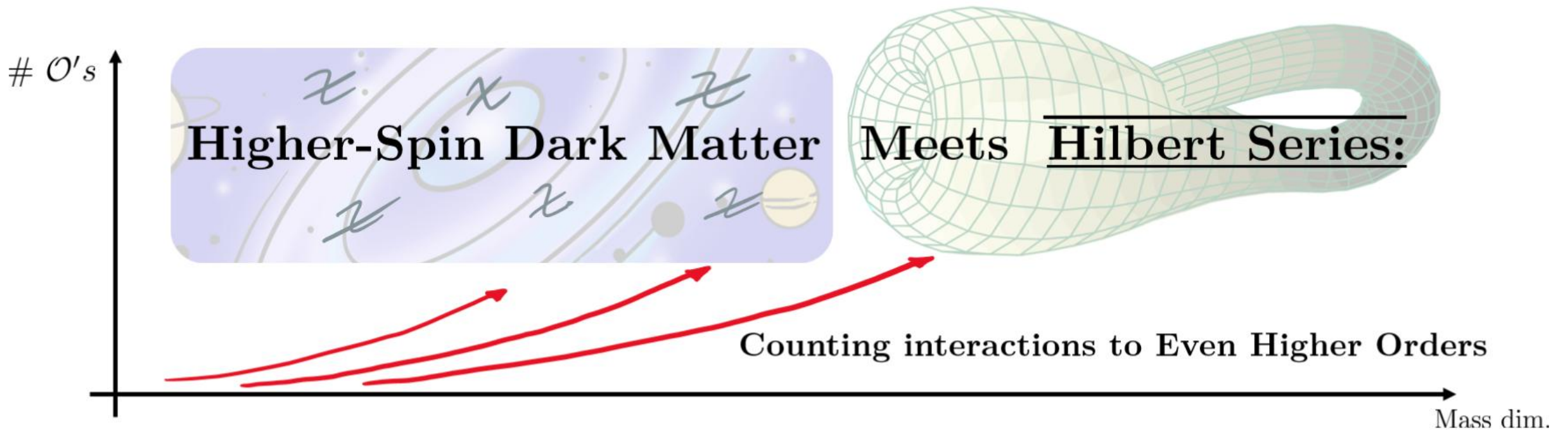




Bruno Siqueira Eduardo
(Universidade de São Paulo/Université de Genève)

work in progress in collaboration with:
Oscar Eboli (Universidade de São Paulo)
Enrico Bertuzzo (Università degli Studi di Modena e Reggio Emilia)

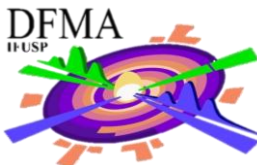


UNIVERSITÉ
DE GENÈVE

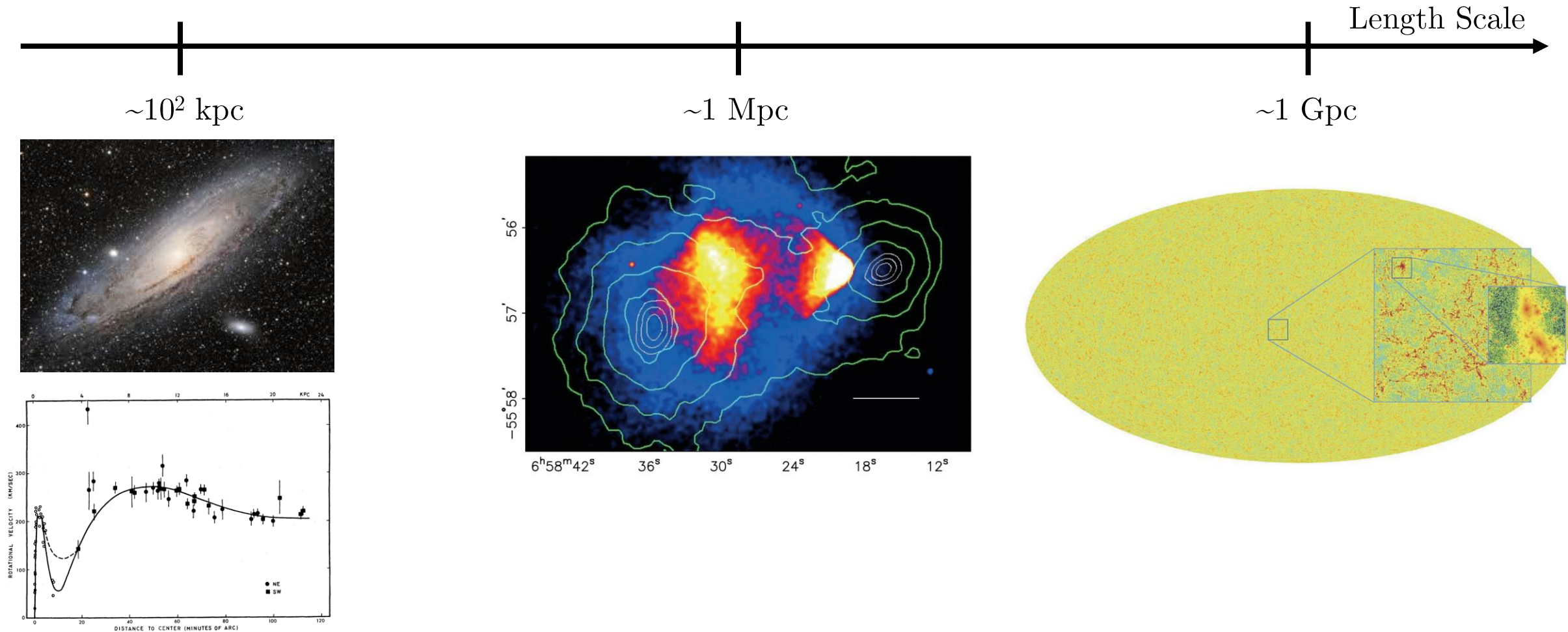
USP



FAPESP



Evidences for Dark Matter



Vera C. Rubin and W. Kent Ford, Jr., "Rotation of the Andromeda Nebula from a Spectroscopic Survey of Emission Regions", 1970.

D. Clowe et al., "A direct empirical proof of the existence of dark matter", 2006.

D. Potter et al., "PKDGRAV3: Beyond Trillion Particle Cosmological Simulations for the Next Era of Galaxy Surveys", 2017.

