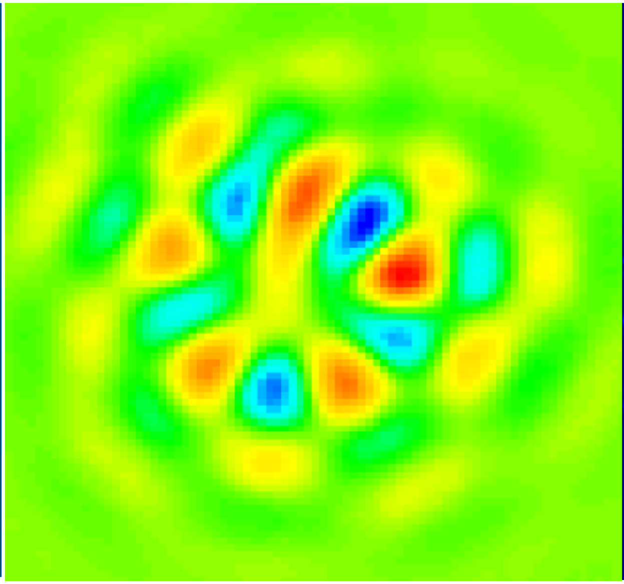
Chromaticity and wakefield “sharing” on the coupling resonance



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Computing
Weed cluster at ASD; Argonne Bebop and Blues clusters at the LCRC, Argonne

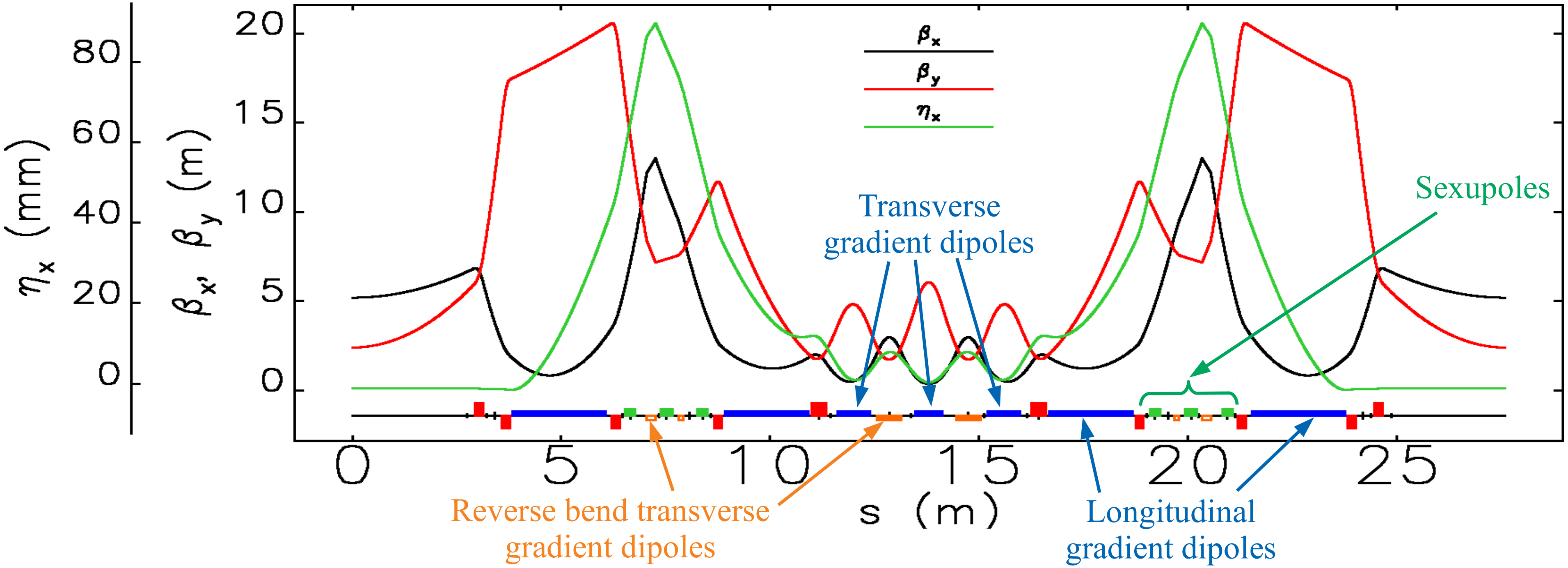
Important ideas and theories have come from many people

Experimental measurements made with the help of
Louis Emery, A.J. Dick, and the APS-U operators

Our tracking simulations all use `elegant`[1]

[1] M.Borland. ANL/APS LS-287, Advanced Photon Source (2000);
Y. Wang and M. Borland. Proc. AAC Workshop, AIP Conf. Proc. **877**, 241 (2006).

The APS underwent a major upgrade to a MBA



Parameter	Value
Energy	6 GeV
(ν_x, ν_y)	(96.1, 36.1)
Coupled $\varepsilon_x = \varepsilon_y$	30 pm
α_c	$4.0\text{x}10^{-5}$
σ_δ	0.135 %
Single rf σ_t	15 ps
Double rf σ_t	100 ps
Total current	200 mA
Single bunch charge	2.27 nC, 15.3 nC

- The APS-Upgrade is a 7-bend hybrid achromat[2]
- Natural emittance $\varepsilon_{x,0}$ = 42 pm with reverse bend transverse gradient dipoles[3] (AKA displaced quads)

[2] L. Farvacque *et al.*, IPAC 2013, pp 79; M. Borland, in *FDR for APS-U* (2019)
[3] J.P. Delahaye and J.P. Potier, PAC 1989, pp. 1611; B. Riemann and A. Streun, PRAB **22**, 021601 (2019).

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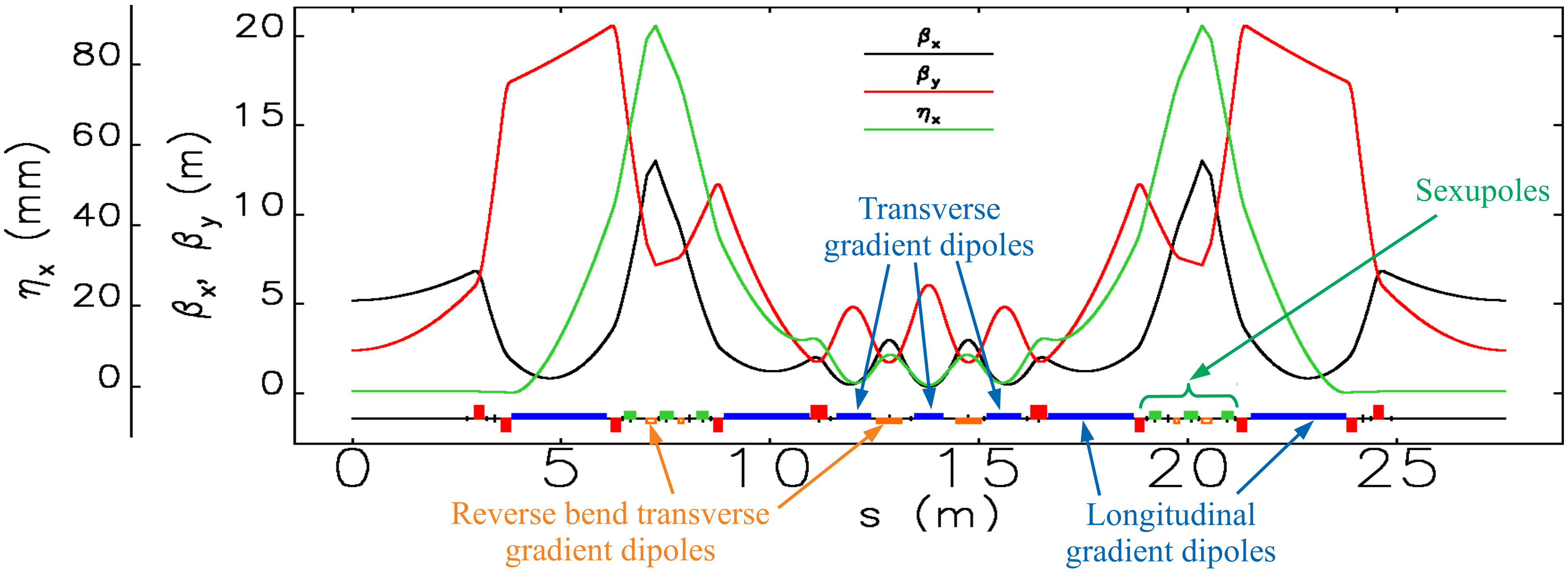

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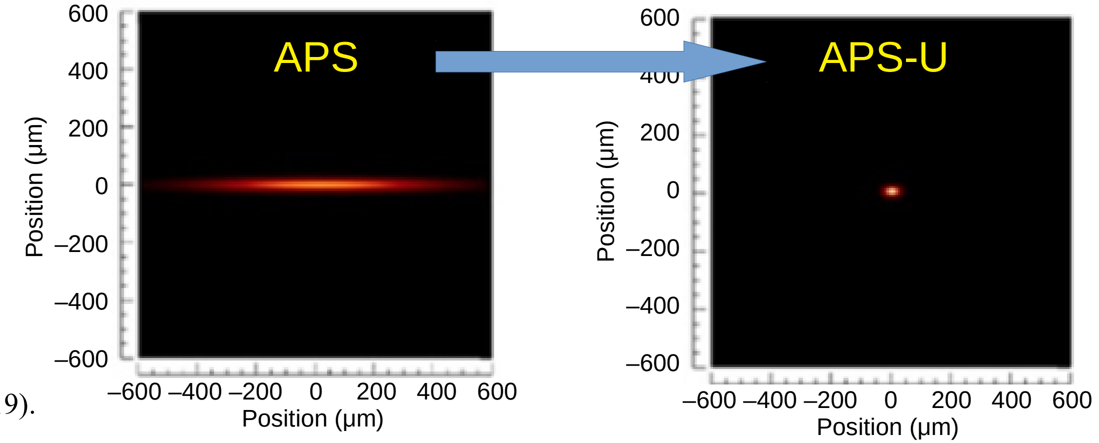
- The APS-Upgrade is a 7-bend hybrid achromat[2]
- Natural emittance $\varepsilon_{x,0} = 42$ pm with reverse bend transverse gradient dipoles[3] (AKA displaced quads)
- Operates on coupling resonance $\nu_x = \nu_y$ for round beams
- Small momentum compaction and natural bunch length



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Collective effects at the APS-U

- The APS-Upgrade is susceptible to all the usual storage ring collective effects
- So far, collective effects have had a relatively minor impact at the APS-U
  - Early evidence of the ion instability that has not resurfaced once the vacuum improved and we adopted a uniform fill with 216 bunches for user operations
  - Some evidence of rf heating in the injection/extraction kickers
    - Present theory is that rf heating bent the blades, which were then hit by synchrotron radiation
    - The resulting temperature runaway has been controlled by shifting the kickers outboard
  - High intensity Robinson instability in the main rf cavities that is stabilized by increasing the main rf voltage
  - Multi-bunch instability that is controlled by cavity temperature
  - Transverse instability that is eliminated by increasing chromaticity and/or using transverse feedback

