What can we see with the FCC-ee?

Sensitivity to ALPs at FCC-ee ZH run at a center of mass energy $\sqrt{s}$ = 240 GeV

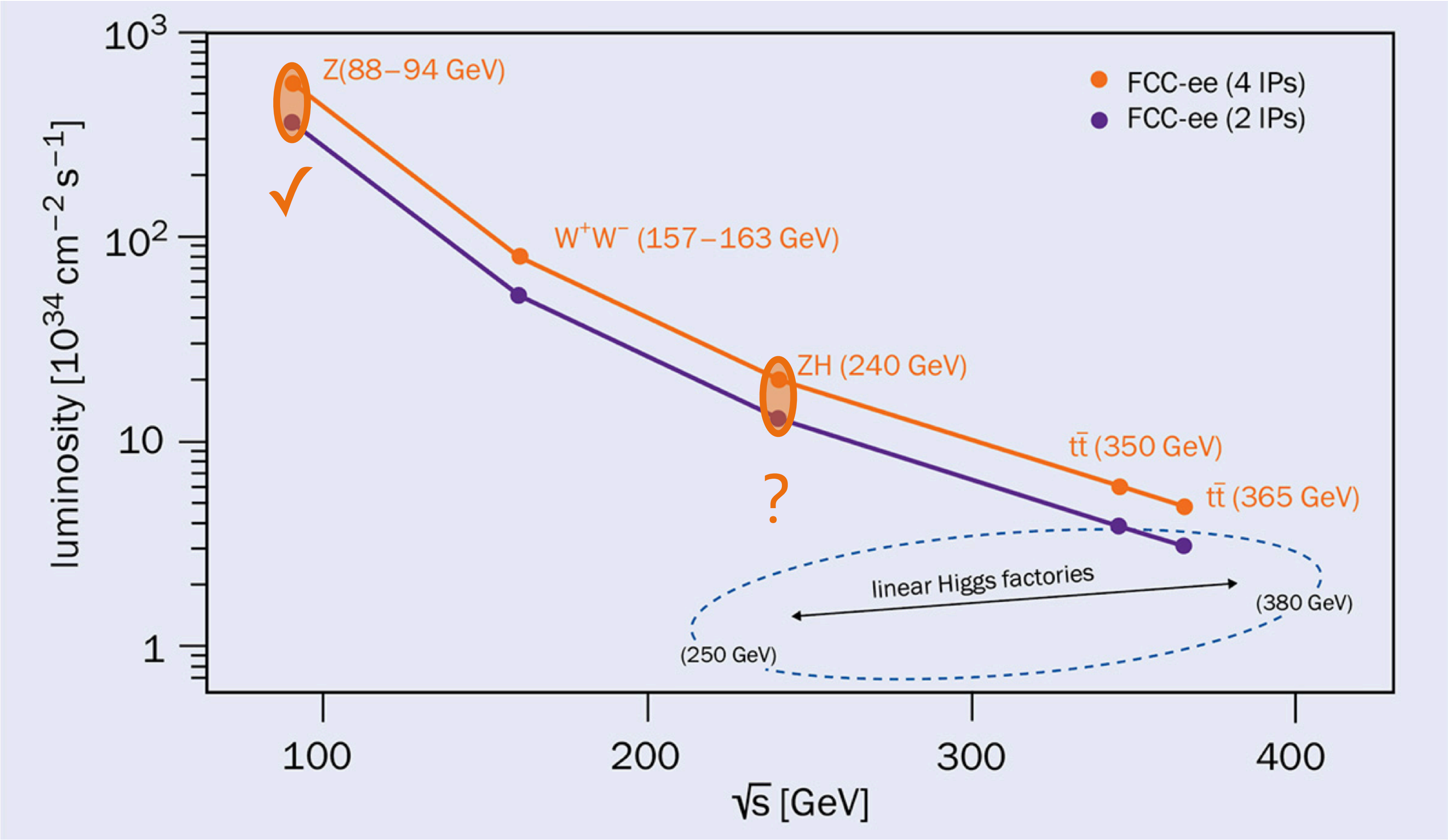
HELMHOLTZ

DESY.

30.06.25

Center of mass energy vs. luminosity

Interaction points of the FCC-ee



Processes at ZH (240 GeV)

On Tuesday, we discussed:

1) $e^+e^- \rightarrow aZ,\ a \rightarrow \gamma\gamma$
2) $e^+e^- \rightarrow aZZ,\ a \rightarrow \gamma\gamma$
3) $e^+e^- \rightarrow aZH,\ a \rightarrow \gamma\gamma$

For reference, I will be using m_a (ALP mass) set to 10 GeV and $c_{\gamma\gamma}$ (coupling strength) set to 1.6.

This corresponds to first row of Table 1 in Elnura's thesis.

Keeping everything the same except for process (see above) and beam energy. Only ran 1 event to start.

m_a[GeV]	$c_{\gamma\gamma}$	σ[pb]
10	1.6	6.764
0.3	1.6	7.014
1.0	1.0	2.739
10.0	0.4	0.423
0.3	0.4	0.438

