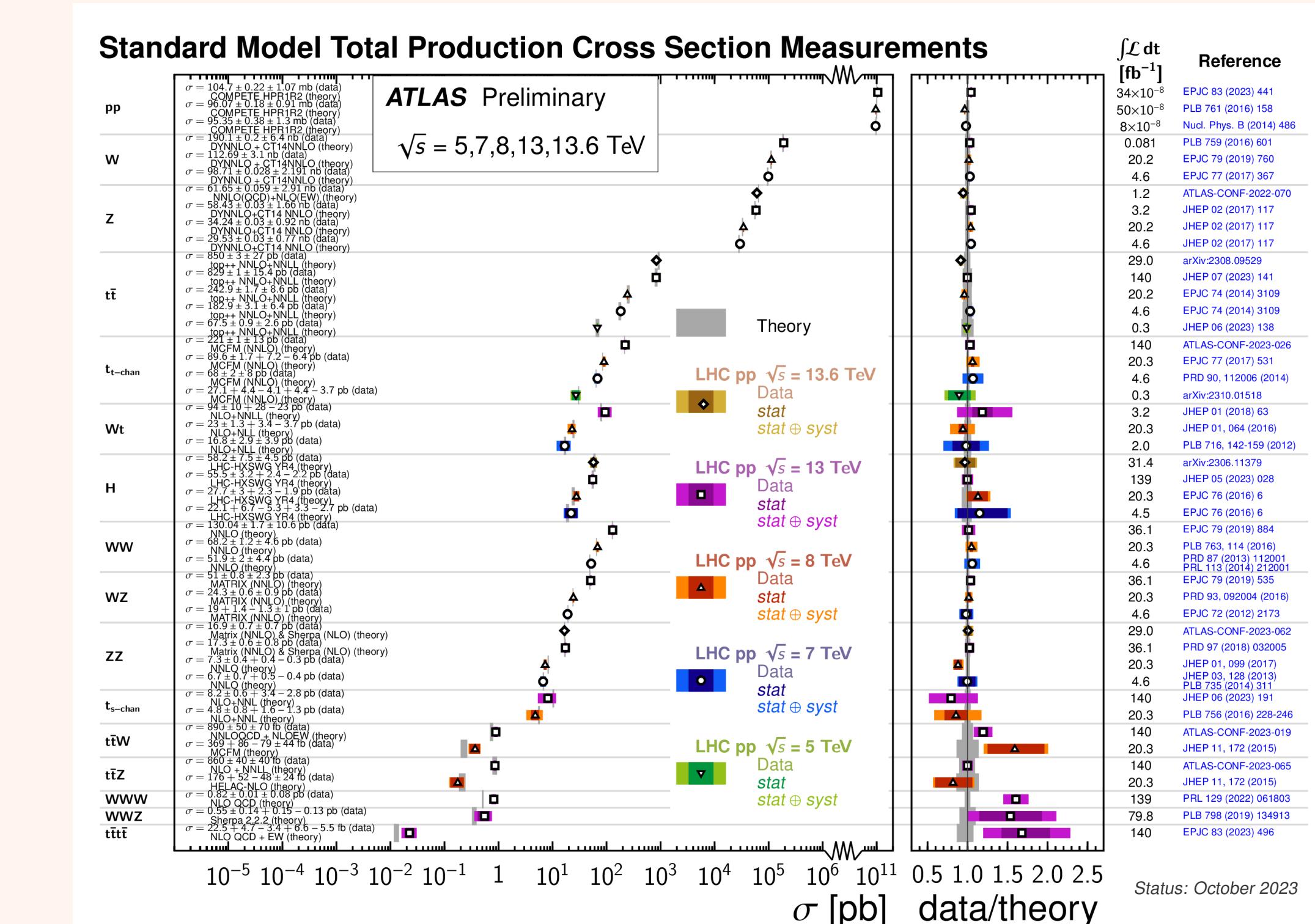
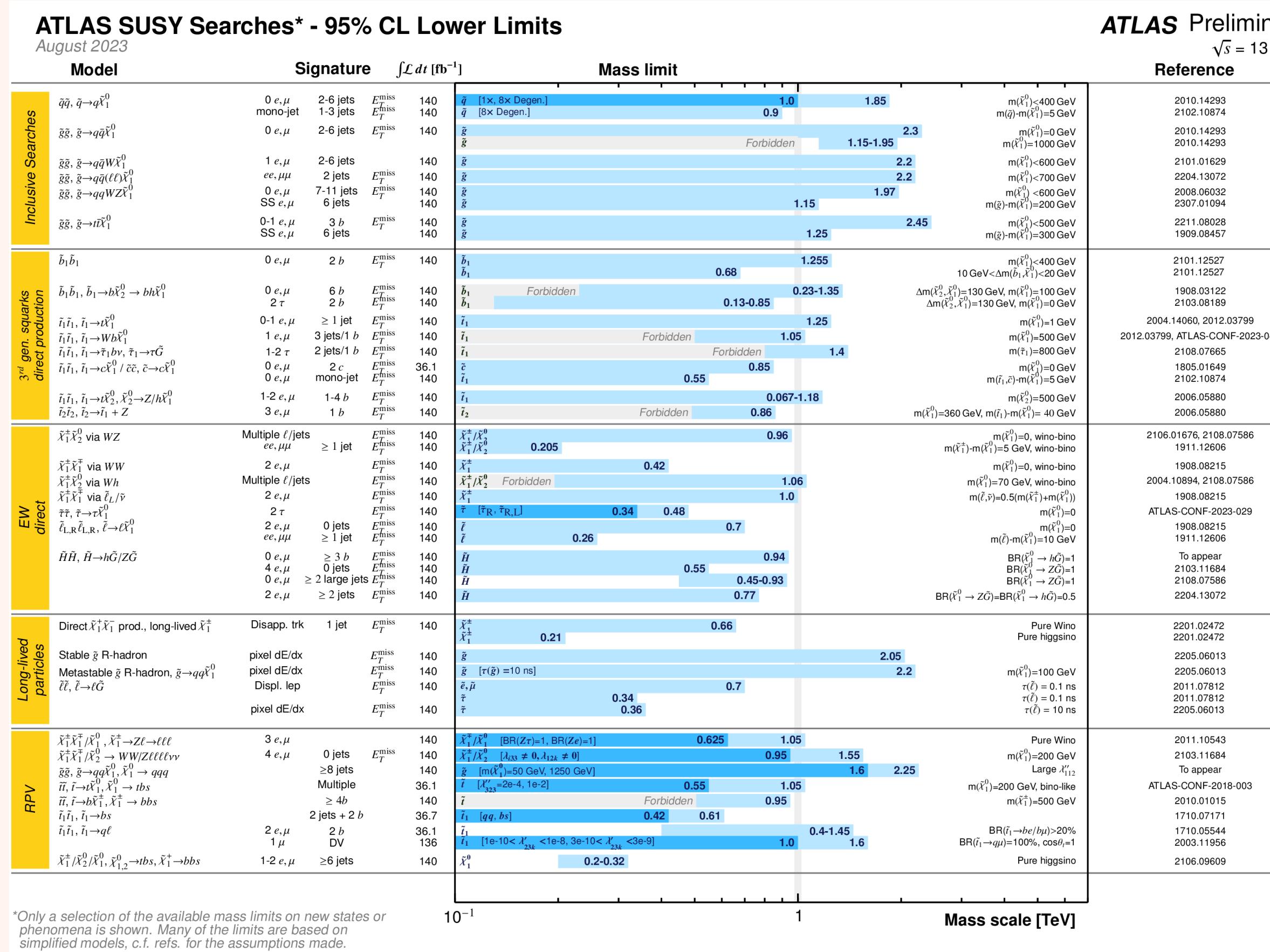


