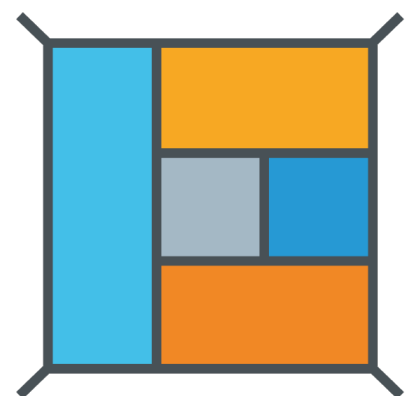


Gravitational Wave Physics from Quantum Field Theory

Gustav Uhre Jakobsen

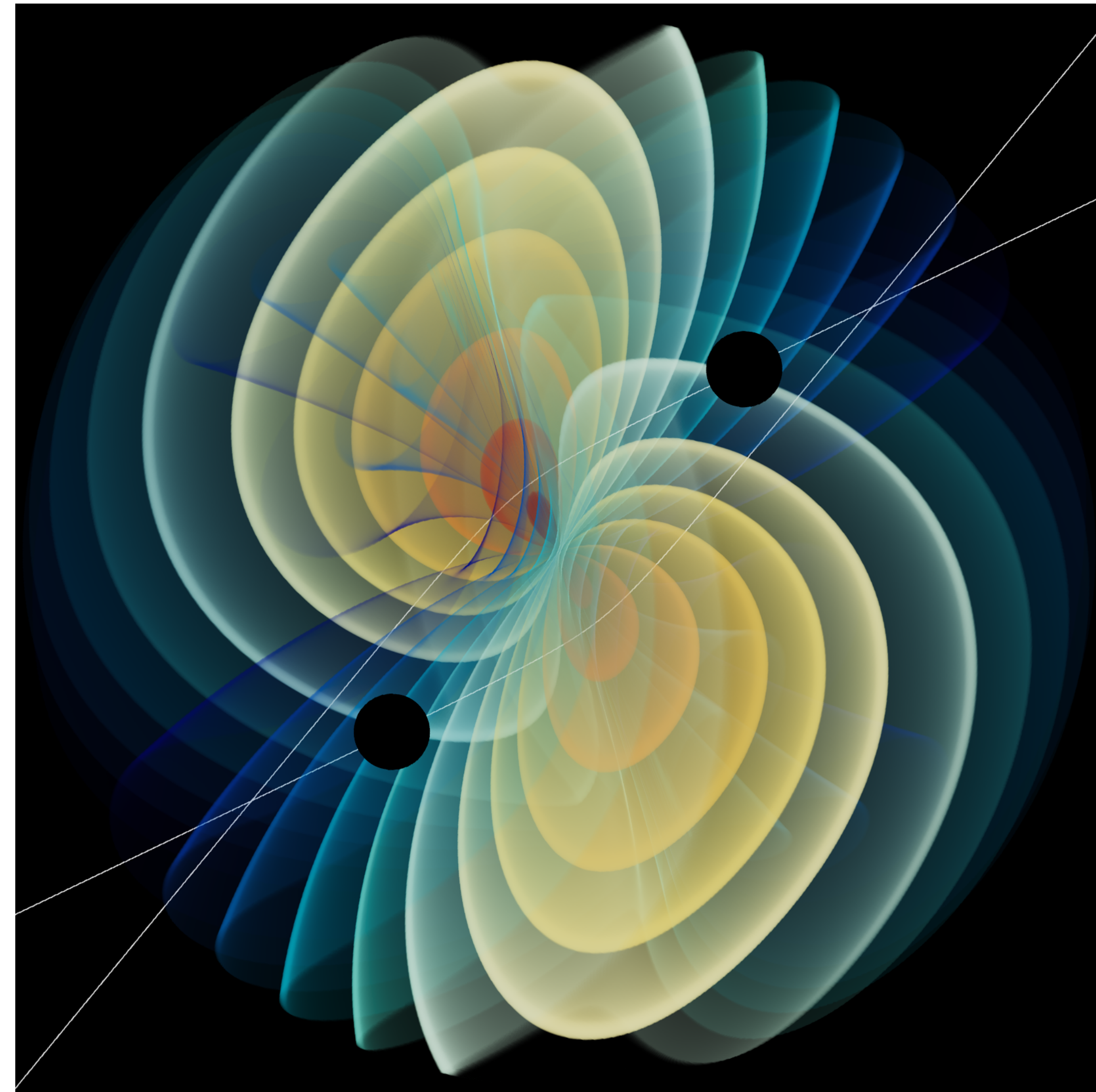
07.10.2025

Astroparticle and Particle Physics Workshop
DESY, Zeuthen - Berlin



RTG 2575:

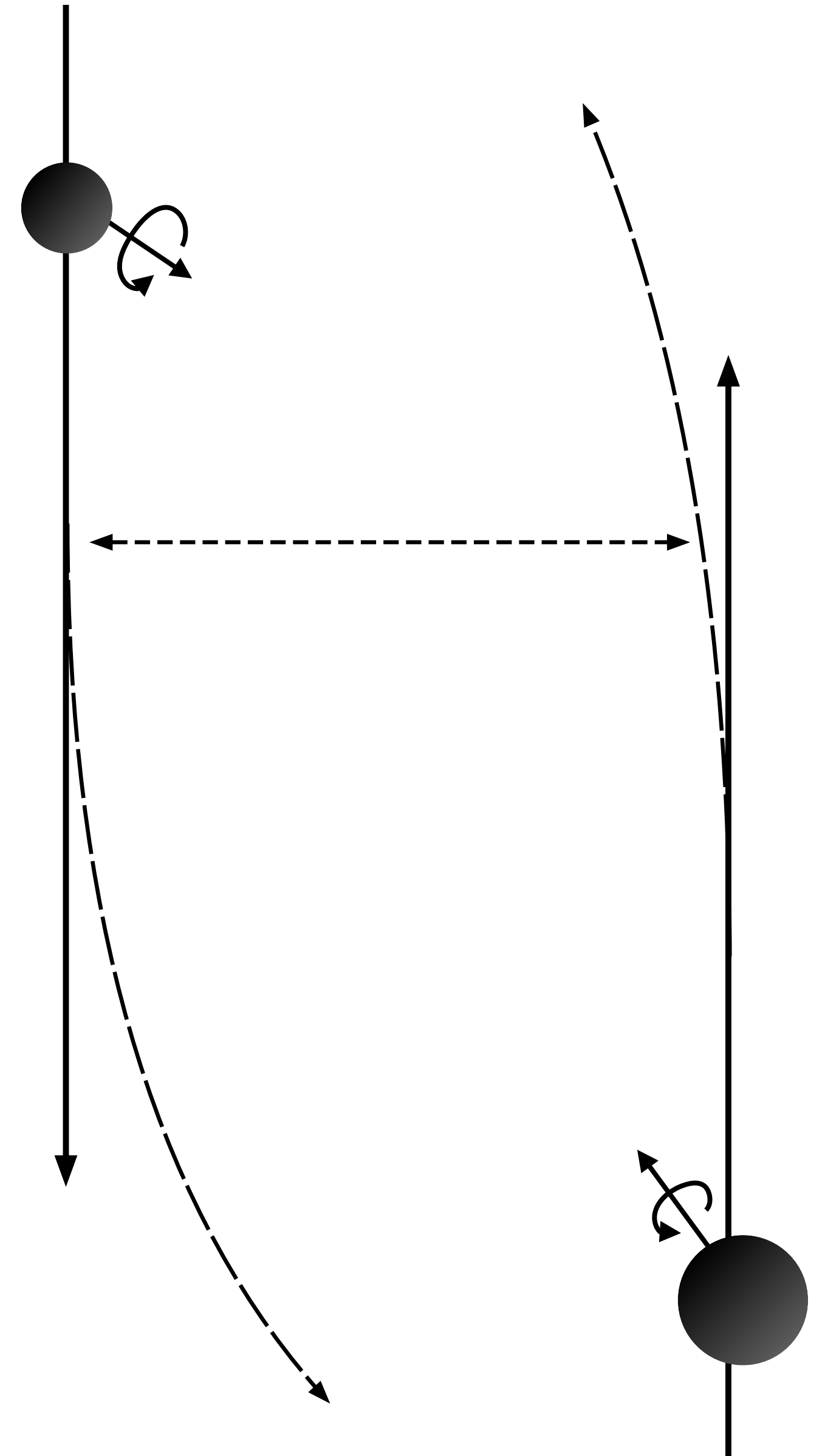
**Rethinking
Quantum Field Theory**



Credit: Mathias Driesse

Outline

- Gravitational wave research at HU:
Worldline quantum field theory
- High-precision, perturbative black hole scattering
- First, motivation:
- Gravitational wave physics
- Black hole two-body problem



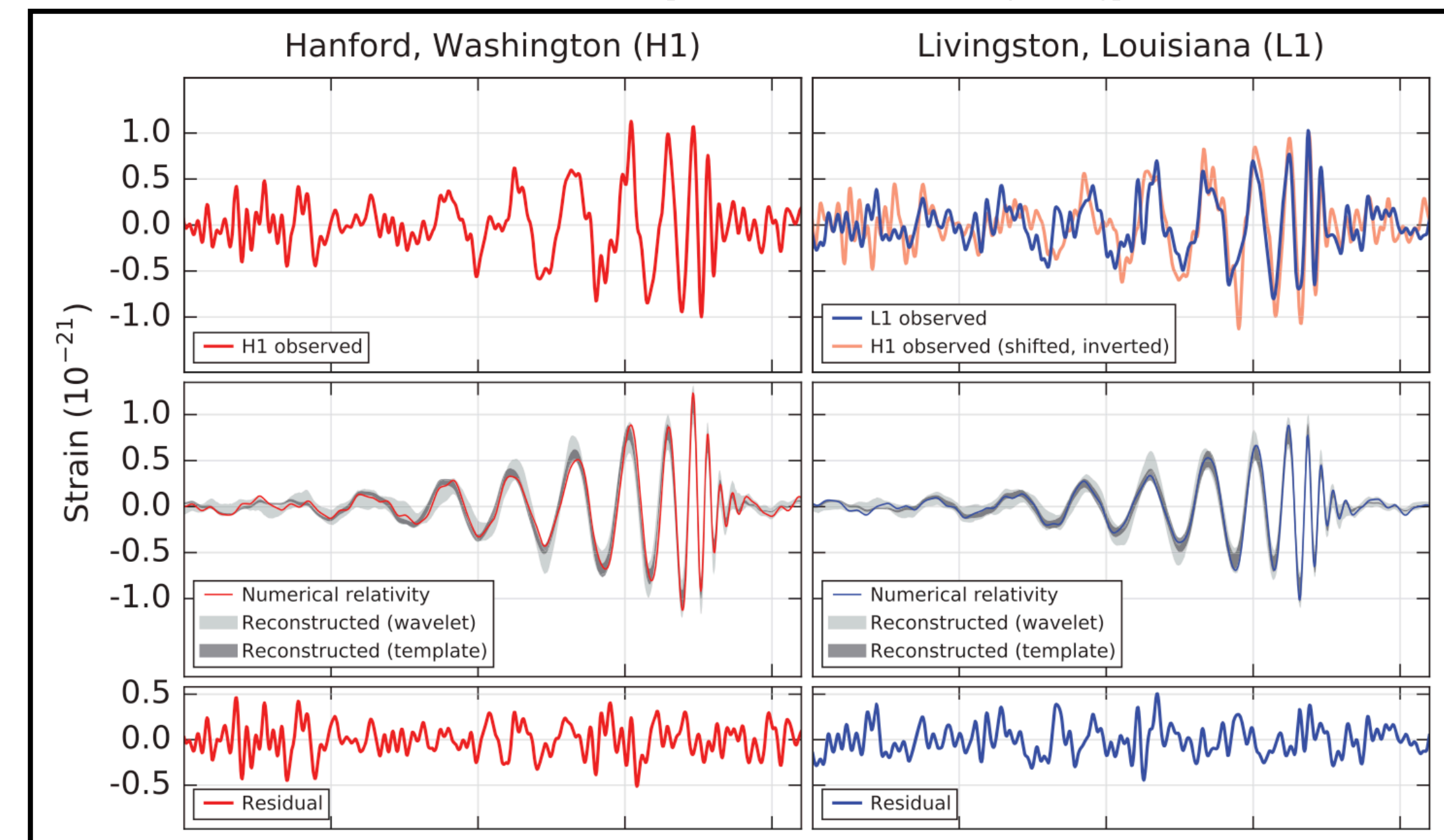
Gravitational Wave Physics

- Hundreds of detections since 2015
- Astrophysical origins: compact binary coalescence
- 2030s detectors: unprecedented sensitivity
- Large banks of theoretical gravitational-wave templates required for data analysis
- Is Einstein's theory of gravity correct?

Virgo, Cascina, Italy

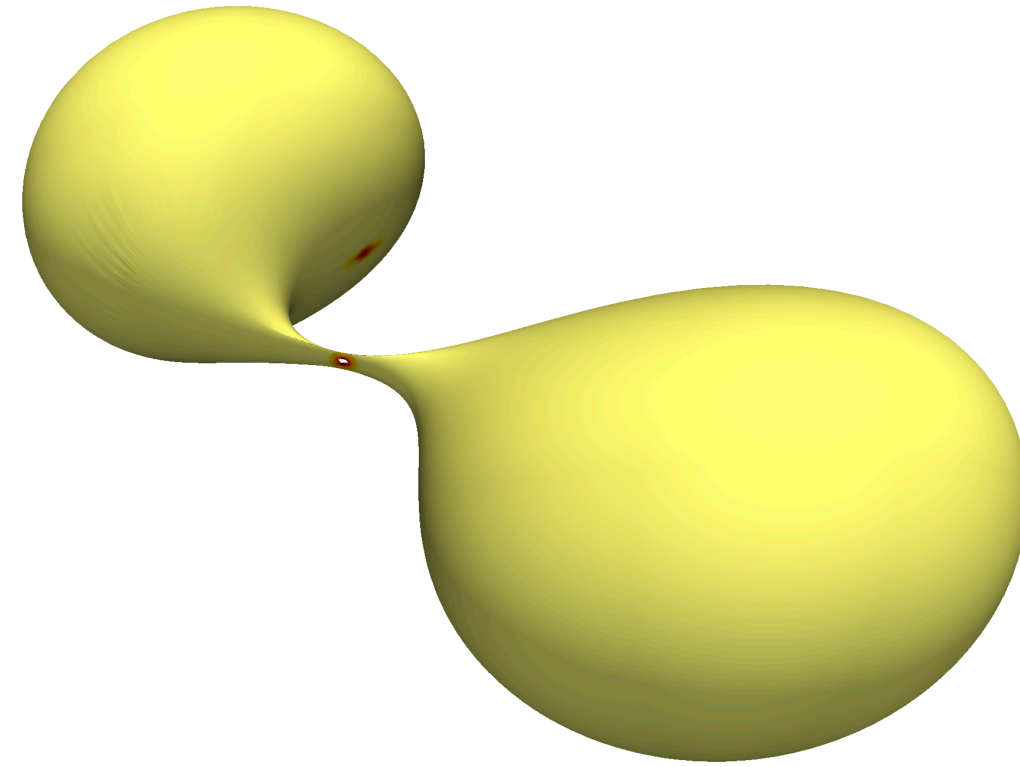


Gravitational strains of GW150914 [PRL 116, 061102 (2016)]



Isolated black holes

- Stationary solutions to $G^{\mu\nu} = 0$
- No-hair theorem, only mass and spin
- Kerr metric
- Event-horizon



Event horizon(s) in numerical relativity simulation
[Bohn, Kidder, Teukolsky 2016]