$e^+e^- \to aZ,\ a \to \gamma\gamma$

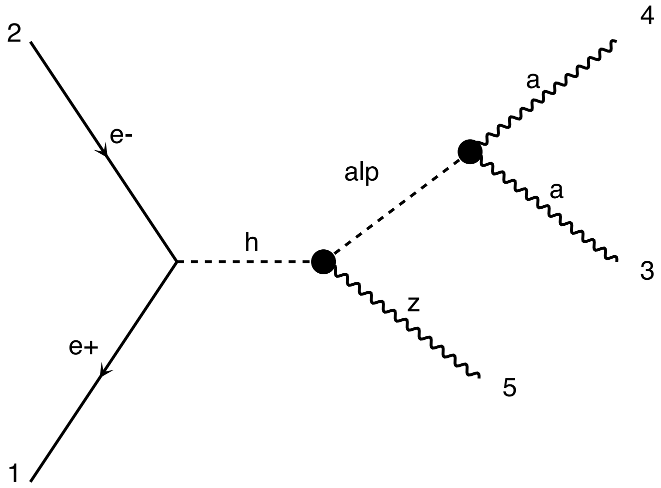


diagram 1 NP=2, QCD=0, QED=3

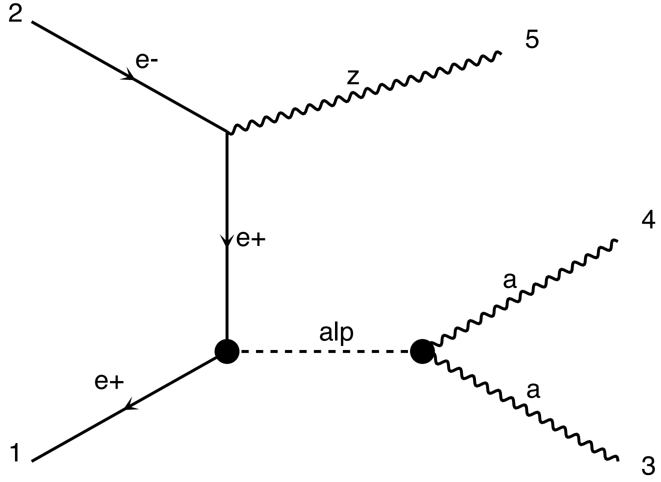


diagram 2 NP=2, QCD=0, QED=3

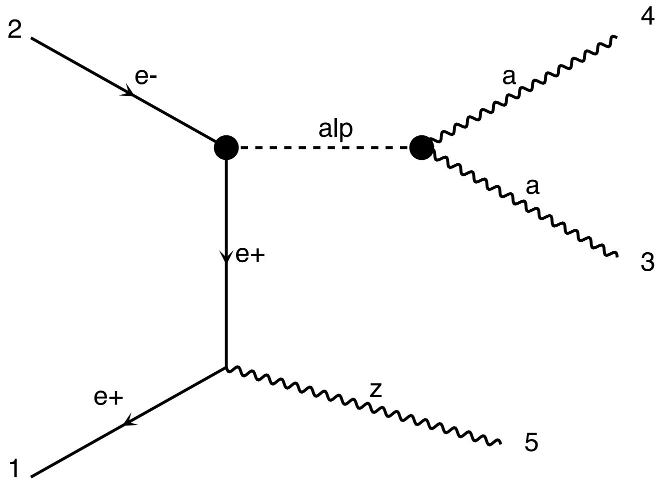


diagram 3 NP=2, QCD=0, QED=3

$e^+e^- \to aZZ,\ a \to \gamma\gamma$

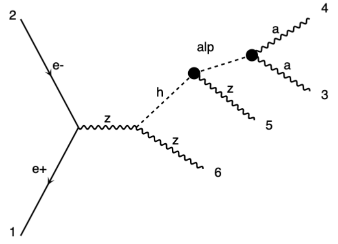


diagram 1 NP=2, QCD=0, QED=4

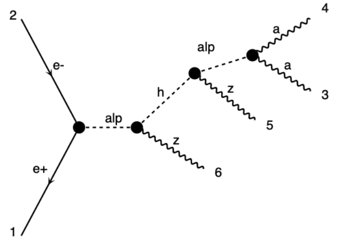


diagram 2 NP=4, QCD=0, QED=2

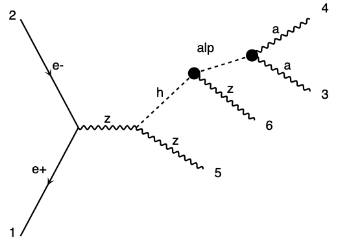


diagram 3 NP=2, QCD=0, QED=4

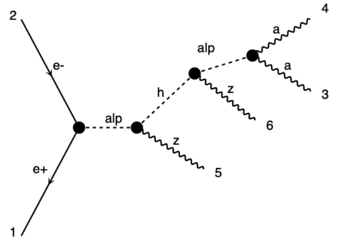


diagram 4 NP=4, QCD=0, QED=2

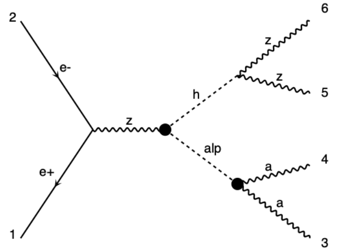


diagram 5 NP=2, QCD=0, QED=4

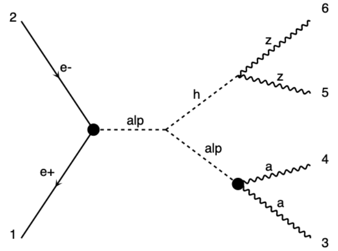


diagram 6 NP=2, QCD=0, QED=4

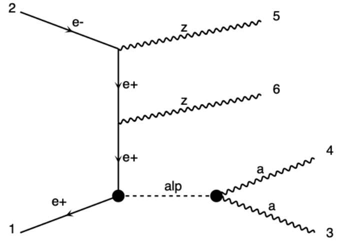


diagram 7 NP=2, QCD=0, QED=4

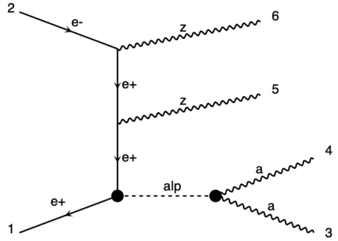


diagram 8 NP=2, QCD=0, QED=4

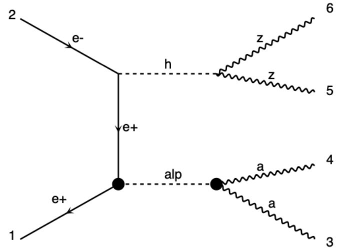


diagram 9 NP=2, QCD=0, QED=4

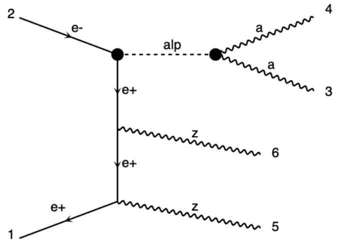


diagram 10 NP=2, QCD=0, QED=4

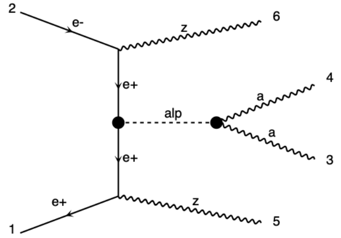


diagram 11 NP=2, QCD=0, QED=4

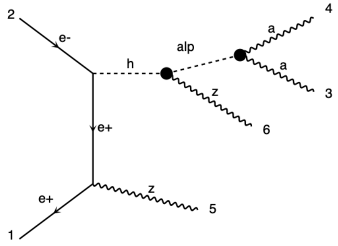


diagram 12 NP=2, QCD=0, QED=4

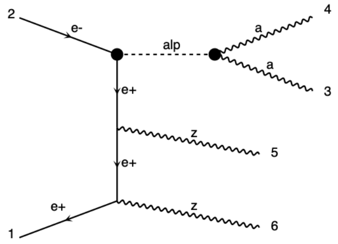


diagram 13 NP=2, QCD=0, QED=4

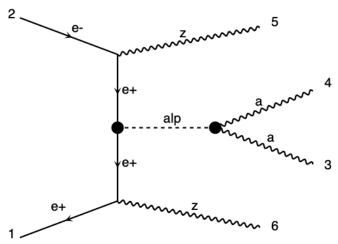


diagram 14 NP=2, QCD=0, QED=4

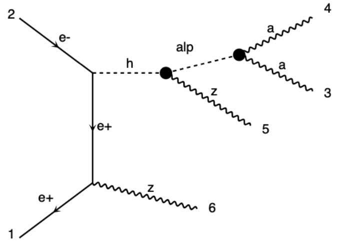


diagram 15 NP=2, QCD=0, QED=4

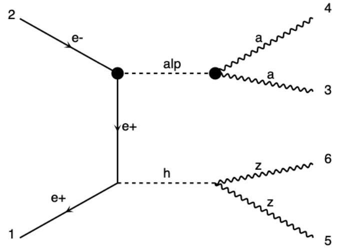


diagram 16 NP=2, QCD=0, QED=4

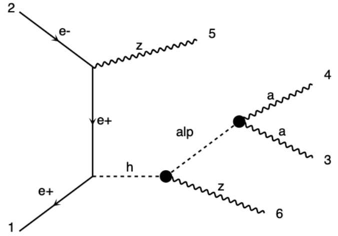


diagram 17 NP=2, QCD=0, QED=4

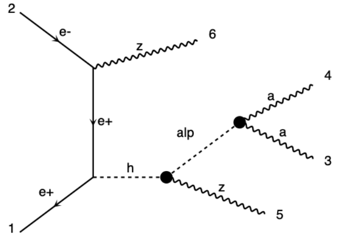
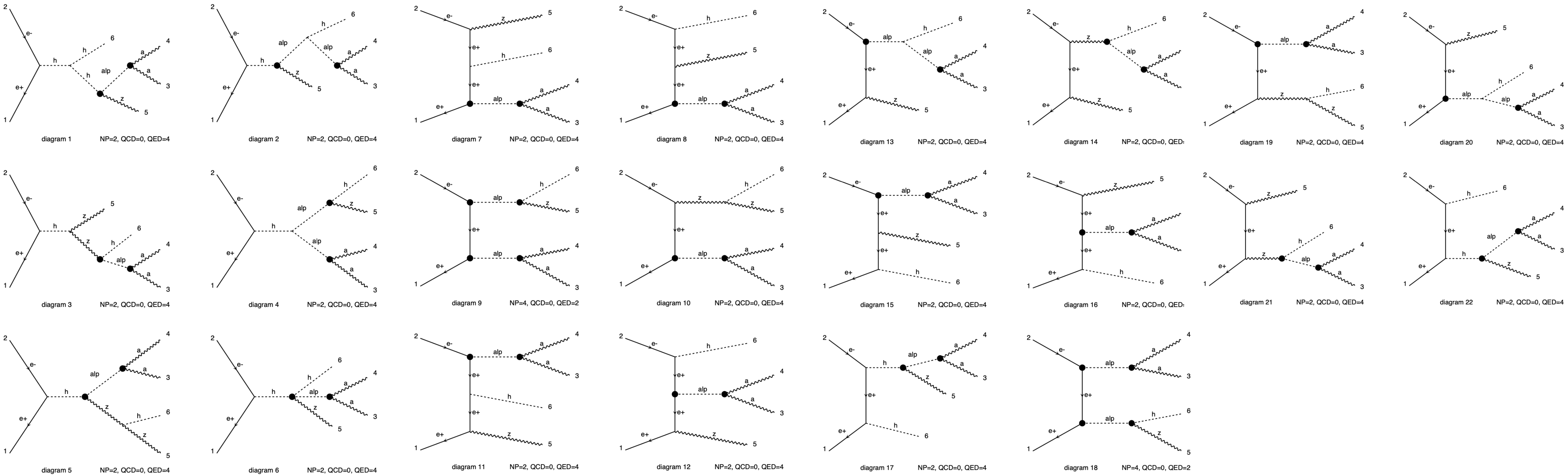


diagram 18 NP=2, QCD=0, QED=4

$e^+e^- \rightarrow aZH,\ a \rightarrow \gamma\gamma$



Couplings

With only the $c_{\gamma\gamma}$ coupling, the processes on the previous three slides give zero cross section.

Maybe we should look into the other couplings a little more closely...

ALP Lagrangian

kinetic energy — mass — fermions

[Georgi, Kaplan, Randall (1986)]

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2} a^2 + \sum_f \frac{c_{ff}}{2} \frac{\partial^\mu a}{\Lambda} \bar{f}\gamma_\mu\gamma_5 f$$

$$+ g_s^2 C_{GG} \frac{a}{\Lambda} G_{\mu\nu}^A \tilde{G}^{\mu\nu,A} + g^2 C_{WW} \frac{a}{\Lambda} W_{\mu\nu}^A \tilde{W}^{\mu\nu,A} + g'^2 C_{BB} \frac{a}{\Lambda} B_{\mu\nu} \tilde{B}^{\mu\nu} , \quad (1)$$

$SU(3)_c$; gluons — $SU(2)_L$; W gauge bosons — $U(1)_Y$; hypercharge gauge bosons

ALP Lagrangian continued

In the broken phase of the electroweak symmetry, the ALP couples to the photon and the Z boson as

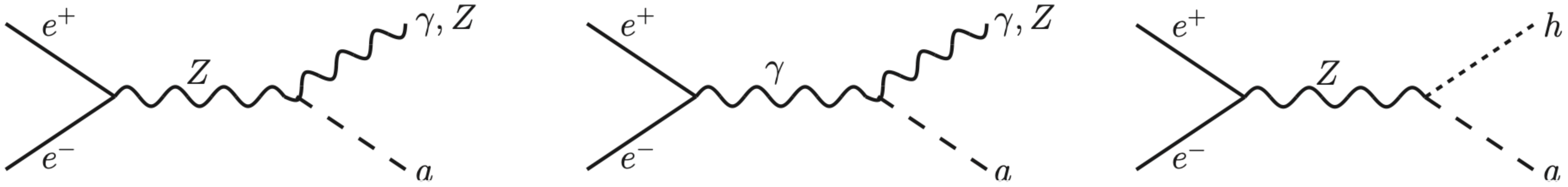
$$\mathcal{L}_{\text{eff}} \ni e^2 \, C_{\gamma\gamma} \, \frac{a}{\Lambda} \, F_{\mu\nu} \, \tilde{F}^{\mu\nu} + \frac{2e^2}{s_w c_w} \, C_{\gamma Z} \, \frac{a}{\Lambda} \, F_{\mu\nu} \, \tilde{Z}^{\mu\nu} + \frac{e^2}{s_w^2 c_w^2} \, C_{ZZ} \, \frac{a}{\Lambda} \, Z_{\mu\nu} \, \tilde{Z}^{\mu\nu} \, . \tag{2}$$

The relevant Wilson coefficients are given by

$$C_{\gamma\gamma} = C_{WW} + C_{BB} , \qquad C_{\gamma Z} = c_w^2 \, C_{WW} - s_w^2 \, C_{BB} , \qquad C_{ZZ} = c_w^4 \, C_{WW} + s_w^4 \, C_{BB} \, , \tag{3}$$

where s_w and c_w are the sine and cosine of the weak mixing angle, respectively. The exotic decay $Z \to \gamma a$ is governed by the Wilson coefficient $C_{\gamma Z}$.

Differential cross sections



$$\frac{d\sigma(e^+e^- \to \gamma a)}{d\Omega} = 2\pi\alpha\alpha^2(s)\frac{s^2}{\Lambda^2}\left(1-\frac{m_a^2}{s}\right)^3\left(1+\cos^2\theta\right)\left(|V_\gamma(s)|^2+|A_\gamma(s)|^2\right), \tag{16}$$

$$\frac{d\sigma(e^+e^- \to Za)}{d\Omega} = 2\pi\alpha\alpha^2(s)\frac{s^2}{\Lambda^2}\lambda^{\frac{3}{2}}\left(x_a, x_Z\right)\left(1+\cos^2\theta\right)\left(|V_Z(s)|^2+|A_Z(s)|^2\right), \tag{17}$$

$$\frac{d\sigma(e^+e^- \to ha)}{d\Omega} = \frac{\alpha}{128\pi\, c_w^2 s_w^2}\frac{|C_{Zh}^{\text{eff}}|^2}{\Lambda^2}\frac{s\, m_Z^2}{(s-m_Z^2)^2}\lambda^{\frac{3}{2}}\left(x_a, x_h\right)\sin^2\theta\left(g_V^2+g_A^2\right), \tag{18}$$

$$V_\gamma(s) = \frac{C_{\gamma\gamma}^{\text{eff}}}{s} + \frac{g_V}{2c_w^2 s_w^2}\frac{C_{\gamma Z}^{\text{eff}}}{s-m_Z^2+im_Z\Gamma_Z}, \qquad A_\gamma(s) = \frac{g_A}{2c_w^2 s_w^2}\frac{C_{\gamma Z}^{\text{eff}}}{s-m_Z^2+im_Z\Gamma_Z}, \tag{19}$$

$$V_Z(s) = \frac{1}{c_w s_w}\frac{C_{\gamma Z}^{\text{eff}}}{s} + \frac{g_V}{2c_w^3 s_w^3}\frac{C_{ZZ}^{\text{eff}}}{s-m_Z^2+im_Z\Gamma_Z}, \qquad A_Z(s) = \frac{g_A}{2c_w^3 s_w^3}\frac{C_{ZZ}^{\text{eff}}}{s-m_Z^2+im_Z\Gamma_Z}, \tag{20}$$

Exotic decay rates

$$\Gamma(Z \to \gamma a) = \frac{\alpha\, \alpha(m_Z)\, m_Z^3}{96\pi^3 s_w^2 c_w^2 f^2} \left|c_{\gamma Z}\right|^2 \left(1 - \frac{m_a^2}{m_Z^2}\right)^3 , \tag{10}$$

$$\Gamma(h \to Za) = \frac{m_h^3 v^2}{64\pi\, f^6} |c_{Zh}|^2 \lambda^{3/2}\left(\frac{m_Z^2}{m_h^2}, \frac{m_a^2}{m_h^2}\right) , \tag{11}$$

$$\Gamma(h \to aa) = \frac{m_h^3\, v^2}{32\pi\, f^4} |c_{ah}|^2 \left(1 - \frac{2m_a^2}{m_h^2}\right)^2 \sqrt{1 - \frac{4m_a^2}{m_h^2}} \, . \tag{12}$$

Couplings

```
# set to electron beams (0 for ele, 1 for proton)
set lpp1 0
set lpp2 0
set ebeam1 45.594
set ebeam2 45.594
# set ALP mass
set Ma ALPMASS
# set ALP couplings
set cWW = 0.0
set CYY = ALPCOUPLING
set cGG = 0.
set cuu = 0.
set cdd = 0.
set ccc = 0.
set css = 0.
set ctt = 0.
set cbb = 0.
set cee = 0.
set cmumu = 0.
set ctautau = 0.
set cah = 0.
set cZh5 = 0.
```

Couplings

Playing around with the couplings for:

1) $e^+e^- \to aZ,\, a \to \gamma\gamma$

- Set all couplings to zero except CYY and cah (0.01) → Survey return zero cross section
- Set all couplings to zero except CYY and **cZh5** (0.01) → 4.768e-11 +- 1.058e-13 pb

How does cross section depend on cZh5?

