Interferometric Measurements and Analysis of Thermal Noise in Solids

Master's degree in Astrophysics and Cosmology
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Thesis Motivation

- Gravitational wave detectors must be able to (and can!) detect strain on the order of $\Delta L/L \approx 10^{-21}$ for compact object mergers
- Reduction of the many noise sources that limit detectors will allow for the detection of a much larger number of sources, and new types of sources
- We investigate one of the main noise sources in detectors: thermal noise

Gravitational Wave Detection

- The current detectors used for GW detection are kilometer-scale Michelson interferometers
- Two test mass mirrors are suspended at the end of each arm
- Tiny ($\Delta L/L \approx 10^{-21}$) relative change of arm lengths can be measured using interferometry

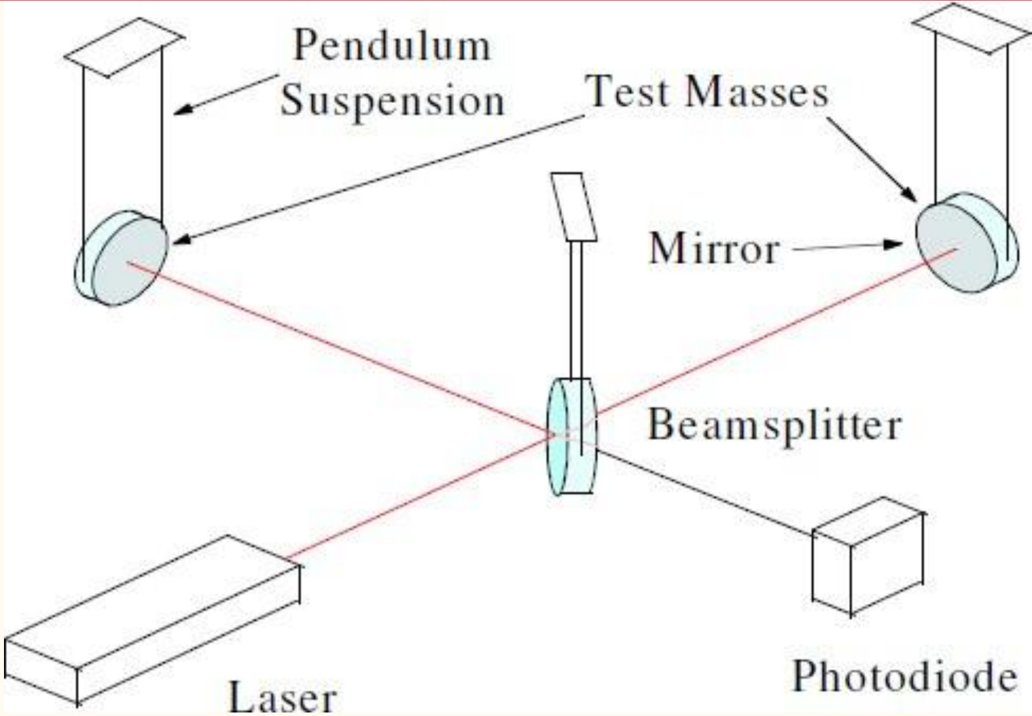


Figure credit: DOI: 10.12942/lrr-2011-5

Noise Sources

- Seismic noise: Ground vibrations
- Newtonian noise: Changes in gravity gradients
- Quantum noise:
  - Shot noise: Randomness of photons arriving at detector
  - Radiation pressure noise: Randomness of photos impacting mirror
- Thermal Noise: Noise due to thermal motion and dissipation

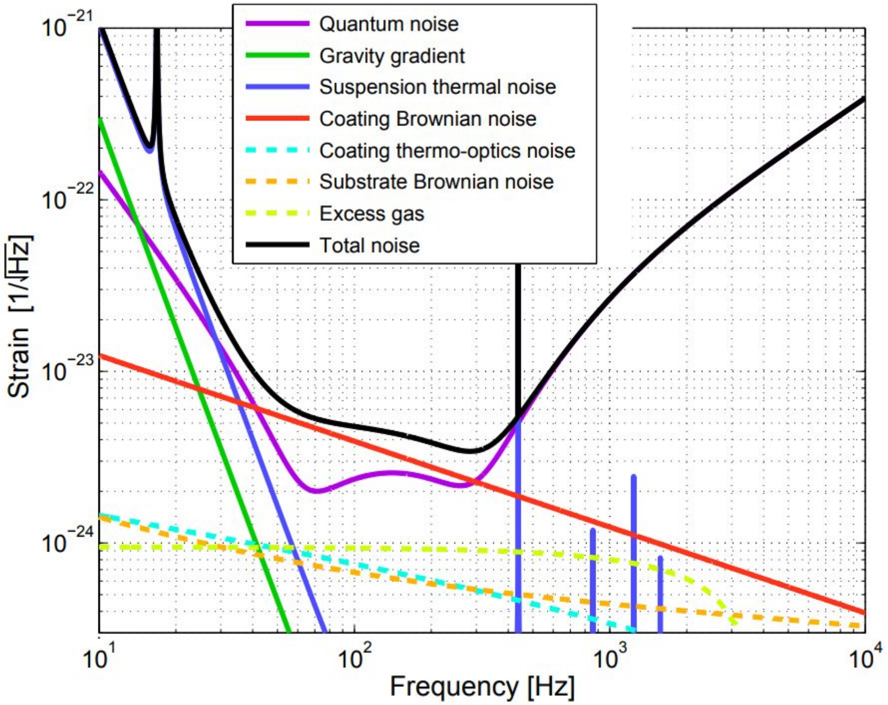


Figure credit: doi: 10.1103/physrevd.89.092004

Measuring Thermal Noise

- Caused by dissipations within the system
- Quantified fluctuation-dissipation theorem in the frequency domain
- In order to measure thermal noise in a macroscopic device we need to be sensitive to thermal vibrations of the size of:

$$\langle \overline{x}_{th}^2 \rangle = \frac{k_B T}{m\omega_0^2}$$

- For T = 300K, m = 0.1kg, and ω_0 = 1kHz; $\langle \bar{x}_{th} \rangle \approx 4 \times 10^{-13}$m

