CERN

Deep Lie-Map Networks

A novel Approach to Infer Nonlinear Synchrotron Optics from Beam Oscillations

Conrad Caliari

Motivation

Magnetic Field Errors

- detrimental to machine performance
  - excite resonances
  - reduce dynamic aperture
  - cause beam loss

- many origins
  - magnet fabrication errors
  - misalignments
  - power supply failures

power corrector magnets for compensation

Require location and magnitude of linear & non-linear field errors!

Established Field Error Identification Procedures

Linear **O**ptics from **C**losed **O**rbits (**LOCO**) [1]

- linear machine model from orbit response
- fit model to measured orbit response
- find dipole & quadrupole errors
- widely employed since 1996
- non-linear field errors not covered

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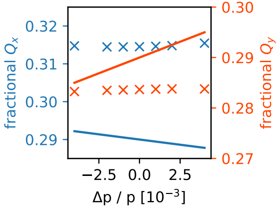
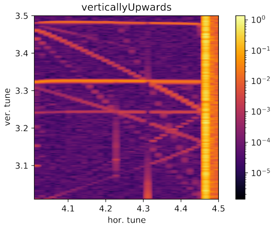
non-linear optics estimation

- different approaches demonstrated
  - resonance-driving terms [2]
  - non-linear tune response matrix (NTRM) [3]
  - Nonlinear optics from off-energy closed orbits (NOECO) [4]
- time demanding measurements
- require structured measurement data (e.g. bumps around machine)
- require accurate linear machine model in advance

SIS18 @ GSI

- 216 m long synchrotron, accelerate heavy ions from protons to uranium
- injector of the future FAIR facility / SIS100 synchrotron
- nominal optics model
  - tunes shifted $\Delta Q_{x,y} = 1 \times 10^{-2}$
  - discrepancies in chromaticity $\Delta\xi_x = 0.13,\ \Delta\xi_y = 0.38$





solid line: MAD-X prediction,
crosses: measurements

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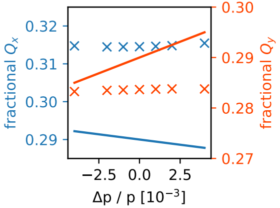
  - indicates sextupole errors



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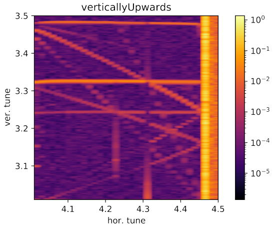

Measured dynamic tunescan in SIS18, vertically upwards.

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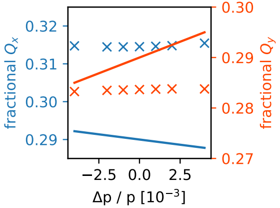

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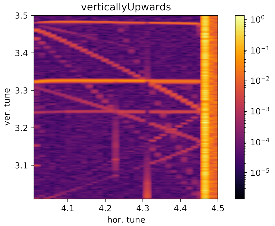


Measured dynamic tunescan in SIS18, vertically upwards.

Use DLMN to identify source of discrepancies

Motivation

- improved & accurate optics description
  $\Longrightarrow$ step towards digital twin
- efficient on beamtime
- independent from existing linear optics model

Propose: **D**eep **L**ie-**M**ap **N**etworks (**DLMN**) [5, 6]

- identify sextupole errors from trajectory data
- machine-learning based approach

Deep Lie-Map Networks

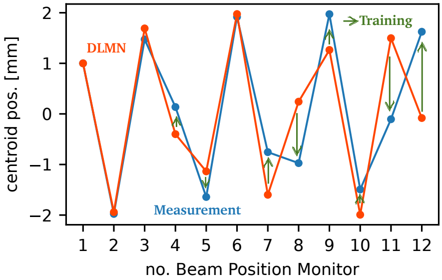
Probe optics with beam centroids oscillations

Approach

Identify magnetic field errors from observed centroid trajectories

Steps

1. compare measurements to model predictions
2. quantify difference by loss $\mathcal{L}$
3. minimize $\mathcal{L}$ by varying multipole strengths of model