Coupling	Cross Section
0.001	4.783e-13 +- 9.372e-16 pb
0.01	4.749e-11 +- 1.097e-13 pb
0.1	4.767e-09 +- 1.054e-11 pb
0.5	1.19e-07 +- 2.743e-10 pb
1.0	4.781e-07 +- 1.557e-09 pb

ALP Lagrangian continued

Interactions with the Higgs boson, ϕ, appear only at dimension-6 and higher,

$$\mathcal{L}_{\text{eff}}^{D\geq 6} = \boxed{\frac{C_{ah}}{\Lambda^2}(\partial_\mu a)(\partial^\mu a)\,\phi^\dagger\phi} + \boxed{\frac{C_{Zh}}{\Lambda^3}(\partial^\mu a)\left(\phi^\dagger\, iD_\mu\,\phi + \text{h.c.}\right)\phi^\dagger\phi} + \dots\,, \tag{5}$$

$h \to aa$ $\quad$ $h \to Za$

where the first operator mediates the decay $h \to aa$, while the second one is responsible for $h \to Za$. Note that a possible dimension-5 operator coupling the ALP to the Higgs current vanishes by the equations of motion. However, in theories where a heavy new particle acquires most of its mass through electroweak symmetry breaking, the non-polynomial dimension-5 operator

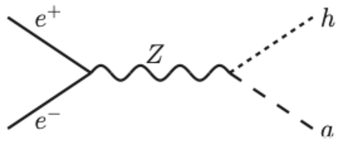
$$\frac{C_{Zh}^{(5)}}{\Lambda}(\partial^\mu a)\left(\phi^\dagger\, iD_\mu\,\phi + \text{h.c.}\right)\ln\frac{\phi^\dagger\phi}{\mu^2} \tag{6}$$

can be present [25, 28, 66, 67]. In our analysis we allow for the presence of such an operator.

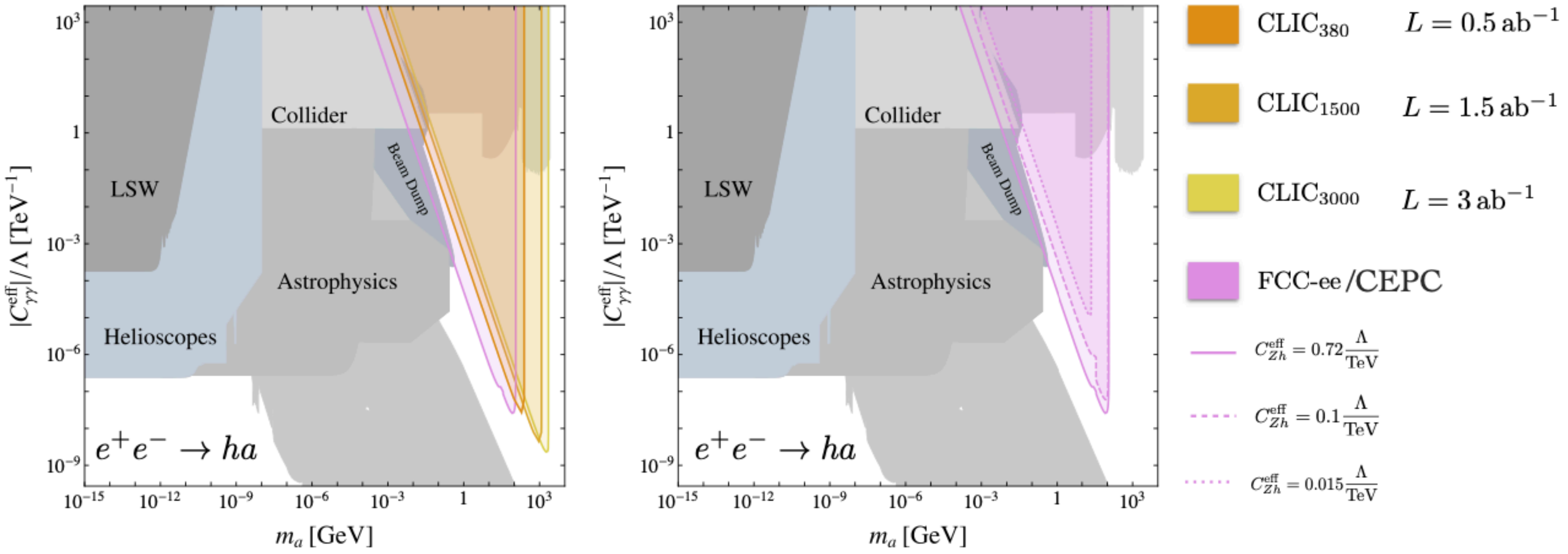
Sensitivity

https://indico.mit.edu/event/876/contributions/2864/attachments/1086/1793/Thamm_ALPs.pdf

- ALP associated production with a H



- ALP decay into photons



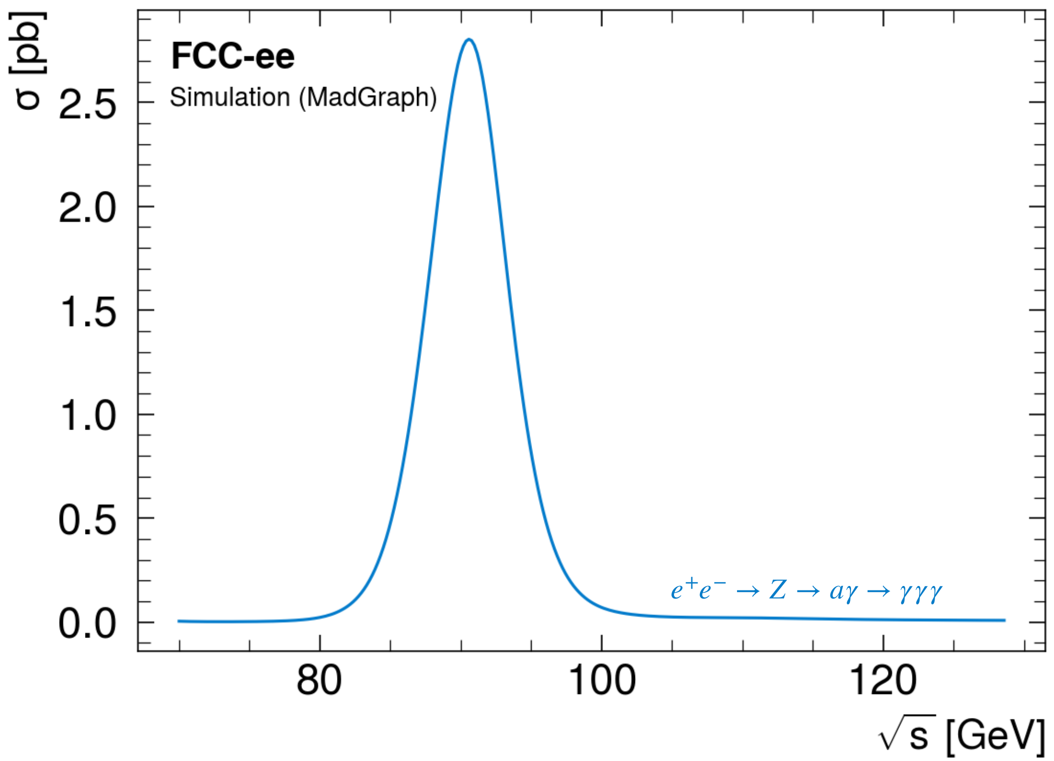
02.07.25

Zero cross section $e^+e^- \rightarrow Z \rightarrow a\gamma \rightarrow \gamma\gamma\gamma$ at higher centre-of-mass energy?

Reran the same process Elnura generated, but at the Z-Higgs associated production center of mass energy (240 GeV). To keep things consistent, only kept $c_{\gamma\gamma}$ coupling. Using m_a of 1.0 GeV and $c_{\gamma\gamma}$ of 1.0

Got zero cross section → What is happening?
Seems to work at 91.188 GeV, but not 240 GeV. Decided to start at 240 GeV and keep lowering centre-of-mass energy until there was a non-zero cross section.

√s/2	Cross Section
35	0.004201 ± 1.205e-05 pb
40	0.0225 ± 6.359e-05 pb
45.594	2.732 ± 0.0074 pb
50	0.07077 ± 0.0001945 pb
55	0.02093 ± 5.927e-05 pb
60	0.01154 ± 2.86e-05 pb
62.5	0.009352 ± 2.504e-05 pb
63.75	0.008561 ± 2.382e-05 pb
64.0625	0.008402 ± 3.167e-05 pb
64.21875	0.008339 ± 2.23e-05 pb
64.296875	0.008304 ± 2.27e-05 pb
64.3	0.008293 ± 2.237e-05 pb
64.31	Survey return zero cross section.



Breit-Wigner distribution cutoff

The Z boson decay $Z \to \gamma a$ is on shell and the process $e^+e^- \to Z \to a\gamma$ is resonant on the Z pole

According to Olivier Mattelaer,

"The run_card parameter bwcutoff defines what is considered to be on-shell s-channel resonances: The resonance if considered to be on-shell if the invariant mass of an s-channel resonance is within M +/- bwcutoff*Gamma, and will then be included as a comment particle in the LHE event file (with status code 2). The value of bwcutoff does not affect the cross section of any processes, except if

1) you use the decay chain formalism: [a b > c d , c > e f, d > g h] Decay chain processes are strictly valid only in the narrow width limit, and we have therefore chosen to allow only production of on-shell particles. This means that the cross section will get smaller as bwcutoff gets smaller, since more of the tails of the Breit Wigner distributions are cut off."

Check.