Thesis Plan

What we know:

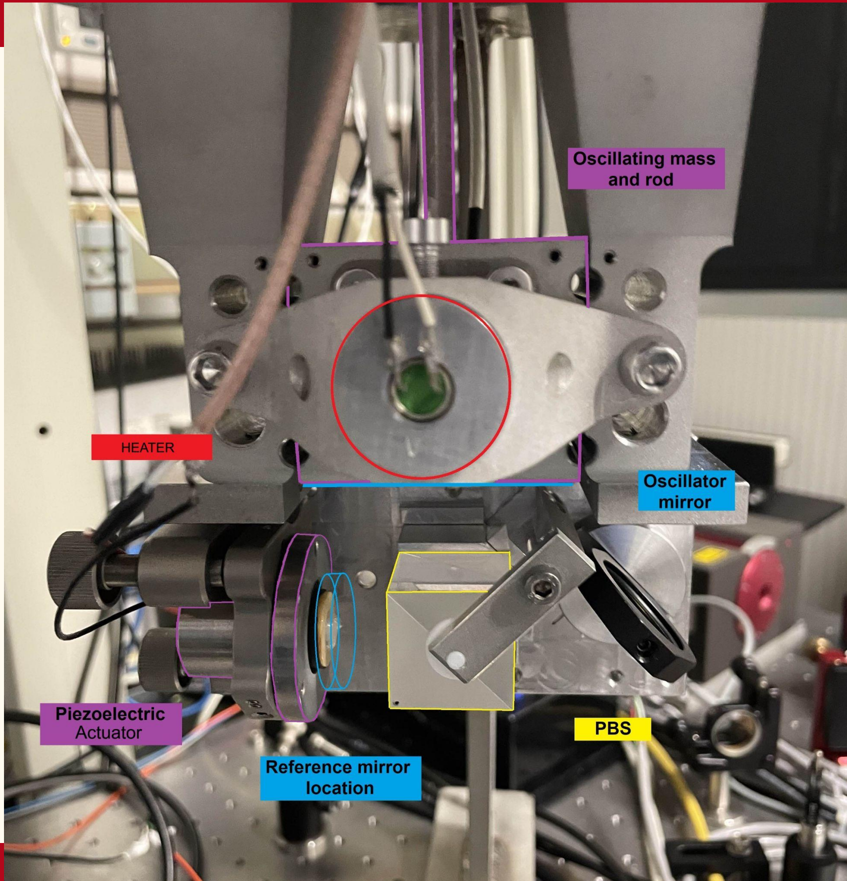
- The fluctuation-dissipation theorem is only valid for describing thermal noise in thermodynamic equilibrium.
- GW interferometers have systems out of thermodynamic equilibrium.
- We want to describe thermal noise out of thermodynamic equilibrium.

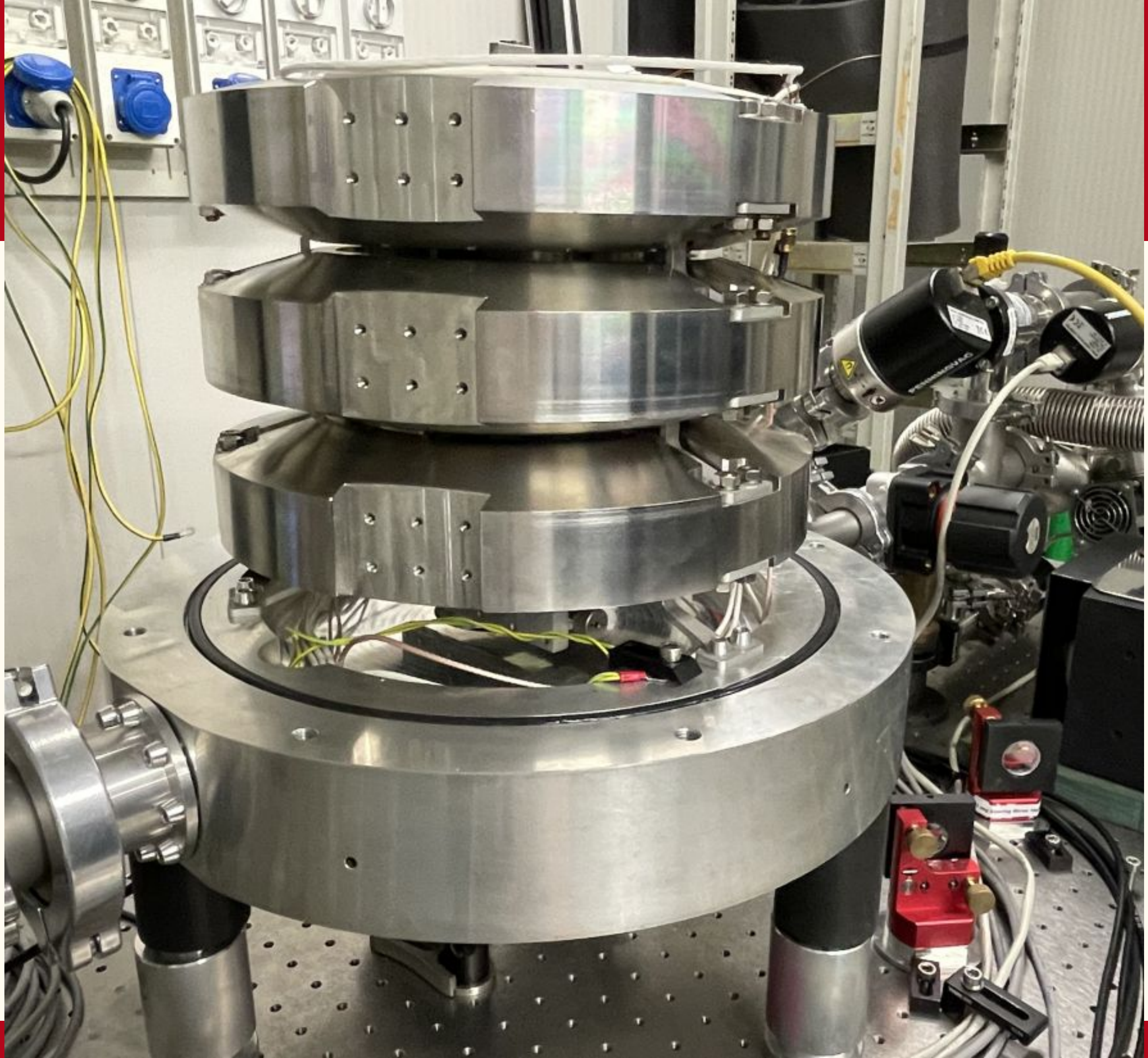
What do we do?: We need to consider an oscillator:

- In thermodynamic equilibrium, where we understand its behavior
- Out of thermodynamic equilibrium, where we have no predicted behavior.

