



umcg



Modeling and benchmarking on Intra-beam Scattering in RF-Track

Paula Desire (CERN-ABP-CAP and University of Groningen), **Andrea Latina** (CERN-ABP-CAP), **Simone Di Mitri** (Univ. Of Trieste, Elettra - Sincrotrone Trieste), **Alexander Gerbershagen** (University of Groningen), **Alexander Malyzhenkov** (CERN).

10th Low Emittance Rings Workshop 2025, DESY, Hamburg.

Date: 10th of October

Table of contents

- **Introduction**
- **Implementation**
- **Results – SwissFEL Studies**
- **Results - FCCee damping rings**
- **Results - Benchmarking against semi-analytical methods**
- **Conclusions and future work**

Introduction

Introduction – State of the art

What is Intrabeam Scattering?

Intrabeam Scattering (IBS) refers to particle-to-particle elastic small-angle collisions by **Coulomb interaction**.

IBS in linear accelerators

IBS has been deeply studied for storage and damping rings, because it has a big impact on their emittance growth.

But most of the existing literature...

- Applies to storage rings.
- Assumes Gaussian beams and/or high energies.
- Almost **never been studied for linear accelerators**.

Why is it now important?

Now, linear accelerators are reaching very high intensities and brilliance (high charge, very low emittances), specially in Free Electron Lasers (FELs). Recent studies, have shown that IBS can be a **determining factor** for FELs as IBS increases the bunch **sliced energy spread** (SES).

Introduction – Implementation in RF-Track

The implementation of IBS in **RF-Track** was motivated by the need to address its impact on the equilibrium emittance in the **final cooling channel of the Muon Collider**.

RF-Track is an optimal simulation tool for Linacs, muon cooling, and FELs because:

- It can handle the relevant collective effects (space-charge, wakefields, beam loading)...
- It can perform time integration, optimal for injectors (space-charge dominated regimes).
- It implements realistic field maps and element overlap.

Since IBS has been identified as main cause of **SES growth** at **SwissFEL**, we performed **experimental benchmark against SwissFEL measurements**.

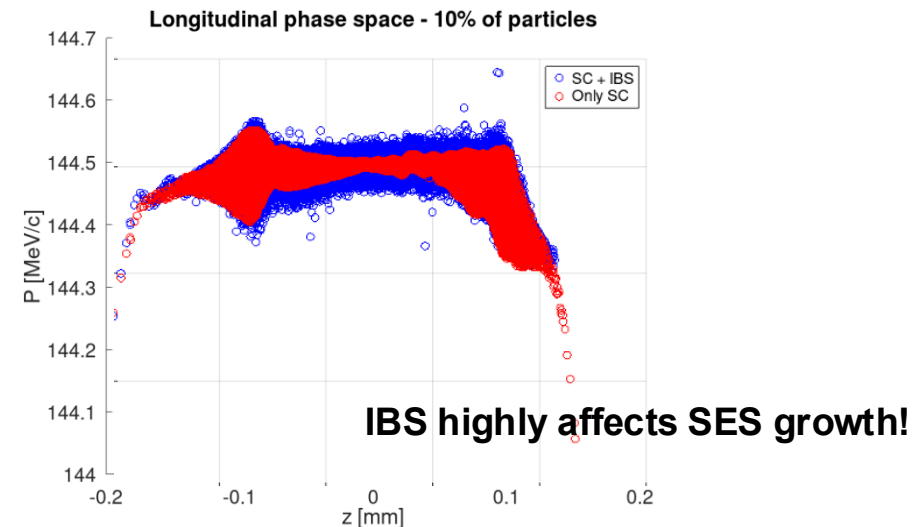
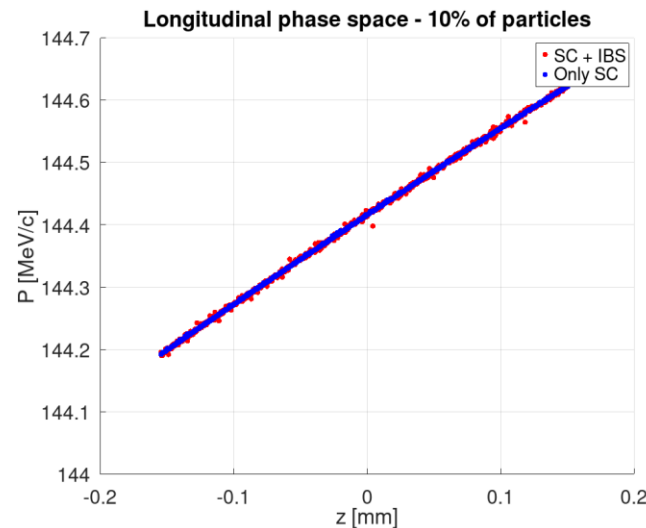
In addition, we are currently working IBS studies in the **FCee damping ring**.

Introduction - Sliced energy spread (SES) calculation

The numerical SES is calculated in different manners: by calculating the energy spread in a central slice, by removing the correlated energy spread from the momentum and getting the energy spread afterwards...

In this work:

1. The longitudinally central **10% of particles** are selected.
2. An **histogram** is done for this 10% of particles.
3. The central bin points $\{Z_a\}$ and the mean momentum in the bin $\{P_a\}$ are stored.
4. An **interpolated curve** from $\{Z_a, P_a\}$ is obtained and **subtracted** to P to obtain the uncorrelated contribution.
5. The **energy spread of the remaining values** is calculated.



Implementation

Implementation – Scheme

IBS implementation follows these steps:

Step 1: Calculation of four 3D grids within the limits of the bunch: density and velocity grids (v_x , v_y , v_z).

The following steps are repeated for all the particles:

Step 2: For a certain particle, an "average particle" is interpolated in its position. This average particle has the interpolated velocity from the velocity 3D grid. A local density is also interpolated.

Step 3: The frame is moved to the average-particle rest frame (IBS kinematics is usually defined in this frame).

Step 4: IBS kinematic parameters are defined (maximum impact parameter, minimum scattering angle, cross section, mean free path per collision, number of collisions per kick...).

