NLO decoherence effects for Quantum Observables at colliders

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Quantum states

- State described by a **density operator** ρ

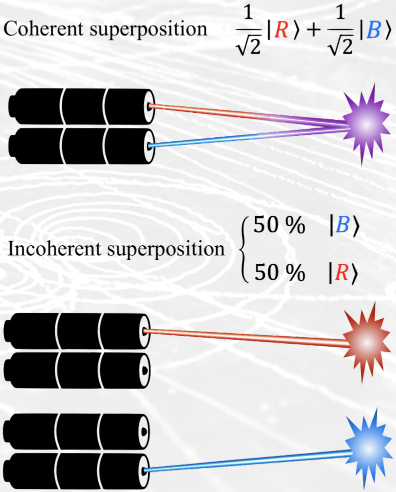
$$\mathrm{tr}[\rho] = 1, \quad \rho = \rho^\dagger, \quad \langle\phi|\rho|\phi\rangle \geq 0 \;\; \forall|\phi\rangle \in \mathcal{H}$$

- A state completely described by a single vector $|\psi\rangle$ is a **pure state**

$$\rho = |\psi\rangle\langle\psi|$$

- A **statistical ensemble** of pure states ($|\psi_i\rangle$ with probability P_i) is a **mixed state**

$$\rho = \sum_i P_i |\psi_i\rangle\langle\psi_i| = \sum_i P_i \rho_i$$



Qubits

- Qubit, 2-level quantum system

up

$|\uparrow\rangle$

down

$|\downarrow\rangle$

superposition

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

- Bipartite qubit (Alice and Bob), $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$

b a

$|\uparrow\rangle_a \otimes |\uparrow\rangle_b$

b a

$|\downarrow\rangle_a \otimes |\uparrow\rangle_b$ $\cdots$

Entanglement

- State ρ_{ab} **not separable** into a (mixture of) product states $\longleftrightarrow$ **Entangled**

$$\rho_{ab} \neq \sum_i P_i \, \rho_a^{(i)} \otimes \rho_b^{(i)}$$

ρ_{ab}

- Use **quantum observables** to **quantify entanglement**
  (e.g. concurrence, negativity, entanglement entropy...)

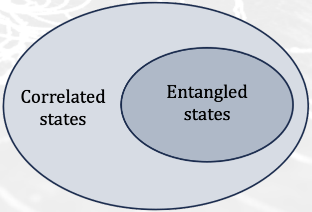
$$\mathcal{C}\left[\rho_{ab}\right] = \begin{cases} 1 \leftrightarrow \text{maximally entangled} \\ 1 > \mathcal{C} > 0 \leftrightarrow \text{entangled} \\ 0 \leftrightarrow \text{separable} \end{cases}$$

Quantum state and correlations

- For a bipartite qubit

$$\rho_{ab} = \frac{1}{4}\left[\mathbb{I}_4 + \sum_{i=1}^{3} B_i^a (\sigma^i \otimes \mathbb{I}_2) + \sum_{i=1}^{3} B_i^b (\mathbb{I}_2 \otimes \sigma^i) + \sum_{i,j=1}^{3} C_{ij} (\sigma^i \otimes \sigma^j)\right]$$

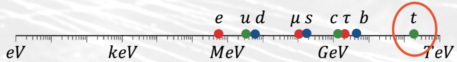
  - B_i^a: Alice's spin polarization
  - B_i^b: Bob's spin polarization
  - C_{ij}: spin correlation (matrix) $\leftarrow$ entanglement



Top quark properties

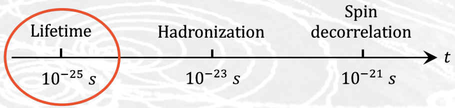
- **Heaviest** known elementary particle

$$m_t \approx 173\,GeV$$



- Short-lived, decays before hadronizing

$$\tau_t \approx 5 \times 10^{-25}\ s$$



- $t\bar{t}$ polarization state is a **bipartite qubit**

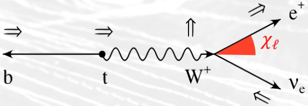
$$|\lambda_t \lambda_{\bar{t}}\rangle \in \mathbb{C}_2 \otimes \mathbb{C}_2$$

