Modeling of Insertion Devices in Xsuite (Ongoing Work)

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Acknowledgments: G. Iadarola, S. Bettoni, M. Boege and PSI team. T. Pieloni, M. Seidel

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Introduction

- Xsuite is a python-based beam simulation software package

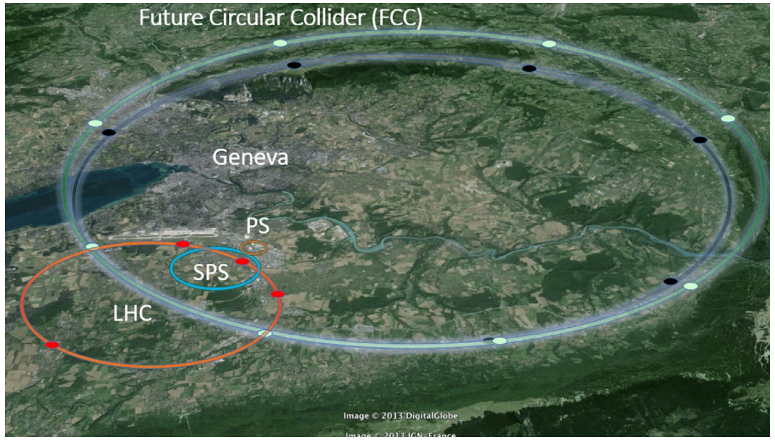
- Currently, insertion devices are not extensively modeled in Xsuite
  - Goal: To build a robust and reproducible model that captures the most important features of an undulator

- We studied the APPLE-Knot undulator in the SLS2.0 in Xsuite
  - Modeled the undulator field
  - Tracked particles through undulator and calculated optical functions
  - Compared to sliced model

- Observed:
  - Beta-beating
  - Coupling due to sextupolar components and longitudinal fields has been observed

Motivation

- Future Circular Collider (FCC-ee)
  - Electron-positron collider
  - Circumference of 90.7 km
  - Beam energy up to 175 GeV
- First stage of acceleration: damping ring
  - Circumference of ~373 m
  - Beam energy of 2.86 GeV
  - Uses wigglers to enhance damping
- Under consideration: Spin polarizer ring
  - Longitudinal magnetic fields cause decoherence [1]
  - **Longitudinal fields may arise in wigglers**



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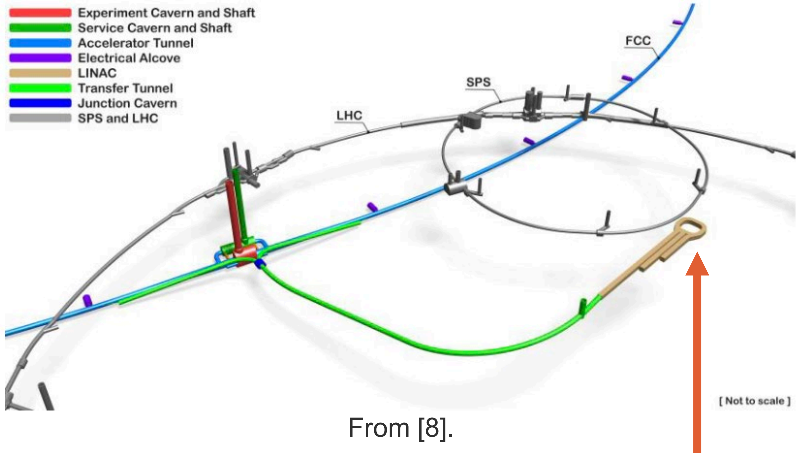


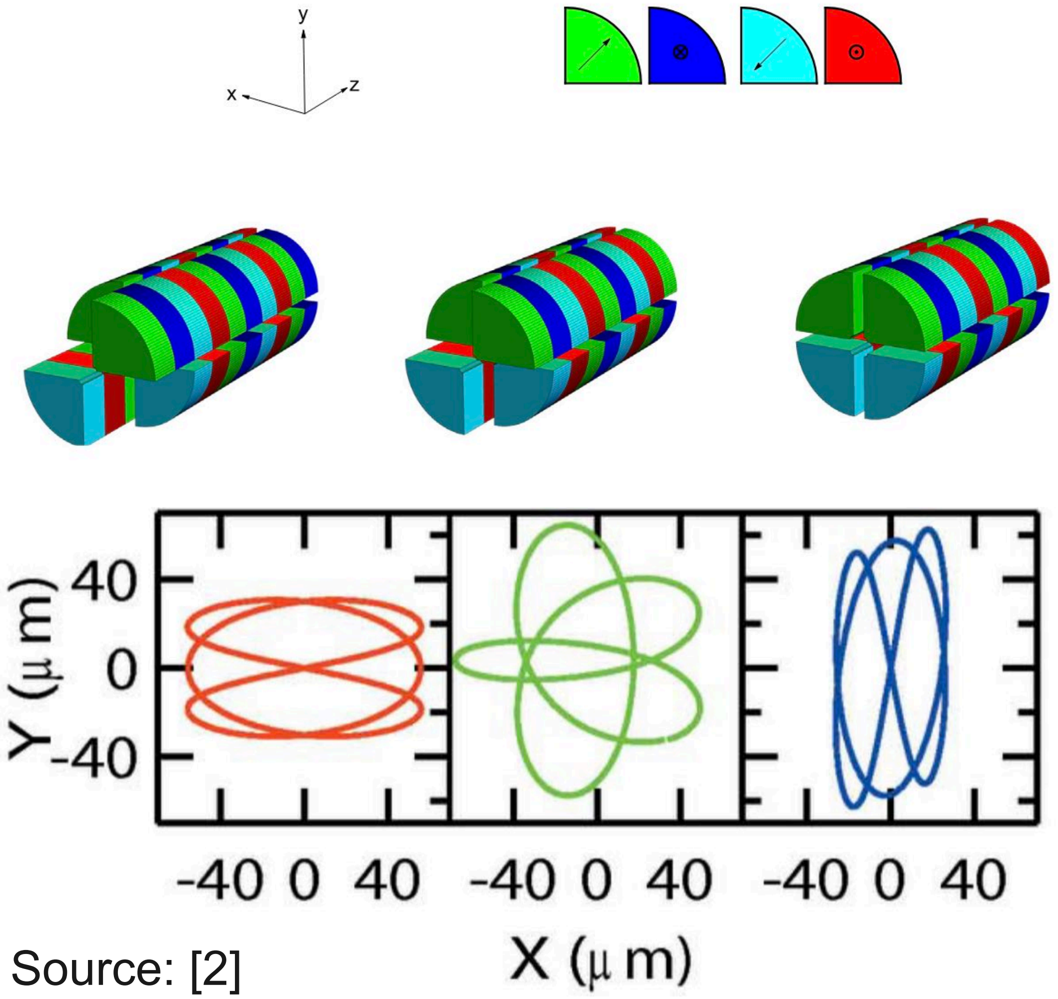
From [8].

Case Study: APPLE-Knot Undulator SLS2.0

- Swiss Light Source (SLS2.0)
  - Electron storage ring
  - Circumference of 288 m
  - Beam energy of 2.4 - 2.7 GeV

