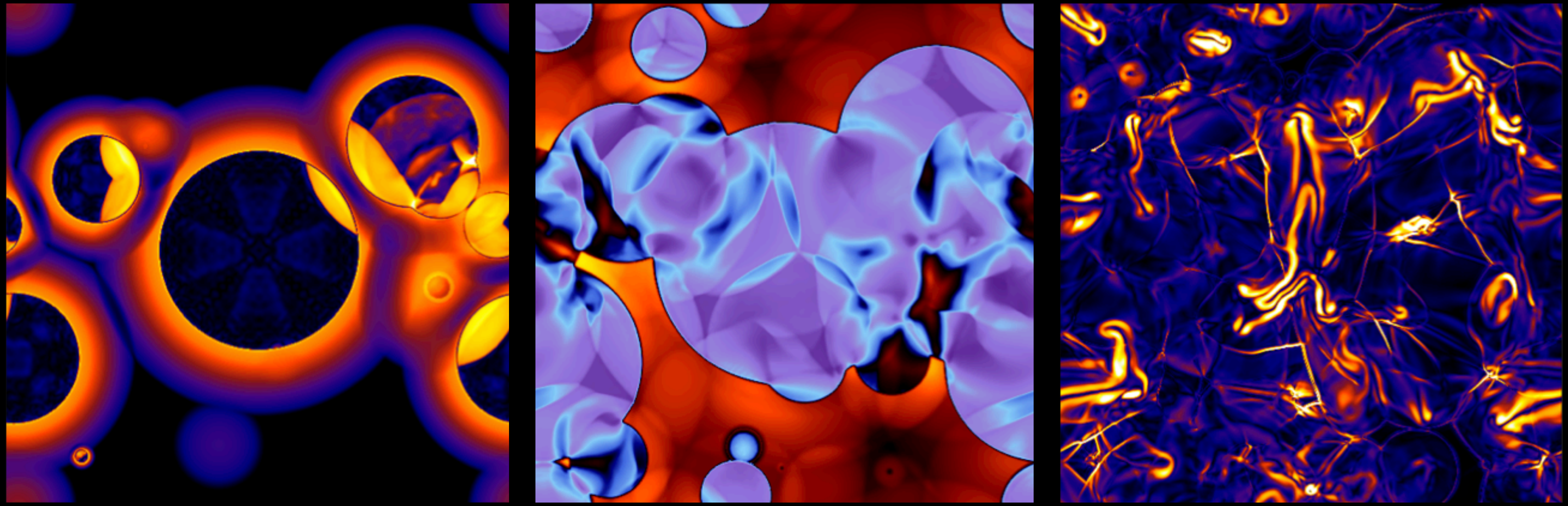
The Higgs boson self-coupling and what we can learn from it

Cédrine Hügli



Astroparticle and Particle Physics Workshop

Zeuthen, October 6th, 2025

We are going to touch the following questions

What does the Higgs self-coupling have to do with gravitational waves?

Were there bubbles in the early universe?

Can we disappear instantly?

Are the current limits we have on the Higgs self-coupling useful?

Why haven't we discovered the Higgs self-coupling yet?

Content

1. Short theory introduction
2. Beginning of the universe
   - Electroweak (EW) phase transition (PT)
   - Primordial Gravitational Waves (PGWs)
3. Current knowledge
   - Introduction and strategy of the $HH \rightarrow bb\gamma\gamma$ analysis
   - Current limits
   - Future limitations and showstoppers
4. Fate of the universe
   - Stability of the universe

Higgs potential and Higgs pairs

The Standard Model (SM) Higgs potential is given by

$$V(\phi) = -\mu\phi^2 + \lambda\phi^4$$

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This form is valid only near the minimum (small h fluctuations)

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$$\mathscr{L}_{Higgs}(h) = \frac{1}{2}m_h^2h^2 + \lambda v h^3 + \frac{1}{4}\lambda h^4$$

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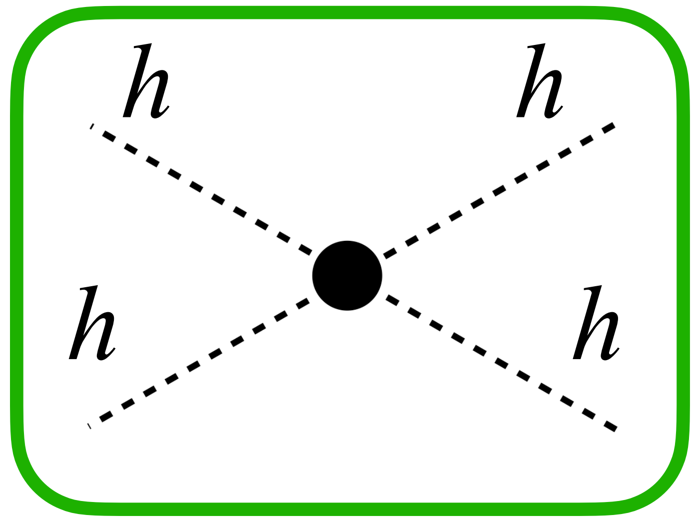

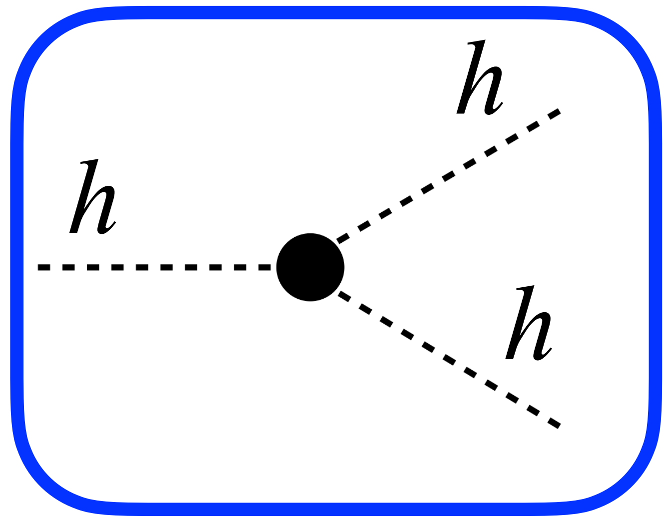

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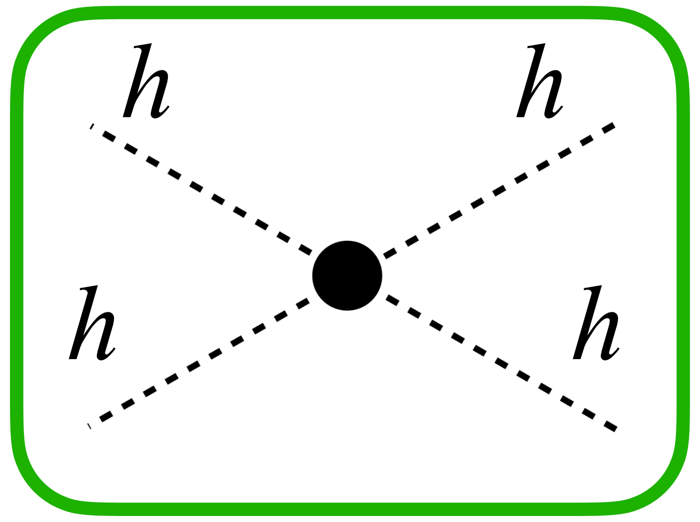

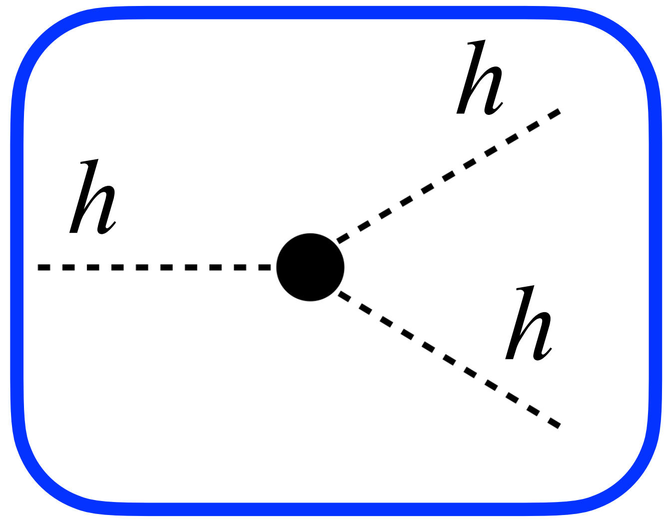

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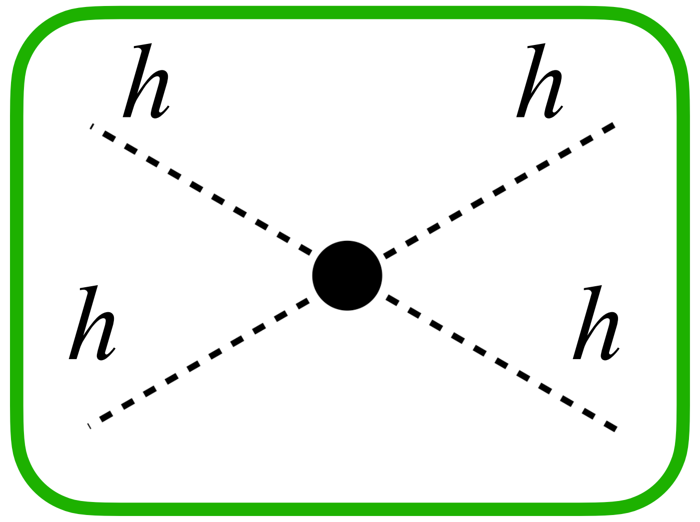
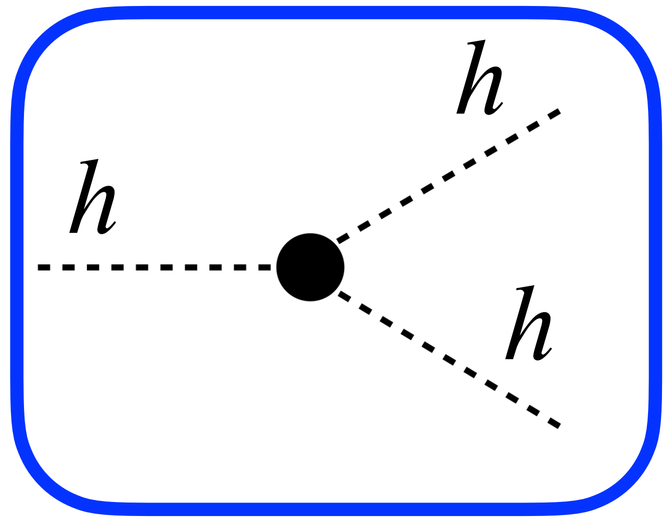


SM: $\lambda = \dfrac{m_h^2}{2v^2} \approx 0.13$

$$\boxed{\kappa_\lambda = \frac{\lambda_{\text{obs.}}}{\lambda_{\text{SM}}}}$$

SM: $\kappa_\lambda = 1$



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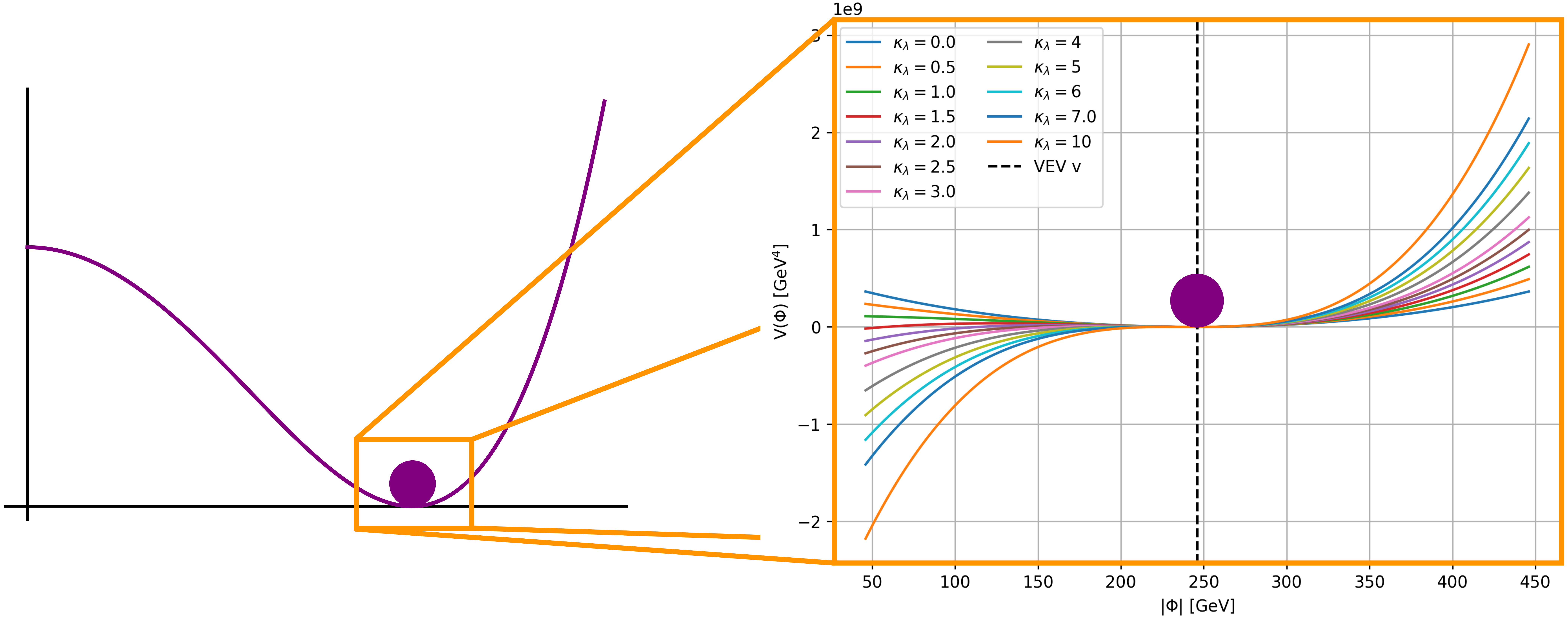
Plot for illustration purpose