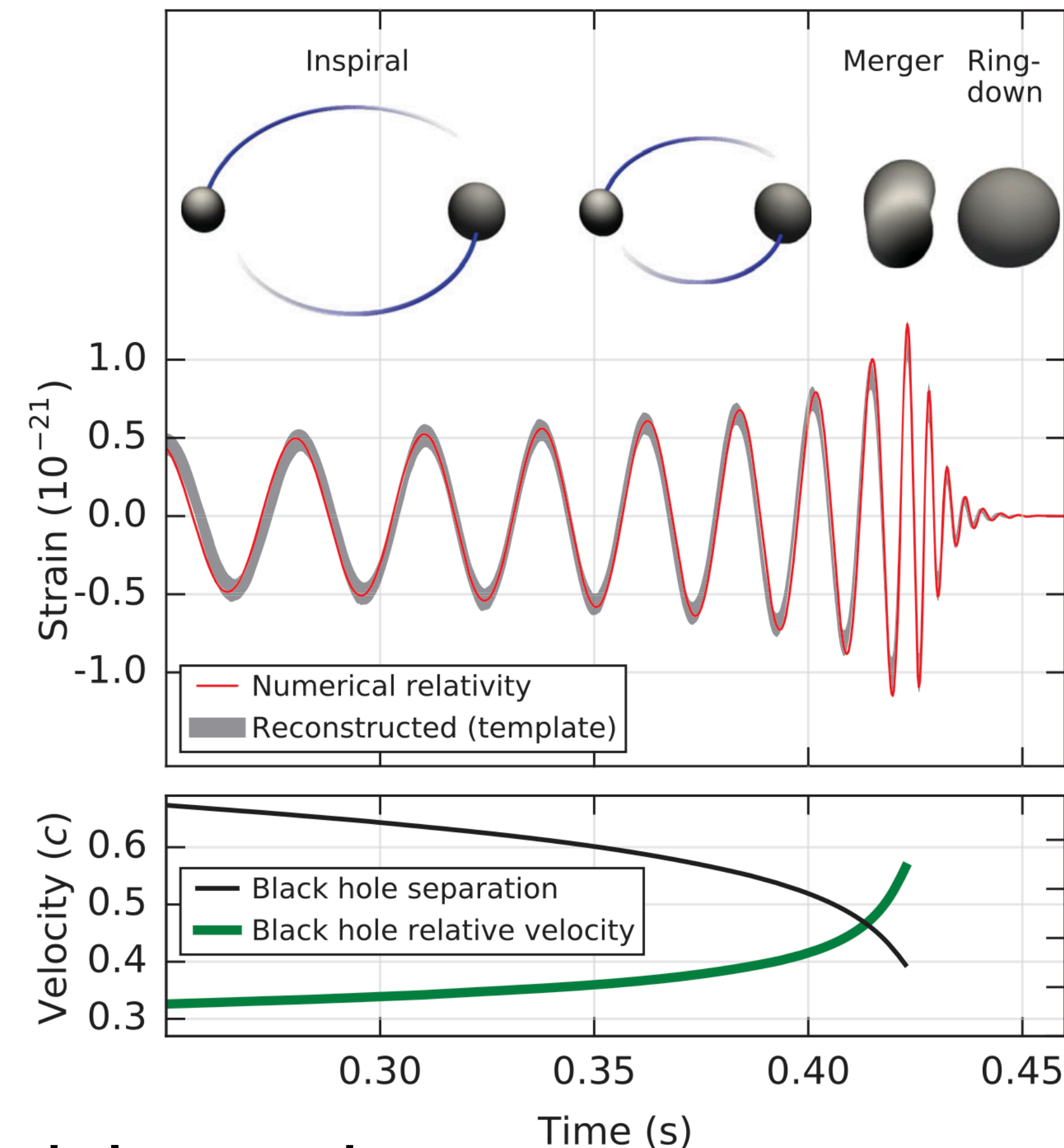
Event Horizon Telescope (2019)



Black hole two-body system

- Non-linear, non-stationary differential equations with two interacting event horizons (black holes)
- Simulated on clusters in numerical relativity (first complete simulation in 2005)
- Perturbative approaches: PN, PM, GSF, BHPT...

Compact Binary Coalescence [PRL 116, 061102 (2016)]



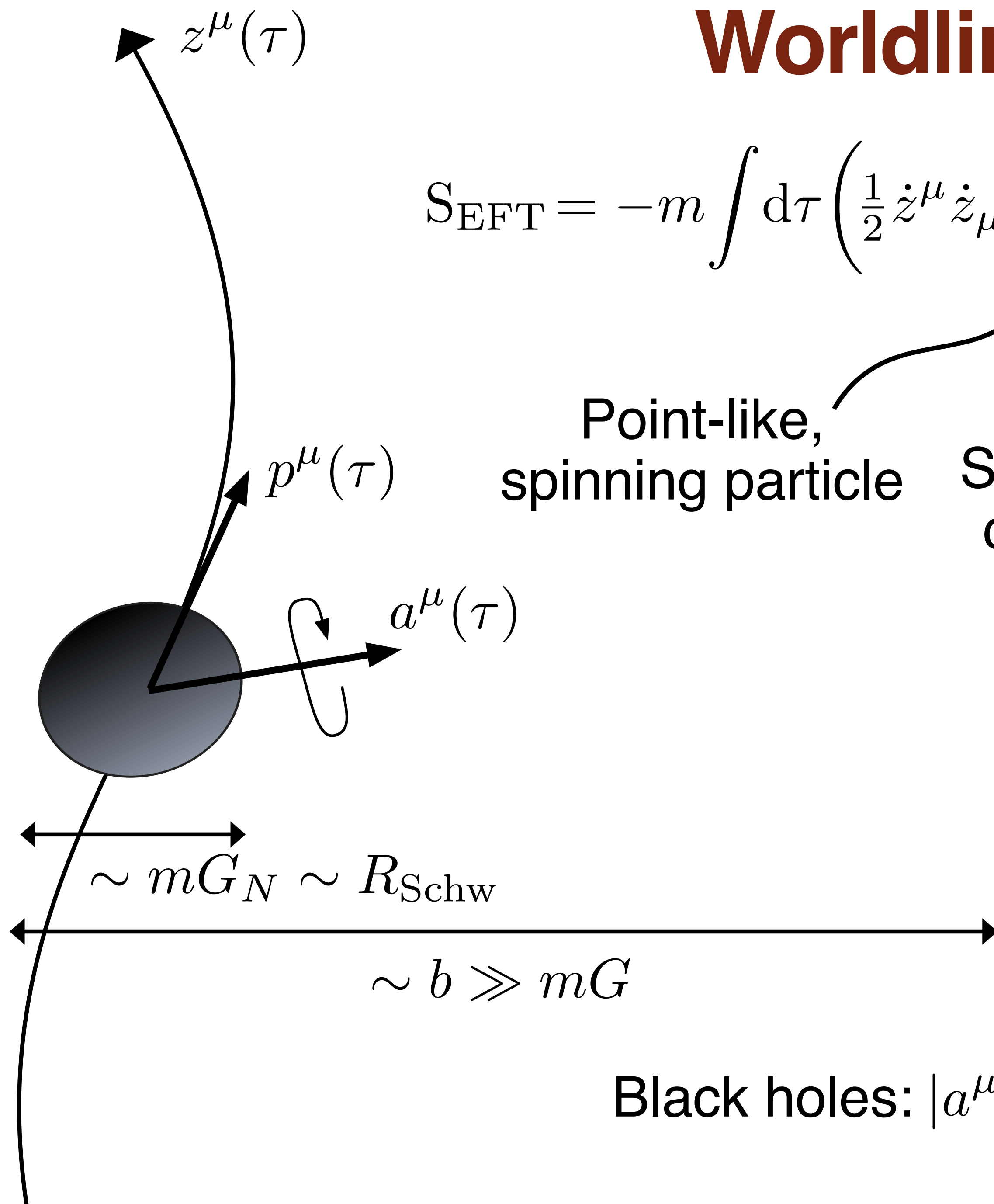
Worldline Effective Field Theory

$$S_{\text{EFT}} = -m \int d\tau \left(\frac{1}{2} \dot{z}^\mu \dot{z}_\mu + i \bar{\alpha}_\mu \frac{D\alpha^\mu}{d\tau} + \frac{1}{4} \lambda_1 a^\mu a^\alpha \dot{z}^\nu \dot{z}^\beta R_{\mu\nu\alpha\beta} + \lambda_2 (mG)^4 R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \dots \right)$$

Point-like,
spinning particle

Spin-induced
quadrupole

Adiabatic tidal
effects



Black holes: $|a^\mu| \leq mG$

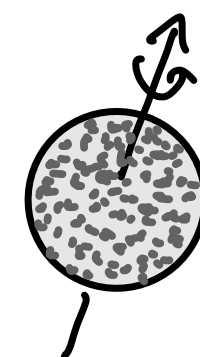
Black Hole



mass, spin

$$\lambda_1 = 1, \lambda_2 = 0$$

Neutron Star



mass, spin, radius,
tidal deformability

$$\lambda_1, \lambda_2 = ??$$

Post-Minkowskian Scattering

- Find perturbative, analytic solutions to **Einstein's field equations** and the **geodesic equations**

$$\begin{aligned} & \frac{1}{2}g^{\alpha\beta}\partial_\alpha\partial_\mu g_{\beta\nu} + \frac{1}{2}g^{\alpha\beta}\partial_\alpha\partial_\nu g_{\mu\beta} - \frac{1}{2}g^{\alpha\beta}\partial_\alpha\partial_\beta g_{\mu\nu} - \frac{3}{2}g^{\alpha\beta}\partial_\mu\partial_\nu g_{\alpha\beta} - \frac{1}{2}g^{\beta\lambda}g^{\alpha\rho}\partial_\alpha g_{\rho\lambda}\partial_\mu g_{\beta\nu} \\ & - \frac{1}{2}g^{\beta\lambda}g^{\alpha\rho}\partial_\alpha g_{\rho\lambda}\partial_\nu g_{\mu\beta} + \frac{1}{4}g^{\beta\lambda}g^{\alpha\rho}\partial_\nu g_{\alpha\lambda}\partial_\mu g_{\rho\beta} + \frac{1}{4|g|}g^{\alpha\beta}\partial_\beta|g|\partial_\nu g_{\mu\alpha} - \frac{1}{4|g|}g^{\alpha\beta}\partial_\beta|g|\partial_\alpha g_{\mu\nu} \\ & - \frac{1}{4|g|}g^{\alpha\beta}\partial_\beta|g|\partial_\mu g_{\alpha\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \end{aligned}$$

$$\ddot{z}^\mu = -\Gamma^\mu_{\alpha\beta}\dot{z}^\alpha\dot{z}^\beta + (\text{spin, tidal...})$$

Ultraviolet divergences!
Use dimensional regularization

Expansion around flat space:

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \sqrt{G_N}h_{\mu\nu}(x)$$

$$z_i^\mu(\tau) = b_i^\mu + \tau v_i^\mu + z_i^{\prime\mu}(\tau)$$

$$\theta \sim \frac{Gm}{b} \ll 1$$

Worldline Quantum Field Theory (WQFT)

- Diagrammatic, algorithmic solution to classical equations of motion
- Solve classical equations of motion as stationary phase of worldline quantum field theory

$$\begin{aligned}
 & \frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\mu g_{\beta\nu} + \frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\nu g_{\mu\beta} - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \partial_\beta g_{\mu\nu} - \frac{3}{2} g^{\alpha\beta} \partial_\mu \partial_\nu g_{\alpha\beta} - \frac{1}{2} g^{\beta\lambda} g^{\alpha\rho} \partial_\alpha g_{\rho\lambda} \partial_\mu g_{\beta\nu} \\
 & - \frac{1}{2} g^{\beta\lambda} g^{\alpha\rho} \partial_\alpha g_{\rho\lambda} \partial_\nu g_{\mu\beta} + \frac{1}{4} g^{\beta\lambda} g^{\alpha\rho} \partial_\nu g_{\alpha\lambda} \partial_\mu g_{\rho\beta} + \frac{1}{4|g|} g^{\alpha\beta} \partial_\beta |g| \partial_\nu g_{\mu\alpha} - \frac{1}{4|g|} g^{\alpha\beta} \partial_\beta |g| \partial_\alpha g_{\mu\nu} \\
 & - \frac{1}{4|g|} g^{\alpha\beta} \partial_\beta |g| \partial_\mu g_{\alpha\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}
 \end{aligned}$$

$$\langle z^\mu(\tau) \rangle = \int \mathcal{D}h_{\mu\nu} \mathcal{D}z^\sigma z^\mu(\tau) e^{iS_{\text{EFT}}/\hbar} \Big|_{\hbar \rightarrow 0}$$

$$\langle h_{\mu\nu}(x) \rangle = \int \mathcal{D}h_{\mu\nu} \mathcal{D}z^\sigma h_{\mu\nu}(x) e^{iS_{\text{EFT}}/\hbar} \Big|_{\hbar \rightarrow 0}$$

$$G^{\mu\nu}(\eta + h) = \textcircled{h} \text{wavy arrow}$$

$$\begin{aligned}
 & - \left(\frac{1}{2} \text{wavy arrow} + \frac{1}{3!} \text{wavy arrow} + \frac{1}{4!} \text{wavy arrow} + \dots \right) \\
 & = T^{\mu\nu} = \sum_i \left(\text{wavy arrow} + \text{wavy arrow} + \frac{1}{2} \text{wavy arrow} + \dots \right)
 \end{aligned}$$

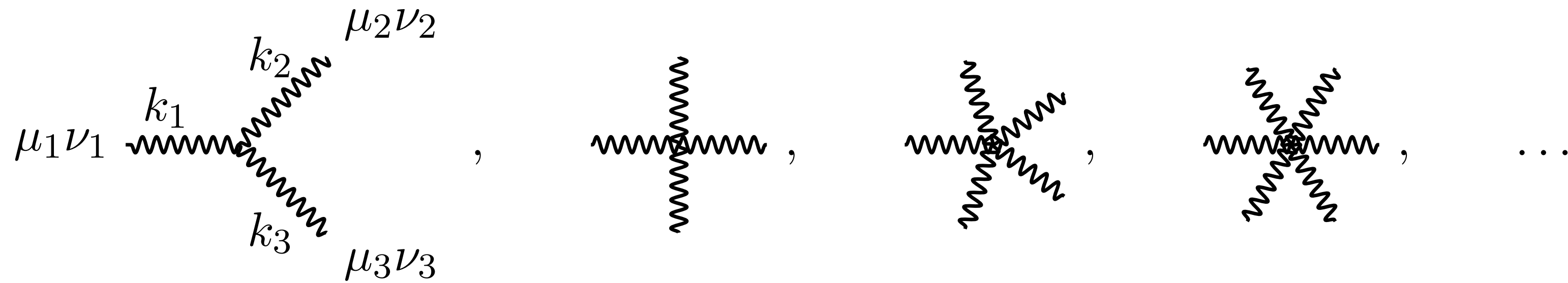
WQFT diagrams

Feynman rules (interactions):

Black hole 1

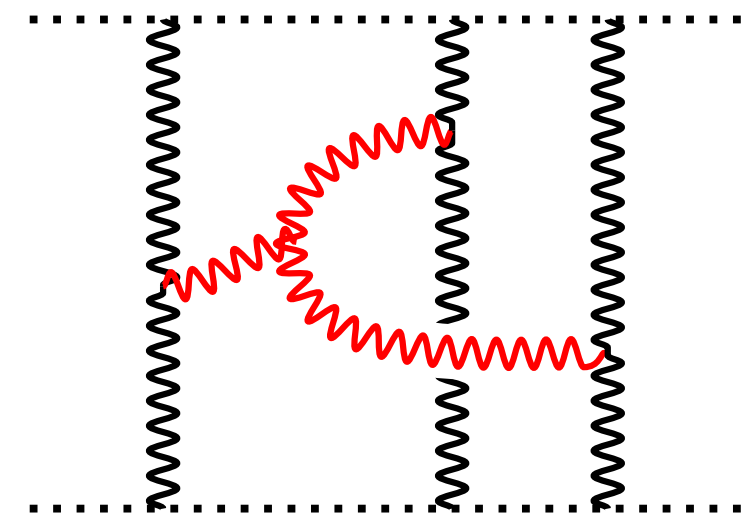
1
2

Example
diagrams:

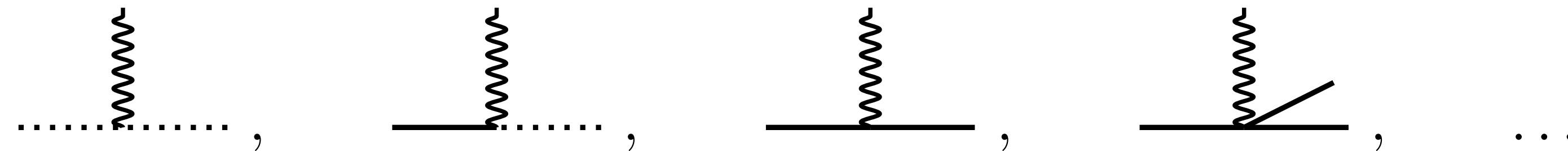


Gravitational
interactions

1
2



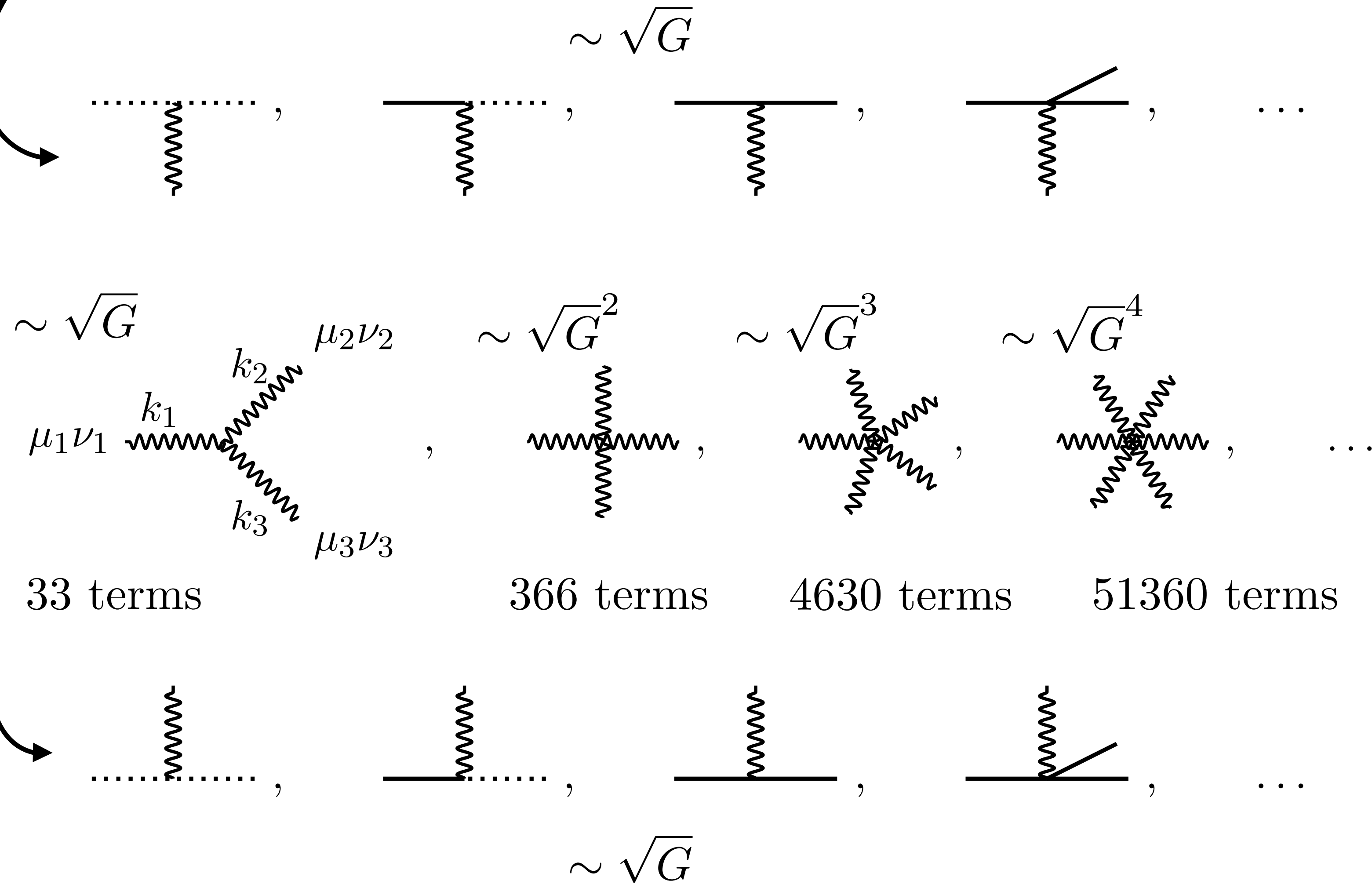
Black hole 2



WQFT diagrams

Worldline effective field theory
(spin, tidal, absorption effects)

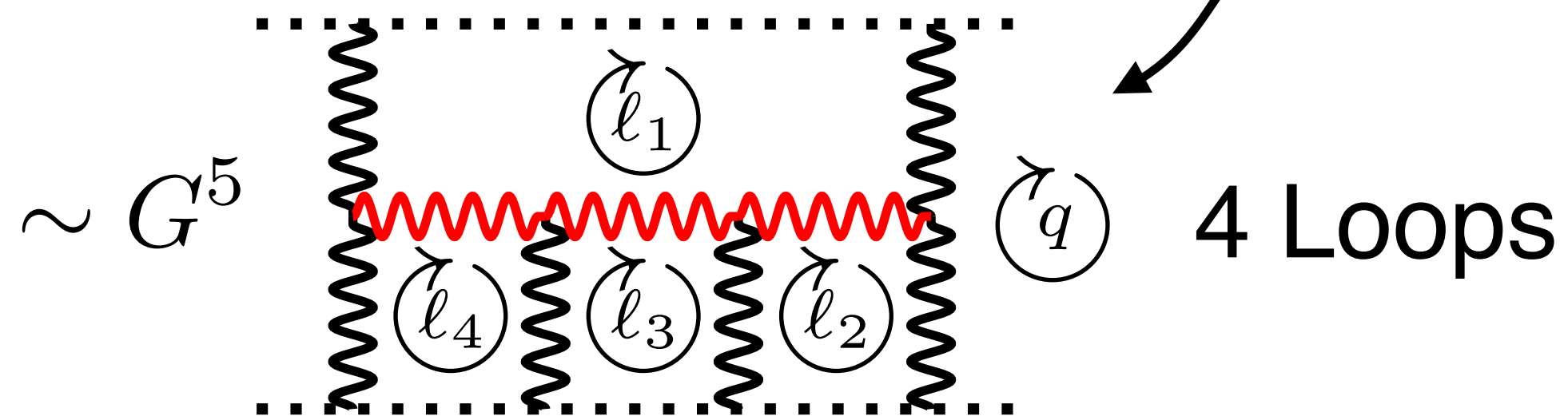
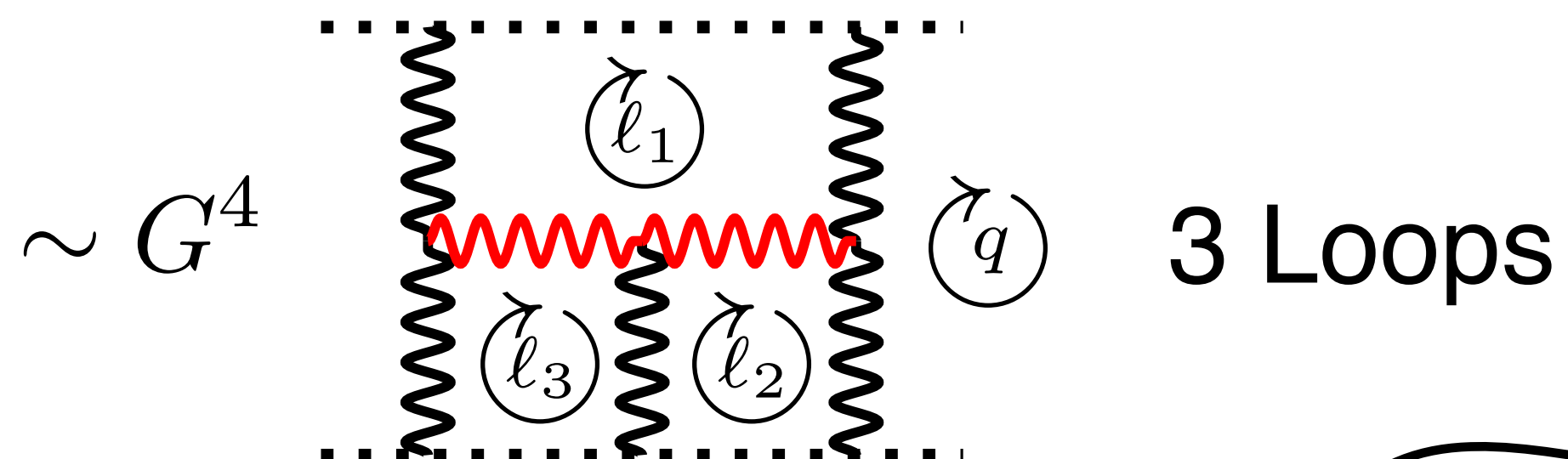
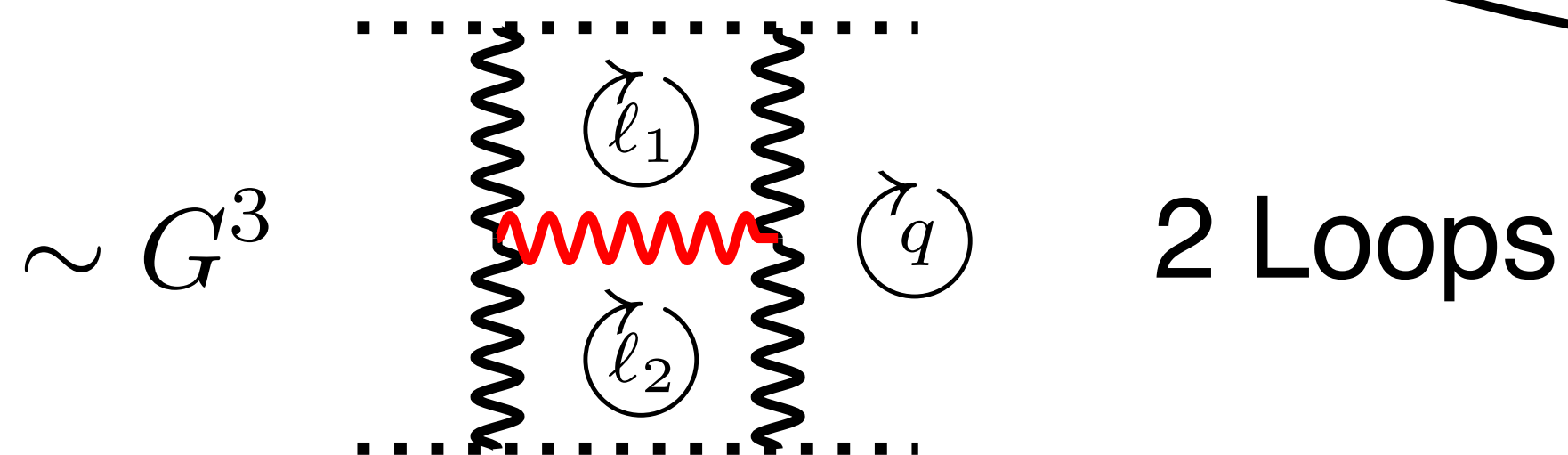
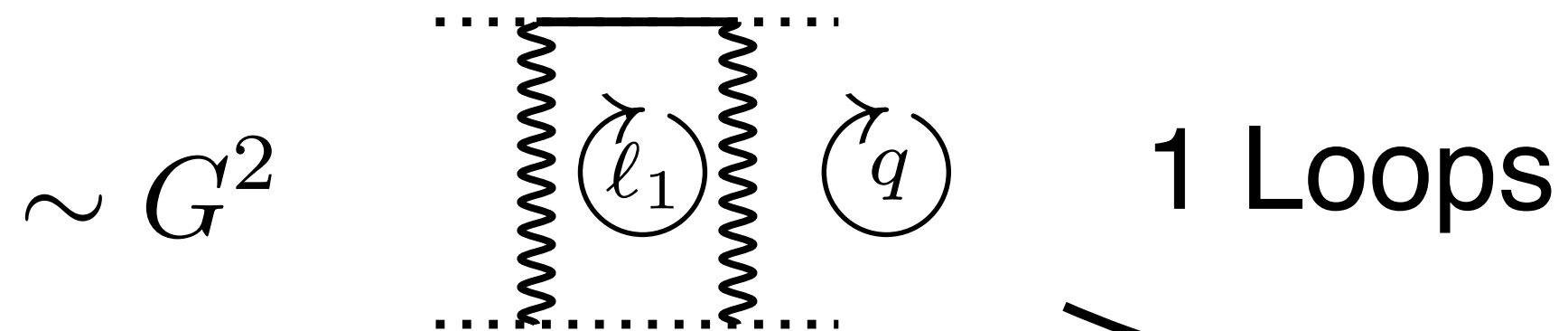
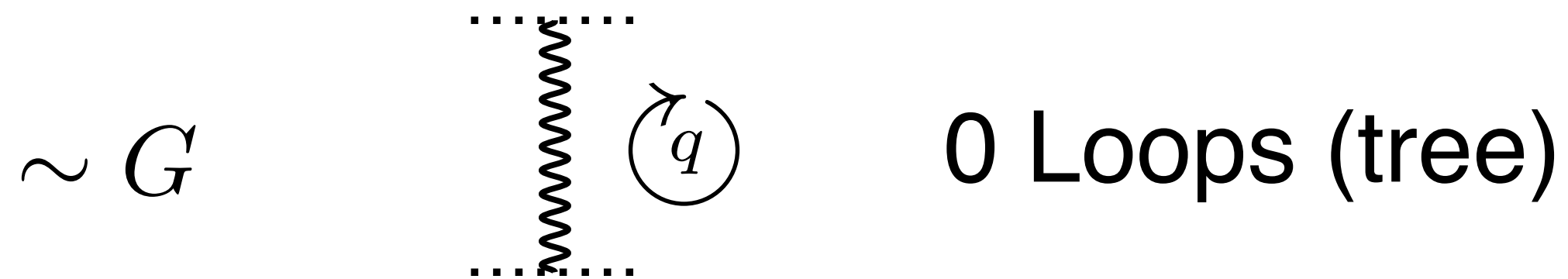
Example
diagrams



Gravitational
interactions

$$33^4 = 1\,185\,921 \text{ terms}$$

Loop Integration



$$\int d^{4-2\epsilon} \ell_1 \frac{\delta(\ell_1 \cdot v_2)}{\ell_1^2 (q - \ell_1)^2 \ell_1 \cdot v_1}$$

10^6 integrals at G^5

$4(4 - 2\epsilon)$ -dimensional integral

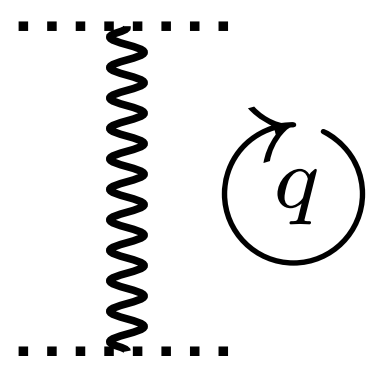
Dimensional regularization

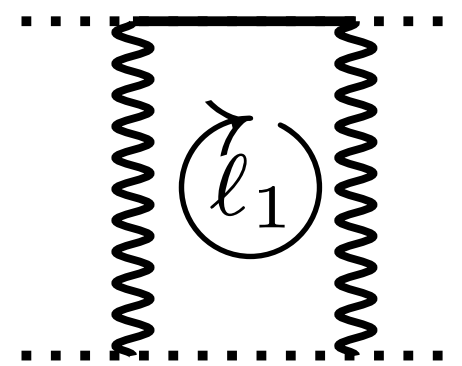
Loop Integration

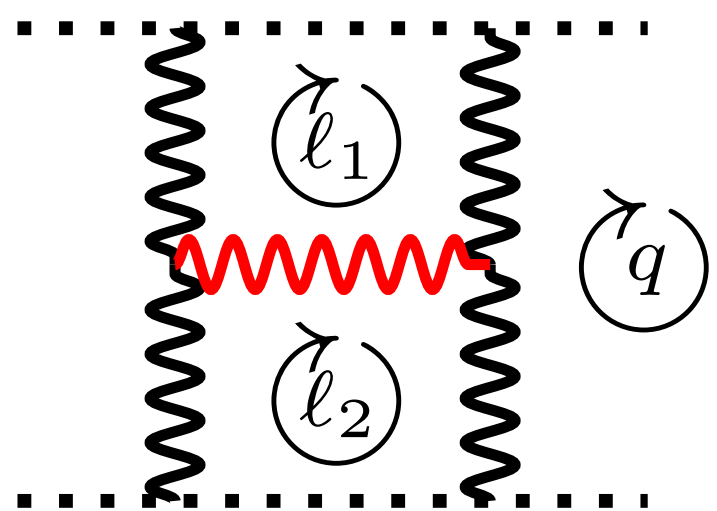
Methods from collider physics:

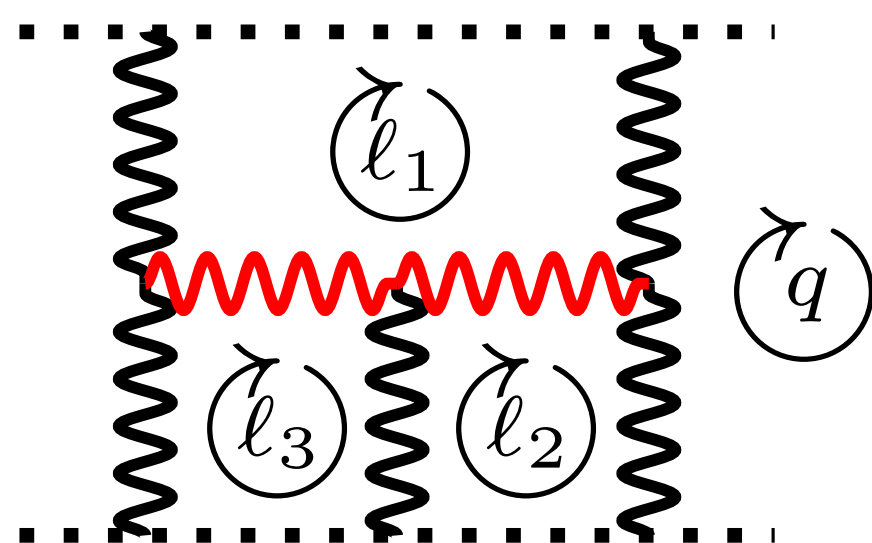
- **Integration-by-parts identities**
- **Differential equations (in canonical form) for master integrals**
- **Method of regions** for asymptotic limits
- **Dimensional regularization**
- Understanding of **special functions** and analytic behaviour

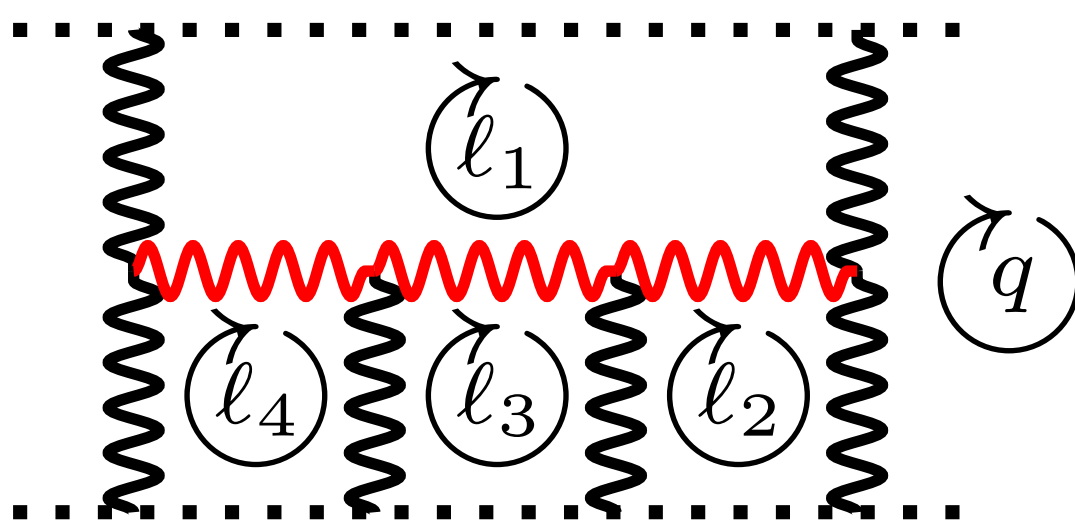
See posters from Mathias Driesse, Jitze Hoogeveen and Kathrin Stoldt for examples!

$\sim G$  0 Loops (tree)

$\sim G^2$  1 Loops

$\sim G^3$  2 Loops

$\sim G^4$  3 Loops

$\sim G^5$  4 Loops

Loop Integration

Methods from collider physics:

- **Integration-by-parts identities**

- Di
- ma

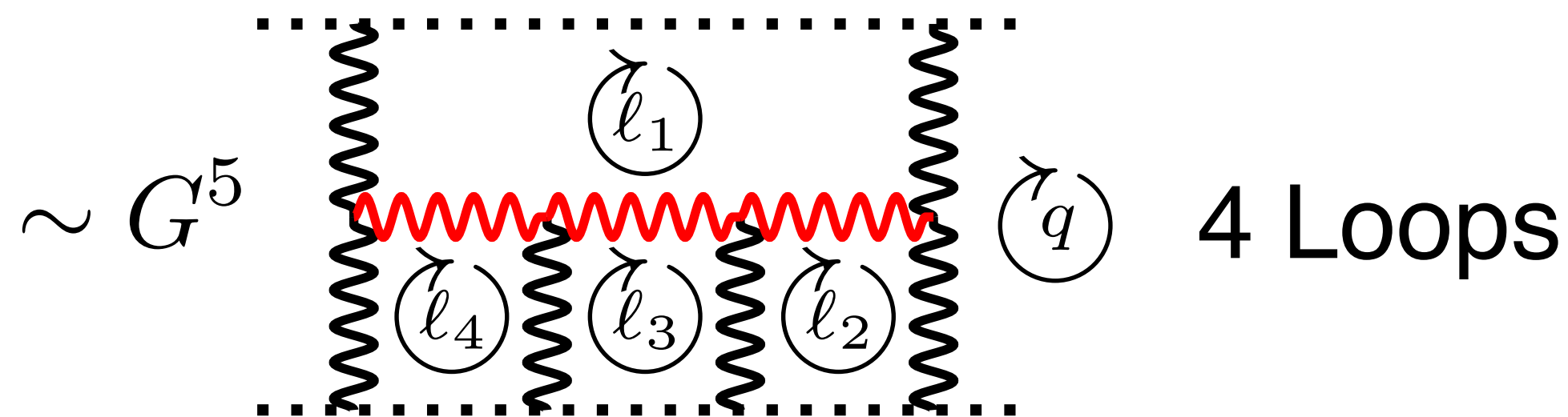
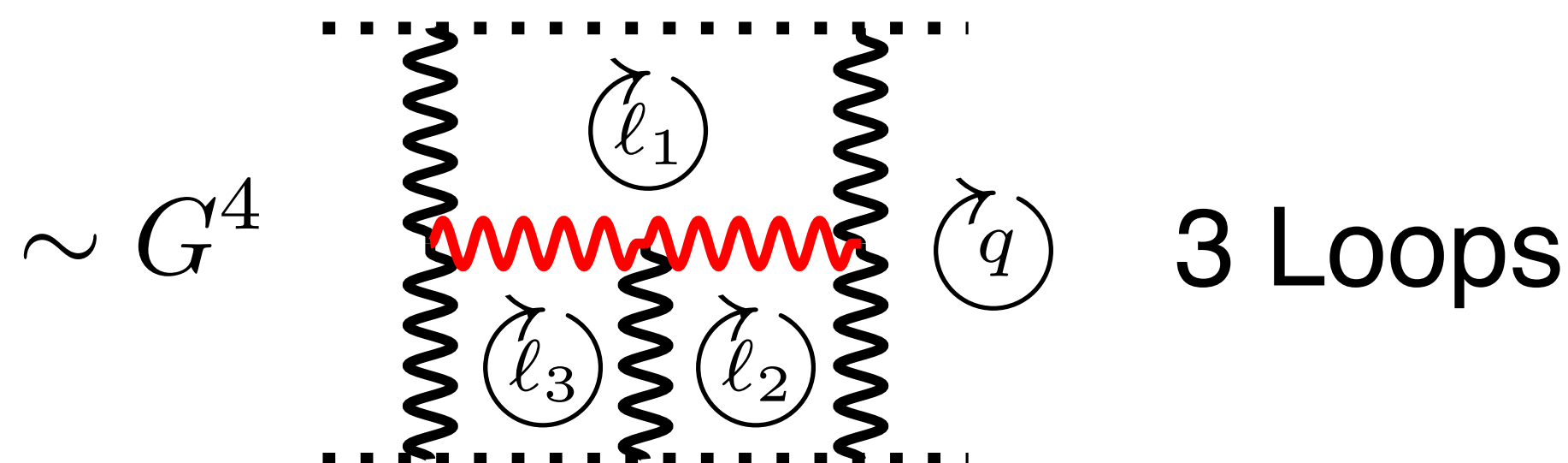
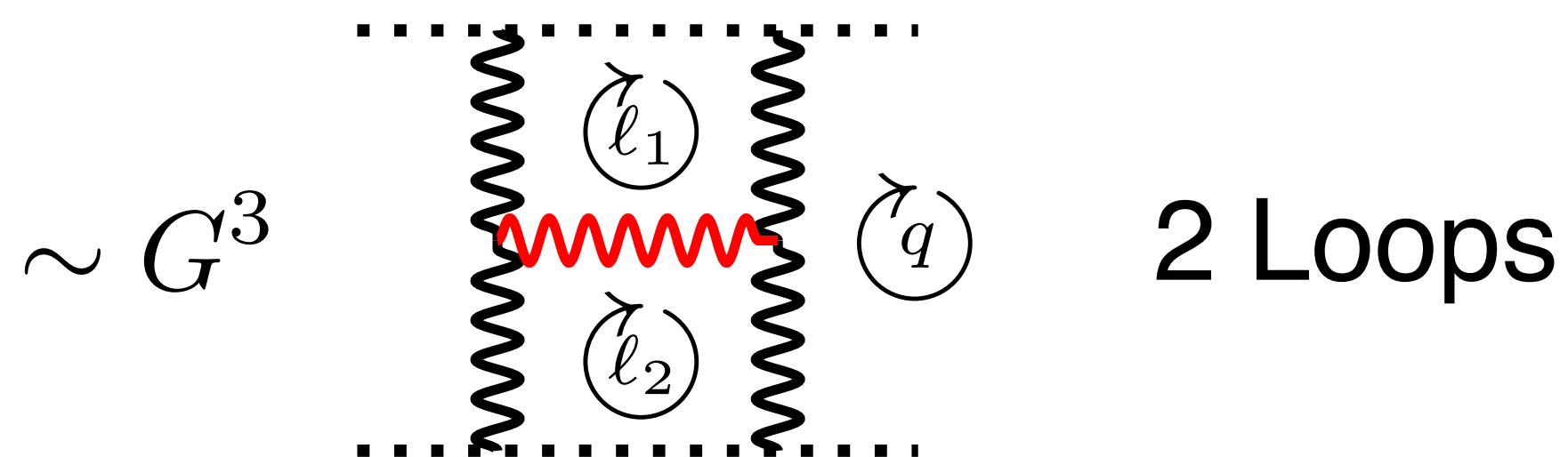
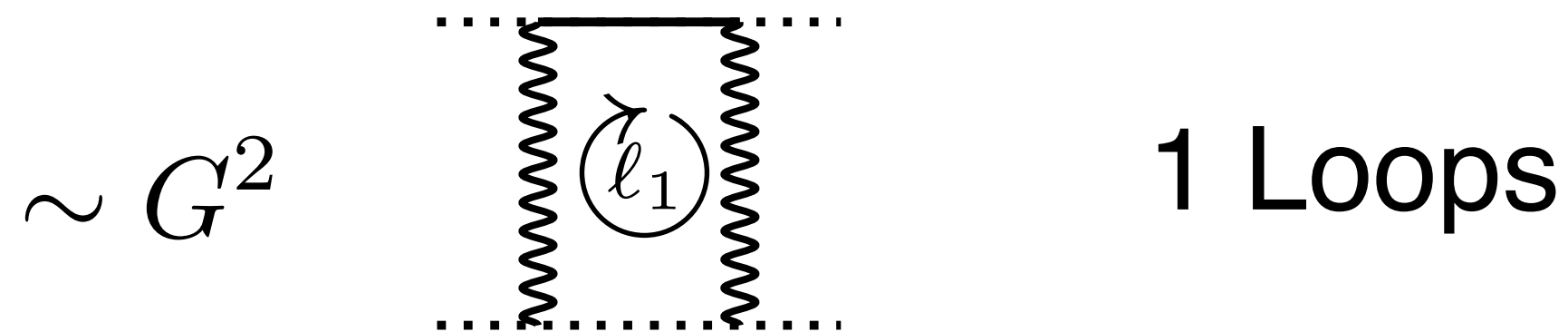
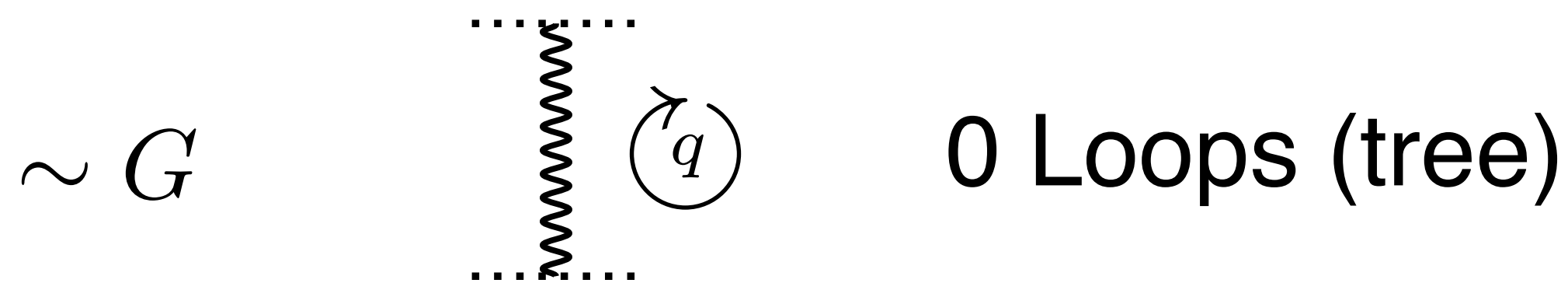
10⁵ core hours of IBP reduction using KIRA
developed by Johann Usovitsch
(~ 4 SpEC simulations)

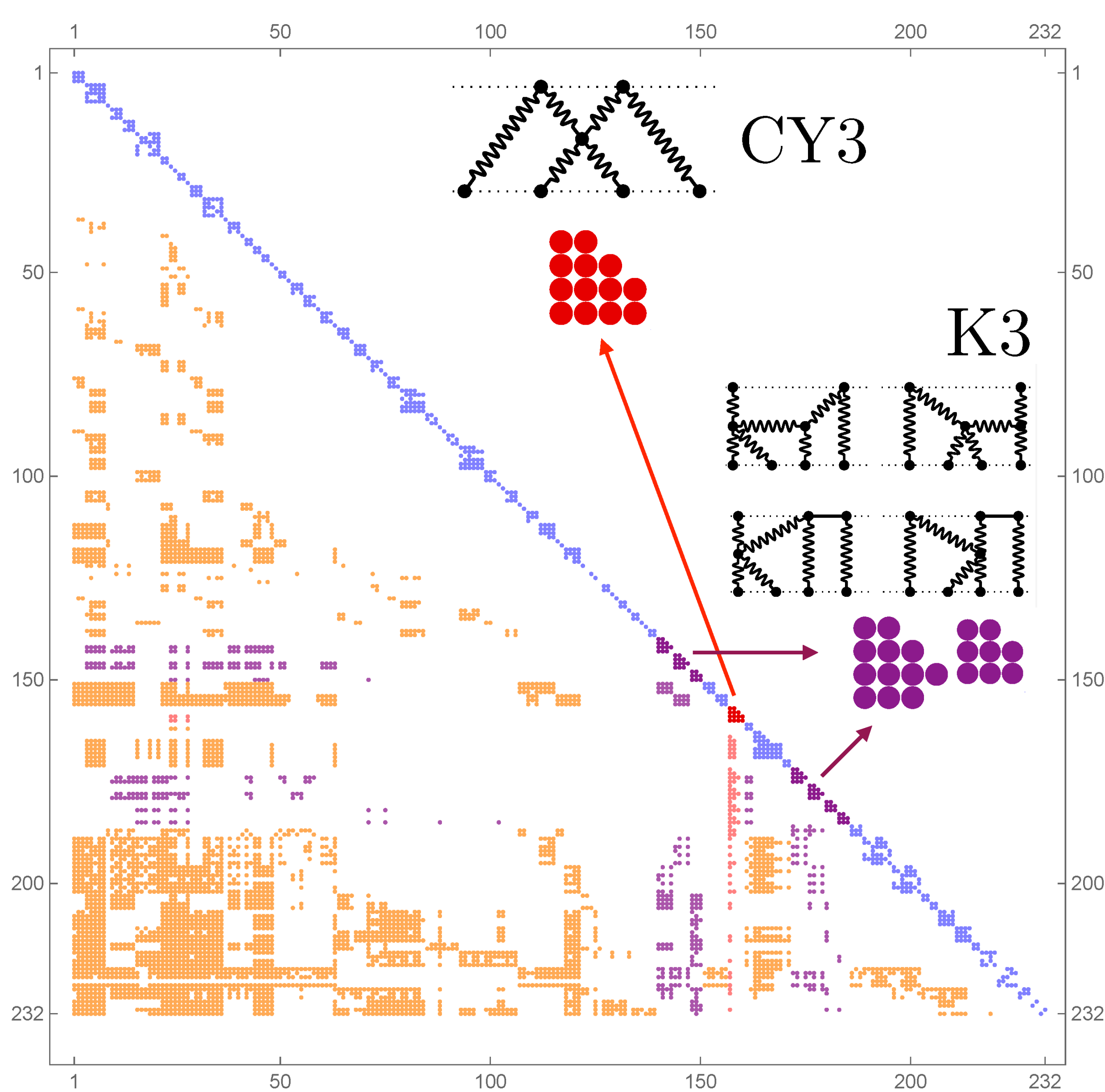
[Lange, Usovitsch, Wu: 2505.20197]

- **Dimensional regularization**

- Understanding of **special functions** and analytic behaviour

See posters from Mathias Driesse, Jitze
Hoogeveen and Kathrin Stoldt for examples!