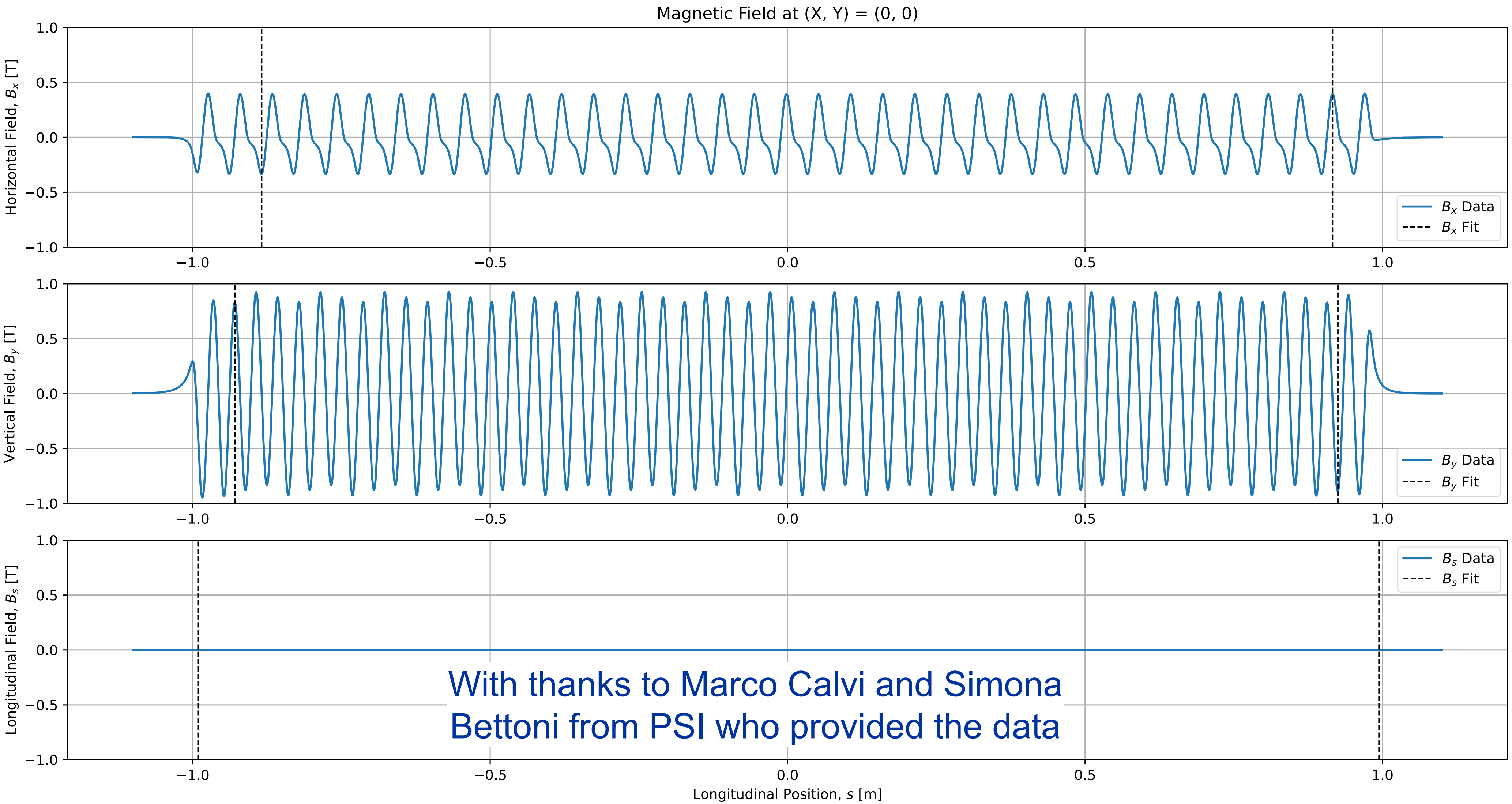
- APPLE-Knot Undulator
  - **A**dvanced **P**lanar **P**olarized **L**ight **E**mitter
  - Control polarization by setting array alignment
  - Knot: Electron makes "knot-like" shape in xy-plane

- **Use as case study to improve understanding of characteristics of wiggler fields**



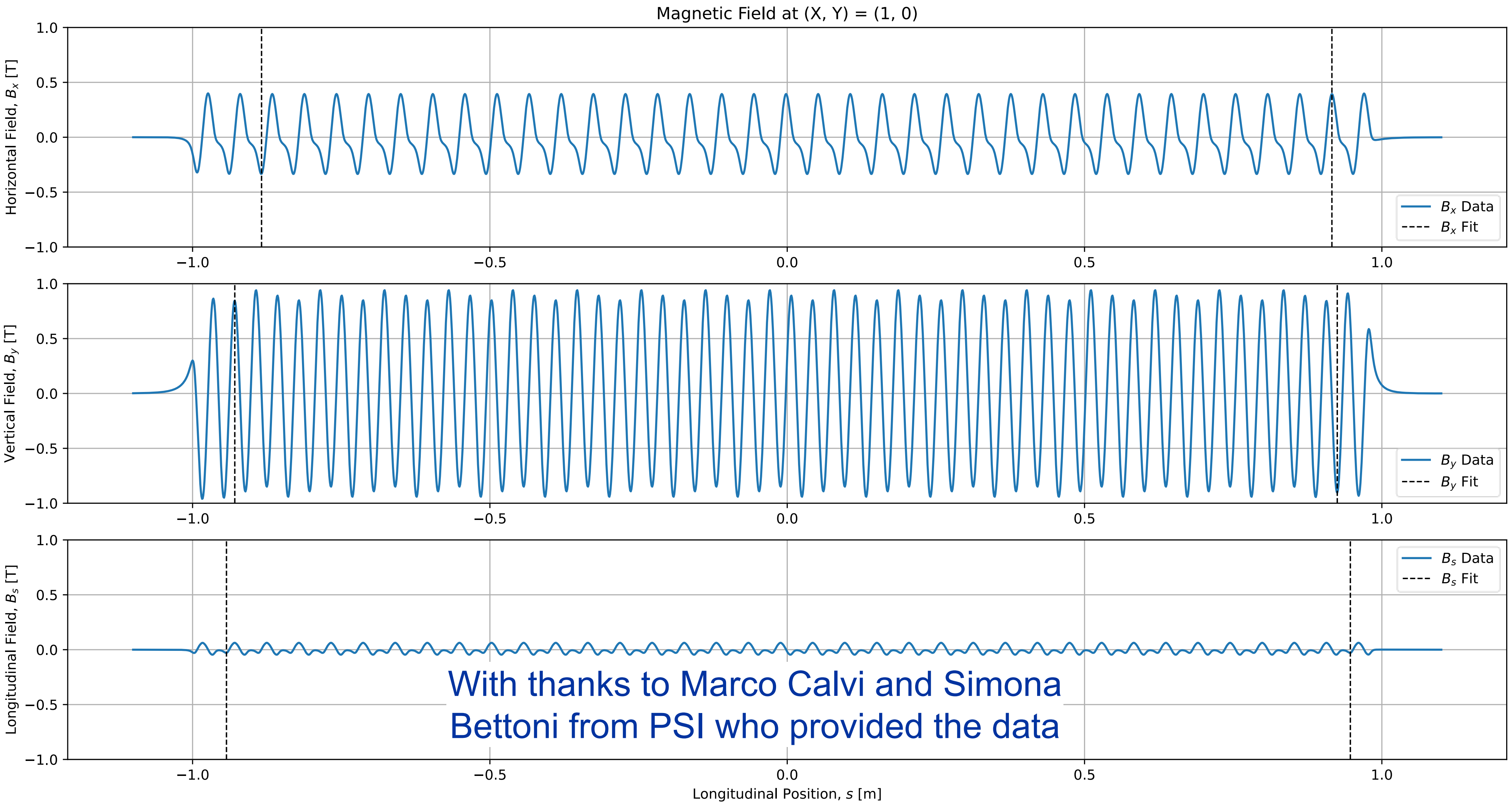
Source: [2]

APPLE-Knot Undulator: $\vec{B}$ at $(x, y) = (0, 0)$ mm



With thanks to Marco Calvi and Simona Bettoni from PSI who provided the data

APPLE-Knot Undulator : $\overrightarrow{B}$ at $(x, y) = (1, 0)$ mm



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Objectives

- Model undulators in Xsuite

- Three-dimensional field map too expensive:
  - $N_x = N_y = 20$, $N_s = 2400$ already gives $2.9 \cdot 10^6$ stored points
  - Need Maxwell-obeying interpolation between these points

- Find a compact and flexible description of $\vec{B}(x, y, s)$
  - Field at any point (x, y, s) can be found from field on one axis, $\vec{B}(0, 0, s)$
  - Field on-axis requires us to store 7200 points
  - Generated function automatically obeys Maxwell's Equations

- Perform tracking and optics calculations in Xsuite

Bpmeth

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- Following method from Ref. [3] and references therein. Define functions

$$a_m(s) = \left.\frac{\partial^{m-1} B_x}{\partial x^{m-1}}\right|_{x=y=0}$$

$$b_m(s) = \left.\frac{\partial^{m-1} B_y}{\partial x^{m-1}}\right|_{x=y=0}$$

$$b_s(s) = B_s(0,0,s)$$

- It can be shown that the scalar potential $\Phi(x,y,s)$ can be expressed as

$$\Phi(x,y,s) = \sum_{n=0}^{\infty} \phi_n(x,s) \frac{y^n}{n!}$$

- Where $\phi_n(x,s)$ depends on $a_m(s)$, $b_m(s)$, $b_s(s)$ and their derivatives. The magnetic field is then found from

$$\vec{B}(x,y,s) = -\nabla\Phi$$

Determining $\vec{B}(x, y, s)$

- So far, bpmeth has mainly been used for fringe fields, but we will apply it to wigglers/undulators
- The full magnetic field $\vec{B}(x, y, s)$ can be found to arbitrary precision
- If $\vec{B}$ does not depend on s, then a_m and b_m reduce to the skew and normal multipole coefficients respectively
- In a sliced multipole model, $a_m(s)$ and $b_m(s)$ are assumed to be approximately constant over a short interval and are thus interpreted as short pure multipole coefficients
- However, this last approach alone will not (correctly) predict a longitudinal component off-axis, as we will see

Transverse Derivatives

- First few coefficients:

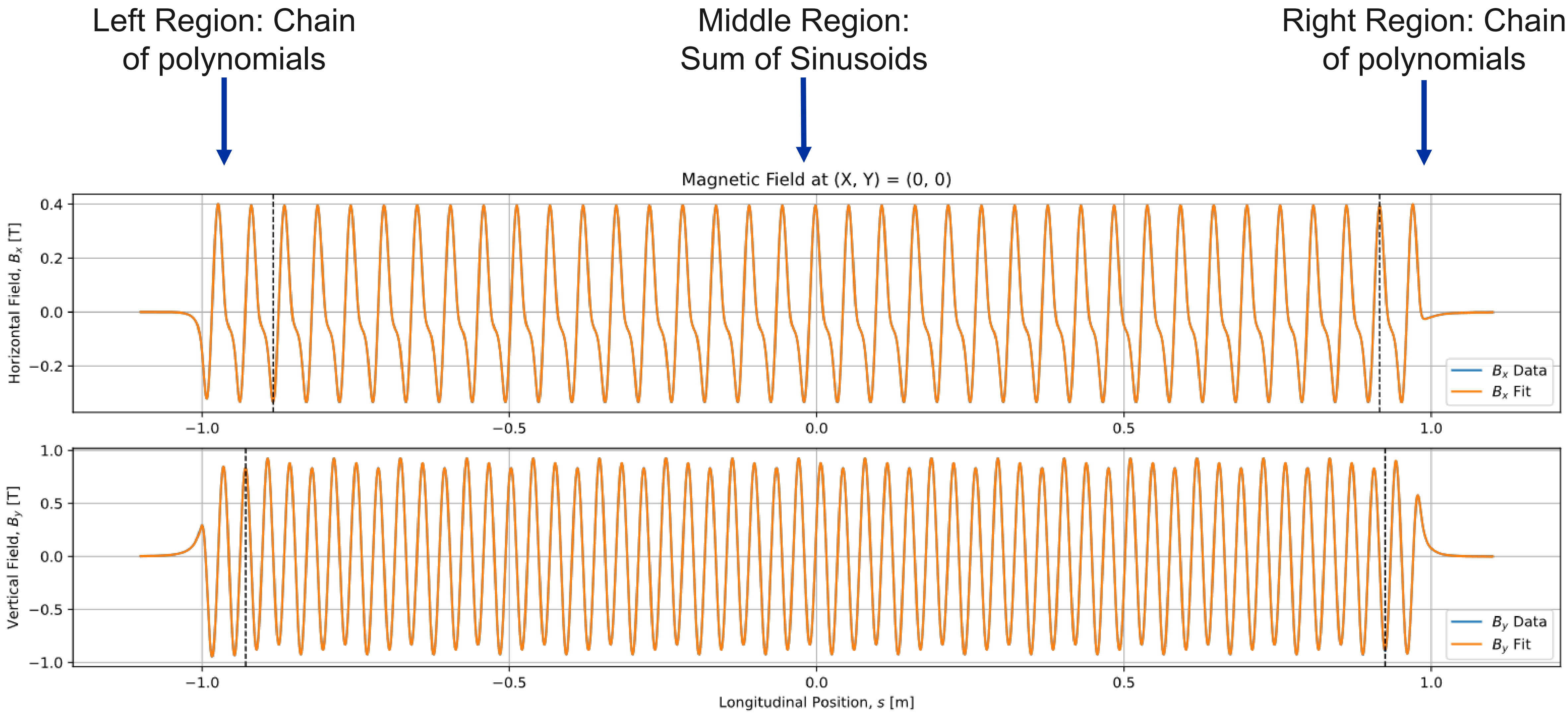
$$a_1(s) = B_x\Big|_{x=y=0}$$

$$a_2(s) = \frac{\partial B_y}{\partial x}\Bigg|_{x=y=0}$$

$$a_3(s) = \frac{\partial^2 B_x}{\partial x^2}\Bigg|_{x=y=0}$$

- So $a_1 \sim$ Dipole, $a_2 \sim$ Quadrupole, $a_3 \sim$ Sextupole
- Transverse functions are fitted up to order $\partial_x^2 B_i$
- First derivative is negligible; $a_2(s) \approx b_2(s) \approx 0$
- Second derivative defines $a_3(s)$ and $b_3(s)$