Our allowed energy range around resonance is: $M_Z - \text{bwcutoff} \times \Gamma_Z < \sqrt{s} < M_Z + \text{bwcutoff} \times \Gamma_Z$

Given:

$M_Z = 91.1880$ GeV

$\Gamma_Z = 2.4955$ GeV

$\text{bwcutoff} = 15$

from run_card.dat

```
#*********************************************************************
# BW cutoff (M+/-bwcutoff*Gamma) ! Define on/off-shell for "$" and decay
#*********************************************************************
  15.0  = bwcutoff ! (M+/-bwcutoff*Gamma)
 #*********************************************************************
```

Putting this together:

$$53.7555 \text{ GeV} < \sqrt{s} < 128.6205 \text{ GeV}$$

MadGraph gives a zero cross section somewhere between 128.6 GeV and 128.62 GeV, so this is consistent!
Note that this only applies when you use decay chain formalism or explicitly forbid s-channel particles to be on shell.

Though we aren't particularly interested in this process at the ZH run, this is still something to be aware of, as it may show up again in calculations of cross sections for other processes.

MadGraph bug?

```
Command "generate_events run_01" interrupted with error:
TypeError : not enough arguments for format string
Please report this bug on https://bugs.launchpad.net/mg5amcnlo
More information is found in '/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/MGOutput/ALP_1p0GeV_cYY1p0/ALP_1p0GeV_cYY1p0_0/run_01_tag_1_debug.log'.
Please attach this file to your report.
```

```
#************************************************************
#*                MadGraph5_aMC@NLO/MadEvent                *
#*                                                          *
#*                *                       *                 *
#*                  *        * *        *                   *
#*                    * * * * 5 * * * *                     *
#*                  *        * *        *                   *
#*                *                       *                 *
#*                                                          *
#*                                                          *
#*         VERSION 3.6.3                 2025-06-12         *
#*                                                          *
#*    The MadGraph5_aMC@NLO Development Team - Find us at   *
#*    https://server06.fynu.ucl.ac.be/projects/madgraph     *
#*                                                          *
#************************************************************
#*                                                          *
#*               Command File for MadEvent                  *
#*                                                          *
#*     run as ./bin/madevent.py filename                    *
#*                                                          *
#************************************************************
generate_events run_01
Traceback (most recent call last):
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/extended_cmd.py", line 1548, in onecmd
    return self.onecmd_orig(line, **opt)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/extended_cmd.py", line 1497, in onecmd_orig
    return func(arg, **opt)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/madevent_interface.py", line 2406, in do_generate_events
    self.run_generate_events(switch_mode, args)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/common_run_interface.py", line 7897, in new_fct
    scan_over_run_card(original_fct, obj, *args, **opts)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/common_run_interface.py", line 7829, in scan_over_run_card
    return original_fct(obj, *args, **opts)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/madevent_interface.py", line 2624, in run_generate_events
    self.exec_cmd('combine_events', postcmd=False,printcmd=False)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/extended_cmd.py", line 1577, in exec_cmd
    stop = Cmd.onecmd_orig(current_interface, line, **opt)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/extended_cmd.py", line 1497, in onecmd_orig
    return func(arg, **opt)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/madevent_interface.py", line 3768, in do_combine_events
    Gdirs = self.remove_empty_events(Gdirs)
  File "/afs/cern.ch/user/c/cdorofee/MG/MG5_aMC_v3_6_3/madgraph/interface/madevent_interface.py", line 6238, in remove_empty_events
    logger.warning('Gdirectory %s has no events.lhe file. (no BW config found %s times)' % G)
TypeError: not enough arguments for format string
```

```python
############################################################################
def remove_empty_events(self, Gdir):
    """return Gdir strip from the one providing empty events.lhe files."""

    reasons = collections.defaultdict(list)
    Gdirs = Gdir[:]
    for G in Gdirs[:]:
        try:
            size = os.path.getsize(pjoin(G, 'events.lhe'))
        except Exception as error:
            size = 0
        if size <10:
            Gdirs.remove(G)
            try:
                log = misc.BackRead(pjoin(G, 'log.txt'))
            except Exception as error:
                log = misc.BackRead(pjoin(G, 'run1_app.log'))

            found = -1
            for line in log:
                if 'Deleting file events.lhe' in line:
                    found = 0
                elif "Impossible BW configuration" in line:
                    reasons['bwconfig'].append(G)
                    break
                elif found < -150:
                    reasons['not found'].append(G)
                    Gdirs.append(G)
                    break
                elif found < 0:
                    found -= 1
                elif 'Loosen cuts or increase max_events' in line:
                    reasons['cuts'].append(G)
                    break
                elif 'all returned zero' in line:
                    reasons['zero'].append(G)
                    break
                elif found > 5:
                    reasons['unknown'].append(G)
                    break
                else:
                    found += 1

    if len(reasons):
        logger.debug('Reasons for empty events.lhe:')
        if len(reasons['unknown']):
            logger.debug('  - unknown: %s' % len(reasons['unknown']))
            logger.log(10, '    DETAIL:' + ','.join(['/'.join(G.rsplit(os.sep)[-2:]) for G in reasons['unknown'][:10]]))
        if len(reasons['not found']):
            logger.debug('  - not found in log: %s' % len(reasons['not found']))
            logger.log(10, '    DETAIL:' + ','.join(['/'.join(G.rsplit(os.sep)[-2:]) for G in reasons['not found'][:10]]))
        if len(reasons['zero']):
            logger.debug('  - zero amplitudes: %s' % len(reasons['zero']))
            logger.log(10, '    DETAIL:' + ','.join(['/'.join(G.rsplit( os.sep)[-2:]) for G in reasons['zero'][:10]]))
        if len(reasons['bwconfig']):
            critical_bwconfig = set()
            for G in reasons['bwconfig']:
                base = G.rsplit('.',1)[0]
                if any(G2.startswith(base) for G2 in Gdirs):
                    continue
                else:
                    critical_bwconfig.add(os.sep.join(base.rsplit(os.sep)[-2:]))
            for G in critical_bwconfig:
                logger.warning('Gdirectory %s has no events.lhe file. (no BW config found %s times)' % G)

            logger.debug('  - impossible BW configuration: %s' % len(reasons['bwconfig']))
            logger.debug('  - channel with no possible BW configuration: %s' %  len(critical_bwconfig))
```

TypeError in remove_empty_events: logger.warning format string expects 2 arguments but gets 1.

This occurs in the remove_empty_events method in madevent_interface.py at line 6238. The issue seems to be that the warning message expects two format arguments, but only one (G) is provided.

As a quick fix, I removed the " %s times" portion from the string. I'm not sure if the Gdirectory is meant to be matched with a specific count, but dropping the second placeholder resolved the crash on my end.

Going through each process systematically

What are the associated cross sections for the following processes?

1) $e^+e^- \rightarrow aH,\ a \rightarrow \gamma\gamma$
2) $e^+e^- \rightarrow aZ,\ a \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow a\gamma,\ a \rightarrow \gamma\gamma$
3) $e^+e^- \rightarrow aZZ,\ a \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow a\gamma\gamma,\ a \rightarrow \gamma\gamma$
4) $e^+e^- \rightarrow aZH,\ a \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow a\gamma H,\ a \rightarrow \gamma\gamma$

First, look at all possible diagrams and determine relevant couplings
MadGraph generates Feynman diagrams before calculating cross sections and using couplings

