Ministero dell'Università e della Ricerca

PRIN — Progetti di Ricerca di Rilevante Interesse Nazionale

Funded by the European Union NextGenerationEU

Bubble wall dynamics at the EW phase transition

Angela Conaci

Università della Calabria & INFN - Cosenza

"Quantum Universe Attract. Workshop"
25/11/2025

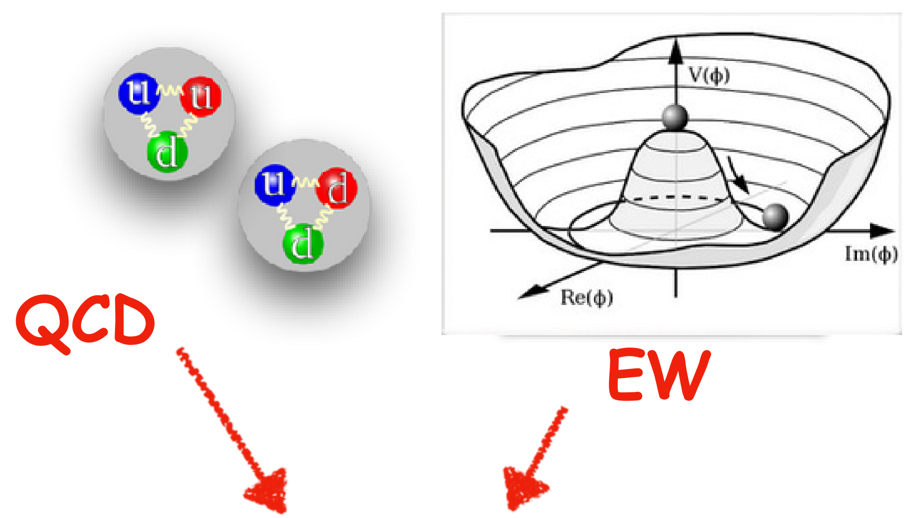
Progetto PRIN 2022 PNRR "SOPHYA"
Progetto PRIN 2022 "Bubble Dynamics in Cosmological Phase Transitions"

In collaboration with L. Delle Rose, C. Branchina, S. De Curtis: arXiv:2504.21213, arXiv:2510.21942

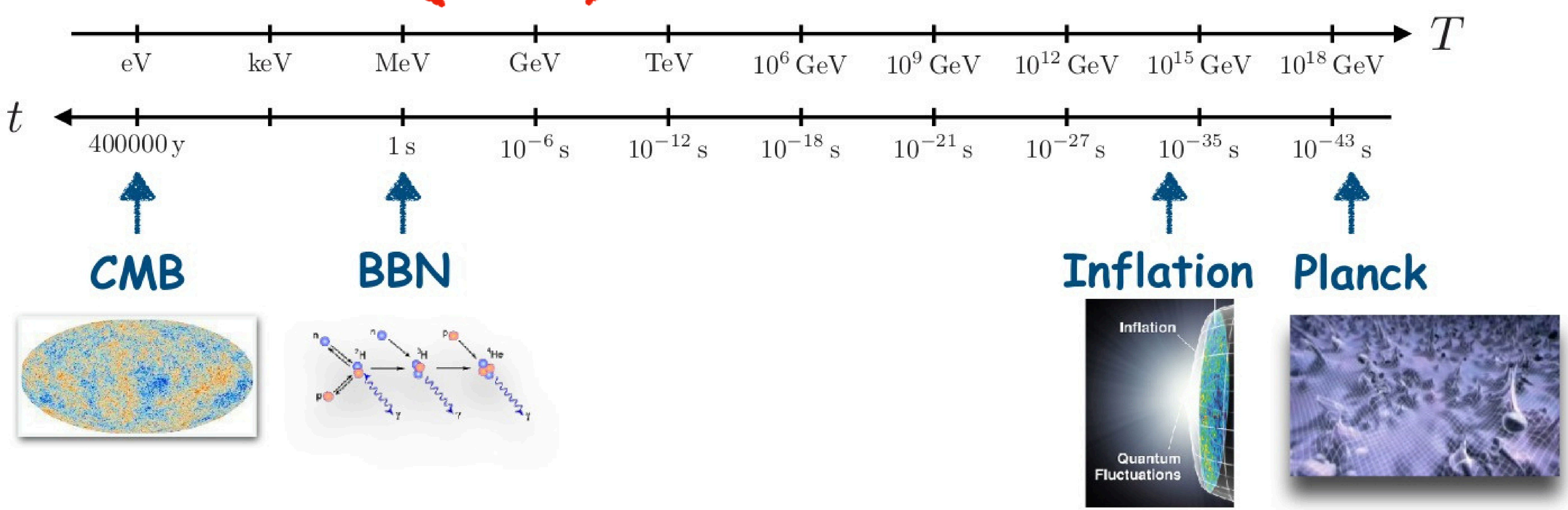
Thermal History of the Universe

Phase transitions are important events in the evolution of the Universe

- the SM predicts two of them (crossover)