Fit for On-Axis Field

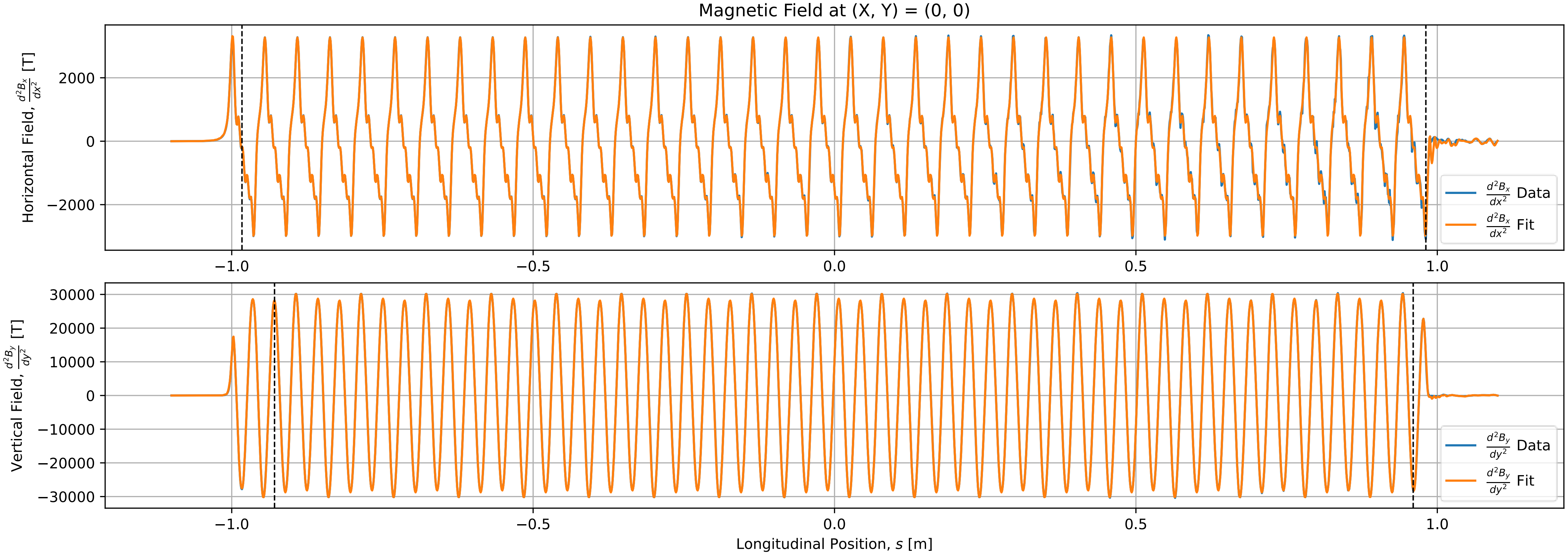


Analytic Expressions for Transverse Derivatives

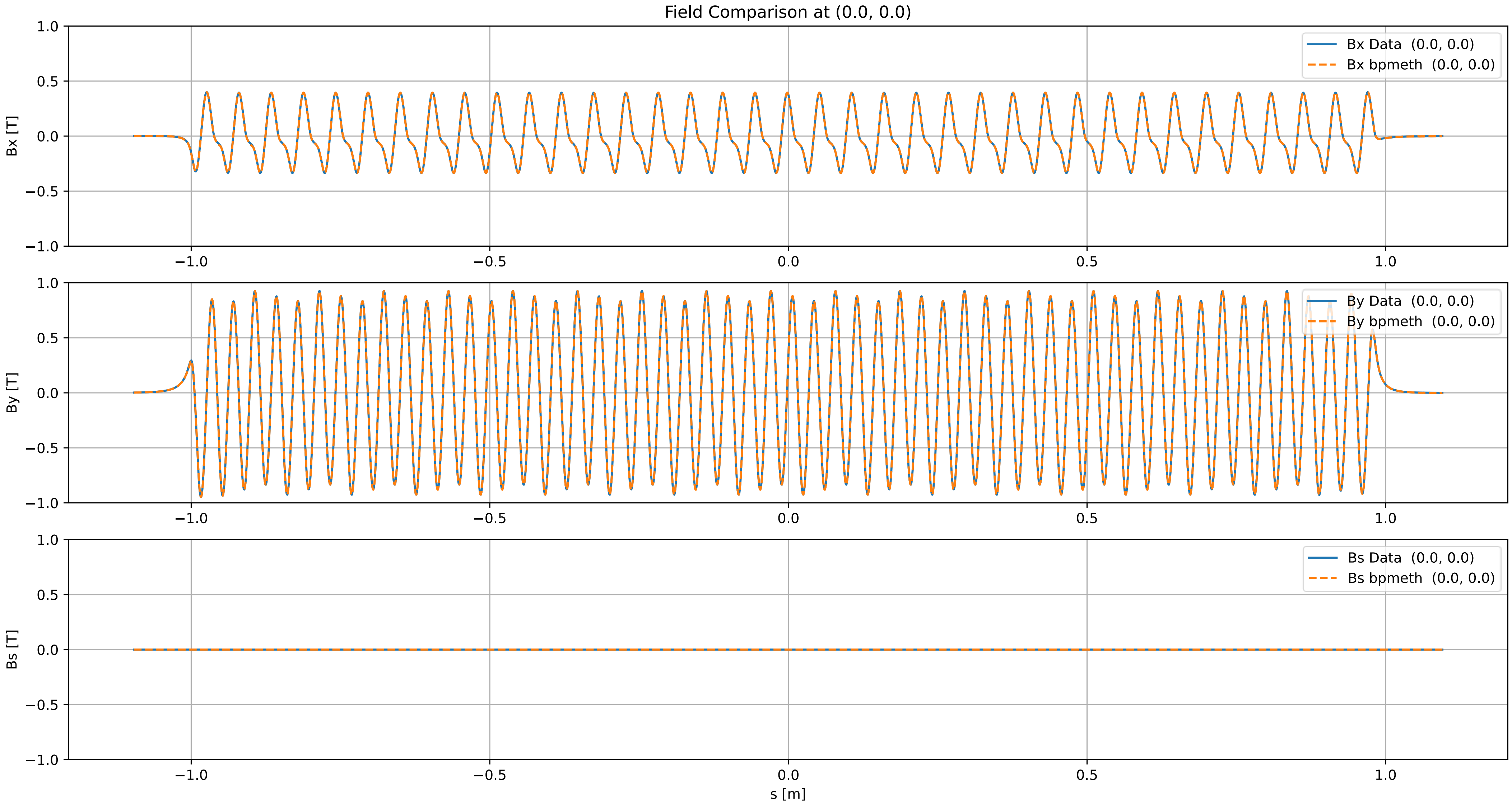
Left Region: Chain of polynomials

Middle Region: Sum of Sinusoids

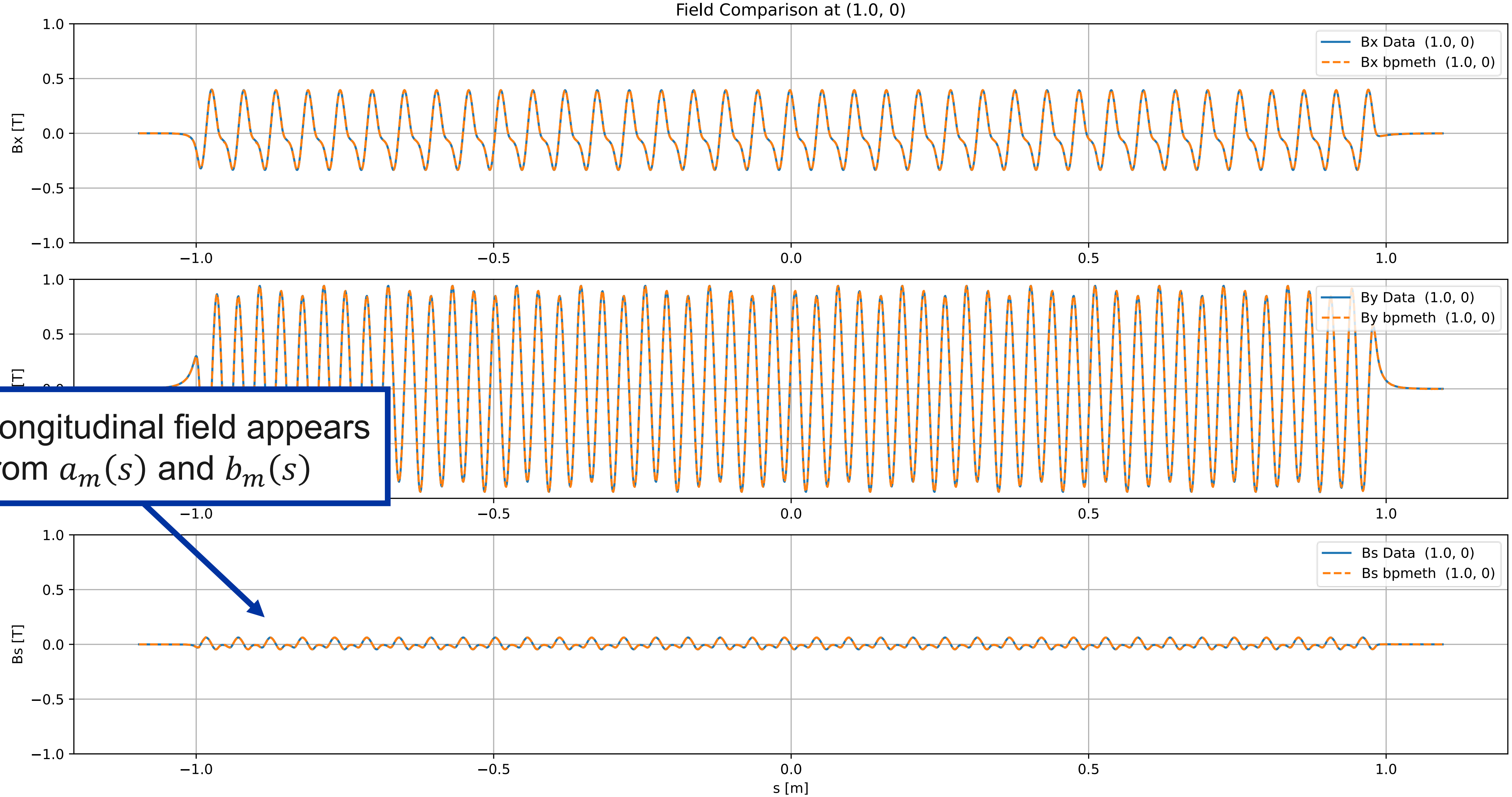
Right Region: Chain of polynomials



Calculate Fields from Outlined Method



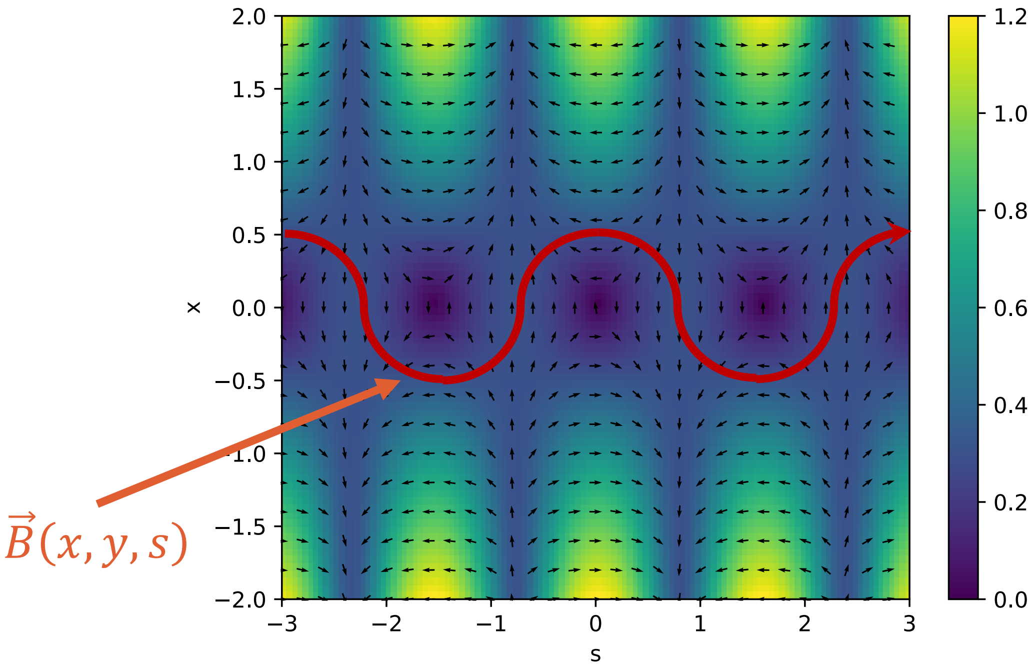
Calculate Fields from Outlined Method

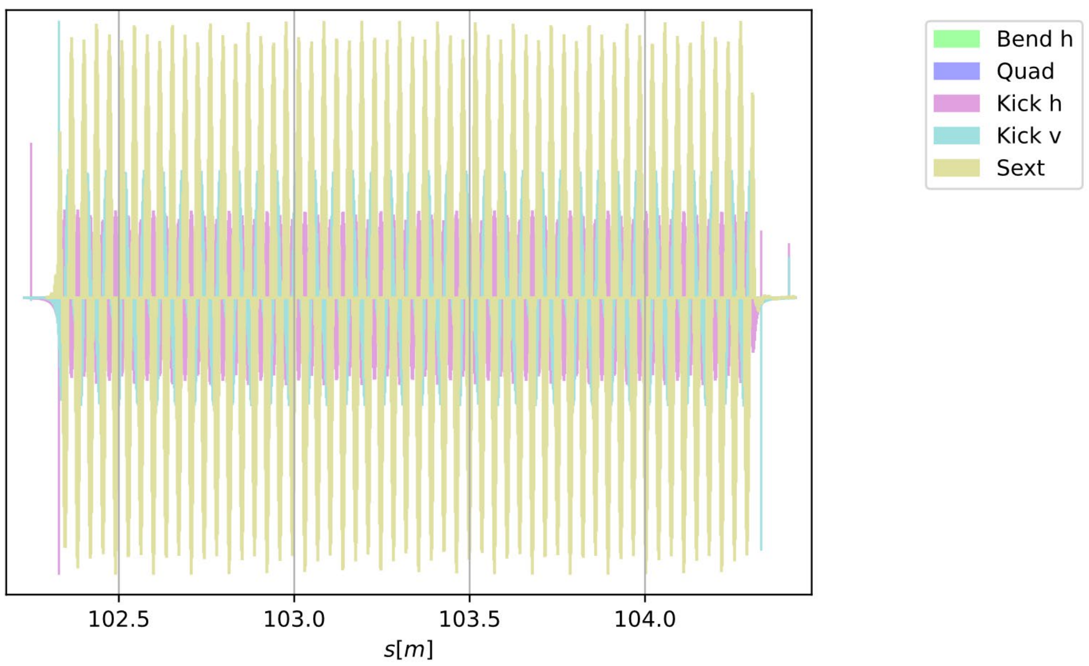


The longitudinal field appears from $a_m(s)$ and $b_m(s)$

Undulator in SLS2.0: Compare Several Methods

- Ring without undulator
- Tracking through undulator: Short-term properties
- Sliced undulator: Long-term studies





Undulator in SLS2.0: Boris Integrator

- Phase-space area conserving integrator [4]
  - Not symplectic
  - One evaluation of $\vec{B}(x, y, s)$ per step