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  - Multi-bunch instability that is controlled by cavity temperature
  - Transverse instability that is eliminated by increasing chromaticity and/or using transverse feedback
- The rest of this talk will present theory, simulations, and recent measurements of the transverse instability threshold with the tuned separated (uncoupled) and together (coupled)
- These studies will help our assess the impact of collective instabilities as we increase the current in the 48 bunch timing mode fill towards our goal of 4.3 mA/bunch[4]

[4] See Joe Calvey's talk earlier today or J. Calvey *et al.*, Proc. of the 2025 NAPAC, MOYN01 (2025) pp. 20

Collective (in)stability on the coupling resonance: Theoretical work from the pre-APS-U era

Single particle dynamics near the coupling resonance[5]

- We let skew quadrupoles weakly couple the horizontal and vertical motion in a ring of circumference C_R
- The single particle motion in the horizontal and vertical planes is described using the complex coordinates

$$u_x(s) = \boxed{\sqrt{\mathscr{J}_x}} e^{i\boxed{\Psi_x}(s)} e^{-2\pi i(\nu_x - \boxed{\Delta_\nu}/2)s/C_R} e^{+i\phi_r/2}$$

$$u_y(s) = \boxed{\sqrt{\mathscr{J}_y}} e^{i\boxed{\Psi_y}(s)} e^{-2\pi i(\nu_y + \boxed{\Delta_\nu}/2)s/C_R} e^{-i\phi_r/2}$$

Canonical particle actions

Canonical angles

Fractional tune difference:
$\Delta_\nu = \{\nu_x - \nu_y\}$

[5] H. Wiedemann, *Particle Accelerator Physics II*, Springer-Verlag (1999).

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Canonical particle actions Canonical angles Fractional tune difference: $\Delta_\nu = \{\nu_x - \nu_y\}$

- The independent degrees of freedom in a weakly coupled (κ << 1) lattice are

$$\begin{pmatrix} v_+ \\ v_- \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

where the "coupling angle" θ satisfies

$$\tan 2\theta = \frac{\kappa}{2\pi(\nu_x - \nu_y)} \text{ and } 0 \le \theta \le \frac{\pi}{2},$$

with κ the normalized coupling strength

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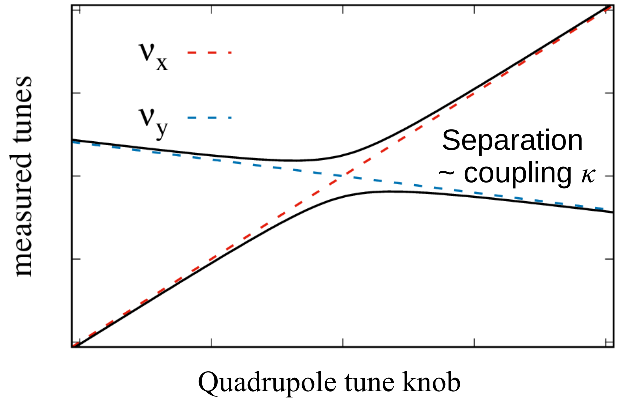
with κ the normalized coupling strength

- The coupled modes leads to the observed tunes

$$\nu_\pm = \frac{\nu_x + \nu_y}{2} \pm \frac{1}{2}\sqrt{(\nu_x - \nu_y)^2 + (\kappa/2\pi)^2}$$



[5] H. Wiedemann, *Particle Accelerator Physics II*, Springer-Verlag (1999).

Operating on the coupling/tune resonance leads to "round" beams with $\varepsilon_y = \varepsilon_y$

Fractional tune difference Δ_ν much larger than the normalized skew coupling strength

$$\frac{2\pi(\{\nu_x\} - \{\nu_y\}) \gg \kappa}{\theta \to 0}$$

$$v_+ \to u_x$$
$$v_- \to u_y$$

Neg. fractional tune difference $-\Delta_\nu$ much larger than the normalized skew coupling strength

$$\frac{2\pi(\{\nu_y\} - \{\nu_x\}) \gg \kappa}{\theta \to \pi/2}$$

$$v_+ \to u_y$$
$$v_- \to -u_x$$

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Fractional tune difference Δ_ν much larger than the normalized skew coupling strength	Fractional tunes equal	Neg. fractional tune difference $-\Delta_\nu$ much larger than the normalized skew coupling strength
$2\pi(\{\nu_x\} - \{\nu_y\}) \gg \kappa$	$\{\nu_x\} = \{\nu_y\}$	$2\pi(\{\nu_y\} - \{\nu_x\}) \gg \kappa$
$\theta \to 0$	$\theta = \pi/4$	$\theta \to \pi/2$
$v_+ \to u_x$	$v_+ = (u_x + u_y)/\sqrt{2}$	$v_+ \to u_y$
$v_- \to u_y$	$v_- = (u_x - u_y)/\sqrt{2}$	$v_- \to -u_x$

$$\begin{pmatrix} v_+ \\ v_- \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

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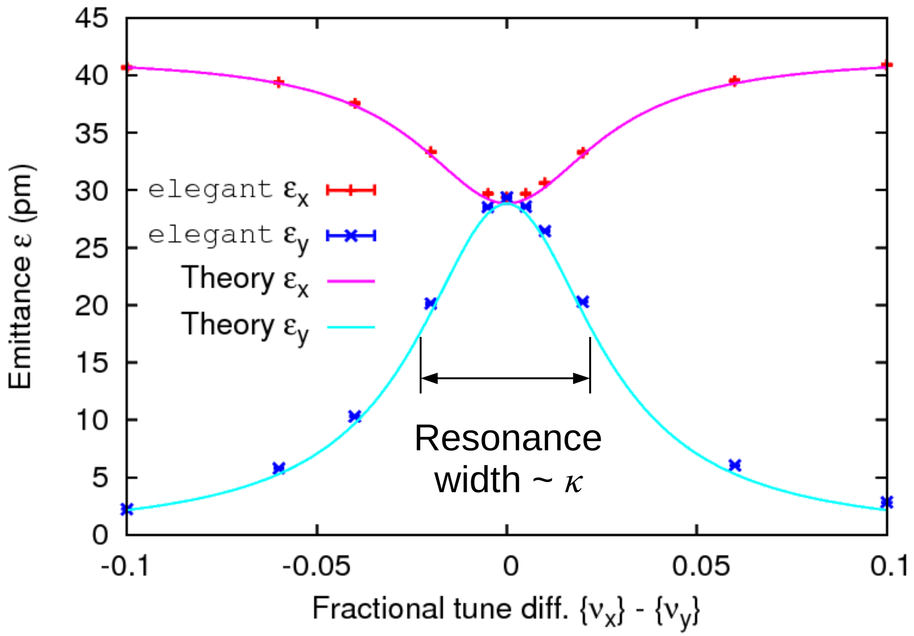

- Round beams maximize electron beam lifetime
- Round beams can be beneficial to some X-ray users
- At present the APS-U typically operates with round beams

[6] M. S. Zisman, S. Chattopadhyay, and J. J. Bisognano ZAP User's Manual. LBL-21270 (1986).
[7] B. Nash, J. Wu, and A. W. Chao. PRST-AB **9**, 032801 (2006).
[8] B. Nash. Stanford PhD Thesis, SLAC-Report-820 (2006).
[9] R. R. Lindberg. AOP-TN2014-003 (2014).

Understanding collective dynamics near the coupling resonance requires the stabilizing effects of chromaticity

- We decouple the transverse DOFs and eliminate the chromatic dependence using the generalization[10]

$$\begin{pmatrix} u_+ \\ u_- \end{pmatrix} = \begin{pmatrix} \cos\theta e^{i(k_{\xi,x}\cos^2\theta + k_{\xi,y}\sin^2\theta)z} & \sin\theta e^{i(k_{\xi,x}\cos^2\theta + k_{\xi,y}\sin^2\theta)z} \\ -\sin\theta e^{i(k_{\xi,x}\sin^2\theta + k_{\xi,y}\cos^2\theta)z} & \cos\theta e^{i(k_{\xi,x}\sin^2\theta + k_{\xi,y}\cos^2\theta)z} \end{pmatrix} \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

"coupling angle" $\tan 2\theta = \dfrac{\kappa}{2\pi(\nu_x - \nu_y)}$ and $0 \leq \theta \leq \dfrac{\pi}{2}$,

Head-tail phases : $\begin{aligned} k_{\xi x} z &= (2\pi\xi_x/\alpha_c \mathcal{C}_R) z \\ k_{\xi y} z &= (2\pi\xi_y/\alpha_c \mathcal{C}_R) z. \end{aligned}$

- Same coupled coordinates in the zero chromaticity limit; usual head-tail phase for uncoupled $\theta = 0$

[10] R. Lindberg, Proc. IPAC21, THPAB075 (2021) pp 3933

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$$\text{“coupling angle”} \quad \tan 2\theta = \frac{\kappa}{2\pi(\nu_x - \nu_y)} \text{ and } 0 \le \theta \le \frac{\pi}{2}, \qquad \text{Head-tail phases}: \quad \begin{aligned} k_{\xi x} z &= (2\pi\xi_x/\alpha_c \mathcal{C}_R)z \\ k_{\xi y} z &= (2\pi\xi_y/\alpha_c \mathcal{C}_R)z. \end{aligned}$$

- Same coupled coordinates in the zero chromaticity limit; usual head-tail phase for uncoupled $\theta = 0$
- The Hamiltonian for the coupled motion is

$$\mathcal{H} = -\frac{i}{2}\sqrt{\kappa^2 + (2\pi\Delta_\nu)^2}\,|v_+|^2 + \frac{i}{2}\sqrt{\kappa^2 + (2\pi\Delta_\nu)^2}\,|v_-|^2 + \text{wakefields}$$

$$\cancel{+ i\pi(\xi_x - \xi_y)\delta\sin 2\theta \left[v_+ v_-^* e^{-i\cos 2\theta(k_{\xi,x} - k_{\xi,y})z} + c.c. \right]}$$

[10] R. Lindberg, Proc. IPAC21, THPAB075 (2021) pp 3933

We can drop this coupling term provided $2\pi \gg \kappa \gg 2\pi(\xi_x - \xi_y)\sigma_\delta \sin 2\theta$

Coupling is weak and the lattice is characterized by uncoupled β_x-functions.