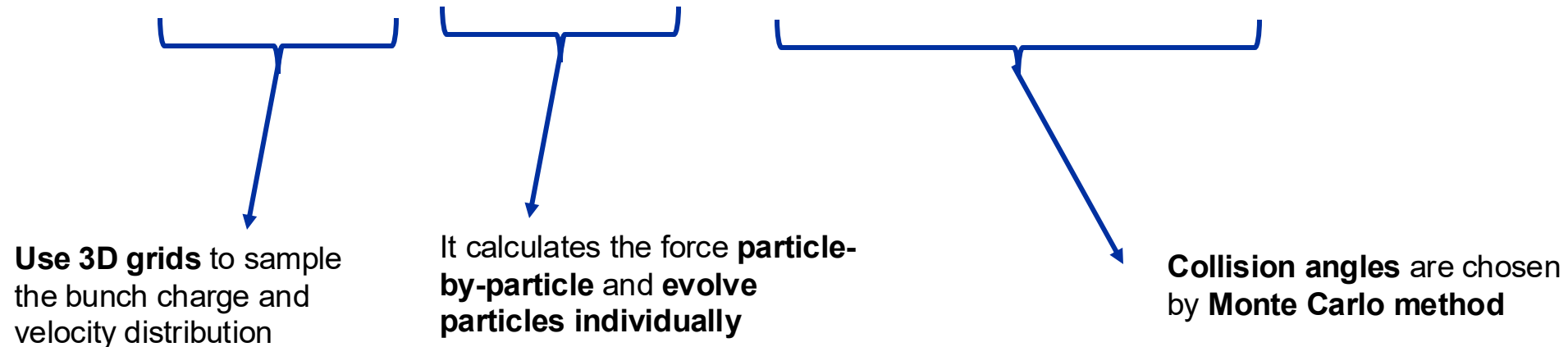
Step 5: Each collision chooses its angles by a Monte Carlo method based on the differential Rutherford cross section.

Step 6: Finally, a change of momentum (a force) is calculated for each particle.

Implementation – Sum up

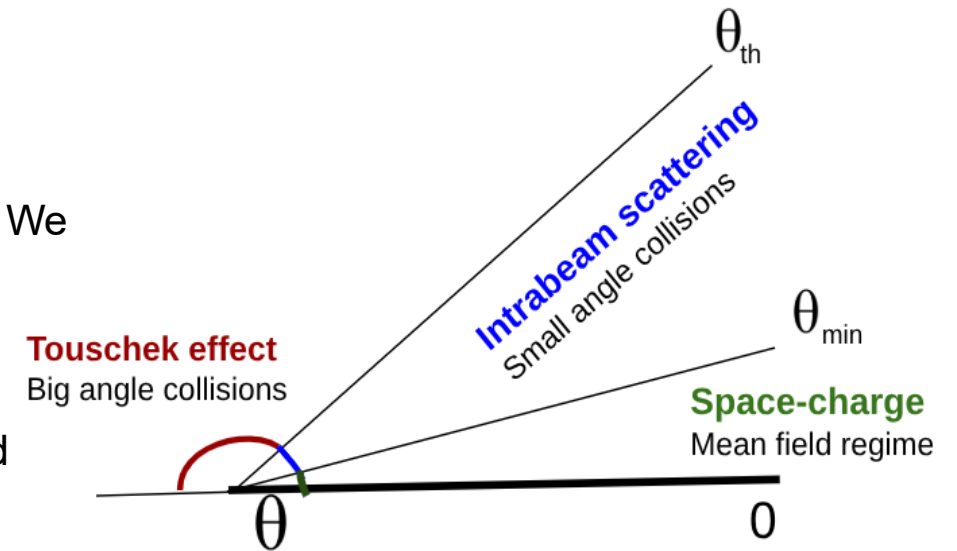
IBS implementation in RF-Track follows a...

Hybrid kinetic Monte Carlo method



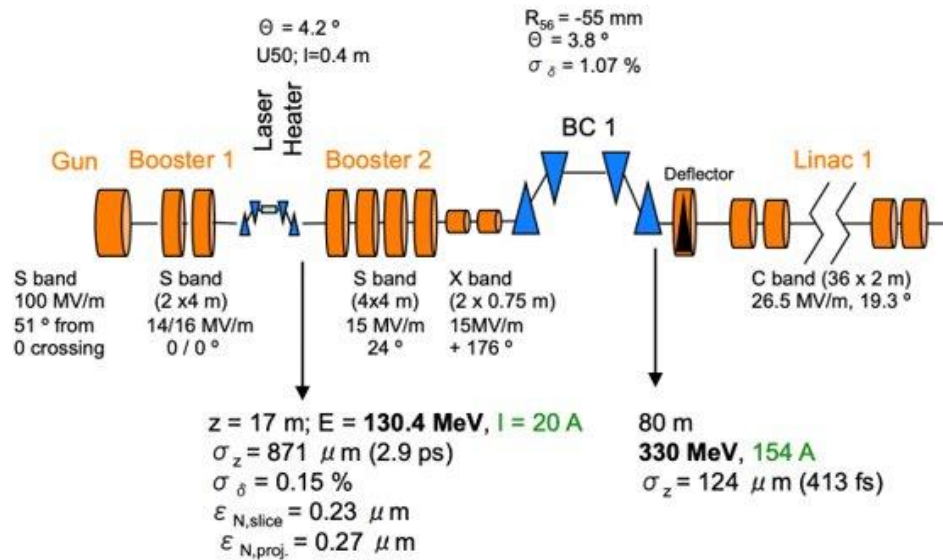
Implementation – Complementary processes

- Touschek effect and IBS are collisional regimes, while space charge (SC) is modelled by a mean field.
- Touschek effect is described by Moller cross section, while IBS is described by Rutherford cross section. This happens because Touschek can push a particle outside the RF/momentum aperture. We are not modelling Touschek effect (by the moment!)
- RF-Track implementation of **IBS and SC are complementary**, and **the discretization in their meshes should be the same!**
 - IBS is restricted to a single cell, the one in which the particle is.
 - SC covers all the cells except the one in which the particle is.



Results – SwissFEL Studies

What was simulated? First 110m of the SwissFEL injector.



Bunch characteristics:

Bunch length: 3.8 ps.

Charge: 192 pC.

Energy: Increases up to 300 MeV.

Spot size: 0.18 mm.

High-charge, low energy and low emittance – Expected to be affected by IBS.

Context: In SwissFEL, a final SES of 6 keV was measured [1]. Simulations considering only SC effects produced a SES that was substantially lower than the measured value.

[1] E. Prat, P. Craievich, P. Dijkstal, S. Di Mitri, E. Ferrari, T. G. Lucas, A. Malyzhenkov, G. Perosa, S. Reiche, T. Schietinger, Energy spread blowup by intrabeam scattering and microbunching at the swissfel injector, *Phys. Rev. Accel. Beams* **25** (2022) 104401.

Results – Comparison against measurements at SwissFEL

OBJECTIVES:

- **Understanding IBS effect** in the SES growth at SwissFEL.
- **Benchmarking** the IBS implementation in RF-Track against measurements.