Higgs potential shape for different κ_λ values

$$\kappa_\lambda = \frac{\lambda_{\text{obs.}}}{\lambda_{\text{SM}}}$$



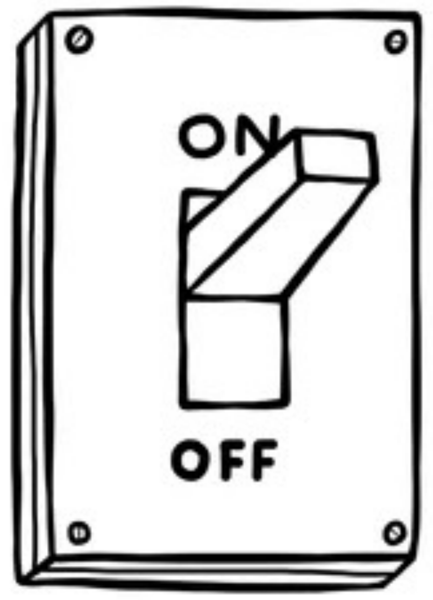
Beginning of the universe

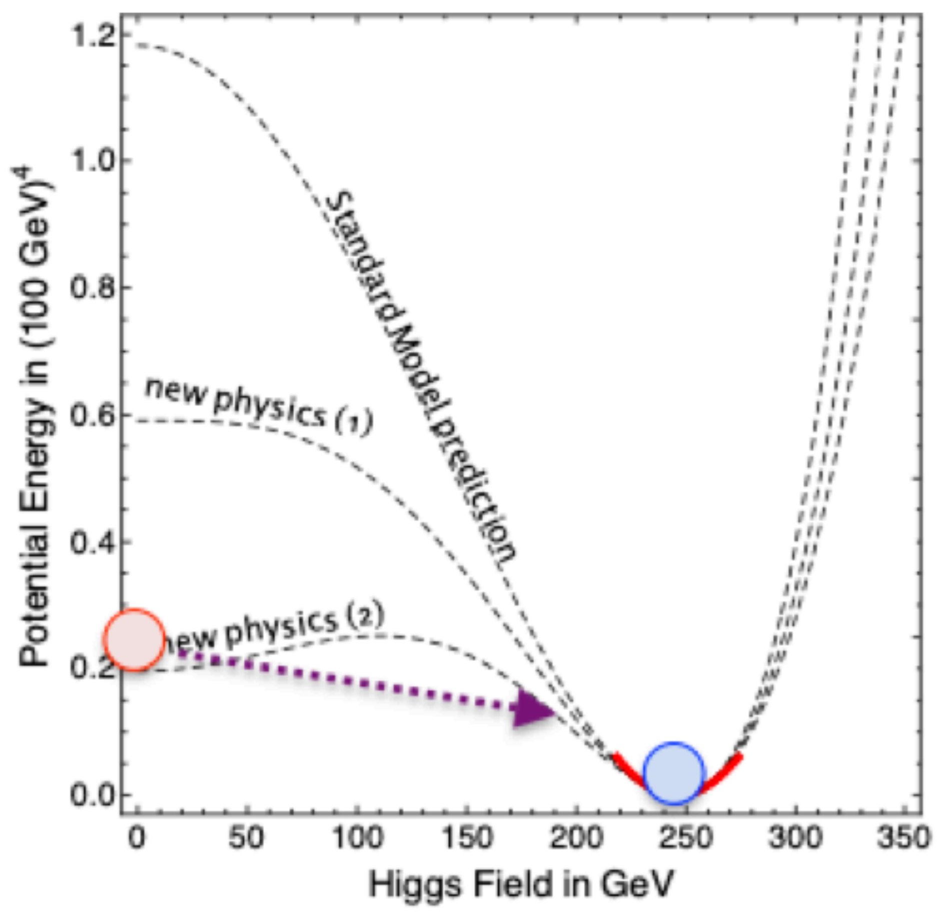
The early universe

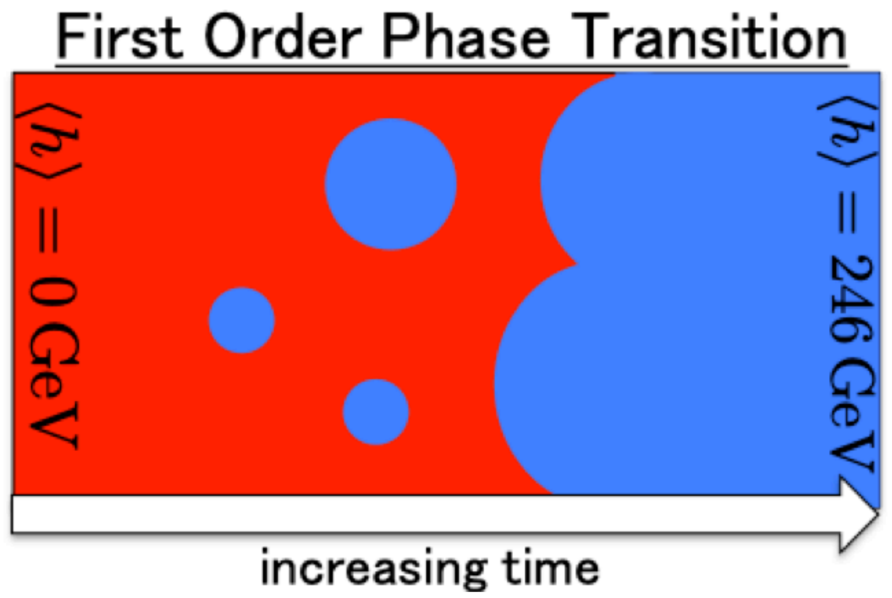
How did the Electroweak symmetry break?

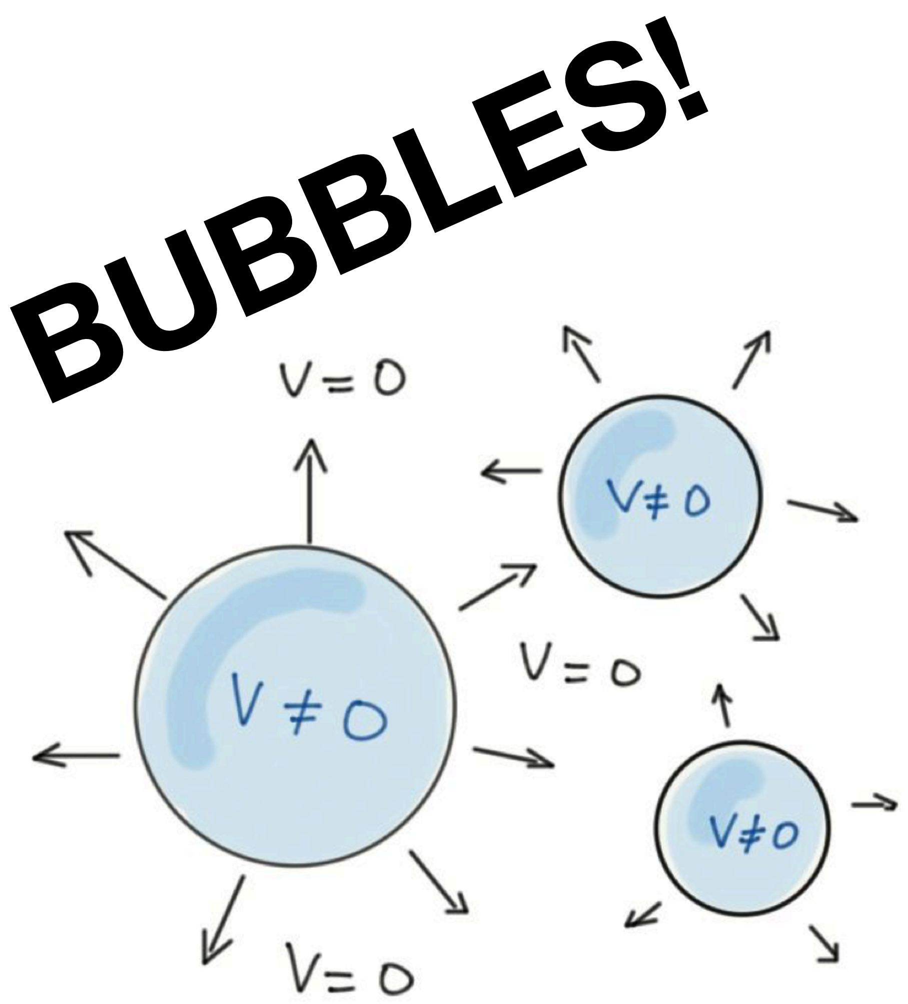
electroweak phase transition = when the Higgs field is "turned on"