Coupling is larger than the difference between the coupled and uncoupled chromatic tune shift

Collective dynamics near the coupling resonance is controlled by "shared" chromaticities and wakefields[11,12]

- Vlasov stability for the '+' degree of freedom is given by[12]

$$\frac{\partial}{\partial\tau}g_+ - \underbrace{\boxed{i\omega_\kappa g_+}}_{\text{Coupled mode frequency}} + \overbrace{\boxed{C_R\alpha_c p_z\frac{\partial}{\partial z}g_+ - \mathcal{C}_R\frac{\partial V_z}{\partial z}\frac{\partial}{\partial p_z}g_+}}^{\text{Longitudinal motion}} = -\frac{ie^2N_e}{2\gamma mc^2}g_0\int d\hat{z}d\hat{p}_z\overbrace{\boxed{g_+(\hat{z},\hat{p}_z;\tau)e^{ik_+(z-\hat{z})}W_+^\beta(z-\hat{z})}}^{\text{Effective wakefield}}$$

$$-\frac{ie^2N_e}{4\gamma mc^2}g_0\int d\hat{z}d\hat{p}_z\underbrace{\boxed{g_-(\hat{z},\hat{p}_z;\tau)e^{i(k_+z-k_-\hat{z})}W_c^\beta(z-\hat{z})}}_{\text{Coupling wakefield (small mostly negligible)}}$$

$$\omega_\kappa = \sqrt{\kappa^2+(2\pi\Delta_\nu)^2}/2\mathcal{C}_R$$

[11] E. Métral, Particle Accelerators **62**, 259 (1999); E. Métral, G. Rumulo, R.R. Steerenberg, and B. Salvant. Proc. PAC07 (2007) pp 4210
[12] R. Lindberg, AOP-TN-2020-039 (2020).

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Longitudinal motion

Effective wakefield

$$\frac{\partial}{\partial\tau}g_+ - \boxed{i\omega_\kappa g_+} + \boxed{C_R\alpha_c p_z\frac{\partial}{\partial z}g_+ - \mathcal{C}_R\frac{\partial V_z}{\partial z}\frac{\partial}{\partial p_z}g_+} = -\frac{ie^2N_e}{2\gamma mc^2}g_0\int d\hat{z}d\hat{p}_z\,\boxed{g_+(\hat{z},\hat{p}_z;\tau)e^{ik_+(z-\hat{z})}W_+^\beta(z-\hat{z})}$$

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Coupled mode frequency

$$\omega_\kappa = \sqrt{\kappa^2+(2\pi\Delta_\nu)^2}/2\mathcal{C}_R$$

Coupling wakefield (small mostly negligible)

- The effective/shared chromaticities:

$$\xi_+ = \xi_x\cos^2\theta + \xi_y\sin^2\theta$$
$$\xi_- = \xi_x\sin^2\theta + \xi_y\cos^2\theta$$

→ Collective stability will increase near the coupling resonance if $\xi_x \neq \xi_y$

- The effective/shared wakefields:

$$W_+^\beta(z) = \cos^2\theta\, W_x^\beta(z) + \sin^2\theta\, W_y^\beta(z)$$
$$W_-^\beta(z) = \sin^2\theta\, W_x^\beta(z) + \cos^2\theta\, W_y^\beta(z)$$

→ Collective stability can be increased near the coupling resonance if $W_x \neq W_y$

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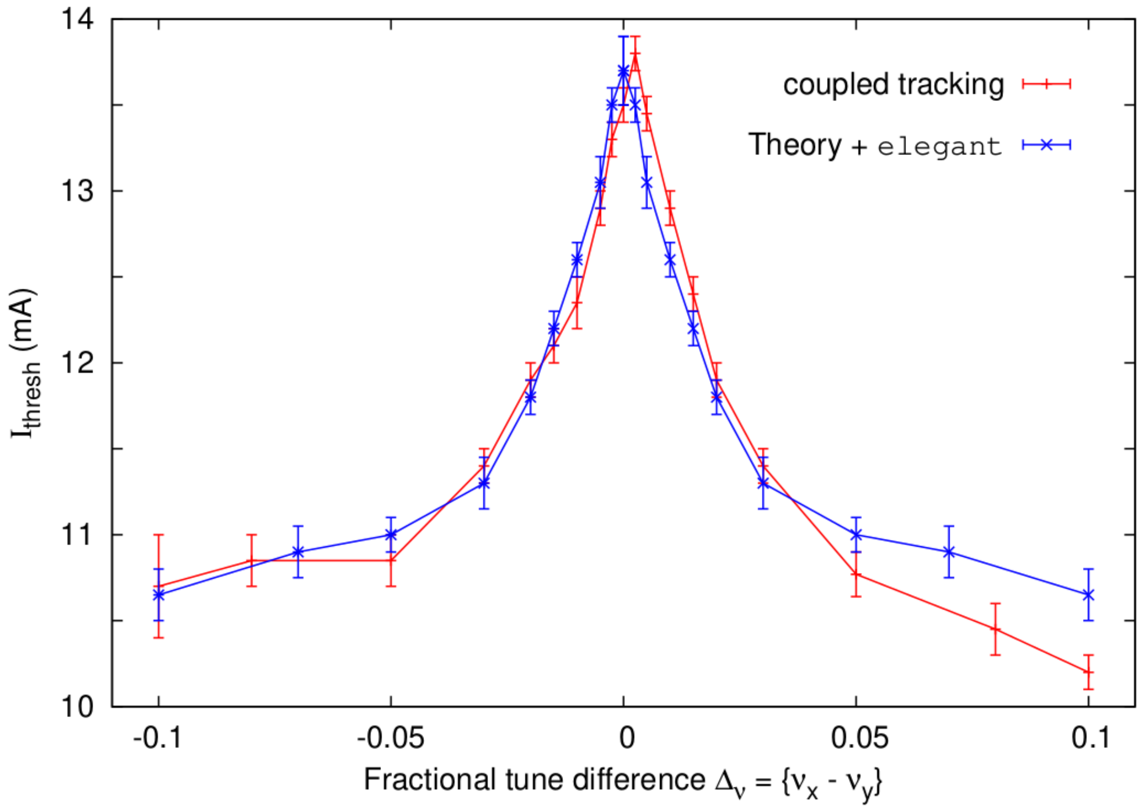
[12] R. Lindberg, AOP-TN-2020-039 (2020).

The theory of chromaticity “sharing”near the coupling resonance was verified by simulations

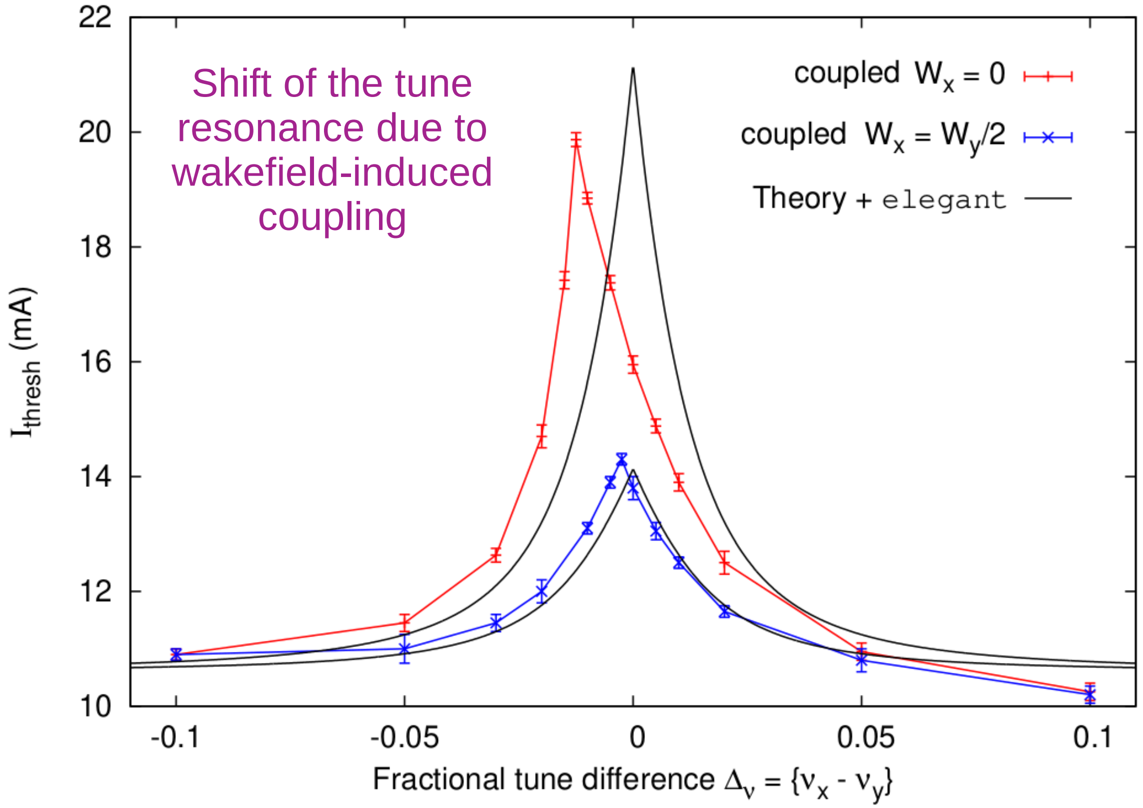
- Test case for the APS-U lattice
  - Optimized lattice with nominal chromaticities ξ_x = 8.1 and ξ_y = 4.7
  - Simplified resistive wall impedance with $W_x = W_y$
  - Harmonic cavity to lengthen bunch but no longitudinal wakefields
- The (vertical) instability threshold increases from ~10.5 mA to ~13.5 mA on the coupling resonance
  - Shared chromaticity 6.4 > ξ_y = 4.7.