Loop Integration

Methods from collider physics:

Integration-by-parts identities

- **Differential equations (in canonical form) for master integrals**

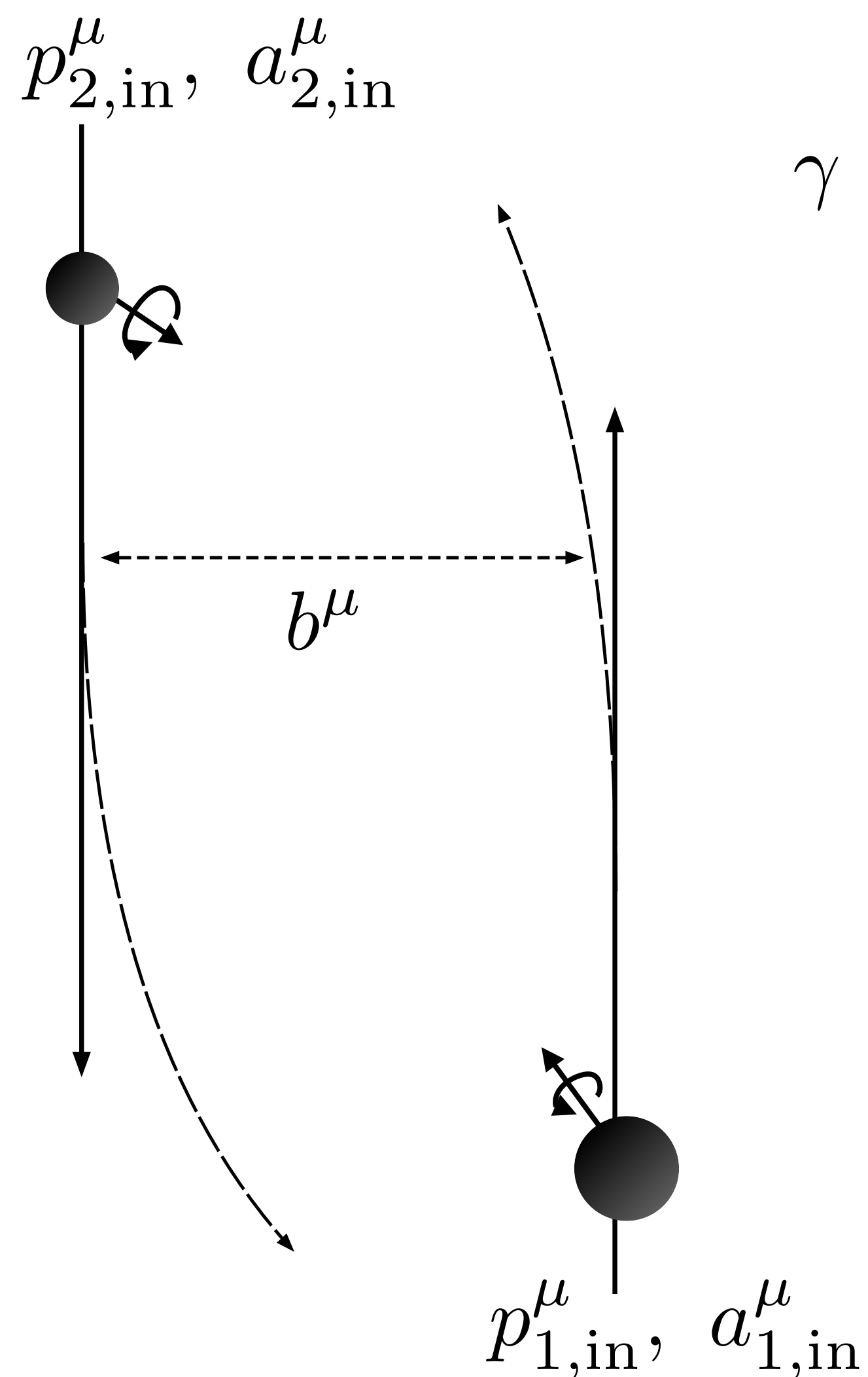
Method of regions for asymptotic computation

Dimensional regularization

Understanding of **special functions** and analytic behaviour

See posters from Mathias Driesse, Jitze Hoogeveen and Kathrin Stoldt for examples!

Kerr black hole scattering at G^5 precision



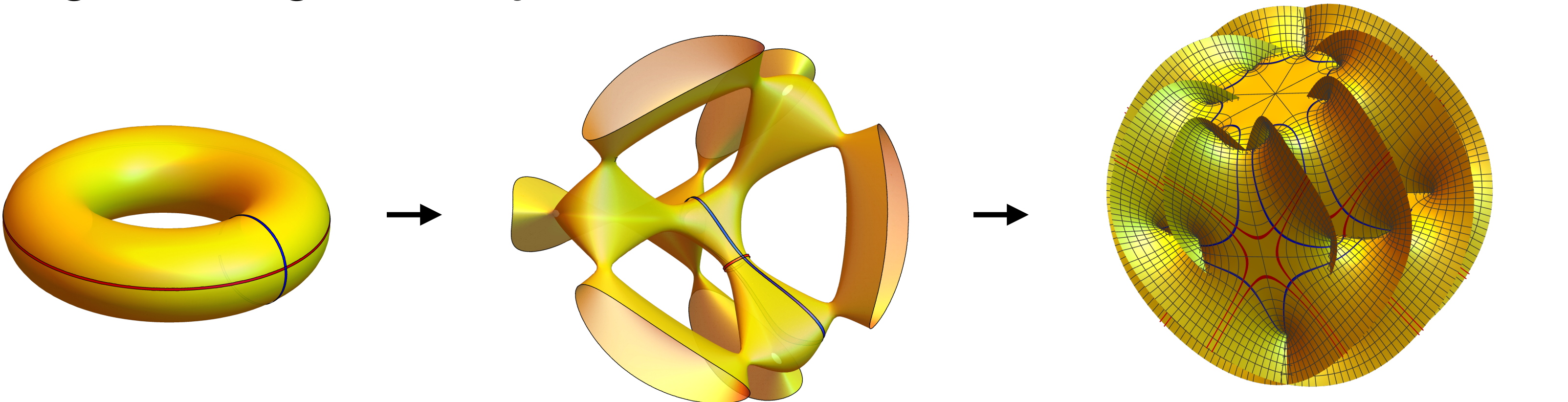
relative Lorentz factor

$$\gamma = \frac{p_{1,\text{in}} \cdot p_{2,\text{in}}}{m_1 m_2} = \frac{1}{2} \left(x + \frac{1}{x} \right)$$

- Analytic results for scattering angle and loss of energy to G^5 order
- Functions of initial data: $p_{i,\text{in}}^\mu, a_{i,\text{in}}^\mu, b^\mu$
- Non-trivial dependence on relative velocity: γ

Kerr black hole scattering at G^5 precision

Algebraic geometry



$$\mathcal{L}^{(2)}$$

$$\mathcal{L}^{(3)} = \text{sym}^2(\mathcal{L}^{(2)})$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(2)} * \mathcal{L}^{(2)}$$

$$K(x)$$

$$K^2(1 - x^2)$$

$$\varpi_0(x)$$

+1 more

+2 more

+3 more + even more

Complete elliptic functions

Calabi-Yau periods

Driesse, GUJ, Klemm, Mogull, Nega, Plefka, Sauer, Usovitsch: [2411.11846]

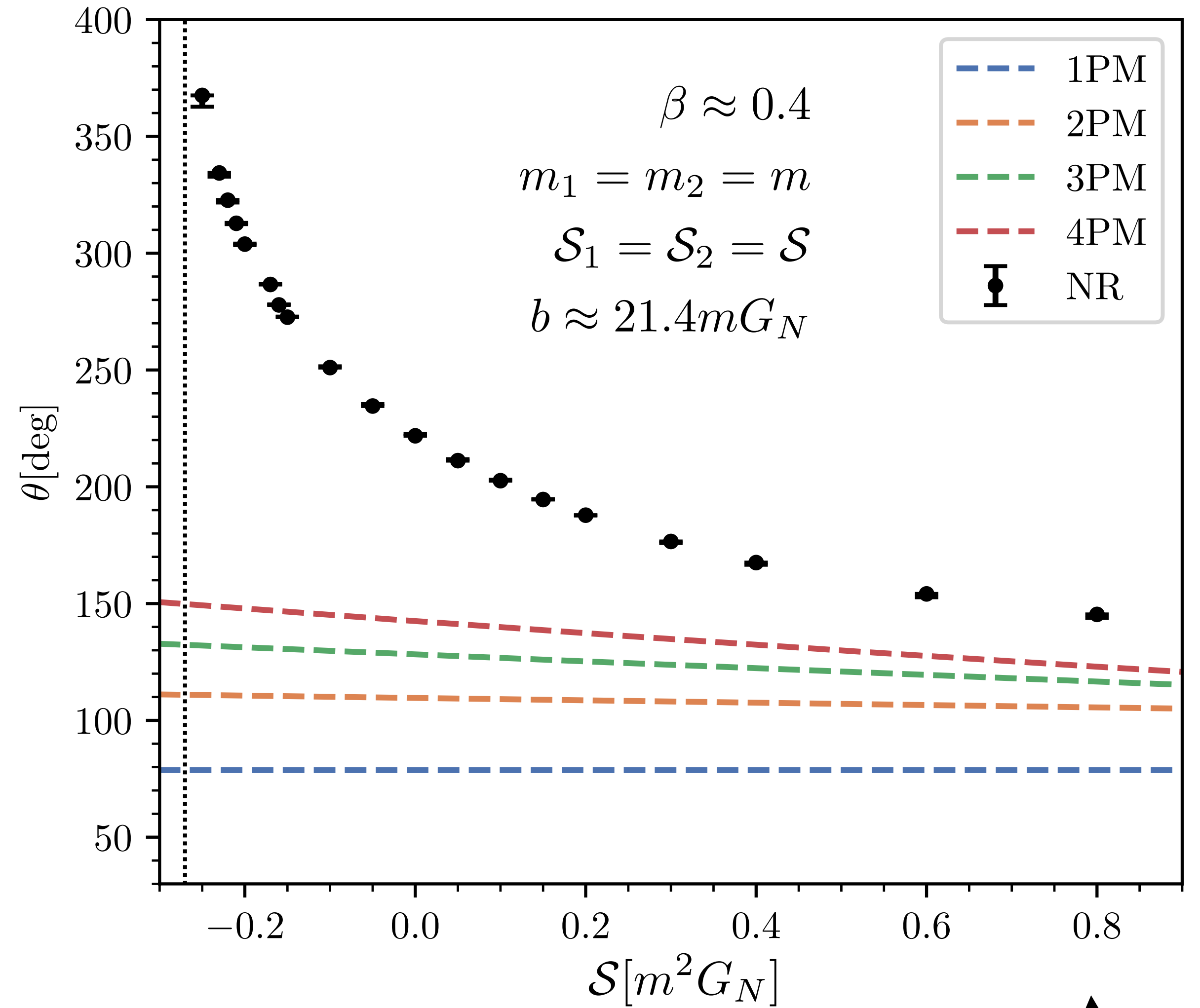
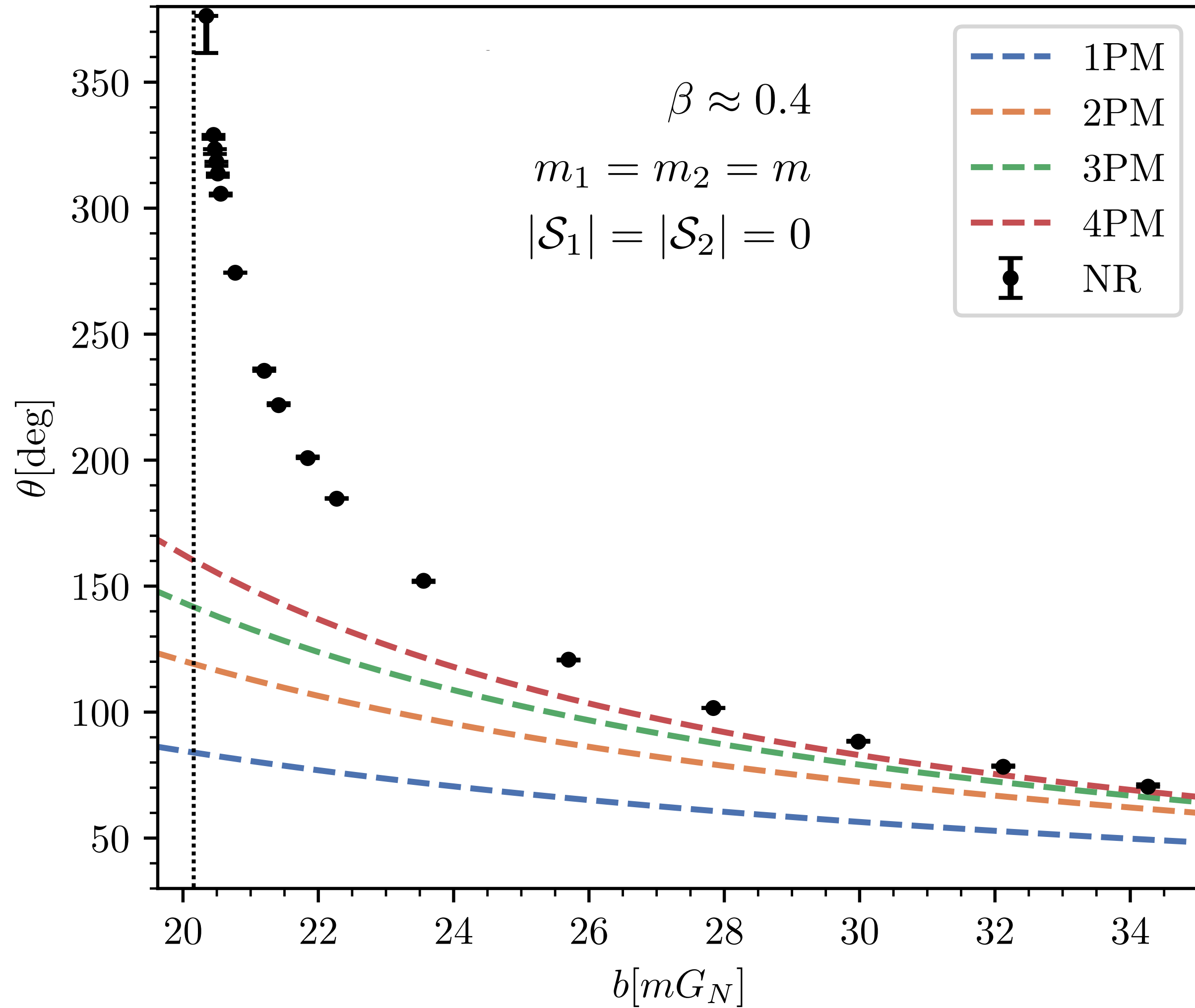