- $\rho_{t\bar{t}}$ can be entangled

Top quark properties

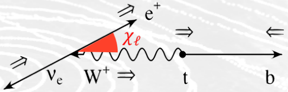
- One main decay channel

$$t \to bW^+ \to bu\bar{d}/b\nu_\ell \ell^+$$



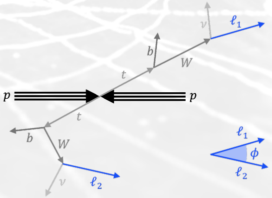
- Info about spin **polarization** passed to the decay products

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\chi_i} = \frac{1}{2}(1 + \alpha_i \cos\chi_i) \qquad \alpha_{\ell^+} = \alpha_{\bar{d}} = 1$$



- $t\bar{t}$ **spin correlations** $\leftrightarrow$ $\ell^+\ell^-/d\bar{d}$ **angular correlations**

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = 1 - \frac{1}{3}\mathrm{tr}(\mathbf{C})\cos\phi$$



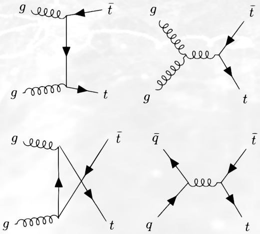
- Entanglement in $t\bar{t}$ spin can be probed experimentally

New strategy

Use **entanglement observables**[a] to **test the SM** at colliders (and **look for New Physics**)

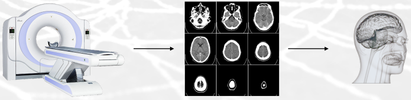
[a] For bipartite qubits only concurrence is independent

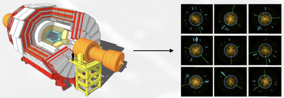
- Predict **concurrence**, QI+SM e.g. in a $t\bar{t}$ at LHC



$\longrightarrow \mathcal{C}[\rho_{t\bar{t}}]$

- Measure concurrence from LHC data via **quantum tomography**

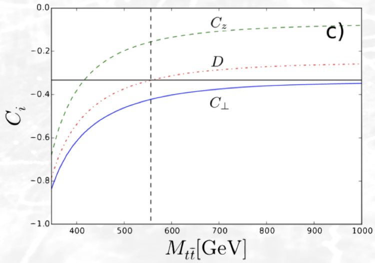




$\longrightarrow \mathcal{C}[\rho_{t\bar{t}}]$

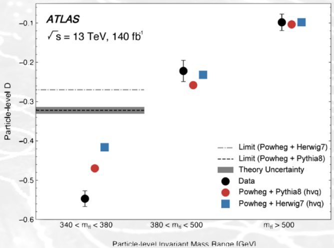
Comparison

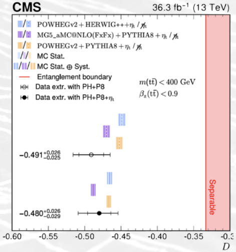
- Entanglement prediction for $pp \to t\bar{t}$



Afik-De Nova prediction (2003.02280)

- Tomography performed at LHC





Atlas (2311.07288)

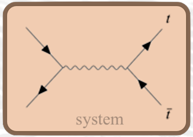
CMS (2406.03976)

First ever detection of entanglement at these energies!

