Higgs boson production at Linear Colliders from a generic 2HDM: the role of triple Higgs self-interactions

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Outline

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A one-slide motivation

Minimal Extensions
of the SM Higgs sector

A one-slide motivation

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$\Updownarrow$

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$SU_L(2)$ doublets

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2HDM

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⇕
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2HDM

- Yukawa-like 3H/4H

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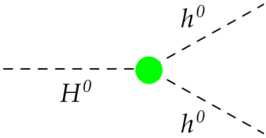
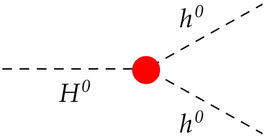
2HDM

♠ Yukawa-like 3H/4H
♠ Unrestricted
Higgs mass spectrum

MSSM

♠ gauge-like 3H/4H
♠ Constrained
Higgs mass spectrum





$$\frac{ie}{M_W\,\sin\theta_W}\left[\sin(\beta-\alpha)\,(M_{h^0}^2 - 2\,M_{H^\pm}^2) - \frac{\cos(\alpha+\beta)}{\sin\,2\beta}\left(2M_{h^0}^2 - \frac{4}{e^2}\,\lambda_5\,M_W^2\,s_W^2\right)\right]$$

$$\frac{-ie\,M_W}{\sin\theta_W}\left[\cos\,2\beta\,\sin(\alpha+\beta)2\,c_W^2 + \sin(\beta-\alpha)\right]$$

Yukawa-like structure

gauge-like structure

A one-slide motivation

Basic goals

- To seek for the most favorable regions for 2H/hZ production, and to correlate them with alternative multiparticle Higgs-boson final states
- To quantify the importance of the quantum effects associated to these processes
- To single out the impact of the 3H self-couplings and their potential enhancements.

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Collaborators & References

♠ **Work in collaboration with N. Bernal (Bonn U. & BCTP), J. Guasch (UB-ICCUB Barcelona), G. Ferrera (Milan U. & INFN), R. N. Hodgkinson (Valencia U. & IFIC), J. Solà (UB-ICCUB Barcelona)**

- G. Ferrera, J. Guasch, D. López-Val and J. Solà, *Phys. Lett.* **B659** (2007) 297-307, arXiv:0707.3162 [hep-ph]
- R. N. Hodgkinson, D. López-Val and J. Solà, *Phys. Lett.* **B673** (2009) 47-56, arXiv:0901.2257 [hep-ph]
- N. Bernal, D. López-Val and J. Solà, *Phys. Lett.* **B677** (2009) 39-47, arXiv:0903.4978 [hep-ph]; *Phys. Rev.* **D81** (2010) 113005, arXiv:1003.4312 [hep-ph];
- D. López-Val and J. Solà, *Phys. Rev.* **D81** (2010) 033003, arXiv:0908.2898 [hep-ph]; Fortsch.Phys.5 (2010) 660; *Phys. Lett.* **B702** (2011) 246, arXiv:1106.3226 [hep-ph]; Nuovo Cim.**C34** S1 (2011) 57 arXiv:1107.1305 [hep-ph]

Outline

The Two-Higgs-Doublet Model: basic settings

- Canonical extension of the SM Higgs sector with a second $SU_L(2)$ doublet with weak hypercharge $Y=1$

- The Higgs potential can be written as:

$$\begin{aligned}
V(\Phi) &= \lambda_1 \left(\Phi_1^\dagger\Phi_1 - \frac{v_1^2}{2}\right)^2 + \lambda_2 \left(\Phi_2^\dagger\Phi_2 - \frac{v_2^2}{2}\right)^2 + \\
&+ \lambda_3 \left(\Phi_1^\dagger\Phi_1 - \frac{v_1^2}{2} + \Phi_2^\dagger\Phi_2 - \frac{v_2^2}{2}\right)^2 + \\
&+ \lambda_4 \left[(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - (\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\right] \\
&+ \lambda_5 \left[\mathrm{Re}(\Phi_1^\dagger\Phi_2) - \frac{v_1 v_2}{2}\right]^2 + \lambda_6 \left[\mathrm{Im}(\Phi_1^\dagger\Phi_2)\right]^2
\end{aligned}$$

