Söding,
On the discovery of the gluon
Eur.Phys.J. H 35 (2010) 3

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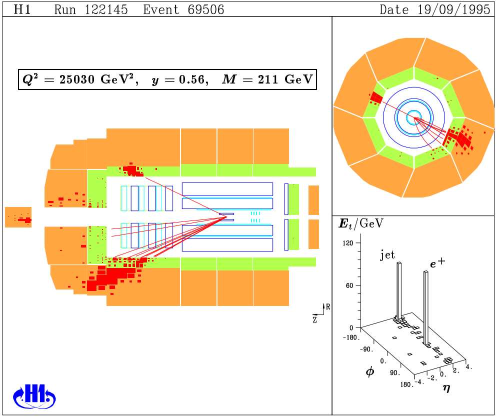

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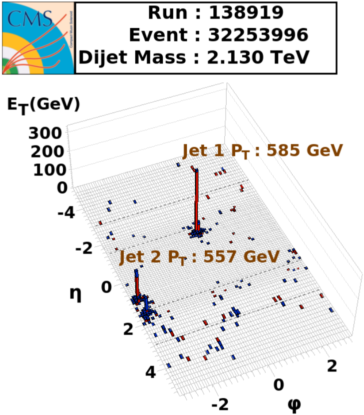
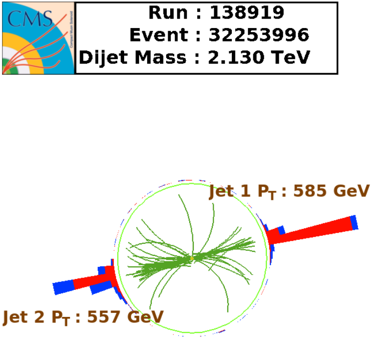
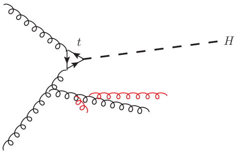




Beyond inclusive final states: hadronic jets

- extend idea of parton-hadron duality: dynamics leading from partons (times $\sim 1/Q$) to final-state hadrons (times $\to \infty$) approx. conserves momentum (hadronisation effects $\sim \mathrm{GeV}$)
- to minimise theory uncertainties:
  - define hadronic jets using an algorithm that is not sensitive to collinear and soft radiation (beyond perturbative control)

"collinear and infrared safe observables"



  - apply to partons in computation, to hadrons in measurement
  - hadronisation corrections should then be moderate and typically decrease with jet p_T estimate using Monte Carlo generators

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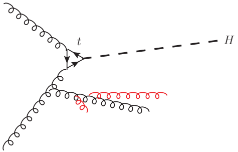

  - different jet definitions are used for different purposes
  - jet substructure observables as tools to reconstruct underlying parton-level dynamics
    $\rightsquigarrow$ active field of research

Summary of Part 2

- perturbative calculations beyond tree level
  only for quantities that are IR and collinear safe
  and hence dominated by large virtualities
- simplest examples: total cross sections/decay rates for colourless initial states
- for differential cross sections/distributions: can have large double logarithms from soft and collinear emissions