no strong breaking of thermal equilibrium
no distinctive experimental signatures

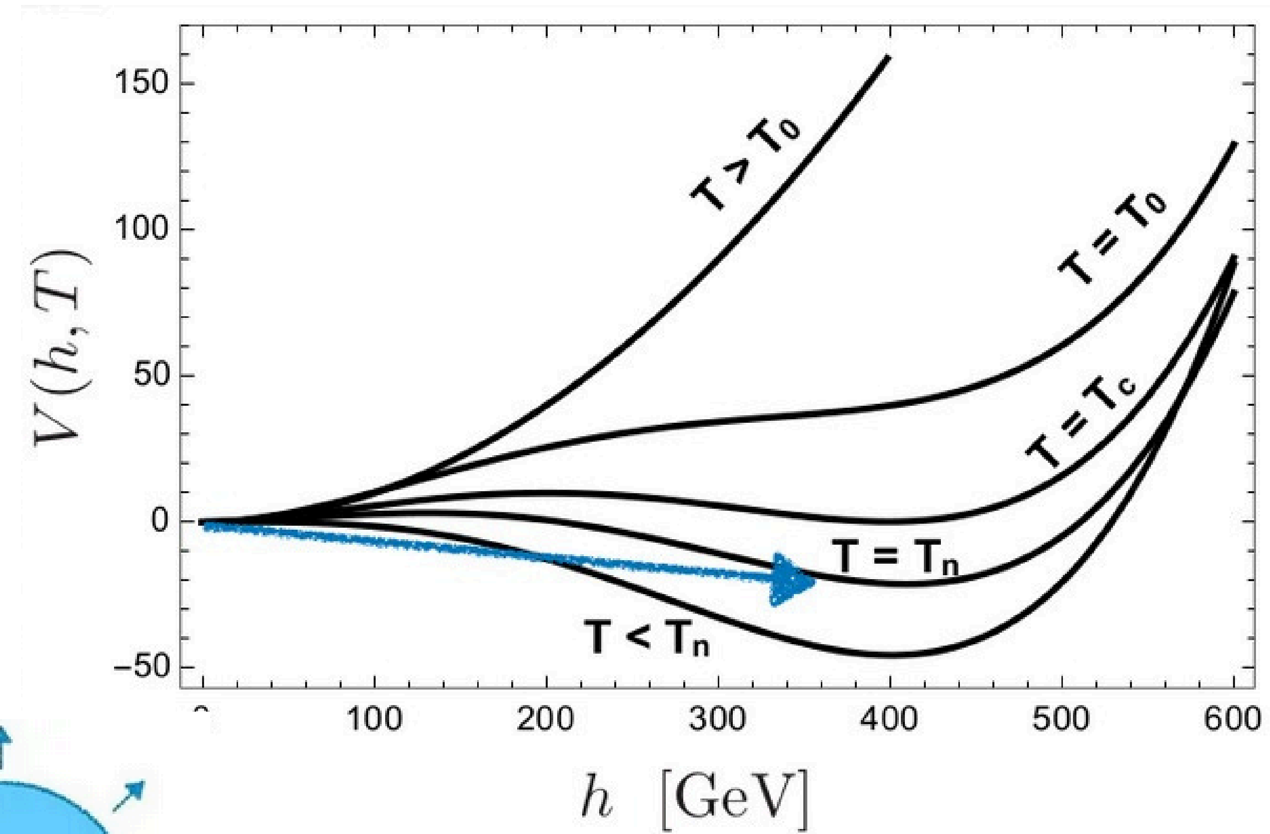


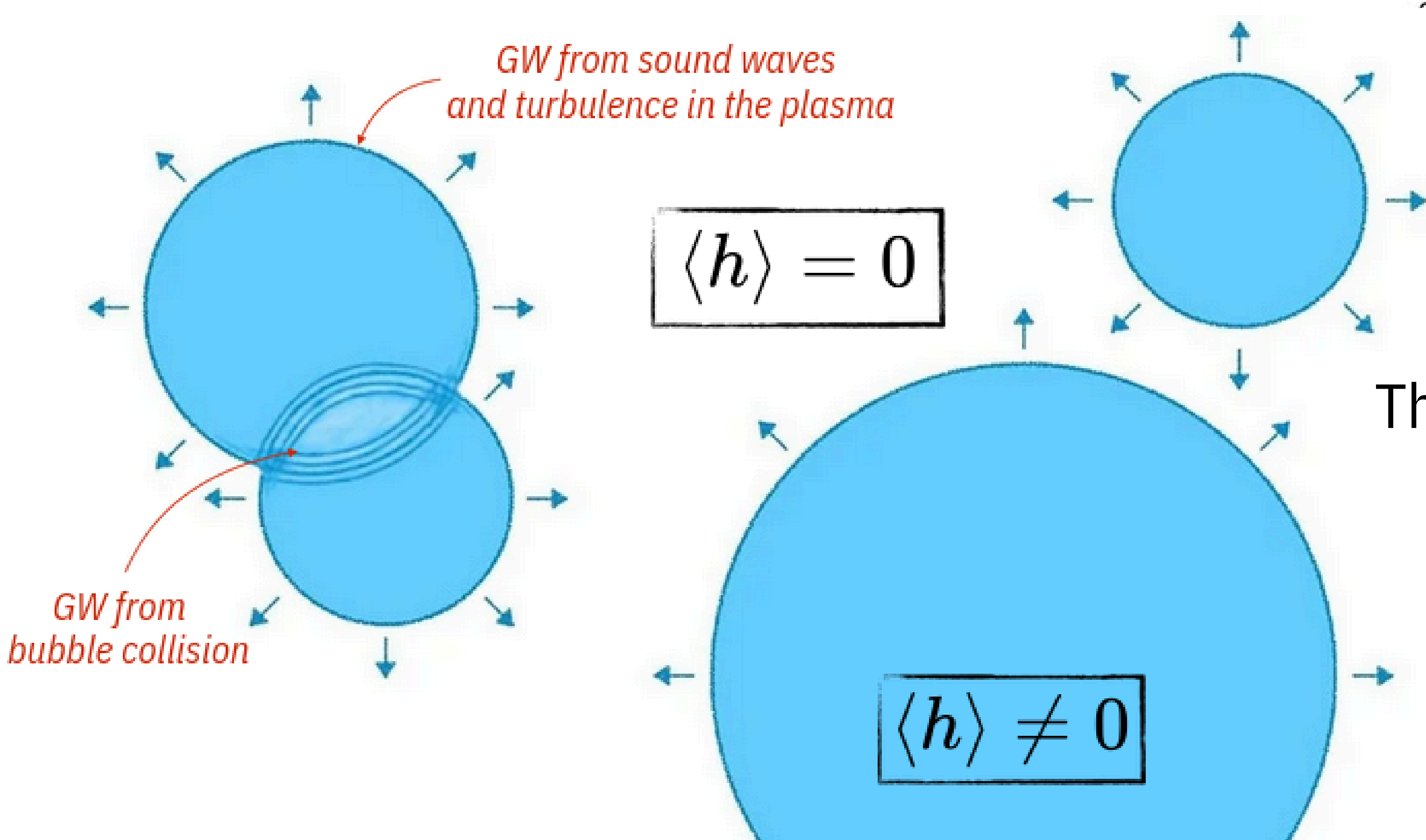
EWPhT and new physics

New physics may provide **first order** phase transitions

- h field **tunnels** from false to true minimum at $T_n < T_c$

- transition proceeds through **bubble nucleation**





The bubble wall in the plasma can produce

1. **breaking of thermal equilibrium** (BG)
2. **experimental signatures** (GW)

Toy example: SSM tree-level potential

Higgs + singlet scalar potential (Z2 symmetric)
in the high-temperature limit

$$V(h,s,T) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_s^2}{2}s^2 + \frac{\lambda_s}{4}s^4 + \frac{\lambda_{hs}}{4}h^2 s^2 + \left(c_h \frac{h^2}{2} + c_s \frac{s^2}{2}\right)T^2$$

important to create a barrier in the potential

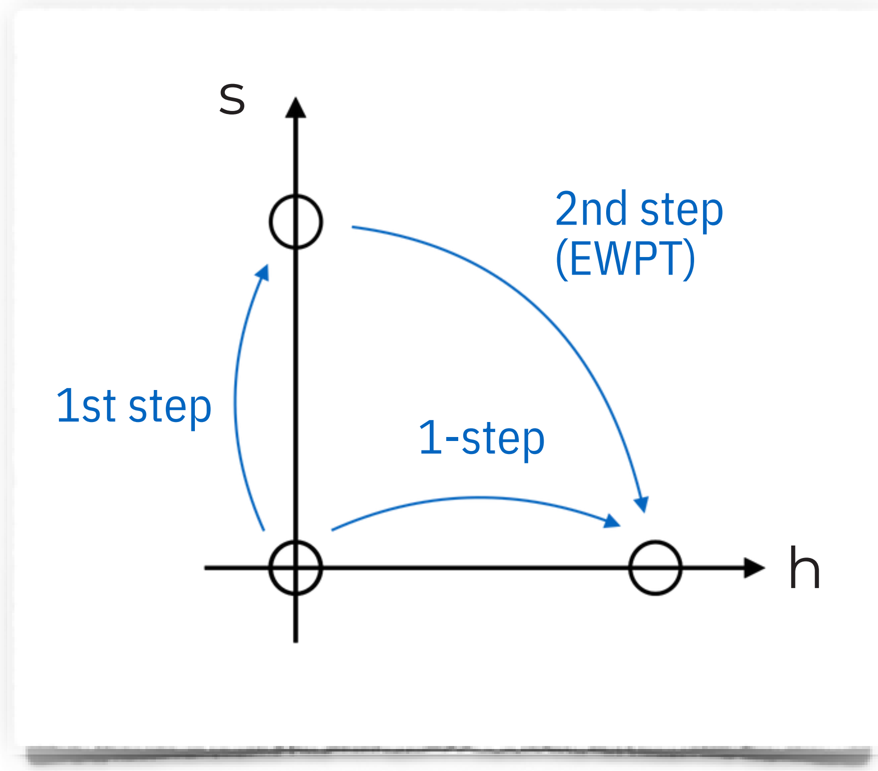
with thermal masses

$$c_h = \frac{1}{48}(9g^2 + 3g'^2 + 12y_t^2 + 24\lambda_h + \lambda_{hs}) \qquad c_s = \frac{1}{12}(2\lambda_{hs} + \lambda_s)$$

- Two interesting patterns of symmetry breaking (as the Universe cools down)

1-step PhT $(0,0) \to (v,0)$

2-step PhT $(0,0) \to (0,w) \to (v,0)$



- 2-step naturally realised since singlet is destabilised before the Higgs

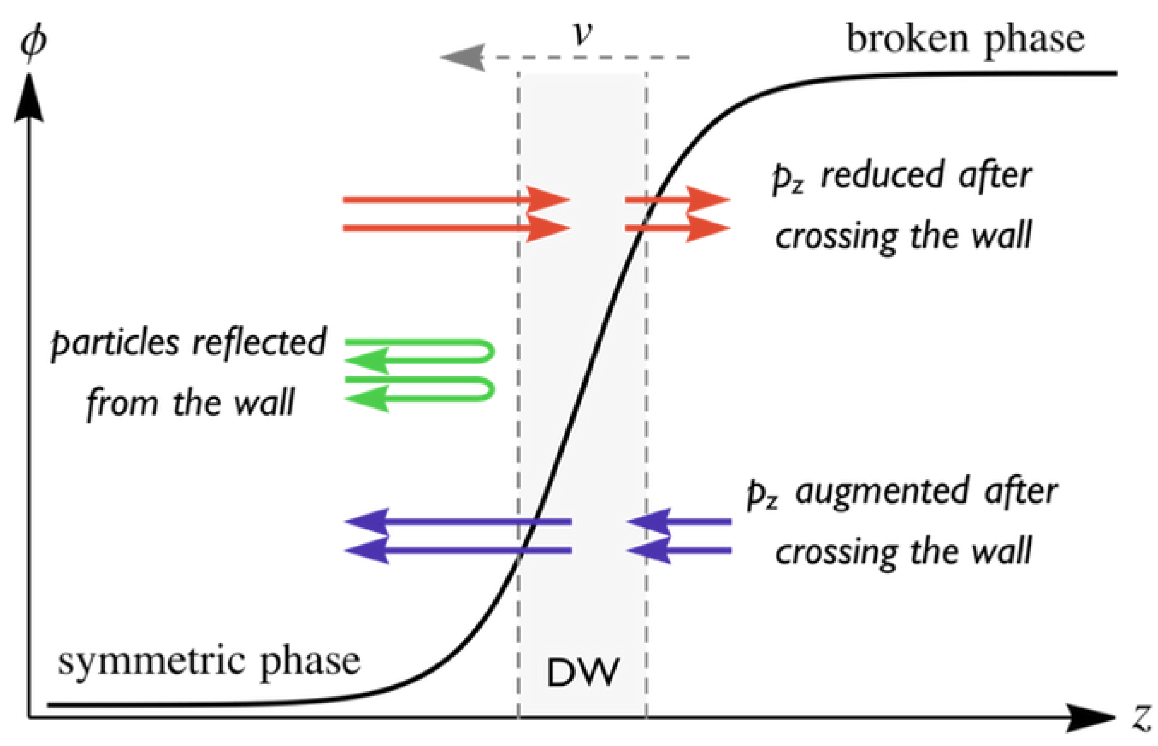
Dynamics of the bubble wall

Plasma conveniently described as a mixture of **three components**

1. Scalar fields participating in the transition
2. Background species: ~ LTE
3. Species directly coupled to the scalars: OOE contributions relevant