→ Changes to DLMN degrees of freedom reveal field errors

Deep Lie Map Networks

Self-implemented tracking code with automatic differentiation

- elements modeled in hard-edge approximation
  $\rightarrow$ piece-wise constant Hamiltonian
- eqn. of motion solved in thin-lens approximation
  - discrete updates to position & momentum
    (drifts & kicks)
  - exact drifts, no truncation
  - transverse magnetic fields up to arbitrary order
- benchmarked against MAD-X [7], SixTrackLib [8]

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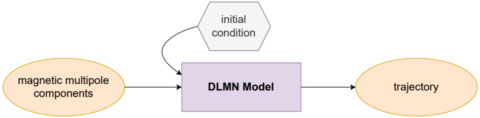
- concatenation of differentiable maps
  - enable differentiation of whole tracking model w.r.t. multipole strengths
  - compute $\frac{\partial \mathcal{L}}{\partial k_{i,j}}$ with $k_{i,j}$ i-th multipole of j-th magnet
- high-dimensional optimization problem
  - derivatives from automatic differentiation
    $\rightarrow$ gradient-based optimization
  - $\left|\frac{\partial \mathcal{L}}{\partial k_{i,j}}\right|$ cover several orders of magnitude

Initial Condition For Tracking



- Beam deflection from stationary state:

$$\Delta p_{x,y} = \frac{\int_s B_{\text{kicker}} ds}{B\rho}$$

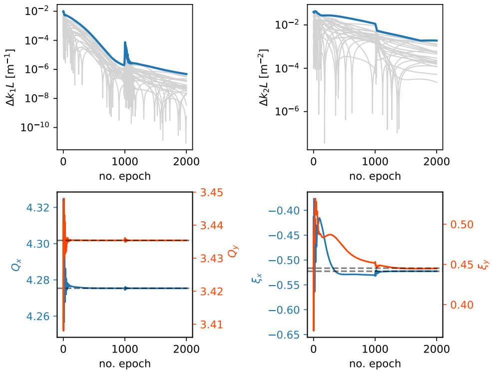
- Momentum change via RF frequency mismatch:

$$\delta = \left(\frac{1}{\gamma^2} - \alpha_C\right) \frac{\Delta f_{\text{rf}}}{f_{\text{rf}}}$$

- Initial condition for DLMN tracking:

$$\vec{z}_0 = [0, \Delta p_x, 0, \Delta p_y, 0, \delta]$$

Simulated Training Results



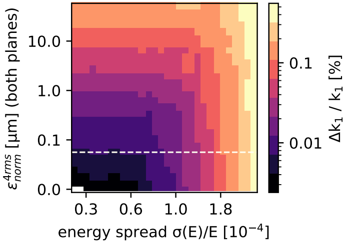
Normal distributed quadrupole & sextupole errors

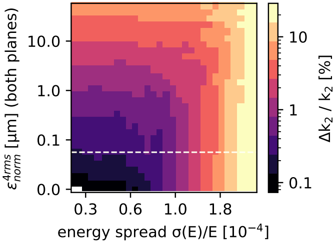
- convergence of magnetic multipole components
  - quadrupole strengths $\Delta k_1 L$
  - sextupole strengths $\Delta k_2 L$
- correct prediction of
  - tunes $Q_{x,y}$
  - chromaticities $\xi_{x,y}$

Only three trajectories required!
$\delta \in [-1 \times 10^{-3}, 0, 1 \times 10^{-3}]$

Chromatic & Amplitude Detuning

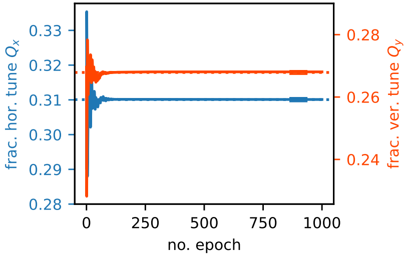
- motion of beam centroid differs from single particle motion
- detuning limits resolution magnetic field errors
  - beam emittance → amplitude detuning
  - momentum spread → chromatic detuning