4-Loop Conservative Scattering Angle

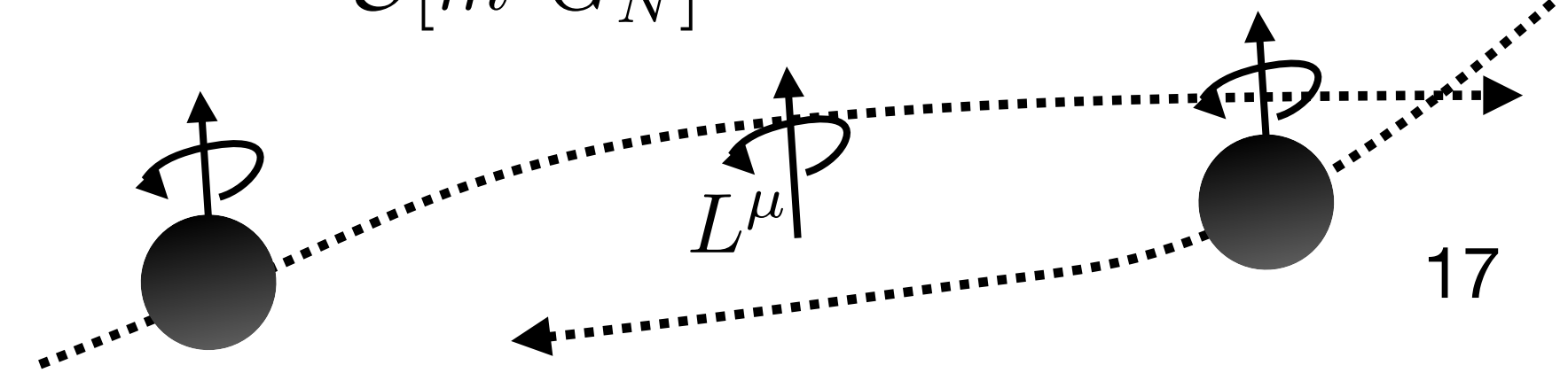
$f_6(\gamma) = 2G(1,y)(G(0,y) - G(1,y) + G(2,y) + \log(2))$	$:(\gamma) = -\frac{651264\gamma^{20} - 7809042\gamma^{18} - 23185512\gamma^{16} + 169295016\gamma^{14} - 315460542\gamma^{12} + 277369170\gamma^{10} - 134264214\gamma^8 + 6510035\gamma^6 - 988015\gamma^4 + 240905\gamma^2 - 18585}{2520\gamma^8(\gamma^2 - 1)^{9/2}}$
$f_7(\gamma) = \frac{1}{2}G(1;y)^2 + (-G(1;y) + G(1-i;y) + G(1+i;y))G(1;y) - G(1,1-i;y) - G(1,1+i;y)$	$:(\gamma) = \frac{6144\gamma^{16} - 587336\gamma^{14} + 4034092\gamma^{12} - 417302\gamma^{10} - 5560073\gamma^8 - 142640\gamma^6 + 35710\gamma^4 - 8250\gamma^2 + 1575}{360(\gamma^2 - 1)^3\gamma^7}$
$f_8(\gamma) = -\frac{1}{2}G(1;y)^2 + G(2;y)G(1;y) + G(0,1;y) - G(1,2;y)$	$c_4(\gamma) = -\frac{\gamma(32768\gamma^8 - 90112\gamma^6 + 1564672\gamma^4 - 1872978\gamma^2 - 7817455)}{336(\gamma^2 - 1)^2}$
$f_9(\gamma) = -G(1;y)G(2;y) + G(0,1;y) + G(1,2;y)$	$c_5(\gamma) = -\frac{491520\gamma^{22} - 2482176\gamma^{20} + 10655064\gamma^{18} - 32742084\gamma^{16} + 17085516\gamma^{14} + 61205662\gamma^{12} - 59068870\gamma^{10} - 5433687\gamma^8 + 1352120\gamma^6 - 330890\gamma^4 + 72450\gamma^2}{840(\gamma^2 - 1)^5\gamma^7}$
$f_{10}(\gamma) = \pi^2 G(1;y)$	$+ \frac{11025}{840(\gamma^2 - 1)^5\gamma^7}$
$f_{11}(\gamma) = -G(1;y)(-G(1;y)^2 - 2G(0;y)G(1;y) + G(2;y)G(1;y) - \log(4)G(1;y) - G(0,1;y) + 4G(1,1-i;y) + 4G(1,1+i;y) - G(1,2;y))$	$c_6(\gamma) = -\frac{24576\gamma^{18} + 213480\gamma^{16} - 1029342\gamma^{14} - 1978290\gamma^{12} + 3752006\gamma^{10} + 816595\gamma^8 - 55260\gamma^6 + 13690\gamma^4 - 3100\gamma^2 + 525}{120\gamma^8(\gamma^2 - 1)^{7/2}}$
$f_{12}(\gamma) = -G(1;y)^3 + 2G(2;y)G(1;y)^2 + (G(1;y)G(2;y) - G(0,1;y) - G(1,2;y))G(1;y) + 4\left(-\frac{1}{6}G(1;y)^3 + G(1,1,1-i;y) + G(1,1,1+i;y)\right)$	$c_7(\gamma) = \frac{198856\gamma^{14} - 689664\gamma^{12} - 154716\gamma^{10} + 666260\gamma^8 - 5091\gamma^6 - 1935\gamma^4 + 155\gamma^2 - 105}{12\gamma^8(\gamma^2 - 1)^{5/2}}$
$f_{13}(\gamma) = \frac{1}{4}G(1;y)^3 - G(2;y)G(1;y)^2 + G(2;y)^2G(1;y) - G(0,1;y)G(1;y) + G(1,2;y)G(1;y) + 2G(0,0,1;y) + G(0,1,1;y) - G(1,1,2;y) - 2G(1,2,2;y)$	$c_8(\gamma) = -\frac{49152\gamma^{18} - 208896\gamma^{16} + 1182464\gamma^{14} - 3741239\gamma^{12} + 3040161\gamma^{10} + 1882567\gamma^8 - 2828161\gamma^6 + 49728\gamma^4 - 2268\gamma^2 + 1260}{42\gamma^6(\gamma^2 - 1)^{7/2}}$
$f_{14}(\gamma) = \left(-\frac{1}{2}G(1;y)^2 + G(2;y)G(1;y) + G(0,1;y) - G(1,2;y)\right)(2G(0;y) - G(1;y) + \log(4))$	$c_9(\gamma) = -\frac{\gamma(525\gamma^8 - 450\gamma^6 + 17700\gamma^4 - 12598\gamma^2 - 5369)}{4(\gamma^2 - 1)^{7/2}}$
$f_{15}(\gamma) = -\frac{1}{6}G(1;y)^3 + G(1,1-i;y)G(1;y) + G(1,1+i;y)G(1;y) + (2G(2;y) - G(1;y))\left(-\frac{1}{2}G(1;y)^2 + G(1,1-i;y) + G(1,1+i;y)\right) - 2G(1,1,1-i;y)$	$c_{10}(\gamma) = -\frac{81920\gamma^6 + 189180\gamma^4 - 1240416\gamma^2 - 199207}{48(\gamma^2 - 1)^{5/2}}$
$- 2G(1,1,1+i;y) + 2G(1,1,2;y) - 2G(1,1-i,2;y) - 2G(1,1+i,2;y)$	$c_{11}(\gamma) = -\frac{128\gamma(2\gamma^2 - 3)(8\gamma^6 - 6\gamma^4 - 51\gamma^2 - 8)}{(\gamma^2 - 1)^4}$
$f_{16}(\gamma) = (-G(1;y)G(2;y) + G(0,1;y) + G(1,2;y))(2G(0;y) - G(1;y) + \log(4))$	$c_{12}(\gamma) = -\frac{\gamma(2\gamma^2 - 3)(2273\gamma^6 - 1851\gamma^4 - 12957\gamma^2 - 2057)}{2(\gamma^2 - 1)^4}$
$f_{17}(\gamma) = 2\left(-\frac{1}{6}G(1;y)^3 + G(0,1,1;y) + G(1,1,2;y)\right) - \frac{3}{2}G(1;y)(-G(1;y)G(2;y) + G(0,1;y) + G(1,2;y))$	$c_{13}(\gamma) = -\frac{\gamma(1575\gamma^6 + 1920\gamma^4 - 5177\gamma^2 - 1182)}{2(\gamma^2 - 1)^{5/2}}$
$f_{18}(\gamma) = \frac{1}{6}G(1;y)^3 - (-G(1;y) + G(1-i;y) + G(1+i;y))(2G(2;y) - G(1;y))G(1;y) - G(1,1-i;y)G(1;y) - G(1,1+i;y)G(1;y)$	$c_{14}(\gamma) = -\frac{1249\gamma^6 - 1083\gamma^4 - 1053\gamma^2 - 9}{(\gamma^2 - 1)^{5/2}}$
$- (G(1;y) - G(1-i;y) - G(1+i;y))(-2G(0;y) + G(1;y) + \log(4))G(1;y) - 4(-G(1;y) + G(1-i;y) + G(1+i;y))\log(2)G(1;y)$	$c_{15}(\gamma) = -\frac{3(225\gamma^6 + 600\gamma^5 - 315\gamma^4 - 1200\gamma^3 + 99\gamma^2 + 56\gamma - 9)}{(\gamma^2 - 1)^{5/2}}$
$+ 2G(1,1,1-i;y) + 2G(1,1,1+i;y) - 2G(1,1,2;y) + 2G(1,1-i,2;y) + 2G(1,1+i,2;y)$	$c_{16}(\gamma) = \frac{\gamma(2100\gamma^6 + 1755\gamma^4 - 6422\gamma^2 - 1209)}{4(\gamma^2 - 1)^{5/2}}$
$+ \left(-\frac{1}{2}G(1;y)^2 + G(1,1-i;y) + G(1,1+i;y)\right)(2G(0;y) - G(1;y) - \log(4)) + 4\left(-\frac{1}{2}G(1;y)^2 + G(1,1-i;y) + G(1,1+i;y)\right)\log(2)$	$c_{17}(\gamma) = \frac{64(160\gamma^{12} - 600\gamma^{10} - 84\gamma^8 + 2058\gamma^6 - 1665\gamma^4 - 72\gamma^2 + 32)}{3(\gamma^2 - 1)^{11/2}}$
$f_{19}(\gamma) = G(1;y)\left(-\frac{1}{2}G(1;y)^2 + G(2;y)G(1;y) + G(0,1;y) - G(1,2;y)\right)$	$c_{18}(\gamma) = \frac{1823\gamma^6 - 1221\gamma^4 - 13155\gamma^2 - 2039}{(\gamma^2 - 1)^{5/2}}$
$f_{20}(\gamma) = (-G(1;y) + G(1-i;y) + G(1+i;y))\left(2G(0,1;y) - \frac{1}{2}G(1;y)^2\right) + (-G(1;y) + G(1-i;y) + G(1+i;y))\left(2(G(1;y)G(2;y) - G(1,2;y)) - \frac{1}{2}G(1;y)^2\right)$	$c_{19}(\gamma) = \frac{\gamma(2\gamma^2 - 3)(1823\gamma^6 - 1221\gamma^4 - 13155\gamma^2 - 2039)}{(\gamma^2 - 1)^4}$
$- 2(-G(0,1,1;y) + G(0,1,1-i;y) + G(0,1,1+i;y)) + 2(-G(1;y)G(1,2;y) + 2G(1,1,2;y) + G(1,2,1-i;y) + G(1,2,1+i;y))$	$c_{20}(\gamma) = \frac{768(8\gamma^6 + 14\gamma^4 - 116\gamma^2 - 19)}{(\gamma^2 - 1)^{5/2}}$
$f_{21}(\gamma) = 2G(1;y)(G(1;y)G(2;y) - G(0,1;y) - G(1,2;y))$	$c_{21}(\gamma) = \frac{3150\gamma^7 + 1846\gamma^6 - 5775\gamma^5 + 198\gamma^4 + 5488\gamma^3 - 7518\gamma^2 - 1935\gamma + 258}{4(\gamma^2 - 1)^4}$
$f_{22}(\gamma) = \frac{1}{2}\left(\frac{1}{6}G(1;y)^3 - G(1,1-i;y)G(1;y) - G(1,1+i;y)G(1;y) + \left(\frac{1}{2}G(1;y)^2 - G(1,1-i;y) - G(1,1+i;y) + 2(-G(1;y)G(2;y) + G(1,2;y) + G(2,1-i;y) + G(2,1+i;y))\right)G(1;y) + 2(-G(0,1,1;y) + G(0,1-i,1;y) + G(0,1+i,1;y)) + 2G(1,1,1-i;y) + 2G(1,1,1+i;y)\right)$	$c_{22}(\gamma) = -\frac{4(1823\gamma^6 + 6459\gamma^4 - 38115\gamma^2 - 6263)}{(\gamma^2 - 1)^{5/2}}$
$f_{23}(\gamma) = \frac{1}{12}G(1;y)^3 - G(2;y)G(1;y)^2 + G(2;y)^2G(1;y) + G(0,1;y)G(1;y) + G(1,2;y)G(1;y) - 2G(0,0,1;y) - G(0,1,1;y) - G(1,1,2;y) - 2G(1,2,2;y)$	$c_{23}(\gamma) = \frac{1050\gamma^6 + 1696\gamma^5 + 389\gamma^4 - 1691\gamma^3 - 2241\gamma^2 - 1041\gamma - 114}{2(\gamma^2 - 1)^{3/2}}$
$f_{24}(\gamma) = \frac{1}{2}G(1;y)^3 - 6G(0,1,1;y) + 4G(0,1,2;y) + 8G(0,2,1;y) - 4(G(1;y)G(1,2;y) - 2G(1,1,2;y)) - 2G(1,1,2;y)$	$c_{24}(\gamma) = -\frac{3\gamma(175\gamma^6 - 1255\gamma^4 + 1985\gamma^2 - 1179)}{4(\gamma^2 - 1)^{5/2}}$
$f_{25}(\gamma) = -\frac{1}{12}G(1;y)^3 + G(0,1,1;y) - 2G(0,1,2;y) + G(1,3,2;y)$	$c_{25}(\gamma) = \frac{3421\gamma^6 - 2067\gamma^4 - 5865\gamma^2 + 111}{2(\gamma^2 - 1)^{5/2}}$
$f_{26}(\gamma) = \frac{1}{2}\left(-\frac{1}{6}G(1;y)^3 - G(1,1-i;y)G(1;y) + G(1,1+i;y)G(1;y) + \left(\frac{1}{2}G(1;y)^2 - G(1,1-i;y) - G(1,1+i;y) + 2(-G(1;y)G(2;y) + G(1,2;y) + G(2,1-i;y) + G(2,1+i;y))\right)G(1;y) + 2(-G(0,1,1;y) + G(0,1-i,1;y) + G(0,1+i,1;y)) + 2G(1,1,1-i;y) + 2G(1,1,1+i;y)\right)$	$c_{26}(\gamma) = \frac{48\gamma(75\gamma^4 - 150\gamma^2 + 7)}{(\gamma^2 - 1)^{5/2}}$
$f_{27}(\gamma) = \frac{1}{6}G(1;y)^3 + \frac{1}{2}(-G(1;y) + G(1-i;y) + G(1+i;y))^2G(1;y) - G(1,1-i;y)G(1;y) - G(1,1+i;y)G(1;y)$	$c_{27}(\gamma) = -\frac{2(4321\gamma^6 + 11973\gamma^4 - 75933\gamma^2 - 12553)}{(\gamma^2 - 1)^{5/2}}$
$- (-G(1;y) + G(1-i;y) + G(1+i;y))\left(\frac{1}{2}G(1;y) - 2G(2;y)\right)\left(2G(0,1;y) - \frac{1}{2}G(1;y)^2\right)$	$c_{28}(\gamma) = \frac{64(160\gamma^{12} - 600\gamma^{10} - 84\gamma^8 + 2058\gamma^6 - 1665\gamma^4 - 72\gamma^2 + 32)}{3(\gamma^2 - 1)^{11/2}}$
$+ G(1,1-i,1-i;y) + G(1,1-i,1+i;y) + G(1,1+i,1-i;y) + G(1,1+i,1+i;y)$	$c_{29}(\gamma) = \frac{64(160\gamma^{12} - 600\gamma^{10} - 84\gamma^8 + 2058\gamma^6 - 1665\gamma^4 - 72\gamma^2 + 32)}{3(\gamma^2 - 1)^{11/2}}$
$f_{28}(\gamma) = G(1;y)^3$	$c_{30}(\gamma) = \frac{64(160\gamma^{12} - 600\gamma^{10} - 84\gamma^8 + 2058\gamma^6 - 1665\gamma^4 - 72\gamma^2 + 32)}{3(\gamma^2 - 1)^{11/2}}$
$f_{29}(\gamma) = G(1;y)^2(-G(1;y) + G(1-i;y) + G(1+i;y))$	$c_{31}(\gamma) = \frac{64(160\gamma^{12} - 600\gamma^{10} - 84\gamma^8 + 2058\gamma^6 - 1665\gamma^4 - 72\gamma^2 + 32)}{3(\gamma^2 - 1)^{11/2}}$
$f_{30}(\gamma) = -\left(G(1;y) - 2G(2;y)\right)\left(2G(0,1;y) - \frac{1}{2}G(1;y)^2\right)$	