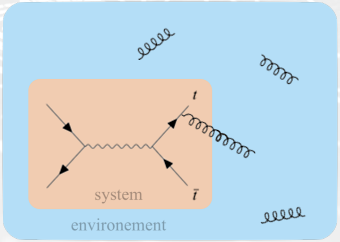
Closed System VS Open System

QFT	**QI**	**Evolution**
LO	Closed	Schrödinger
NLO, NNLO, ...	Open	Lindblad

LO ≈ closed



NLO ≈ open



- Prediction done at LO, system is **closed**
- Closed system is **unphysical**, background of undetected interactions = environment
- Incorporate **higher-order corrections** to study $t\bar{t}$ as **open system**

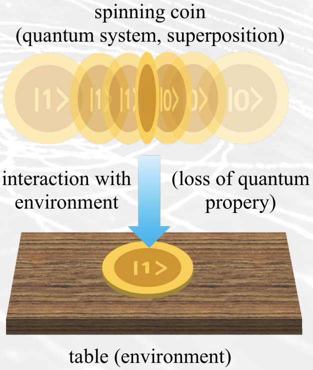
Objectives

Project goal

Describe **decoherence effects at colliders**, starting with the $t\bar{t}$ production

Why decoherence?

- Loss of quantum properties due to interaction with environment
- It's an issue in all the applications of QM
- Decoherence at colliders is a **new topic**
- We are collaborating with experimentalists to **propose a measurement**



Previous Work

First article about decoherence at colliders

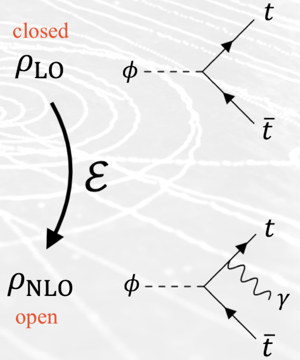
Aoude, Barr, Maltoni, Satrioni, arXiv:2504.07030v1

- Heavy scalar boson decay $\phi \to t\bar{t}$
- **Established the formalism** to describe decoherence
- Decoherence as **quantum maps** to relate ρ_{LO} and ρ_{NLO}

$$\mathcal{E}: \quad \rho_{\mathrm{LO}} \mapsto \mathcal{E}[\rho_{\mathrm{LO}}] = \rho_{\mathrm{NLO}}$$

For open systems we need **Kraus operators** K_i

$$\mathcal{E}[\rho_{\mathrm{LO}}] = \sum_i K_i \rho_{\mathrm{LO}} K_i^\dagger$$

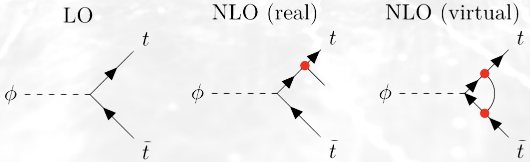


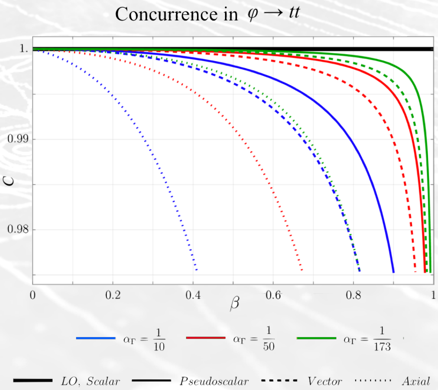
Previous Work

First article about decoherence at colliders

Aoude, Barr, Maltoni, Satrioni, arXiv:2504.07030v1

- Heavy scalar boson decay $\phi \to t\bar{t}$
- Environment reduces entanglement, **quantum decoherence**