Results – Comparison against measurements at SwissFEL

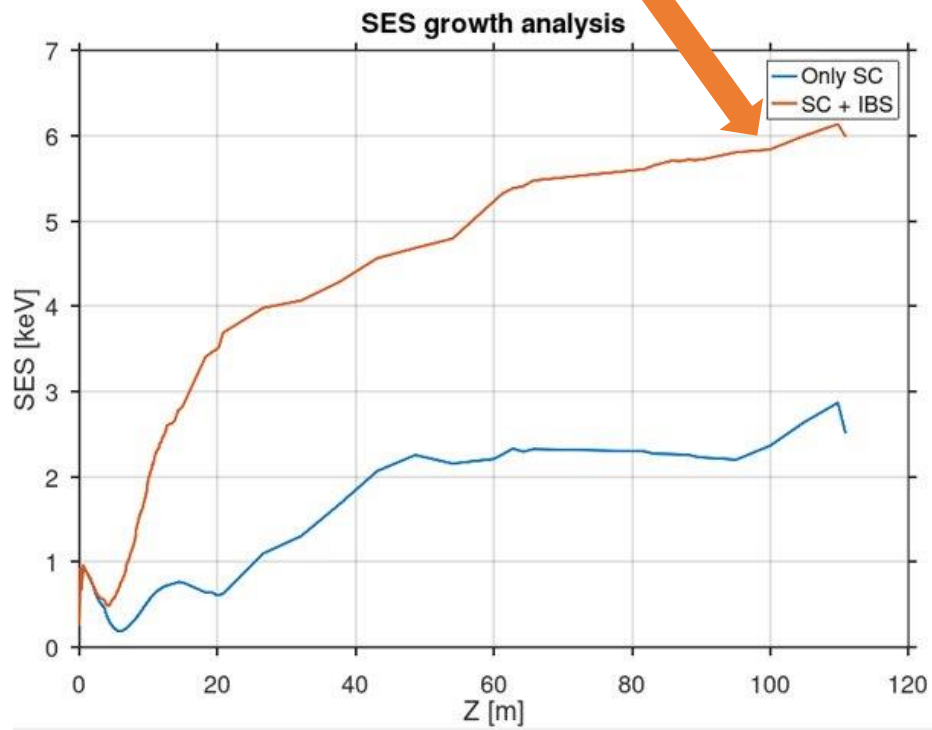
OBJECTIVES:

- **Understanding IBS effect** in the SES growth at SwissFEL.
- **Benchmarking** the IBS implementation in RF-Track against measurements.

STEPS:

1. Translation of the SwissFEL lattice from ASTRA to RF-Track.
2. RF-Track simulation including Only SC.
3. RF-Track simulation including SC.
4. Optimization of the parameters to find tune.
 - In the beam: Some of the beam parameters (bunch size, bunch length...) had not been published, neither stored, and were set as free parameters in the optimization.
 - In the lattice: Gun phase, gun solenoid.
 - The optimization was set to find the minimum emittance after the gun (3m) within some limits of the parameters.
 - The final values of the free parameters found after the optimization, were approved by SwissFEL.

Exact match of the experimental measurement!!

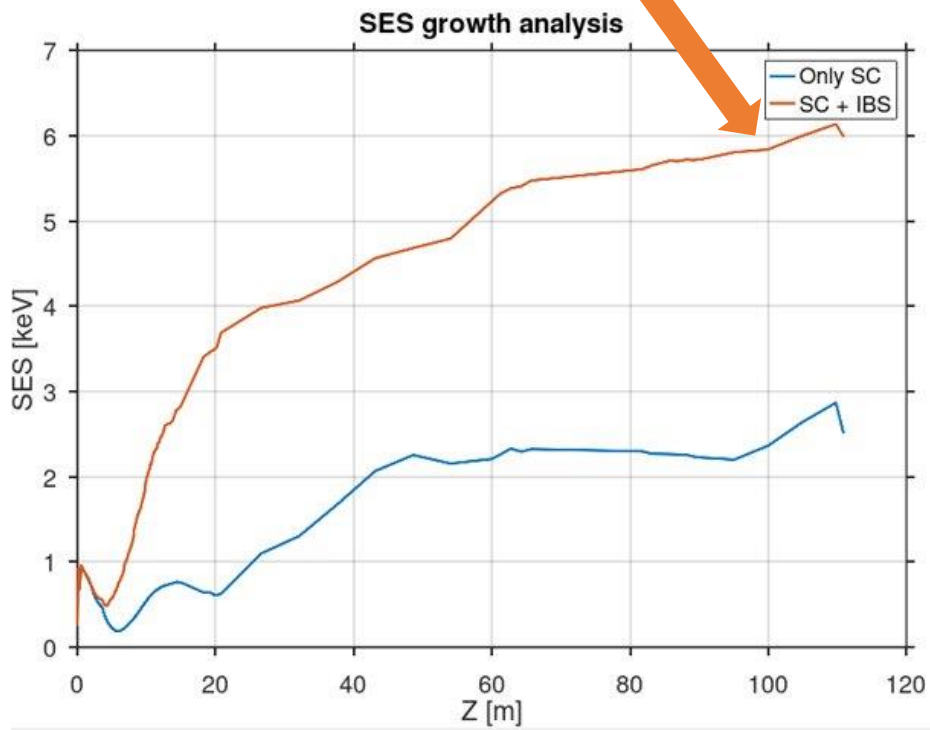


The results showed an **excellent agreement between the measurements of SwissFEL** and the simulations of RF-Track, providing an excellent benchmark.

This example proves that RF-Track:

- Allows to understand the exact effect of IBS in SES growth.
- Could be used for optimization of the machine, avoiding SES growth.
- Could be used in other FELs:
 - European XFEL (DESY).
 - Photo-Injector Test Facility, PITZ (DESY)
 - FERMI (ELETTRA)

Exact match of the experimental measurement!!