Weak-field to Strong-field

Dynamical Capture



Buonanno, GUJ, Mogull: [2402.12342]
 Damour, Guercilena, Hinder, Hopper, Nagar, Rezzolla: [1402.7307]
 Rettegno, Pratten, Thomas, Schmidt, Damour: [2307.06999]



Construct (Effective-one-body) potential

From weak-field (post-Minkowskian) data

$$V(r, p_\infty, L) = \frac{G_N c_1(p_\infty, L)}{r} + \frac{G_N^2 c_2(p_\infty, L)}{r^2} + \frac{G_N^3 c_3(p_\infty, L)}{r^3} + \frac{G_N^4 c_4(p_\infty, L)}{r^4} + \mathcal{O}(G_N^5)$$

$$p_r^2 = \frac{1}{(1 + B_{\text{np}}^{\text{Kerr}})} \left[\frac{1}{A^{\text{Kerr}}} \left(E_{\text{Kerr}} - \frac{2MLa}{r^3 + a^2(r + 2M)} \right)^2 - \left(\mu^2 + \frac{L^2}{r^2} + B_{\text{npa}}^{\text{Kerr}} \frac{L^2 a^2}{r^2} \right) \right]$$
$$= p_\infty^2 - V(r, p_\infty, L)$$

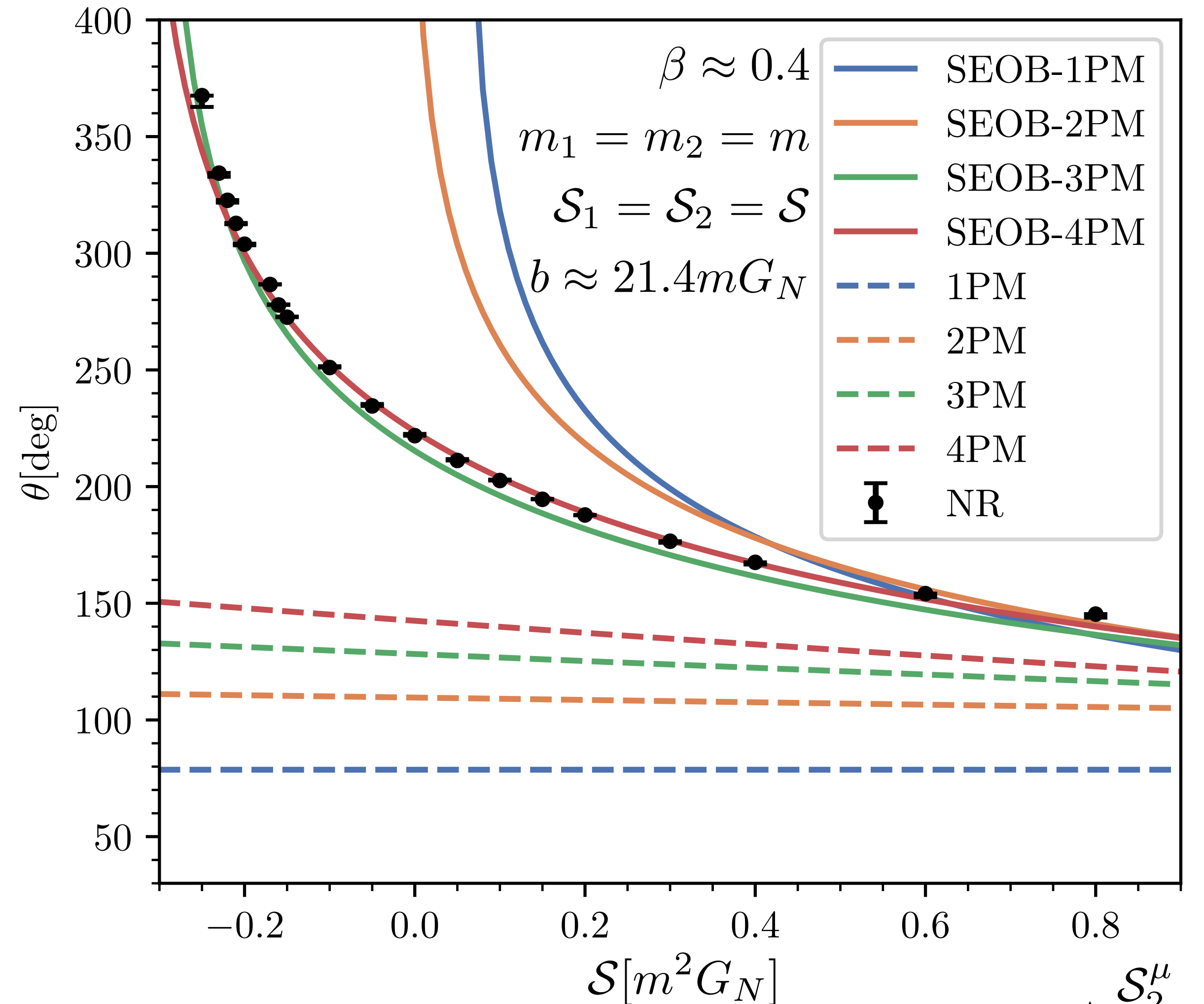
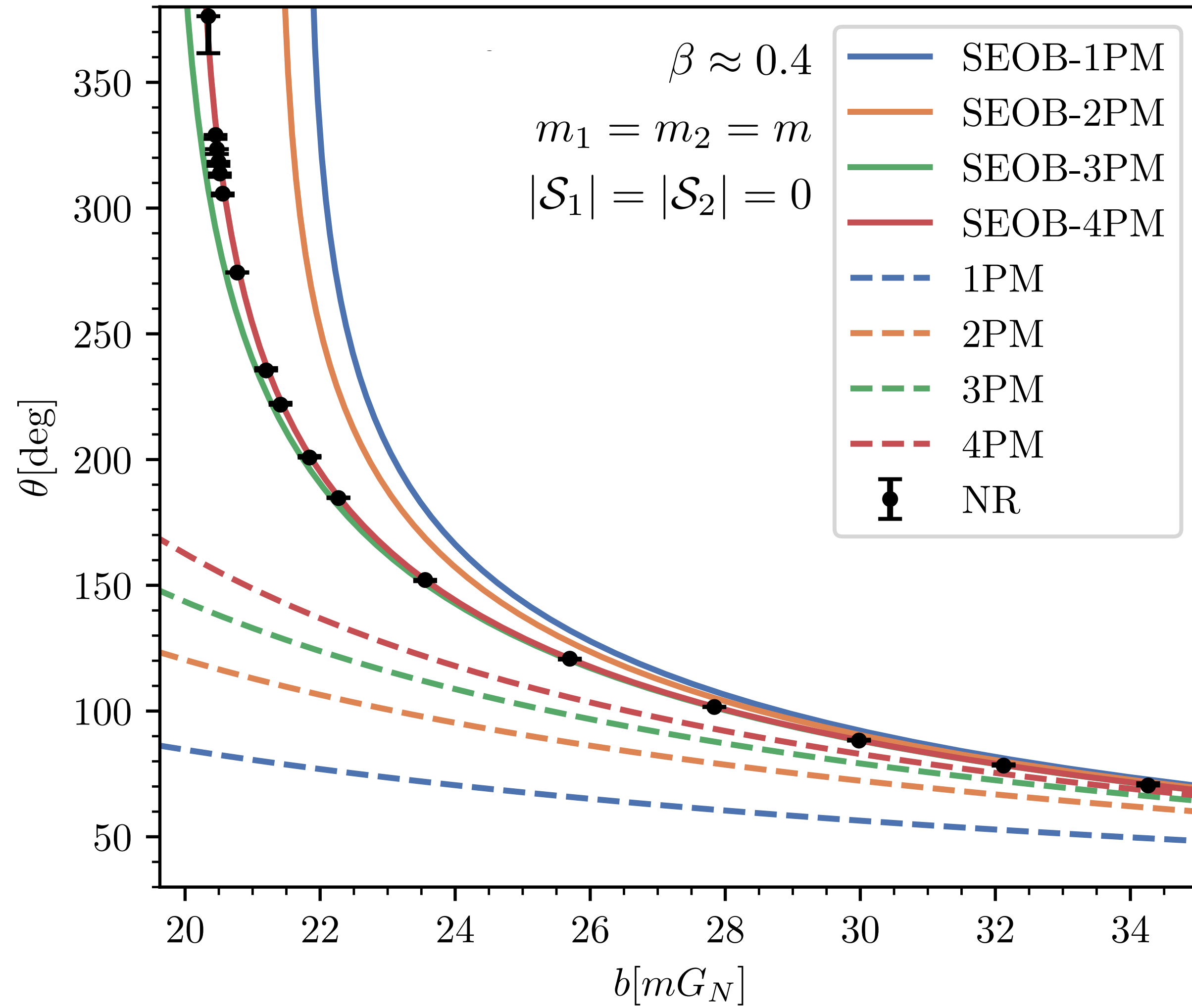
$$A^{\text{Kerr}} = \frac{1 - 2u + \chi^2 u^2}{1 + \chi^2 u^2 (2u + 1)}$$

$$B_{\text{np}}^{\text{Kerr}} = \chi^2 u^2 - 2u$$

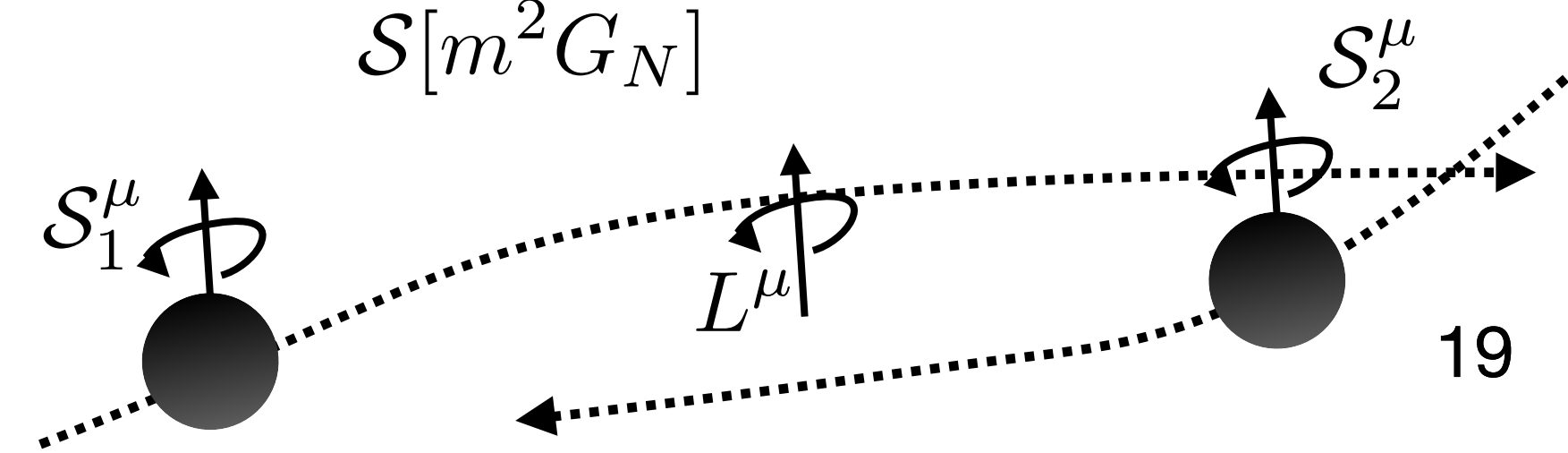
From geodesic limit (effective-one-body formalism)

$$B_{\text{npa}}^{\text{Kerr}} = -\frac{1 + 2u}{r^2 + a^2(1 + 2u)}$$

SEOB-PM resummation model



Buonanno, GUJ, Mogull: [2402.12342]
 Damour, Guercilena, Hinder, Hopper, Nagar, Rezzolla: [1402.7307]
 Rettegno, Pratten, Thomas, Schmidt, Damour: [2307.06999]