$\phi \to t\bar{t}g$

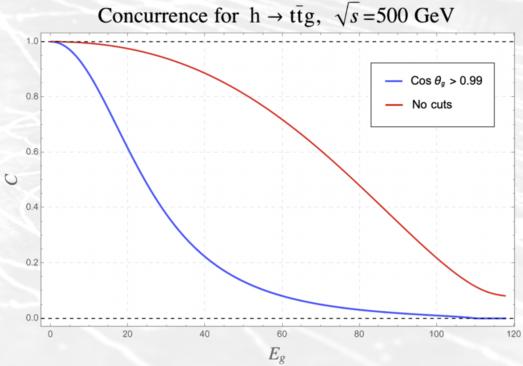
- At LO the $t\bar{t}$ is in a **Bell singlet state**

$$\rho_{t\bar{t}} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = |\Psi^-\rangle\langle\Psi^-|$$

maximally entangled $\mathcal{C}[\rho_{t\bar{t}}] = 1$

- At NLO the state is a function of the gluon phase-space

$$\rho_{t\bar{t}g} = \rho_{t\bar{t}g}(E_g, \cos\theta_g)$$

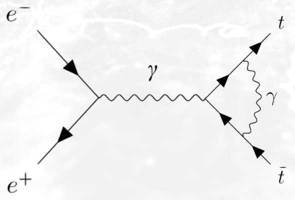


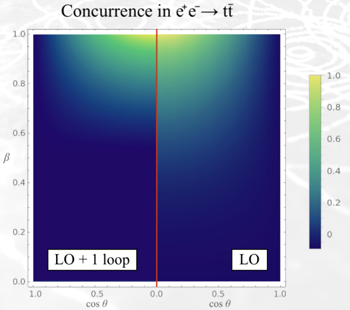
Ongoing work

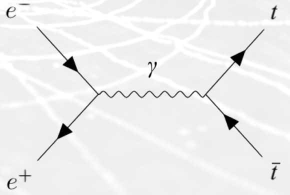
Ongoing work: $e^+e^- \to t\bar{t}$

- Realistic system (e.g. FCC-ee)
- Intermediate step for *pp*

- Adding 1-loop correction







Future plans

Next step: $pp \to t\bar{t}$

- Technical challenges:
  - Mixed initial states and multiple production channels ($q\bar{q}$, gg)
  - Radiation couples to the initial state

Next-to-next step: further developments

- Extend the formalism to **all orders** (resummation, dealing with logs)
- Generalize to **other systems** (e.g. qutrits, quitrit$\otimes$quibit, ...)
- Explore extensions **BSM** (e.g. effective theories like SMEFT)

Thank you for your attention!

Bell states

Bell states

Bell states are **maximally entangled** states of a bipartite qubit system:

$$|\Psi^{\pm}\rangle \equiv \frac{1}{\sqrt{2}}\left(|01\rangle \pm |10\rangle\right) \ ,$$
$$|\Phi^{\pm}\rangle \equiv \frac{1}{\sqrt{2}}\left(|00\rangle \pm |11\rangle\right) \ .$$

- $|\Psi^{-}\rangle$ is a singlet
- $|\Psi^{+}\rangle$, $|\Phi^{\pm}\rangle$ are triplets

$$|\Psi^{\pm}\rangle\langle\Psi^{\pm}| = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm1 & 0 \\ 0 & \pm1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$|\Phi^{\pm}\rangle\langle\Psi^{\pm}| = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & \pm1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \pm1 & 0 & 0 & 1 \end{pmatrix}$$

Generalized spin sums

- Useful spinor identities: Michel-Bouchiat formulae

$$\mathbb{P}_u^{hh'} \equiv u_{h'}(p)\bar{u}_h(p) = \frac{1}{2}\left(\delta_{h'h}\mathbb{I} + \gamma^5 \not{s}^a [\sigma^a]_{h'h}\right)(\not{p} + m),$$

$$\mathbb{P}_v^{hh'} \equiv v_h(p)\bar{v}_{h'}(p) = \frac{1}{2}\left(\delta_{h'h}\mathbb{I} + \gamma^5 \not{s}^a [\sigma^a]_{h'h}\right)(\not{p} - m).$$