- Scalar field EOMs

$$\phi' \Box \phi - V_T' = \sum N_i \frac{dm^2}{dz} \int \frac{d^3p}{(2\pi)^3 2E_p} \delta f(p)$$



We assume a planar wall and a steady state regime

Dynamics of the bubble wall

Plasma conveniently described as a mixture of **three components**

1. Scalar fields participating in the transition
2. Background species: ~ LTE
3. Species directly coupled to the scalars: OOE contributions relevant

- ## Hydrodynamics of the plasma

Equations for the plasma are obtained from the **conservation of the stress-energy tensor**

$$T^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{eq} + T^{\mu\nu}_{out}$$

scalar field; plasma at equilibrium; plasma out of equilibrium

$$\partial_z T^{zz} = \partial_z T^{z0} = 0$$ planar limit

Hydrodynamic equations provide solutions to the **temperature and velocity profiles**

$$T^{30} = w\gamma^2 v + T^{30}_{\mathrm{OOE}} = c_1$$

$$T^{33} = \frac{(\partial_z\phi)^2}{2} - V(\phi, T) + w\gamma^2 v^2 + T^{33}_{\mathrm{OOE}} = c_2$$

Constants determined from the hydrodynamic regimes: Detonation, Deflagration or Hybrid.

Dynamics of the bubble wall

Plasma conveniently described as a mixture of **three components**

1. Scalar fields participating in the transition
2. Background species: ~ LTE
3. Species directly coupled to the scalars: OOE contributions relevant

- The Boltzmann equation

$$\left(\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right)(f_v + \delta f) = -C[f_v + \delta f]$$

Assumptions on the plasma:

1. High temperature, **weakly coupled** plasma
2. Only **2 - 2 processes** are considered
3. Plasma made of **Top quark** and **W/Z bosons** (other SM particles are assumed to be in LE)

The Collision Term

The collision term is the challenging part of the Boltzmann Equation

$$C[f_v + \delta f] = \frac{1}{4N_i E_i} \sum_j \int \frac{d^3k\, d^3p'\, d^3k'}{(2\pi)^5 2E_k\, 2E_{p'}\, 2E_{k'}} |\mathcal{M}_j|^2 \mathcal{P}[f_v + \delta f]\, \delta^4(p + k - p' - k')$$

The Boltzmann Equation is an **integro-differential** equation

Typical setup:

- friction contributions from the top quark and W/Z bosons
- background is not perturbed
- infrared divergences regularised by thermal masses
- only leading-log terms are considered

Examples of processes:

process	$\lvert\mathcal{M}\rvert^2$
$t\bar{t} \to gg$	$\frac{128}{3} g_s^4 \left[\frac{ut}{(t - m_q^2)^2} + \frac{ut}{(u - m_q^2)^2} \right]$
$tg \to tg$	$-\frac{128}{3} g_s^4 \frac{su}{(u - m_q^2)^2} + 96 g_s^4 \frac{s^2 + u^2}{(t - m_g^2)^2}$
$tq \to tq$	$160 g_s^4 \frac{s^2 + u^2}{(t - m_g^2)^2}$

Determination of the wall speed

- we solve the EOM of the scalar fields

$$E_h = -\partial_z^2 h + \frac{\partial V}{\partial h} + \frac{F(z)}{h'} = 0$$
$$E_s = -\partial_z^2 s + \frac{\partial V}{\partial s} = 0$$

approx solutions $\longrightarrow$

$$h(z) = \frac{h_-}{2}\left(1 + \tanh\left(\frac{z}{L_h}\right)\right)$$
$$s(z) = \frac{s_+}{2}\left(1 + \tanh\left(\frac{z}{L_s} + \delta s\right)\right)$$

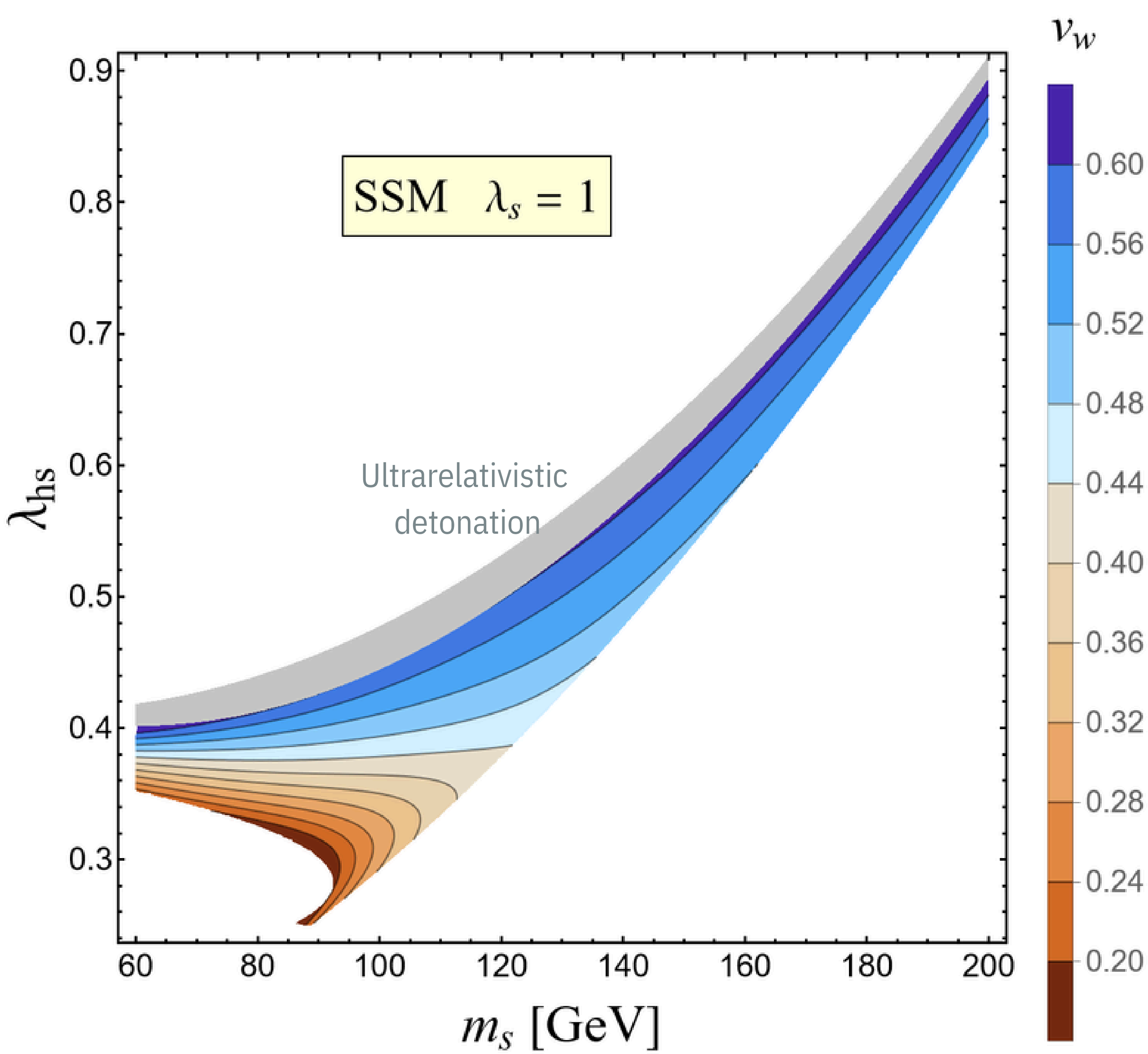
four parameters to be determined: v_w, L_h, L_s, δ_s