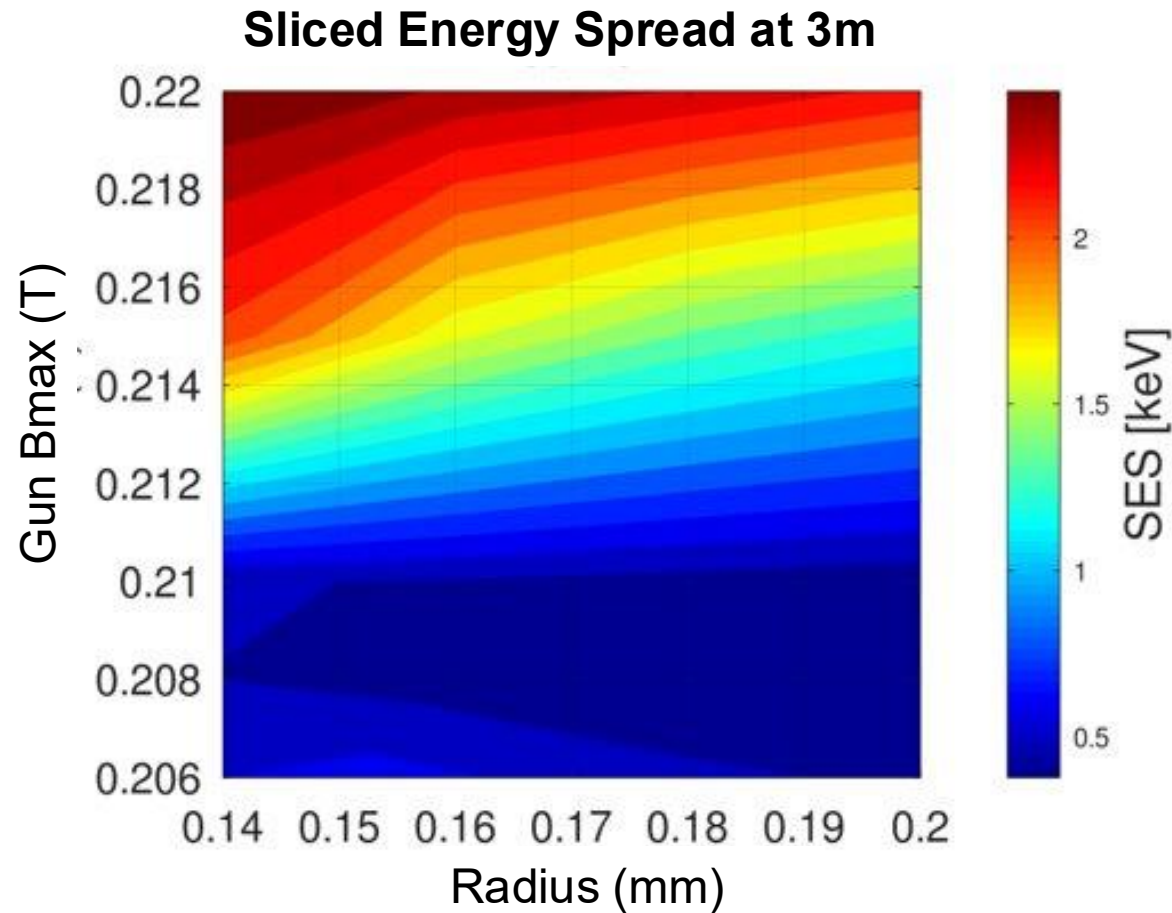


2D scans of the spot size and maximum gun solenoid, analyzing the SES after 110m, were performed.

The results showed an **excellent agreement between the measurements of SwissFEL** and the simulations of RF-Track, providing an excellent benchmark.

This example proves that RF-Track:

- Allows to understand the exact effect of IBS in SES growth.
- Could be used for **optimization of the machine**, avoiding SES growth.
- Could be used in other FELs:
 - European XFEL (DESY).
 - Photo-Injector Test Facility, PITZ (DESY)
 - FERMI (ELETTRA)



The 2D scans provide a simulation of the SES at the 3m (crucial point for the optimization of the rest of the lattice) varying the gun solenoid and the beam size.

Possible next objectives:

- Using these 2D scans for the optimization of the machine.
- Performing 2D scans with other parameters such as the bunch length or the gun phase, that can also determine the SES growth.
- Benchmarking the 2D Scans against measurements.

Results – FCC-ee Damping Rings

Preliminary results – FCCee Damping Rings Description

Recently, FCCee Damping Rings have been simulated with RF-Track. The lattice by A. De Santis et al [2] has been used (a preliminary lattice still subjected to changes). *More information in the talk given by A.Latina yesterday.*

OBJECTIVE: Testing if IBS will have an impact on emittance growth in FCCee Damping Rings.

RF-Track was used to simulate the FCCee Damping rings:

1. Firstly, with only incoherent synchrotron radiation (ISR).
2. Secondly, with ISR + IBS.

BEAM CHARACTERIZATION – Likely to be affected by IBS

- 2.86 GeV.
- Positron beam.
- Initial (from positron source) 13 mm.mrad in the horizontal plane, 0.002 mm.mrad in the vertical plane.
- 5 nC bunch charge. However, the case of 1 nC has also been studied, to understand how much IBS is affected by different bunch charges.

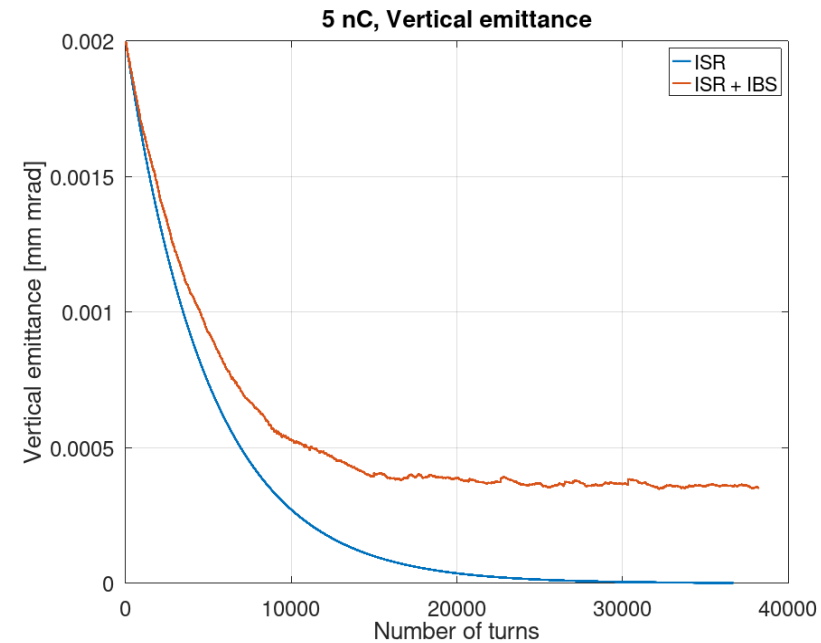
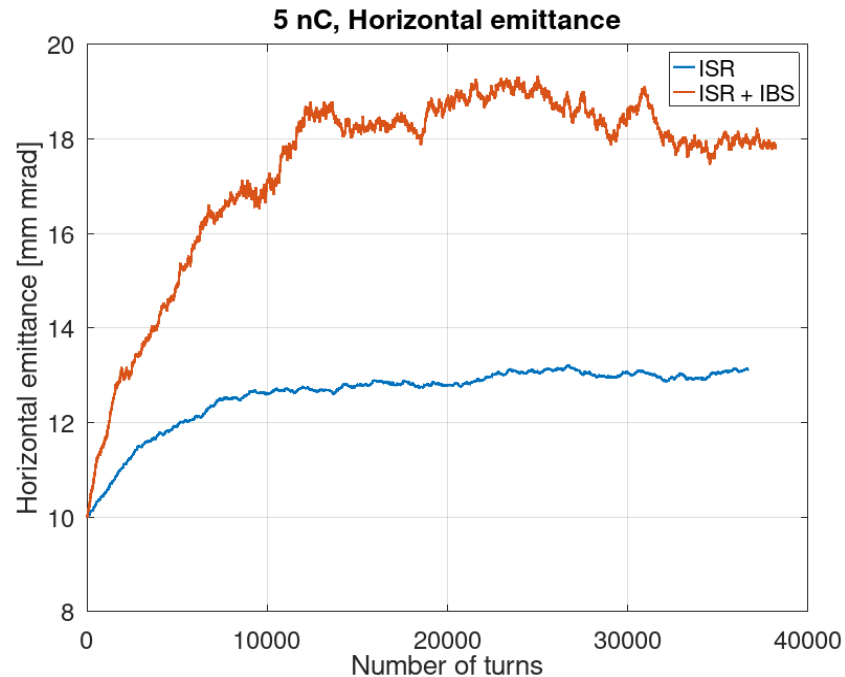
[2] Antonio De Santis, Catia Milardi, Ozgur Etisken. 2025. FCC-ee Injector: New DR at 2.86 GeV. FCC Week 2025, Vienna.