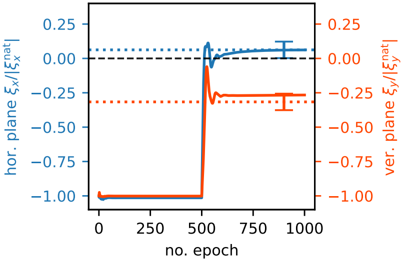




Proof-of-Principle Experiment

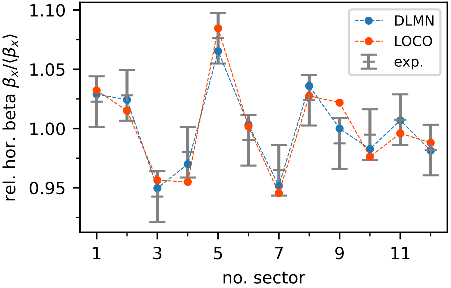
Reconstructed Tune & Chromaticity





DLMN predicted tunes & chromaticities consistent within measurement uncertainty

Predicted Beta Function



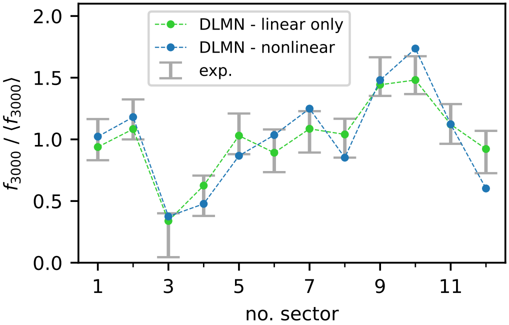
Default SIS18 doublet optics during magnetic flattop, (Q_x, Q_y)=(4.29, 3.29)

Relative beta function

- is predicted in good agreement between
  - Deep Lie Map Networks
  - LOCO-fit of orbit response matrix
  - spectral analysis of BPM turn-by-turn data
- is local quantity depending on quadrupole error distribution

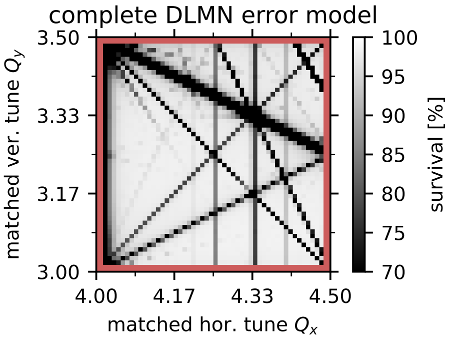
  → demonstrates that field errors can be quantified & localized

Predicted Nonlinear Optics



- Observe sextupoles via resonance driving terms
  - f_{3000} is dominant at working point (Q_x, Q_y)=(4.29, 3.29)
- Chromaticity corrected in control system,
  afterwards turn-off single sextupole in Sector 9
- DLMN predicted f_{3000} matches analysis of BPM spectra

Simulated Static Tunescans



- DLMN training results yield effective accelerator model in terms of magnetic multipole components
- refined optics model enables further use in
  simulation tools & tracking codes
- one application: simulating tune scans

  → analyze sources of resonance excitation
  → compute compensation schemes

Conclusion & Outlook

Conclusion

Deep Lie Map Networks

- new approach to identify magnetic field errors
  - → key observable: centroid oscillations in time-domain
- identify linear & nonlinear magnetic field errors in parallel
  - → step towards digital twin
- time-efficient
  - → only three trajectories required
- yields optics model in terms of physical quantities
  - → enables follow-up usage in analytical and numerical studies

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Proof-of-Principle Experiment

- recover linear optics
  - single trajectory
  - recover tunes, beta-functions, phase advances, dispersion
- recover second order optics
  - three trajectories
  - chromaticities, resonance driving term f_{3000}
- method applicable under realistic conditions at hadron synchrotron

Outlook

Application to optics reconstruction in LHC

- investigate large beta-beating / errors in beam energy [10]
- application to in arcs in single-pass mode
- replace single-turn kicker magnet with AC dipole excitation

Investigate forward-mode automatic differentiation

- Generalized Truncated Power Series Algebra (GTPSA)
  - efficient generalization of forward-mode AD to high number of input variables
  - reduced memory consumption as no intermediate results need to be kept
- implemented in MAD-NG / PyMAD-NG [11]
  - fully benchmarked against PTC [12]

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[1] J. Safranek, "Experimental determination of storage ring optics using orbit response measurements," *Nucl. Instrum. Meth. A*, vol. 388, pp. 27–36, 1997.