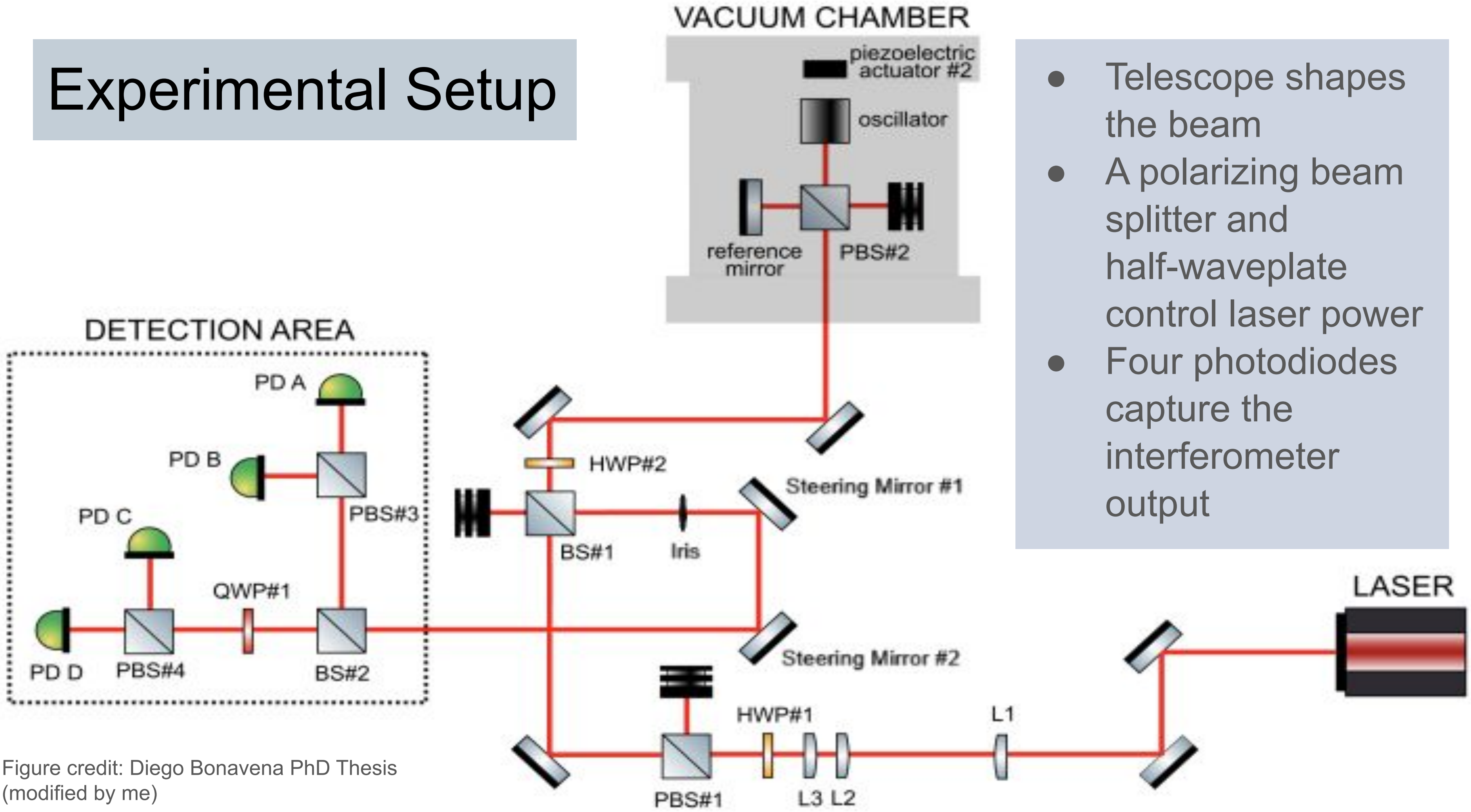
Experimental Setup

- The oscillator is a single machined piece of AL5056 aluminum alloy in the form of 5cm cube mass and a square rod with a 5.5mm cross section with a 100mm rod.
- The resonant frequency of the first longitudinal mode is ~1400Hz.
- Suspended in a vacuum chamber at ~10^{-6} mbar.
- We use a quadrature phase differential interferometer (QPDI) to acquire the interferometric output.