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♠ Most general form compatible with i) **gauge symmetry**; ii) **renormalizability**; iii) ***CP*-invariance**; iv) **discrete symmetry** $\phi_i \to (-1)^i\,\phi_i$

The Higgs sector in the 2HDM: degrees of freedom & parameters

- 5 **physical** Higgs fields: 2 $\mathcal{CP}$-even Higgs bosons (h^0, H^0), 1 $\mathcal{CP}$-odd Higgs boson A^0 and 2 charged Higgs bosons $H^{\pm}$
- 7 free parameters:
  - 6 couplings in the Higgs potential, $\lambda_i \qquad i = 1 \ldots 6$
  - 2 VEV's, v_1, v_2 with one constraint: $v^2 \equiv v_1^2 + v_2^2 = G_F^{-1}/\sqrt{2}$
- The masses of the physical Higgs particles ($M_{h^0}, M_{H^0}, M_{A^0}, M_{H^{\pm}}$)
- The ratio of the two VEV's, $\tan\beta \equiv \frac{\langle H_2^0 \rangle}{\langle H_1^0 \rangle} \equiv \frac{v_2}{v_1}$
- The mixing angle α between the two neutral *CP*-even states
- The coupling λ_5
- Higgs-fermion Yukawa sector: $\Rightarrow$ **Types I/II 2HDM**

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Phenomenological and Theoretical constraints

- $H^{\pm}$ contribution to $\mathcal{B}(b \to s\gamma)$ & $B_d^0 - \bar{B}_d^0$ $\Rightarrow$ restrictions over the $\tan\beta, M_{H^\pm}$ plane [Mahmoudi, Stal '07.]
- Extra Higgs-boson one-loop corrections to $\delta\rho$: $|\delta\rho_{2HDM}| \lesssim 10^{-3}$ [Barbieri & Maiani, '83]
- Perturbativity on the Higgs-quark Yukawa couplings:
  $Y_t \propto \frac{m_t}{v \tan\beta}$ $\quad Y_b \propto \frac{m_b \tan\beta}{v}$ $\Rightarrow$ [El Kaffas, Osland & Greid '07]
  $0.3 < \tan\beta \lesssim 60$
- Vacuum stability [Kanemura, Kasai & Okada '09]; + Unitarity Kanemura, Kubota & Takasugi ['93]; Akeroyd, Arhrib & Naimi, ['00]; Horejsi & Kladiva, ['06]

Restrictions over combinations of the Higgs self-couplings

$\lambda_5 < 0; |\lambda_5| < 10$

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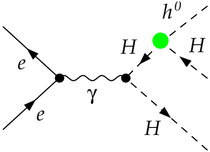
Restrictions over combinations of the Higgs self-couplings

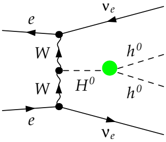
$$\lambda_5 < 0; |\lambda_5| < 10$$

Probing the 3H self-couplings at Linear Colliders

$e^+e^- \to HHH$



$e^+e^- \to (V^*V^*) \to HH + 2l$



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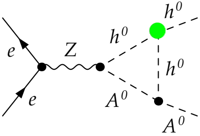
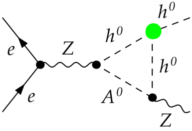
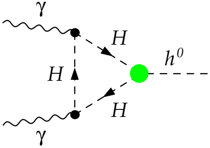


$e^+e^- \to hZ^0$



$\gamma\gamma \to h$



Hodgkinson, Ferrera, Guasch, DLV, Solà ['07], DLV, Solà ['09], DLV, Solà ['09], Bernal, DLV, Solà ['10]. For a recent update see Solà, DLV, arXiv:1107.1305 [hep-ph], in the LC10 proceedings

Renormalization of the Higgs sector

- Higgs fields: 1 WF constant per $SU_L(2)$ doublet

$$\begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix} \to Z_{\Phi_1}^{1/2} \begin{pmatrix} \Phi_1^+ \\ \Phi_1^0 \end{pmatrix}, \qquad \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix} \to Z_{\Phi_2}^{1/2} \begin{pmatrix} \Phi_2^+ \\ \Phi_2^0 \end{pmatrix},$$