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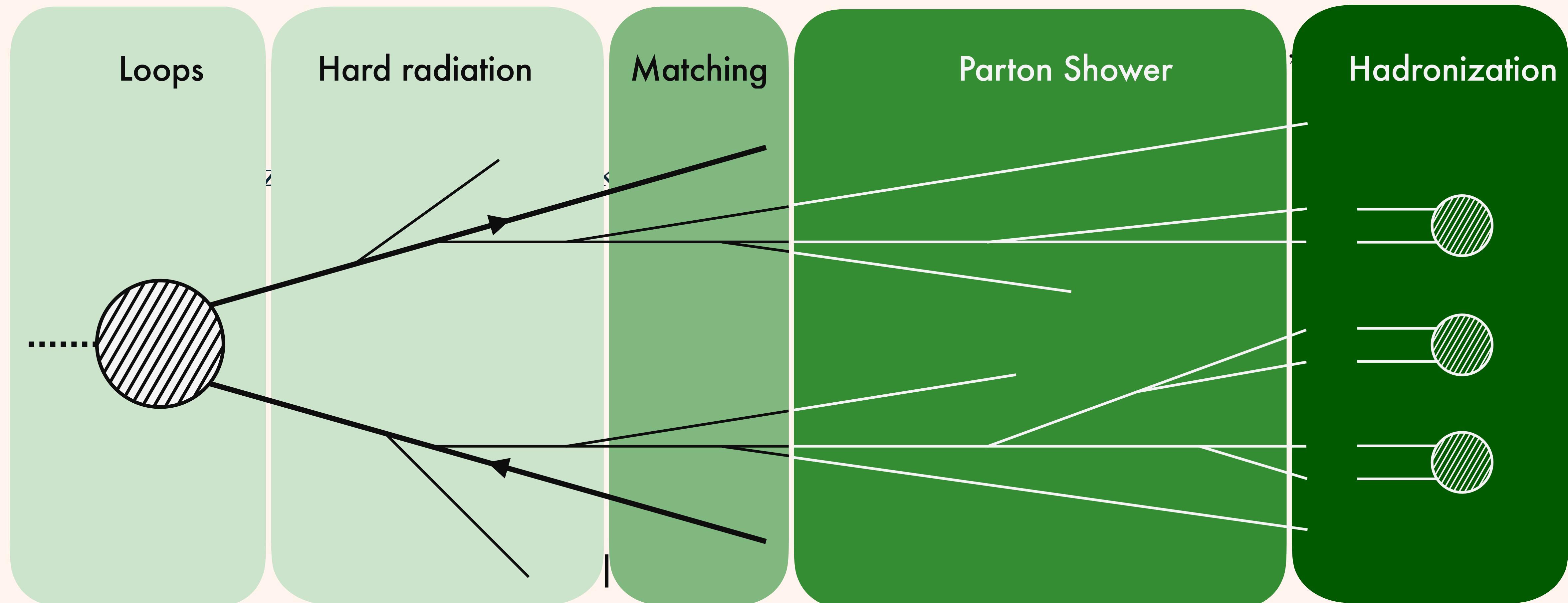
THEORY CHALLENGES AT HL-LHC AND BEYOND