Dark Matter Frameworks

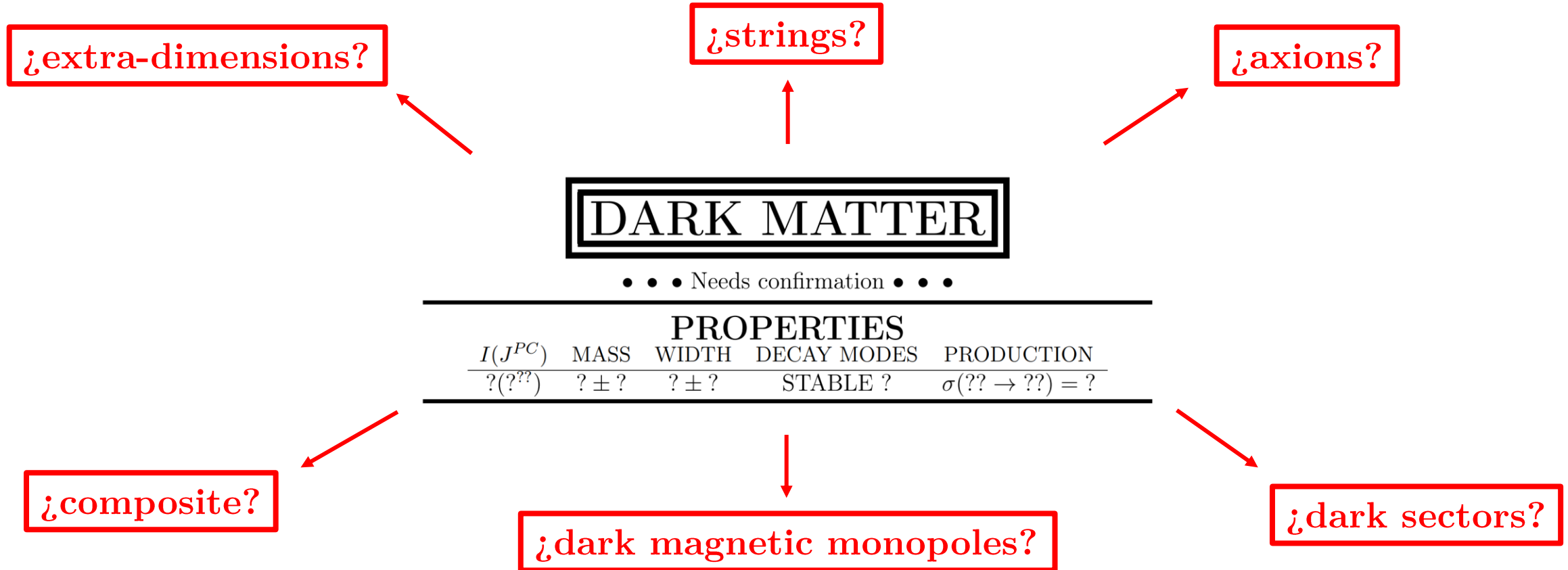
DARK MATTER

• • • Needs confirmation • • •

PROPERTIES

$I(J^{PC})$	MASS	WIDTH	DECAY MODES	PRODUCTION
$\frac{?}{?^{(??)}}$	$? \pm ?$	$? \pm ?$	STABLE ?	$\sigma(?? \rightarrow ??) = ?$

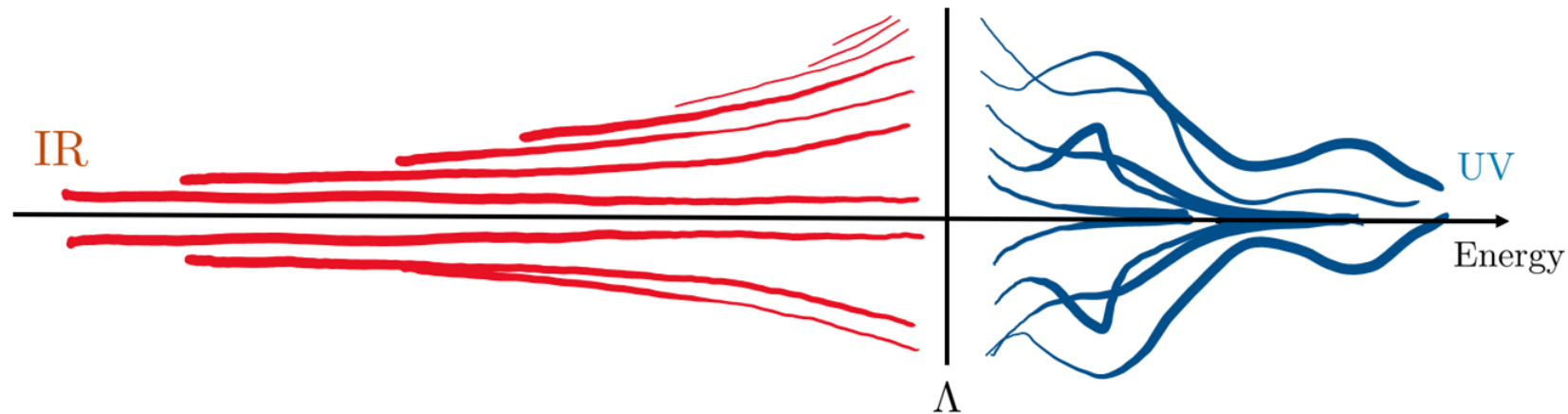
Dark Matter Frameworks



Effective field theories

“All physical theories are effective theories”

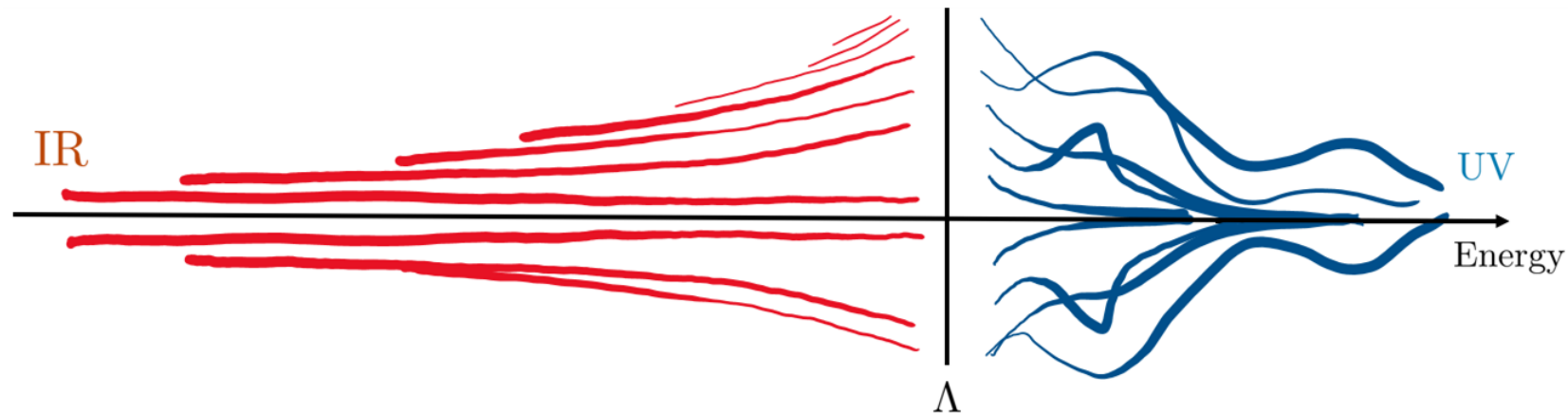
Riccardo Penco, “An Introduction to Effective Field Theories”, 2020.



Effective field theories

“All physical theories are effective theories”

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Parametrizes our ignorance by including all allowed terms in

$$\mathcal{L} = \mathcal{L}_0 + \sum_j \frac{c_j}{\Lambda^{\Delta_j - d}} \mathcal{O}_j$$

Which EFT?

SM + DM candidate



Which EFT?

SM + DM candidate

✓ massive

Which EFT?

SM + DM candidate

- ✓ massive
- SM gauge singlets

Which EFT?

SM + DM candidate

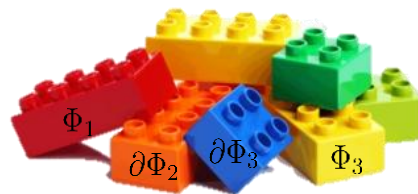
- ✓ massive
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- higher spin (3/2 and 2)

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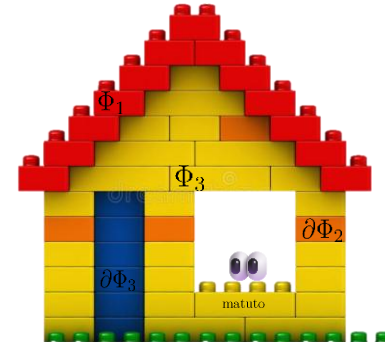
Problem:



EOM, IBP, Fierz...



