Me

Crow TAO

Shanghai

[Something fun]

Edinburgh

[Cobordysm hypothesis]

[Quantum Information& Typicality]

Amsterdam

[Something fun]

~~Gravity,~~ Chaos, Matrix Model

Symmetry Breaking

Crow TAO

Supervisor: Diego M. Hofman

University of Amsterdam

Based on the paper by

Tarek Anous & Diego M. Hofman

Michael Winer & Brian Swingle

And many more…

Minimal introduction of Quantum chaos

- “Quantum Chaos” is a name for a collection of different but related phenomenas of non-integrable quantum mechanical systems
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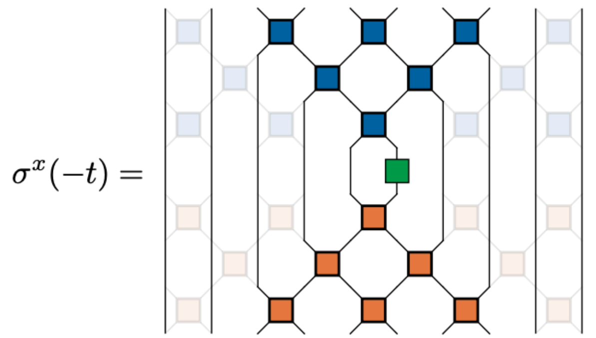
$$\mathscr{C}(r,t) \sim \frac{1}{N}\exp(\lambda t)$$

-



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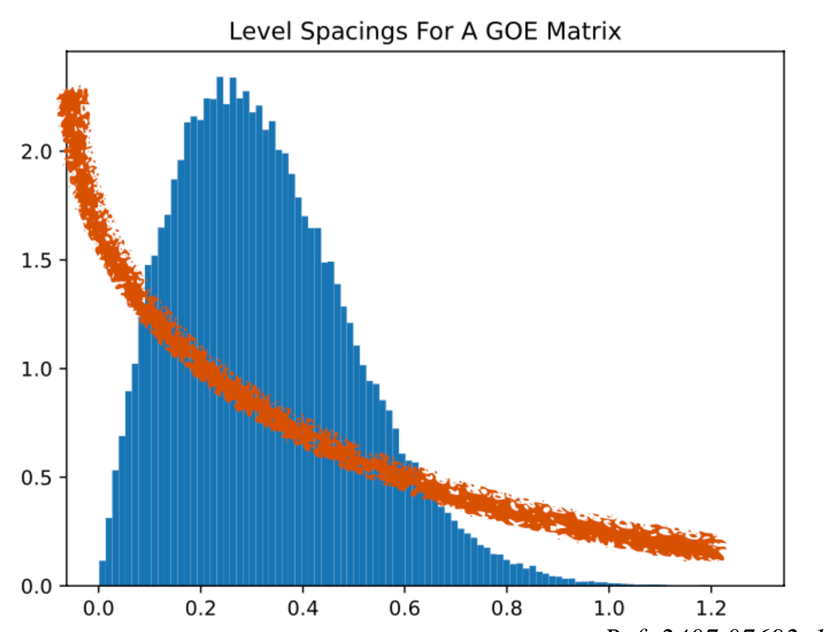

Ref: 2407.07692v1

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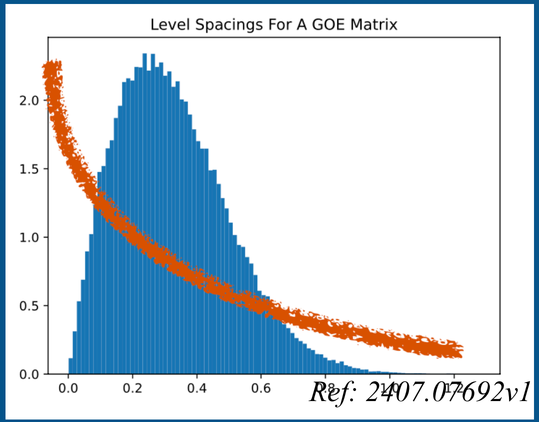

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Eigenvalue thermalization hypothesis

ergodicity hierarchy

unitary design

complexity

entanglement dynamics

…

Gravity

Two “Slogens”