Bad Honnef, December 21, 2025

It is very likely that in the foreseeable future, exploration of fundamental physics at HL-LHC and beyond will only be possible through precision studies.



Accessing the full potential of precision physics at future colliders will (probably) require a foundational change in how simulations for precision collider physics are performed, to make them **reliable and sustainable**.



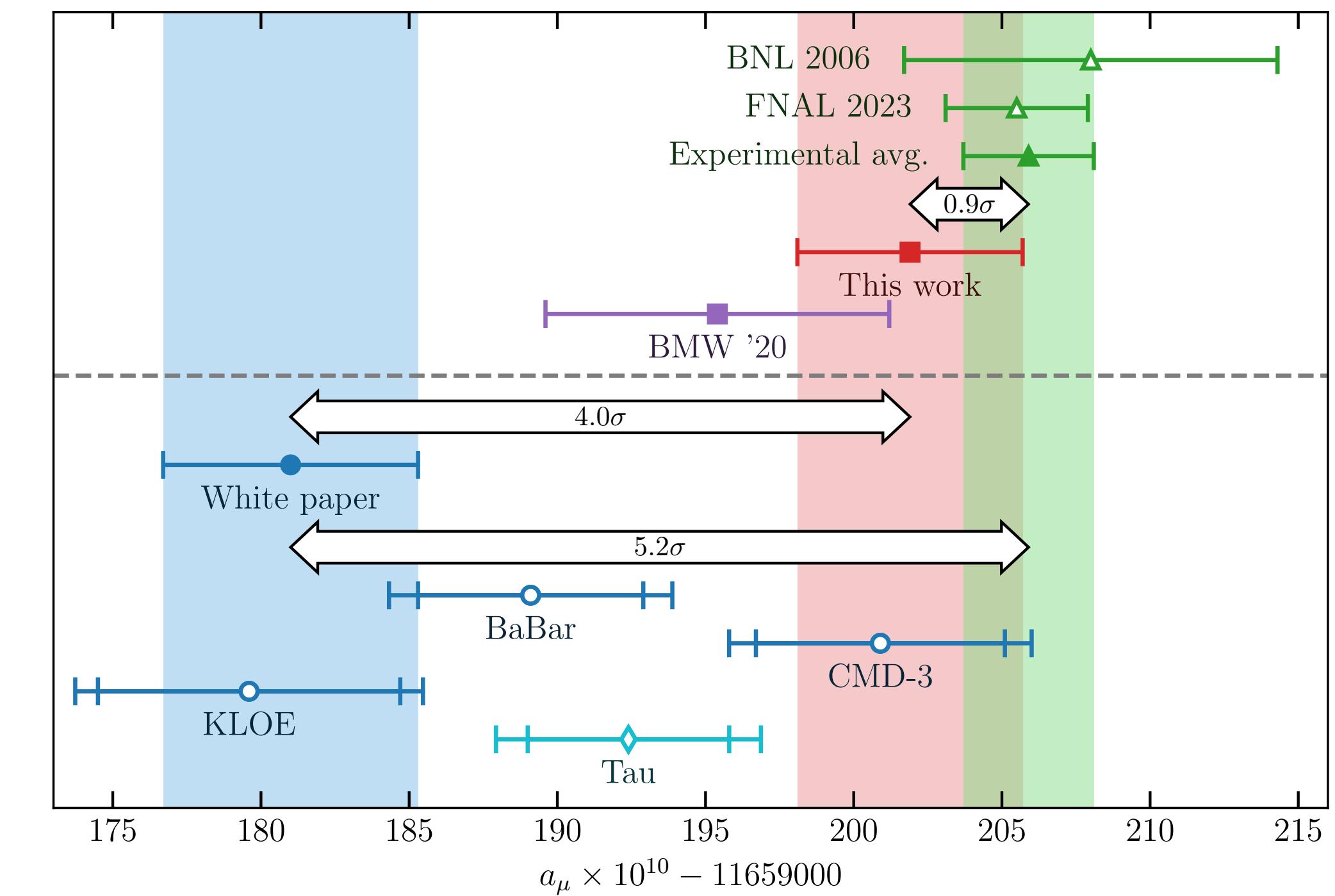
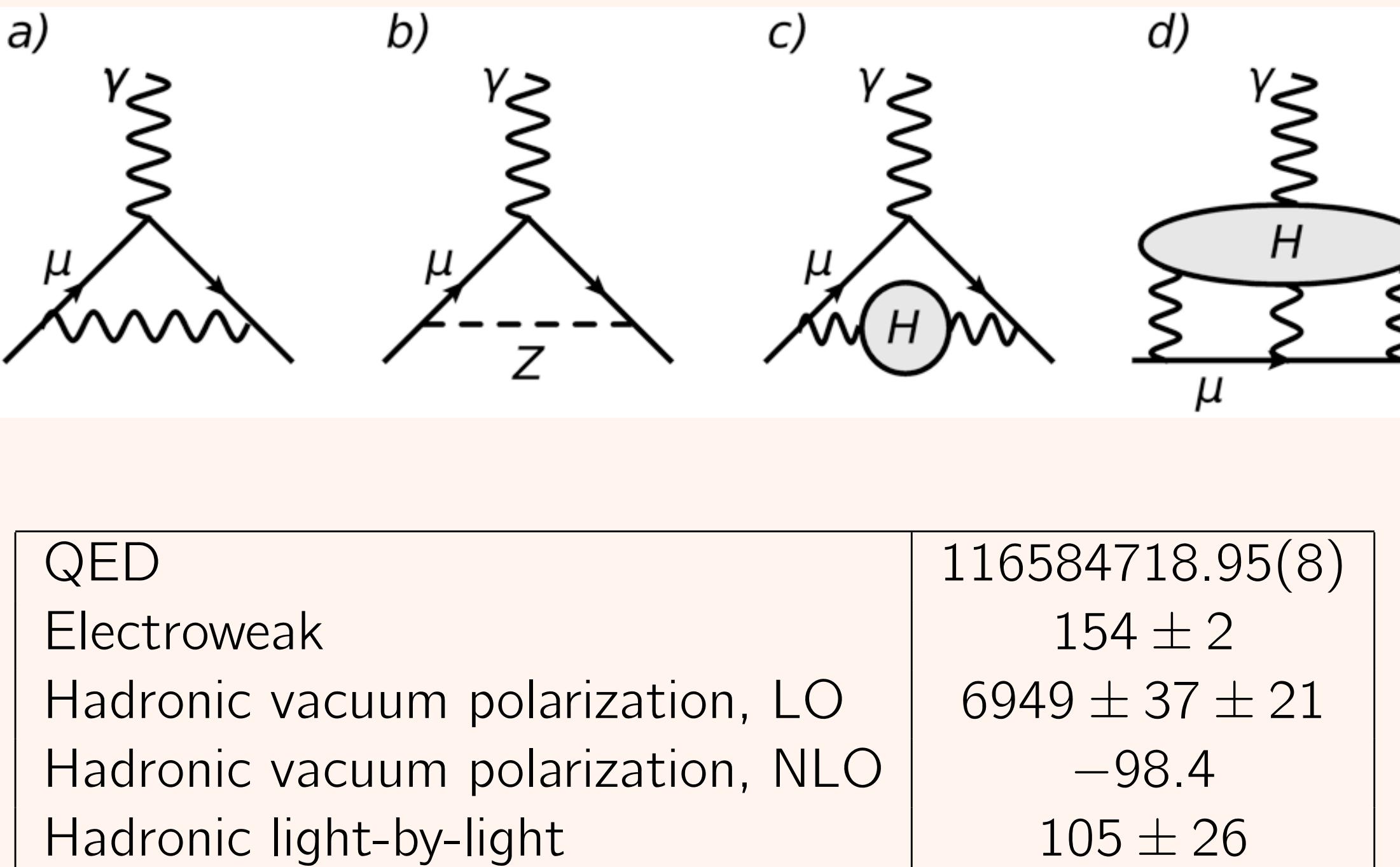
Conceptually, precision physics is a relatively simple thing:

- 1) identify a quantity that can be accurately measured and reliably computed in the context of a fundamental theory, e.g. the Standard Model of particle physics.
- 2) compute and measure this quantity;
- 3) compare the results;
- 4) If results agree, increase the accuracy of the computation and the measurement and repeat the comparison, or move to a new quantity;
- 5) if they disagree, we have to make a difficult choice and, perhaps, the fundamental theory has to go... **This is the ultimate goal of the ``precision physics'' that we are after.**
- 6) Yet, as the precision increases, it becomes more and more difficult to access the credibility of claims that precision can really be controlled at the required level, complicating the whole approach.

Because of high stakes associated with potential outcomes of such precision physics studies, the observables and the theoretical tools that are needed to describe them should be as transparent and simple as possible.