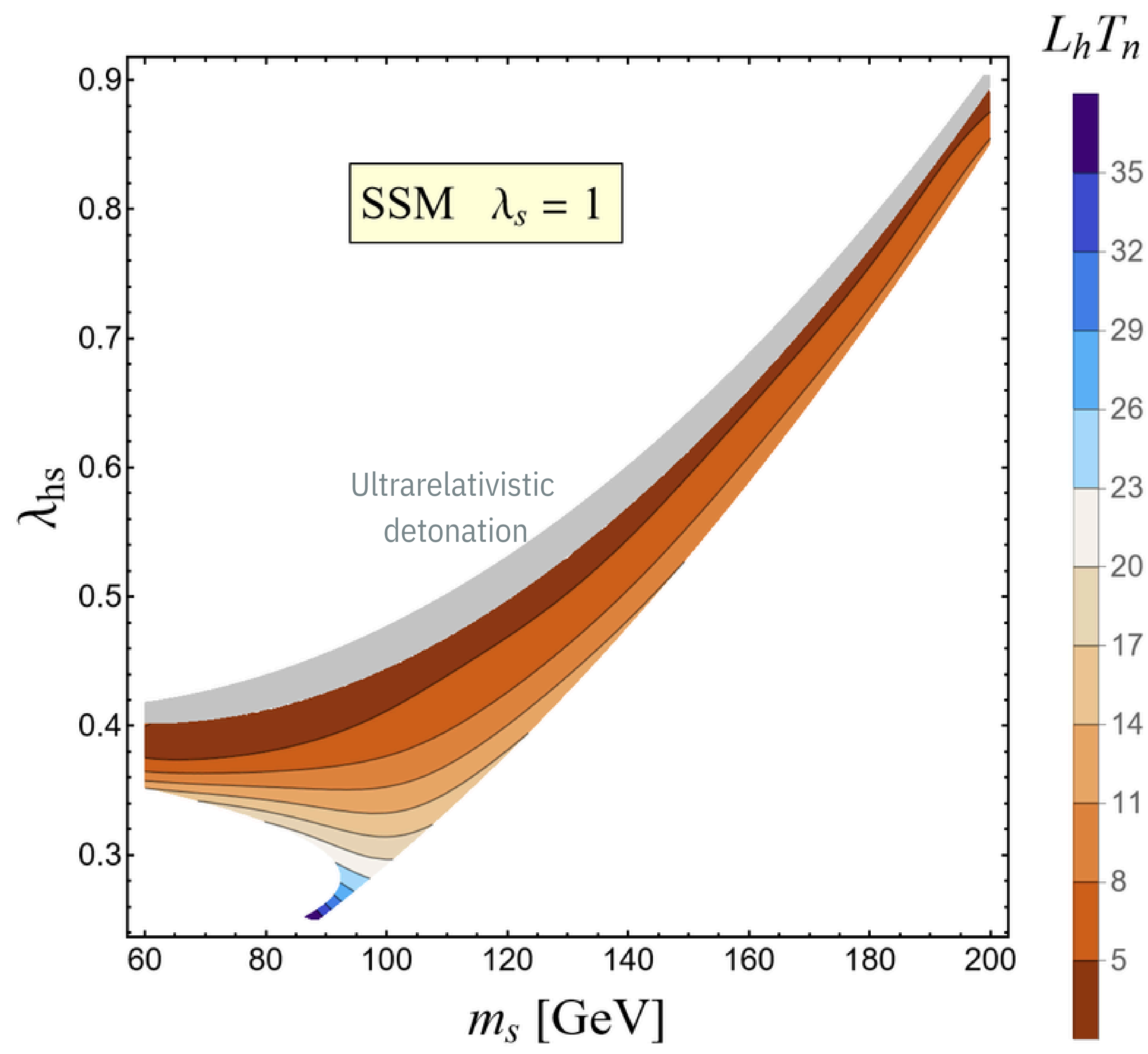
- the four parameters are determined by taking momenta of the EOMs

$$P_h = \int dz E_h h' = 0, \quad G_h = \int dz E_h \left(\frac{2h}{h_-}\right) h' = 0,$$
$$P_s = \int dz E_s s' = 0, \quad G_s = \int dz E_s \left(\frac{2s}{s_+}\right) s' = 0.$$

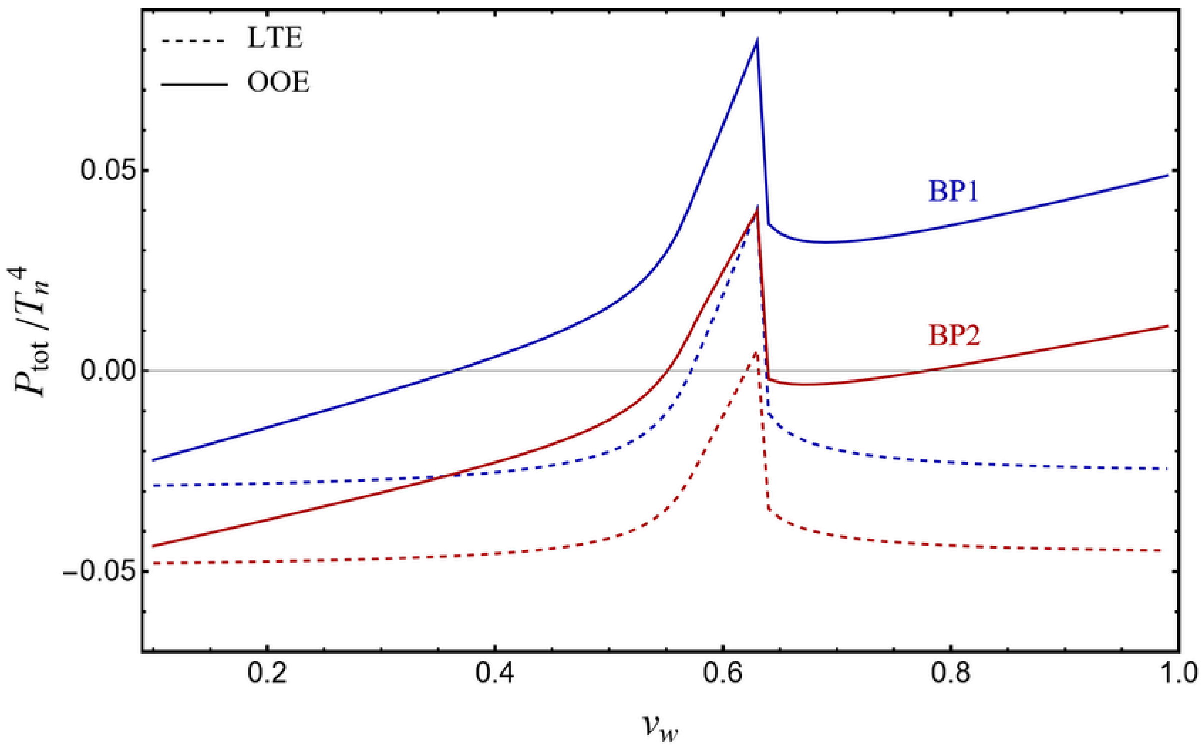
Numerical results

- Computation of bubble wall velocity, including OOE contributions, and width for the SSM scenario