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[3] A. Parfenova and G. Franchetti, "Experimental benchmarking of nonlinear tune response matrix with several controlled sextupolar errors," *Nucl. Instrum. Meth. A*, vol. 646, pp. 7–11, 2011.

[4] D. K. Olsson, A. Andersson, and M. Sjöström, "Nonlinear optics from off-energy closed orbits," *Phys. Rev. Accel. Beams*, vol. 23, p. 102803, Oct 2020.

[5] C. Caliari, A. Oeftiger, and O. Boine-Frankenheim, "Identification of magnetic field errors in synchrotrons based on deep lie map networks," *Phys. Rev. Accel. Beams*, vol. 26, p. 064601, Jun 2023.

References (II)

[6] C. Caliari, A. Oeftiger, and O. Boine-Frankenheim, “Beam-based identification of magnetic field errors in a synchrotron using deep lie map networks,” *Phys. Rev. Accel. Beams*, vol. 28, p. 024601, Feb 2025.

[7] R. De Maria, L. Deniau, J. Dilly, J. M. Gray, A. Latina, F. Schmidt, P. Skowronski, J. S. Berg, and T. Gläßle, “Status of MAD-X V5.09,” *JACoW IPAC*, vol. 2023, p. WEPL101, 2023.

[8] M. Schwinzerl, R. De Maria, K. Paraschou, H. Bartosik, G. Iadarola, and A. Oeftiger, “Optimising and Extending A Single-Particle Tracking Library For High Parallel Performance,” 2021.

[9] D. P. Kingma and J. Ba, “Adam: A method for stochastic optimization,” 2014.

[10] J. M. Gray, “Beam energy discrepancy: What can the optics mismatch reveal?,” in *15th HL-LHC Collaboration Meeting, CERN*, (CERN, Geneva, Switzerland), October 2025.
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References (III)

[11] L. Deniau, “MAD-NG, a standalone multiplatform tool for linear and non-linear optics design and optimisation,” 2025.

[12] P. Skowronski, F. Schmidt, and E. Forest, “Advances in MAD-X using PTC,” in *2007 IEEE Particle Accelerator Conference (PAC)*, pp. 3381–3383, 2007.

Thank you for your Attention!

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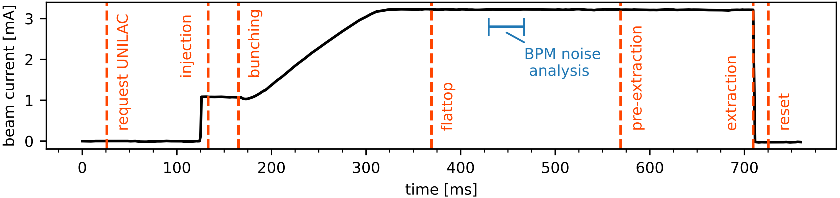
Setup

Trajectory measurement

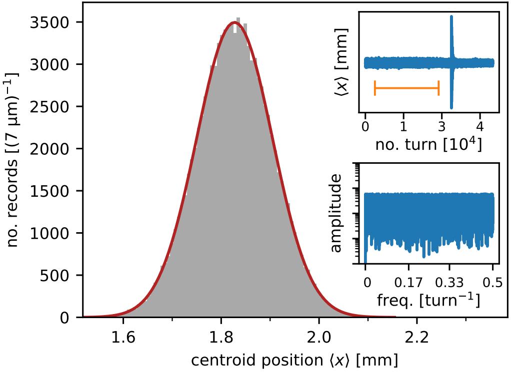
- vary momentum
  $\delta \in [-1 \times 10^{-3}, 0, 1 \times 10^{-3}]$
- record at magnetic flattop, 5.5 T m