1) $e^+e^- \to aH,\ a \to \gamma\gamma$

coupling(s): CYY, cZh5

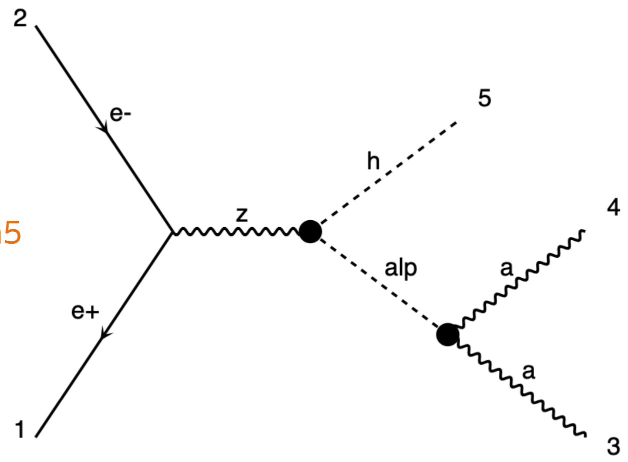


diagram 1 NP=2, QCD=0, QED=3

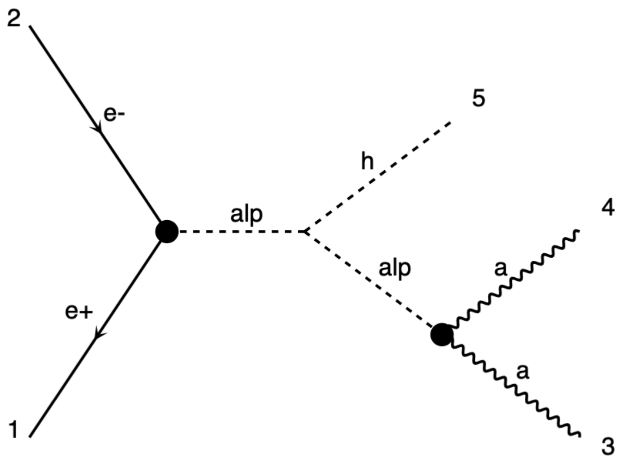


diagram 2 NP=2, QCD=0, QED=3

coupling(s): CYY, cah

coupling(s): CYY, cee

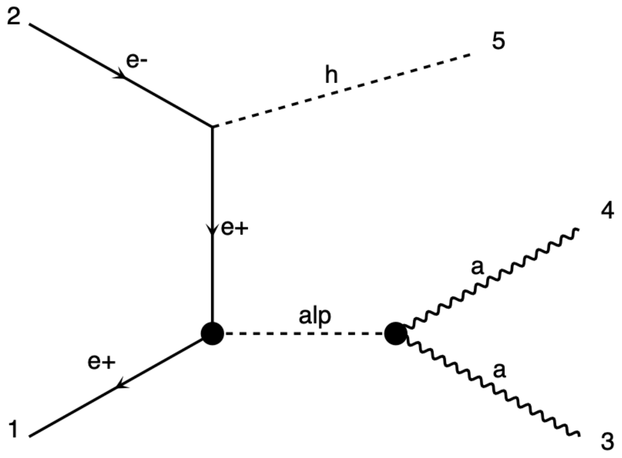


diagram 3 NP=2, QCD=0, QED=3

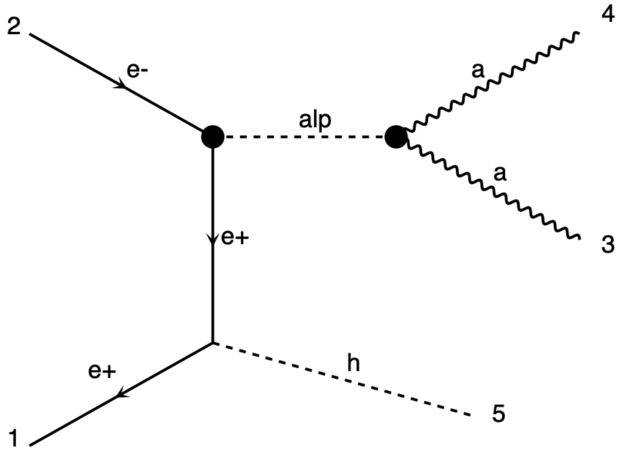


diagram 4 NP=2, QCD=0, QED=3

coupling(s): CYY, cee

2a) $e^+e^- \to aZ,\ a \to \gamma\gamma$

coupling(s): CYY, cZh5

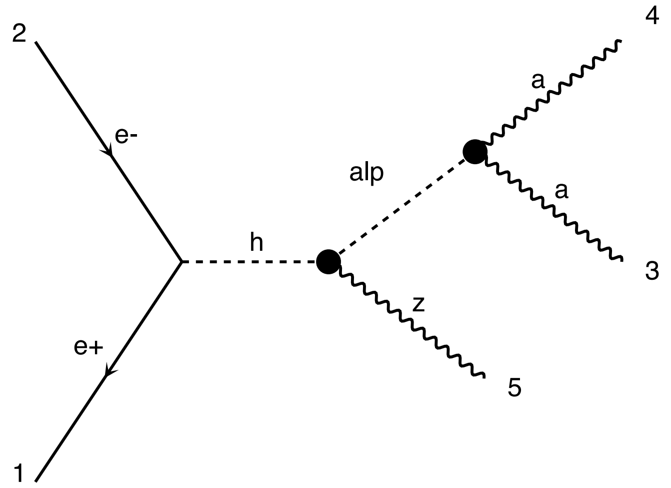


diagram 1 NP=2, QCD=0, QED=3

coupling(s): CYY, cee

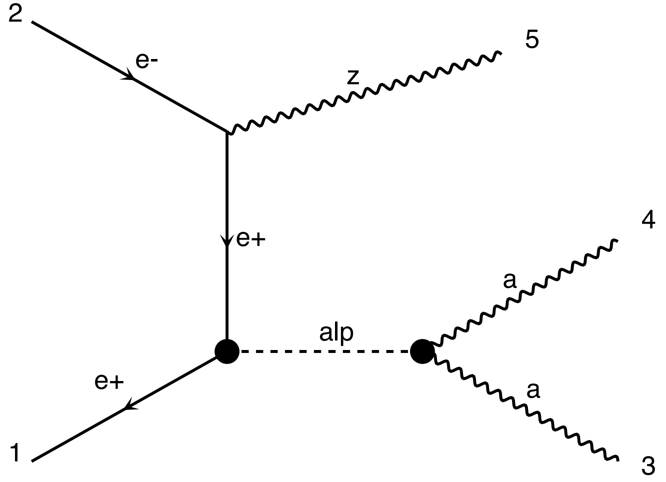


diagram 2 NP=2, QCD=0, QED=3

coupling(s): CYY, cee

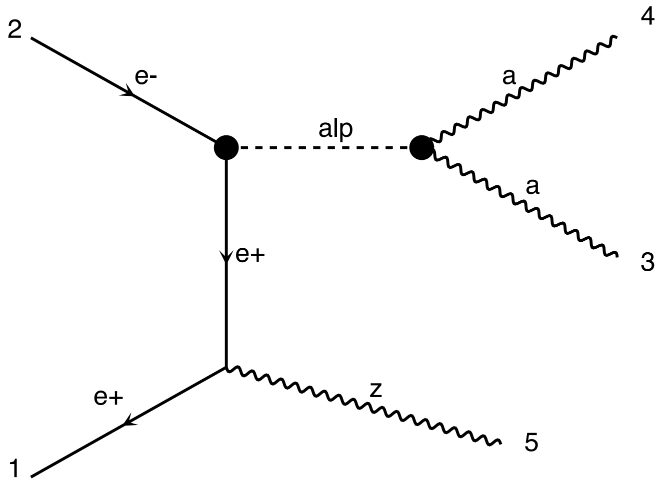


diagram 3 NP=2, QCD=0, QED=3

2b) $e^+e^- \to a\gamma,\ a \to \gamma\gamma$

coupling(s): CYY, cee

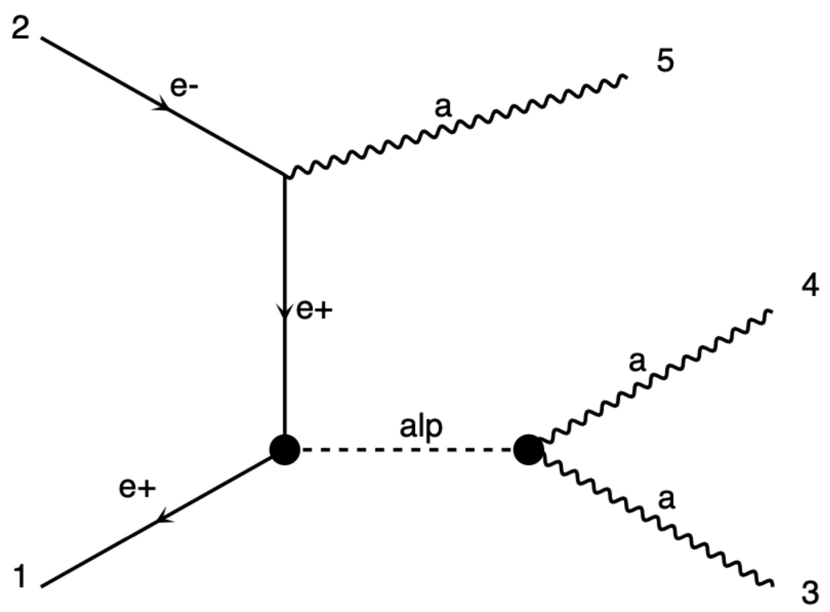


diagram 1 NP=2, QCD=0, QED=3

coupling(s): CYY, cee

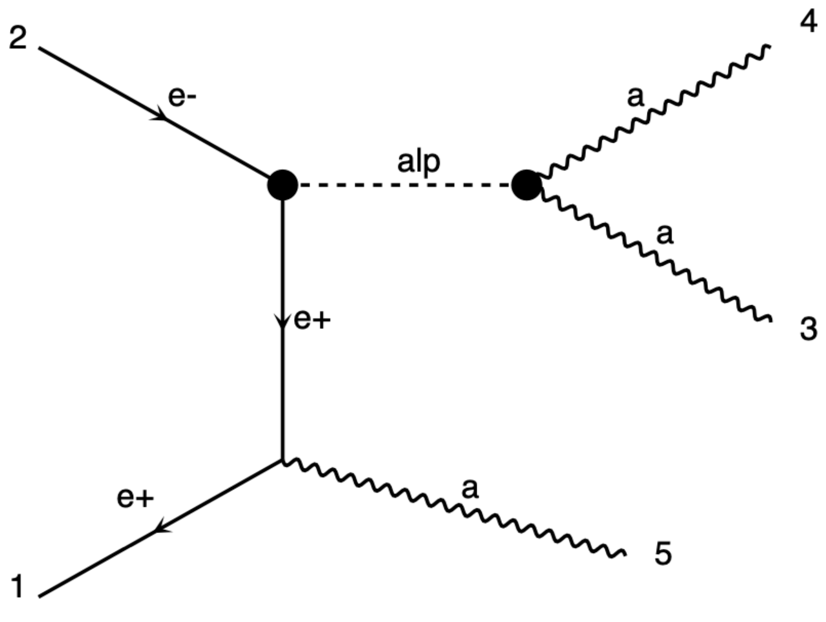


diagram 2 NP=2, QCD=0, QED=3

3a) $e^+e^- \to aZZ,\ a \to \gamma\gamma$

coupling(s): CYY, cZh5

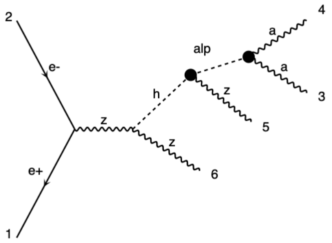


diagram 1 NP=2, QCD=0, QED=4

coupling(s): CYY, cee, cZh5

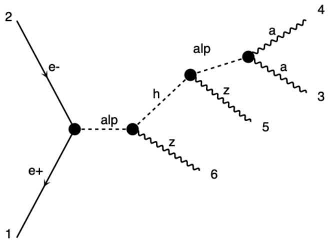


diagram 2 NP=4, QCD=0, QED=2

coupling(s): CYY, cee

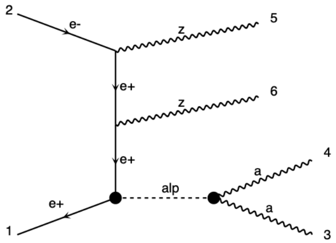


diagram 7 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

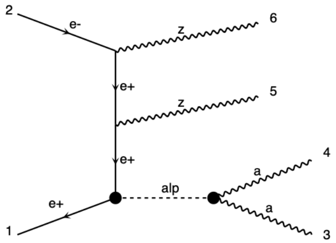


diagram 8 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

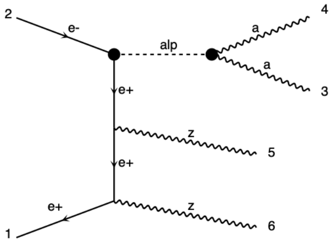


diagram 13 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

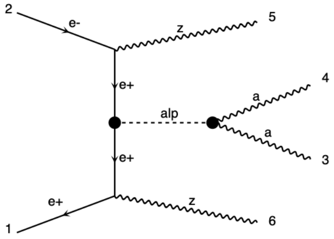


diagram 14 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

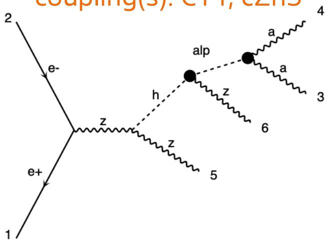


diagram 3 NP=2, QCD=0, QED=4

coupling(s): CYY, cee, cZh5

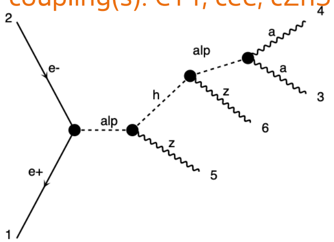