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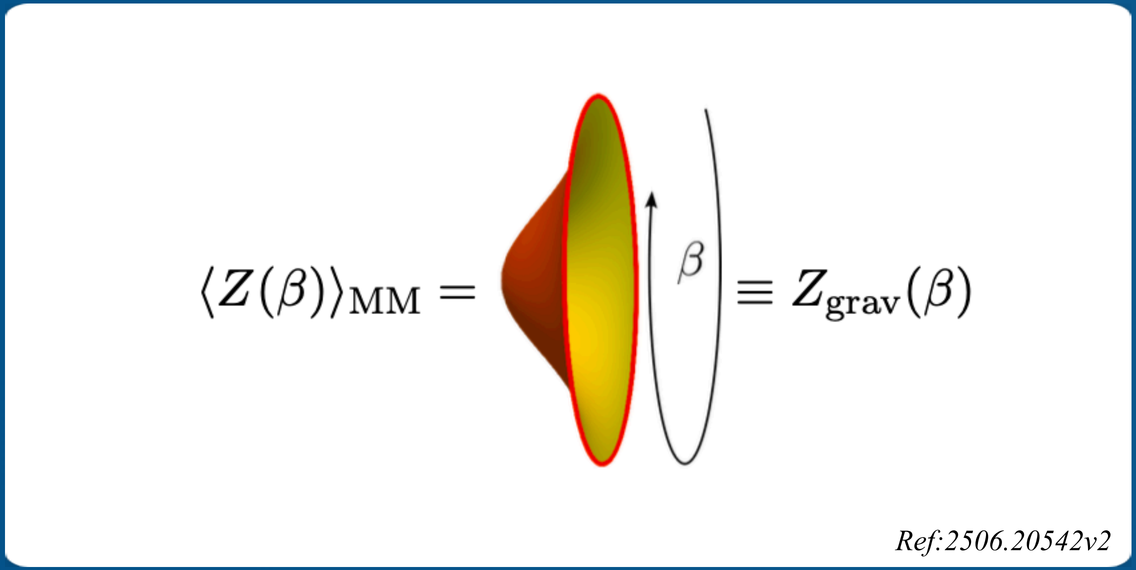
- 

Spectral Form Factor

Given hamiltonian H, can define density of states $\rho(E) = \sum_\lambda \delta(E - \lambda)$

The "loop operator" is defined through Fourier transform $Z(iT) \equiv \int_{-\infty}^{\infty} e^{-iET}\rho(E)dE = \sum_\lambda e^{-i\lambda T} = \mathrm{tr}\, e^{-iHT}$

The SFF is defined through as the square of loop operator $\mathrm{SFF}(T) = Z(iT)Z(-iT) = \mathrm{tr}\, e^{-iHT}\, \mathrm{tr}\, e^{iHT} = \sum_{\lambda_1,\lambda_2} e^{-i(\lambda_1 - \lambda_2)T}$

"Wiggliness" at resolution 1/T

Spectral Form Factor

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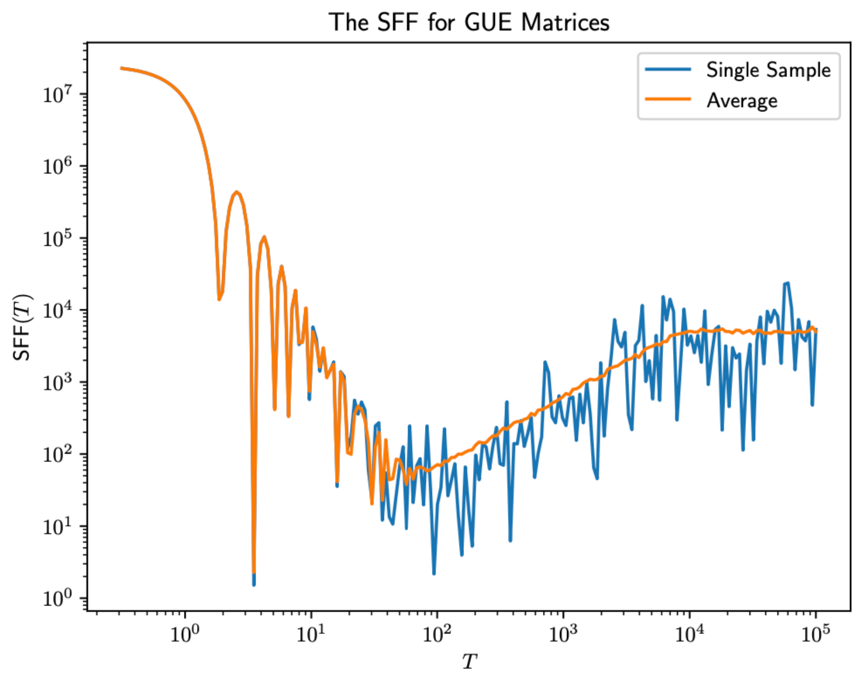
$$\langle \cdot \rangle_{\mathrm{MM}} = \frac{1}{\mathcal{N}} \int \mathrm{d}H(\,\cdot\,)e^{-\mathrm{Tr}\, V(H)}$$



