QCD

Part 3

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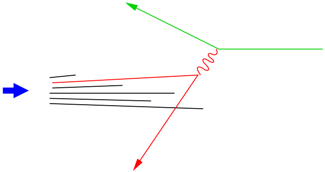
HELMHOLTZ

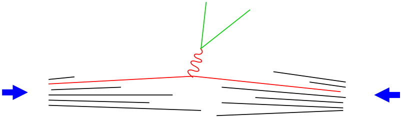
DESY.

The parton model

- describes deep inelastic scattering, Drell-Yan process, etc.
  - fast-moving hadron
    $\approx$ set of free partons $(q, \bar{q}, g)$ with low transverse momenta
  - physical cross section
    $=$ cross section for partonic process $(\gamma^* q \to q,\ q\bar{q} \to \gamma^*)$
    $\times$ parton densities



Deep inelastic scattering (DIS): $\ell p \to \ell X$



Drell-Yan: $pp \to \ell^+ \ell^- X$

Nobel prize 1990 for
Friedman, Kendall, Taylor

The parton model

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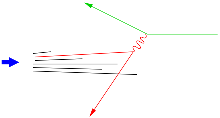
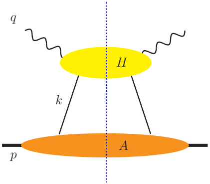
Factorisation

- implements and corrects parton-model ideas in QCD
  - conditions and limitations of validity
    kinematics, processes, observables
  - corrections: partons interact
    α_s small at large scales $\rightsquigarrow$ perturbation theory
  - define parton densities in field theory
    derive their general properties
    make contact with non-perturbative methods

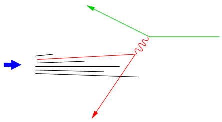
Factorisation: physics idea and technical implementation

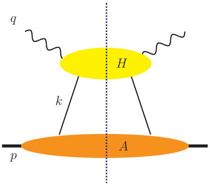




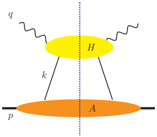
- idea: separation of physics at different scales
  - high scales: quark-gluon interactions
    $\rightsquigarrow$ compute in perturbation theory
  - low scale: proton $\rightarrow$ quarks, antiquarks, gluons
    $\rightsquigarrow$ parton densities
- requires hard momentum scale in process
  large photon virtuality $Q^2 = -q^2$ in DIS

Factorisation: physics idea and technical implementation





- implementation: separate process into
  - "hard" subgraph H with particles far off-shell
    compute in perturbation theory
  - "collinear" subgraph A with particles moving along proton
    turn into definition of partOn density

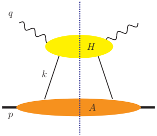


Collinear expansion

- graph gives $\int d^4k\, H(k) A(k)$; simplify further
- light-cone coordinates: $v^{\pm} = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$, $\boldsymbol{v} = (v^1, v^2)$

  more detail ⇝ blackboard

Collinear expansion



- graph gives $\int d^4k\, H(k)A(k)$; simplify further
- in hard graph neglect small components of external lines
  ⇝ Taylor expansion

$$H(k^+, k^-, k_T) = H(k^+, 0, 0) + \text{corrections}$$

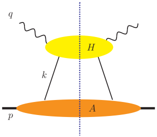
⇝ loop integration greatly simplifies:

$$\int d^4k\; H(k)\, A(k) \approx \int dk^+\, H(k^+, 0, 0) \int dk^- d^2k_T\, A(k^+, k^-, k_T)$$

- in hard scattering treat incoming/outgoing partons as exactly collinear $(k_T = 0)$ and on-shell $(k^- = 0)$
- in collin. matrix element integrate over k_T and virtuality
  ⇝ collinear (or k_T integrated) parton densities
  only depend on $k^+ = xp^+$

further subtleties related with spin of partons, not discussed here

Definition of parton distributions



- matrix elements of quark/gluon operators

$$f_q(x) = \int \frac{dz^-}{2\pi} \, e^{ixp^+z^-} \, \langle p \big| \bar{\psi}(0) \, \tfrac{1}{2}\gamma^+ \psi(z) \big| p \rangle \Big|_{z^+=0,\, z_T=0}$$