diagram 4 NP=4, QCD=0, QED=2

coupling(s): CYY, cee

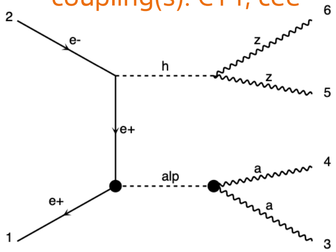


diagram 9 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

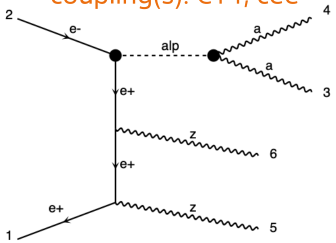


diagram 10 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

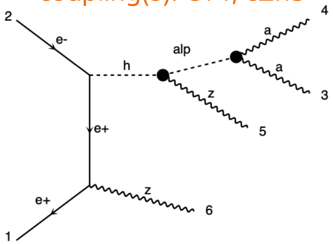


diagram 15 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

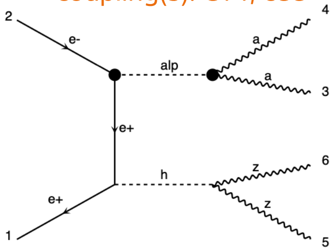


diagram 16 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

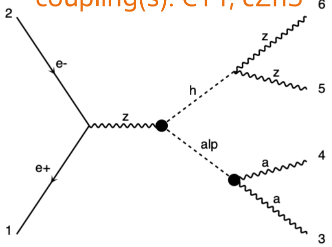


diagram 5 NP=2, QCD=0, QED=4

coupling(s): CYY, cee, cah

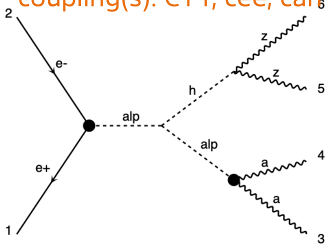


diagram 6 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

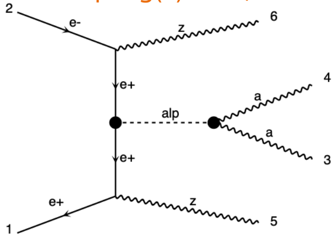


diagram 11 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

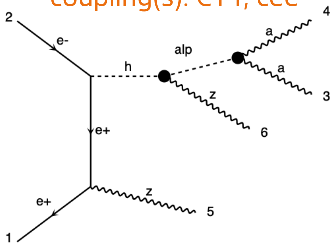


diagram 12 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

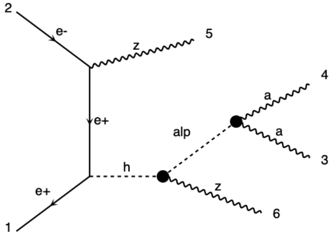


diagram 17 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

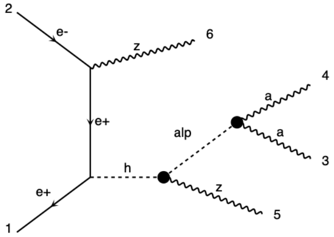


diagram 18 NP=2, QCD=0, QED=4

3b) $e^+e^- \to a\gamma\gamma,\ a \to \gamma\gamma$

coupling(s): CYY, cee

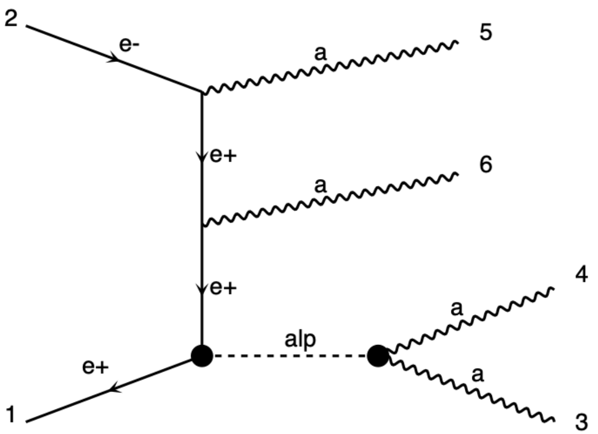


diagram 1 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

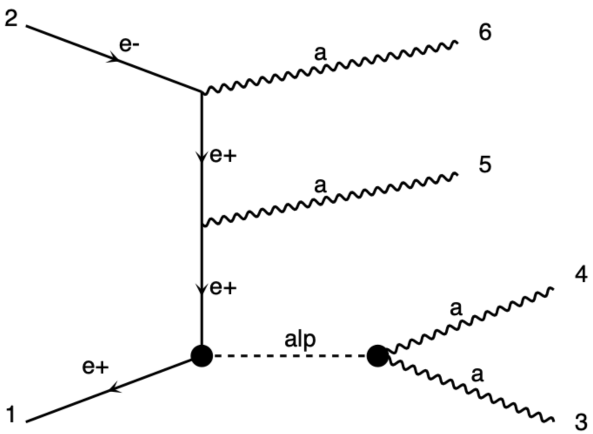


diagram 2 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

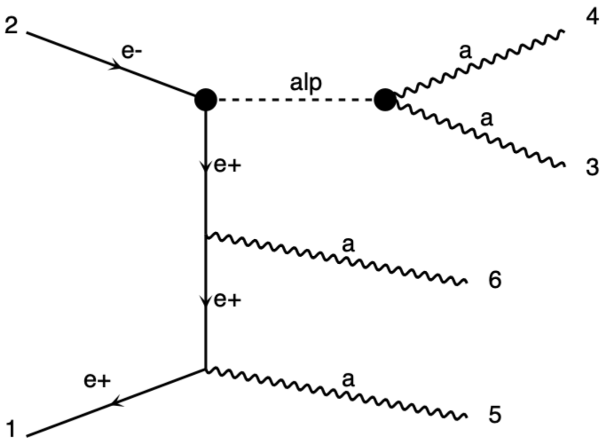


diagram 3 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

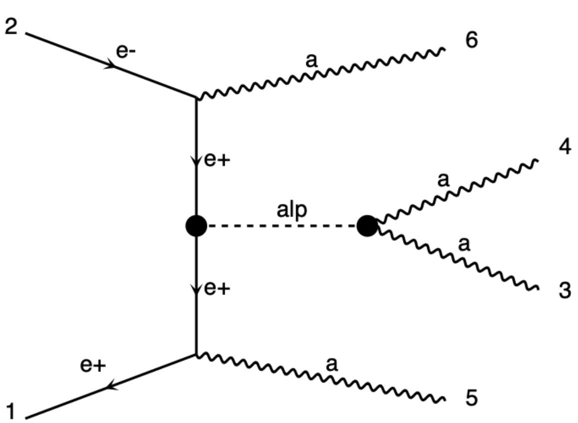


diagram 4 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

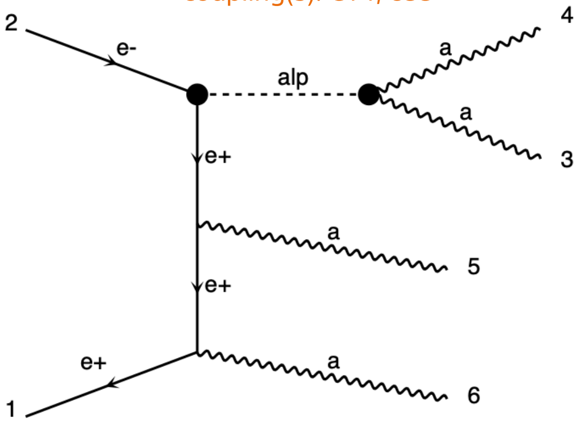


diagram 5 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

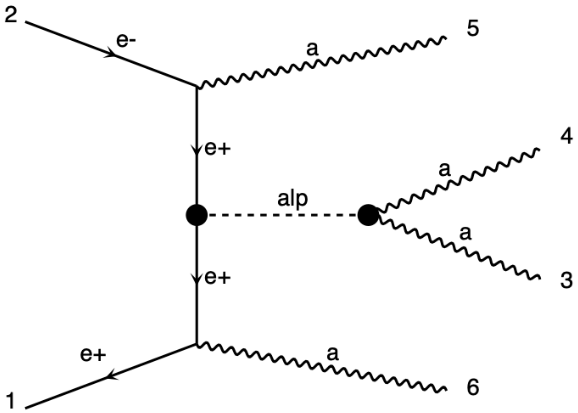
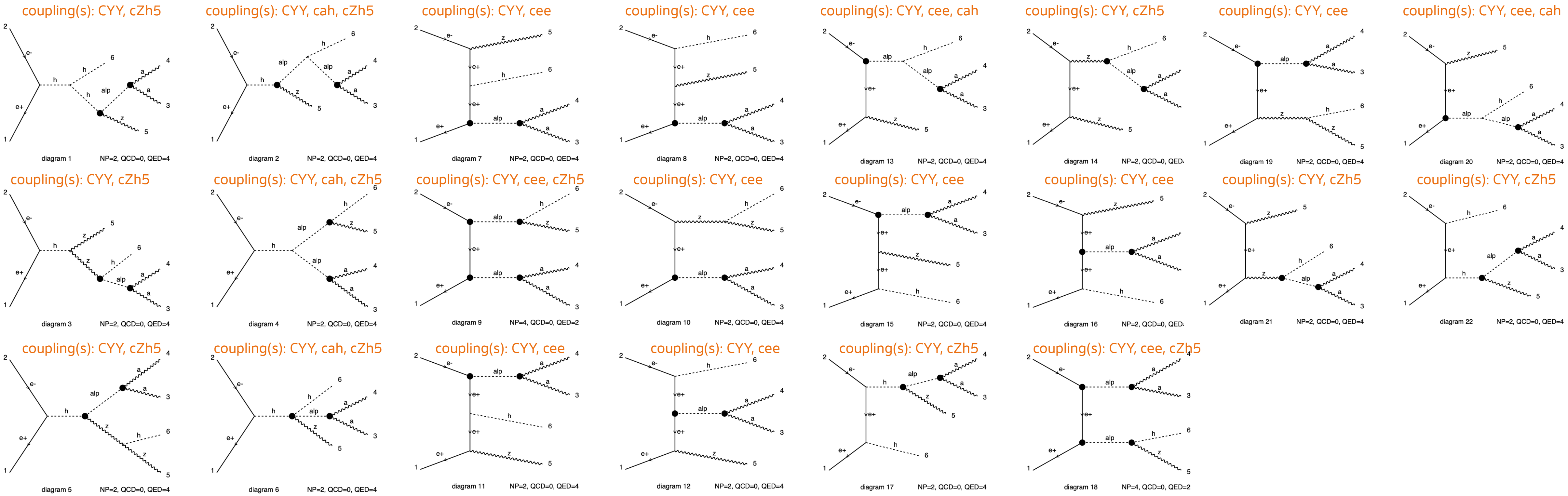