“Theory + `elegant`” means we use uncoupled `elegant` tracking results obtained using the theoretical shared chromaticity

The theory of wakefield "sharing"near the coupling resonance was verified by simulations

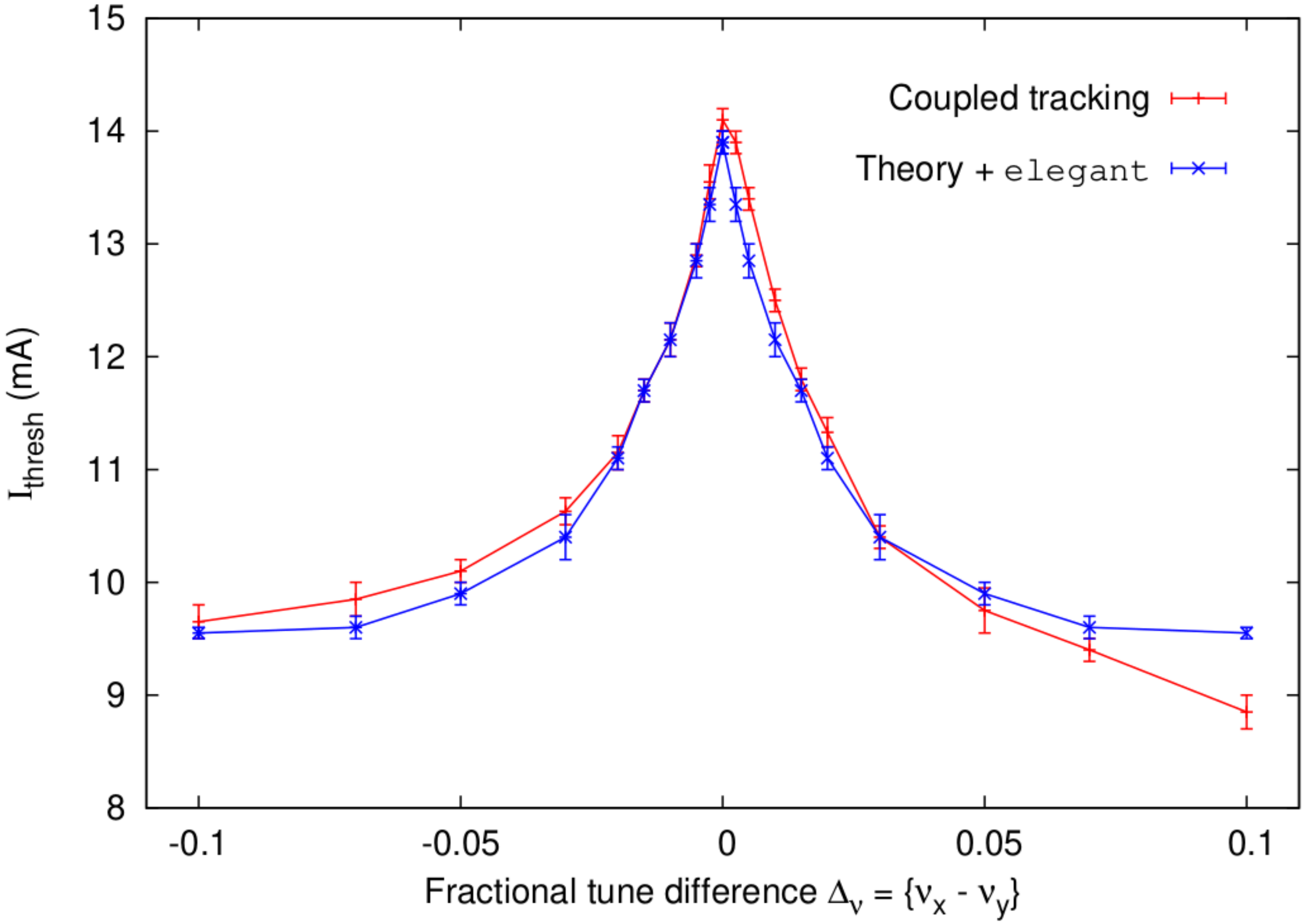
- Test case for the APS-U lattice
  - Assumes equal chromaticities with $\xi_x = \xi_y = 4.7$
  - Resistive wall impedance with an artificial scaling to the horizontal W_x
  - Harmonic cavity to lengthen bunch but no longitudinal wakefields
- The (vertical) instability threshold increases from ~10.5 mA to up to 20 mA on the coupling resonance
  - Shared wakefield < W_y



"Theory + `elegant`" means we use uncoupled `elegant` tracking result of 10.6 mA scaled by the wakefield ratio $W_y/W_{\rm share}$

The optimized APS-U lattice was predicted to benefit from both chromaticity and wakefield sharing

- Simulations of the optimized APS-U show benefits from both chromaticity and wakefield "sharing" near the coupling resonance
  - Lattice optimization $\rightarrow \xi_x > \xi_y$, so that coupling increases shared chromaticity
  - Narrow-gap IDs $\rightarrow W_y > W_x$, so that coupling reduces the shared wakefield
- Instability threshold increased by ~40% on the tune resonance



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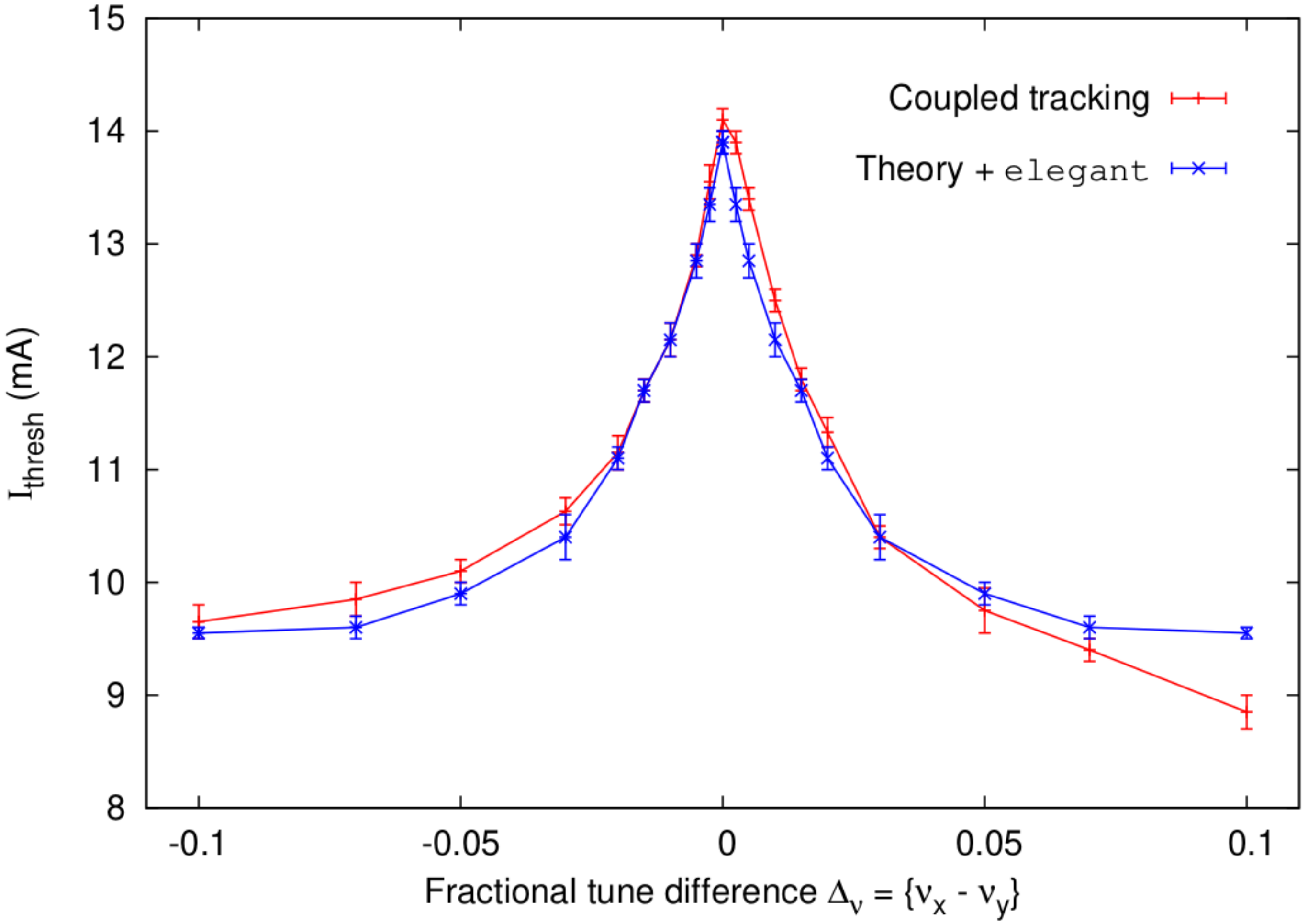
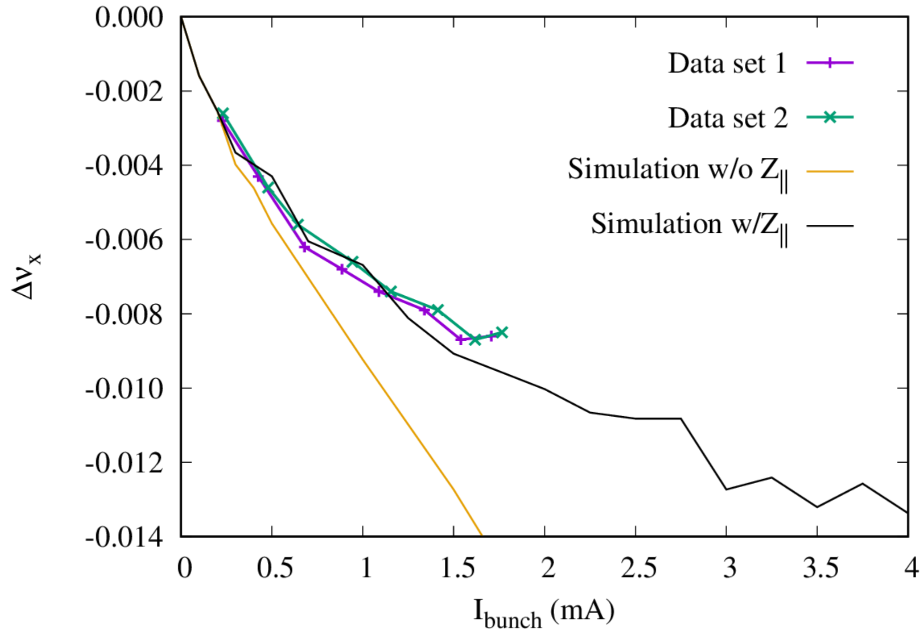
- Instability threshold increased by ~40% on the tune resonance
- Simulations include the following
  - Coupled lattice model including lowest-order nonlinearities in 6D phase space
  - APS-U impedance model including all geometric and resistive contributions.
  - Passive higher harmonic rf cavity tuned to overstretch the bunch
  - Longitudinal wakefield scaled to its value at the nominal 4.2 mA/bunch

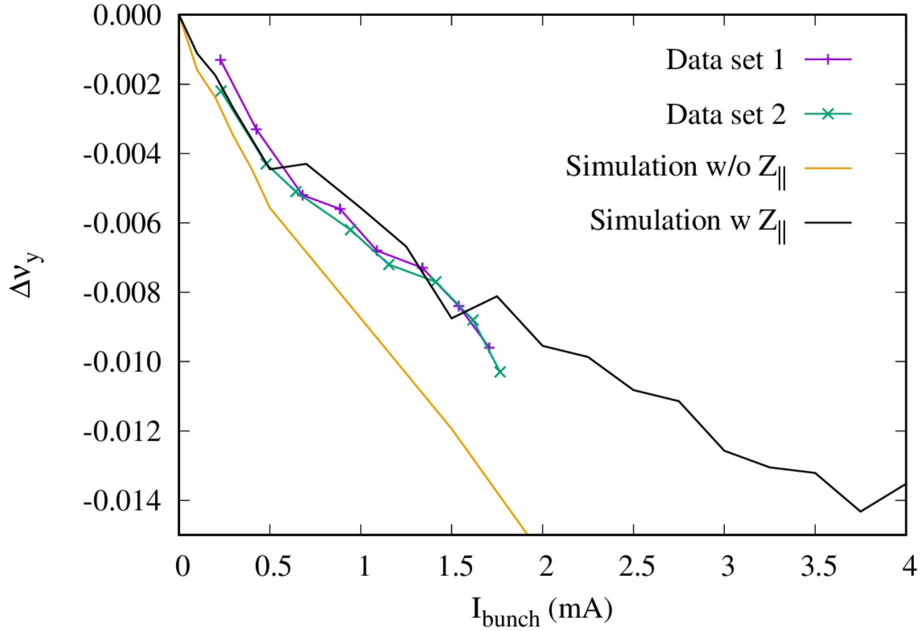


Collective (in)stability on the coupling resonance: Simulations and measurements during the APS-U era

