# Symplectic duality and quiver algorithms

## Higgs mechanisms in supersymmetric quivers

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QuantumUniverse Attract.Workshop
November 25, 2025

# Supersymmetric quantum field theories

Quantum field theory (QFT) = mathematical framework for diverse physics phenomena

- experimentally verified
- intractable for strong coupling

# Supersymmetric quantum field theories

Quantum field theory (QFT) = mathematical framework for diverse physics phenomena

- experimentally verified
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$\Rightarrow$ Introduce supersymmetry

$\rightsquigarrow$ more control over quantum corrections

# Review of 3d $\mathcal{N} = 4$ theories

Large, highly symmetric moduli space of vacua
$\rightsquigarrow$ has two maximal branches:

## Higgs branch

- classical
- Hyper-Kähler quotient

## Coulomb branch

- quantum
- general symplectic singularity

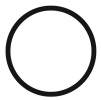
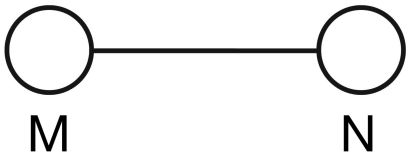
# Review of 3d $\mathcal{N} = 4$ theories

- Both are singular spaces: singularities where gauge symmetry partially restored
  $\rightsquigarrow$ new massless states

- Both admit stratification into symplectic leaves $\mathcal{L}$
  $\rightsquigarrow$ partial order in Hasse diagram

# Quivers in supersymmetric QFTs

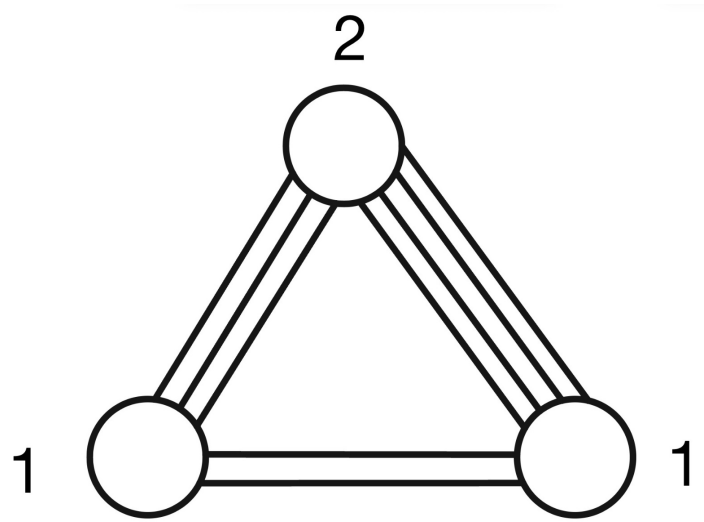
- graphical depiction of 3d $\mathcal{N} = 4$ field content

| Quiver component | Field interpretation |
|---|---|
|   | $\mathcal{N} = 4$ vector multiplet |
|   | $\mathcal{N} = 4$ hypermultiplet |

# Higgs mechanism on the Higgs branch

- parametrized by scalar fields
  $\rightsquigarrow$ Higgsed in multiple ways
- 2 algorithms :
  - Quiver subtraction [arXiv:2409.16356]
  - Ext-quivers [Crawley-Boevey '01], [Nakajima '94]

# From now on: Example




$$A = \begin{pmatrix} -2 & 3 & 2 \\ 3 & -2 & 4 \\ 2 & 4 & -2 \end{pmatrix}$$

$$K = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Real roots:

$R_R = \{(1,0,0),(0,1,0),(0,0,1)\}$

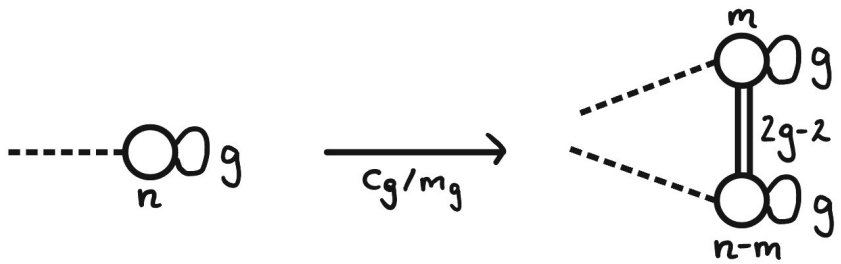
Imaginary roots:

$I_R = \{(0,1,1),(0,2,1),(1,0,1),(1,1,0),(1,1,1),(1,2,1)\}$

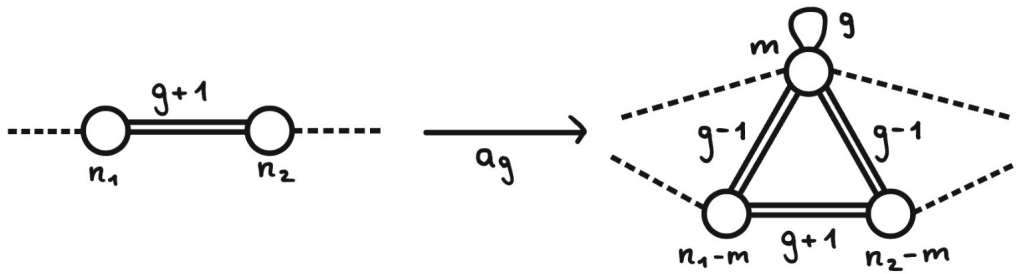
# Hasse diagram via quiver subtraction [2409.16356]

Rules of quiver subtraction:

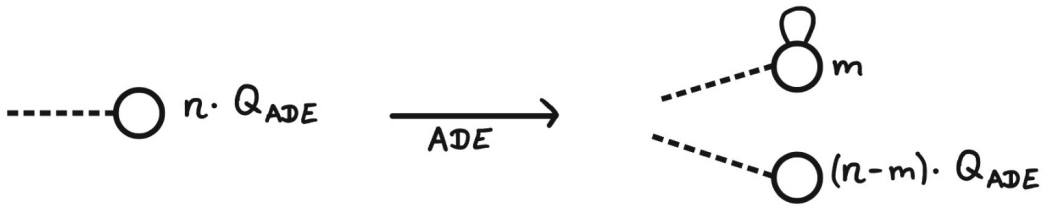
- Rule 1:



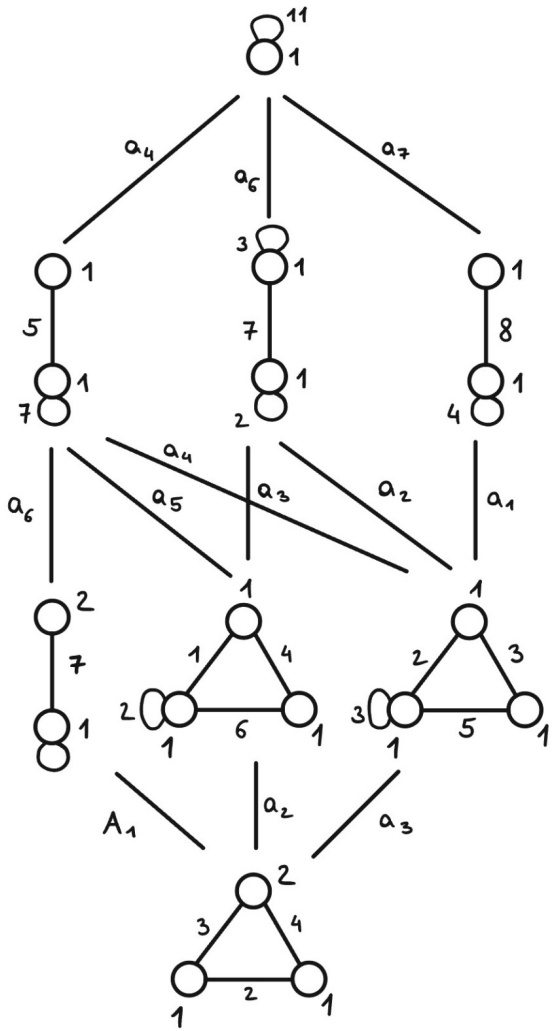

- Rule 2:




- Rule 3:



# Hasse diagram via quiver subtraction [2409.16356]



# Building the Ext-quiver [Crawley-Boevey '01]

Representation type of $\mathcal{L} : \tau = (n_1, \beta^{(1)}; \ldots; n_k, \beta^{(k)})$

$n_i$ = multiplicity

$\beta^{(i)}$ = dimension vector

so that $K = \sum_{i=1}^{k} n_i \beta^{(i)}$

- symplectic leaves labelled by $\tau$ of K
- construct local quiver on a leaf = Ext-quiver

# Building the Ext-quiver

Representation types of K:

$$1 \cdot (1,0,0) + 2 \cdot (0,1,0) + 1 \cdot (0,0,1)$$
$$1 \cdot (1,2,1)$$
$$1 \cdot (1,1,1) + 1 \cdot (0,1,0)$$
$$1 \cdot (1,1,0) + 1 \cdot (0,1,0) + 1 \cdot (0,0,1)$$
$$1 \cdot (1,0,1) + 2 \cdot (0,1,0)$$
$$1 \cdot (0,2,1) + 1 \cdot (1,0,0)$$
$$1 \cdot (0,1,1) + 1 \cdot (1,0,0) + 1 \cdot (0,1,0)$$
$$1 \cdot (0,1,1) + 1 \cdot (1,1,0)$$

$\rightsquigarrow$ Each quiver is the same as in quiver subtraction Hasse diagram

# Hasse diagram via Ext-quiver

Goal:

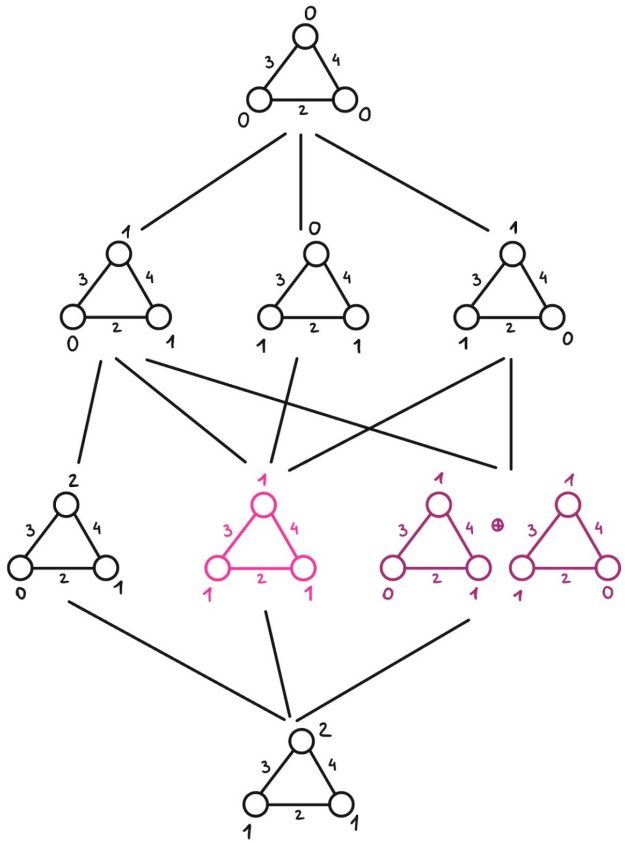
- partial order on $I_R$ to obtain Hasse diagram
  $\rightsquigarrow$ same as for quiver subtraction
- Relation between quiver subtraction rules and change of decompositions

# Higgs mechanism on the Coulomb branch

Obtain Hasse diagram by Decay & Fission algorithm
[arXiv:2401.08757]

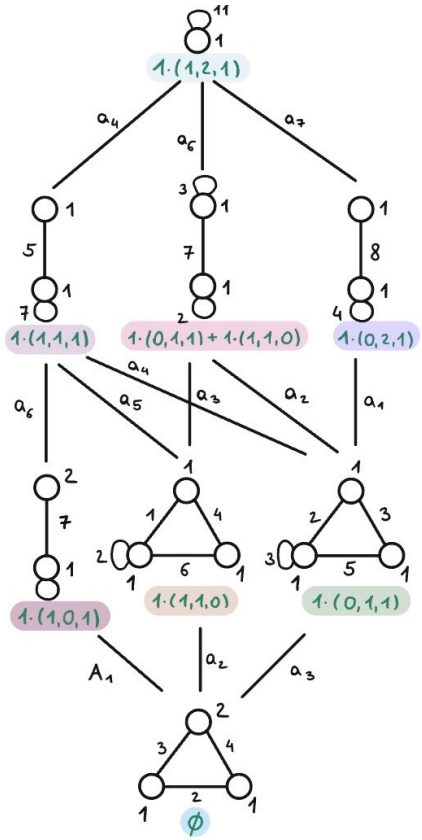
Decay: smaller rank

Fission: splits into two parts but total rank preserved

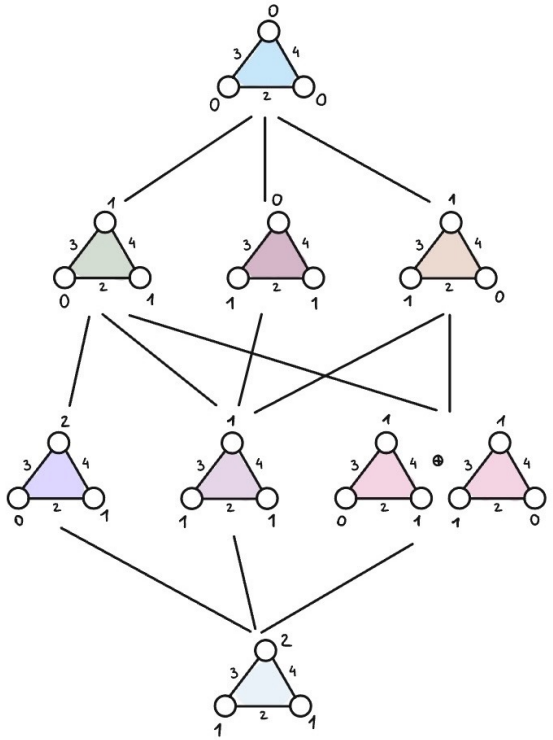


# Comparison of both branches

## Higgs branch (HB)




## Coulomb branch (CB)



# Comparison of both branches

Goal:

- map between partial order on Higgs branch and Coulomb branch
- Symplectic duality?:
  - Same amount of leaves on HB and CB
  - isometries of HB describe resolutions of CB and vice versa

Thank you!