... SYSTEMATICALLY!
... to ALL ORDERS!

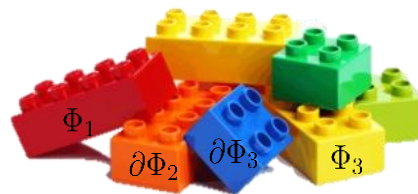


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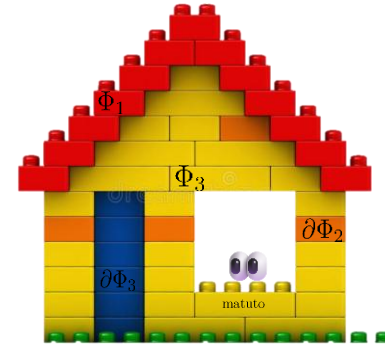
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Remedy:

⚡ HILBERT SERIES ⚡

Hilbert Series 101

Field content: $\{\Phi_1, \dots, \Phi_N\}$

Hilbert Series 101

Field content: $\{\Phi_1, \dots, \Phi_N\}$

$$H(\mathcal{D}, \{\phi_i\}) \equiv \sum_{n=0}^{\infty} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} \cdots \sum_{r_N=0}^{\infty} d_{n,r_1,r_2,\dots,r_N} \mathcal{D}^n \phi_1^{r_1} \phi_2^{r_2} \cdots \phi_N^{r_N}$$

Hilbert Series 101

Field content: $\{\Phi_1, \dots, \Phi_N\}$

$$H(\mathcal{D}, \underbrace{\{\phi_i\}}_{\text{complex variables ("spurions")}}) \equiv \sum_{n=0}^{\infty} \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} \cdots \sum_{r_N=0}^{\infty} d_{n,r_1,r_2,\dots,r_N} \mathcal{D}^n \phi_1^{r_1} \phi_2^{r_2} \cdots \phi_N^{r_N}$$

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Hilbert Series 101

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Hilbert Series 101

Example: SMEFT (1 generation)

$$H(\mathcal{D}, \{\phi_i\}) = H(\mathcal{D}, Q, Q^\dagger, L, L^\dagger, H, H^\dagger, u, u^\dagger, d, d^\dagger, e, e^\dagger, B_L, B_R, W_L, W_R, G_L, G_R)$$

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$$\text{Dim-5: } H^2 L^2 + H^{\dagger 2} L^{\dagger 2}$$

Hilbert Series 101

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Dim-6:

$$\begin{aligned} & H^3 H^{\dagger 3} + u^\dagger Q^\dagger H H^{\dagger 2} + 2Q^2 Q^{\dagger 2} + Q^{\dagger 3} L^\dagger + Q^3 L + 2Q Q^\dagger L L^\dagger + L^2 L^{\dagger 2} + u Q H^2 H^\dagger \\ & + 2u u^\dagger Q Q^\dagger + u u^\dagger L L^\dagger + u^2 u^{\dagger 2} + e^\dagger u^\dagger Q^2 + e^\dagger L^\dagger H^2 H^\dagger + 2e^\dagger u^\dagger Q^\dagger L^\dagger + e L H H^{\dagger 2} + e u Q^{\dagger 2} \\ & + 2e u Q L + e e^\dagger Q Q^\dagger + e e^\dagger L L^\dagger + e e^\dagger u u^\dagger + e^2 e^{\dagger 2} + d^\dagger Q^\dagger H^2 H^\dagger + 2d^\dagger u^\dagger Q^{\dagger 2} + d^\dagger u^\dagger Q L \\ & + d^\dagger e^\dagger u^{\dagger 2} + d^\dagger e Q^\dagger L + d Q H H^{\dagger 2} + 2d u Q^2 + d u Q^\dagger L^\dagger + d e^\dagger Q L^\dagger + d e u^2 + 2d d^\dagger Q Q^\dagger + d d^\dagger L L^\dagger \\ & + 2d d^\dagger u u^\dagger + d d^\dagger e e^\dagger + d^2 d^{\dagger 2} + u^\dagger Q^\dagger H^\dagger G_R + d^\dagger Q^\dagger H G_R + H H^\dagger G_R^2 + G_R^3 + u Q H G_L \\ & + d Q H^\dagger G_L + H H^\dagger G_L^2 + G_L^3 + u^\dagger Q^\dagger H^\dagger W_R + e^\dagger L^\dagger H W_R + d^\dagger Q^\dagger H W_R + H H^\dagger W_R^2 + W_R^3 \\ & + u Q H W_L + e L H^\dagger W_L + d Q H^\dagger W_L + H H^\dagger W_L^2 + W_L^3 + u^\dagger Q^\dagger H^\dagger B_R + e^\dagger L^\dagger H B_R \\ & + d^\dagger Q^\dagger H B_R + H H^\dagger B_R W_R + H H^\dagger B_R^2 + u Q H B_L + e L H^\dagger B_L + d Q H^\dagger B_L + H H^\dagger B_L W_L \\ & + H H^\dagger B_L^2 + 2Q Q^\dagger H H^\dagger \mathcal{D} + 2L L^\dagger H H^\dagger \mathcal{D} + u u^\dagger H H^\dagger \mathcal{D} + e e^\dagger H H^\dagger \mathcal{D} + d^\dagger u H^2 \mathcal{D} + d u^\dagger H^{\dagger 2} \mathcal{D} \\ & + d d^\dagger H H^\dagger \mathcal{D} + 2H^2 H^{\dagger 2} \mathcal{D}^2 \end{aligned}$$

Computing the Hilbert Series

Fields Φ_i transform under (Lorentz/gauge) group irreps V_i

$$V_i \otimes V_j = \bigoplus_k V_k$$

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$$\int d\mu_G(g) \overline{\chi_i^G(g)} \chi_j^G(g) = \delta_{ij}, \quad g \in G$$