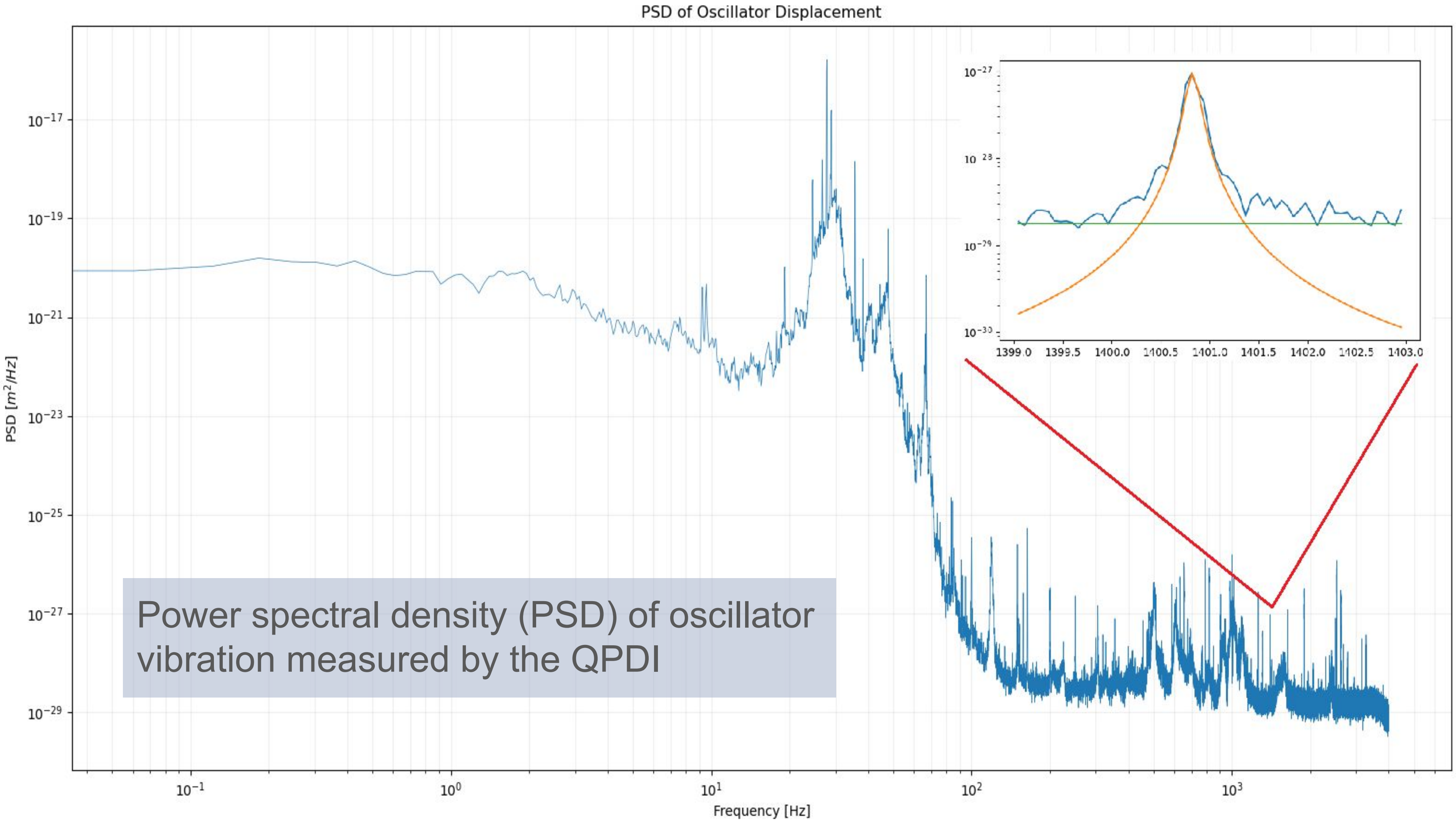


Experimental Setup



- Telescope shapes the beam
- A polarizing beam splitter and half-waveplate control laser power
- Four photodiodes capture the interferometer output

Figure credit: Diego Bonavena PhD Thesis (modified by me)


PSD of Oscillator Displacement
PSD [m^2/Hz]
Frequency [Hz]
10^{-17}
10^{-19}
10^{-21}
10^{-23}
10^{-25}
10^{-27}
10^{-29}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Power spectral density (PSD) of oscillator vibration measured by the QPDI

Effective Temperature as an estimate for Thermodynamic Temperature

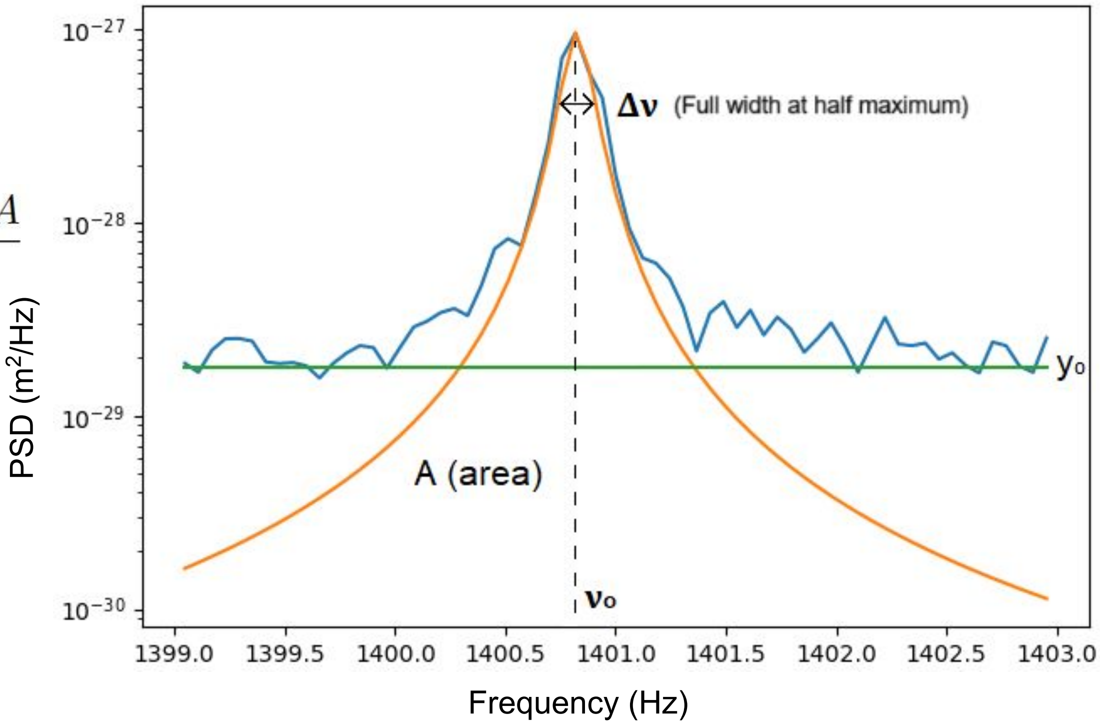
We estimate the amount of thermal energy driving the oscillator using T_{eff}:

$$T_{eff} = \frac{m \cdot (2\pi\nu_0)^2 \langle x^2(t) \rangle}{k_b} = \frac{m \cdot (2\pi\nu_0)^2 A}{k_b}$$