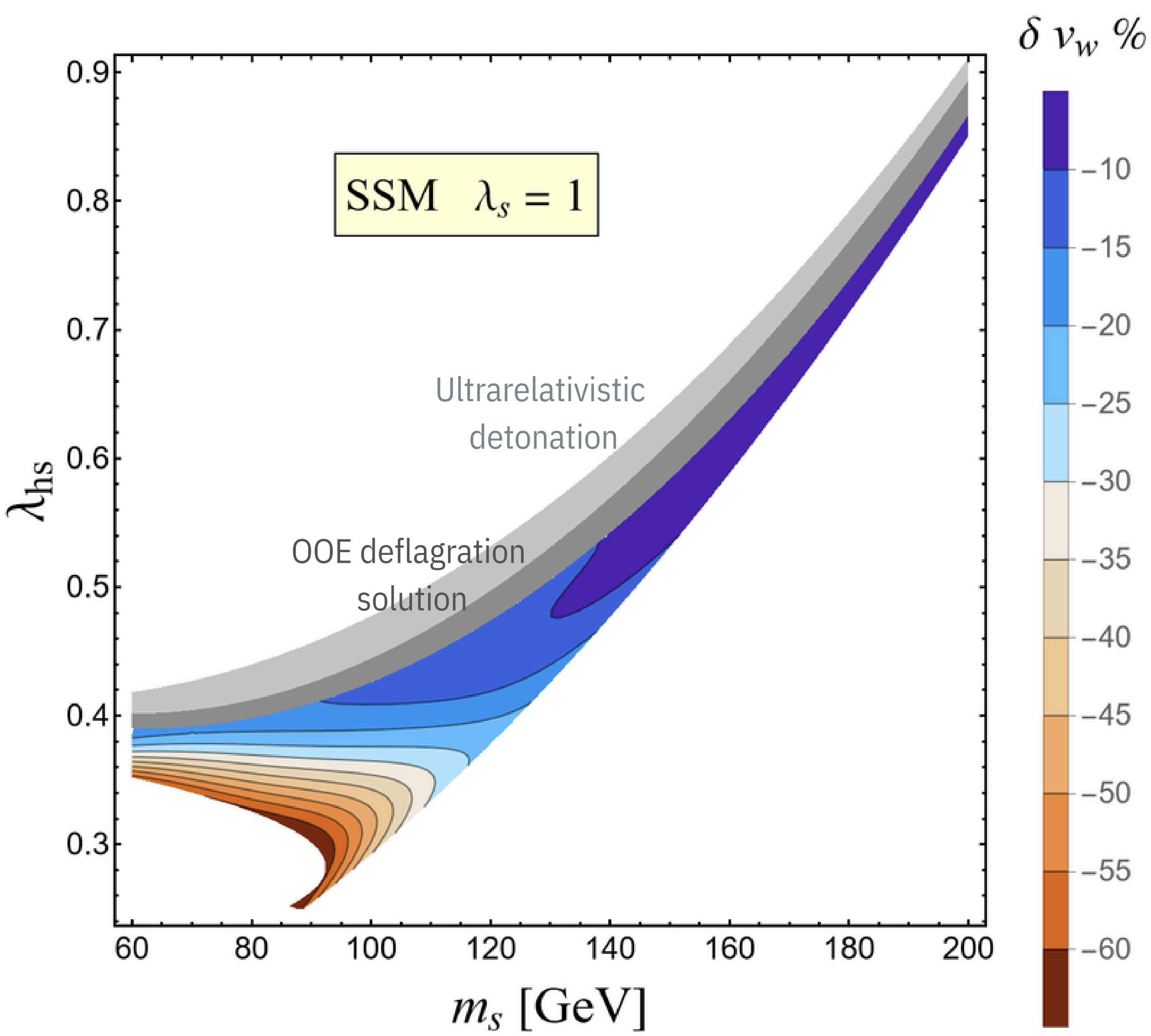
Numerical results



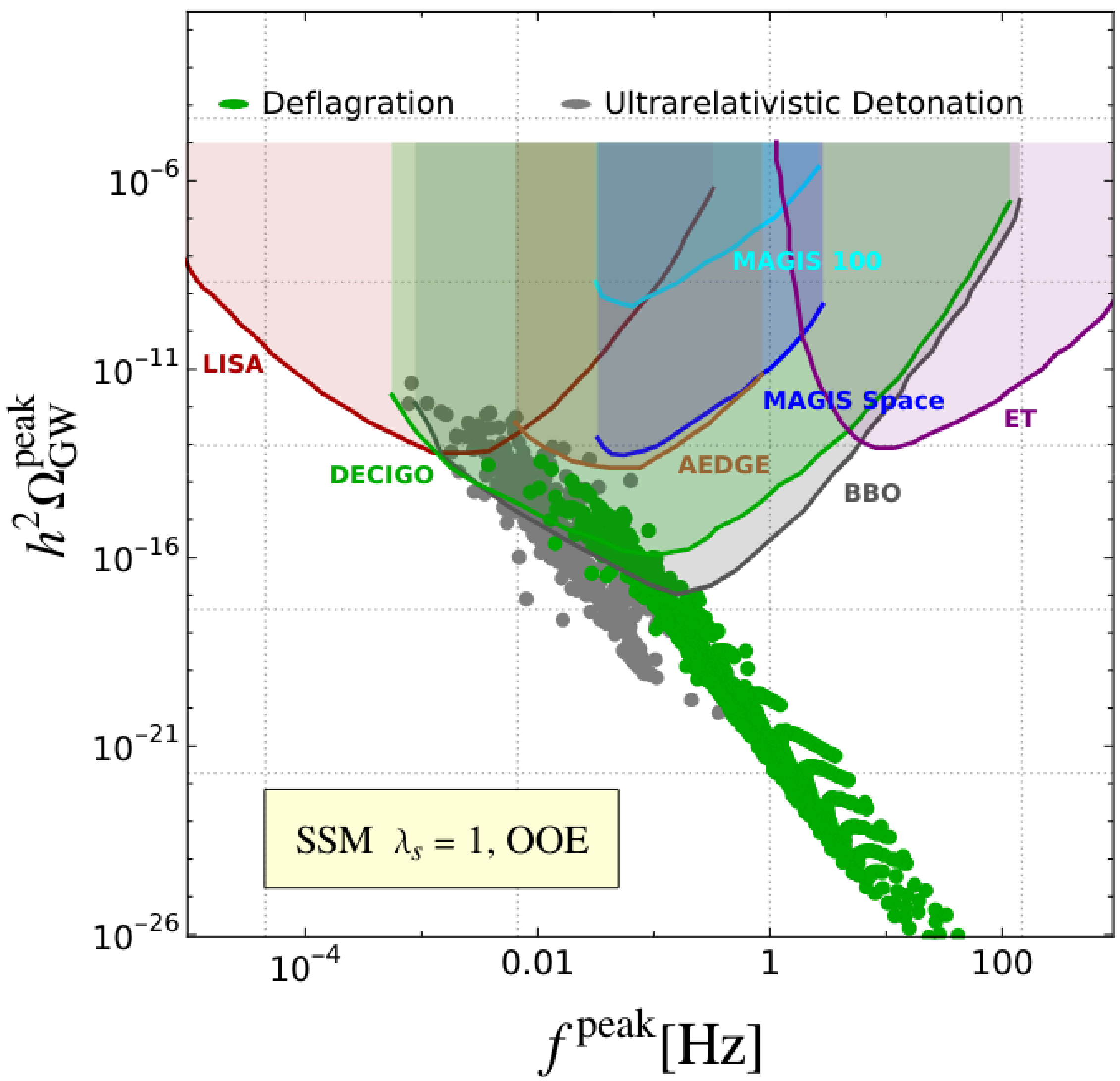
- The behavior of the **total pressure** changes following the inclusion of the OOE contributions

$$P_{\mathrm{tot}} = P_h + P_s$$

- The **percentage difference** between the bubble wall velocity calculation in the LTE regime and OOE, respectively.