Computing the Hilbert Series

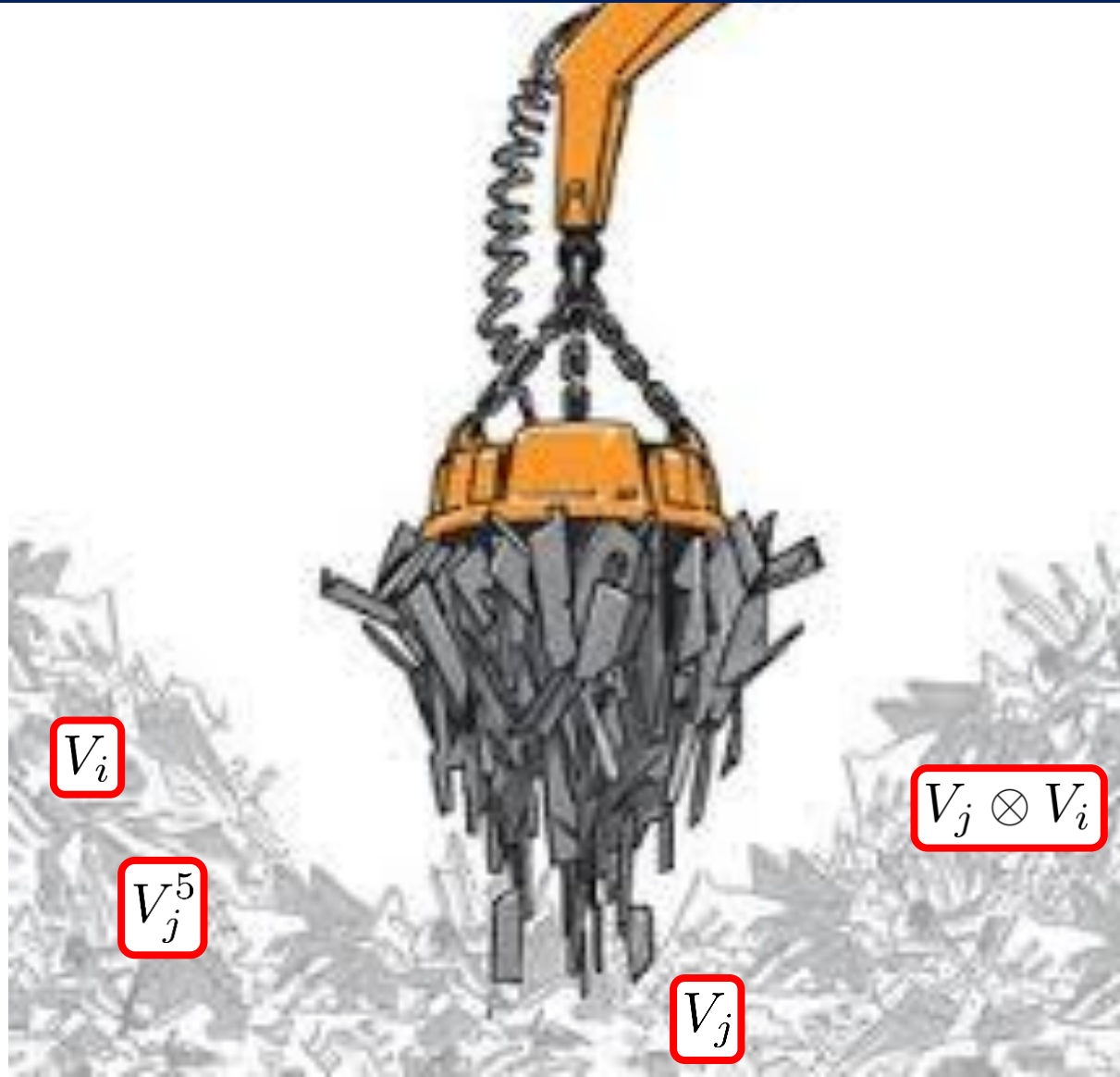
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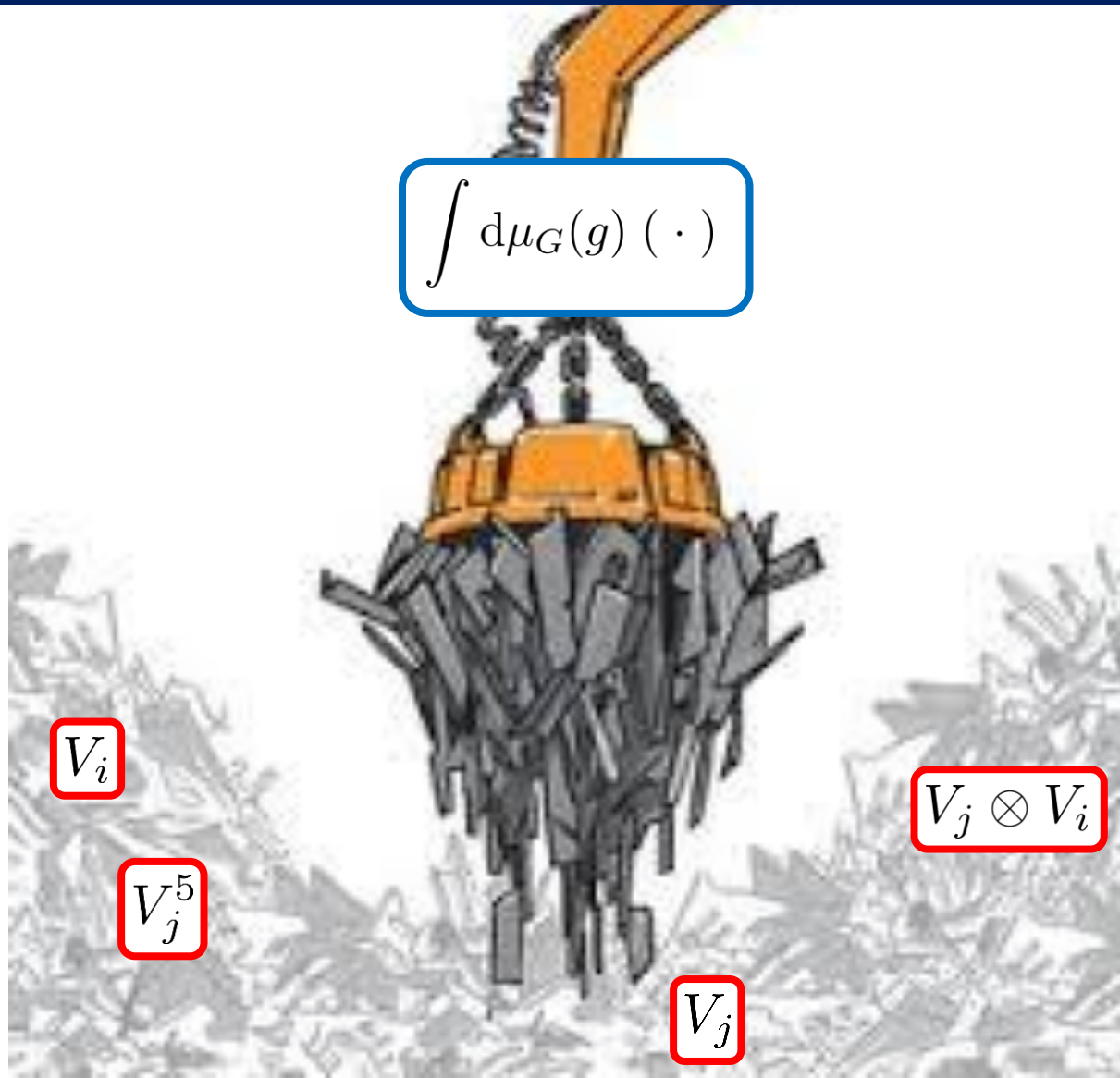
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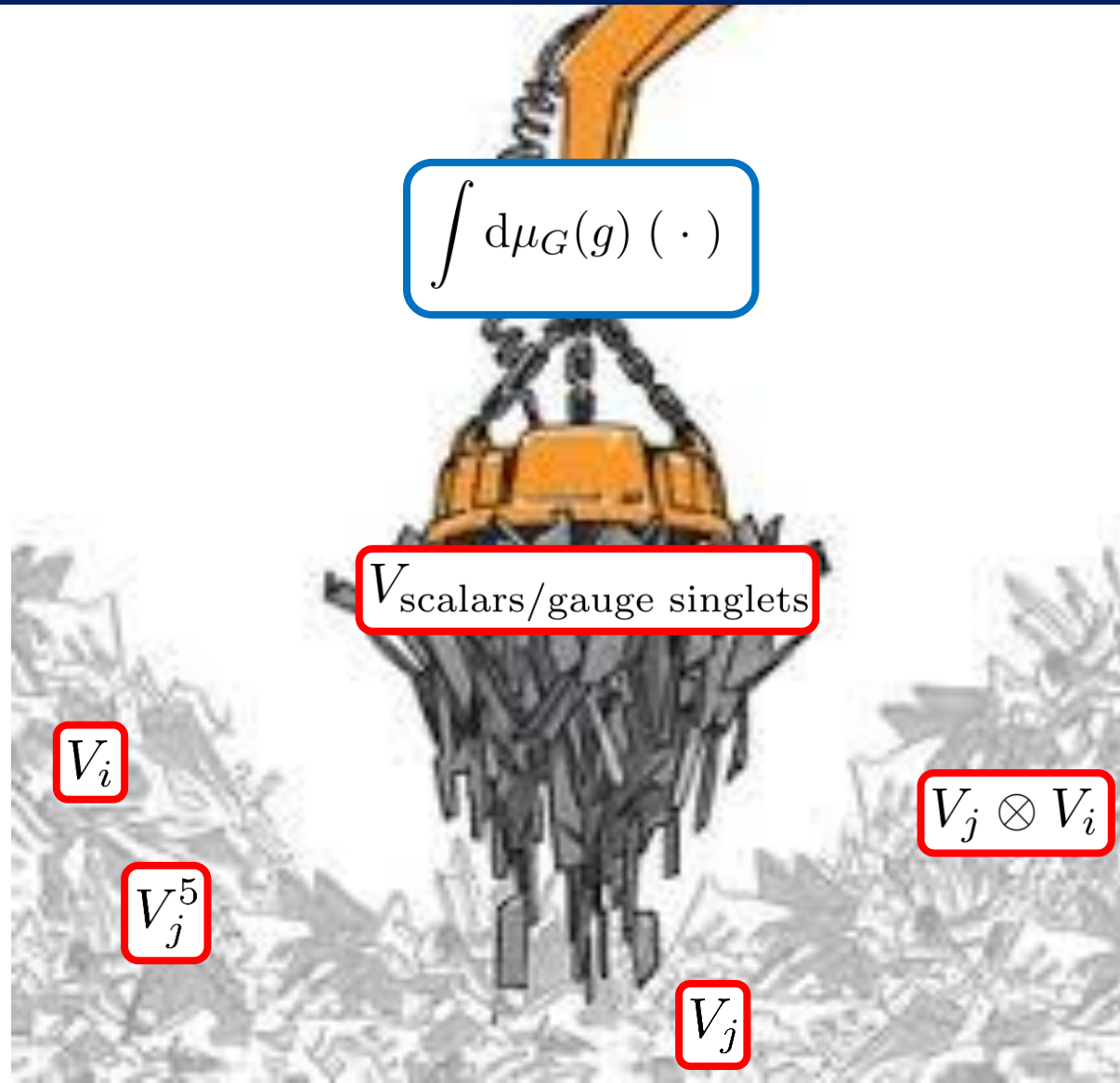
Character orthogonality