- 1) Perturbation theory is the best possible tool that we have, but the perturbative expansion alone is almost never sufficient.
- 2) Standard Model precision physics and perturbative Standard Model physics are certainly not the same thing, even at the energy frontier. It is only the question of the requested precision, that the non-perturbative physics starts playing a role.
- 3) The question is one of balance. If parton showers or lattice methods, or other ways to estimate non-perturbative effects are critical for claiming discrepancy (or agreement) with the Standard Model, it is inevitable that the reliability of such complex theoretical tools will be scrutinized.

A very instructive example of how precision physics works is the muon anomalous magnetic moment. A significant deviation between theory and experiment that existed for more than twenty years, seems to disappear thanks to new results of the BMW and CMD-3 collaborations. But experts still cannot pinpoint issues in other analyses of the hadronic VP.



A used-to-be deviation: $\mathcal{O}(3\sigma)$

Z. Fodor, BMW collaboration, ICHEP 2024

The magnitude of various contributions, in units of 10^{-11} .

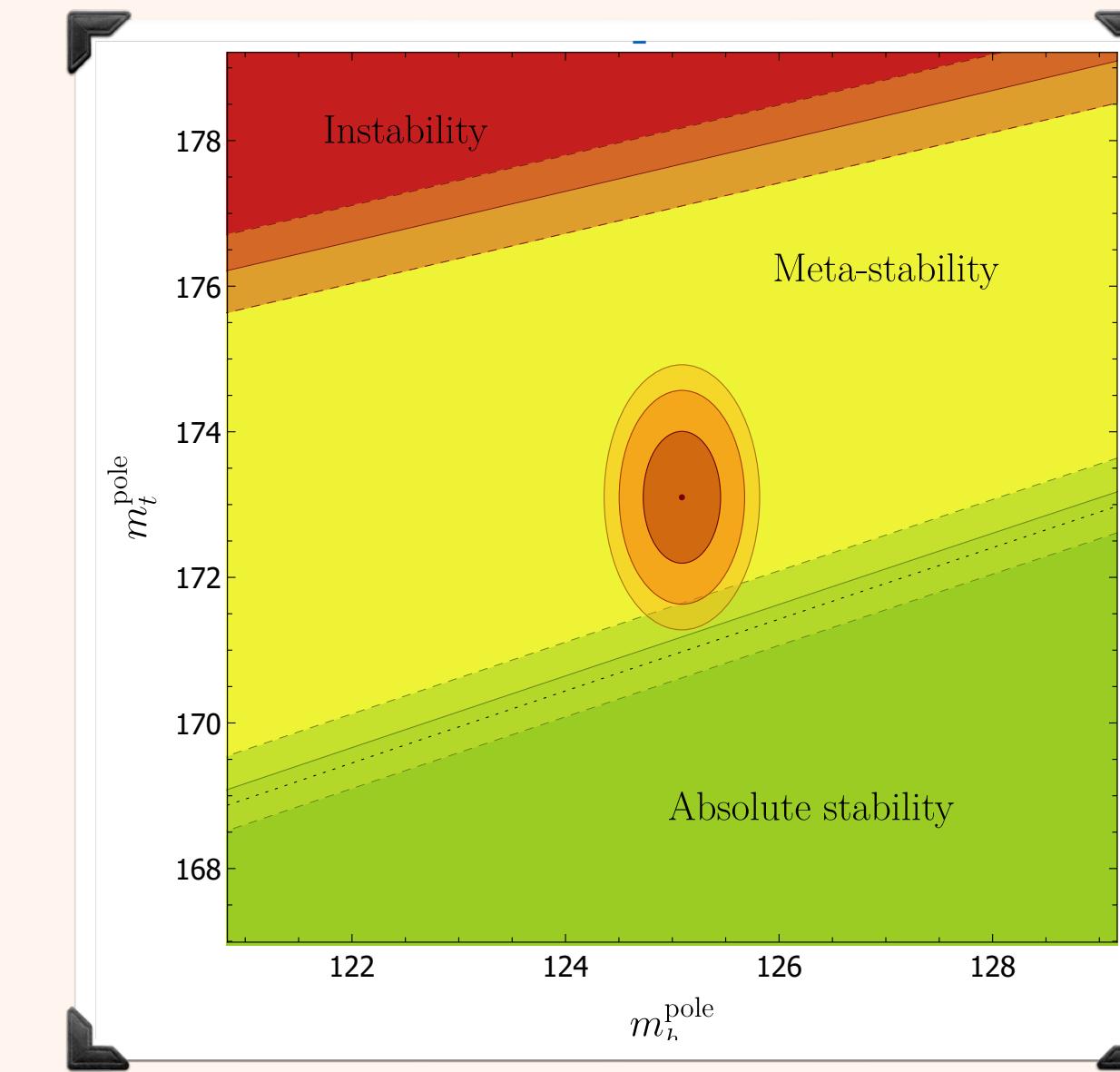
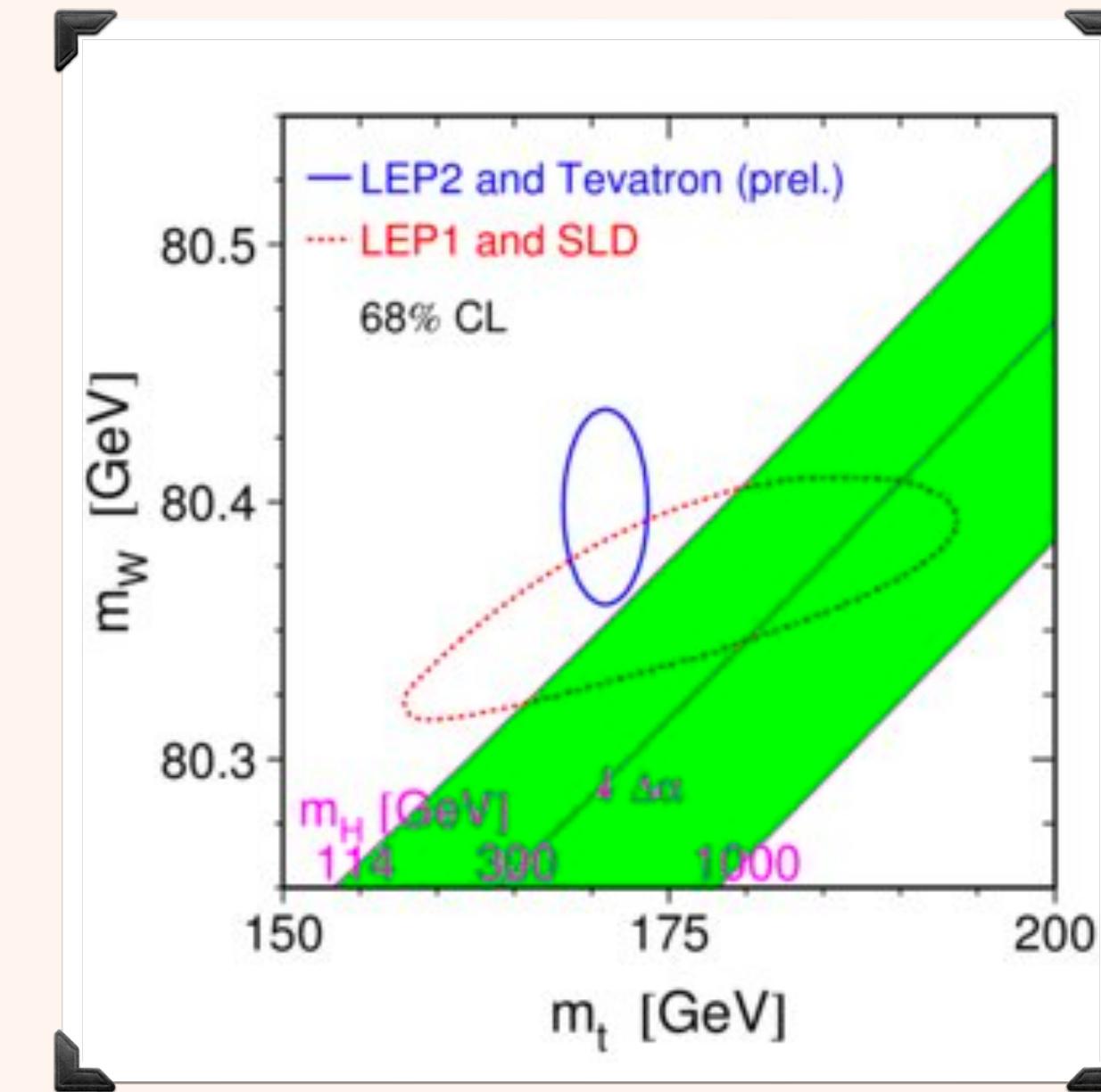
Values of the **strong coupling constant** obtained from fits to event shapes and from more global observables and the lattice measurements, disagree.

$$\alpha_s(M_Z) = \begin{cases} 0.1179 \pm 0.0010, & \text{PDG} \\ 0.1135 \pm 0.0010, & \text{thrust} \\ 0.1123 \pm 0.0015, & C - \text{parameter} \end{cases}$$

LEP event shapes receive significant (?) contribution from 3-jet events which are affected by their own non-perturbative corrections. These corrections can be “estimated”, but the computation of such effects is certainly not a first-principles computation.

Hence, the fate of this one percent precision measurement of a fundamental quantity from a high-energy observable, and the possible resolution of a decade-long stalemate hinges on a subtle non-perturbative effects that we (currently, at least) do not fully control.

Another quantity where non-perturbative effects play a role and may obscure the outcome, is the **top quark mass**. According to PDG, top quarks have different masses (pole, MS, MC..) which are all quoted there.