  - Per-step error of $\mathcal{O}(h^3)$, integrated error of $\mathcal{O}(h^2)$
  - Error oscillates, does not blow up

- For h such that the error approaches machine precision, Boris integrator is effectively symplectic

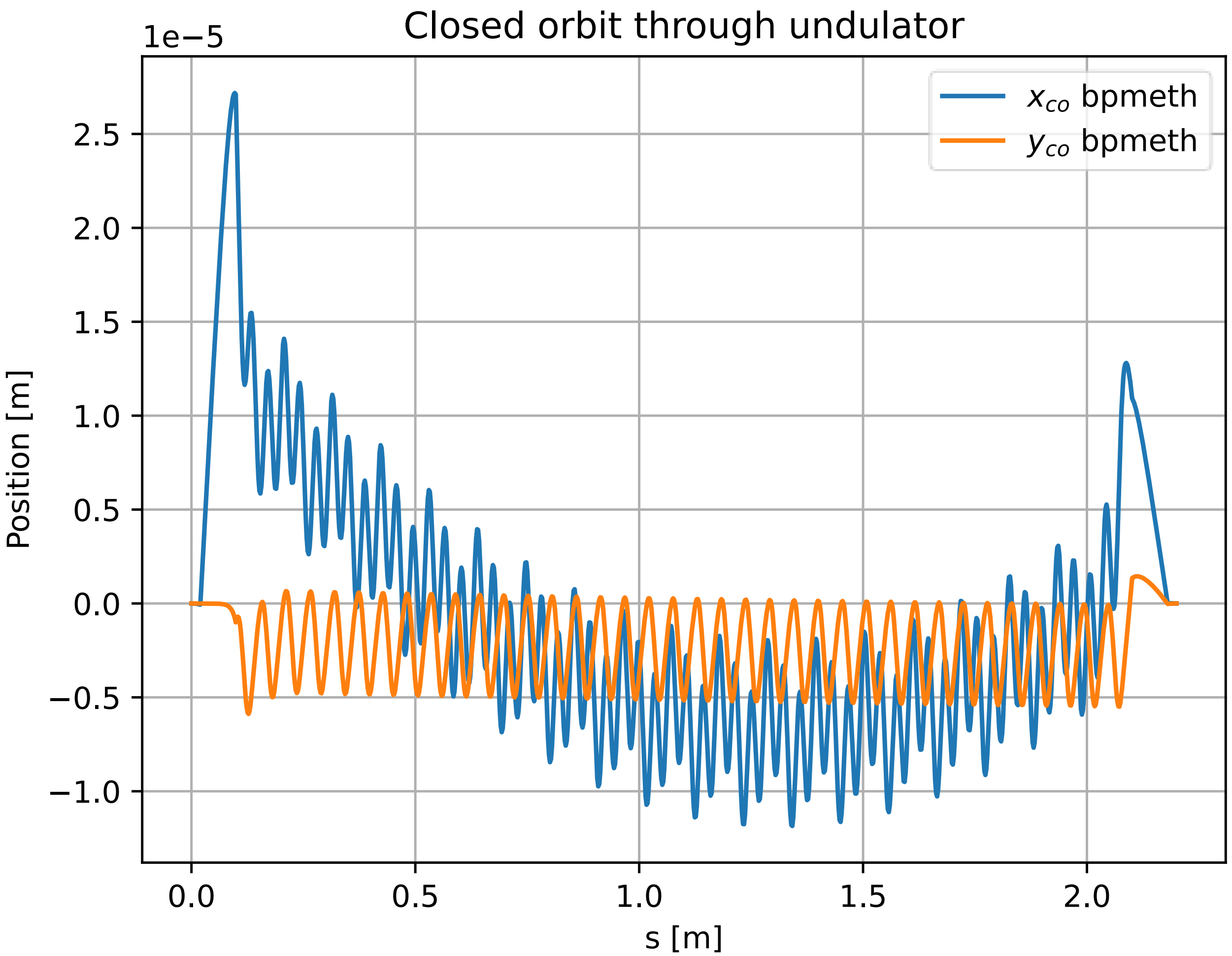
- Xsuite calculates Twiss parameters with finite differences
  - Can calculate Twiss from tracking
  - Investigate beta-beating and coupling effects

- Sliced undulator constructed from $\vec{B}(x, y, s)$
  - Compare the sliced undulator with the tracked one: On-axis and $5 \cdot 10^{-4}$ m displacement
  - Work on Boris integrator has just started; Investigate its viability

Undulator in SLS2.0: Closed Orbit

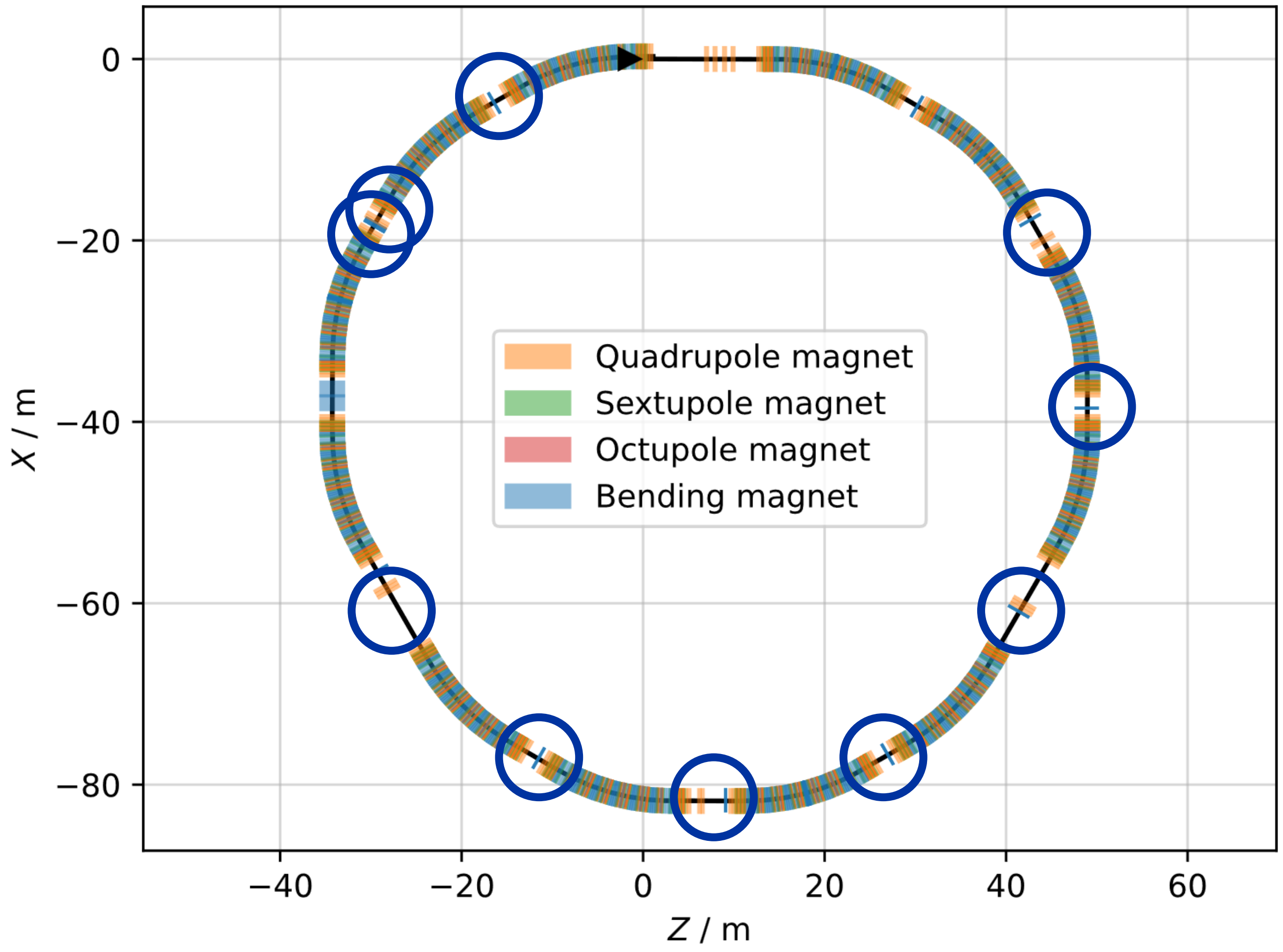
- Orbit might look unexpected
  - Cross-check with undulator simulation expert at PSI