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- In the mass-eigenstate basis:

$$\begin{aligned}
\delta Z_{h^0 h^0} &= \sin^2\alpha\, \delta Z_{\Phi_1} + \cos^2\alpha\, \delta Z_{\Phi_2} \\
\delta Z_{h^0 H^0} &= \sin\alpha\, \cos\alpha\, (\delta Z_{\Phi_2} - \delta Z_{\Phi_1}) \\
\delta Z_{A^0 A^0} &= \sin^2\beta\, \delta Z_{\Phi_1} + \cos^2\beta\, \delta Z_{\Phi_2} \\
\delta Z_{A^0 - Z^0} &= \frac{1}{2} \left(\delta Z_{A^0 - G^0} + \sin 2\beta \, \frac{\delta \tan\beta}{\tan\beta} \right)
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- **$\tan\beta$ renormalization:**

$$\frac{\delta v_1}{v_1} = \frac{\delta v_2}{v_2}$$

$$t_{h^0H^0} + \delta t_{h^0H^0} = 0$$

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- $\tan\beta$: $\left.\begin{array}{c} \frac{\delta v_1}{v_1} = \frac{\delta v_2}{v_2} \\ t_{h^0H^0} + \delta t_{h^0H^0} = 0 \end{array}\right\} \Rightarrow \boxed{\frac{\delta \tan\beta}{\tan\beta} = \frac{1}{2}\,(\delta Z_{\Phi_2} - \delta Z_{\Phi_1})}$

Renormalization of the Higgs sector

Higgs fields (OS scheme) Dabelstein ['94], Chankowski et al ['95]

$$\mathrm{Re}\,\hat{\Sigma}'_{A^0 A^0}(q^2)\Big]_{q^2=M^2_{A^0}} = 0 \quad , \quad \mathrm{Re}\,\hat{\Sigma}_{A^0 Z^0}(q^2)\Big]_{q^2=M^2_{A^0}} = 0$$

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$$\delta Z_{\Phi_1} = -\text{Re}\,\Sigma'_{A^0A^0}(M^2_{A^0}) - \frac{1}{M_Z \tan\beta}\,\text{Re}\,\Sigma_{A^0Z^0}(M^2_{A^0})$$

$$\delta Z_{\Phi_2} = -\text{Re}\,\Sigma'_{A^0A^0}(M^2_{A^0}) + \frac{\tan\beta}{M_Z}\,\text{Re}\,\Sigma_{A^0Z^0}(M^2_{A^0})$$

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$$\begin{aligned} \delta Z_{\Phi_1} &= -\mathrm{Re}\,\Sigma'_{A^0 A^0}(M^2_{A^0}) - \frac{1}{M_Z \tan\beta}\,\mathrm{Re}\,\Sigma_{A^0 Z^0}(M^2_{A^0}) \\ \delta Z_{\Phi_2} &= -\mathrm{Re}\,\Sigma'_{A^0 A^0}(M^2_{A^0}) + \frac{\tan\beta}{M_Z}\,\mathrm{Re}\,\Sigma_{A^0 Z^0}(M^2_{A^0}) \end{aligned}$$

Physical content of the OS conditions:

- On-shell A^0 renormalized propagators have unit residue, $1/[1+\mathrm{Re}\,\Sigma'_{A^0}(M^2_{A^0})] = 1$
- No $A^0 - Z^0$ (nor $A^0 - G^0$) mixing occurs for on-shell A^0 bosons at any order in perturbation theory.

Renormalization of the Higgs sector

Higgs masses (OS scheme)

$\mathrm{Re}\,\hat{\Sigma}_{h^0h^0}\,(M^2_{h^0}) = 0$; $\mathrm{Re}\,\hat{\Sigma}_{H^0H^0}\,(M^2_{H^0}) = 0$; $\mathrm{Re}\,\hat{\Sigma}_{A^0A^0}\,(M^2_{A^0}) = 0$;
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Renormalization of the mixing angle α