A) A first order phase transition

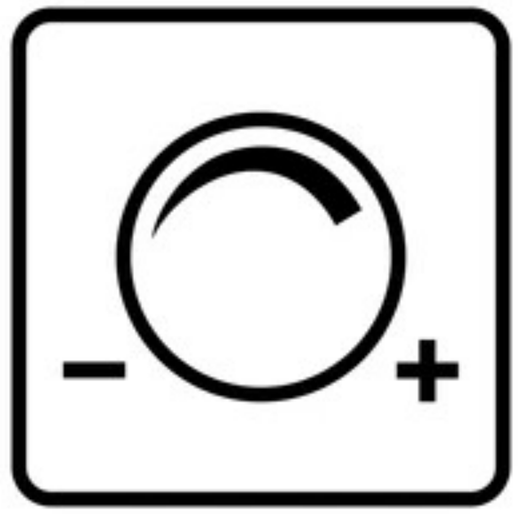


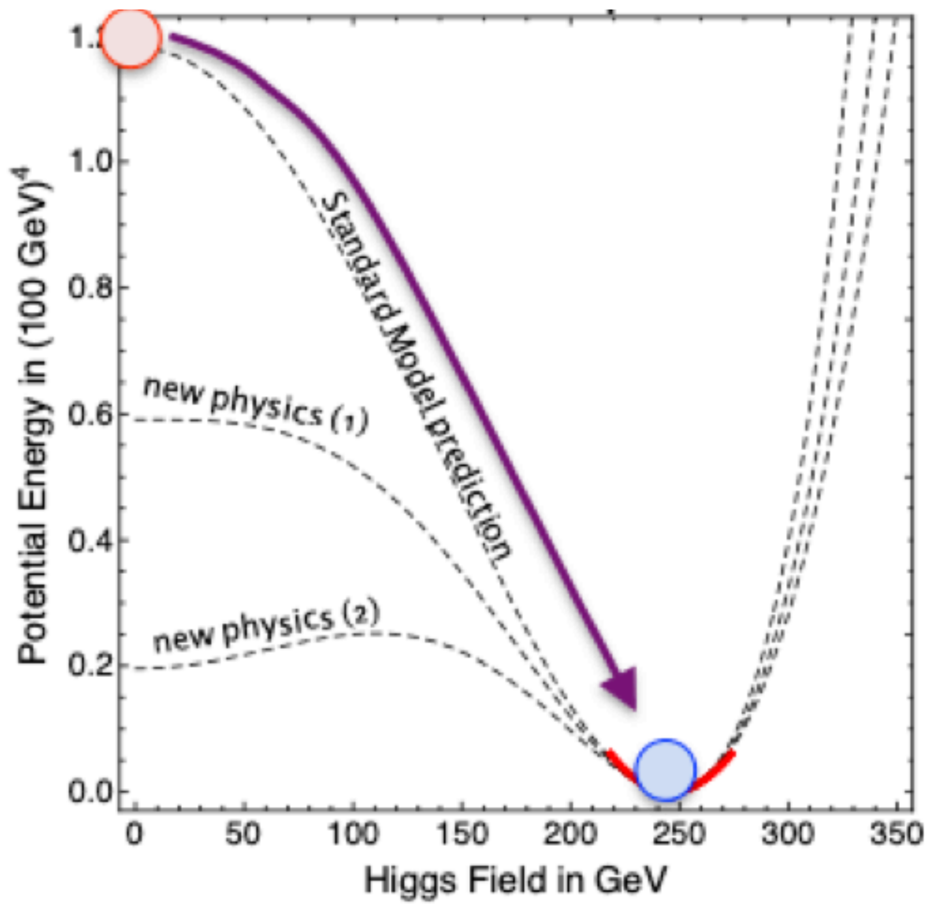


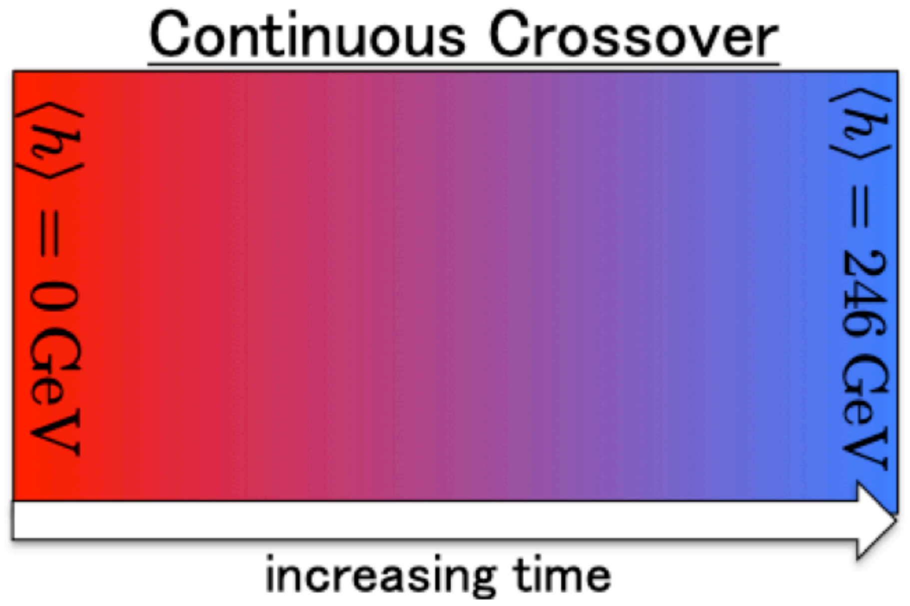




B) A higher order phase transition







Cédrine Hügli (DESY), Astroparticle and Particle Physics Workshop

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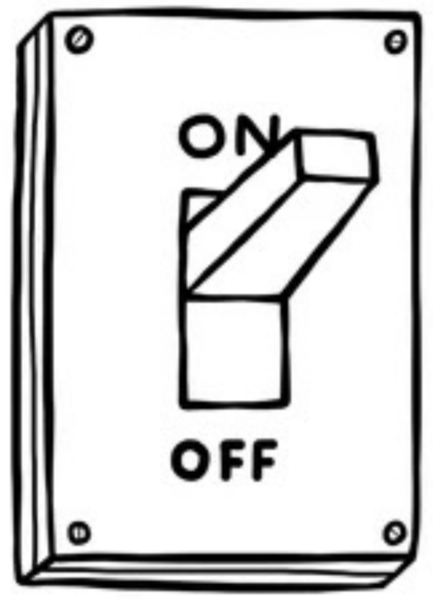
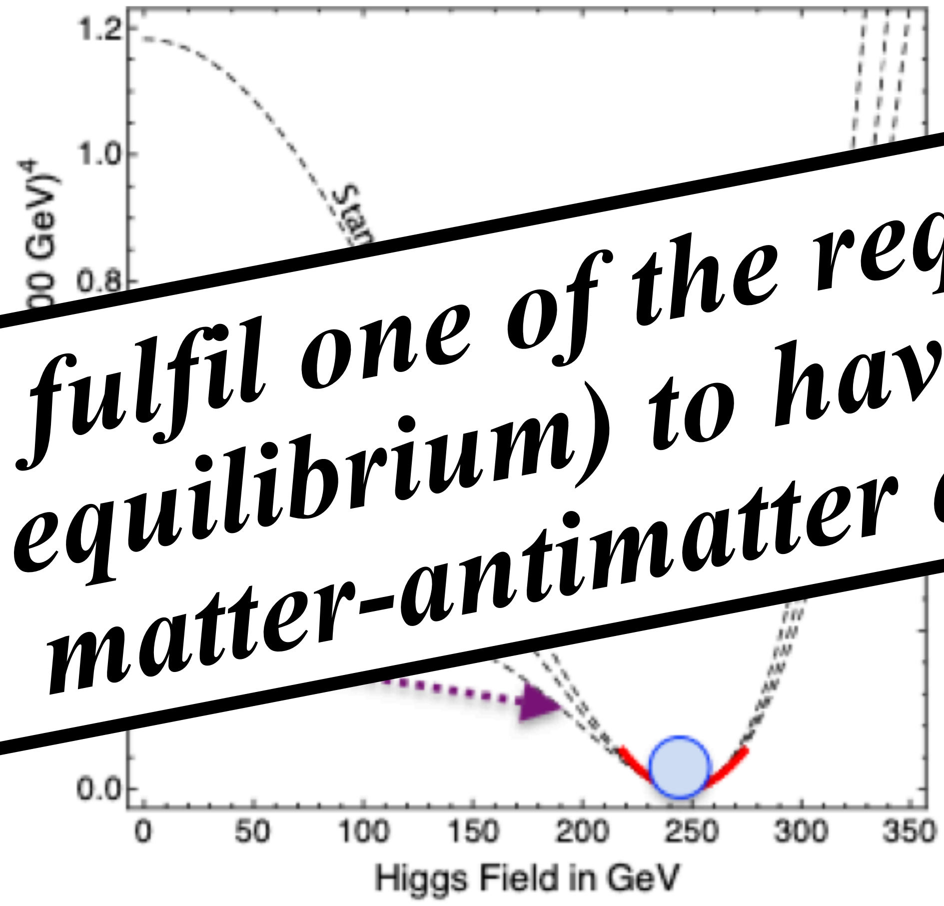
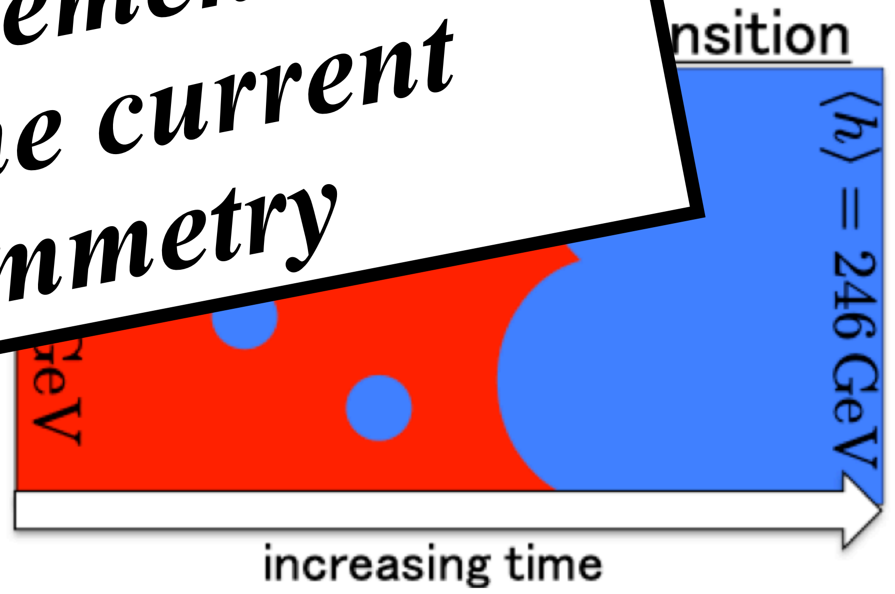
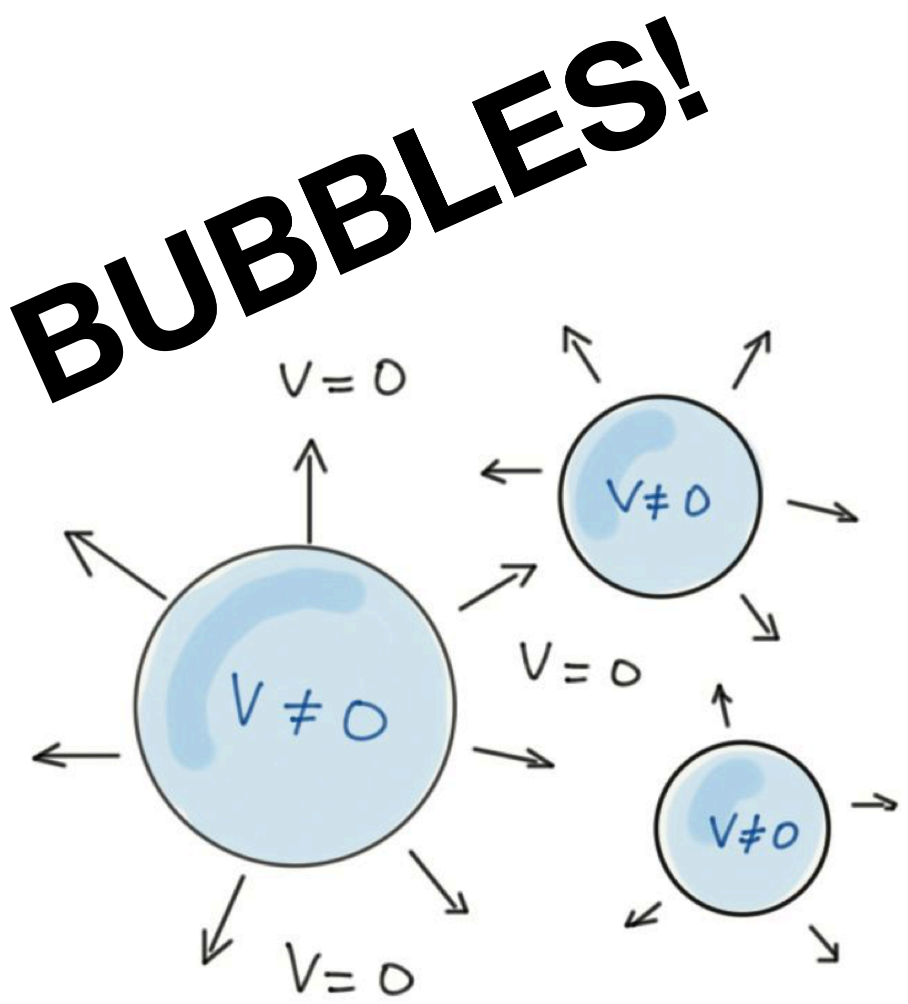