- Dipole kicks added at the edges to avoid closed orbit leakage outside the wiggler

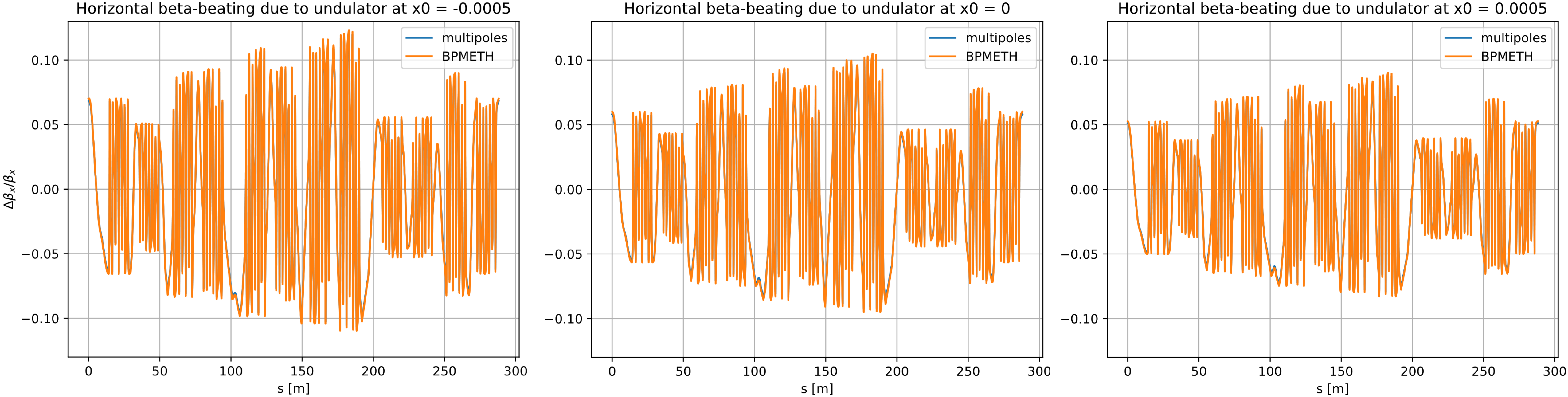


Undulator in SLS2.0: Closed Orbit

- Insert 10 such wigglers in the ring.
- Real SLS2.0 does not have ten APPLE-Knot undulators
- Does provide understanding of some important features



Undulator in SLS2.0: Beta Beating from Wigglers



- Large portion of beta-beating likely artifact of distorted closed orbit
- Weak effect due to quadrupole as a result of feed-down from the sextupole components
- Good agreement between tracking and slicing

Undulator in SLS2.0: Coupling

- In the presence of coupling:
  - Eigenmodes (1, 2) of one-turn matrix do not align with the modes in the chosen coordinates (x, y) [7]
  - Amplitude ratios:

$$r_1 = |A_{1,x}|/|A_{1,y}|$$
$$r_2 = |A_{2,x}|/|A_{2,y}|$$

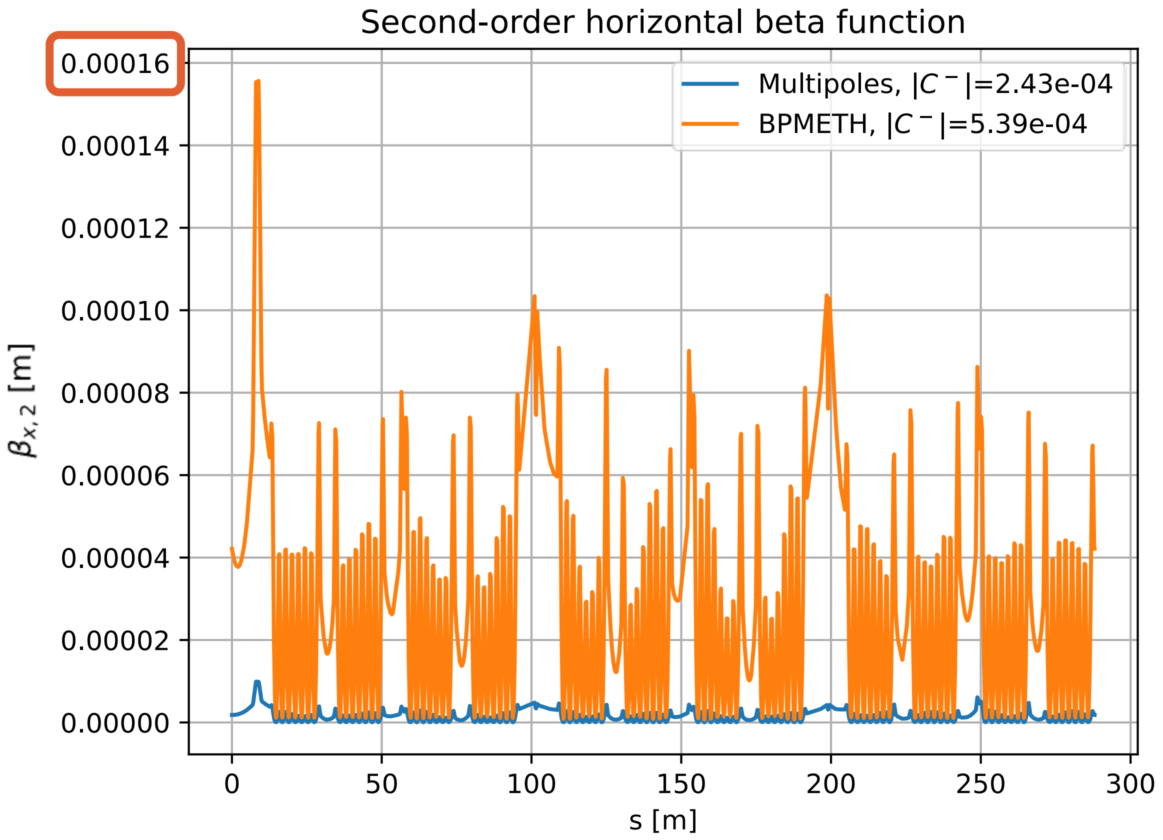
  - Tunes of eigenmodes, Q_1, Q_2
  - Coupling parameter:

$$|C^-| = \frac{2\sqrt{r_1 r_2}|Q_1 - Q_2|}{1 + r_1 r_2}$$

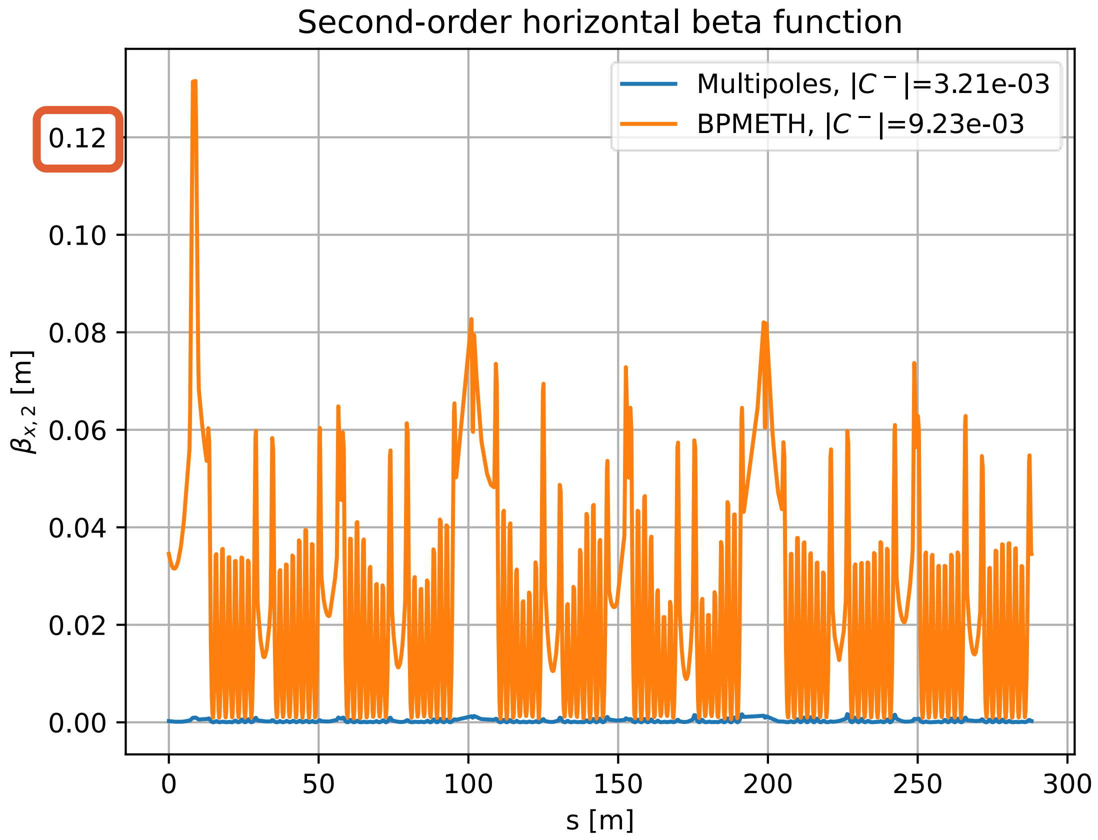
- Parameter arises (among others) from skew quadrupoles or longitudinal fields

Undulator in SLS2.0: Coupling

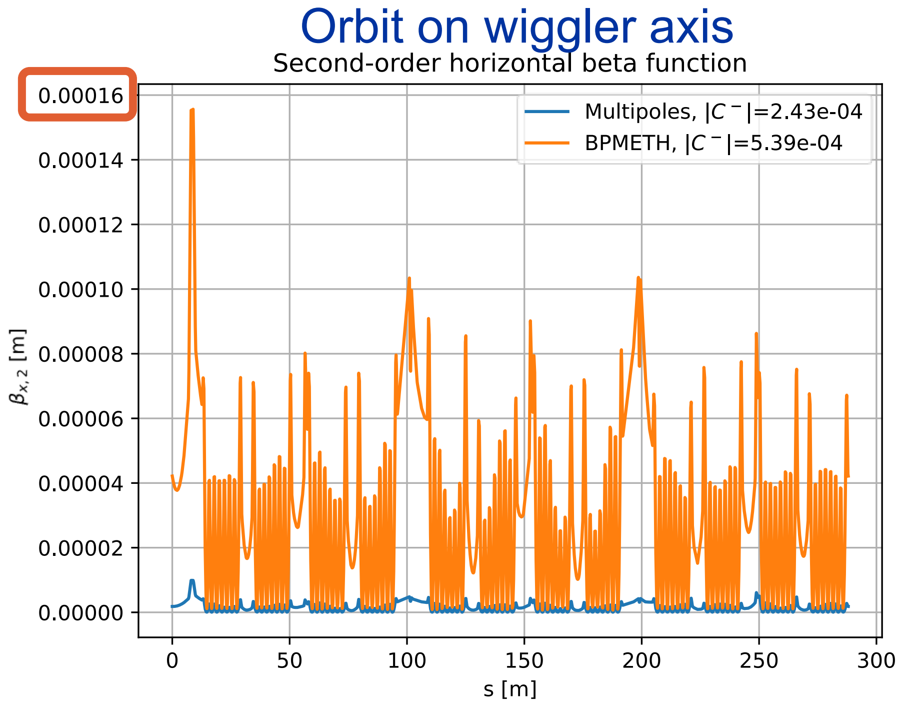
Orbit on wiggler axis

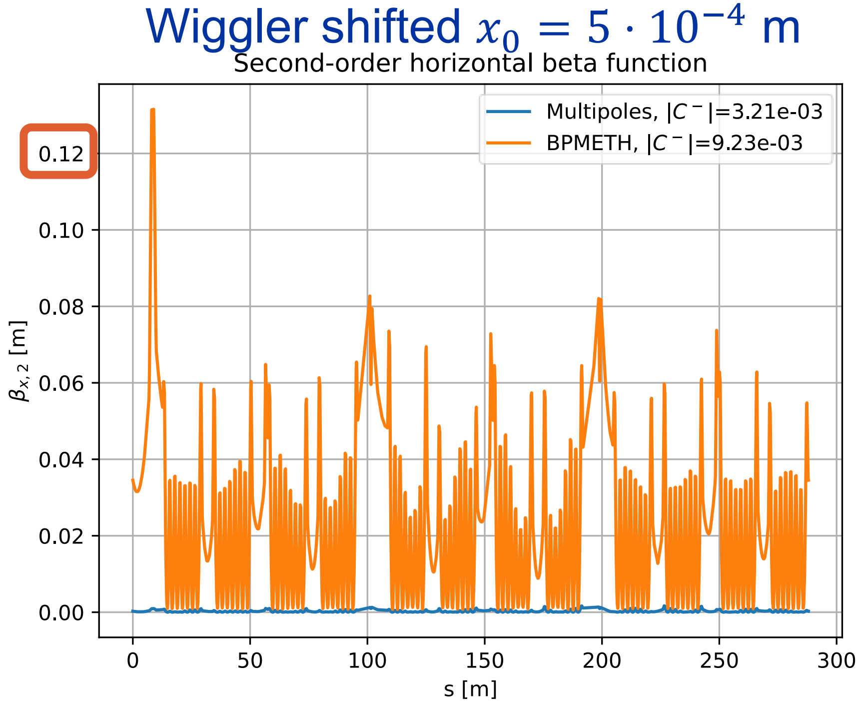


Wiggler shifted $x_0 = 5 \cdot 10^{-4}$ m



Undulator in SLS2.0: Coupling





- Coupling much stronger in misaligned wiggler
- Off-centre sextupoles contribute ~1 order of magnitude
- Longitudinal fields contribute ~ factor 3

Conclusions and Outlook

- We have a way of modeling insertion devices with arbitrary field distributions in Xsuite
  - Model is based on field and its derivatives, only on-axis
  - Bpmeth can extrapolate magnetic field to arbitrary points, such that it satisfies Maxwell’s Equations

- Undulator model, constructed this way, can be integrated into an Xsuite lattice
  - For now: Can be used for short-term tracking and Twiss calculations
  - Demonstrated that we can compute impact on beta-beating and coupling
  - Already observed that wiggler constructed from multipoles does not correctly account for coupling

- Next steps:
  - Improve performance of field calculation (present implementation is quite naïve)
  - Make fitting procedure more robust and versatile
  - Validate spin-tracking through map
  - Ensure reliable long-term tracking simulation (investigate using symplectic integrators)
  - Implement existing map models from other codes and compare with existing maps [6][7]

References

[1] Y. Wu, D. Barber, F. Carlier, E. Gianfelice-Wendt, T. Pieloni, & L. van Riesen-Haupt (2023). *Spin-Polarization Simulations for the Future Circular Collider ee using BMAD*

[2] X. Liang, M. Calvi, M.E. Couprie, R. Ganter, C. Kittel, N. Sammut, & K. Zhang (2021). Analysis of the first magnetic results of the PSI APPLE X undulators in elliptical polarisation.

[3] S. Van der Schueren et al. (2024) Magnetic field modelling and symplectic integration of magnetic fields on curved reference frames for improved synchrotron design: first steps.

[4] Stoltz, P. H., Cary, J. R., Penn, G., & Wurtele, J. (2002). Efficiency of a Boris-like integration scheme with spatial stepping.

[5] Wiedemann, H. (2015). *Particle accelerator physics*.

[6] Elleaume, P. (1992). A new approach to the electron beam dynamics in undulators and wigglers.

[7] Bahrdt, J., & Wüstefeld, G. (2011). Symplectic tracking and compensation of dynamic field integrals in complex undulator structures.