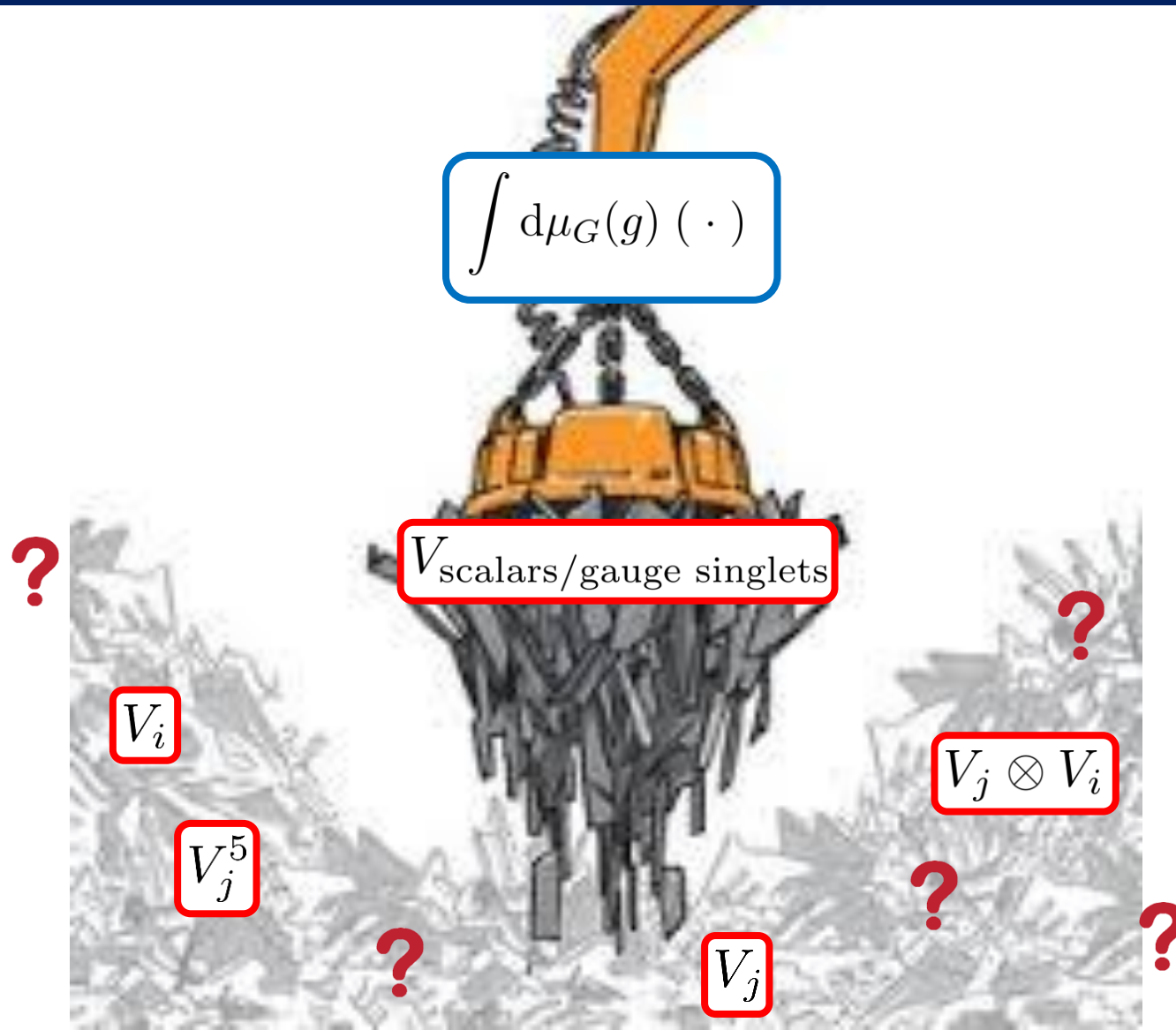
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The plethystic exponential

complete operator basis

$\Leftrightarrow$

selecting $V_{\text{scalars/gauge singlets}}$ from a representation that contemplates **ALL** possible **(anti)symmetric tensor powers** of the (fermionic) bosonic field irreps.

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$$\prod_{i=1}^N \text{PE}_i[\phi_i \chi_i] \equiv \prod_{i=1}^N \sum_{n_i=0}^{\infty} \phi_i^{n_i} \cdot \begin{cases} S^{n_i}(\chi_i), & \text{if } \Phi_i \text{ is a boson} \\ \Lambda^{n_i}(\chi_i), & \text{if } \Phi_i \text{ is a fermion} \end{cases}$$

Projecting out scalars (+ removing IBP)

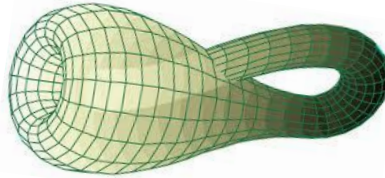
IBP relations:

total divergences ~ 0

Projecting out scalars (+ removing IBP)

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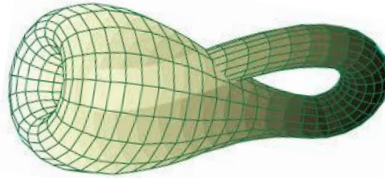
cohomology...