Preliminary results – 5 nC FCCee Damping Rings

Results for the 5 nC case:

- This case corresponds to the designed charge.
- Number of turns simulated: 40.000 turns (enough to see emittance stabilization).

The results show a **big effect of IBS on the emittance growth**, modifying the equilibrium emittance.



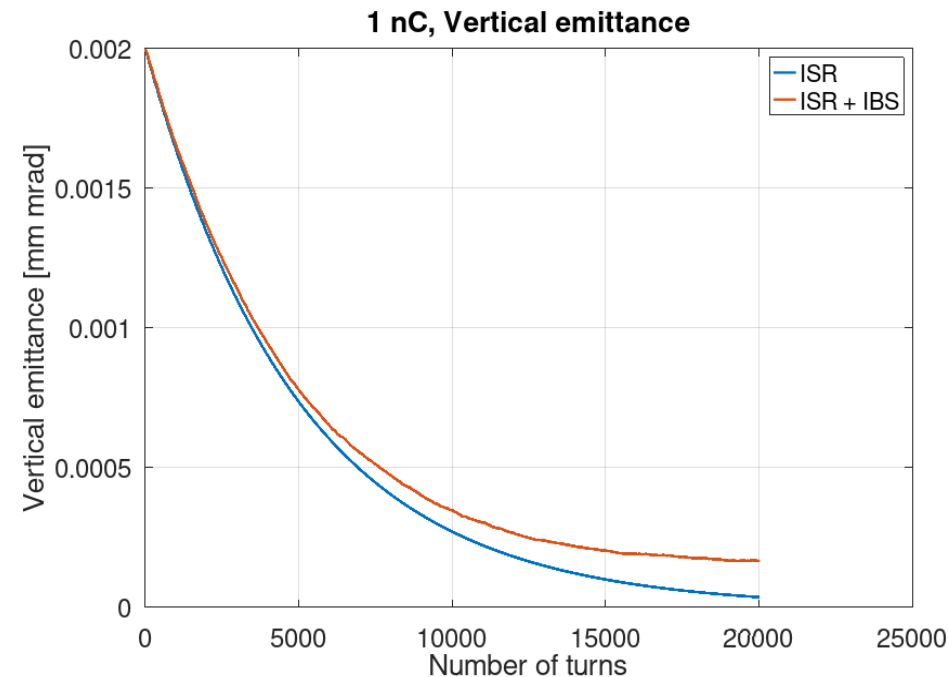
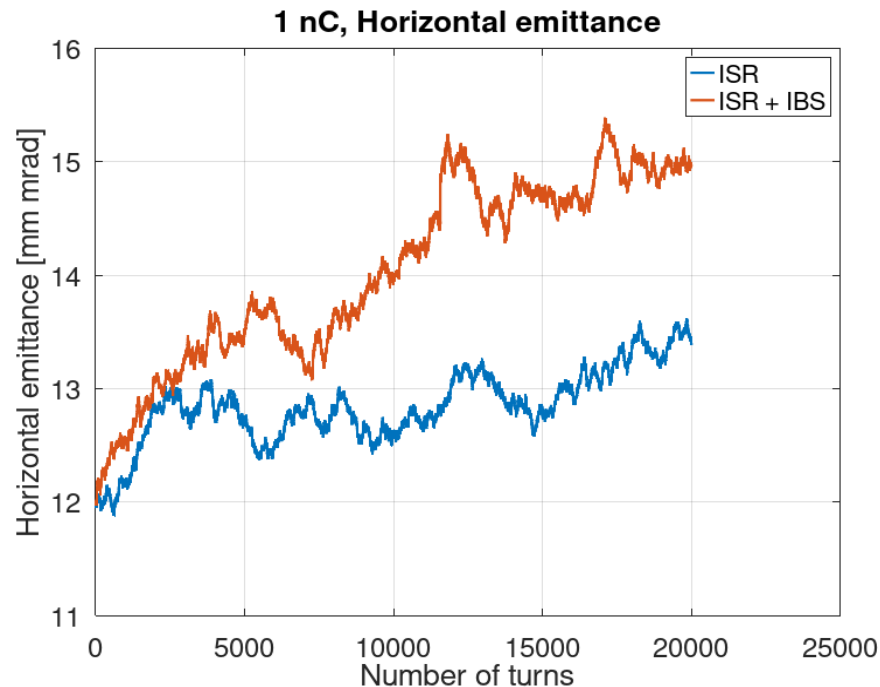
Note that the horizontal plane and the vertical plane have very different behaviours due to quantum excitation.

Preliminary results – 5 nC FCCee Damping Rings

Results for the 1 nC case:

- This case corresponds to a smaller charge, expecting to have a smaller IBS effect.
- Number of turns simulated: 20.000 turns (enough to see emittance stabilization vertically).

The results still show a **big effect of IBS on the emittance growth**, modifying the equilibrium emittance.



Note that the horizontal plane and the vertical plane have very different behaviours due to quantum excitation.