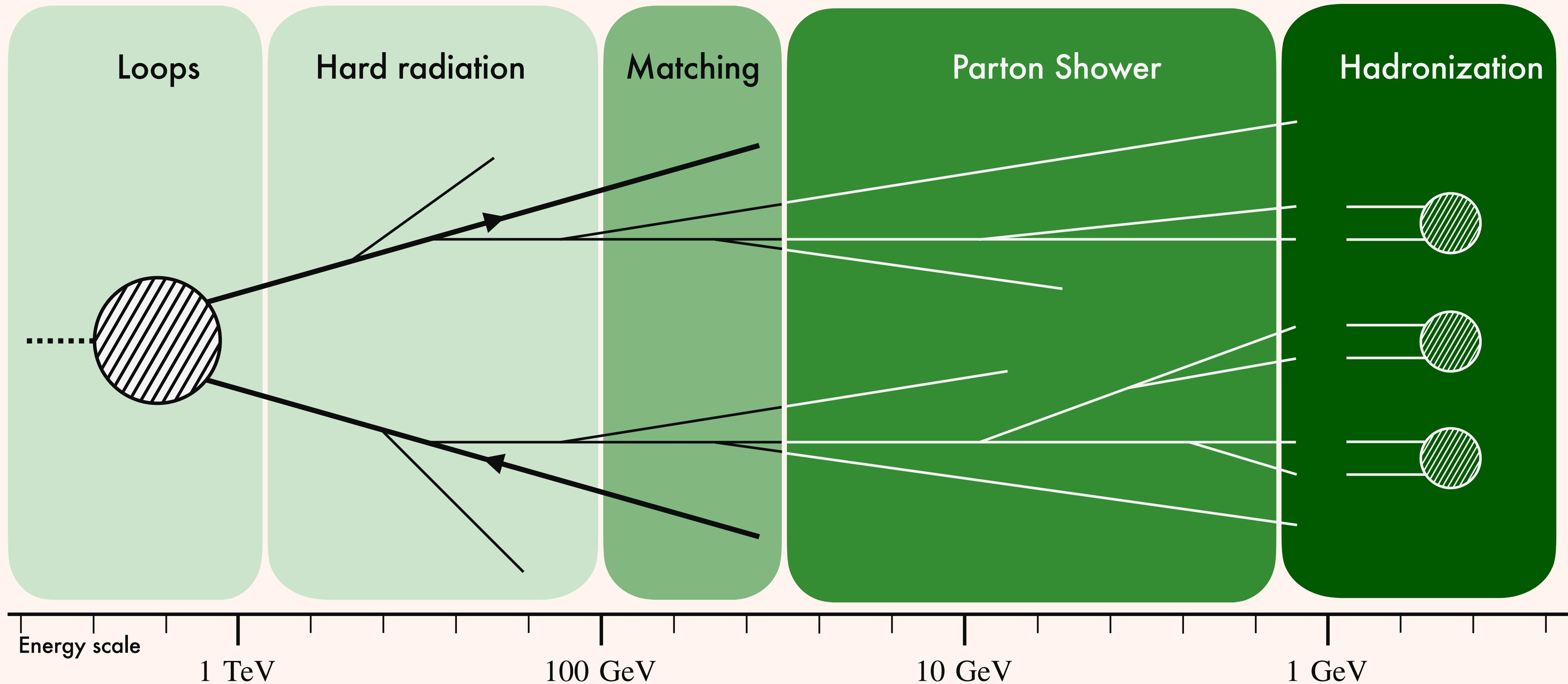


- 1) The MC top quark mass is a complex issue. Probably it is a combination of a technical issue (hard cut-off in the event generators) and the physics question of how energy of heavy quark jets is calibrated.
- 2) The pole mass of a top quark **cannot** be measured from a $t\bar{t}$ production cross section with the accuracy that is better than $\mathcal{O}(\Lambda_{\text{QCD}})$.

The discussion of non-perturbative effects may have important consequences for the Z-pole physics at future colliders where one talks about precision at the level of $O(10^{-5})$ for quantities such as Z-width, heavy-quark forward-backward asymmetry, strong coupling constant etc.

One can perhaps argue that non-perturbative effects in inclusive quantities are small. But whether or not they are sufficiently small in quantities affected by fiducial cuts, is not obvious.

Tomography of a High-Energy Process



Loops and hard radiation are supposed to describe short-distance physics. Yet, these contributions are divergent in the infra-red. Although these divergencies are similar and are known to cancel in the sum of the two, they are handled very differently in practice.

Loops: mostly analytic computation of “loop integrals”, extraction of divergencies.

Real radiation: subtraction (or slicing away) of singular limits of amplitudes and phase-spaces.

Analytic loop computations have been great for problems with small number of kinematic invariants; but for high-multiplicity processes they become increasingly contrived. The scaling with the number of loops and with multiplicities is very poor. At the same time, (integrated) divergencies of such integrals are known for arbitrary processes, through N3LO.

Methods to address arbitrary real-emissions processes (subtraction and slicing) are now available for NLO and NNLO computations. It is expected that slicing methods at N3LO will be formulated sooner rather than later (all this for this massless final states).