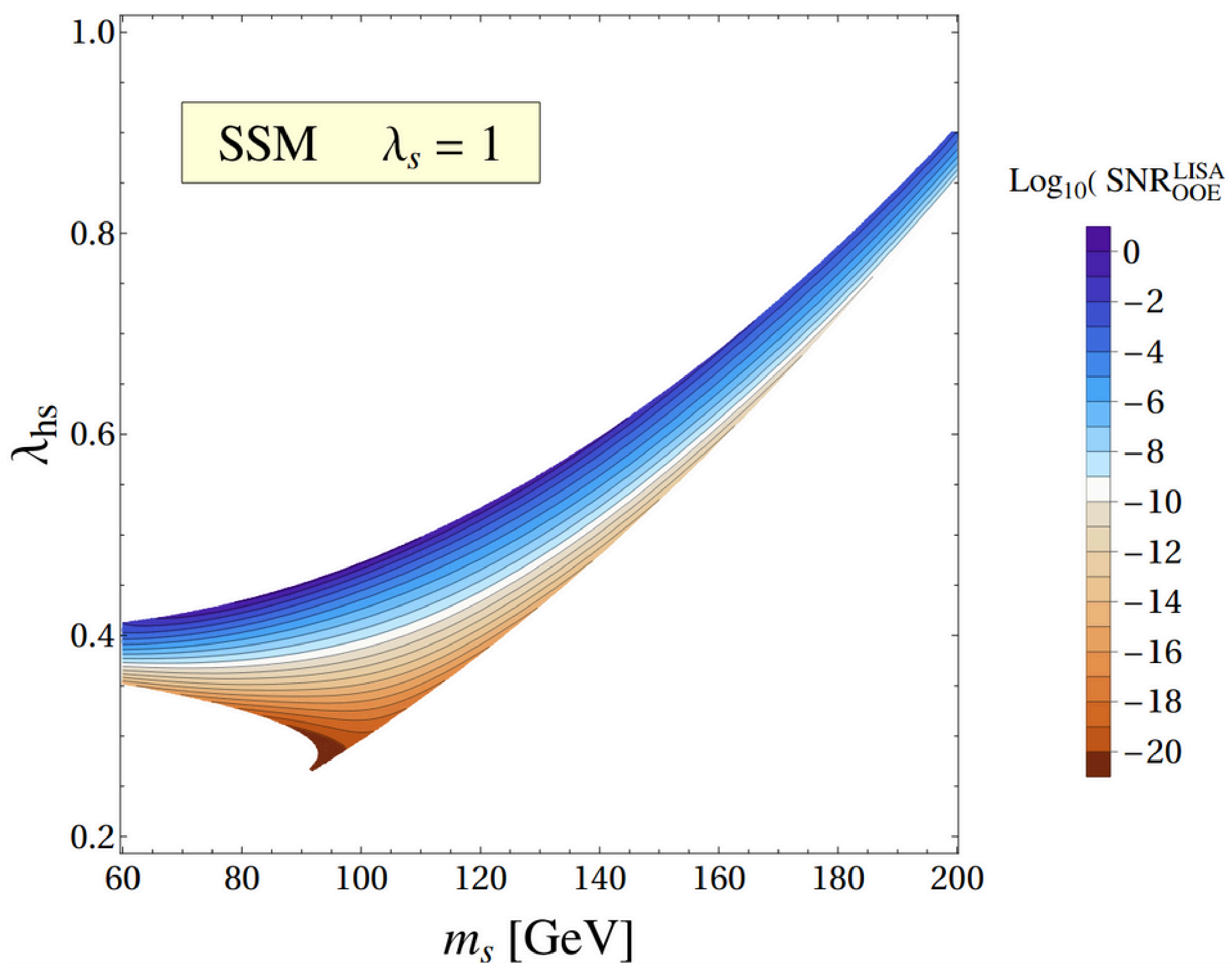
Applications: GW spectrum



The resulting GW signal is dominated by sound wave contributions, and the **spectrum at the peak** is given by

$$h^2\Omega_{\rm sw} = 2.56\times10^{-6}\left(\frac{k_{sw}\alpha_n}{1+\alpha_n}\right)^2\left(\frac{H}{\beta}\right)\left(\frac{100}{3}\right)^{\frac{1}{3}}(H\tau_{sw})v_{sw}$$

$$f_{\rm sw} = 1.9\times10^{-5}{\rm Hz}\left(\frac{1}{v_w}\right)\left(\frac{\beta}{H}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}\left(\frac{T_n}{100\ {\rm GeV}}\right)$$



To assess the detectability of the predicted GW background, we computed the **signal to noise ratio**

$$\text{SNR} = \sqrt{t_{\rm obs}\int_{f_{\rm min}}^{f_{\rm max}} df\left[\frac{h^2\Omega_{\rm GW}(f)}{h^2\Omega_{\rm Sens}(f)}\right]^2}$$

$$\Omega_{\rm Sens}(f) = \frac{2\pi^2}{3H_0^2}f^3S_h(f)$$

Conclusions and outlook

Conclusions:

- First order EWPT: theoretically and experimentally compelling
- Strategy put forward to provide full solution of the (steady-state) wall dynamics
- In LTE + OOE: complete solution in the parameter space of SSM models
- OOE effects have a significant impact on the wall dynamics across the parameter space
- The determination of the SNR of GW has shown that OOE contributions reduce the probability of observation with respect to the LTE estimate

Outlook:

- Study of the effects of W bosons in the collision integral
- Evaluation of the effect of velocity and wall width in the calculation of baryogenesis

Back up slides

Key features of a first-order PhT

- the nucleation temperature T_n
- the strength α
- the (inverse) time duration of the transition β/H

} *equilibrium quantities*

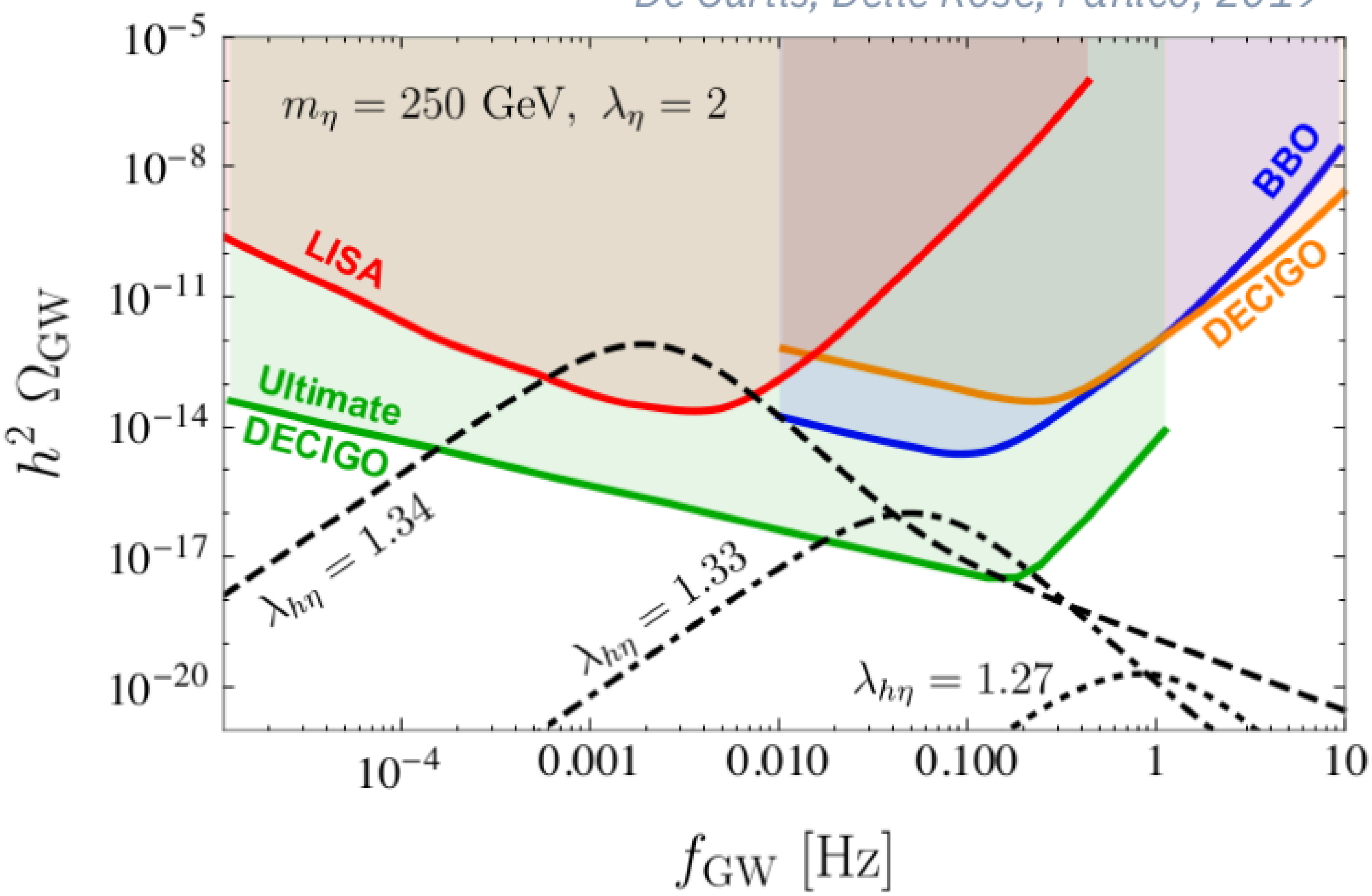
- the speed of the bubble wall v_w
- the thickness of the bubble wall L_w