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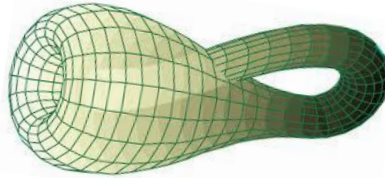


$\Rightarrow$ operator basis consists of
not co-exact 0-forms

Projecting out scalars (+ removing IBP)

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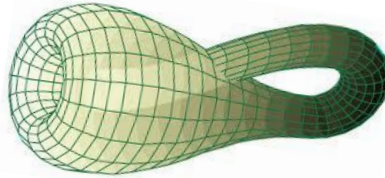
$\Rightarrow$ operator basis consists of
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$$\begin{aligned} & \#(\text{not co-exact 0-forms}) \\ &= \sum_{k=0}^d (-\mathcal{D})^k \#(k\text{-forms}) - \sum_{k=1}^d (-\mathcal{D})^k \#(\text{co-closed not co-exact } k\text{-forms}) \end{aligned}$$

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cohomology...



$\Rightarrow$ operator basis consists of
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$$\begin{aligned} & \#(\text{not co-exact 0-forms}) && \text{mass dim.} \leq d \\ &= \sum_{k=0}^d (-\mathcal{D})^k \#(k\text{-forms}) - \sum_{k=1}^d (-\mathcal{D})^k \#(\text{co-closed not co-exact } k\text{-forms}) \end{aligned}$$

Further projecting out scalars (+removing IBP)

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with cohomology...
 $\Rightarrow$

operator basis consists of
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$$\#(\text{not co-exact 0-forms}) \approx \sum_{k=0}^d (-\mathcal{D})^k \#(k\text{-forms})$$

A k -form transforms as $\Lambda^k \square$ under $SO(d)$:

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Removing EOM redundancies

Single Particle Module (SPM)

$$R_\Phi = \begin{pmatrix} \Phi \\ \partial_\mu \Phi \\ \partial_{\{\mu_1} \partial_{\mu_2\}} \Phi \\ \vdots \end{pmatrix}$$

Removing EOM redundancies

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Removing EOM redundancies

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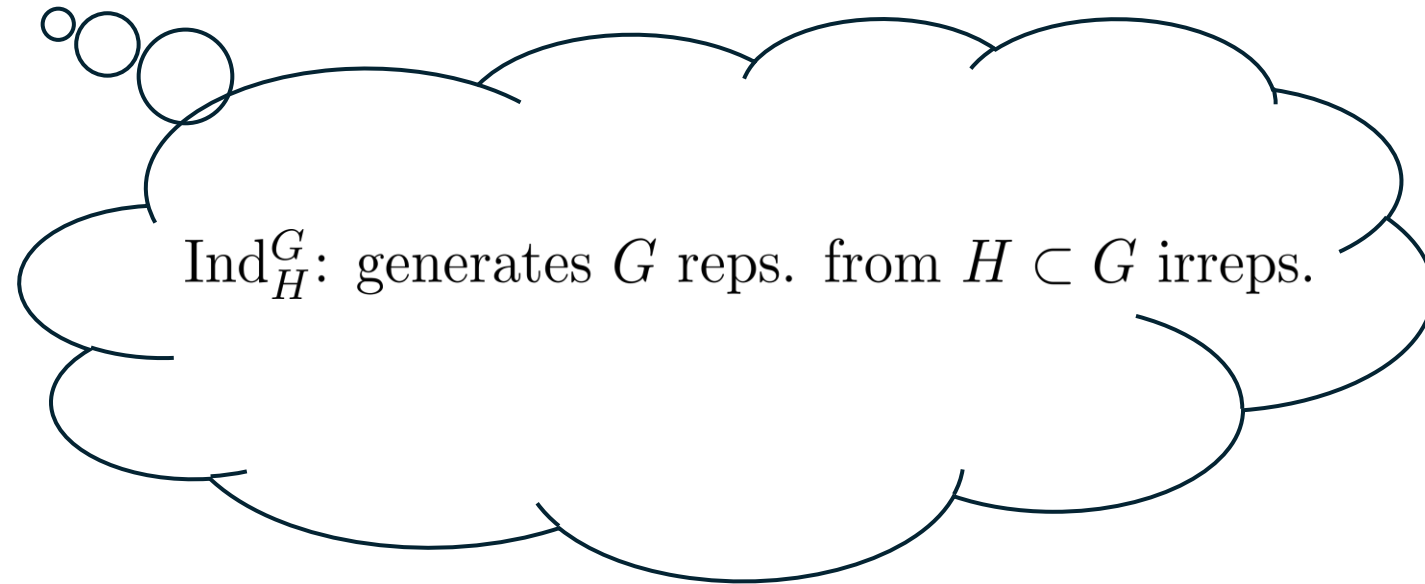
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$$\chi_{R_\Phi} ???$$

Removing EOM redundancies

Single Particle Module (SPM)

$$R_\Phi = \text{Ind}_{SO(3)}^{SO(4)}$$



Removing EOM redundancies

Single Particle Module (SPM)

$$R_{\Phi} = \text{Ind}_{SO(3)}^{SO(4)} = \bigoplus_{n=0}^{\infty} \bigoplus_{m=-k}^k (k+n, m)$$

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spin, $SO(3)$ irrep.

Removing EOM redundancies

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$$\Rightarrow \chi_{R_{\Phi}} = \sum_{n=0}^{\infty} \sum_{m=-k}^k \mathcal{D}^{\Delta_{m,n}} \chi_{(k+n,m)}$$

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$$\Rightarrow \chi_{R_\Phi} = \sum_{n=0}^{\infty} \sum_{m=-k}^k \mathcal{D}^{\Delta_{m,n}} \chi_{(k+n, m)}$$

smallest j such that
 $\partial^j \Phi \supset (k+n, m)$

Higher-spin characters

spin	$s = 3/2$
field	ψ_μ (Rarita-Schwinger)
Lorentz rep. - $SU(2) \times SU(2)$	$\left(1, \frac{1}{2}\right) \oplus \left(\frac{1}{2}, 1\right)$
Lorentz rep. - $SO(4)$	$\left(\frac{3}{2}, \frac{1}{2}\right) \oplus \left(\frac{3}{2}, -\frac{1}{2}\right)$
SPM character	$P(1 - \mathcal{D}) \left[\chi_{(3/2, 1/2)} + \chi_{(3/2, -1/2)} \right. \\ \left. - \mathcal{D} \left(\chi_{(1/2, 1/2)} + \chi_{(1/2, -1/2)} \right) \right]$

Higher-spin characters

spin	$s = 2$
field	$h_{\mu\nu}$
Lorentz rep. - $SU(2) \times SU(2)$	$(1, 1)$
Lorentz rep. - $SO(4)$	$(2, 0)$
SPM character	$P(1 - \mathcal{D}^2)(\chi_{(2,0)} - \mathcal{D}\chi_{(1,0)})$