Measured tune-shift-with-charge agrees reasonably well with tracking simulations

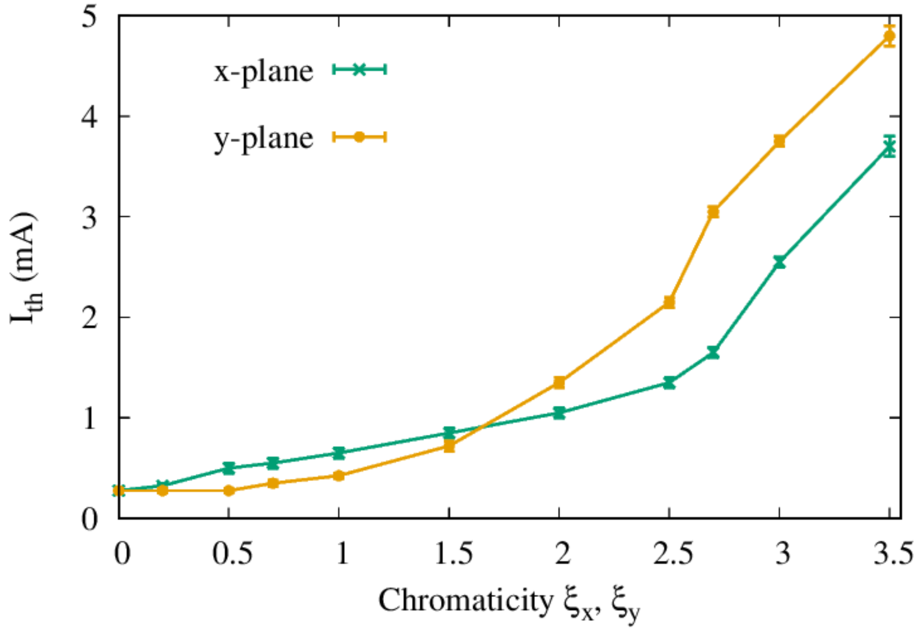
- Tune shift with charge was measured in single-bunch mode
  - Single bunch charge was controlled with the number of bunches injected into the particle accumulator ring
  - Current steps were ~0.4 mA
- Measurements were shifted by the (unknown) zero-current tune to match simulations at ~0.4 mA
- Tracking reproduces the same shape of the tune shift curve up to the maximum measured ~1.8 mA
- Curvature is primarily due to bunch lengthening
- On this study we were unfortunately limited to about 2 mA single bunch current (typically we can get up to ~3.2 mA)
- Observations indicate that we have identified the major sources of transverse impedance





Simulations of transverse stability at the APS-U

- We simulate transverse stability without the harmonic cavity to compare to single bunch measurements
- Stability in each plane was first determined for an uncoupled lattice
  - Horizontal stability (x-plane) found by setting $\xi_y = 5$
  - Vertical stability (y-plane) found by setting $\xi_x = 5$



Parameter	Value
Horizontal tune ν_y	95.14
Vertical tune ν_y	36.14
Rf voltage V_{rf}	4.2 MV
Circumference	1103.6 m
Momentum compaction α_c	3.85×10^{-5}
Energy loss/turn	2.935 MeV

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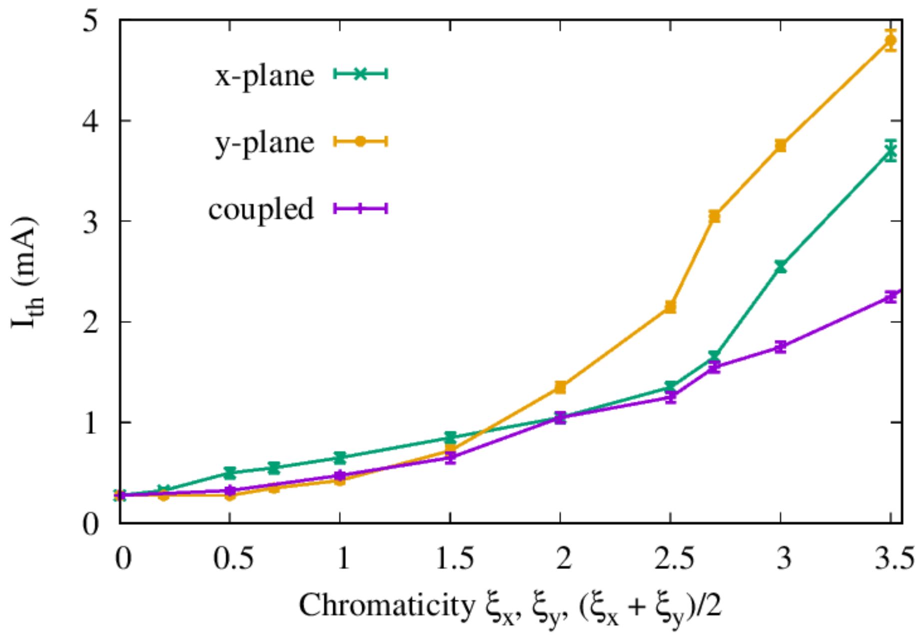
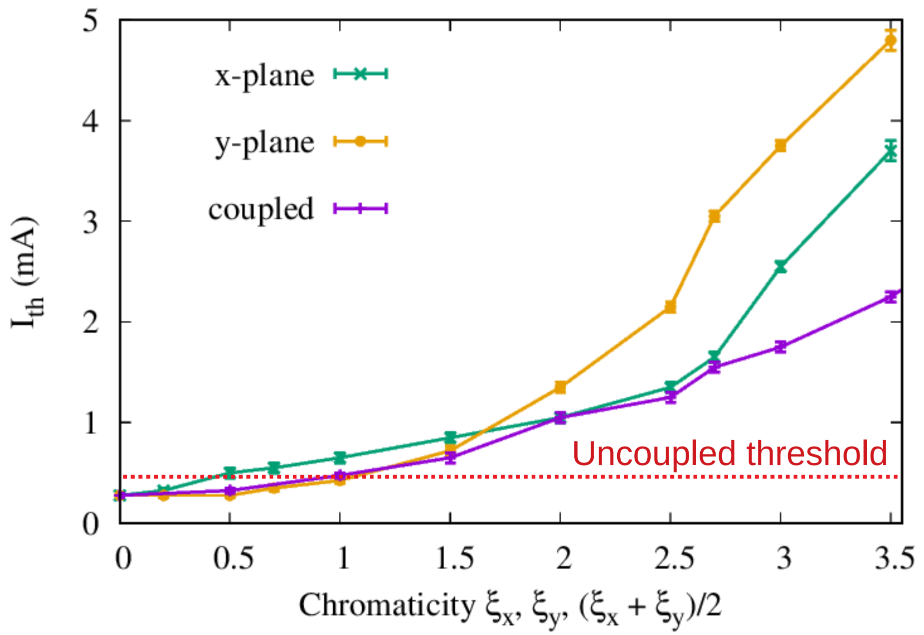
- Stability in each plane was first determined for an uncoupled lattice
  - Horizontal stability (x-plane) found by setting $\xi_y = 5$
  - Vertical stability (y-plane) found by setting $\xi_x = 5$
- Coupled lattice was made by adding weak skew quadrupoles and simulating on the coupling resonance where $\nu_x = \nu_y = 0.14$.
- Coupled lattice simulations then fix the vertical chromaticity $\xi_y = 1$ and vary the shared chromaticity by changing only ξ_x.
- Naive expectation is that the coupled threshold will lie between the two uncoupled predictions
  - Simulations show it follows the lower limit when $\xi_x < 4.4$
  - Stability for the coupled lattice continues to increase for $\xi_x > 4.4$, but less than uncoupled results might indicate



Parameter	Value
Horizontal tune ν_y	95.14
Vertical tune ν_y	36.14
Rf voltage $V_{\rm rf}$	4.2 MV
Circumference	1103.6 m
Momentum compaction α_c	3.85×10^{-5}
Energy loss/turn	2.935 MeV

Simulations of transverse stability at the APS-U

- We simulate transverse stability without the harmonic cavity to compare to single bunch measurements
- Stability in each plane was first determined for an uncoupled lattice
  - Horizontal stability (x-plane) found by setting $\xi_y = 5$
  - Vertical stability (y-plane) found by setting $\xi_x = 5$
- Coupled lattice was made by adding weak skew quadrupoles and simulating on the coupling resonance where $\nu_x = \nu_y = 0.14$.
- Coupled lattice simulations then fix the vertical chromaticity $\xi_y = 1$ and vary the shared chromaticity by changing only ξ_x.
- Naive expectation is that the coupled threshold will lie between the two uncoupled predictions
  - Simulations show it follows the lower limit when $\xi_x < 4.4$
  - Stability for the coupled lattice continues to increase for $\xi_x > 4.4$, but less than uncoupled results might indicate
  - The coupled threshold is significantly larger than the uncoupled threshold $I_{\rm th}$ = 0.47 mA when $\xi_x > 1$
  - The APS-U's nonlinear lattice and shared wakefield displays more complicated stability characteristics than our theoretical model