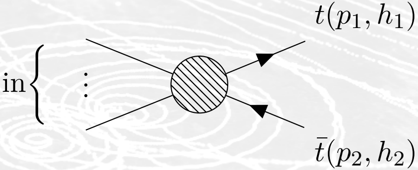
- Generalization of the completeness relation

$$\sum_i u_h(p)\bar{u}_h(p) = (\not{p} - m)$$

Density matrix for $t\bar{t}$

- Consider the scattering from an initial state ”in” to final state $t\bar{t}$ (with momenta p_i and helicities h_i)

$$R_{t\bar{t}}^{h_1 h_2, h_1' h_2'} = \frac{1}{N_{\text{in}}} \sum_{\text{in.dof}} \mathcal{A}^{h_1 h_2}_{\text{in}\to t\bar{t}} \bar{\mathcal{A}}^{h_1' h_2'}_{\text{in}\to t\bar{t}}$$



- Normalize to find the quantum state

$$\rho_{t\bar{t}} = \frac{R_{t\bar{t}}}{\text{tr}[t\bar{t}]}$$

Density matrix for $t\bar{t}$

- Start from amplitude

$$\mathcal{A} \sim \underbrace{K_\mu}_{\text{in. state}} \left(\bar{u}(p_1, h_1) T^\mu v(p_2, h_2)\right)$$

- The R matrix is

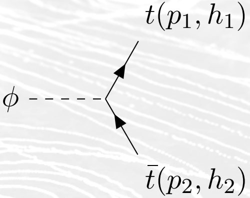
$$R^{h_1 h_2, h_1' h_2'} \sim K_\mu \bar{K}_\nu \left(\bar{u}(p_1, h_1) T^\mu v(p_2, h_2) \bar{v}(p_2, h_2') \bar{T}^\nu u(p_1, h_1')\right)$$

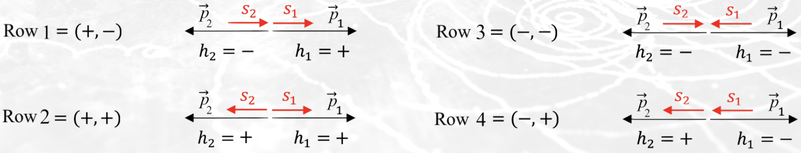
- Apply Michel-Bouchiat formulae

$$R^{h_1 h_2, h_1' h_2'} \sim K_\mu \bar{K}_\nu \,\mathrm{tr}\left[\mathbb{P}_u^{h_1 h_1'} T^\mu \mathbb{P}_v^{h_2 h_2'} \bar{T}^\nu\right]$$

- Decay of a massive scalar boson ϕ with $p^2 = m_\phi^2$

$$[R_{\mathrm{LO}}]^{h_1 h_2, h_1' h_2'} = \frac{4 N_C y_t^2 m_t^2 \beta^2}{1-\beta^2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



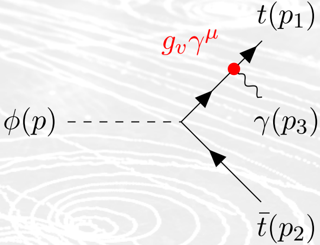


- LO is a **Bell singlet** $\rho_{\mathrm{LO}} = |\Psi^-\rangle\langle\Psi^-|$ which is **maximally entangled** $\mathcal{C}[\rho_{\mathrm{LO}}] = 1$

- Add **real emission corrections** (e.g. vector radiation)

- Collinear **reduced** density matrix

$$R_{\text{col}} = \begin{pmatrix} R^{11}_{\text{col}} & 0 & 0 & 0 \\ 0 & R^{22}_{\text{col}} & -R^{22}_{\text{col}} & 0 \\ 0 & -R^{22}_{\text{col}} & R^{22}_{\text{col}} & 0 \\ 0 & 0 & 0 & R^{11}_{\text{col}} \end{pmatrix}$$



$$a = 1 - \frac{R^{11}_{\text{col}}}{R^{11}_{\text{LO}} + R^{11}_{\text{col}} + R^{22}_{\text{col}}} \sim 1 - \mathcal{O}(\alpha_v)$$