Counting the effective operators

$$\begin{aligned} &3H^\dagger L^\dagger \psi_\mu \mathcal{D} \\ &+3HL\psi_\mu \mathcal{D} \\ &+2H^\dagger H \psi_\mu^2 \end{aligned}$$

dim 5, at least one spin 3/2 field
Total: 8

Counting the effective operators

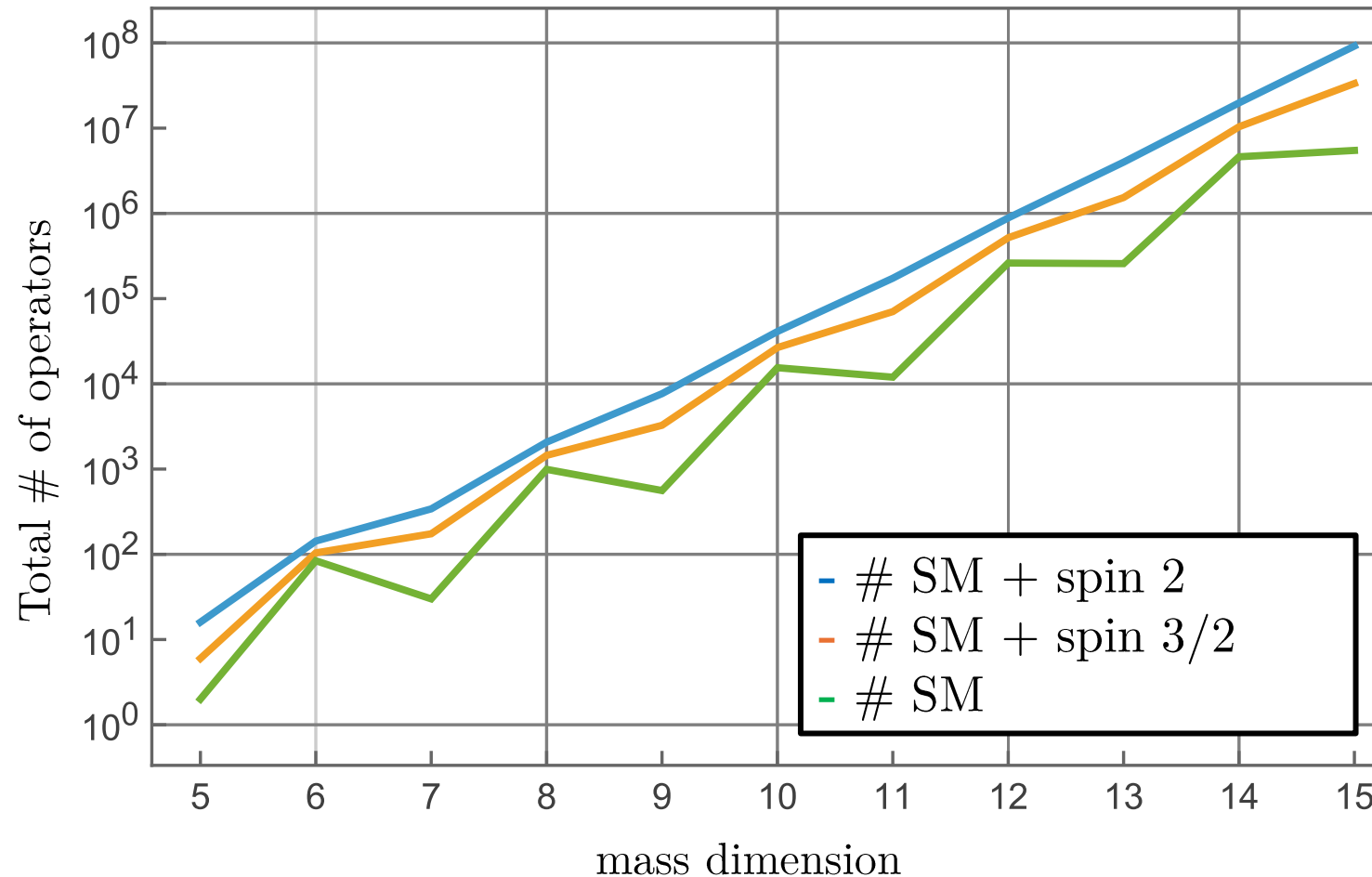
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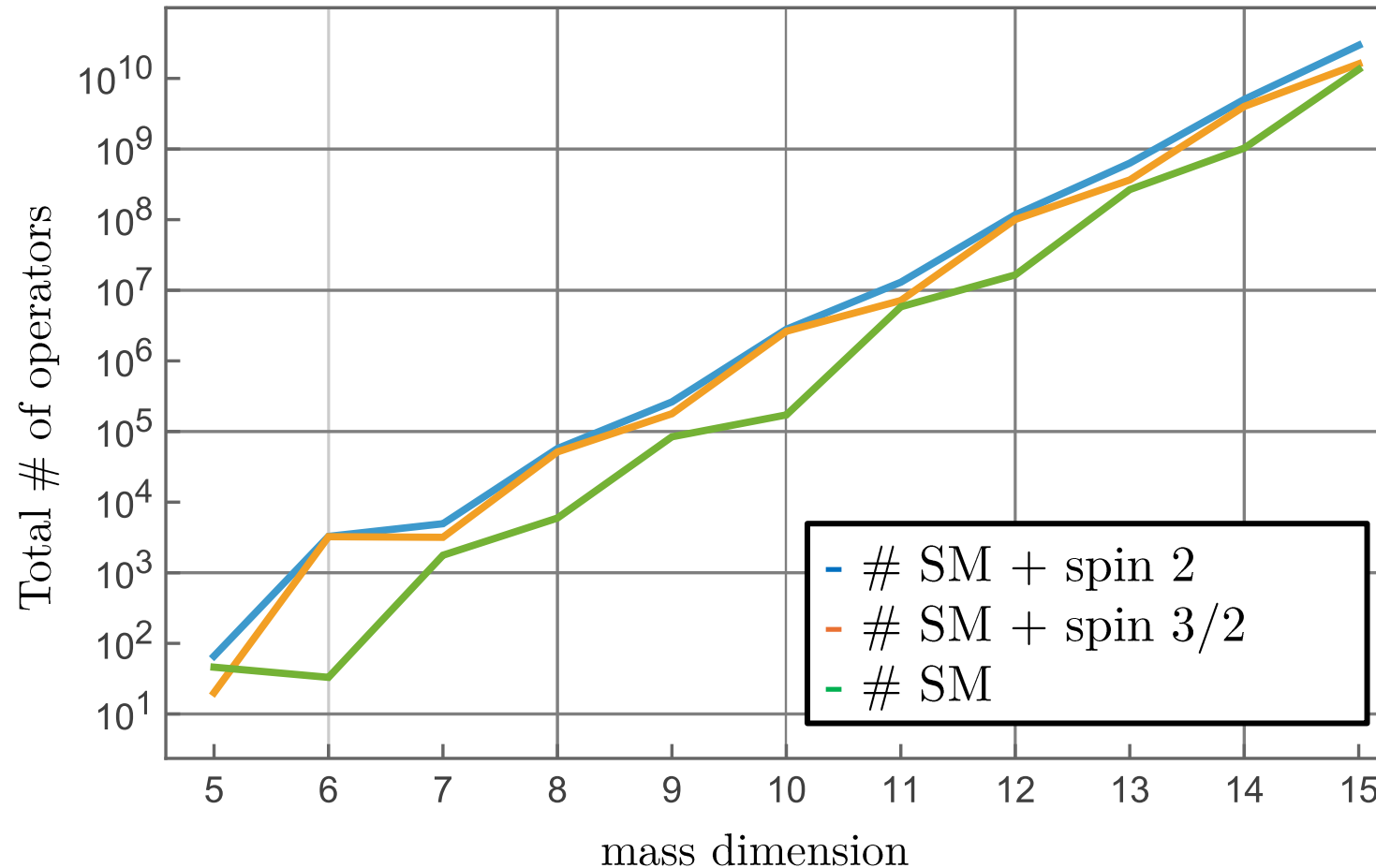
$$\begin{aligned} & 3BH^\dagger \psi_\mu L^\dagger + 3BHL\psi_\mu + 27eud^\dagger \psi_\mu \\ & + 9dd^\dagger \psi_\mu^2 + 9d^\dagger \psi_\mu (Q^\dagger)^2 + 27de^\dagger \psi_\mu u^\dagger \\ & + 9dQ^2 \psi_\mu + 3\mathcal{D}e^\dagger (H^\dagger)^2 \psi_\mu + 9ee^\dagger \psi_\mu^2 \\ & + 3eH^2 \mathcal{D}\psi_\mu + 3WH^\dagger \psi_\mu L^\dagger + H\mathcal{D}H^\dagger \psi_\mu^2 \\ & + 3HLW\psi_\mu + 9L\psi_\mu^2 L^\dagger + 27Q\psi_\mu L^\dagger u^\dagger \\ & + 27Lu\psi_\mu Q^\dagger + 9Q\psi_\mu^2 Q^\dagger + 9u\psi_\mu^2 u^\dagger + 4\psi_\mu^4 \end{aligned}$$