} *non-equilibrium quantities*

GW from a first-order PhT

First-order PhTs produce stochastic background of gravitational waves

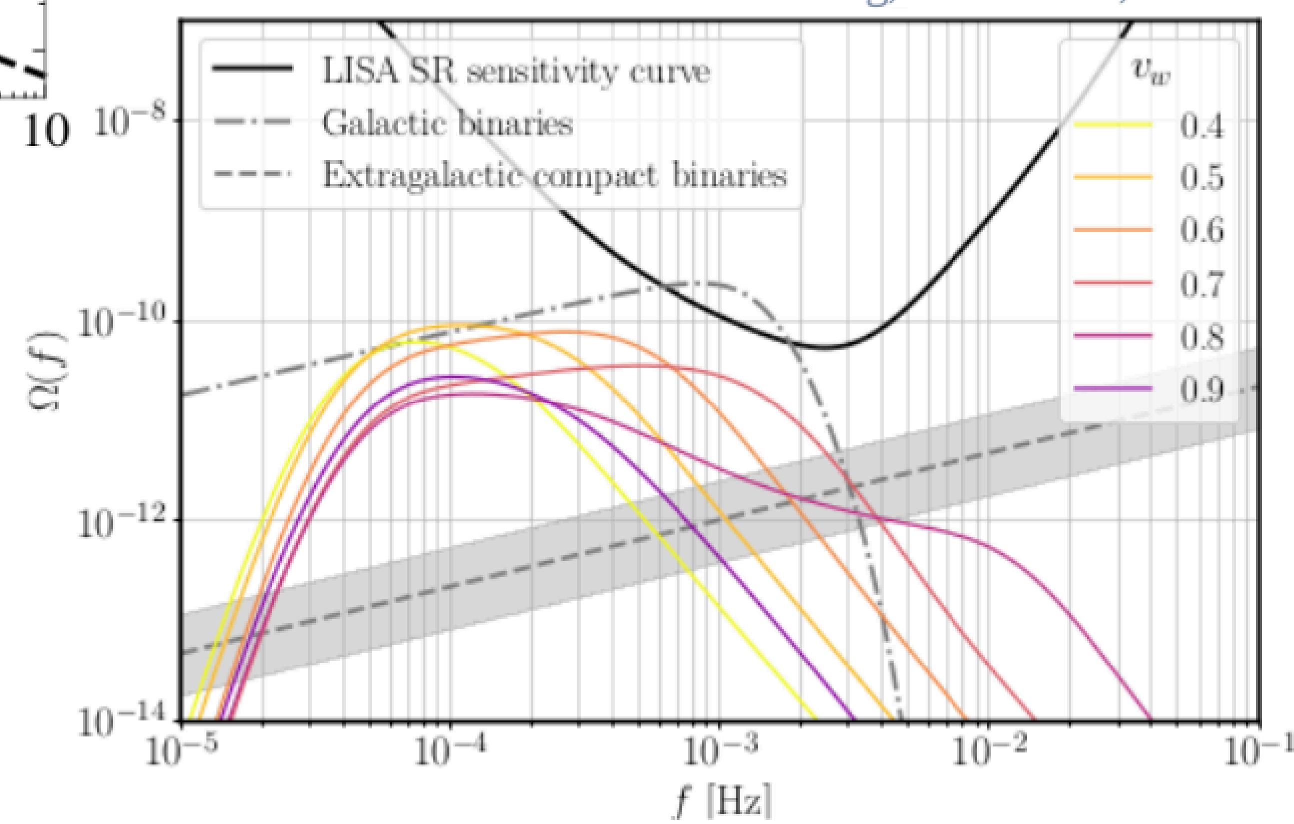
De Curtis, Delle Rose, Panico, 2019



for the EWPhT the peak frequency is within the range of future experiments

- wall speed has a strong effect on the shape of the power spectrum
- wall speed will be the best determined parameter

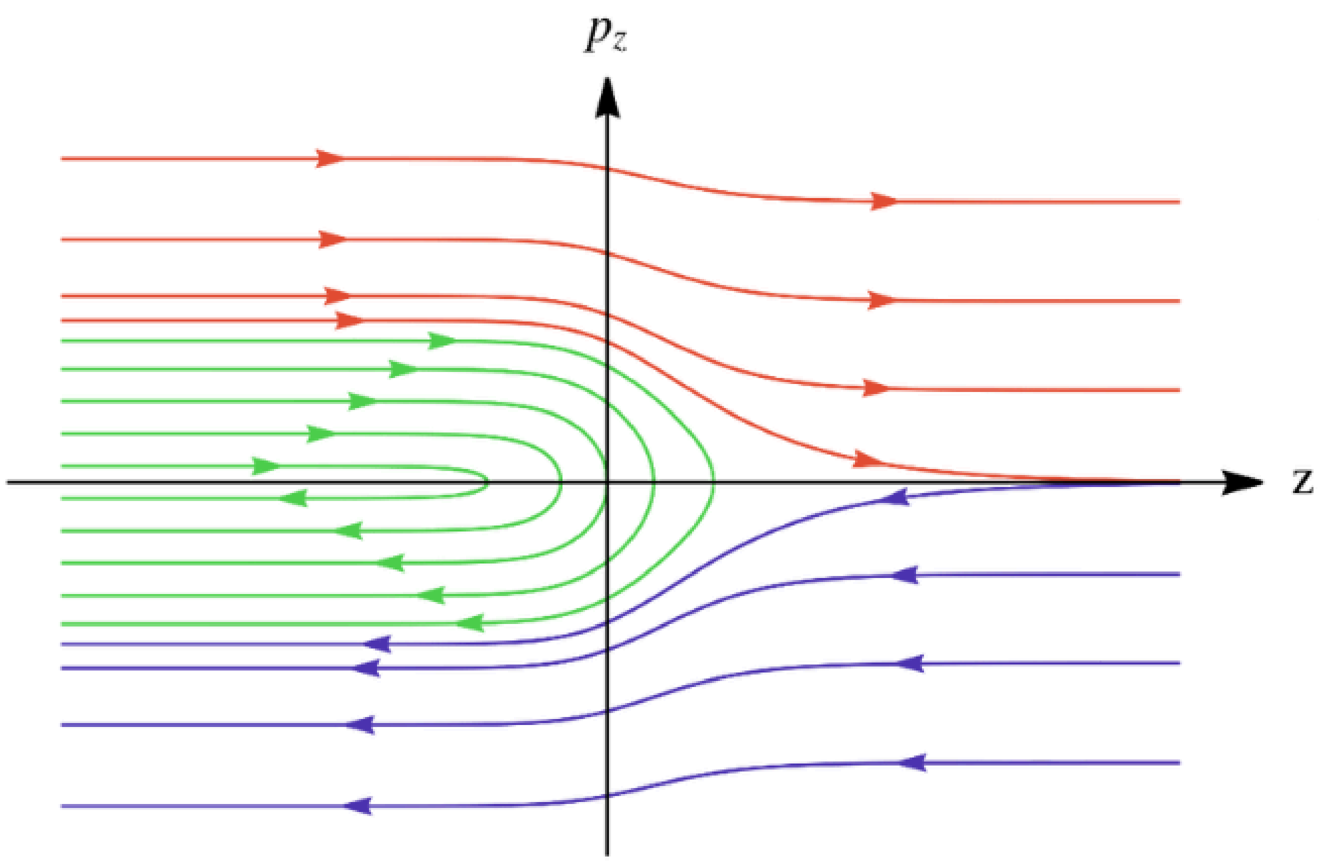
Gowling, Hindmarsh, 2019

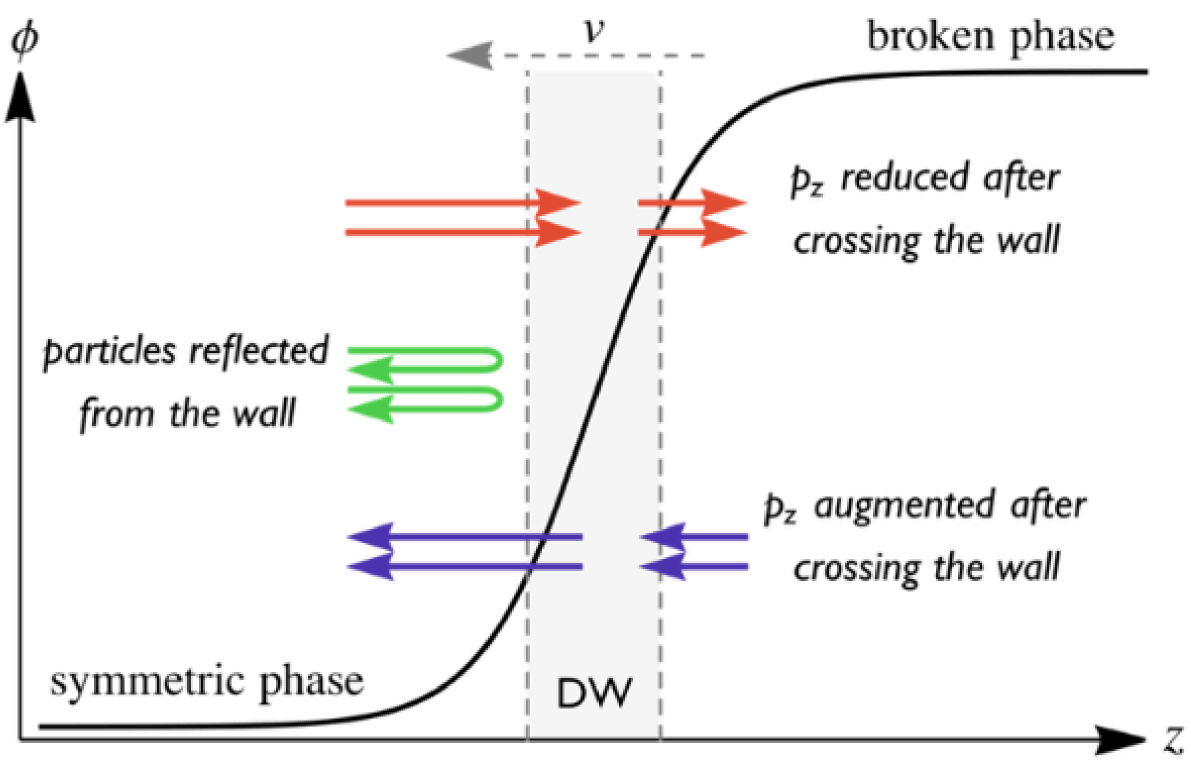


The Liouville operator

Liouville operator is a derivative along flow paths

$$\mathcal{L}[f] = \left(\frac{p_z}{E}\partial_z - \frac{(m_z^2)'}{2E}\partial_{p_z}\right) f \longrightarrow \frac{p_z}{E}\frac{df}{dz}$$