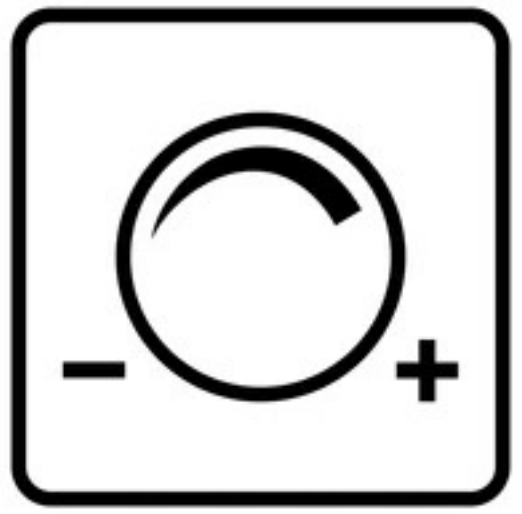


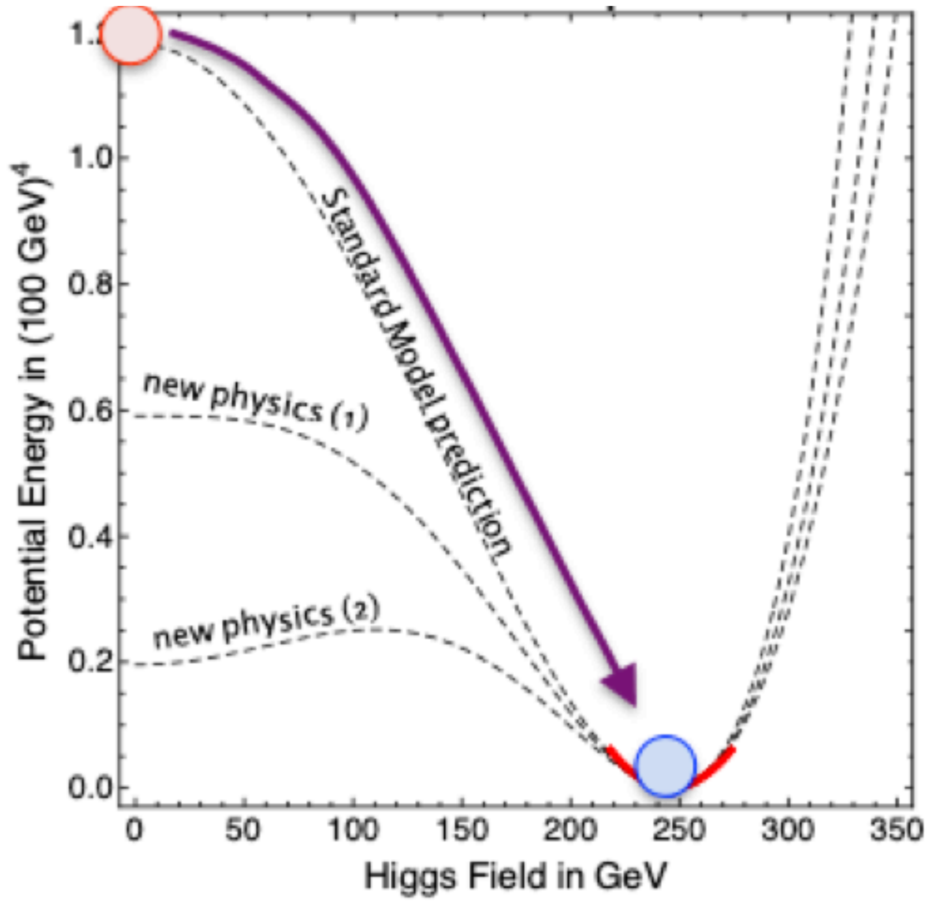


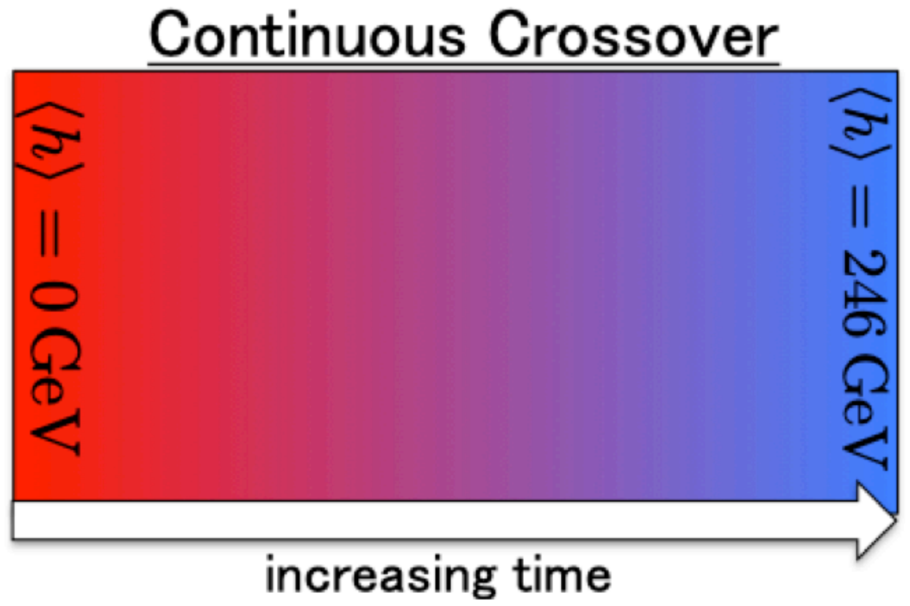


Would fulfil one of the requirements (out-of equilibrium) to have the current matter-antimatter asymmetry

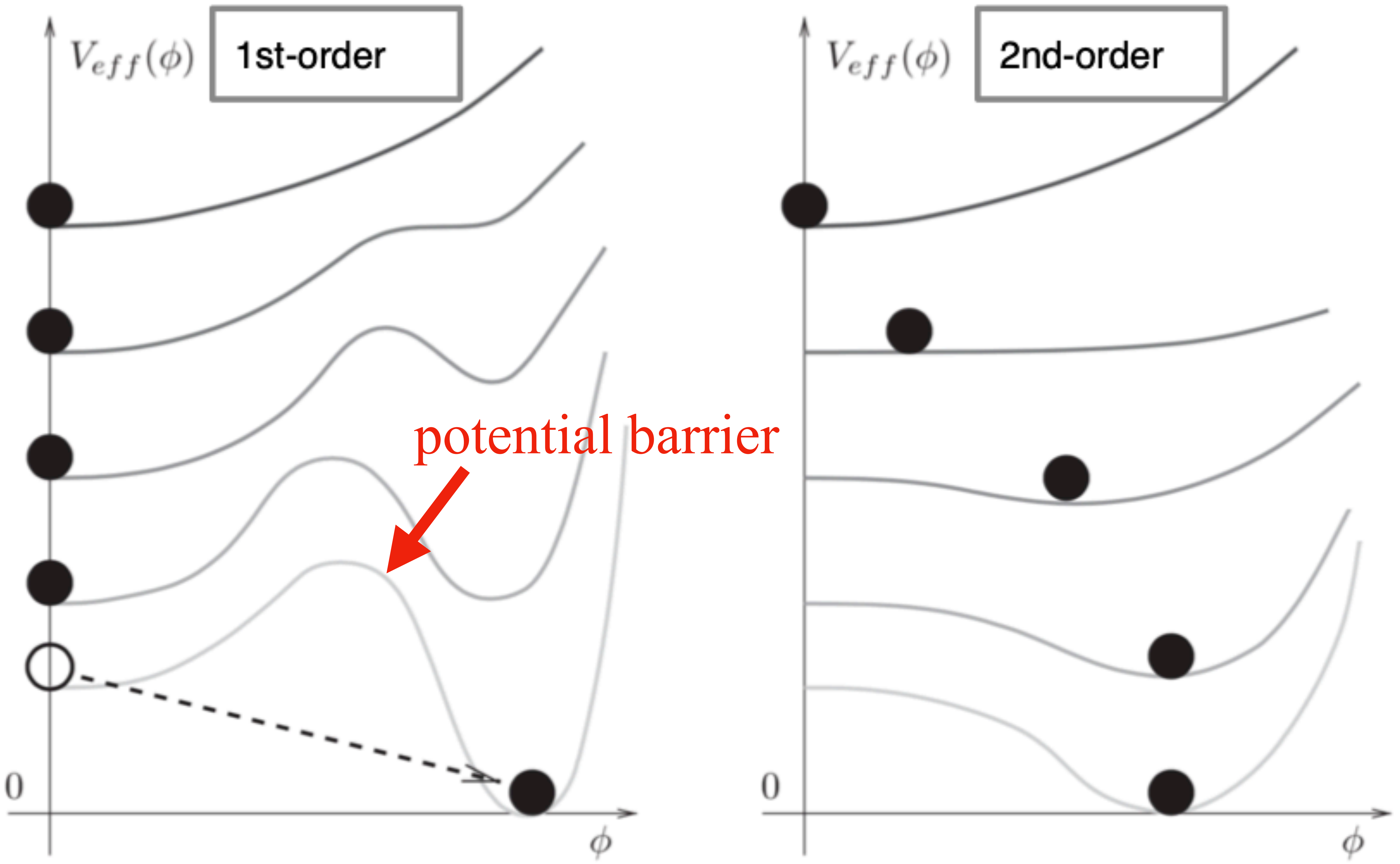
B) A higher order phase transition







Phase transition and Higgs self-coupling?



The potential barrier depends on the Higgs self-coupling

A strong deviation from the SM value would be a hint of a strong first-order EWPT

The SM does not predict a first order phase transition

First order is only allowed in BSM models

G. Weiglein, Higgs Pairs 2025

SM = Standard Model, BSM = Beyond Standard Model

What about gravitational waves?

Primordial Gravitational Waves

<u>**Two types of gravitation waves (GWs):**</u>

1. **Astrophysical GWs:** from black holes/neutron star merging, what we can detect now
2. **Primordial (cosmological) GWs (PGWs):** from the early universe (produced during inflation, cosmic strings, or first-order phase transitions)

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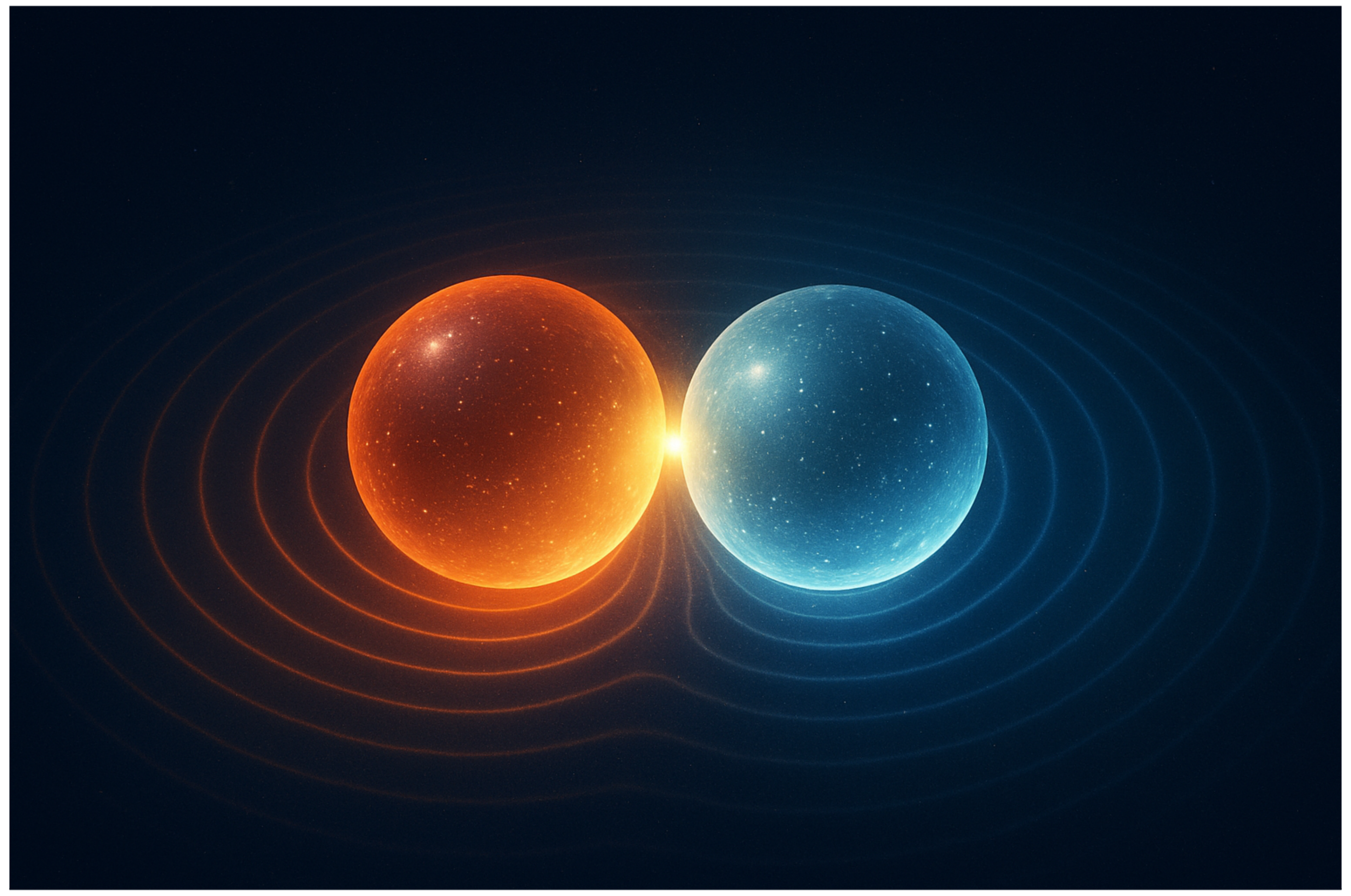
A possible origin of PGWs:

- If the phase transition is first order → bubbles → bubbles collisions → shake spacetime → producing gravitational waves



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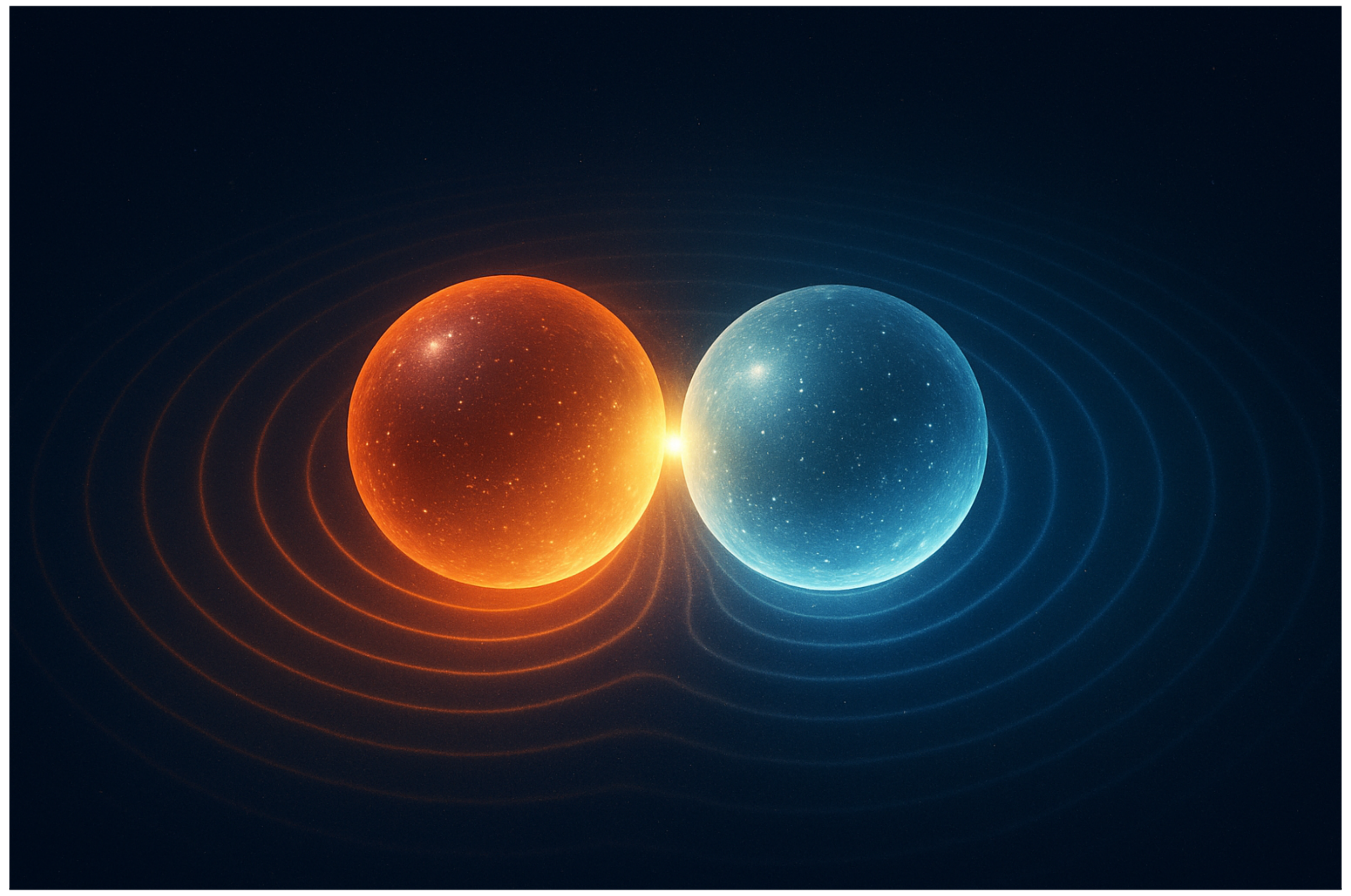


Properties of PGWs:

- should present as a stochastic gravitational wave background, a random "hum" present everywhere

Gravitational Waves and the Higgs boson

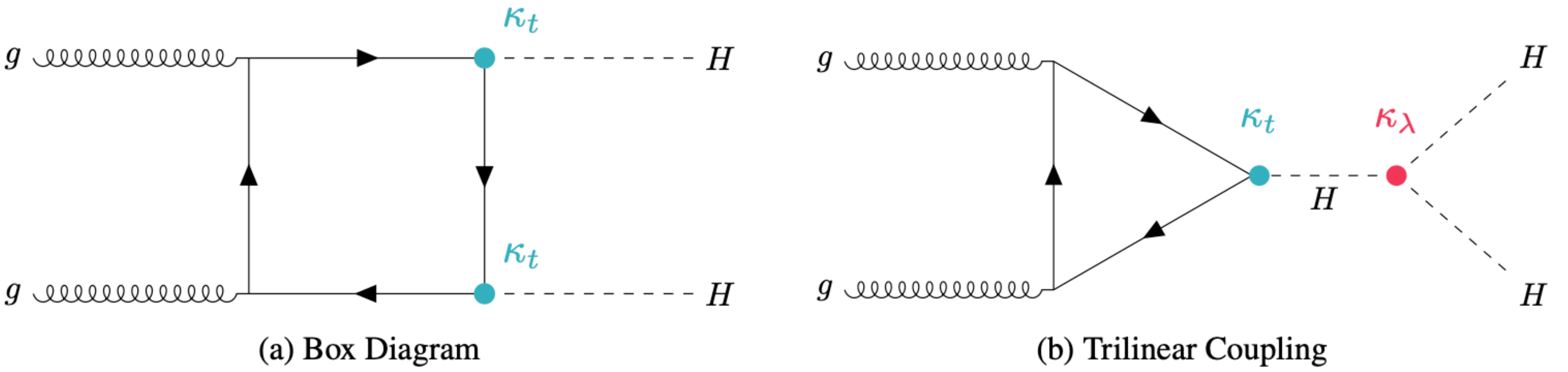
Gravitational Waves		Higgs boson
	EW phase transition	
PGWs	⟷	Shape of the potential

The connection origins from the EW phase transition with is connected to both the Higgs bosons self-coupling and to the primordial gravitational wave production