Building upon the real-emission example, it is natural to ask if it is possible to remove divergences from **loop integrands** by subtracting universal quantities from them, and arriving at finite integrands directly in momentum space for arbitrary processes?

Can these subtractions be formulated in the language of amplitudes, rather than individual Feynman diagrams, in an explicitly gauge-invariant way?

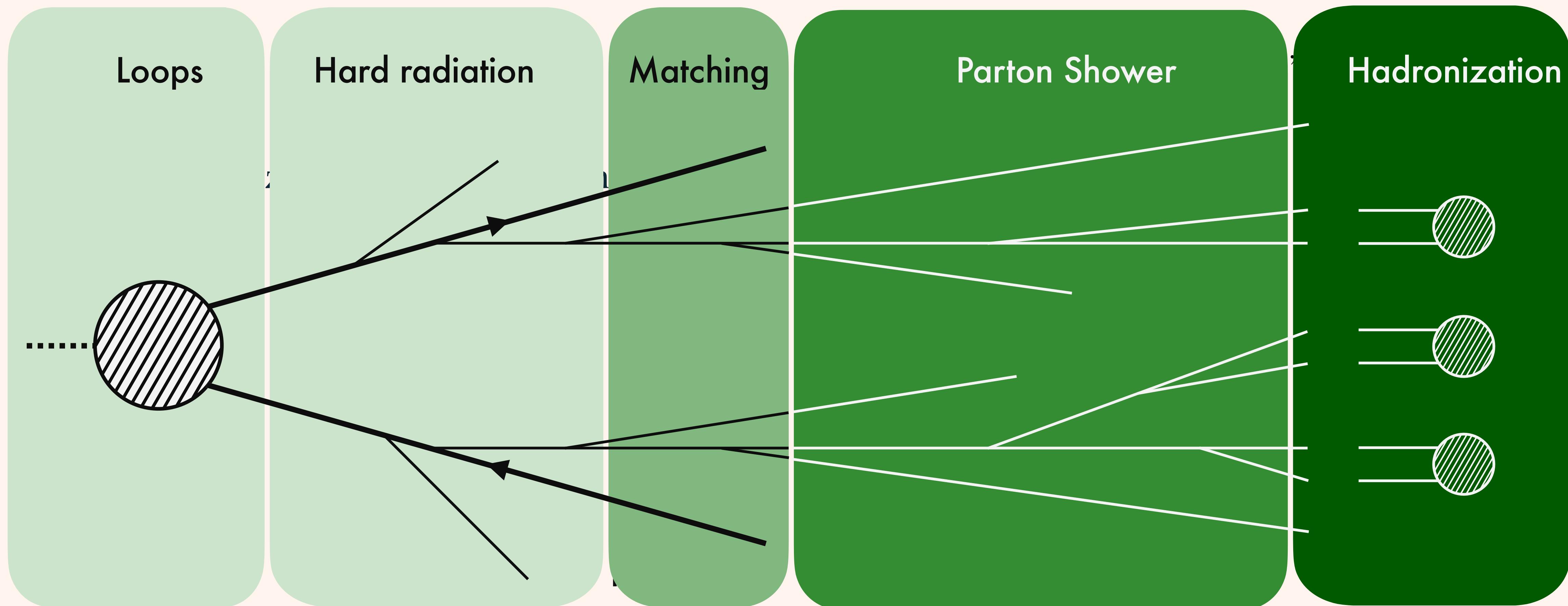
Can one even drop subtractions altogether and integrate over loop momenta and real emission momenta **in one go**, cancelling singularities on the fly?

If this were possible, perturbative computations in quantum field theory, at least through NNLO and, perhaps N3LO, will be formulated as a problem of computing finite multidimensional integrals.

Improving on the efficiency and quality of numerical integrations is very challenging, but also necessary to make sure that perturbative computations have acceptable “efficiency” scaling with final-state multiplicities.

Extending Monte-Carlo methods to sample over perturbative orders, (well-defined) quantum histories etc. is another possibility to (perhaps) keep explosion of a computational cost in perturbative calculations under control.

The need to combine parton showers with fixed-order computations may affect the logarithmic accuracy and introduce unphysical features, such as negative probabilities. This emphasizes the need to rethink the strict separation into different parts, shown below, and perhaps calls for an approach that has a stronger unifying power from the start.



To conclude, there are (at least) three major theoretical challenges in the context of precision physics for HL-LHC and future colliders, that we need to address.

- 1) Designing a framework for perturbative computations that is sustainable both with respect to partonic multiplicities, and perturbative order;
- 2) Find a way to combine fixed-order computations with parametrically-accurate parton showers in a natural and seamless way, to allow efficient and physical generation of multi-particle final-states.
- 3) Develop a theory of non-perturbative corrections, primarily the linear ones, to ensure that ultra-precise measurements of important SM quantities and key observables are indeed possible with the highest precision.