$E, \; p_\perp$ and $c = \sqrt{p_z^2 + m_z^2}$ are conserved along the flow paths

Structure of the collision integral

The linearised collision integral

$$\bar{\mathcal{C}}[\delta f_i] = \frac{1}{2N_i E_i}\sum_j \int \frac{d^3k\, d^3p'\, d^3k'}{(2\pi)^5\, 2E_k 2E_{p'} 2E_{k'}} |\mathcal{M}_j|^2 \bar{\mathcal{P}}[f] \delta^4(p+k-p'-k')$$

the population factor

$$\bar{\mathcal{P}}[f] = f_v(p) f_v(k)(1 \pm f_v(p'))(1 \pm f_v(k')) \sum \mp \frac{\delta f}{f_v'}$$

the collision integral yields two classes of terms:

$$\bar{\mathcal{C}}[\delta f] = \mathcal{Q}\frac{\delta f}{f_v'(p)} + (\langle \delta f(k)\rangle - \langle \delta f(p')\rangle - \langle \delta f(k')\rangle)$$

- the perturbation does not appear inside the integral: easy to handle
- perturbation is integrated (*bracket*): very challenging

Full solution to the Boltzmann equation

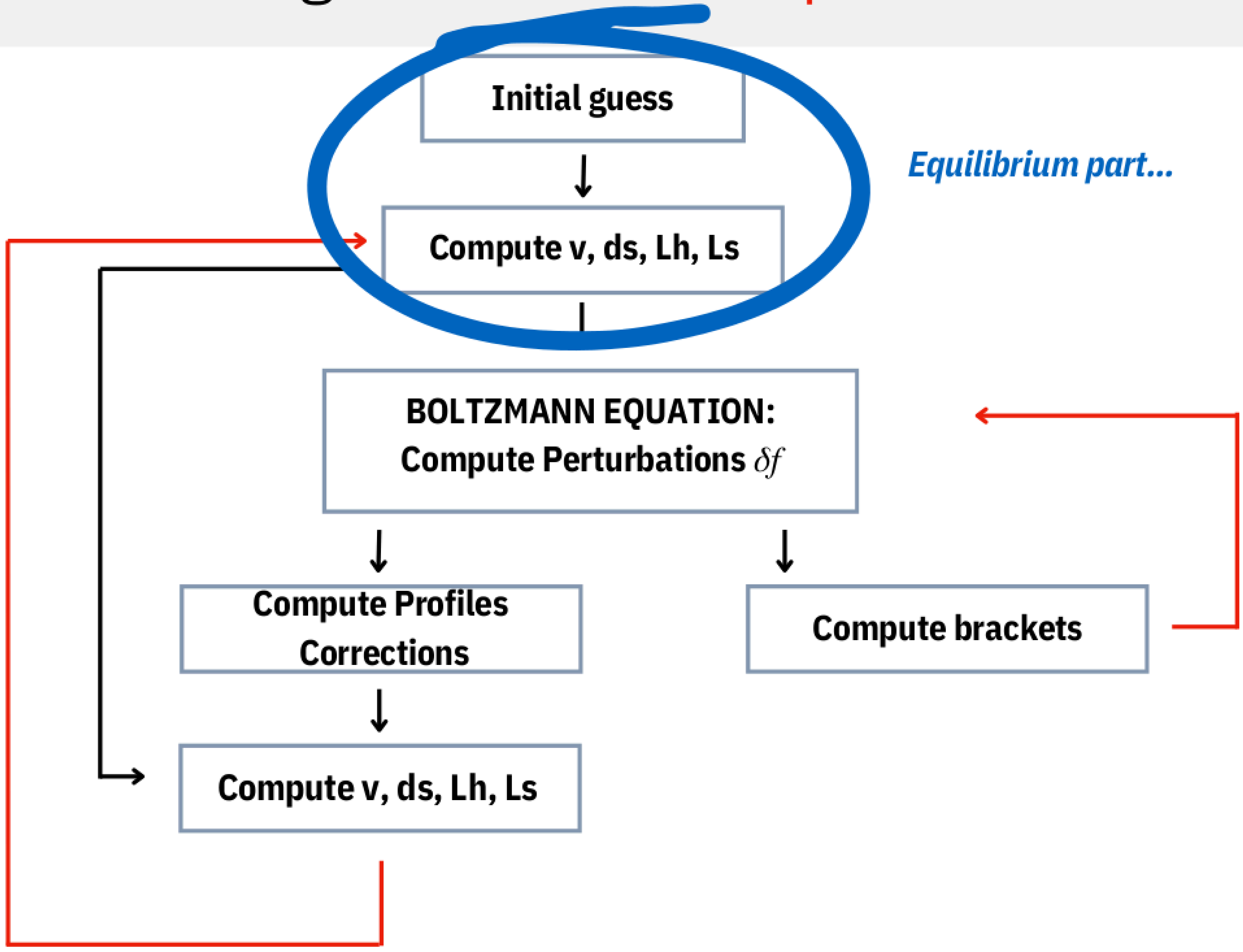
We propose a new method to solve the Boltzmann equation
without imposing any ansatz for δf

De Curtis, Delle Rose, Guiggiani, Gil Muyor, Panico, 2022

Key features

- No term in the Boltzmann equation is neglected
- New approach to deal with collision integrals
- Iterative routine where convergence is achieved in few steps

Work flow algorithm: iterative procedure



Spectral decomposition of the collision integral

Structure of the collision integral: *the bracket*

$$\langle \delta f \rangle = -\frac{f_v(p/\beta(z))}{\beta(z)E_p} \int |\bar{\mathbf{k}}| d|\bar{\mathbf{k}}| d\cos\theta_{\bar{k}} f_0(|\bar{\mathbf{k}}|) \tilde{\mathcal{K}}(|\bar{\mathbf{p}}|, \cos\theta_{\bar{p}}, |\bar{\mathbf{k}}|, \cos\theta_{\bar{k}}) \frac{\delta f(k_\perp/\beta(z), k_z/\beta(z), z)}{f_0'(|\bar{\mathbf{k}}|)}$$

the bracket can be seen as the an application of a hermitian operator on the perturbations

$$\mathcal{O}[g] \equiv \int \mathcal{D}\bar{k} \tilde{\mathcal{K}}_{\bar{p},\bar{k}} g(|\bar{\mathbf{k}}|, \cos\theta_{\bar{k}})$$

main idea: decompose the bracket operator into its eigenfunctions ψ

$$\tilde{\mathcal{K}}_{\bar{p},\bar{k}} = \sum_l \lambda_l \psi_l(|\bar{\mathbf{p}}|, \cos\theta_{\bar{p}}) \psi_l(|\bar{\mathbf{k}}|, \cos\theta_{\bar{k}})$$

- kernels can be (numerically) evaluated only once
- huge improvement in time performance (~ 2 orders of magnitude)

De Curtis, Delle Rose, Guiggiani, Gil Muyor, Panico, 2023

Full solution to the Boltzmann equation

Structure of the Boltzmann equation

$$\frac{d}{dz}\delta f - \frac{\mathcal{Q}}{p_z}\frac{\delta f}{f_v'} = \frac{(m^2)'}{2p_z}\partial_{p_z} f_v + (\langle \delta f(k)\rangle - \langle \delta f(p')\rangle - \langle \delta f(k')\rangle)$$

Iterative procedure

- initial guess of the perturbation $\delta f0$
- next step of the iteration is found by solving

$$\frac{d}{dz}\delta f_n - \frac{\mathcal{Q}}{p_z}\frac{\delta f_n}{f_v'} = \frac{(m^2)'}{2p_z}\partial_{p_z} f_v + (\langle \delta f_{n-1}(k)\rangle - \langle \delta f_{n-1}(p')\rangle - \langle \delta f_{n-1}(k')\rangle)$$

De Curtis,Delle Rose , Guiggiani, Gil Muyor, Panico, 2022