dim 6, at least one spin 3/2 field
Total: 194

Operator growth ($N_f = 1$)

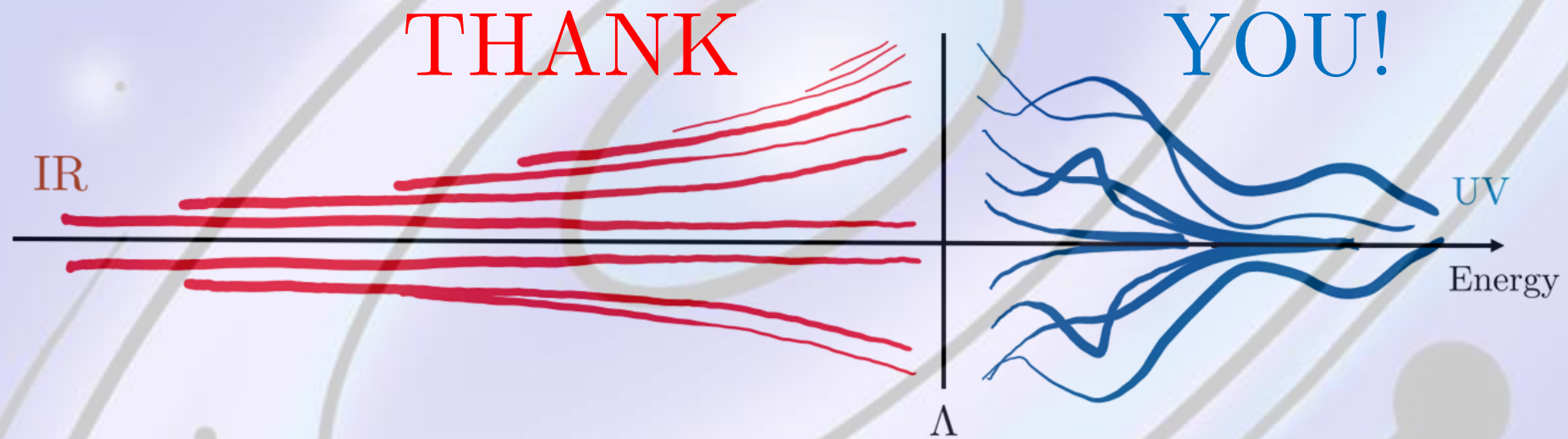


Operator growth ($N_f = 3$)



Conclusions

- Compelling evidence for DM and a plethora of models to describe it;
- EFTs is a powerful model independent tool;
- Hilbert Series can address IBP and EOM redundancies;
- Higher-spin fields can be embedded into the Hilbert Series formalism;
- Higher-spin fields $\Rightarrow$ more operators allowed.



EXTRA

Counting the effective operators

$$\begin{aligned} & 3H^\dagger L^\dagger \psi_\mu \mathcal{D} \\ & + 3HL\psi_\mu \mathcal{D} \\ & + 2H^\dagger H \psi_\mu^2 \end{aligned}$$

dim 5, spin 3/2

$$\begin{aligned} & 3h_{\mu\nu}^3 \mathcal{D}^2 + h_{\mu\nu}^5 \\ & + h_{\mu\nu}(G^2 + W^2 + B^2) \\ & + 9h_{\mu\nu} \mathcal{D}(e^\dagger e + L^\dagger L + d^\dagger d + u^\dagger u + q^\dagger q) \\ & + h_{\mu\nu} \mathcal{D}^2 H^\dagger H \\ & + h_{\mu\nu}^3 H^\dagger H \end{aligned}$$

dim 5, spin 2

Counting the effective operators

$$\begin{aligned}
 & 3BH^\dagger\psi_\mu L^\dagger + 3BHL\psi_\mu + 27eud^\dagger\psi_\mu \\
 & + 9dd^\dagger\psi_\mu^2 + 9d^\dagger\psi_\mu (Q^\dagger)^2 + 27de^\dagger\psi_\mu u^\dagger \\
 & + 9dQ^2\psi_\mu + 3\mathcal{D}e^\dagger (H^\dagger)^2\psi_\mu + 9ee^\dagger\psi_\mu^2 \\
 & + 3eH^2\mathcal{D}\psi_\mu + 3WH^\dagger\psi_\mu L^\dagger + H\mathcal{D}H^\dagger\psi_\mu^2 \\
 & + 3HLW\psi_\mu + 9L\psi_\mu^2 L^\dagger + 27Q\psi_\mu L^\dagger u^\dagger \\
 & + 27Lu\psi_\mu Q^\dagger + 9Q\psi_\mu^2 Q^\dagger + 9u\psi_\mu^2 u^\dagger + 4\psi_\mu^4
 \end{aligned}$$

dim 6, spin 3/2

$$\begin{aligned}
 & 5B^2h_{\mu\nu}^2 + 9Qd^\dagger h_{\mu\nu}^2 H^\dagger + 27d\mathcal{D}d^\dagger h_{\mu\nu}^2 \\
 & + 9dHh_{\mu\nu}^2 Q^\dagger + 9Le^\dagger h_{\mu\nu}^2 H^\dagger + 27e\mathcal{D}e^\dagger h_{\mu\nu}^2 \\
 & + 9eHh_{\mu\nu}^2 L^\dagger + 5G^2h_{\mu\nu}^2 + 2Hh_{\mu\nu}^4 H^\dagger \\
 & + 6H\mathcal{D}^2h_{\mu\nu}^2 H^\dagger + 9uh_{\mu\nu}^2 H^\dagger Q^\dagger \\
 & + 9HQh_{\mu\nu}^2 u^\dagger + 27L\mathcal{D}h_{\mu\nu}^2 L^\dagger + 3h_{\mu\nu}^6 \\
 & + 11\mathcal{D}^2h_{\mu\nu}^4 + 27\mathcal{D}Qh_{\mu\nu}^2 Q^\dagger + 27\mathcal{D}uh_{\mu\nu}^2 u^\dagger \\
 & + 5W^2h_{\mu\nu}^2 + H^2h_{\mu\nu}^2 (H^\dagger)^2
 \end{aligned}$$

dim 6, spin 2