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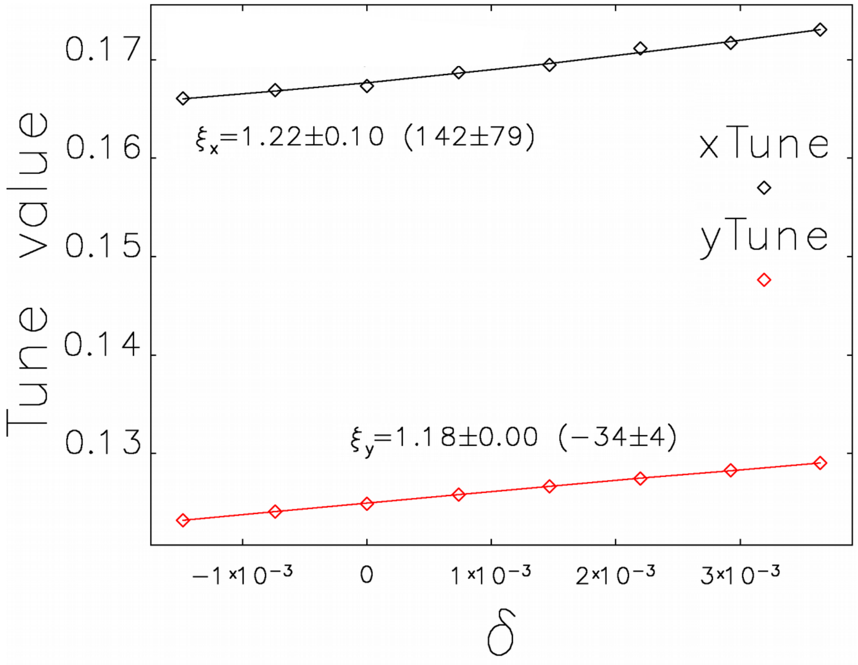
Experimental steps to measure the transverse instability threshold on the coupling (tune difference) resonance

I. Restore standard user lattice and separate the fractional tunes such that $\nu_x = 0.168$ and $\nu_y = 0.125$.

II. Adjust skew quadrupoles to a modest coupling
  - Negligible effect when the tunes are separated
  - Reproducible, fully-coupled lattice when tunes are knobbed together with $\nu_x = \nu_y = 0.142$

III. Set starting chromaticities such that $\xi_x \approx \xi_y \approx 1$



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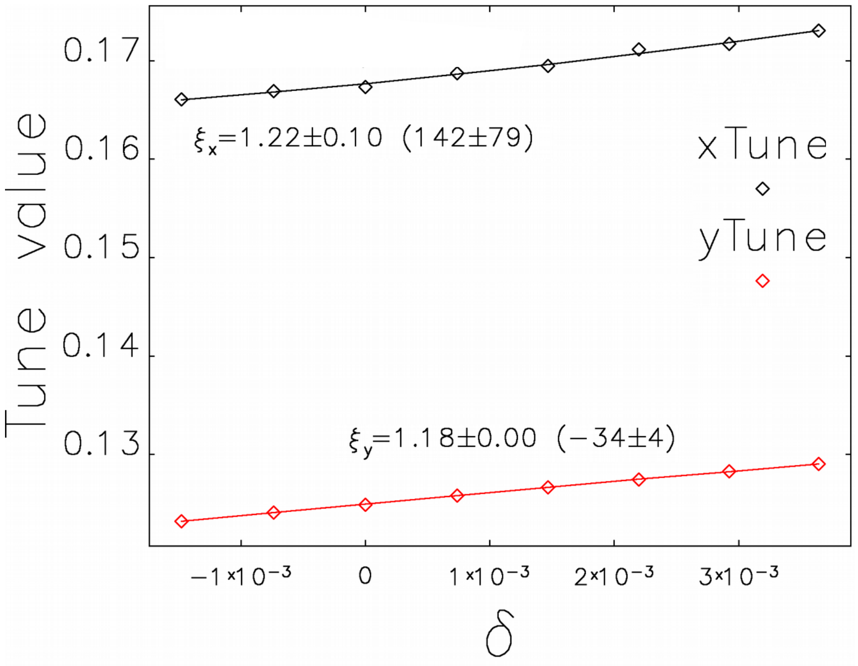
IV. Determine transverse threshold current for given ξ_x
  1. Measure chromaticities with tunes separated
  2. Inject increasing charge by increasing the number of linac pulses (optimized injector + top up → 0.3 mA steps)
  3. Monitor stability using Dimtel feedback pickup
  4. Repeat stability measurement with tunes together

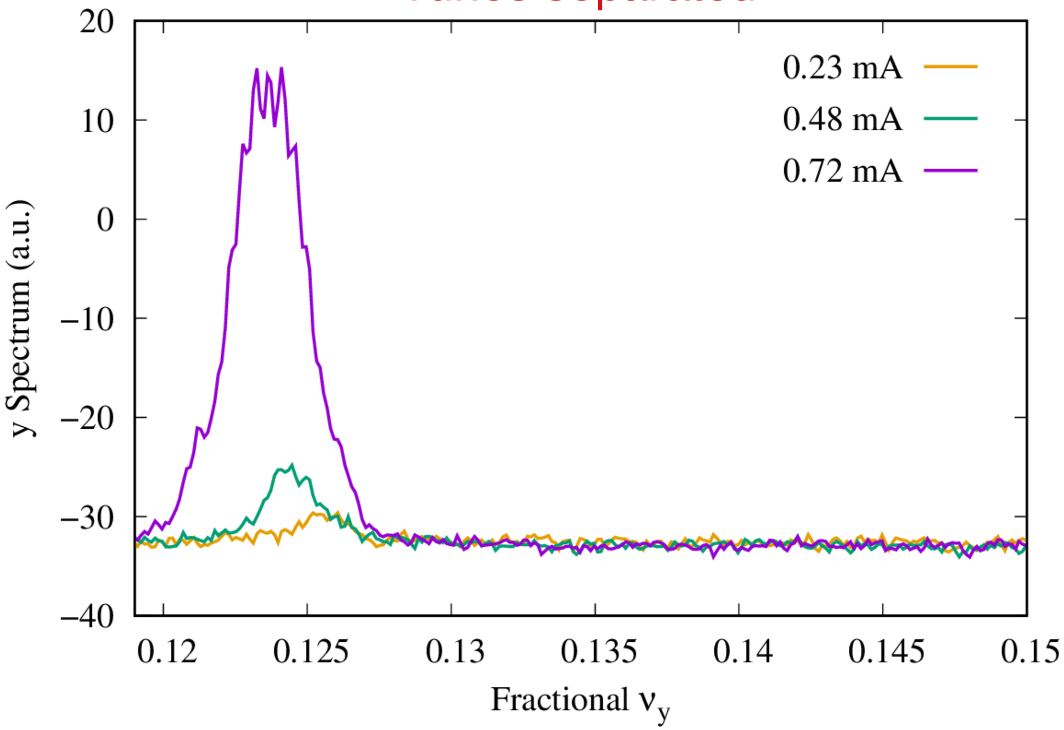
V. Increase horizontal chromaticity and repeat



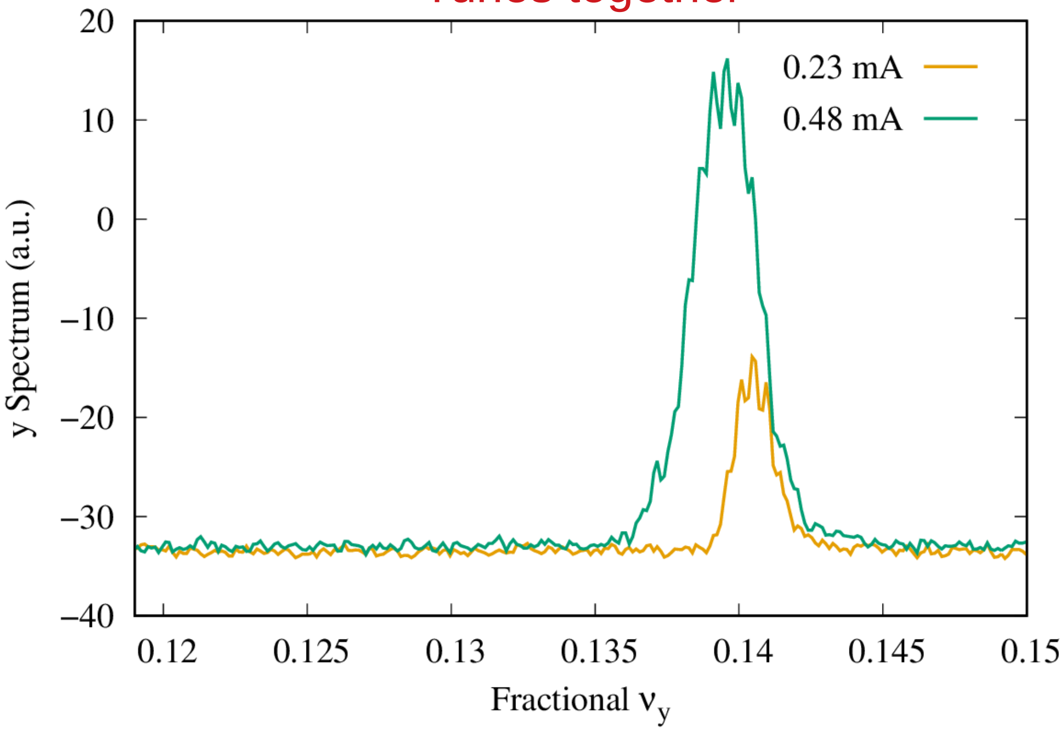
Coupled and uncoupled thresholds for $\xi_x \approx \xi_y \approx 1$

The coupled lattice has a
lower instability threshold!