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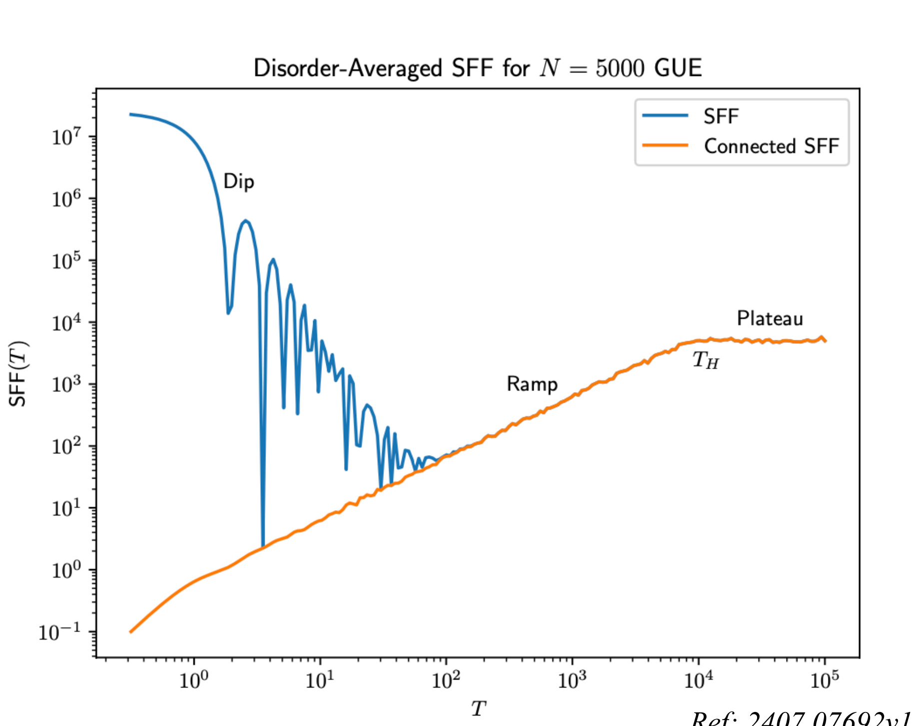

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Three regions: Slope, Ramp, Plateau

Slope: Controlled by disconnected part, not universal, lack of constructive interference

Ramp: Linear in time, reflection of level repulsion

Plateau: Finiteness, discreteness

Two point function that saturates to a constant value at late time!

WHYY?

$\langle Z(\beta) \rangle = 0$

GUE matrix model corresponds to an effective theory that describes the symmetry broken phase

Sum Rule for one point function

$$\langle Z(it)\rangle_{\text{MM}} = \langle \text{Tr}\, e^{-iHt}\rangle$$

Potential independent, relies only on the finiteness of the model

At t = 0 $\langle Z(0)\rangle_{\text{MM}} = \langle \text{Tr}\, \mathbb{I}\rangle_{\text{MM}} = L$

At t ~ L $\langle Z(it)\rangle_{\text{MM}} = \int_{-\infty}^{\infty} \mathrm{d}E e^{-L(iE\tilde{t}+v(E))} \times$ (polynomial in E)

Saddle point $\langle Z(iL\tilde{t}\,)\rangle_{\text{MM}} \propto e^{-L\left(iE^*\tilde{t} + v(E^*)\right)} = e^{-L\left(v(E^*) - E^* v'(E^*)\right)}$

GUE potential $\langle Z(it)\rangle_{\text{MM}} \underset{t\sim L}{\propto} e^{-\frac{(t\lambda)^2}{2L} +}$ *subleading...*

$$\langle Z(it)\rangle_{\text{MM}} \underset{t\sim L}{\propto} e^{-t\Lambda_{IR}(t)}$$