Beam properties

- ion: Au^{65+}, $150\,\mathrm{MeV\,u^{-1}}$
- Emittances $\epsilon^{\text{4-rms}}_{x,\text{norm}} = 20\,\mu\mathrm{m}$, $\epsilon^{\text{4-rms}}_{y,\text{norm}} = 4\,\mu\mathrm{m}$
- Momentum Spread $\sigma_\delta = 4.8 \times 10^{-4}$

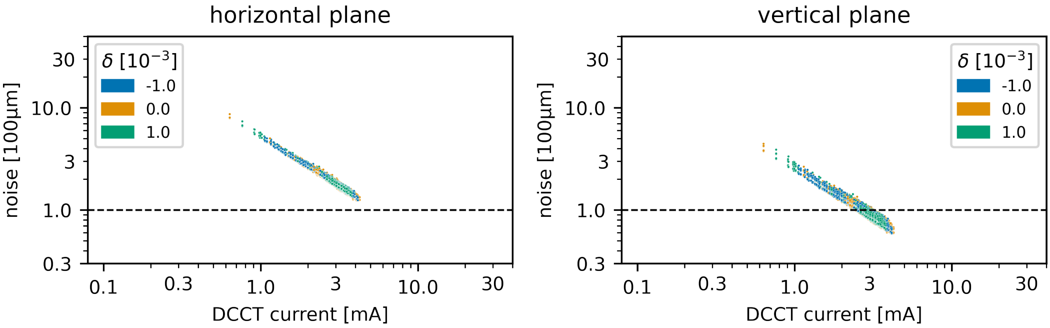


Determination BPM Noise Level



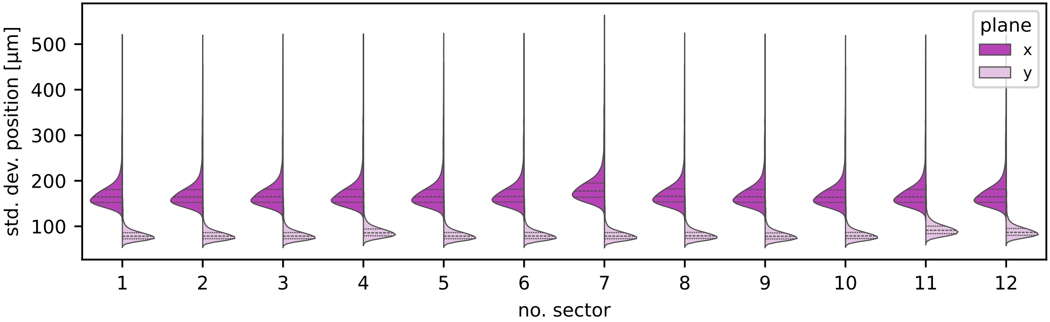
- beam in equilibrium state / analyze position over 25 000 turns
- residual fluctuations follow normal distribution
  - mean: closed orbit
  - standard deviation: BPM noise level
  - no significant deviation from normal distribution according to Shapiro-Wilk test
- white noise
  - flat frequency spectrum
  - no residual synchrotron oscillations

BPM Noise Level vs. Current

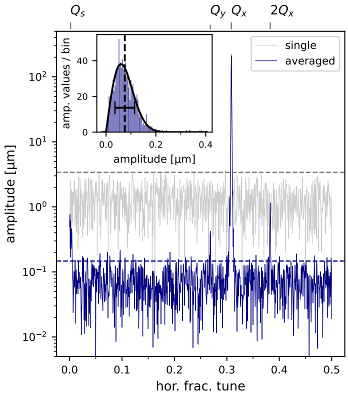


δ: fractional momentum offset
induced by rf frequency mismatch, dpfrev setting in Paramodi

BPM Noise Level per Sector



Spectral Noise Floor



Possible Resonances