- As usual, we introduce a 1-loop counterterm $\alpha^{(0)} = \alpha + \delta\,\alpha$
- Renormalization condition: $\text{Re}\,\hat{\Sigma}_{h^0H^0}(q^2 = M^2_{h^0}) = 0 \Rightarrow$

$$\delta m^2_{h^0H^0} = \Sigma_{h^0H^0}(M^2_{h^0}) + \underbrace{\delta\,Z_{h^0H^0}\,(M^2_{h^0} - M^2_{H^0})}_{(M^2_{h^0} - M^2_{H^0})\,\delta\alpha}$$

- The same condition ensures that the *CP*-even Higgs final state is on-shell, as the *physical* masses must fulfill

$$\left(p^2 - M^2_{H^0} + \hat{\Sigma}_{H^0H^0}\right)\left(p^2 - M^2_{h^0} + \hat{\Sigma}_{h^0h^0}\right) - \hat{\Sigma}^2_{h^0H^0} = 0\,.$$

Computation setup

♠ Analytical calculation & Numerical evaluation : FEYNARTS, FORMCALC, LOOPTOOLS T. Hahn; T. Hahn, M. Pérez-Victoria

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♠ MSSM Higgs spectrum : ⇒ FEYNHIGGS v2.8 T. Hahn et al. (2-loop accuracy)

Outline

Exclusive 2H production: *CP*-allowed 2H channels

$$\left.\begin{array}{lcc} e^+e^- & \to & H^\pm H^\mp \\ e^+e^- & \to & h^0A^0 \\ e^+e^- & \to & H^0A^0 \end{array}\right\}$$

LO: tree-level diagrams ($\mathcal{O}(\alpha^2_{ew})$, $\sigma \sim 0.01 - 0.1$ pb.)

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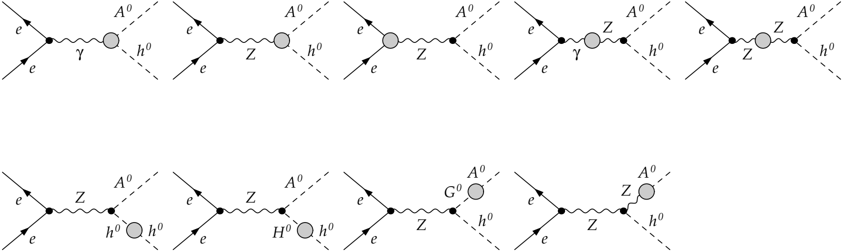
♠ There is no dynamical distinction between the general 2HDM and the MSSM
♠ Dedicated studies on radiative corrections in 2H processes are hence mandatory: Chankowski, Pokorski, Driesen, Hollik, Rosiek ['96]; Arhrib, Moultaka ['98]; Guasch, Hollik, Kraft ['01]; Heinemeyer et al. ['01]

Quantum corrections to $e^+e^- \to h^0A^0$: an overview



i) loop-induced $\gamma h^0 Z^0$ interaction; ii) $h^0A^0Z^0$ vertex corrections; iii) $e^+e^-Z^0$ vertex corrections; iii) $\gamma - Z^0, Z^0 - Z^0$ self-energy insertions; iv) WF corrections to the external Higgs boson legs; v) Box-type diagrams

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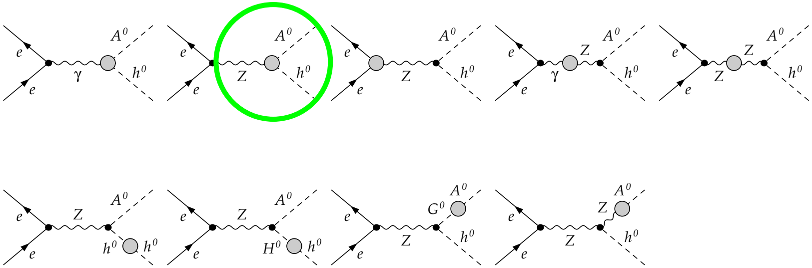

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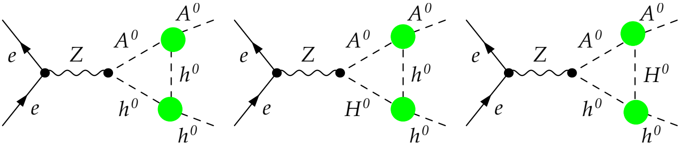
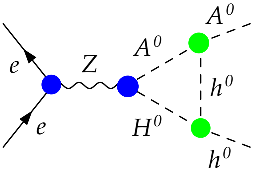
The $Z^0 A^0 h^0$ interaction at 1-loop