$\psi(z) =$ quark field operator: annihilates quark

$\bar{\psi}(0) =$ conjugate field operator: creates quark

$\frac{1}{2}\gamma^+ =$ matrix in Dirac space: sums over quark spin

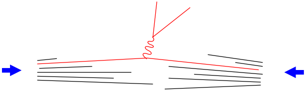
$(2\pi)^{-1} \int dz^- \, e^{ixp^+z^-}$ projects on quarks with $k^+ = xp^+$

- analogous definitions for polarised quarks, antiquarks, gluons
- analysis of factorisation used Feynman graphs
  but here provide non-perturbative definition

further subtleties related with choice of gauge, not discussed here

Factorisation for pp collisions

- example: Drell-Yan process $pp \to \gamma^* + X \to \mu^+\mu^- + X$ where X = any number of hadrons
- one parton distribution for each proton $\times$ hard scattering $\rightsquigarrow$ deceptively simple physical picture



Factorisation for pp collisions

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- "spectator" interactions produce additional particles
  which are also part of unobserved system X ("underlying event")
- need not calculate this thanks to unitarity
  as long as cross section/observable sufficiently inclusive
- but must calculate/model if want more detail on the final state
  $\rightsquigarrow$ multiparton interactions

More complicated final states

- production of W, Z or other colourless particle (Higgs, etc) same treatment as Drell-Yan
- jet production in ep or pp: hard scale provided by p_T
- heavy quark production: hard scale is m_c, m_b, m_t

Importance of factorisation concept

- describe processes for study of electroweak and BSM physics, e.g.
  - W mass measurement
  - determination of Higgs boson properties
  - signal and background in new physics searches
- determine parton densities as a tool to make predictions and to learn about proton structure
  - requires many processes to disentangle quark flavors and gluons

A closer look at one-loop corrections

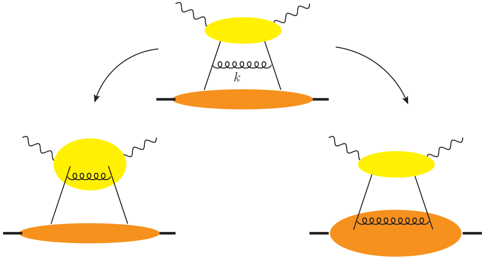
- example: DIS

- UV divergences removed by standard renormalisation
- soft divergences cancel in sum over graphs
- collinear div. do not cancel, have integrals

$$\int_0 \frac{dk_T^2}{k_T^2}$$

what went wrong?

- hard graph should not contain internal collinear lines
  collinear graph should not contain hard lines
- must not double count $\rightsquigarrow$ factorisation scale μ



- with cutoff: take $k_T > \mu$ — take $k_T < \mu$
  $1/\mu \sim$ transverse resolution

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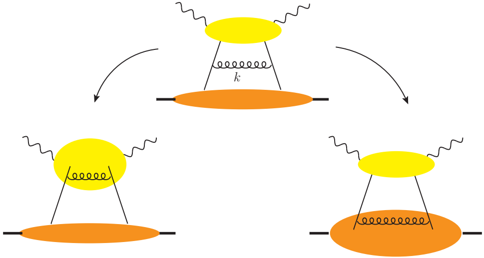
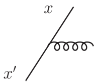


- with cutoff: take $k_T > \mu$ take $k_T < \mu$
  $1/\mu \sim$ transverse resolution
- in dim. reg.:
  subtract collinear divergence subtract ultraviolet div.

The evolution equations

- DGLAP equations

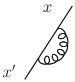
$$\frac{d}{d\log\mu^2}\, f(x,\mu) = \int_x^1 \frac{dx'}{x'}\, P\Big(\frac{x}{x'}\Big)\, f(x',\mu) \;=\; \big(P\otimes f(\mu)\big)(x)$$



- $P =$ splitting functions
  - have perturbative expansion

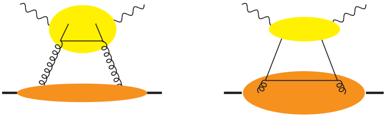
$$P(x) = \alpha_s(\mu)\, P^{(0)}(x) + \alpha_s^2(\mu)\, P^{(1)}(x) + \alpha_s^3(\mu)\, P^{(2)}(x) \ldots$$

known to $\mathcal{O}(\alpha_s^3)$, in part to $\mathcal{O}(\alpha_s^4)$ Moch, Vermaseren, Vogt