diagram 6 NP=2, QCD=0, QED=4

4a) $e^+e^- \to aZH,\ a \to \gamma\gamma$



4b) $e^+e^- \rightarrow a\gamma H,\ a \rightarrow \gamma\gamma$

coupling(s): CYY, cee

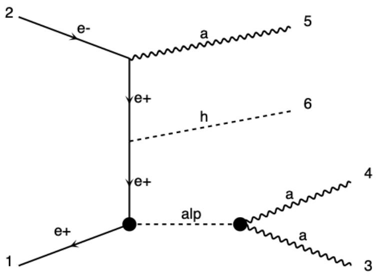


diagram 1 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

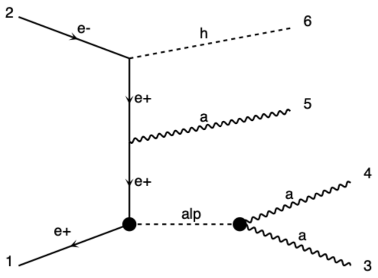


diagram 2 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

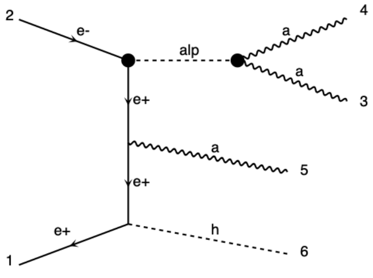


diagram 7 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

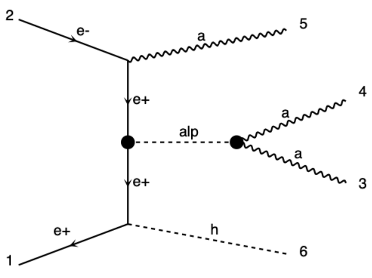


diagram 8 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

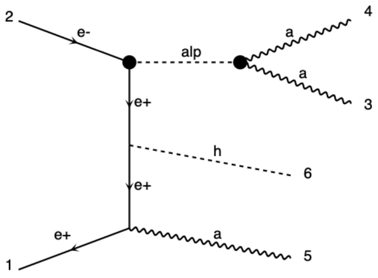


diagram 3 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

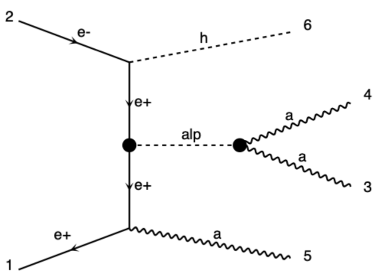


diagram 4 NP=2, QCD=0, QED=4

coupling(s): CYY, cee, cah

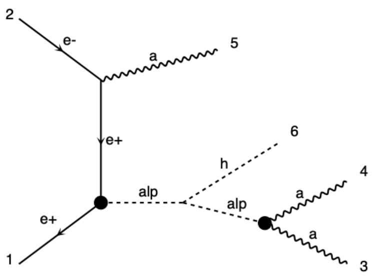


diagram 9 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

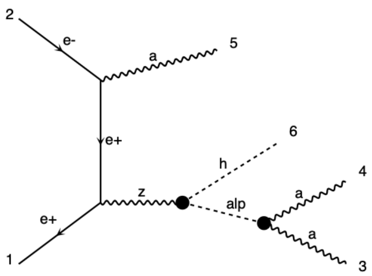


diagram 10 NP=2, QCD=0, QED=4

coupling(s): CYY, cee, cah

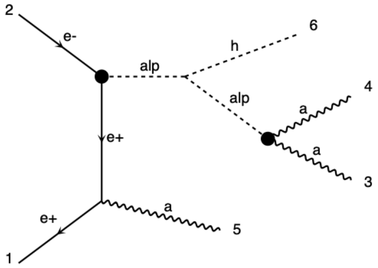


diagram 5 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

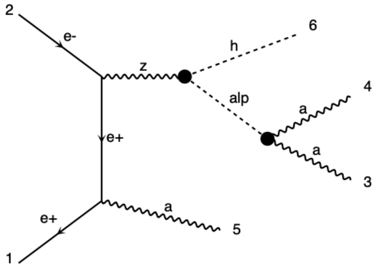


diagram 6 NP=2, QCD=0, QED=4

Cross sections

Code in the table corresponds to couplings:

Code **ABCD**:
A: coupling to photons **CYY**
B: coupling to electrons **cee**
C: coupling to ALP and Higgs **cah**
D: coupling to Z and Higgs **cZh5**

Example: 1001
CYY set to 1.
cee set to 0.
cah set to 0.
cZh5 set to 1.

If coupling is enabled, it is set to 1.0
CYY is always enabled since we specify decay of ALP to two photons

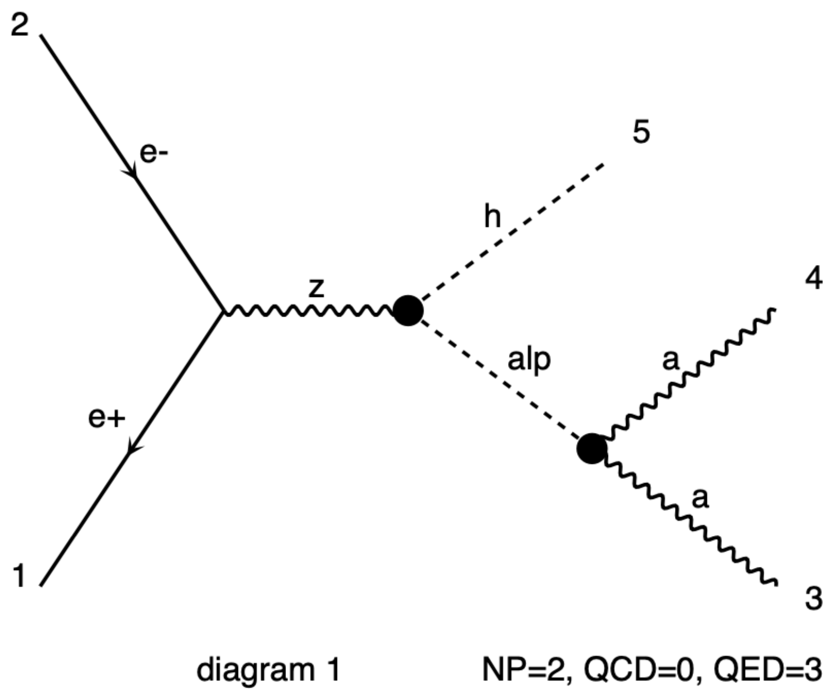
Mass of ALP also set to 1.0 GeV for all processes

Process	Code	Cross Section
$e^+e^- \to aH, a \to \gamma\gamma$	1001	165.6 ± 0.6018 pb
	1010	Survey return zero cross section.
	1100	2.208e-07 ± 2.196e-09 pb
$e^+e^- \to aZ, a \to \gamma\gamma$	1001	4.805e-07 ± 1.045e-09 pb
	1100	6.243e-07 ± 2.044e-09 pb
$e^+e^- \to a\gamma, a \to \gamma\gamma$	1100	6.301e-07 ± 1.438e-09 pb
$e^+e^- \to aZZ, a \to \gamma\gamma$	1001	5697 ± 12.04 pb
	1100	5.762e-11 ± 4.292e-13 pb
	1101	4247 ± 9.412 pb
	1110	1.042e-05 ± 1.069e-07 pb
$e^+e^- \to a\gamma\gamma, a \to \gamma\gamma$	1100	2.747e-08 ± 6.527e-10 pb
$e^+e^- \to aZH, a \to \gamma\gamma$	1001	0.003187 ± 2.133e-05 pb
	1011	0.003182 ± 1.456e-05 pb
	1100	1.674e-12 ± 6.996e-15 pb
	1101	0.002328 ± 2.152e-05 pb
	1110	1.005e-07 ± 4.229e-10 pb
$e^+e^- \to a\gamma H, a \to \gamma\gamma$	1001	3.502 ± 0.02914 pb
	1100	4.326e-09 ± 3.305e-11 pb
	1110	0.003682 ± 4.148e-05 pb

Cross section $e^+e^- \to aH,\ a \to \gamma\gamma$

165.6 ± 0.6018 pb	
1001: CYY, cZh5	

Cross section if cZh5 set to limit from paper (0.72): 86.02 pb



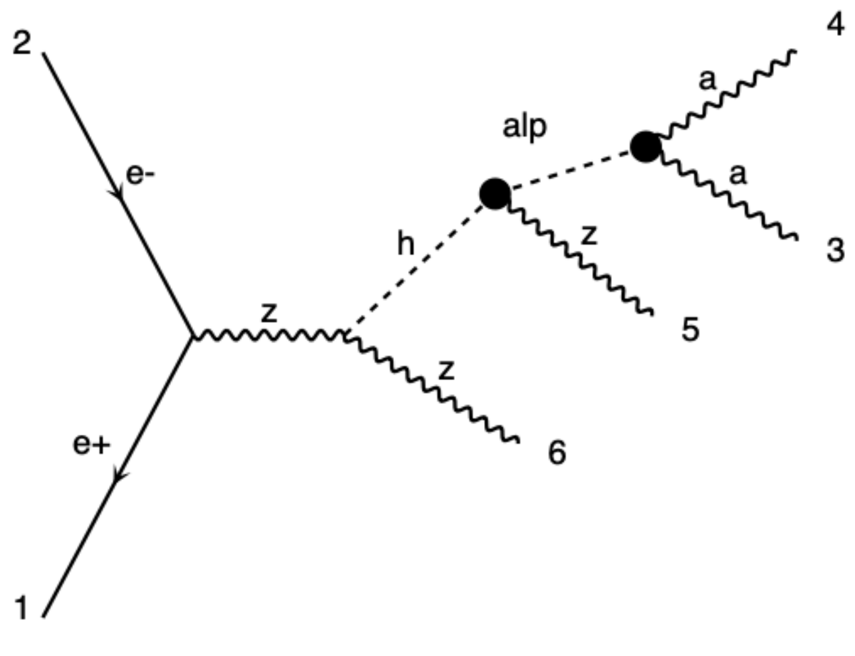
s= 165.61 ± 0.602 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.7	165.6	0.602	7.009	391.0	2.36
G4	0	0	0.0	1.0	0
G3	0	0	0.0	1.0	0
G2.8	0	0	0.0	0.0	0
G2.7	0	0	0.0	1.0	0
G1.8	0	0	0.0	0.0	0

Cross section $e^+e^- \to aZZ,\ a \to \gamma\gamma$

5697 ± 12.04 pb
1001: CYY, cZh5

Cross section if cZh5 set to limit from paper (0.72): 2939 pb



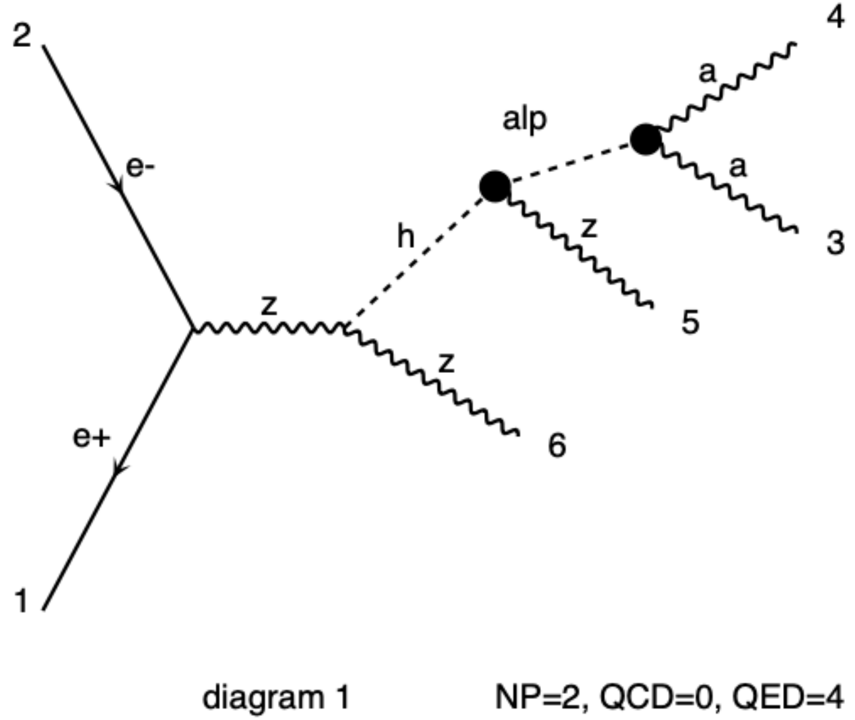
s= 5697.1 ± 12 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1	5697	12	7.009	384.0	0.0674
G5.25	0.007277	5.65e-05	7.009	403.0	5.54e+04
G17	8.197e-17	9.7e-19	7.009	421.0	5.14e+18
G12	8.138e-17	8.88e-19	7.009	392.0	4.82e+18
G16.02	0	0	0.0	1.0	0
G11	0	0	0.0	1.0	0
G10	0	0	0.0	1.0	0
G9.06	0	0	0.0	1.0	0
G7	0	0	0.0	1.0	0
G6.26	0	0	0.0	0.0	0
G6.25	0	0	0.0	1.0	0
G5.26	0	0	0.0	0.0	0
G2.26	0	0	0.0	0.0	0
G2.25	0	0	0.0	1.0	0
G2.23	0	0	0.0	0.0	0
G2.22	0	0	0.0	1.0	0