Current knowledge

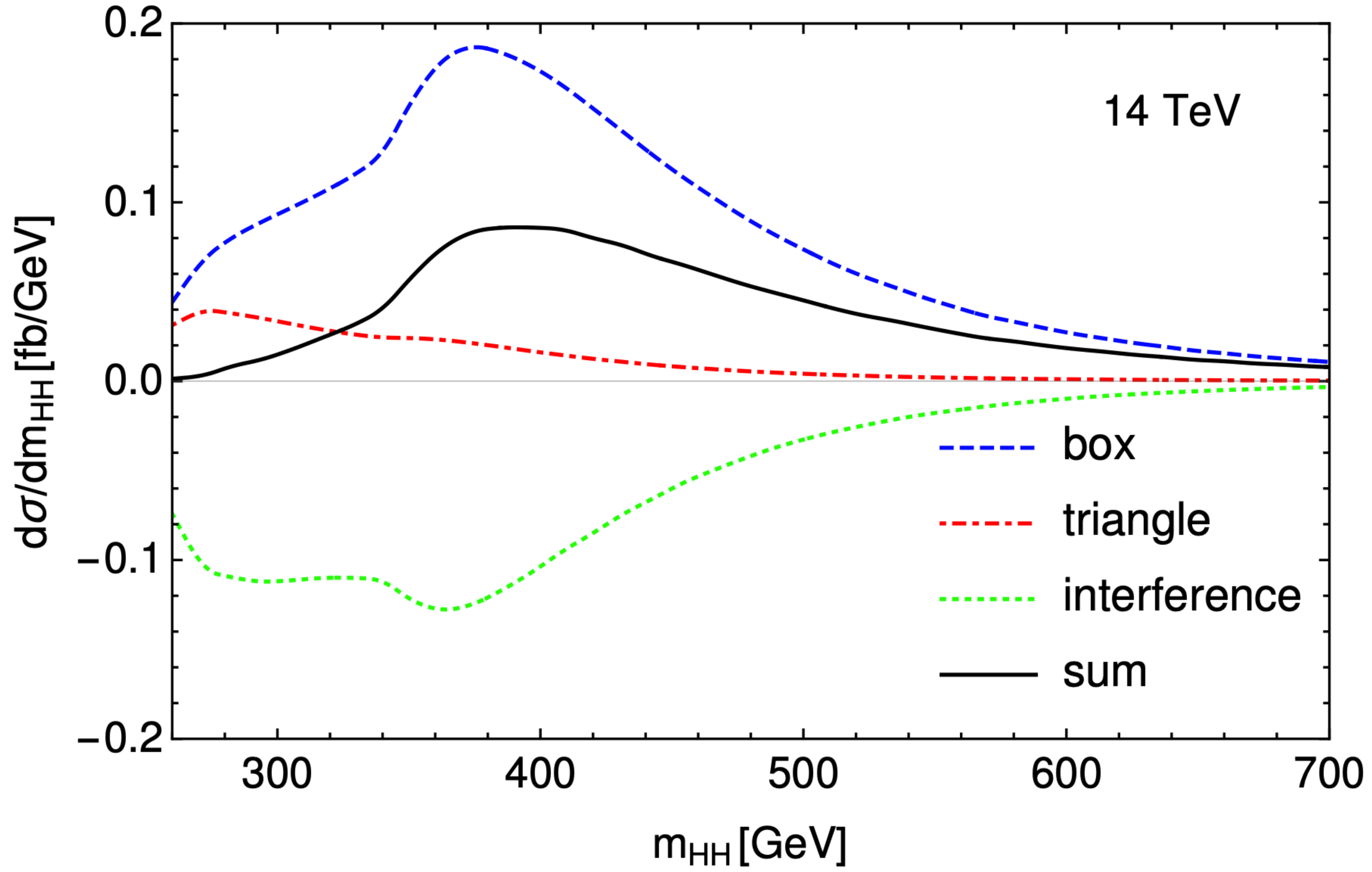
arXiv:1910.00012

Di-Higgs production at the LHC



Leading order Feynman diagrams for the dominant gluon–gluon fusion production

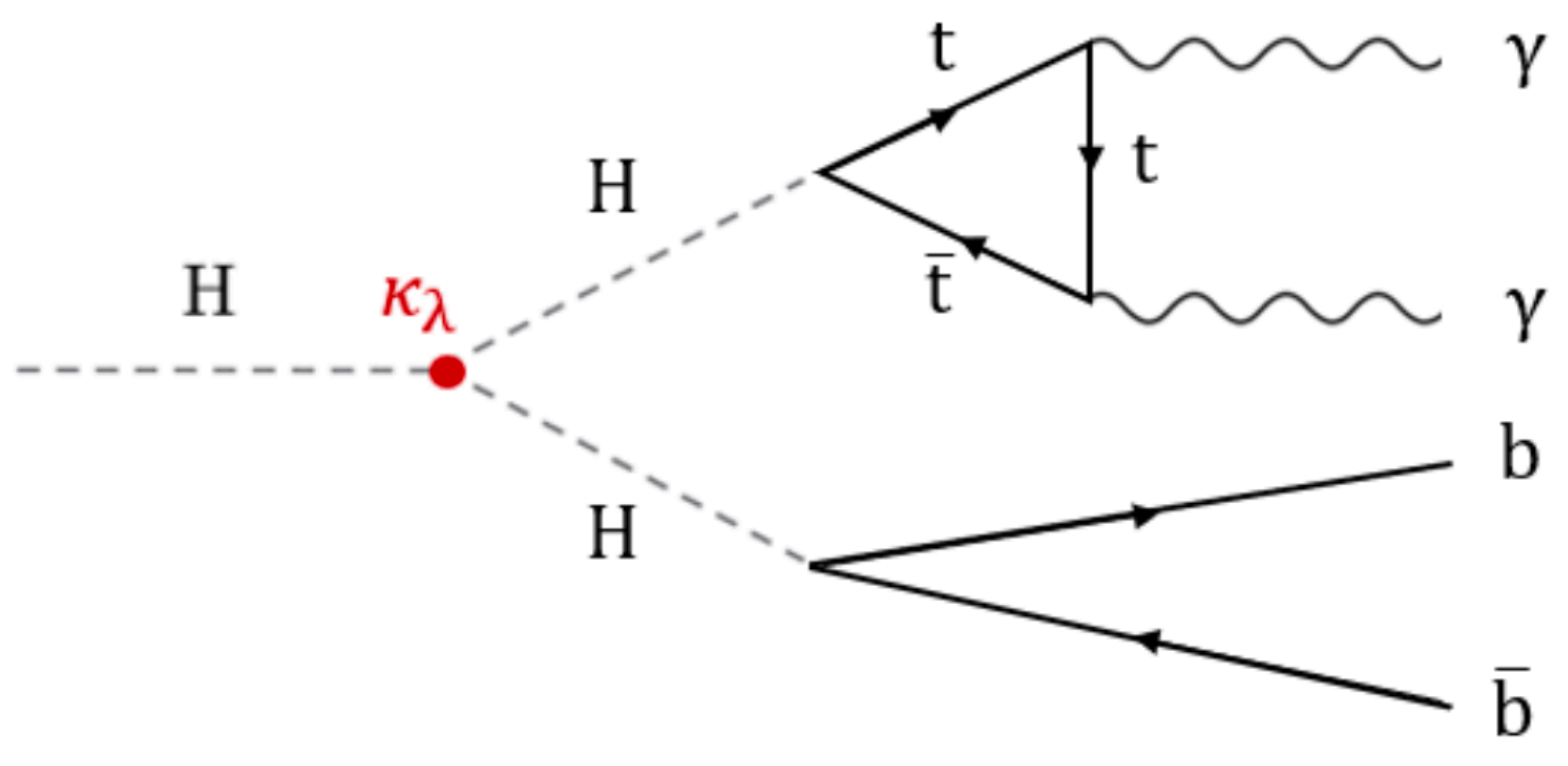
These two diagrams interfere destructively!



Consequences:

- Low amount of events: very rare process
- Amount of events depends on κ_λ

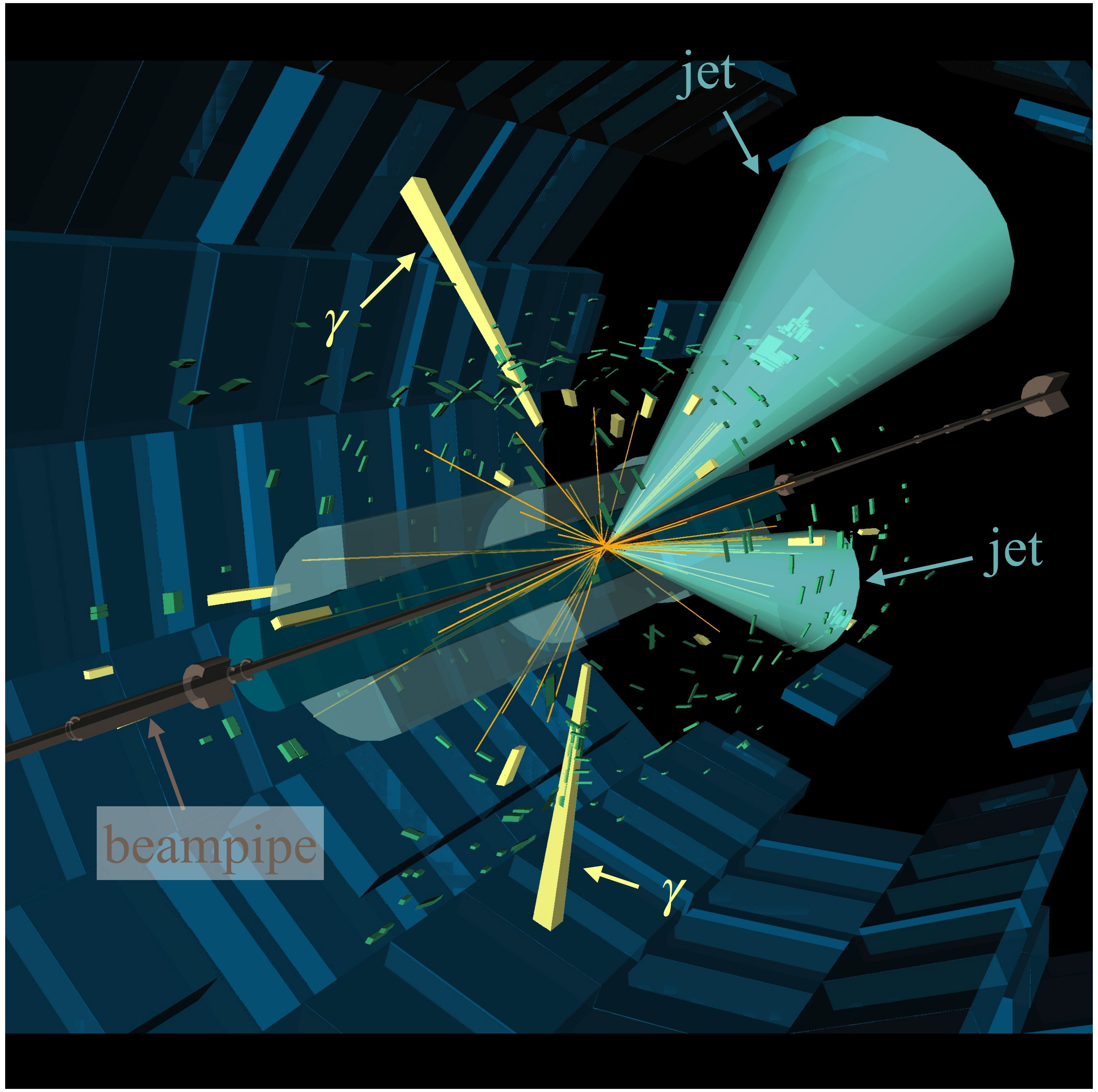
The $HH \rightarrow bb\gamma\gamma$ channel



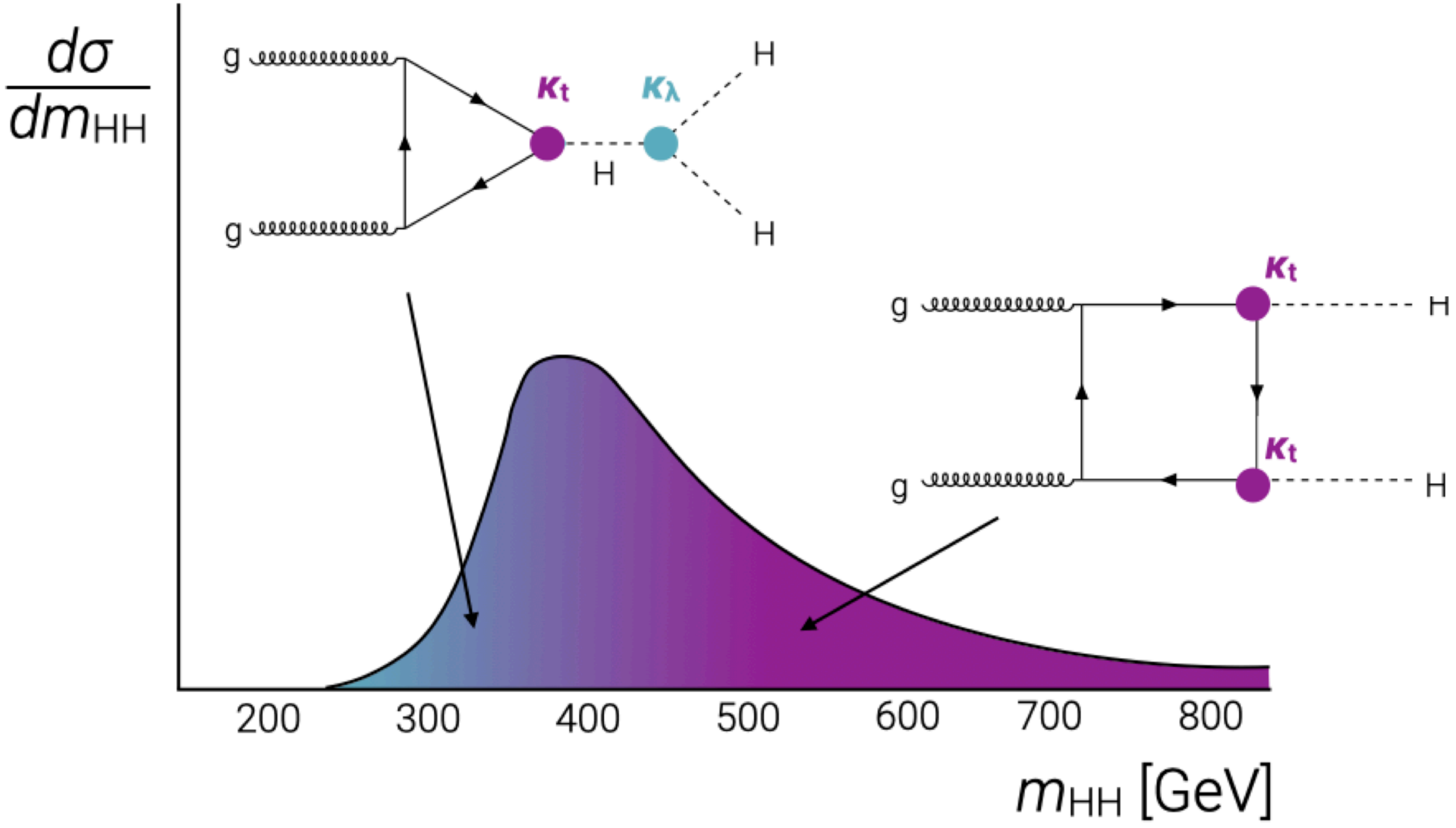
Final state particles observed in the ATLAS detector: photons and b-jets

⇒ photon reconstruction and calibration

⇒ b-jet reconstruction and calibration



Why are we doing this analysis?



We can access low di-Higgs masses thanks to the photons (that have low thresholds in the trigger)