The Lorentzian + flat contribution of the fit of the resonant peak is:

$$y(\nu) = y_0 + \frac{2}{\pi} A \frac{\Delta\nu}{4(\nu - \nu_0)^2 + \Delta\nu^2}$$

T_{eff} should be equal to thermodynamic temperature



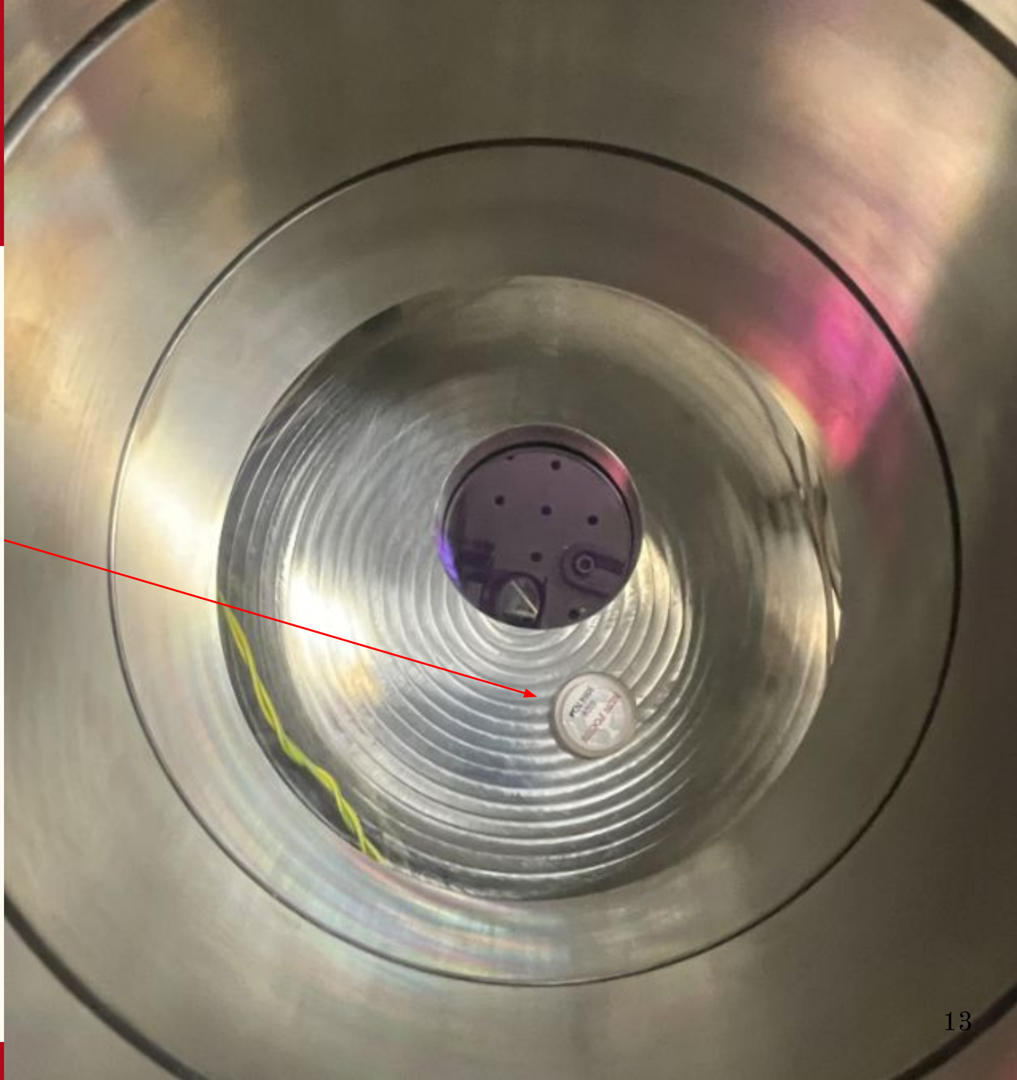
Thesis Data Collection

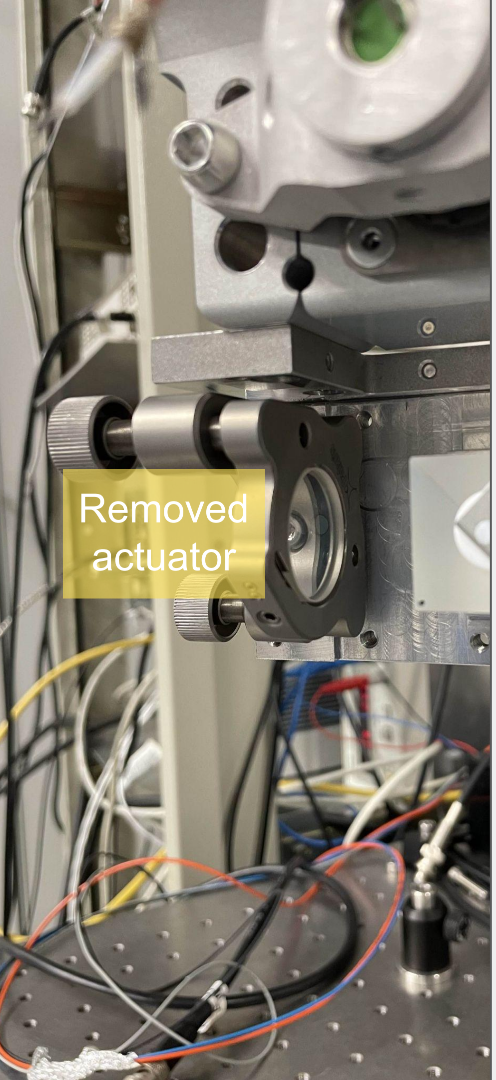
What we know:

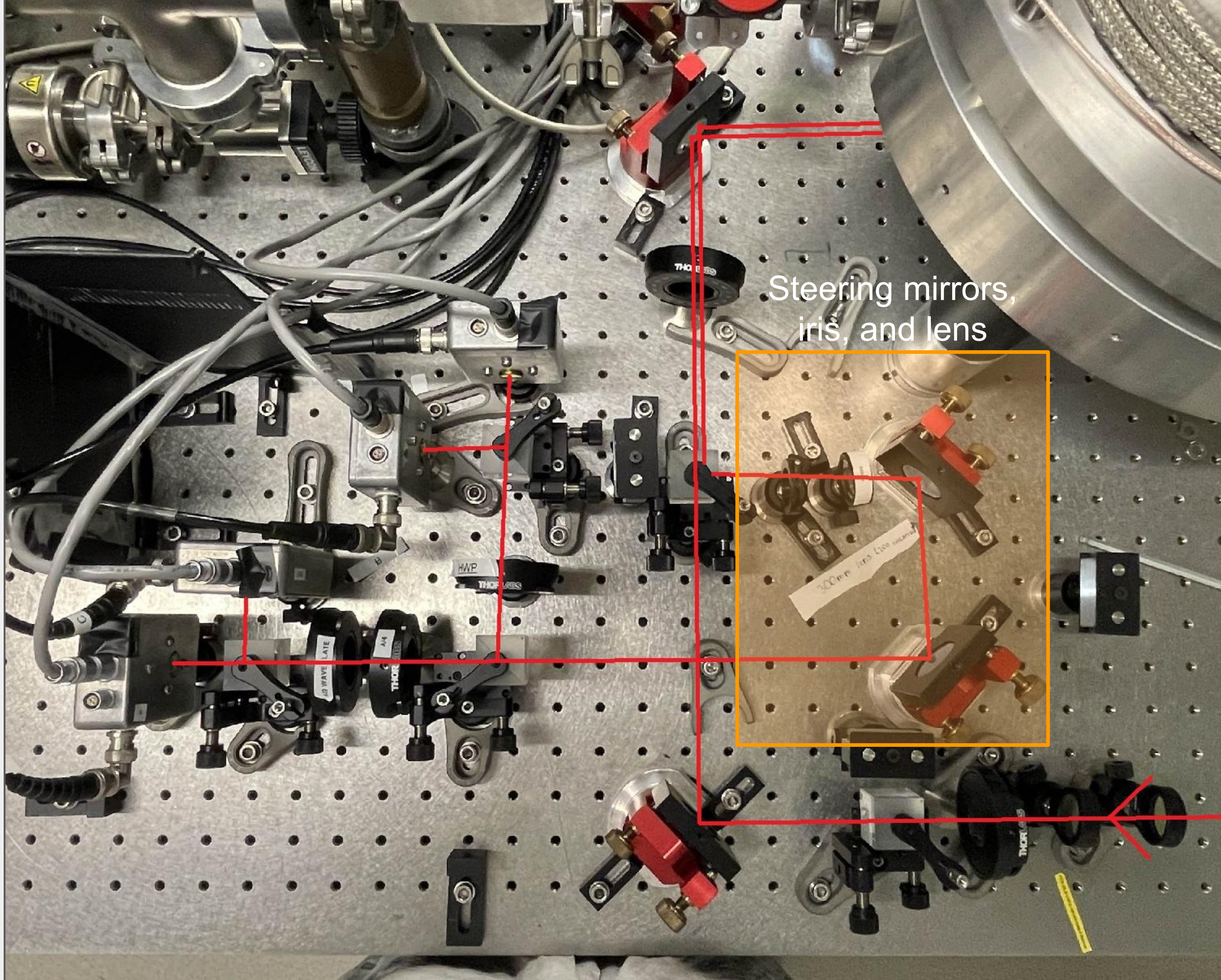
- We measure the thermal energy T_{eff} by measuring the PSD
- Previous work has shown that there is no stationarity of T_{eff}
- We observed that T_{eff} is consistently lower than the measured thermodynamic temperature

Diagnosing Problems

- The vacuum chamber had been opened to adjust the alignment of the reference mirror
- Further disassembly showed that the reference mirror had become improperly attached due to heating, eventually detaching
- Changes were made to the experiment to prevent this from happening again
- This increased the stability of T_{eff}