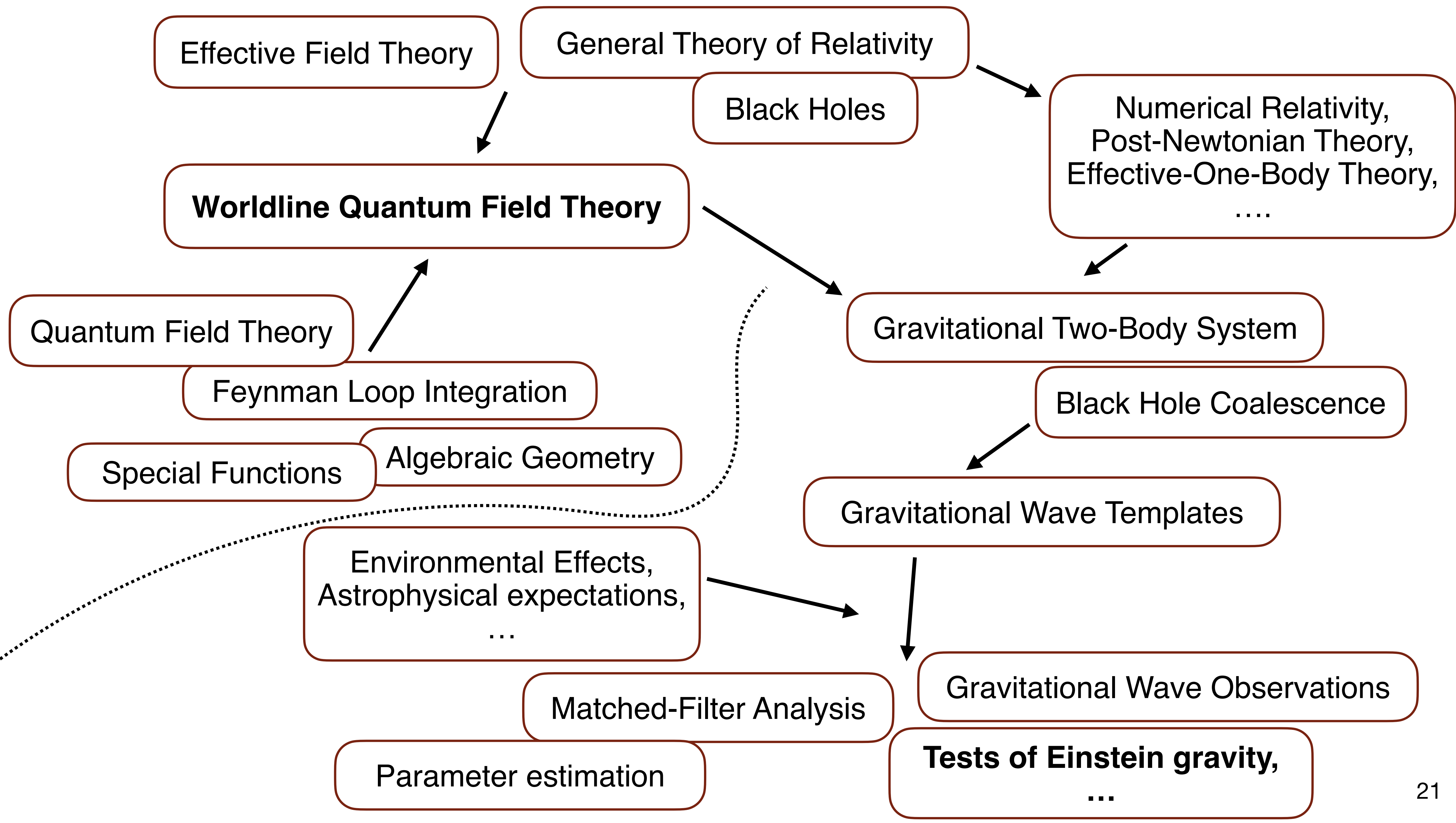


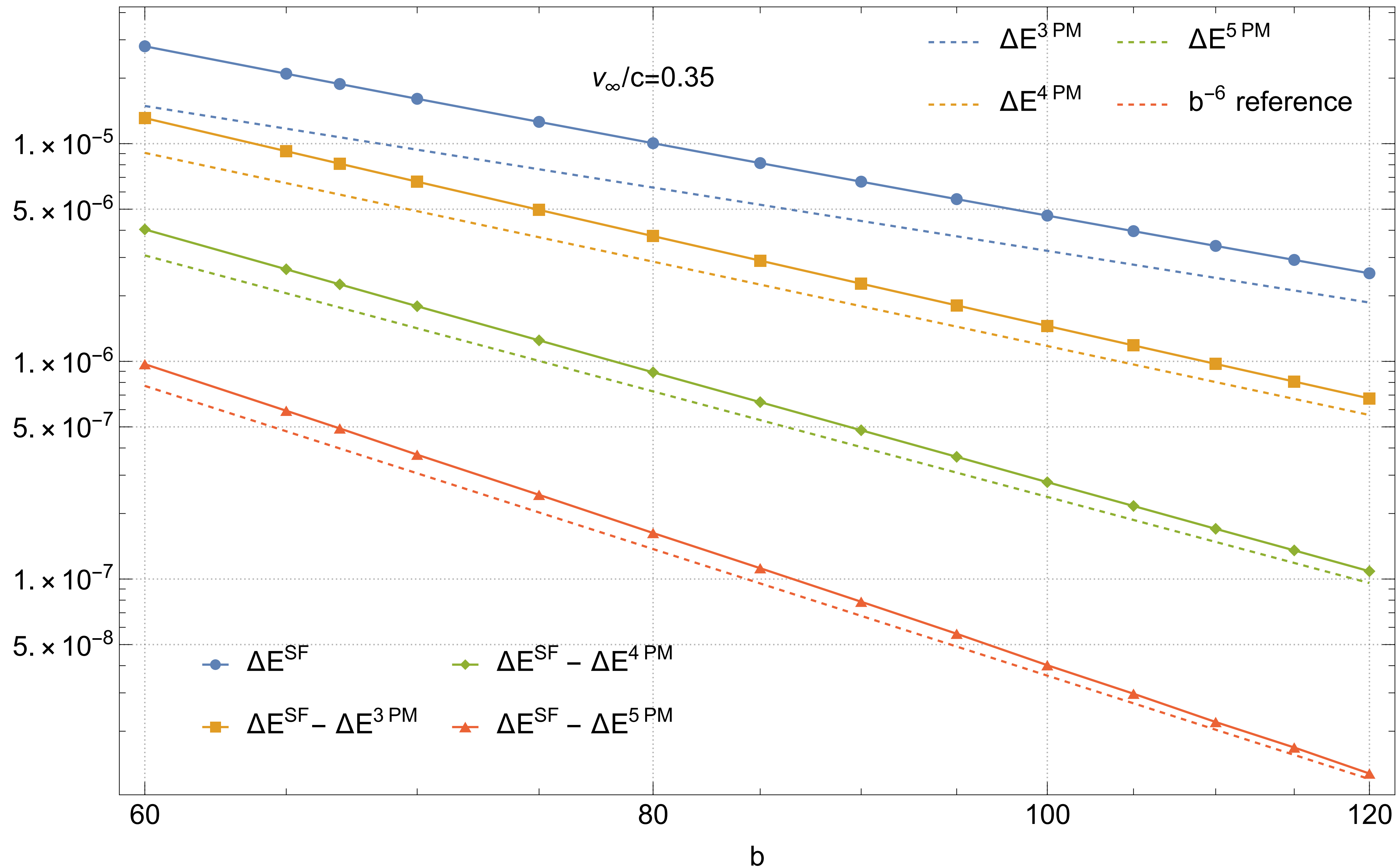
Outlook

See posters from Mathias Driesse, Jitze Hoogeveen and Kathrin Stoldt for more!

- Improve resummation models
- Import our results to bound waveform models
Buonanno, Mogull, Patil, Pompili: [2405.19181]
- Continue to ever higher orders?

- Wish for more numerical relativity simulations
- Wish for experimental black hole scattering data





Black Hole Coalescence

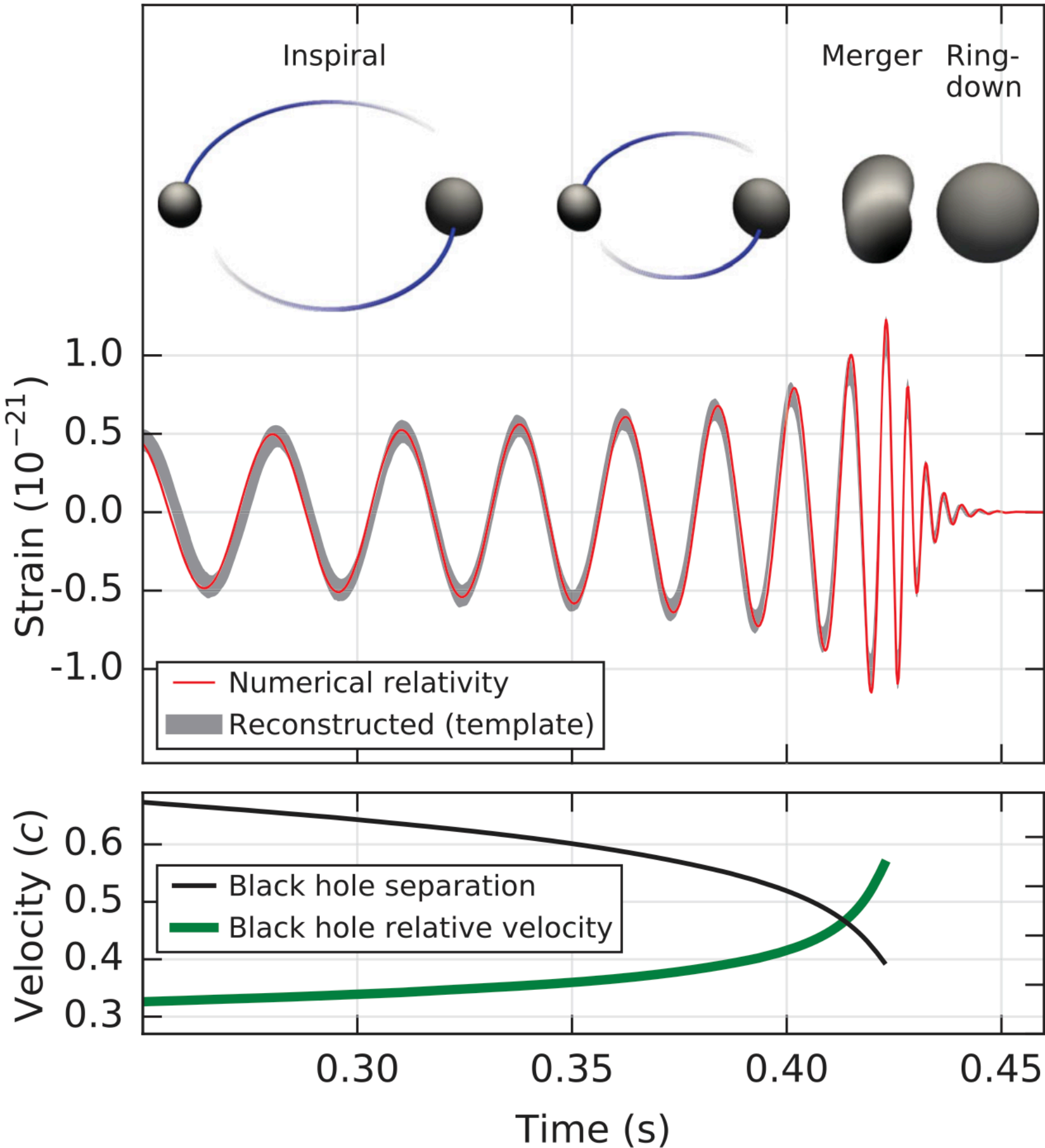
Weak field approximations
and resummations

$$\dot{\mathbf{r}} = \frac{\partial H}{\partial \mathbf{p}}$$
$$\dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{r}} + \vec{\mathcal{F}}_{\text{r.r.}}$$

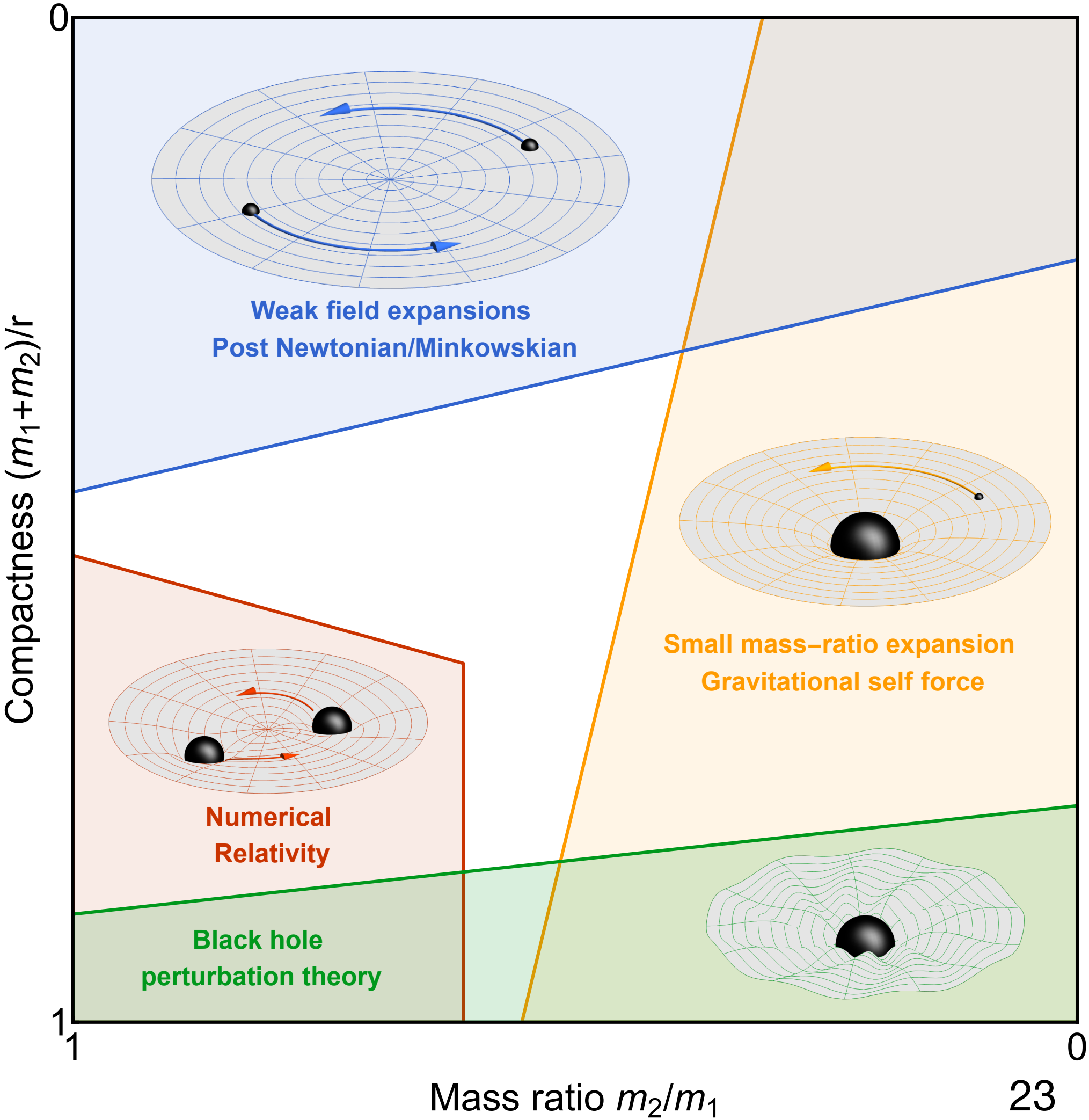
Post-Newtonian,
Hamiltonian
Formulation

Numerical Relativity

Black Hole
Perturbation Theory



Compact Binary Coalescence [PRL 116, 061102 (2016)]



Weak field expansion

$$H = E_1 + E_2 + V(\mathbf{r}, \mathbf{p})$$

$$V(\mathbf{r}, \mathbf{p}) = - \frac{m_1 m_2 G_N}{|\mathbf{r}|}$$

$$+ \frac{|\mathbf{p}|^2 G_N}{|\mathbf{r}|} + \frac{G_N^2}{|\mathbf{r}|^2}$$

$$+ \frac{|\mathbf{p}|^4 G_N}{|\mathbf{r}|} + \frac{|\mathbf{p}|^2 G_N^2}{|\mathbf{r}|^2} + \frac{G_N^3}{|\mathbf{r}|^3}$$

$$+ \frac{|\mathbf{p}|^6 G_N}{|\mathbf{r}|} + \frac{|\mathbf{p}|^4 G_N^2}{|\mathbf{r}|^2} + \frac{|\mathbf{p}|^2 G_N^3}{|\mathbf{r}|^3} + \frac{G_N^4}{|\mathbf{r}|^4}$$

$$+ \frac{|\mathbf{p}|^8 G_N}{|\mathbf{r}|} + \frac{|\mathbf{p}|^6 G_N^2}{|\mathbf{r}|^2} + \frac{|\mathbf{p}|^4 G_N^3}{|\mathbf{r}|^3} + \frac{|\mathbf{p}|^2 G_N^4}{|\mathbf{r}|^4} + \frac{G_N^5}{|\mathbf{r}|^5}$$

$$+ \underbrace{\dots\dots\dots}_{1\text{PM}} \underbrace{\hspace{1cm}}_{2\text{PM}} \underbrace{\hspace{1cm}}_{3\text{PM}} \underbrace{\hspace{1cm}}_{4\text{PM}} \underbrace{\hspace{1cm}}_{5\text{PM}}$$

$$\dot{\mathbf{r}} = \frac{\partial H}{\partial \mathbf{p}}$$

$$\dot{\mathbf{p}} = - \frac{\partial H}{\partial \mathbf{r}} + \vec{\mathcal{F}}_{\text{r.r.}}$$

Weak field expansion

$$H = E_1 + E_2 + V(\mathbf{r}, \mathbf{p})$$

$$\dot{\mathbf{r}} = \frac{\partial H}{\partial \mathbf{p}}$$

$$\dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{r}} + \vec{\mathcal{F}}_{\text{r.r.}}$$

$$V(\mathbf{r}, \mathbf{p}) = - \frac{m_1 m_2 G_N}{|\mathbf{r}|}$$

$$+ \frac{|\mathbf{p}|^2 G_N}{|\mathbf{r}|} + \frac{G_N^2}{|\mathbf{r}|^2}$$

$$+ \frac{|\mathbf{p}|^4 G_N}{|\mathbf{r}|} + \frac{|\mathbf{p}|^2 G_N^2}{|\mathbf{r}|^2} + \frac{G_N^3}{|\mathbf{r}|^3}$$

$$+ \frac{|\mathbf{p}|^6 G_N}{|\mathbf{r}|} + \frac{|\mathbf{p}|^4 G_N^2}{|\mathbf{r}|^2} + \frac{|\mathbf{p}|^2 G_N^3}{|\mathbf{r}|^3} + \frac{G_N^4}{|\mathbf{r}|^4}$$

$$+ \frac{|\mathbf{p}|^8 G_N}{|\mathbf{r}|} + \frac{|\mathbf{p}|^6 G_N^2}{|\mathbf{r}|^2} + \frac{|\mathbf{p}|^4 G_N^3}{|\mathbf{r}|^3} + \frac{|\mathbf{p}|^2 G_N^4}{|\mathbf{r}|^4} + \frac{G_N^5}{|\mathbf{r}|^5}$$

$$+ \underbrace{\dots\dots\dots}_{1\text{PM}} \quad \underbrace{\dots\dots\dots}_{2\text{PM}} \quad \underbrace{\dots\dots\dots}_{3\text{PM}} \quad \underbrace{\dots\dots\dots}_{4\text{PM}} \quad \underbrace{\dots\dots\dots}_{5\text{PM}}$$

Newton's potential

0PN

Einstein-Infeld-Hoffmann potential

1PN

2PN

3PN

4PN

Radiation-reaction force starts at 2.5PN

Non-local interactions appear!