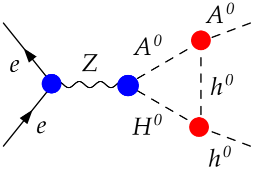
♠ Due to its sensitivity to 3H self-couplings, the strenght of the $Z^0A^0h^0$ interaction (which is purely gauge-like at the leading-order) may be largely enhanced at the quantum level:

$$\begin{aligned} \Gamma^0_{A^0\,h^0\,Z^0} &\rightarrow Z^{1/2}\left(\Gamma^0_{A^0\,h^0\,Z^0} + \Gamma^1_{A^0\,h^0\,Z^0}\right) \\ &\simeq Z^{1/2}\left(\Gamma^0_{A^0\,h^0\,Z^0} + \Gamma^{1,3H}_{A^0\,h^0\,Z^0}\right) \\ \Gamma^{eff}_{A^0\,h^0\,Z^0} &\sim \Gamma^0_{A^0\,h^0\,Z^0}\,\frac{\lambda^2_{3H}}{16\pi^2\,s}\,f(M^2_{h^0}/s, M^2_{A^0}/s) \end{aligned}$$

Trilinear couplings: gauge vs Yukawa-like



$$C[\mathsf{h}^0\mathsf{h}^0\mathsf{H}^0] = \frac{-ie\cos(\beta-\alpha)}{2\,M_W\ \sin\theta_W\ \sin\,2\beta}\Big[\Big(2M_{\mathsf{h}^0}^2+M_{\mathsf{H}^0}^2\Big)\sin\,2\alpha -2\,\frac{1}{e^2}\lambda_5\,M_W^2\ \sin^2\theta_W\,(3\sin 2\alpha-\sin 2\beta)\Big]$$

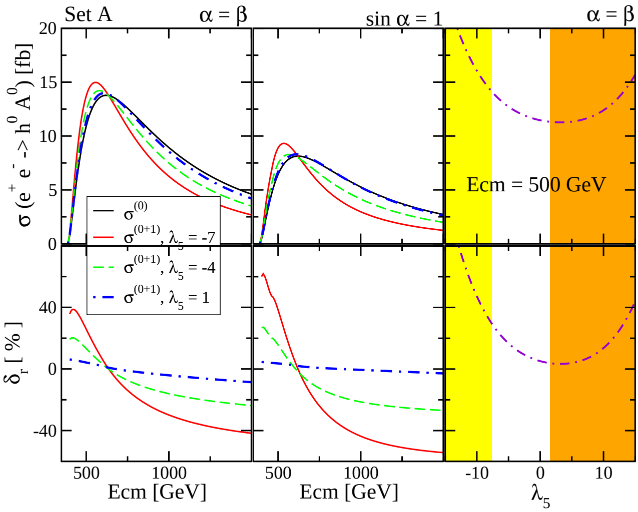


$$C_{\mathsf{MSSM}}[\mathsf{h}^0\mathsf{h}^0\mathsf{H}^0] = \frac{ie\,M_W}{2\ \cos\,\theta_W\ \sin\theta_W}\,(\cos 2\alpha\ \cos(\alpha+\beta) -2\sin 2\alpha\ \sin(\alpha+\beta))$$

$e^+ e^- \to A^0 h^0$ at one loop: phenomenology in a nutshell

$$\delta_r \equiv \frac{\sigma^{(0+1)} - \sigma^{(0)}}{\sigma^{(0)}}$$

M_{h^0} [GeV]	M_{H^0} [GeV]	M_{A^0} [GeV]	$M_{H^\pm}$ [GeV]
130	200	260	300



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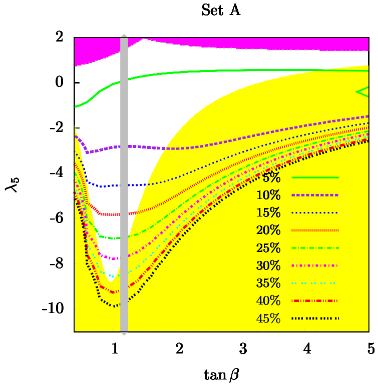