Cross section $e^+e^- \rightarrow aZZ,\ a \rightarrow \gamma\gamma$

4247 ± 9.412 pb
1101: CYY, cee, cZh5

Cross section if cZh5 set to limit from paper (0.72): 2207 pb



s= 4246.7 ± 9.41 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1	4246	9.41	7.009	386.0	0.0909
G2.22	0.8363	0.00278	7.009	400.0	478
G10	0.1002	0.00129	7.009	380.0	3.79e+03
G7	0.0997	0.00143	7.009	390.0	3.91e+03
G2.25	0.05683	0.002	15.009	391.0	6.88e+03
G11	0.01657	0.000189	7.009	425.0	2.57e+04
G5.25	0.004842	3.57e-05	7.009	388.0	8.01e+04
G12	5.062e-07	2.73e-09	7.009	386.0	7.63e+08
G17	5.042e-07	2.59e-09	7.009	384.0	7.62e+08
G9.06	9.707e-12	1e-13	7.009	390.0	4.02e+13
G16.02	9.68e-12	1.75e-13	7.009	398.0	4.11e+13
G6.26	0	0	0.0	0.0	0
G6.25	0	0	0.0	1.0	0
G5.26	0	0	0.0	0.0	0
G2.26	0	0	0.0	0.0	0
G2.23	0	0	0.0	0.0	0

08.07.25

ALPnlo

recently obtained from Bauer by Abu Dabi colleagues, no known problem, except lack of phase space factors in decays into fermions

```
set gPU 0
set gPd 0
set gPl 0
set cah 0
set cZh5 0
set cWW 0.
set cBB 1.
set cGG 0
set Lambda 1000.
set mh 125
set MALP 1.
set WALP auto
```

ALP

obtained from Thamm in 2019 only works if mh is put high

```
set gPU 0
set gPd 0
set gPl 0
set cah 0
set cZh5 0
set cAA 1.
set cZA -0.223
set cGG 0
set Lambda 1000.
set mh 1000
set MALP 1.
set WALP auto
```

ALP_NLO_UFO

used for Snowmass report, available in LLP git, seems to yield sensible results only for CWW=0

```
set Ma 1.
set cWW = 0.0
set CYY = 1.
set cGG = 0.
set cuu = 0.
set cdd = 0.
set ccc = 0.
set css = 0.
set ctt = 0.
set cbb = 0.
set cee = 0.
set cmumu = 0.
set ctautau = 0.
set cah = 0.
set cZh5 = 0.
set falp = 6.33
set WALP auto
```

ALP_linear_UFO

Brivio et al.

From Giacomo:

In order to match the input paramters for this UFO
(ALP_linear_UFO) to the ones for the Bauer et al. UFO (ALP), the
following formulas
are useful:

CBtil = CBB * 4\pi\alpha/cw2 = CBB * 0.128215343
CWtil = CWW * 4\pi\alpha/sw2 = CWW * 0.420418893

CBB = -CZA + CAA*(1-sw2)
CWW = CZA + CAA*sw2

where the input parameters for the W/gamma sector in card are

Mimasu: CBtil, CWtil
Thamm: CAA, CZA

The correspondance of other parameters is

Mimasu	Thamm
------	-----
fa	Lambda
Ma	MALP
Wax (*)	WALP

(*) In the original version of the UFO the width of the ALP
was hardcoded to zero. I modified the UFO files in order to
get it parametric.

set pta 0.
set ptl 0.
set ptj 0.
set ptb 0.
set etaa 2.6
set draa 0.0
set ea 0.0
set CGtil 0.
set CWtil 0.
set **CBtil 0.1282155343**
set CaPhi 0
set fa 1000.
set **Ma 1**
set Wax auto

Cross section calculation

$$\frac{d\sigma(e^+e^- \to \gamma a)}{d\Omega} = 2\pi\alpha\alpha^2(s)\frac{s^2}{\Lambda^2}\left(1-\frac{m_a^2}{s}\right)^3 \left(1+\cos^2\theta\right)\left(|V_\gamma(s)|^2+|A_\gamma(s)|^2\right), \qquad (16)$$

Based on formula 16 of Bauer et al. arXiv:1808.10323

```
import math
#-----------------------------------------------------------------
# Calculate production cross-section for e+e-->\gamma a
# based on formula 16 of Bauer et al. arXiv:1808.10323
# GP 06/07/25
#-----------------------------------------------------------------
# electroweak constants
#-----------------------------------------------------------------
XSZpole=58500 # pb LO XS at sqrts=91.2 GeV
MZ =91.1876 # GeV
MZ2=MZ**2
GamZ=2.4952 # GeV
aEWM1=127.9
aEWM1Z=128.952
Gf=0.0000116637 # GeV-2
aEW=1/aEWM1
aEWZ=1/aEWM1Z
MW=math.sqrt(MZ**2/2. + math.sqrt(MZ**4/4. - (aEW*math.pi*MZ**2)/(Gf*math.sqrt(2))))
sw2=1 - MW**2/MZ**2
cw2=1 - sw2
gv=2*sw2-0.5
ga=-0.5
#-----------------------------------------------------------------
```

```
# e+e-->\gamma a cross-section calculaton
def calc_xs_ga(sqs,ma,cgg,ggz,Lam):
  s=sqs**2
# vector and axial coupling, formula 19
  Vg=cgg/s+gv*cgz/2/sw2/cw2/complex(s-MZ2,MZ*GamZ)
  Vg2=(Vg*Vg.conjugate()).real
  Ag=ga*cgz/2/sw2/cw2/complex(s-MZ2,MZ*GamZ)
  Ag2=(Ag*Ag.conjugate()).real
# formula 16
  const=2*math.pi*aEW*aEWZ**2*s**2/Lam**2*(1-ma**2/s)**3*(Vg2+Ag2)
# integral of phase space integrate on (1+cos^2\theta))
  phsp=2*math.pi*((1+1/3)-(-1-1/3))
# convert using hc^2 into picobarns
  xs=const*phsp*0.389379*1e9
  return xs
#-----------------------------------------------------------------
# width of Z into gamma alp (GeV), formula 12
def gam_galp(ma,cgz,Lam):
  gamalp=0
  if(ma<MZ):
    gamalp=8*math.pi*aEW*aEWZ*MZ**3*cgz**2*(1-ma**2/MZ**2)**3/3/sw2/cw2/Lam**2
  return gamalp
#-----------------------------------------------------------------
# transform fron cww, cbb to cgg, cgz, czz, formula 3
def calc_coeff(cbb,cww):
    cgg=cww+cbb
    cgz=cw2*cww-sw2*cbb
    czz=cw2**2*cww+sw2**2*cbb
    return cgg,cgz,czz
```

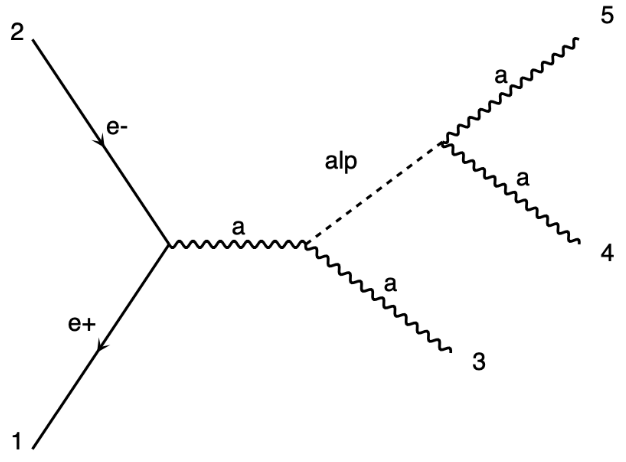
Cross sections

Test each model and compare cross sections

Process: e+ e- > a, (a > a ALP, (ALP > a a))

Particle decaying to itself so potentially some issues with infinite recursion?

$\sqrt{s}$, m_a, $c_{\gamma\gamma}$	Model	Cross Section
91.188, 1, 1	ALPnlo	0.01919 +- 5.113e-05 pb
	ALP	0.01921 +- 5.29e-05 pb
	ALP_NLO_UFO	0.01914 +- 5.157e-05 pb
	ALP_linear_UFO	0.01919 +- 4.968e-05 pb
240, 1, 1	ALPnlo	0.01915 +- 6.019e-05 pb
	ALP	0.01919 +- 5.259e-05 pb
	ALP_NLO_UFO	0.01916 +- 7.818e-05 pb
	ALP_linear_UFO	0.01918 +- 5.112e-05 pb



Direct calculation (crossalp.py)

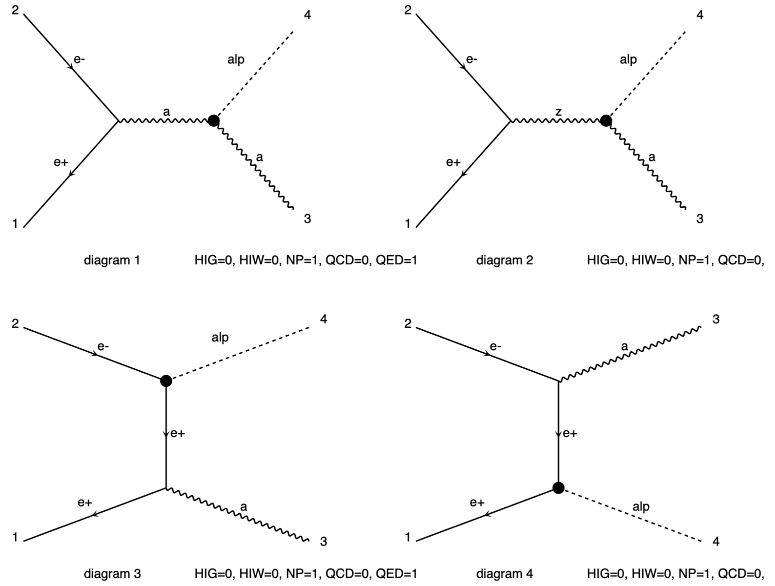
Inputs: ma= 1 GeV; Lambda= 1000 GeV; sqrts= **91.188 GeV**
cww= 0 cbb= 1
cgg= 1 cgz= -0.2337
xs from formula **2.769765571070452 pb**

Inputs: ma= 1 GeV; Lambda= 1000 GeV; sqrts= **240 GeV**
cww= 0 cbb= 1
cgg= 1 cgz= -0.2337
xs from formula **0.02304513132546023 pb**

Cross sections

Test each model and compare cross sections

Process: e+ e- > a ALP



$\sqrt{s}$, m_a, $c_{\gamma\gamma}$	Model	Cross Section
91.188, 1, 1	ALPnlo	Zero cross section
	ALP	2.469 +- 0.006559 pb
	ALP_NLO_UFO	Zero cross section
	ALP_linear_UFO	2.778 += 0.006595 pb
240, 1, 1	ALPnlo	Zero cross section
	ALP	0.02082 +- 4.045e-05 pb
	ALP_NLO_UFO	Zero cross section
	ALP_linear_UFO	0.02115 +- 4.063e-05 pb

Direct calculation (crossalp.py)

Inputs: ma= 1 GeV; Lambda= 1000 GeV; sqrts= **91.188 GeV**
cww= 0 cbb= 1
cgg= 1 cgz= -0.2337
xs from formula **2.769765571070452 pb**

Inputs: ma= 1 GeV; Lambda= 1000 GeV; sqrts= **240 GeV**
cww= 0 cbb= 1
cgg= 1 cgz= -0.2337
xs from formula **0.02304513132546023 pb**

Use Brivio et al. model? Unfortunately the coupling of the ALP to Higgs and fermions has not been mapped yet

Preliminary plot

Plot made by Giacomo's student