- Full state is different from LO

$$\rho_{\text{NLO}} = \frac{R_{\text{LO}} + R_{\text{col}}}{\text{tr}[R_{\text{LO}} + R_{\text{col}}]} = a|\Psi^-\rangle\langle\Psi^-| + \frac{1-a}{2}\Big(|00\rangle\langle 00| + |11\rangle\langle 11|\Big)$$

- Build a quantum map to reproduce the NLO emission

$$\rho_{\rm LO} = |\Psi^-\rangle\langle\Psi^-| \quad \xrightarrow{?} \quad \rho_{\rm NLO} = a|\Psi^-\rangle\langle\Psi^-| + \frac{1-a}{2}\Big(|\Phi^+\rangle\langle\Phi^+| + |\Phi^-\rangle\langle\Phi^-|\Big)$$

- Look at K_{00}, K_{10}, K_{20} with $P_{00} = a$ and $P_{i0} = \frac{1-a}{2}$

$$\left.\begin{array}{lccc} K_{00}: & |\Psi^-\rangle\langle\Psi^-| & \xrightarrow{K_{00}} & P_{00}|\Psi^-\rangle\langle\Psi^-| \\ K_{10}: & |\Psi^-\rangle\langle\Psi^-| & \xrightarrow{K_{10}} & \frac{1-a}{2}|\Phi^-\rangle\langle\Phi^-| \\ K_{20}: & |\Psi^-\rangle\langle\Psi^-| & \xrightarrow{K_{20}} & \frac{1-a}{2}|\Phi^+\rangle\langle\Phi^+| \end{array}\right\} \quad \mathcal{E}_{\rm NLO}[\bullet] = \sum_{i=0,1,2} K_{i0} \bullet K_{i0}^\dagger$$

Kraus operators

- Basic Kraus operators transform **a Bell state into another one**

- $K_{00} = \sqrt{P_{00}}\,\sigma^0 \otimes \sigma^0$:

$$|\Psi^-\rangle\langle\Psi^-| \longmapsto P_{00}|\Psi^-\rangle\langle\Psi^-|$$

- $K_{20} = \sqrt{1-P_{20}}\,\sigma^2 \otimes \sigma^0$:

$$|\Psi^-\rangle\langle\Psi^-| \longmapsto (1-P_{20})\,|\Phi^+\rangle\langle\Phi^+|$$

- $K_{10} = \sqrt{1-P_{10}}\,\sigma^1 \otimes \sigma^0$:

$$|\Psi^-\rangle\langle\Psi^-| \longmapsto (1-P_{10})\,|\Phi^-\rangle\langle\Phi^-|$$

- $K_{30} = \sqrt{1-P_{30}}\,\sigma^3 \otimes \sigma^0$:

$$|\Psi^-\rangle\langle\Psi^-| \longmapsto (1-P_{30})|\Psi^+\rangle\langle\Psi^+|$$

Kraus operators

- There are also **linear combinations** of the Pauli matrices

$$\sigma^{\pm} = \frac{1}{2}(\sigma^1 \pm i\sigma^2) \qquad \sigma^{L/R} = \frac{1}{2}(\sigma^0 \pm \sigma^3)$$

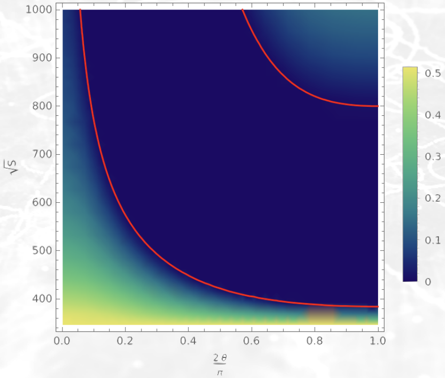
- These Kraus operators transform **a Bell state into a separable state**