Preliminary results – Future Benchmark

The above examples can be used for an additional **benchmark of IBS implementation of RF-Track**, focused in the transverse plane.

Comparison with analytical formulas.

- Models of Intrabeam Scattering: Piwinski, Bjorken and Mtingwa, Nagaitsev, Bane.
- These models contain certain approximations such as Gaussian Beams or High Energy regime, which may limit the comparisons.

Comparison with other codes such as X-Suite:

- X-Suite is another code developed at CERN focused on the simulation of rings.
- X-Suite implements IBS.
- Work actually in progress, currently performing the input lattice conversion.

Results – Benchmarks with semi-analytical methods.

Results – Benchmark against semi-analytical method

IBS implementation in RF-Track has been benchmarked against different **semi-analytical methods** proposed by **S. Di Mitri**.

The effect of IBS in the final SES given by the analytical expression and the code RF-Track, have been compared.

Studied **lattices**:

TEST 1: A 20m long drift without acceleration.

TEST 2: A 18m long FODO, first without acceleration, then with an acceleration of 3.3 MV/m, and then an acceleration of 20 MV/m.

Limitations in the comparison:

- S. Di Mitri's semi-analytical methods are based on a Bane's IBS high-energy formula, however, in order to see an increment on the SES, we should stay in a low energy regime.
- The semi-analytical method calculates the final expression assuming constant parameters such as the beta functions along the 20m long simulations. On the other hand, the tracking code, updates the quantities after each kick, and the beta functions are affected by IBS themselves.

Results – SA Benchmark, TEST 1 (Drift, no acceleration)

TEST 1: A total of 20 cases varying parameters that highly affect IBS contribution were performed:

- 4 different configurations were tested, with different values of **emittance** and **beam energy**.
- For each configuration, a **charge** scan was performed, using {0.1, 0.5, 1, 5, 10} nC – Total of 20 cases.

As it was previously explained, the **final SES obtained by the two IBS methods** has been compared.

	TEST 1	TEST 2	TEST 3	TEST 4
E (MeV)	100	500	100	500
$\epsilon_{\perp}$ (mm·mrad)	0.1	0.1	0.5	0.5
$\beta_{\perp}$ (m)			20	
$\alpha_{\perp}$			1	
FWHM(z) (ps)			5	
σ_E (keV)			1	
Long. profile			Parab.	

Results – SA Benchmark, TEST 1 (Drift, no acceleration)

TEST 1: A total of 20 cases varying parameters that highly affect IBS contribution were performed:

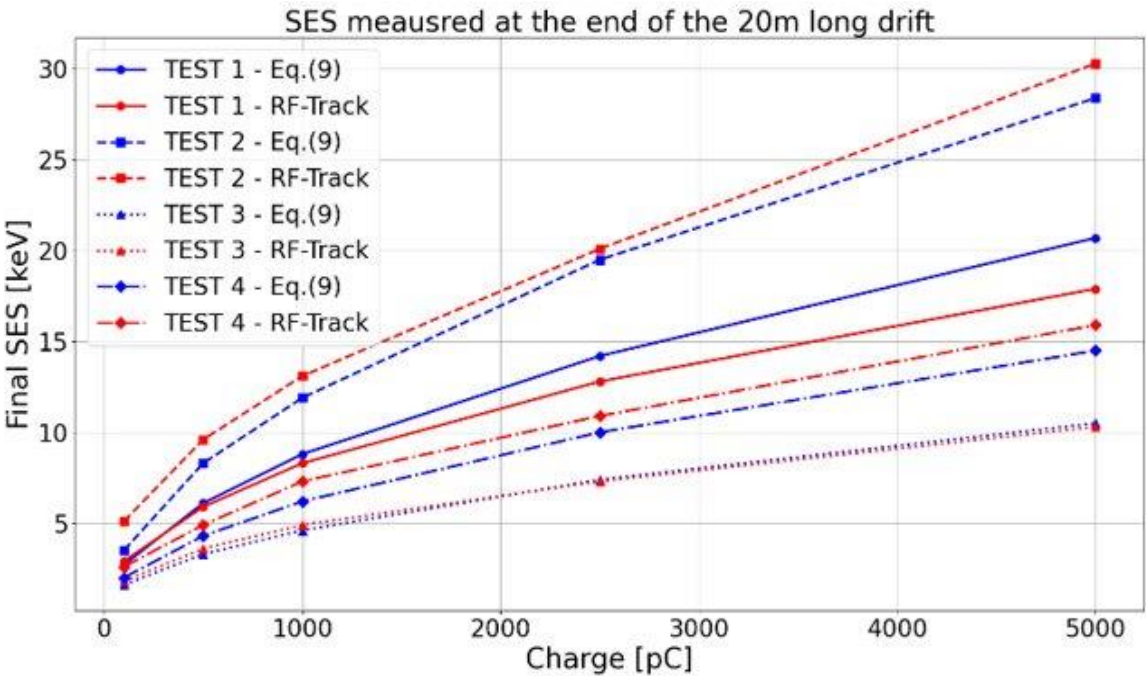
- 4 different configurations were tested, with different values of **emittance** and **beam energy**.
- For each configuration, a **charge** scan was performed, using {0.1, 0.5, 1, 5, 10} nC – Total of 20 cases.

	TEST 1	TEST 2	TEST 3	TEST 4
E (MeV)	100	500	100	500
$\epsilon_{\perp}$ (mm· mrad)	0.1	0.1	0.5	0.5
$\beta_{\perp}$ (m)			20	
$\alpha_{\perp}$			1	
FWHM(z) (ps)			5	
σ_E (keV)			1	
Long. profile			Parab.	

As it was previously explained, the **final SES obtained by the two IBS methods** has been compared.

The results showed an **excellent agreement**, and the measured differences have been attributed to the already discussed points.

S. Di Mitri found a common scale factor to fine-tuned his models on RF-Track results. Later, he found the same scale factor when he compared his model against measurements in FERMI-ELETTRA.



Results – SA Benchmark, TEST 2 FODO (acceleration)

TEST 2: The accelerations are applied by a constant electric field in the longitudinal direction.

	TEST 1	TEST 2	TEST 3
Initial beam energy (MeV)	100	100	100
Final beam energy (MeV)	100	160	460
Accelerating gradient (MV/m)	0	3.33	20

There are **three semi-analytical methods** that handle the acceleration in a different way. One of them, is less accurate because assumes a constant energy along the lattice (the average).

Why a FODO? In a FODO, the beam is more confined than in a drift, reducing the discrepancy regarding the mean beta functions along the lattices, which are assumed to be constant by the semi-analytical method, but are affected by IBS.