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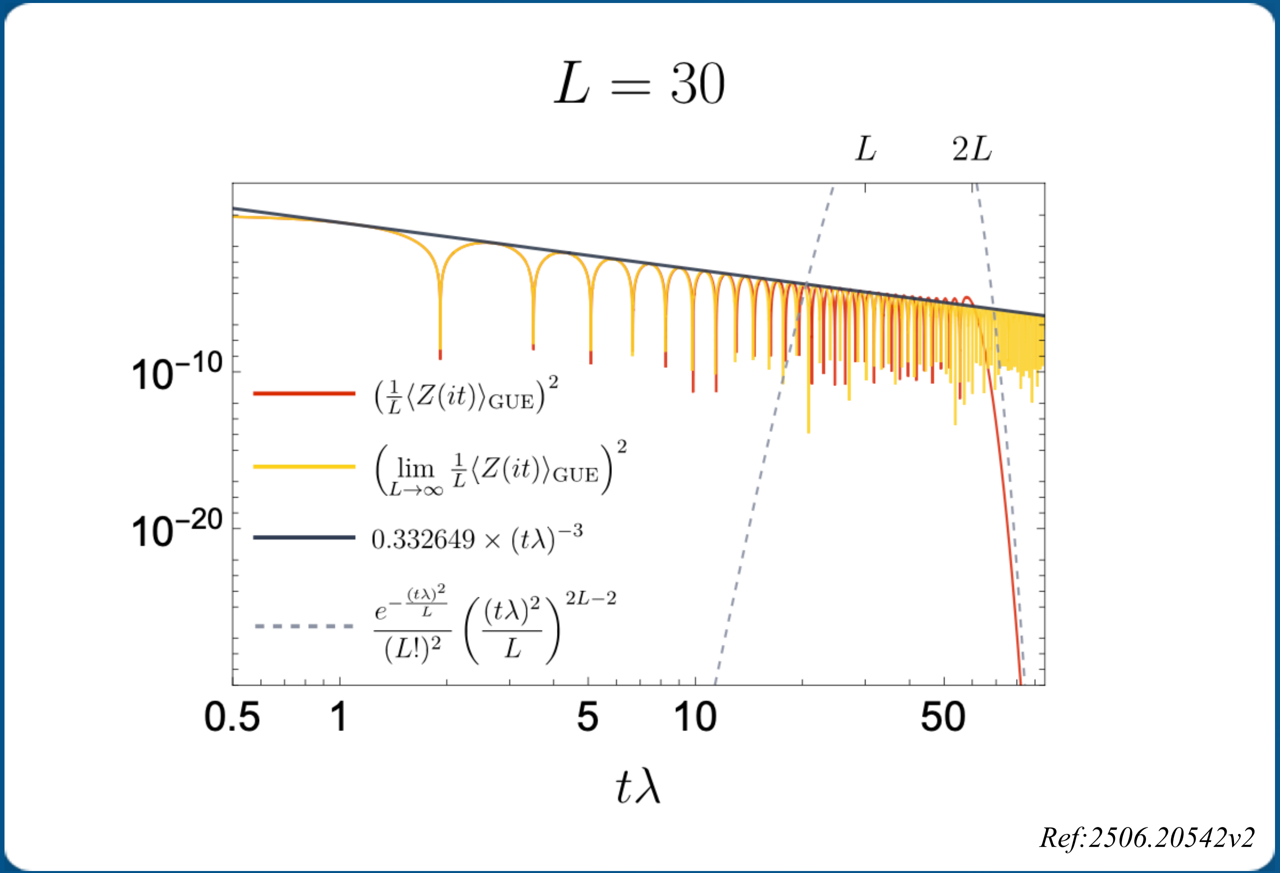

Coincide at early time

Start to perform differently at volume scale

At late time exponential decay

$$\langle Z(\beta)\rangle_{\mathrm{GUE}} = \frac{1}{\mathcal{N}_{\mathrm{GUE}}}\int [\mathrm{d}H]\left[\mathrm{Tr}\, e^{-\beta H}\right] e^{-\frac{L}{2\lambda^2}\mathrm{Tr}\, H^2}$$

Sum Rule for two point function

$$\mathrm{S}(t) \equiv \langle Z(it)Z(-it)\rangle_{\mathrm{MM}}$$

At t = 0 $\mathrm{S}^c(0) = \langle Z(0)Z(0)\rangle^c_{\mathrm{MM}} = -L$

At t ~ L $\mathrm{S}^c(t) \underset{t\sim L}{\propto} -e^{-2(V(E^*) - E^*V'(E^*))}$

GUE potential $\mathrm{S}^c(t) \propto -e^{-\frac{(t\lambda)^2}{L}+}$ *subleading...*

$$\mathrm{S}^c(t) \underset{t\sim L}{\propto} -e^{-2t\Lambda_{IR}(t)}$$

Questions?

Thank you for your attention.