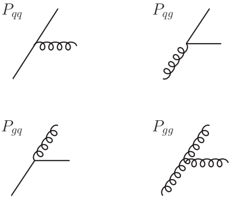
  - contains terms $\propto \delta(1-x)$ from virtual corrections

- quark and gluon densities mix under evolution:



- matrix evolution equation

$$\frac{d}{d\log\mu^2}\, f_i(x,\mu) = \sum_{j=q,\bar{q},g} \big(P_{ij} \otimes f_j(\mu)\big)(x) \qquad (i,j = q,\bar{q},g)$$



more transitions possible at higher orders in α_s

- parton content of proton depends on resolution scale μ

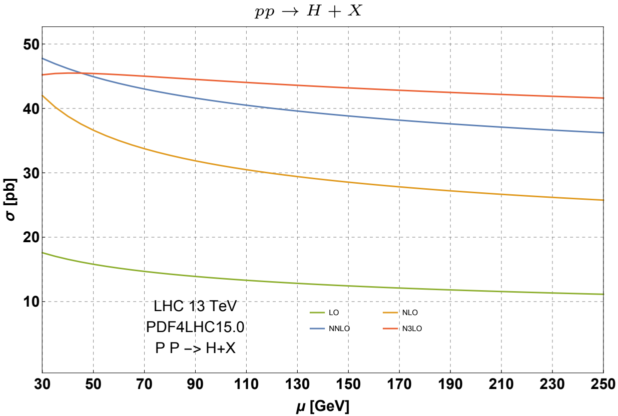
Factorisation formula

- example: $p + p \to H + X$

$$\sigma(p+p \to H+X) = \sum_{i,j=q,\bar{q},g} \int dx_i\, dx_j\; f_i(x_i,\mu_F)\, f_j(x_j,\mu_F) \times \hat{\sigma}_{ij}\big(x_i, x_j, \alpha_s(\mu_R), \mu_R, \mu_F, m_H\big) + \mathcal{O}\bigg(\frac{\Lambda^2}{m_H^4}\bigg)$$

  - $\hat{\sigma}_{ij}$ = cross section for hard scattering $i + j \to H + X$
    m_H provides hard scale
  - μ_R = renormalisation scale, μ_F = factorisation scale
    may take different or equal
  - μ_F dependence in C and in f cancels up to higher orders in α_s
    similar discussion as for μ_R dependence
  - accuracy: α_s expansion and power corrections $\mathcal{O}(\Lambda^2/m_H^2)$
- can make σ and $\hat{\sigma}$ differential in kinematic variables, e.g. p_T of H

Scale dependence



Mistlberger, arXiv:1802.00833

$$\mu_F = \mu_R = \mu$$

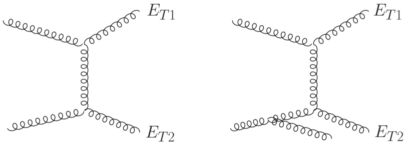
LO, NLO, and higher

- instead of varying scale(s) may estimate higher orders by comparing N^{n}LO result with N^{n-1}LO
- caveat: comparison NLO vs. LO may not be representative for situation at higher orders

often have especially large step from LO to NLO

- certain types of contribution may first appear at NLO
  e.g. terms with gluon density $g(x)$ in DIS, $pp \to Z + X$, etc.
- final state at LO may be too restrictive

  e.g. in $\dfrac{d\sigma}{dE_{T1}\, dE_{T2}}$ for dijet production



Summary of Part 3

- implements ideas of parton model in QCD
  - perturbative corrections (NLO, NNLO, ...)
  - field theoretical def. of parton densities
    $\rightsquigarrow$ bridge to non-perturbative QCD
- valid for sufficiently inclusive observables
  and up to power corrections in Λ/Q or $(\Lambda/Q)^2$
  which are in general not calculable
- must in a consistent way
  - remove collinear kinematic region in hard scattering
  - remove hard kinematic region in parton densities
    $\leftrightarrow$ UV renormalisation

  procedure introduces factorisation scale μ_F
  - separates "collinear" from "hard", "object" from "probe"