- **Optimal rates:** $\sigma \sim \mathcal{O}(10-50)$ fb (@ 0.5 TeV)
- Enhanced $|\delta_r| \sim \mathcal{O}(50)\%$ (for $|\lambda_5| \sim 5-10$, $\tan\beta \sim \mathcal{O}(1)$)
- quantum effects as a **boost** VS **suppression**:

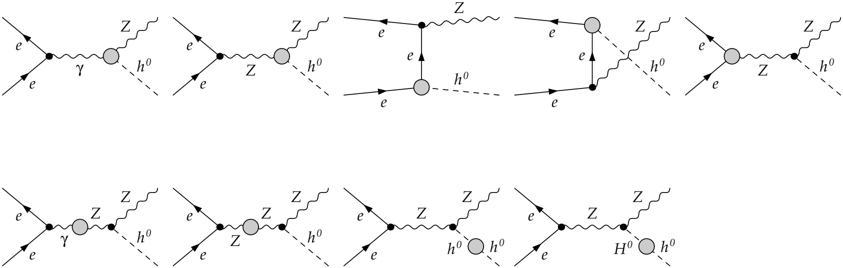
$$\delta_r > 0 \; (\sqrt{s} \sim 0.5\,\text{TeV})$$

$$\delta_r < 0 \; (\sqrt{s} \gtrsim 1\,\text{TeV})$$

- Fairly generic pattern – indep. of Higgs masses, type-I/II . . .

Outline

Quantum corrections to $e^+e^- \to h^0Z^0$: an overview



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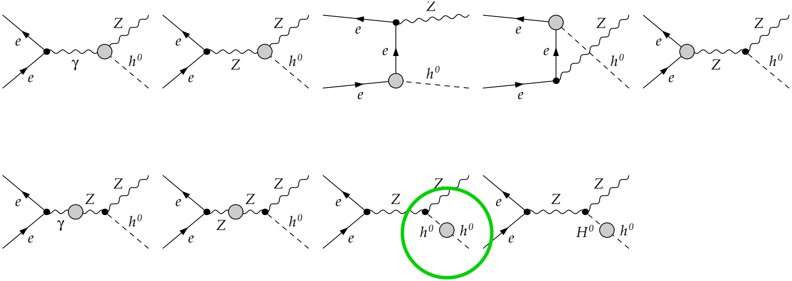

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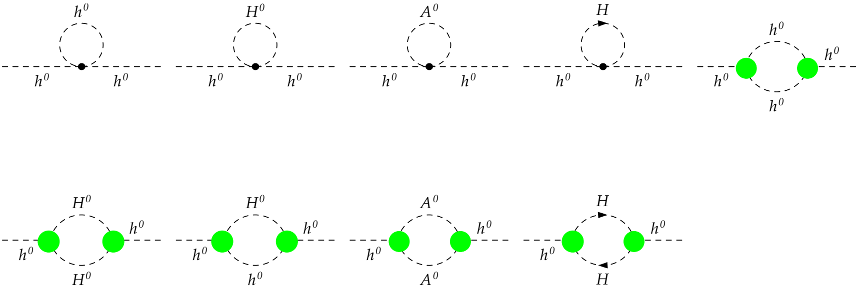

$$M^{(0+1)}_{e^+e^- \to h^0 Z^0} = M^{(0)}_{e^+e^- \to h^0 Z^0} + M^{(1)}_{e^+e^- \to h^0 Z^0} + \delta\, M^{(1)}_{e^+e^- \to h^0 Z^0} + M^{\mathrm{WF}}_{e^+e^- \to h^0 Z^0}$$

$M^{\mathrm{WF}}_{e^+e^- \to h^0 Z^0}$ $\quad \mathcal{O}(\alpha_{ew}\, \lambda^2_{3H})$