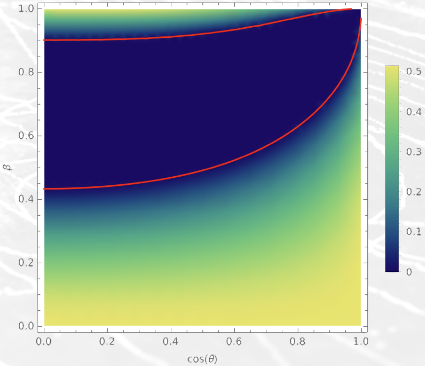
  - $K_{+0}\ :\ |\Psi^-\rangle\langle\Psi^-| \xrightarrow{K_{+0}} \frac{1-P_{+0}}{2}|11\rangle\langle 11|$
  - $K_{R0}\ :\ |\Psi^-\rangle\langle\Psi^-| \xrightarrow{K_{R0}} \frac{1-P_{R0}}{2}|01\rangle\langle 01|$
  - $K_{-0}\ :\ |\Psi^-\rangle\langle\Psi^-| \xrightarrow{K_{-0}} \frac{1-P_{-0}}{2}|00\rangle\langle 00|$
  - $K_{L0}\ :\ |\Psi^-\rangle\langle\Psi^-| \xrightarrow{K_{L0}} \frac{1-P_{L0}}{2}|10\rangle\langle 10|$

$\phi \to t\bar{t}$

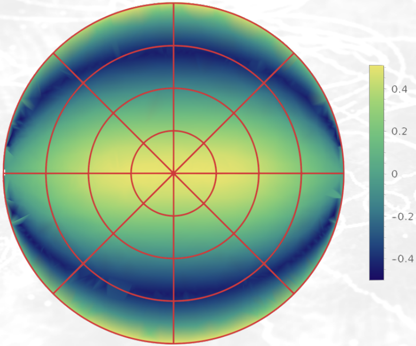
- **Virtual** correction $\mathcal{E}_V[\rho_{\rm LO}] = p_V \mathbb{I}\rho_{\rm LO}\mathbb{I}$
  - Contains UV divergencies, removed via renormalization
  - Contains IR divergencies, cancelled by those in the $\mathbb{I}$ operator **from the Real map**
- **Real** correction $\mathcal{E}_R[\rho_{\rm LO}] = (p_R^{\rm soft} + p_R^{\rm hard})\mathbb{I}\rho_{\rm LO}\mathbb{I} + q_R^{\rm hard}\sum_{j\neq\mathbb{I}} K_j\rho_{\rm LO}K_j$
  - No collinear divergencies due the top mass m_t
  - Contains IR divergencies in $p_R^{\rm soft}$, cancelled by those in the Virtual map
  - Additional IR finite terms (from σ^3) for **parity odd** radiation $q_R^5\sum_j K_j\rho_{\rm LO}K_j^\dagger$
- **Full map** $\mathcal{E}_{\rm full}[\rho_{\rm LO}] = p_\mathbb{I}\mathbb{I}\rho_{\rm LO}\mathbb{I} + \sum_j q_j K_j\rho_{\rm LO}K_j$
  - $p_\mathbb{I} = p_{\rm LO} + p_V + p_R^{\rm soft} + p_R^{\rm hard}$
  - $q_j = q_R^{\rm hard} + q_R^5$