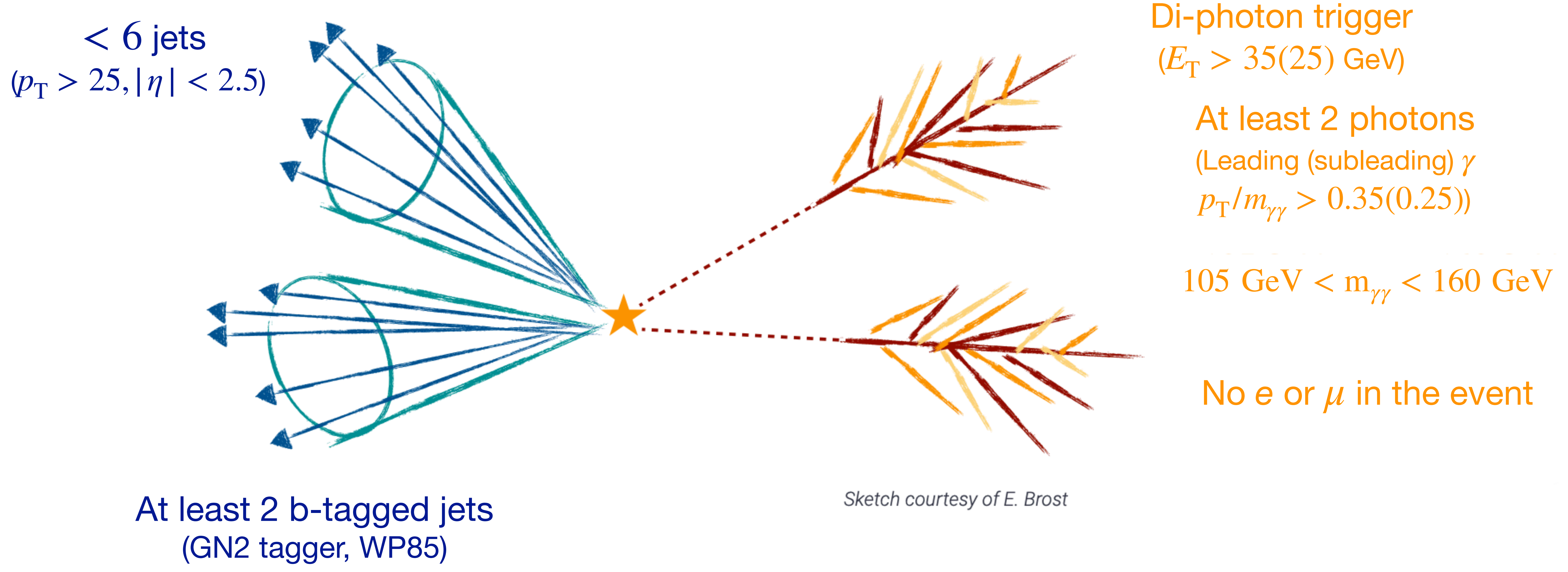
This region is more dependent on κ_λ

$\Rightarrow$ better sensitivity to κ_λ despite the lower statistics

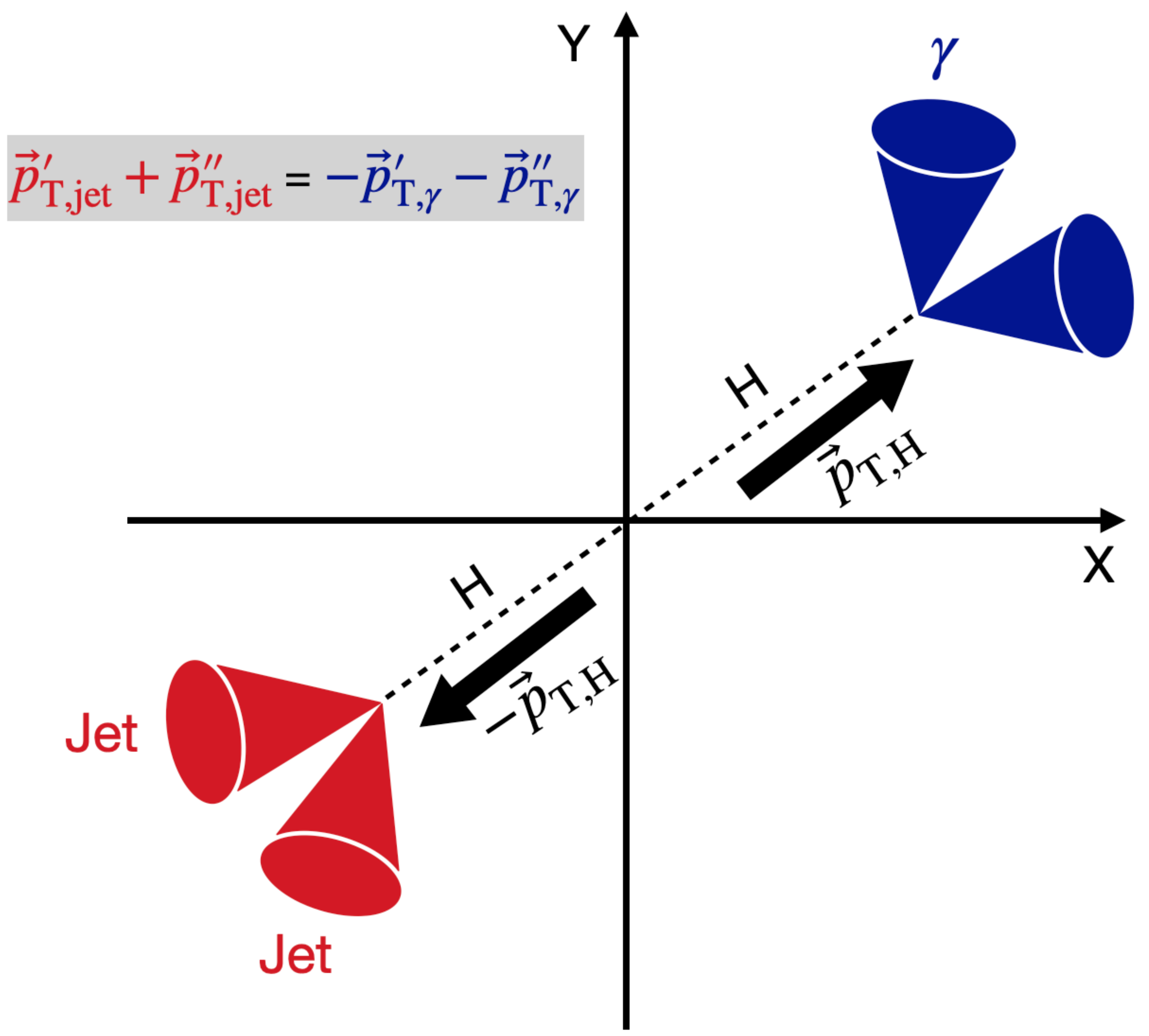
Simplified Analysis Strategy

Prepare the objects (photons and b-jets)

Object and event selection



Kinematic fit to improve m_{bb} resolution



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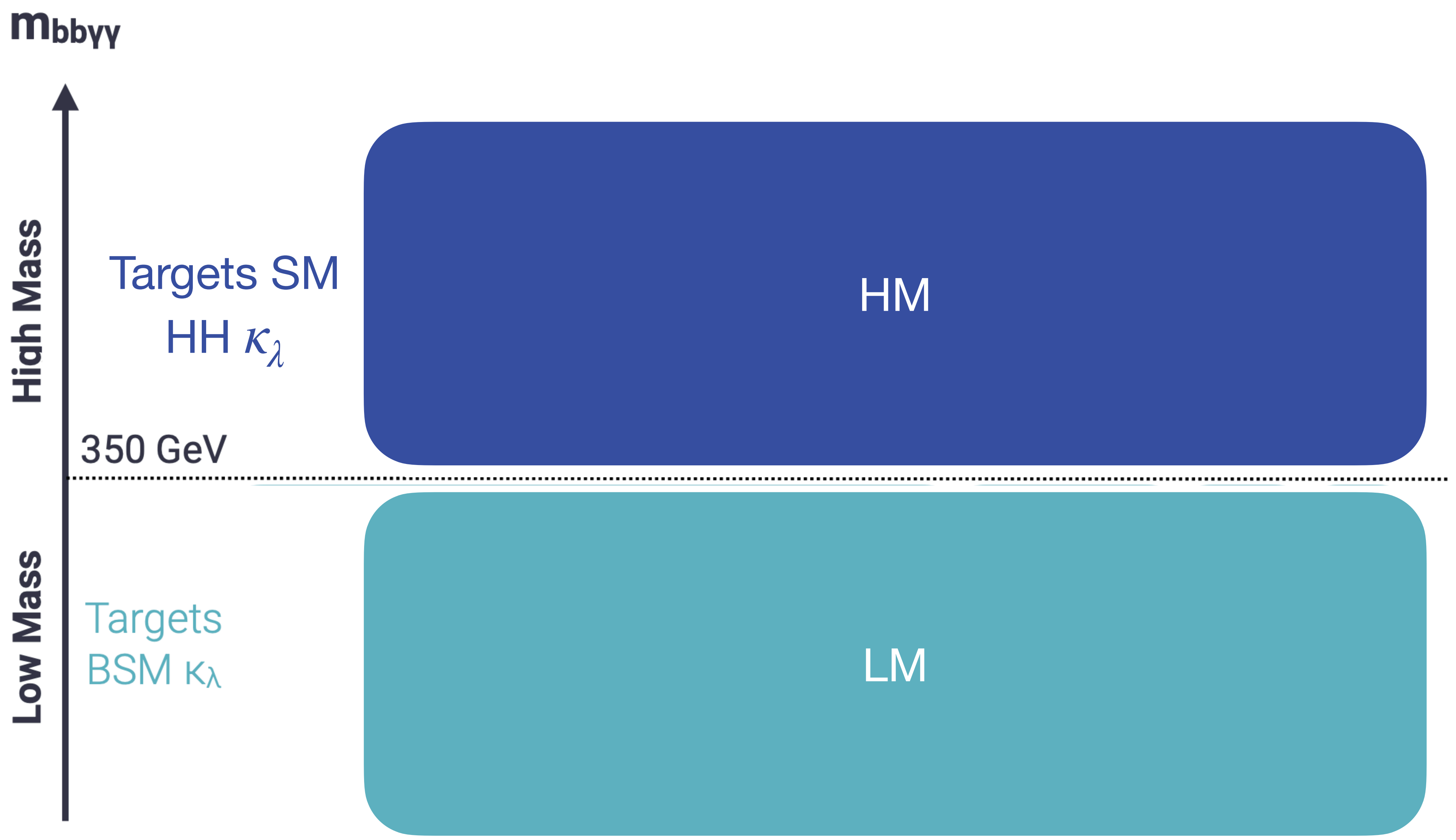
Prepare the objects (photons and b-jets)

⇓

Split into high mass region and low mass region



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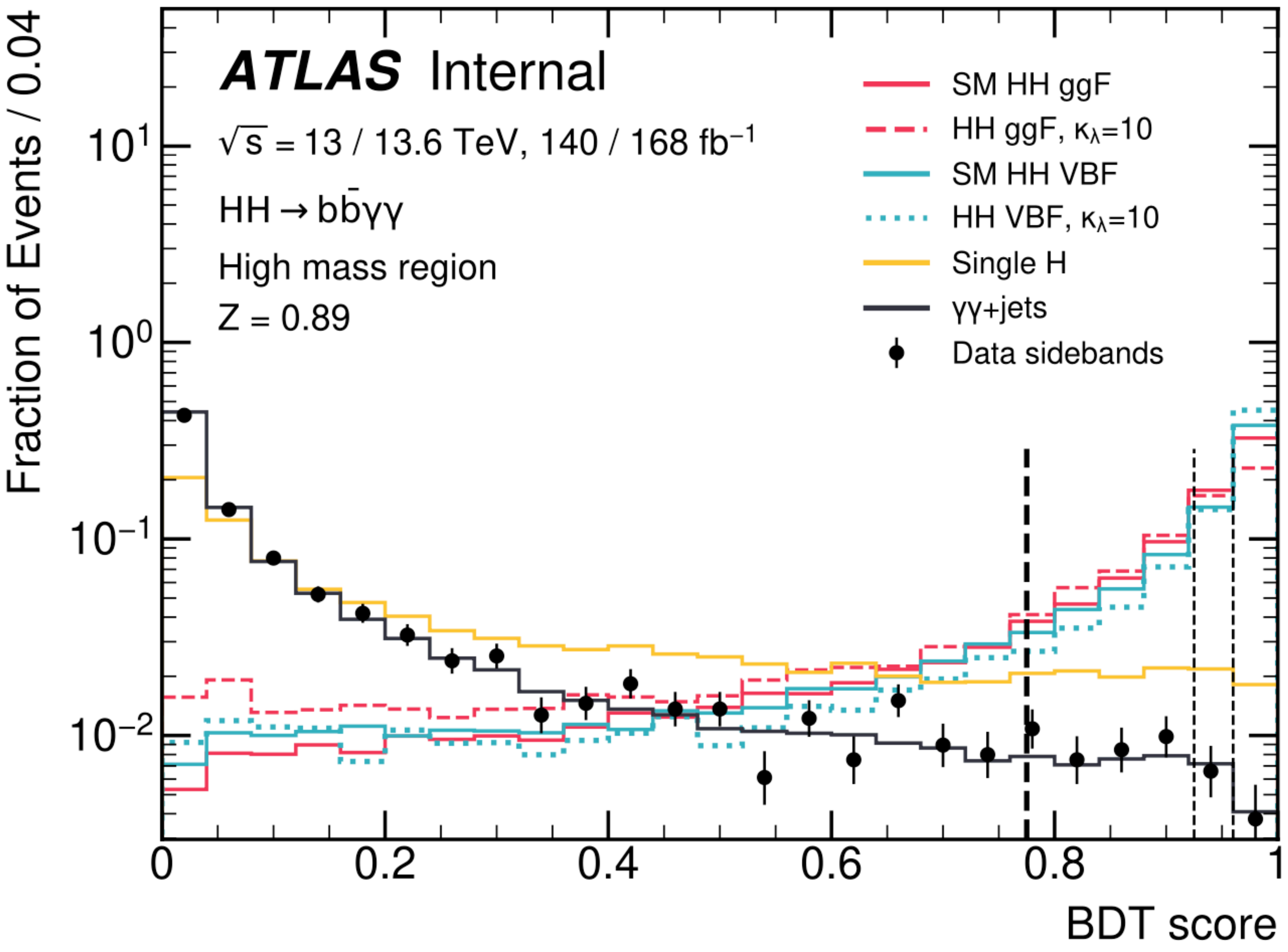
Train Boosted Decision Trees (BDTs) to distinguish signal from backgrounds