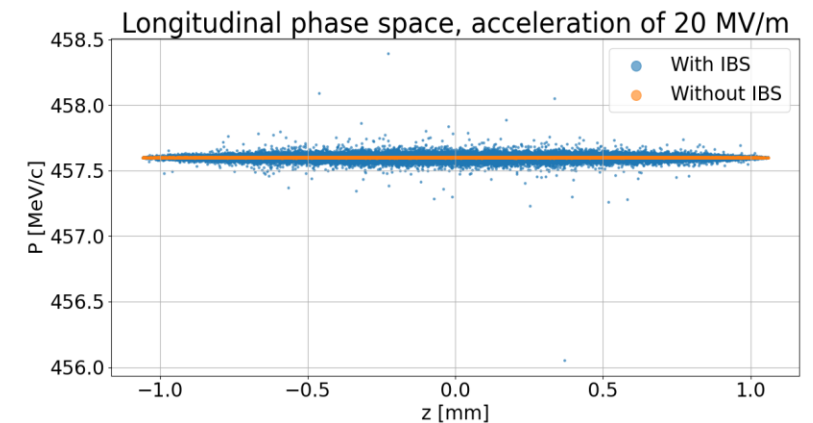
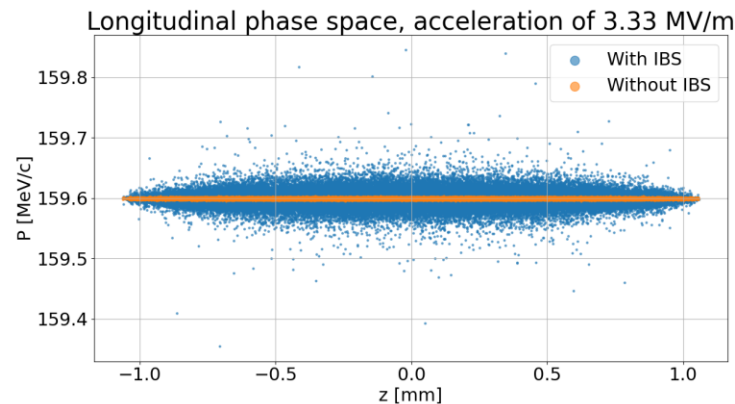
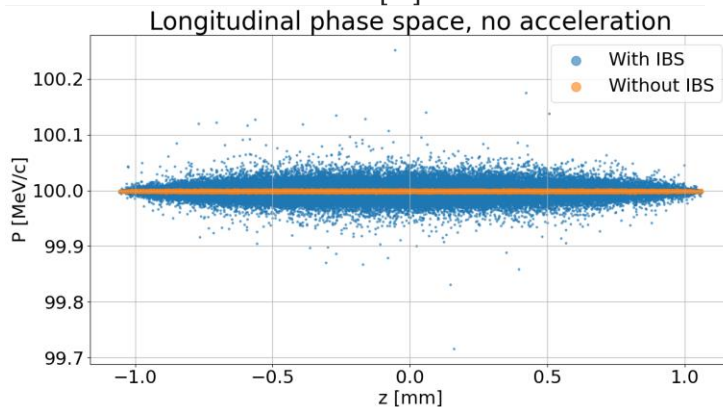
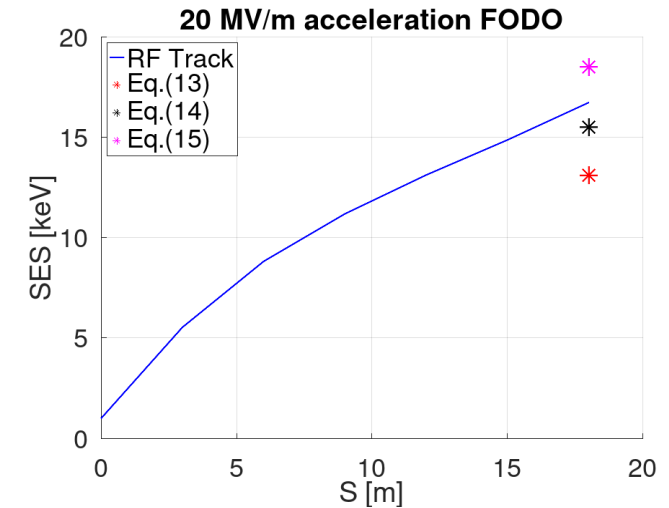
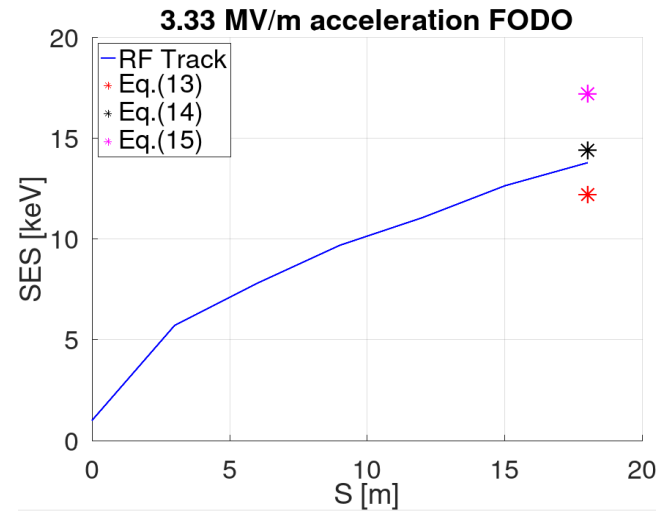
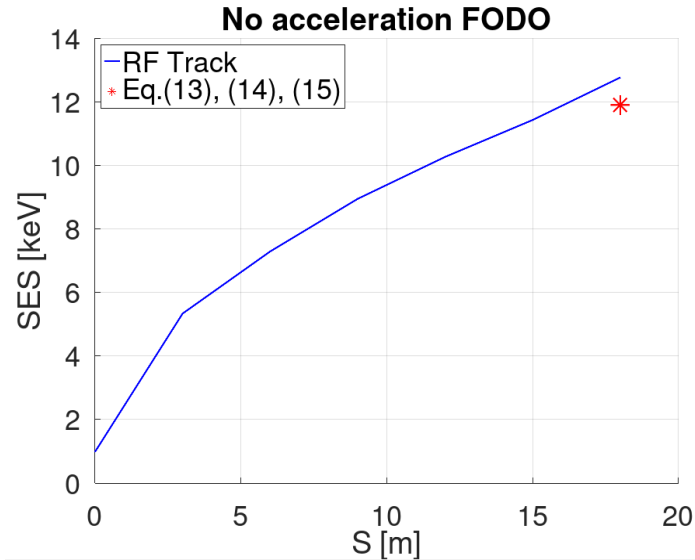
FODO characterization	
Number of cells	6
Length of cell	3
Length of quadrupoles	0
k_{1L} of quadrupoles (1/m)	0.943
Phase advance per cell	90°
Beam characterization	
β_x (m)	5.121
β_y (m)	0.879
α_x (m)	0
α_y (m)	0
$\epsilon_{\perp}$ (mm mrad)	0.2
Bunch charge (nC)	1
Peak current (A)	200
σ_E (keV)	1
FWHM(z) (ps)	5
Initial beam energy (MeV)	100

Results The next slide shows:

- The final SES measured by RF-Track including IBS, and the final values provided by the 3 methods.
- The final longitudinal phase space, with and without IBS.