[8] P. Craievich (on behalf of the CHART/FCC-ee Injector Study collaboration). (2025, May 21). *FCC-ee injector overview and outlook* [Conference presentation]. *FCC week*. URL

With thanks to S. van der Schueren, R. de Maria, G. Iadarola, S. Bettoni and M. Calvi

Textbook Wiggler/Undulator

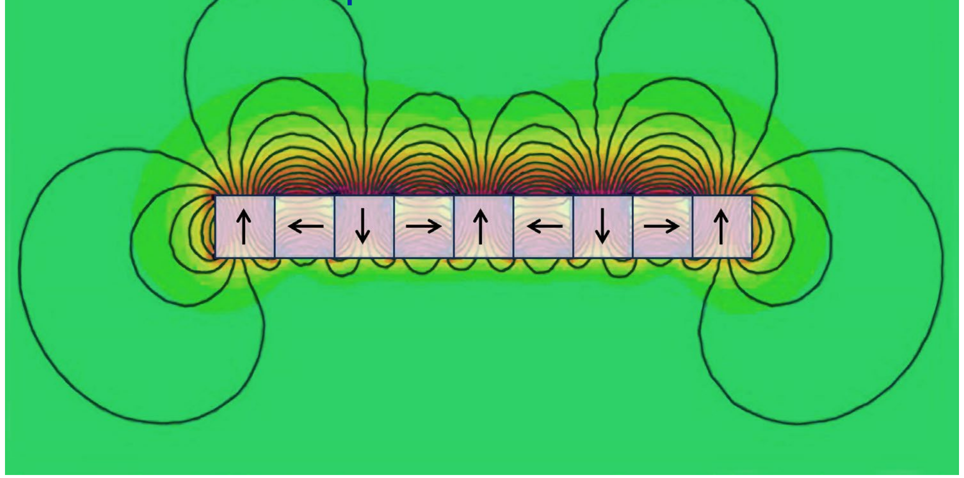
Cite: Wikipedia
https://www.researchgate.net/figure/Schematic-of-a-wiggler-undulator-structure-adopted-from-77_fig5_231521467
Wille for equation

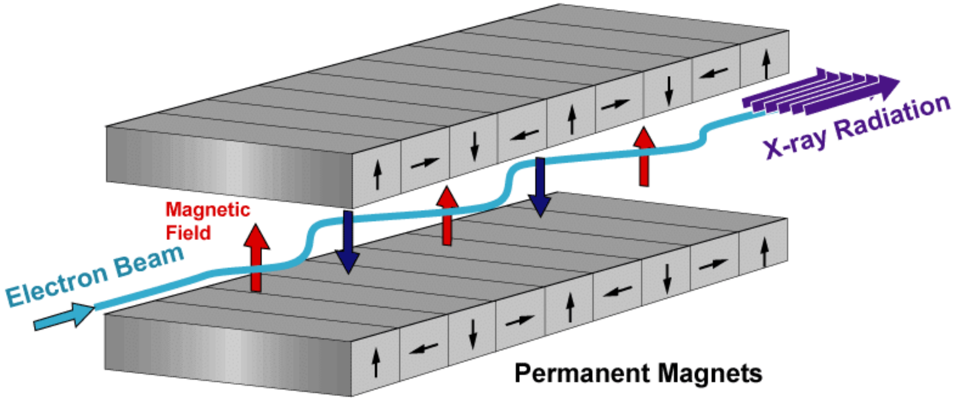
- Halbach array
  - Arrangement of permanent dipole magnets
  - Enhances the field on one side

- Two Halbach arrays generate a field of the form

$$B_y = B_0 \cosh(k\,y)\sin(k\,s)$$
$$B_s = B_0 \sinh(k\,y)\cos(k\,s)$$

- Important features:
  - Field periodic along axis;
  - No horizontal component ($B_x = 0$)
  - Longitudinal field zero on-axis
  - **Off-axis, B_s appears**





The Bpmeth(od)

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- Using Maxwell's Equations, full $\vec{B}(x, y, s)$ can be recovered from $\vec{B}(0, 0, s)$, $\left.\frac{\partial B_x}{\partial x}\right|_{x=y=0}$, $\left.\frac{\partial B_y}{\partial x}\right|_{x=y=0}$ and higher derivatives

- Method outlined in Ref. [SILKE] and references therein

$$\vec{B} = -\nabla\Phi$$

- Harmonic magnetic field can be derived from a scalar potential Φ that obeys:

$$\nabla^2\Phi = 0$$

- The scalar potential can be expanded as:

$$\Phi(x, y, s) = \sum_{n=0}^{\infty} \phi_n(x, s)\frac{y^n}{n!}$$

The Bpmeth(od)

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- Substituting this into the Laplace equation gives the recurrence relation

$$\phi_{n+2} = -\frac{1}{1+hx}\left[\partial_x\big((1+hx)\,\partial_x\phi_n\big) + \partial_s\left(\frac{1}{1+hx}\partial_s\phi_n\right)\right]$$

- This means we only need two ϕ_n to determine all others

- Choose the first two ϕ_n; ϕ_0 and ϕ_1 and expand these in x

$$\phi_0 = -a_0(s) - \sum_{m=1}^{\infty} a_m(s)\frac{x^m}{m!}$$

$$\phi_1 = -\sum_{m=1}^{\infty} b_m(s)\frac{x^{m-1}}{(m-1)!}$$

The Bpmeth(od)

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- Then, applying $\vec{B} = -\nabla\Phi$ gives

$$B_x(x,0,s) = -\partial_x \phi_0 = \sum_{m=1}^{\infty} a_m(s) \frac{x^{m-1}}{(m-1)!}$$

$$B_y(x,0,s) = -\phi_1 = \sum_{m=1}^{\infty} b_m(s) \frac{x^{m-1}}{(m-1)!}$$

$$B_s(0,0,s) = -\partial_s a_0 = b(s)$$

- From this, we find that

$$a_m(s) = \left.\frac{\partial^{m-1} B_x}{\partial x^{m-1}}\right|_{x=y=0}$$

$$b_m(s) = \left.\frac{\partial^{m-1} B_y}{\partial x^{m-1}}\right|_{x=y=0}$$

The Bpmeth(od)

Table 1: First Terms of the Expansion of the Magnetic Field

	1	x	y	x^2	xy	y^2
B_x	a_1	a_2	b_2	$\frac{a_3}{2}$	b_3	$\frac{-a_3-h(a_2-a_1h-2b_s')+b_sh'-a_1''}{2}$
B_y	b_1	b_2	$-b_s'-a_1h-a_2$	$\frac{b_3}{2}$	$-a_3-h(a_2-a_1h-2b_s')+b_sh'-a_1''$	$-\frac{b_3+hb_2+b_1''}{2}$
B_s	b_s	$-b_sh+a_1'$	b_1'	$b_sh^2-a_1'h+\frac{a_2'}{2}$	$-hb_1'+b_2'$	$-\frac{a_1h'+ha_1'+b_s''+a_2'}{2}$

Boris Integrator

- State vector:

$$w = \begin{bmatrix} p_x \\ p_y \\ U/c \end{bmatrix}$$

- Space step:

$$\frac{dw}{dz} = Mw + b$$

- Where:

$$M = \frac{q}{p_z}\begin{bmatrix} 0 & B_z & E_x/c \\ -B_z & 0 & E_y/c \\ E_x/c & E_y/c & 0 \end{bmatrix}, \qquad b = q\begin{bmatrix} -B_y \\ B_x \\ E_z/c \end{bmatrix}$$

- Transform w half-step by only b, transform full-step with M, transform w with final half-step using only b

Undulator in SLS2.0: Damping and Equilibrium Emittance ($x_0 = 0$)

	Bpmeth	Sliced	No Wiggler
α_x [s^{-1}]	184.87	184.87	169.73
α_y [s^{-1}]	107.85	107.85	92.706
α_ζ [s^{-1}]	138.67	138.67	108.39
$\langle\epsilon_x\rangle$ [pm]	123.1	123.1	133.3
$\langle\epsilon_y\rangle$ [pm]	0.02021	0.02111	0.0

Undulator in SLS2.0: Tune and Chromaticity

	Q_x	Q_y	dQ_x	dQ_y
Base Ring (bend model)	39.3737	15.2251	1.03928	1.53434
Without Wiggler	39.3700	15.2200	0.999197	1.57039
Analytic Wiggler (aligned)	39.4182	15.2222	1.38289	1.57962
Sliced Wiggler (aligned)	39.4171	15.1841	1.37377	1.39486
Analytic Wiggler (misaligned)	39.4135	15.2270	1.34358	1.60337
Sliced Wiggler (misaligned)	39.4124	15.1888	1.33558	1.42358

- Observations:
  - Sliced wiggler has difference on the order of 10^{-1} in vertical chromaticity w.r.t. integrated wiggler
    - Induced by sextupoles, but partially canceled by longitudinal fields? Need to investigate.
  - Misalignment had 10^{-2} effect on chromaticity, $10^{-3}/10^{-4}$ effect on tunes
  - Tune differences on the order of 10^{-3} depending on slicing/integrator

SLS2.0: Beta Function