Tunes separated



Tunes together



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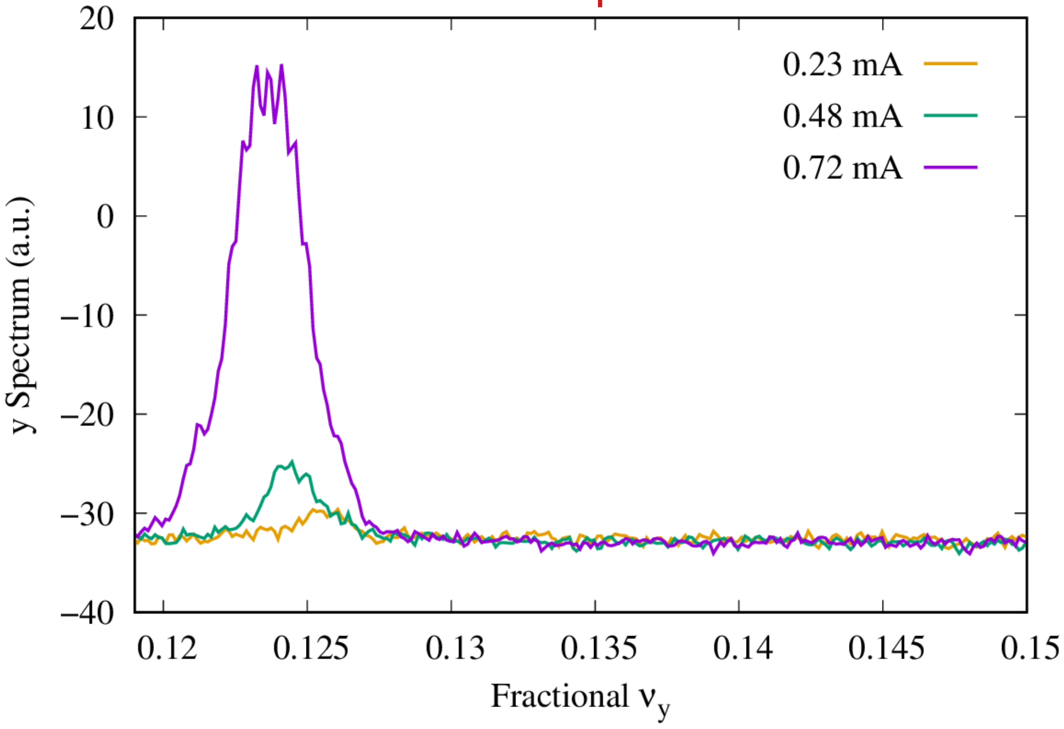
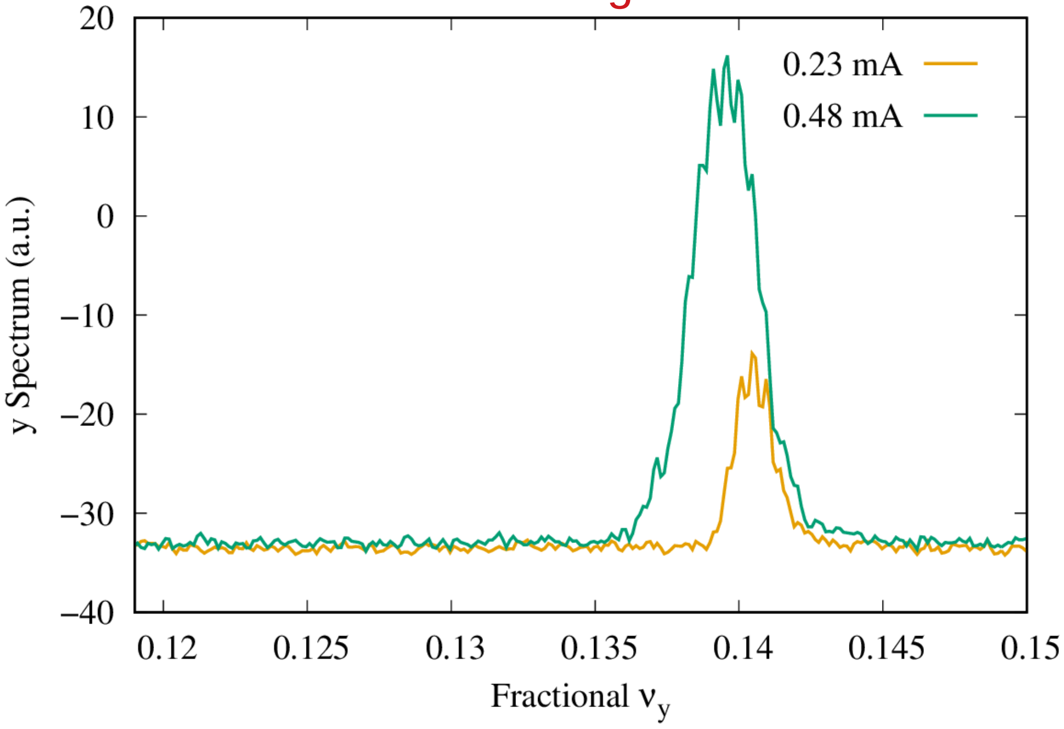


Tunes together

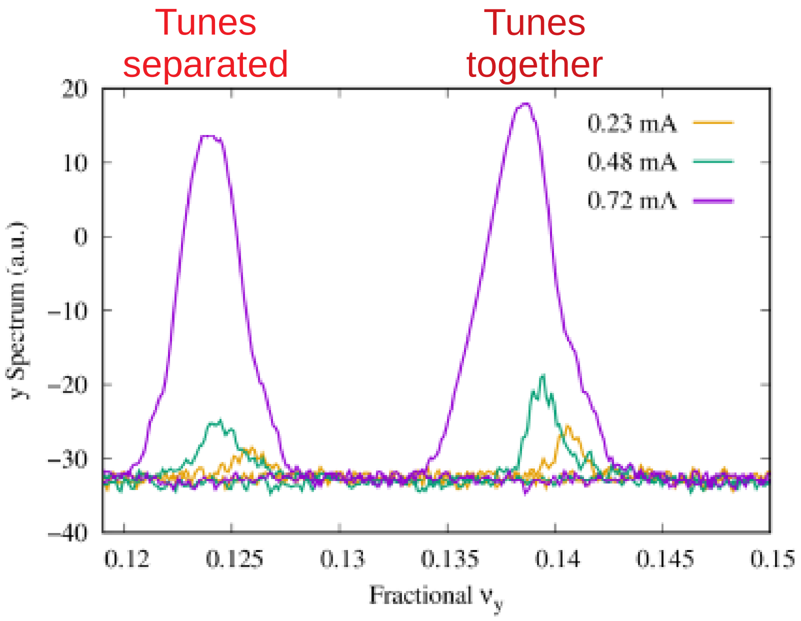


- The uncoupled lattice has I_{th} = 0.60 ± 0.12 mA
- The coupled lattice with shared chromaticity ≤ 2.1 (horizontal $\xi_x \leq 3.2$) has I_{th} = 0.36 ± 0.13 mA

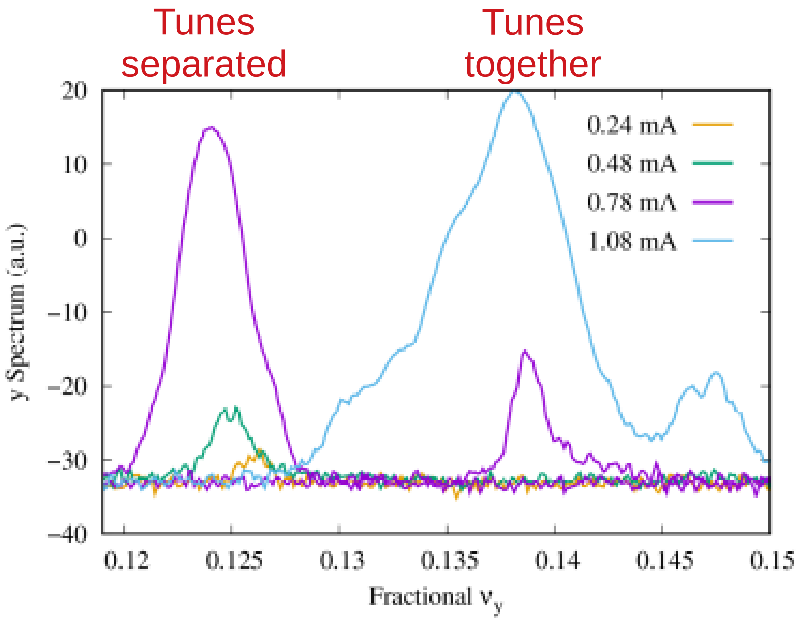
The coupled lattice has a smaller instability threshold current at low chromaticity.

While surprising, this is perhaps not shocking, since we have seen that our simple linear model does not cover all the physics

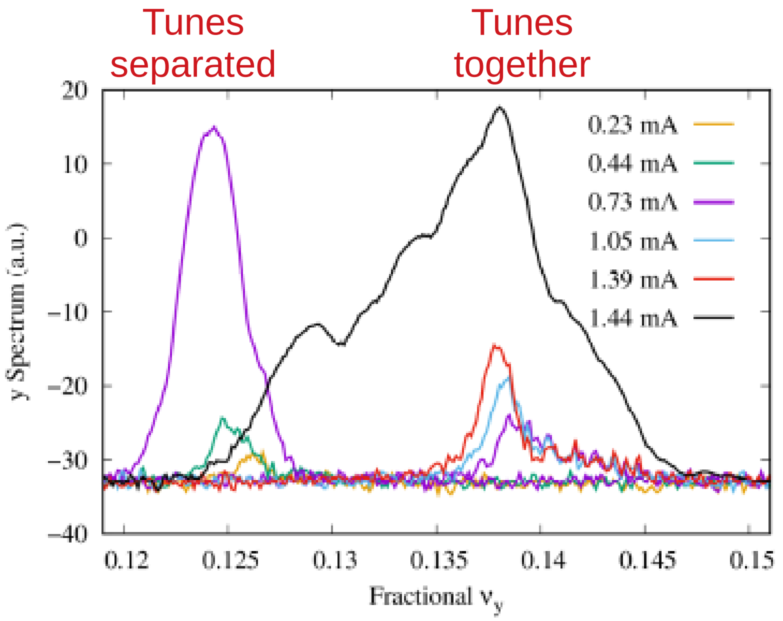
Further increasing ξ_x eventually raises the coupled threshold above that of the uncoupled lattice



Coupled and uncoupled lattices have the same I_{th} when the shared chromaticity is 2.4 ($\xi_x = 3.9$, $\xi_y = 1$)



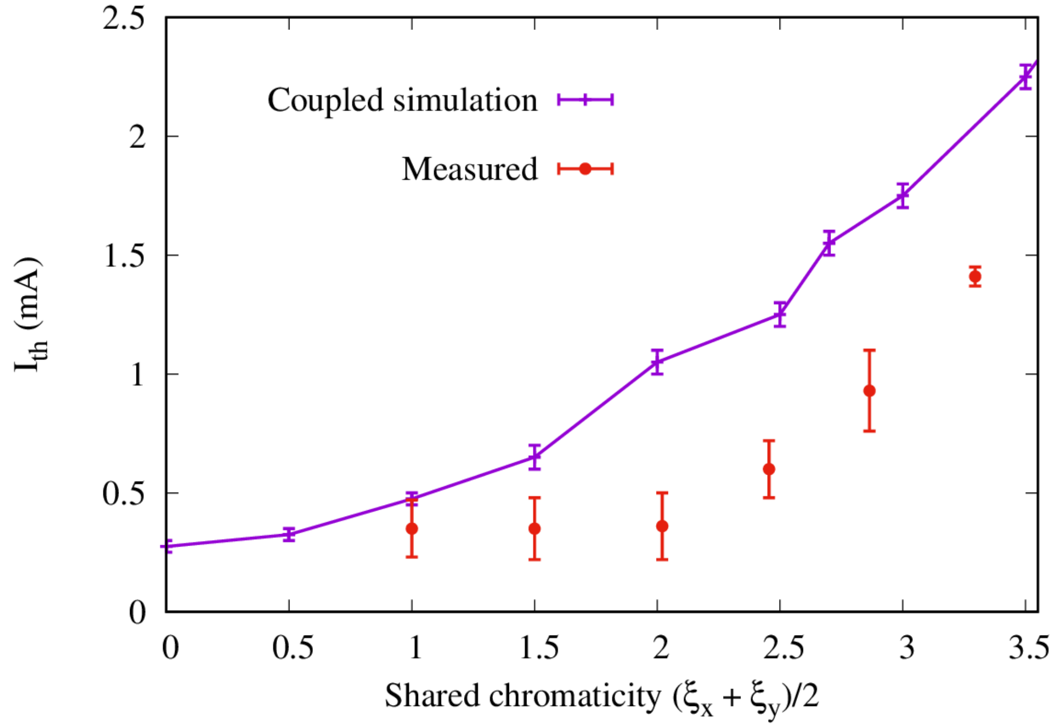
Coupled I_{th} is larger than the uncoupled threshold when the shared chromaticity is 2.9 ($\xi_x = 4.7$, $\xi_y = 1$)



Coupled I_{th} more than triples when the shared chromaticity increases to 3.3 ($\xi_x = 5.6$, $\xi_y = 1$)

Measurements and simulations of the transverse instability threshold for a coupled lattice qualitatively agree

- The measured and simulated thresholds for the coupled lattice are pretty close when $\xi_x = \xi_y = 1$
- The simulated threshold increases steadily when the shared chromaticity increases above 1.
- The measured threshold stays essentially constant until the shared chromaticity is above 2.