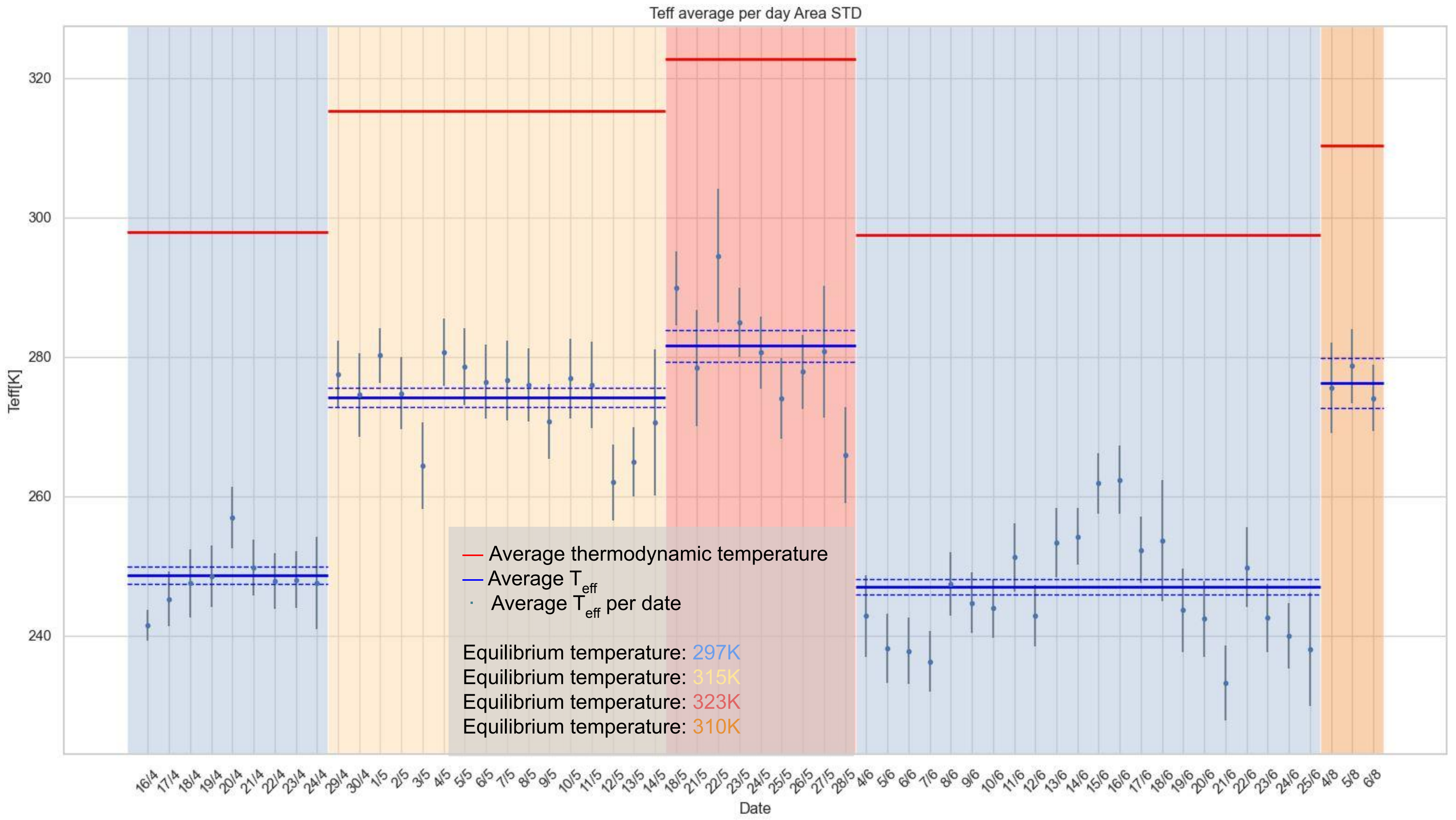

Removed
actuator


Steering mirrors,
iris, and lens
HWP

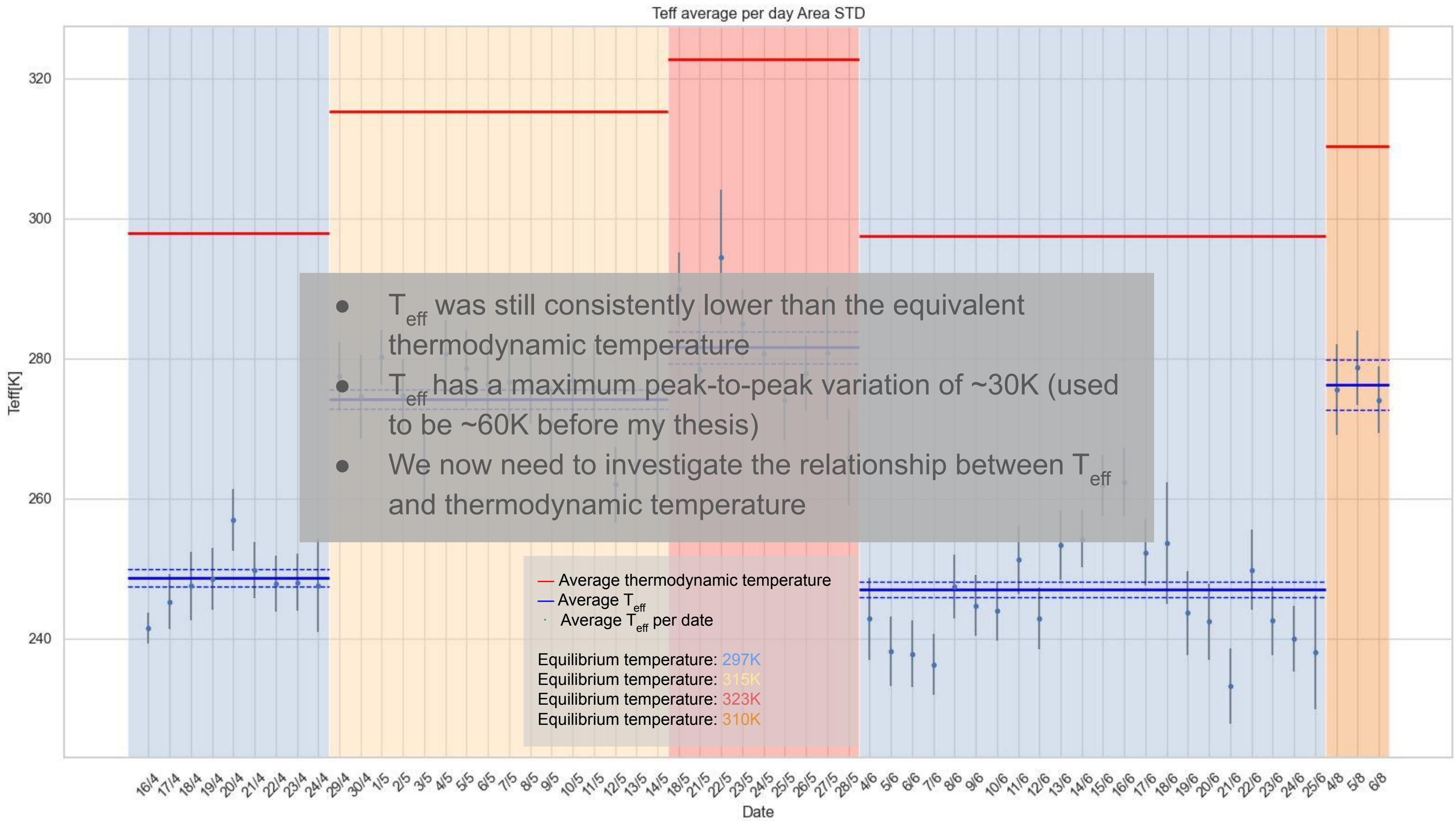
Thesis Data Analysis

What do we do?: Collect data at different thermodynamic temperatures to see how T_{eff} changes.