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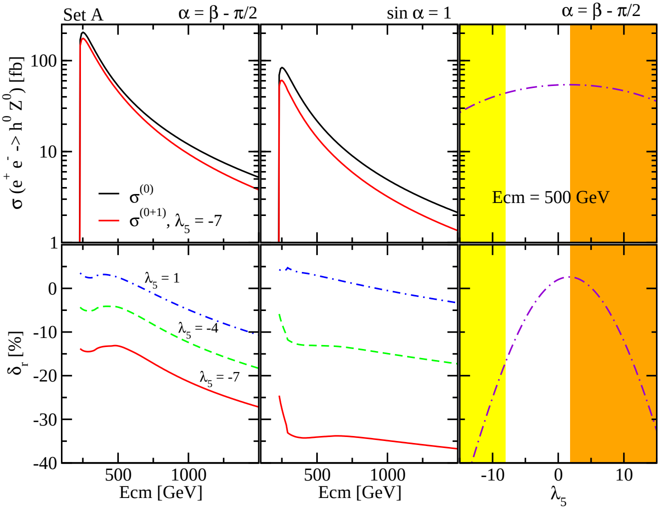

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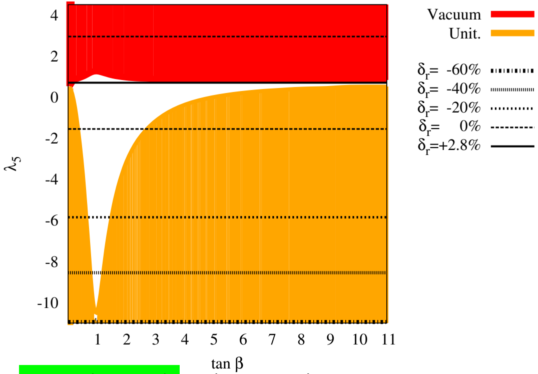

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- systematic $\delta_r < 0$ $\Rightarrow$ 3H self-couplings & Z_h^{WF}

Correlating different multi-Higgs production modes at the LC

Phenomenological highlights

- Enhanced Higgs production rates
- Enhanced quantum effects
- Correlation of hZ, 2H and 3H signatures at different $\sqrt{S}$, unmatched to the MSSM

$\tan\beta$	α	λ_5	M_{h^0} [GeV]	M_{H^0} [GeV]	M_{A^0} [GeV]	$M_{H^\pm}$ [GeV]
1	β	-10	130	150	200	160

Process	$\sigma(\sqrt{s} = 0.5\,\text{TeV})$[fb]	$\sigma(\sqrt{s} = 1.0\,\text{TeV})$[fb]	$\sigma(\sqrt{s} = 1.5\,\text{TeV})$[fb]
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$h^0H^0A^0$	0.02	5.03	3.55
$H^0H^+H^-$	0.17	11.93	8.39
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Hodgkinson, Ferrera, Guasch, DLV, Solà ['07], DLV, Solà ['09], DLV, Solà ['09], Bernal, DLV, Solà ['10]. For a recent update see Solà, DLV, arXiv:1107.1305 [hep-ph], in the LC10 proceedings

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Summary

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- Higgs boson self-interactions constitute a disclosing piece of the 2HDM dynamics $\Rightarrow$ unrelated to the gauge symmetry $\leftrightarrow$ potentially very large.
- Observables which are sensitive to Higgs self-couplings, either directly or through quantum corrections, gives rise to trademark imprints – very distinctive of non-SUSY vs SUSY multi-doublet Higgs sectors.
- Phenomenological portray of 2HDM Higgs boson production @LC for large Higgs boson self-couplings:
  - **Large Higgs boson production event rates** in a variety of channels, both from direct processes ($e^+e^- \to 3H, e^+e^- \to 2H + X$) and loop-induced processes ($\gamma\gamma \to h$, $e^+e^- \to \gamma^*\gamma^* \to h + e^+e^-$)
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  - **Large quantum effects** ($\delta_r \sim \pm 50\%$) ($e^+e^- \to 2h, e^+e^- \to hZ^0$)
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THANK YOU !!