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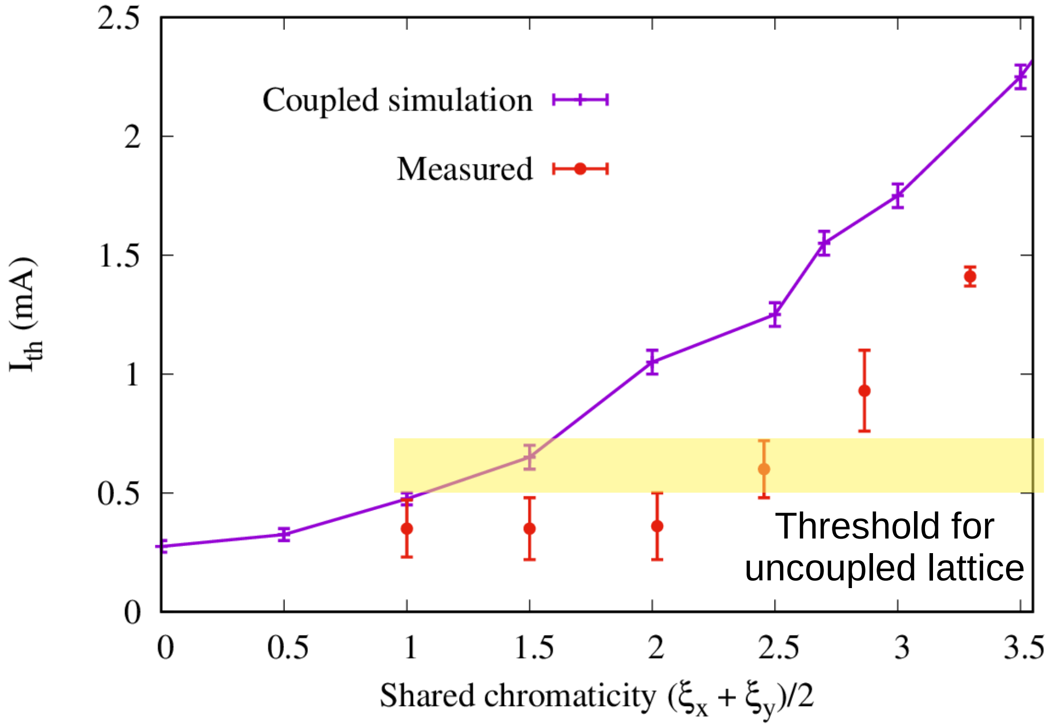
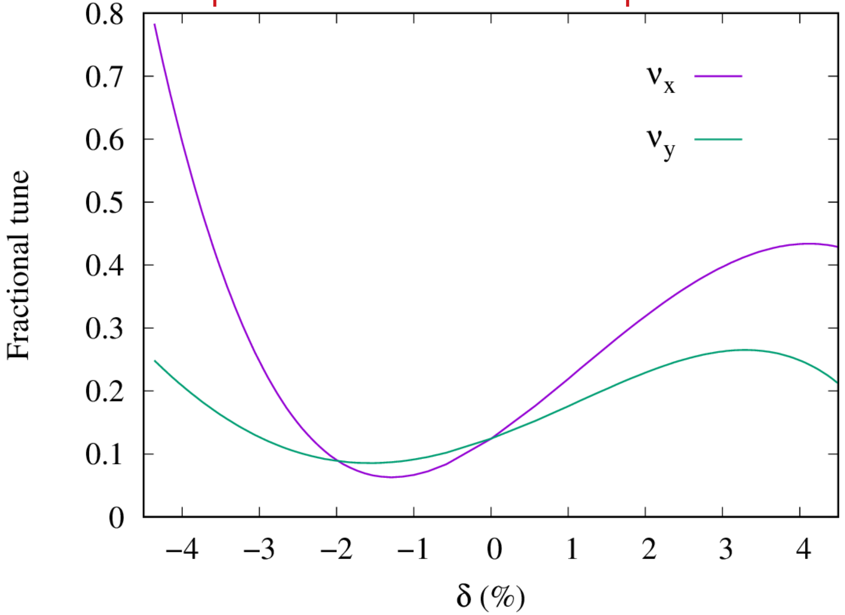
- The simulated threshold increases steadily when the shared chromaticity increases above 1.
- The measured threshold stays essentially constant until the shared chromaticity is above 2.
- Once chromatic effects play a role, the measured slope of $I_{\mathrm{th}}(\xi_x+\xi_y)$ is similar to the simulated one
- The coupled threshold is measured to be larger than the uncoupled one for shared chromaticities > 2.9
- Future measurements will target higher chromaticities and a finer current resolution by (de)tuning the injector



The linear theory may be too simple to describe APS-U collective instability for all cases

- The MOGA-optimized APS-U lattice has a strongly nonlinear chromatic tune footprint
  - Both the second-order and third-order chromaticities are play a role over energies < bucket height
  - The lifetime is maximized at rather large linear chromaticities (ξ_x, ξ_y) = (8.1, 4.7).
- For the optimized bunch-lengthening harmonic cavity settings and the optimized ξ_y = 4.7, the head-tail phase $k_{\xi_y}\sigma_z = \frac{2\pi\xi_y}{\alpha_c C_R}\sigma_z \sim 19$, and the second order chromaticity only reduces the stability threshold by ~10%.

Optimized chromatic tune dependence



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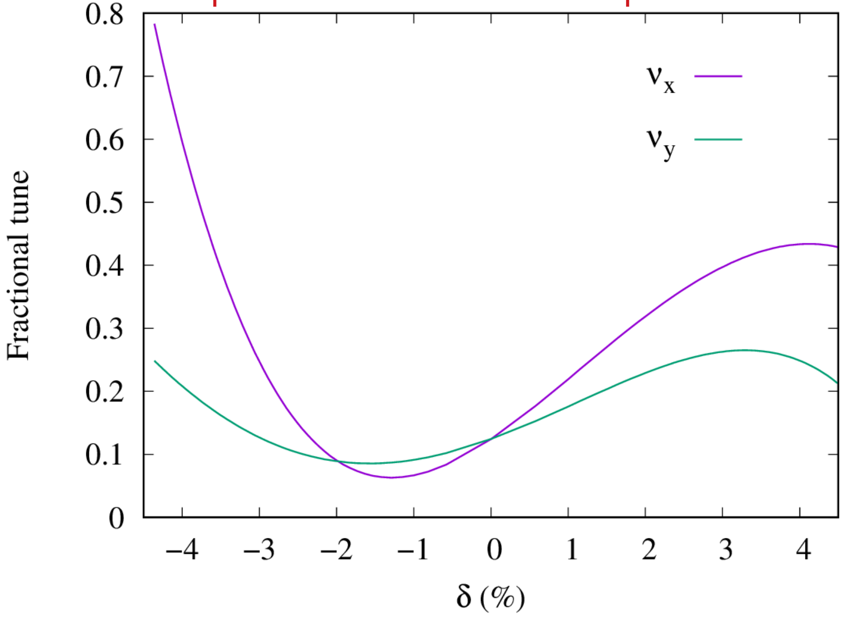
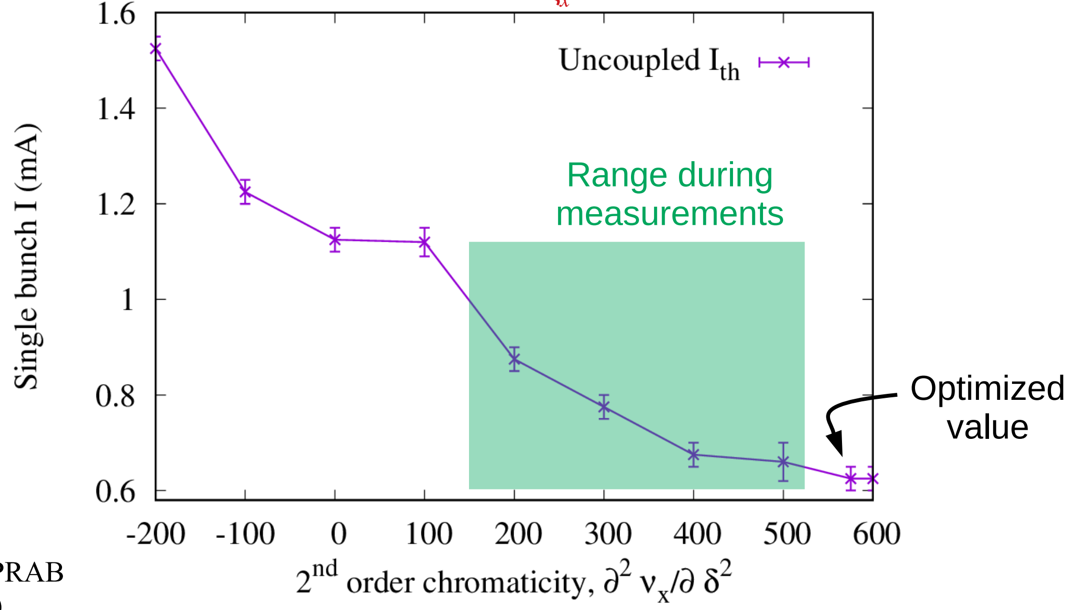
- For the low chromaticity and short bunches of the experiments, the second-order chromaticities in both planes can play an important role for coupled stability[13]

Optimized chromatic tune dependence



Simulations for ξ_x = 1.5



[13] M. Schenk *et al.*, PRAB **21**, 084401 (2018)

Summary and outlook

- Collective stability near the coupling resonance depends upon lattice parameters in both planes
- On the coupling resonance we need to consider
  - Shared chromaticity $\xi_\pm = \frac{1}{2}\left(\xi_x + \xi_y\right)$
  - Shared wakefield $W_\pm = \frac{1}{2}\left(W_x + W_y\right)$
- Simulations and measurements at APS-U show qualitative agreement, and both demonstrate the essential aspects of the theory
- Future work requires more detailed lattice characterization

Thanks for your attention!