QCD Phase Transitions at Imaginary Chemical Potential

Using Perturbation Theory

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QCD: The Ingredients

Quantum Chromodynamics(QCD) describes how quarks and gluons interact under the strong force within the standard model of particle physics.

- Matter: N_{f} flavors of quarks(dirac fermions), come in N_{c} charges(colors)
- Force mediators: gluons(gauge bosons), come in $N_{\mathrm{c}}^2 - 1$ charges(colors)
- In our universe: $N_{\mathrm{f}} = 6$ and $N_{\mathrm{c}} = 3$

The QCD Phase Diagram

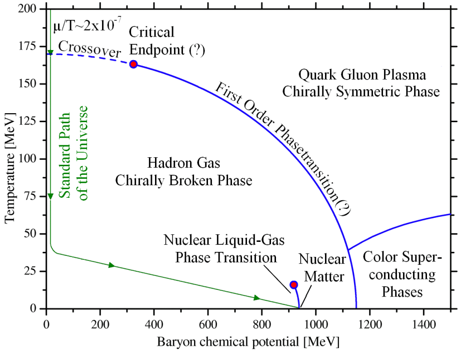


Figure: QCD Phase Diagram, taken from [Boeckel and Schaffner-Bielich, 2012].

- Studied extensively using lattice field theory (e.g. Guenther [2021], Pasztor [2024], de Forcrand [2009])
- non-zero chemical potential → lattice encounters sign problem :(
  $\implies$ move to purely *imaginary* chemical potential [Roberge and Weiss, 1986]

(Imaginary) Chemical Potential

- Moving to non-zero chemical potential

$$Z(\beta) \to Z(\beta, \mathrm{i}\theta) \quad \text{with } \beta = 1/T \text{ and } \theta \in [0, 2\pi)\,.$$

- Need to modify action

$$S_{\mathrm{QCD}}^{\mathrm{E}}\left[A, \bar{\psi}, \psi\right] = \int_{[0,\beta]} d\tau \left[\int_{\mathbb{R}^3} d^3x \left\{\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + \bar{\psi}\left[i\not{D} - M\right]\psi\right\} + \frac{i\theta}{\beta}\int_{\mathbb{R}^3} d^3x \psi^\dagger \psi\right]$$

$\implies$ chemical potential equivalent to constant shift in the background (gauge-)field

$$A_0 \to A_0 + \frac{\theta}{g\beta}\mathbf{1}_{N\times N}$$

Roberge-Weiss Phase Transition:
First order phase transition of QCD when varying the *imaginary chemical potential* at a high but *fixed Temperature* $T \geq T_c^{\mathrm{RW}}$. Do *not* occur at low temperatures $T \leq T_c^{\mathrm{RW}}$ [Roberge and Weiss, 1986]. Numerics suggest $T_c^{\mathrm{RW}} \approx$ 200MeV[Czaban et al., 2016].

Finding Roberge-Weiss Transitions for SU(N_c) QCD

- Order parameter $\rightarrow$ Polyakov loop-angle

$$L\left(\vec{y}\right) = \mathrm{Tr}_{\mathrm{color}}\left\{\mathscr{P}\exp\left[ig\int_0^\beta d\tau A_0(\tau,\vec{y})\right]\right\}$$

  Expectation value will simplify to

$$\langle L\rangle = N_c e^{i2\pi k/N_c} \implies k \text{ labels } N_c \text{ different phases.}$$

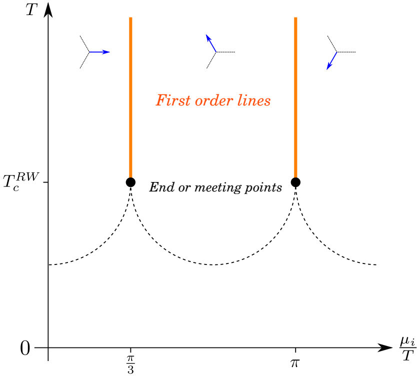
- Effective potential in QFT

$$\left.\frac{dV_{\mathrm{eff}}\left(\varphi\right)}{d\varphi}\right|_{\varphi=\varphi_0} \stackrel{!}{=} 0 \implies \langle\text{Loop-angle}\rangle = \varphi_0$$

- high temperature $\implies$ One-loop approximation

$$V_{\mathrm{eff}}\left(\varphi\right) \approx V_{\mathrm{eff}}^{\text{one-loop}}\left(\varphi\right).$$

Roberge-Weiss Phase Transitions for $N_\mathrm{f} = 1$ and SU(3)



[Czaban et al., 2016]

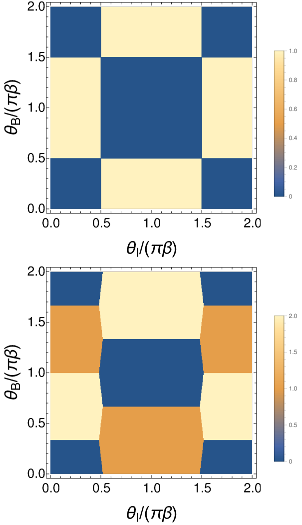
Research Question:
What happens if we consider multiple chemical potential degrees of freedom?

$$Z\left(\beta, \mathrm{i}\theta\right) \rightarrow Z\left(\beta, \mathrm{i}\theta_1, \cdots, \mathrm{i}\theta_{N_\mathrm{f}}\right)$$

Multi-fermion case for SU(2) and SU(3)

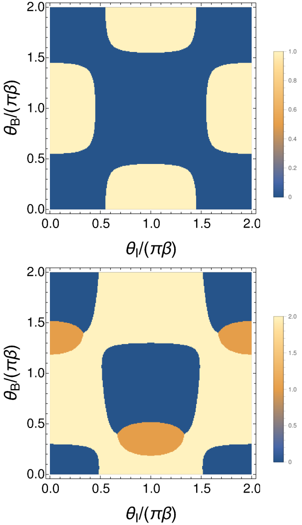
upper: SU(2)
lower: SU(3)

$N_\mathrm{f} = 2$



(Lattice: Brandt et al. [2024])

$N_\mathrm{f} = 3$ at $\theta_\mathrm{S} = 2\pi\beta/5$



new paper coming soon!

Conclusions

- QCD has a rich phase-structure with many planes to explore(multi-flavor, imag. chem. pot., etc.)
- Phase transitions at imaginary chemical potential at high temperatures
- Can be verified using lattice field theory simulations

Thank You for Your Attention!

Appendix: QCD at Finite Temperature

$$S^{\mathrm{E}}_{\mathrm{QCD}}\left[A,\bar{\psi},\psi\right]=\int_{[0,\beta]}d\tau\int_{\mathbb{R}^3}d^3x\left\{\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\bar{\psi}\left[i\not{D}-M\right]\psi\right\}$$

$$Z\left(\beta\right)=\int_{\mathrm{PBC}}\mathscr{D}A_\mu\int_{\text{anti-PBC}}\mathscr{D}\psi\mathscr{D}\bar{\psi}\;\delta\left(G\left[A\right]\right)\det\left(D^a_\mu\frac{\partial G}{\partial A^a_\mu}\right)\exp\left\{-S^{\mathrm{E}}_{\mathrm{QCD}}\left[A,\bar{\psi},\psi\right]\right\}$$

- Euclidean Gauge Theory on $\mathbb{R}^3\times S^1$
- Bosons: periodic boundary conditions
- Fermions: anti-periodic boundary conditions

(Review: e.g. Laine and Vuorinen [2016])

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