BACKUP

SLIDES

Renormalization of the Higgs sector

Physical content of the OS conditions:

- On-shell A^0 renormalized propagators have unit residue, $1/[1 + \mathrm{Re}\,\Sigma'_{\mathsf{A}^0}(M^2_{\mathsf{A}^0})] = 1$
- No $\mathsf{A}^0 - \mathsf{Z}^0$ mixing occurs for on-shell A^0 bosons at any order in perturbation theory.
- Likewise, the absence of $\mathsf{A}^0 - G^0$ mixing is guaranteed by the Slavnov-Taylor identity:

$$q^2\hat{\Sigma}_{\mathsf{A}^0\mathsf{Z}^0}(q^2) + M_Z\,\hat{\Sigma}_{\mathsf{A}^0G^0}(q^2)\Big]_{q^2=M^2_{\mathsf{A}^0}} = 0$$

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- Likewise, the absence of $\mathsf{A}^0 - G^0$ mixing is guaranteed by the Slavnov-Taylor identity:

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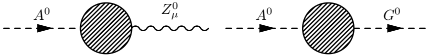

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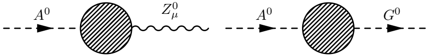
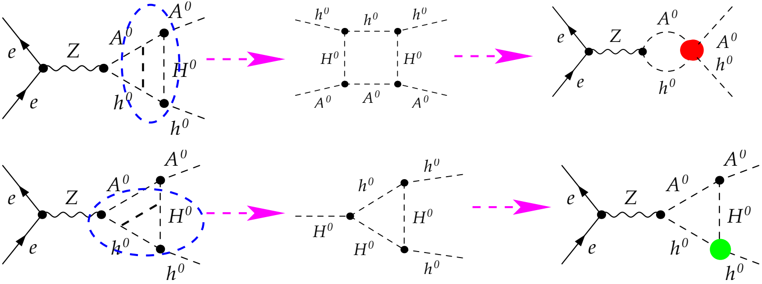
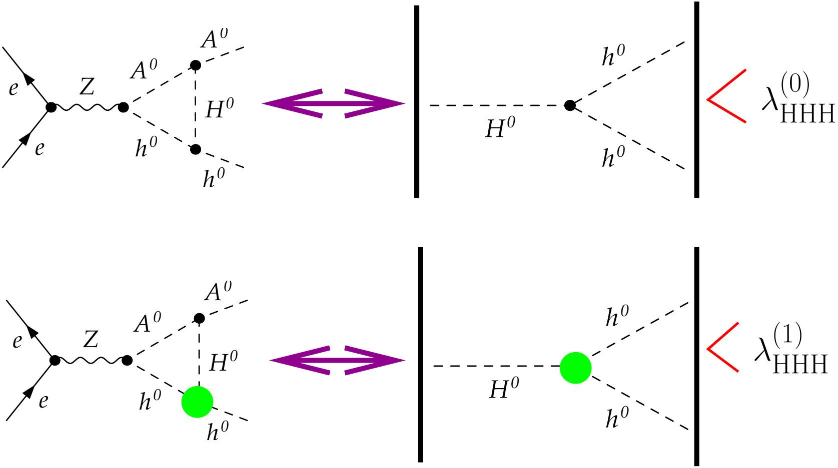


Higher-order corrections to $e^+e^- \to h^0 A^0$



Interplay of perturbativity and unitarity in the 2HDM



$e^+e^- \to h^0Z^0$ at one loop: some analytical properties

The leading quantum effects can be estimated as:

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M^{(1)}_{e^+e^-\to h^0Z^0} \quad &\simeq \quad M^{\mathrm{WF}}_{e^+e^-\to h^0Z^0} \simeq -\frac{1}{2}\, M^{(0)}_{e^+e^-\to h^0Z^0}\, \mathrm{Re}\, \Sigma'_{h^0h^0}(M^2_{h^0}) \\
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&\sim \quad -\frac{1}{2}\, \frac{|\lambda_{3H}|^2}{16\pi^2}\, M^{(0)}_{e^+e^-\to h^0Z^0}\, B'_0(M^2_{h^0}, M^2, M^2)
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♠ $\delta_r = \frac{\sigma^{(0+1)}-\sigma^{(0)}}{\sigma^{(0)}} = \frac{\langle 2\, M^{(0)}\, M^{(1)}\rangle}{\langle |M^{(0)}|^2\rangle} \simeq -\frac{|\lambda_{3H}|^2}{16\pi^2\, M^2_H}\, f(M^2_{h^0}, M^2, M^2)$