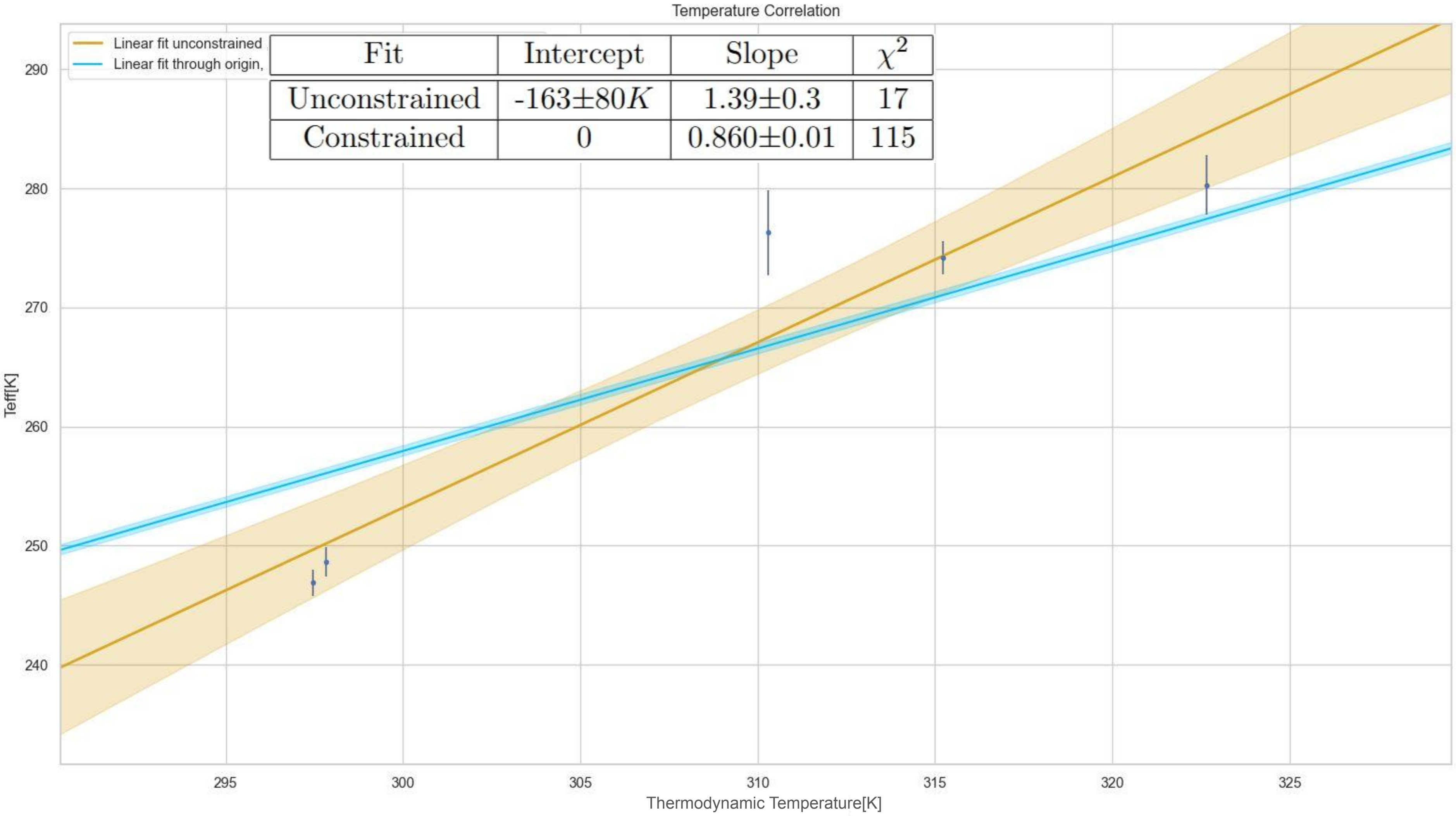
The expectation is for T_{eff} and thermodynamic temperature to be equal; we want to test if our measurements are compatible with this.



Teff average per day Area STD
Teff[K]
Date
320
300
280
260
240
16/4 17/4 18/4 19/4 20/4 21/4 22/4 23/4 24/4 29/4 30/4 1/5 2/5 3/5 4/5 5/5 6/5 7/5 8/5 9/5 10/5 11/5 12/5 13/5 14/5 18/5 21/5 22/5 23/5 24/5 25/5 26/5 27/5 28/5 4/6 5/6 6/6 7/6 8/6 9/6 10/6 11/6 12/6 13/6 14/6 15/6 16/6 17/6 18/6 19/6 20/6 21/6 22/6 23/6 24/6 25/6 4/8 5/8 6/8
Average thermodynamic temperature
Average T_{eff}
Average T_{eff} per date
Equilibrium temperature: 297K
Equilibrium temperature: 315K
Equilibrium temperature: 323K
Equilibrium temperature: 310K



- T_{eff} was still consistently lower than the equivalent thermodynamic temperature
- T_{eff} has a maximum peak-to-peak variation of ~30K (used to be ~60K before my thesis)
- We now need to investigate the relationship between T_{eff} and thermodynamic temperature



Fit	Intercept	Slope	χ^2
Unconstrained	$-163\pm80K$	1.39 ± 0.3	17
Constrained	0	0.860 ± 0.01	115

Conclusions + Future Work

- I improved the experimental setup so that it is significantly easier to align the system, and is less sensitive to thermal distortions
- T_{eff} is more stable than in previous work, ~30K peak-to-peak variation instead of ~60K.
- Investigated relationship between T_{eff} and thermodynamic temperature and found expected behavior within ~2σ of the expectation.
- After this evidence of confirmation of the expectation, we can consider conducting measurements in non-equilibrium states.