Results – SA Benchmark, TEST 2 FODO (acceleration) II

A good agreement can be observed, as well as a big impact of IBS in the long.final phase space.



Conclusions and Future Work

Conclusions and Future work

Conclusions

- RF-Track has reproduced the experimental measurements of SES in SwissFEL and has helped to understand which part of the contribution corresponded to IBS.
- RF-Track could be used as a tool for understanding SES contribution in other FELs, and optimizing its performance.
- RF-Track is currently being used to understand the IBS effect on FCC-ee Damping Rings.
- IBS implementation in RF-Track has been benchmarked against semi-analytical methods.

Future work

- Add Touschek effect to RF-Track.
- Complete the study of emittance growth in FCCee DR by adding space-charge.
- Complete the benchmarks in the transverse plane with the FCCee Damping Rings as described.
- Study the effect of IBS in the muon cooling of the Muon Collider. WORK ON GOING.

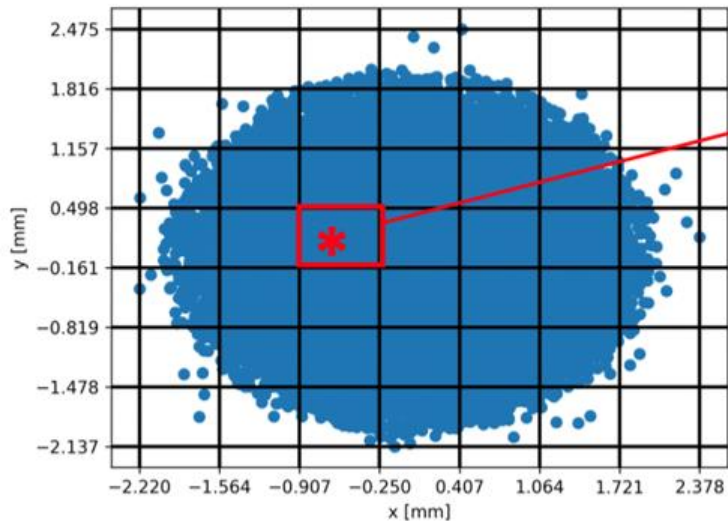
Thank you

Backup // Implementation - Grid & Interpolation

STEP 0 – Grid Creation

For each kick, two 3D Grids are created with the discretization chosen by the user $\{N_x, N_y, N_z\}$:

- Average velocity (as a 3D vector) - $Grid3V()$
- Average density of the cell – $Grid3dp()$



Iteration over all particles.

From now on, the next steps will be repeated for all the particles. Let p be a particle of the beam, with this position .

$$p_{lab} = (p_x, p_y, p_z);$$

lab refers to the lab frame.

STEP 1 - Interpolation: For each particle, a new virtual particle, the average particle, is created from the interpolation at the location of the real one. We will define its velocity and local density.

$$q_{v,lab} = (q_{vx}, q_{vy}, q_{vz}) = Grid3V(p_x, p_y, p_z);$$

$$\rho_{lab} = Grid3\rho(p_x, p_y, p_z);$$

Backup // Implementation – Frame change, collisions computation

STEP 2 - Average particle rest frame

Scattering processes are often **modelled in a frame in which one of the particles is at rest.**, in this case, the average particle.

The velocity was updated by a Lorentz Boost, the local density suffers length contraction.

$$\mathbf{q}_v = \vec{0};$$

$$\rho = \rho_{lab}/\gamma;$$

$$\mathbf{v} = \text{LorentzBoost}(\mathbf{v}_{lab});$$

STEP 3 - Calculation of the number of collisions per kick

3.1 Maximum impact parameter, b_{max} : Max. perpendicular distance between the particles. It is calculated as the radius of a circle with the same transverse area as a mesh cell.

$$b_{max} = \sqrt{h_x h_y / \pi},$$

3.2 Minimum scattering angle: Obtained from b_{max} , otherwise the cross section diverges.

$$\theta_{min} = 2 \arctan \left(\frac{q^2}{4\pi\epsilon_0} \frac{1}{2E_{kin} b_{max}} \right),$$

3.3 Total cross section: Rutherford cross section between the min. scattering angle and a threshold set to $\pi/4$, to limit **Touschek effect**.

$$\frac{d\sigma}{d\Omega} = \left(\frac{q^2}{4\pi\epsilon_0} \right)^2 \left(\frac{1}{4E_{kin}} \right)^2 \frac{1}{\sin^4(\theta/2)}. \quad \longrightarrow \quad \sigma = \int_0^{2\pi} d\phi \int_{\theta_{min}}^{\theta_{th}} \sin(\theta) d\theta \frac{d\sigma}{d\Omega} = 4\pi \left(\frac{q^2}{4\pi\epsilon_0} \right)^2 \left(\frac{1}{4E_{kin}} \right)^2 \left[\frac{1}{\sin^2(\theta/2)} \right]_{\theta_{th}}^{\theta_{min}},$$

Backup // Implementation - Collisions and force calculation

STEP 3 - Calculation of the number of collisions per kick

3.4 Mean free path, λ : Average distance travelled after a collision. $\lambda = \frac{1}{\rho\sigma},$

3.5 Number of collisions per kick, N_{col}^* : Ratio between the integration step and the mean free path. $N_{col} = dS / \lambda,$

However, the user can set a maximum number of collisions per kick (N_{col}^*) to limit the computational time.

The explanation of N_{col}^ extends beyond the limits of this presentation but can be found in the appendix!*

STEP 4 - Computation of collisions – Monte Carlo Method.

- For the azimuthal angle, select a random number between 0 and 2π .
- For the scattering angle, select a random number between the minimum scattering angle and the Touschek effect threshold, from a PDF with the differential Rutherford cross section.
- Compute the new momentum and repeat N_{col} times.

STEP 5 – Force calculation.

For each particle, the change of momentum is stored, or in other words, a force.

The previous steps are repeated among all the particles to get the force associated to a kick.