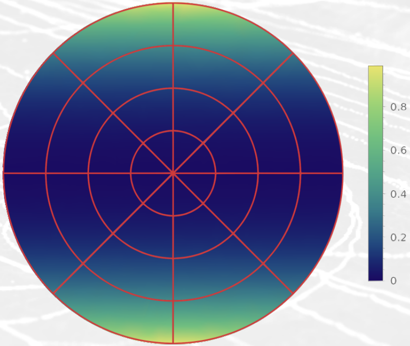
- Concurrence for $gg \to t\bar{t}$





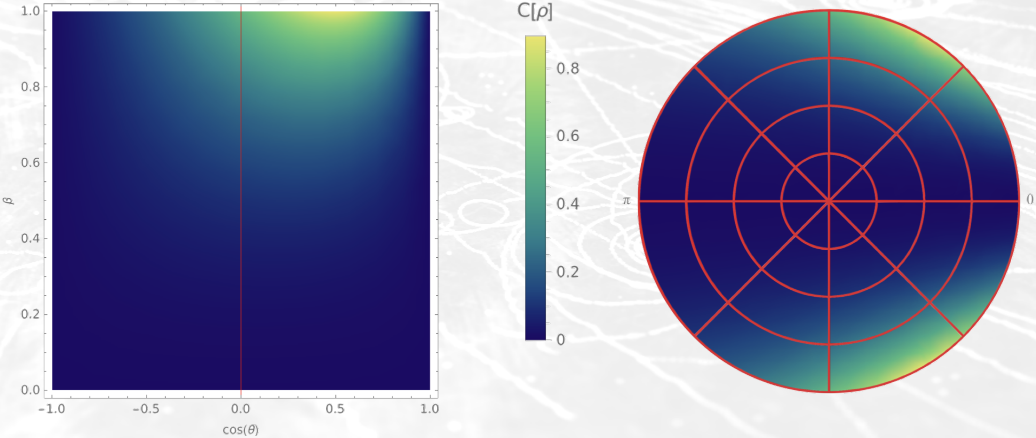
- Concurrence for $gg \to t\bar{t}$ (left) and $q\bar{q} \to t\bar{t}$ (right)





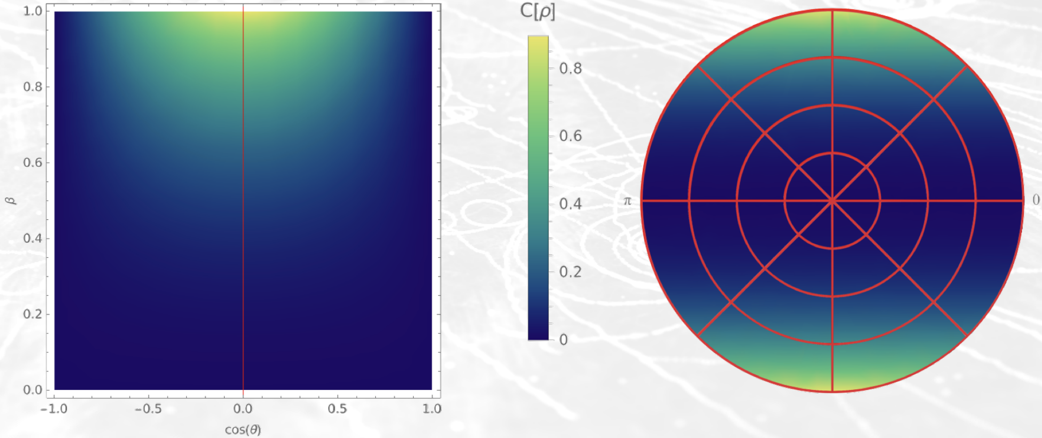
polar coordinates: radius is $\beta = \sqrt{1 - 4\frac{m_t^2}{m_{t\bar{t}}^2}}$, angle is the scattering angle θ

- Concurrence for $e^+e^- \to t\bar{t}$ (both γ and Z)



polar coordinates: radius is $\beta = \sqrt{1 - 4\frac{m_t^2}{m_{t\bar{t}}^2}}$, angle is the scattering angle θ

- Concurrence for $e^+e^- \to t\bar{t}$ (only γ)



polar coordinates: radius is $\beta = \sqrt{1 - 4\frac{m_t^2}{m_{t\bar{t}}^2}}$, angle is the scattering angle θ