High Mass region



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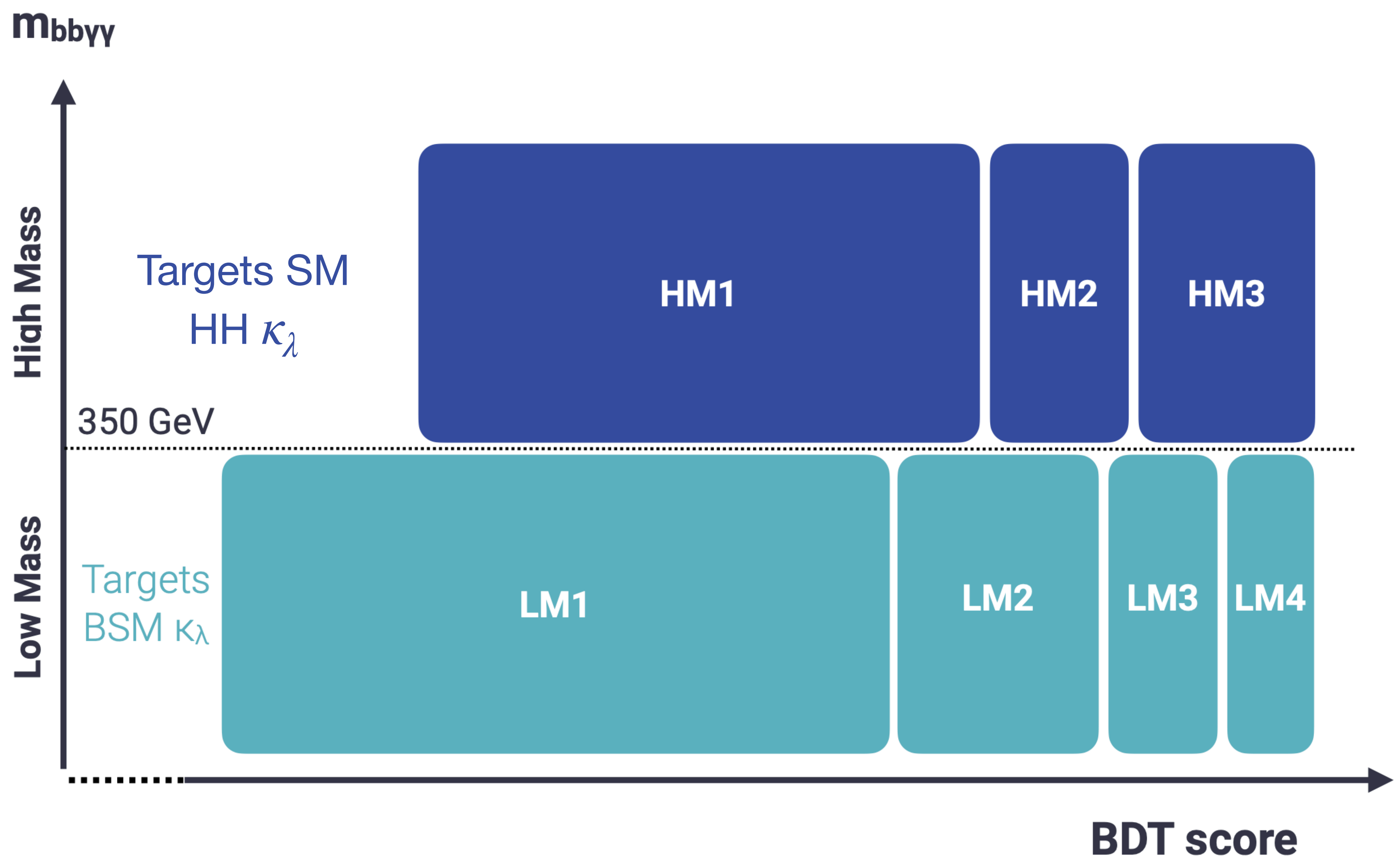

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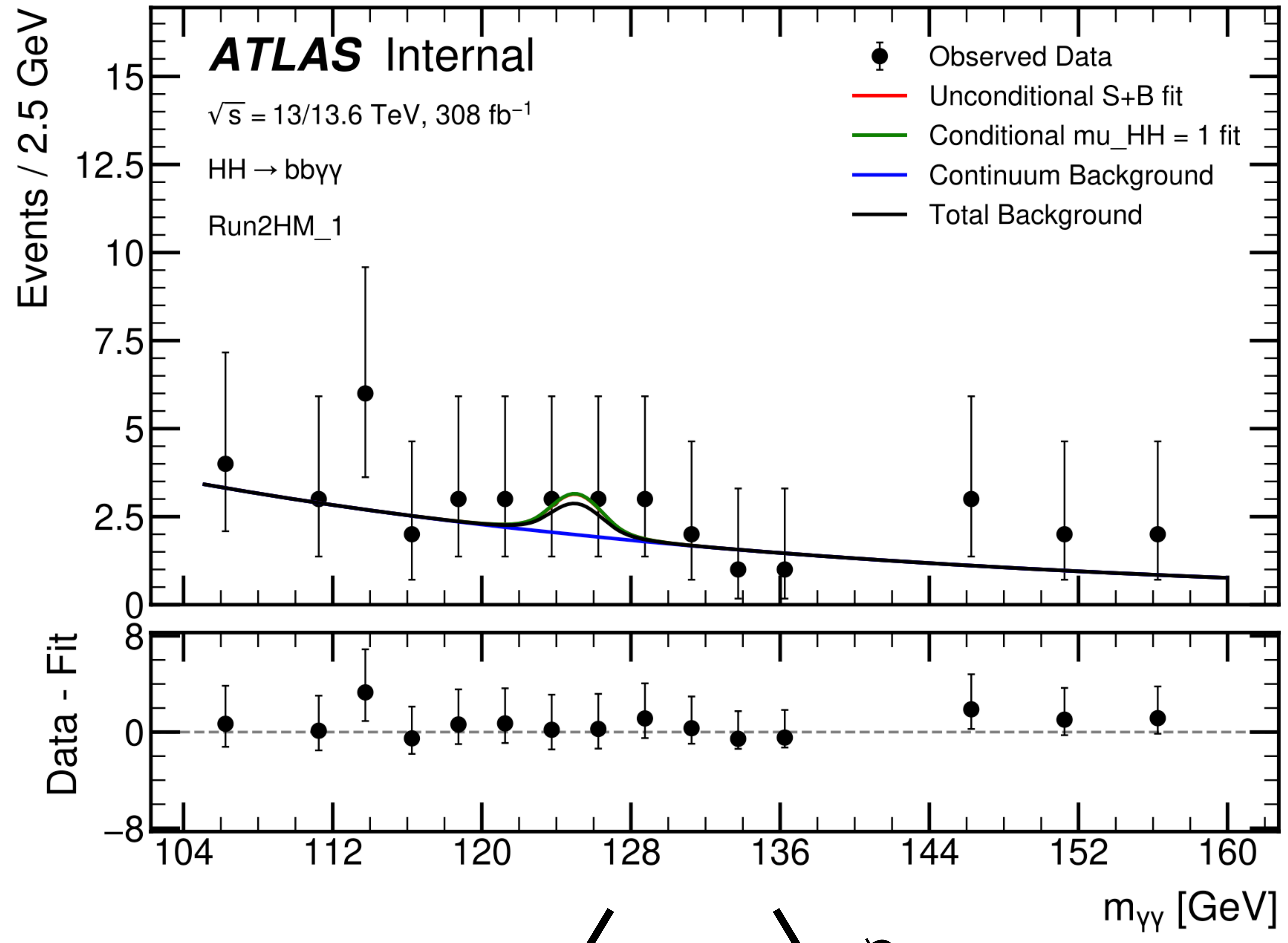
Make simultaneous fits to $m_{\gamma\gamma}$

Run 2 HM1



fit A

fit B

$$\mu = \frac{\sigma_{\text{obs.}}}{\sigma_{\text{SM}}}$$

$$\kappa_\lambda = \frac{\lambda_{\text{obs.}}}{\lambda_{\text{SM}}}$$

Simplified Analysis Strategy

Prepare the objects (photons and b-jets)

⇓

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⇓

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⇓

Make categories based on the BDTs outputs

⇓

Make a simultaneous fit to $m_{\gamma\gamma}$

⇓

Get the upper limit on the signal strength from fit A and the constraints on κ_λ from fit B

fit A:

Observed significance of SM HH: 0.8σ

Expected significance: 1.0σ

Simplified Analysis Strategy

Prepare the objects (photons and b-jets)

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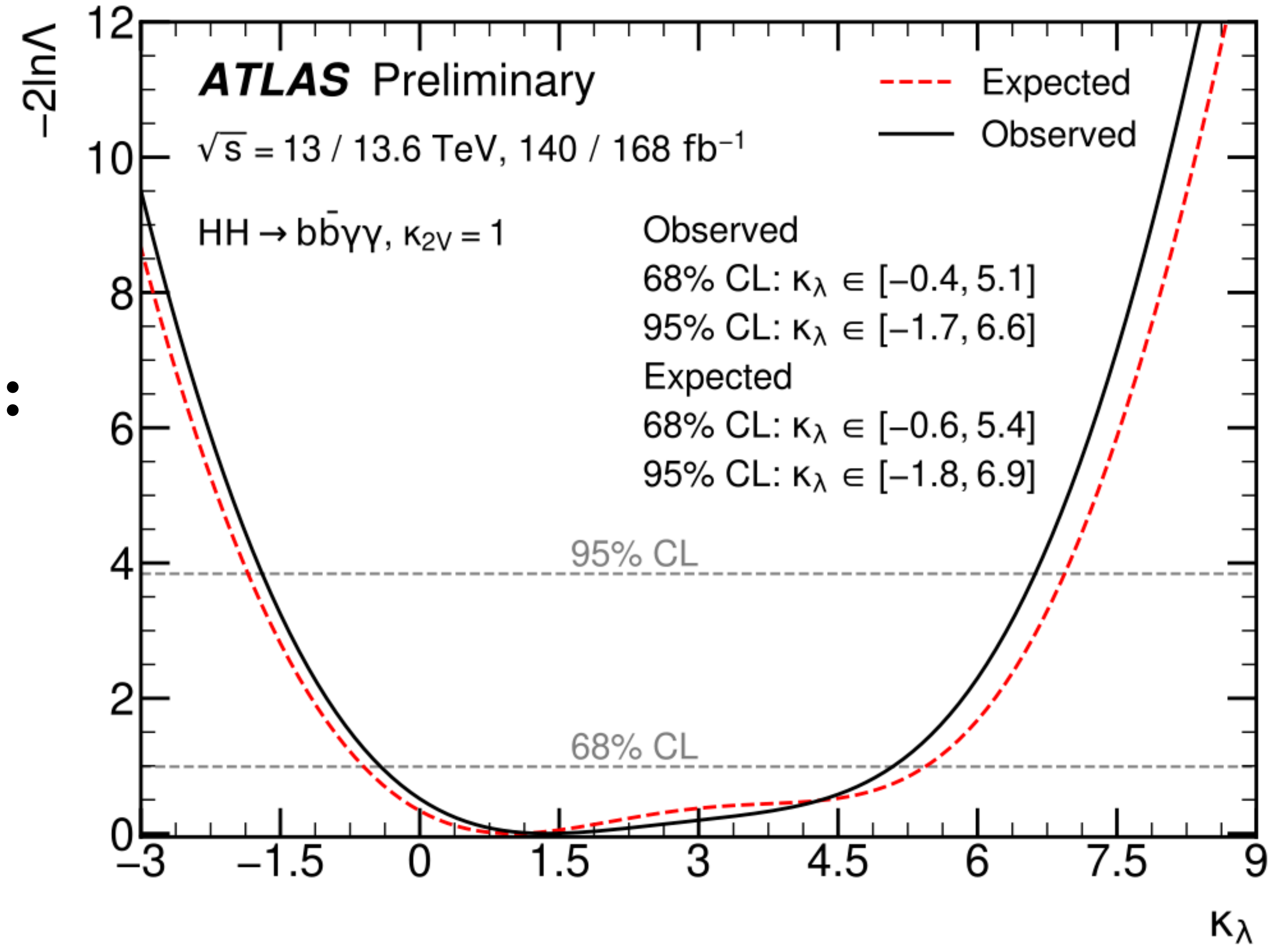

Simplified Analysis Strategy

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⇓

Split into high mas

Train Boosted
distinguish si

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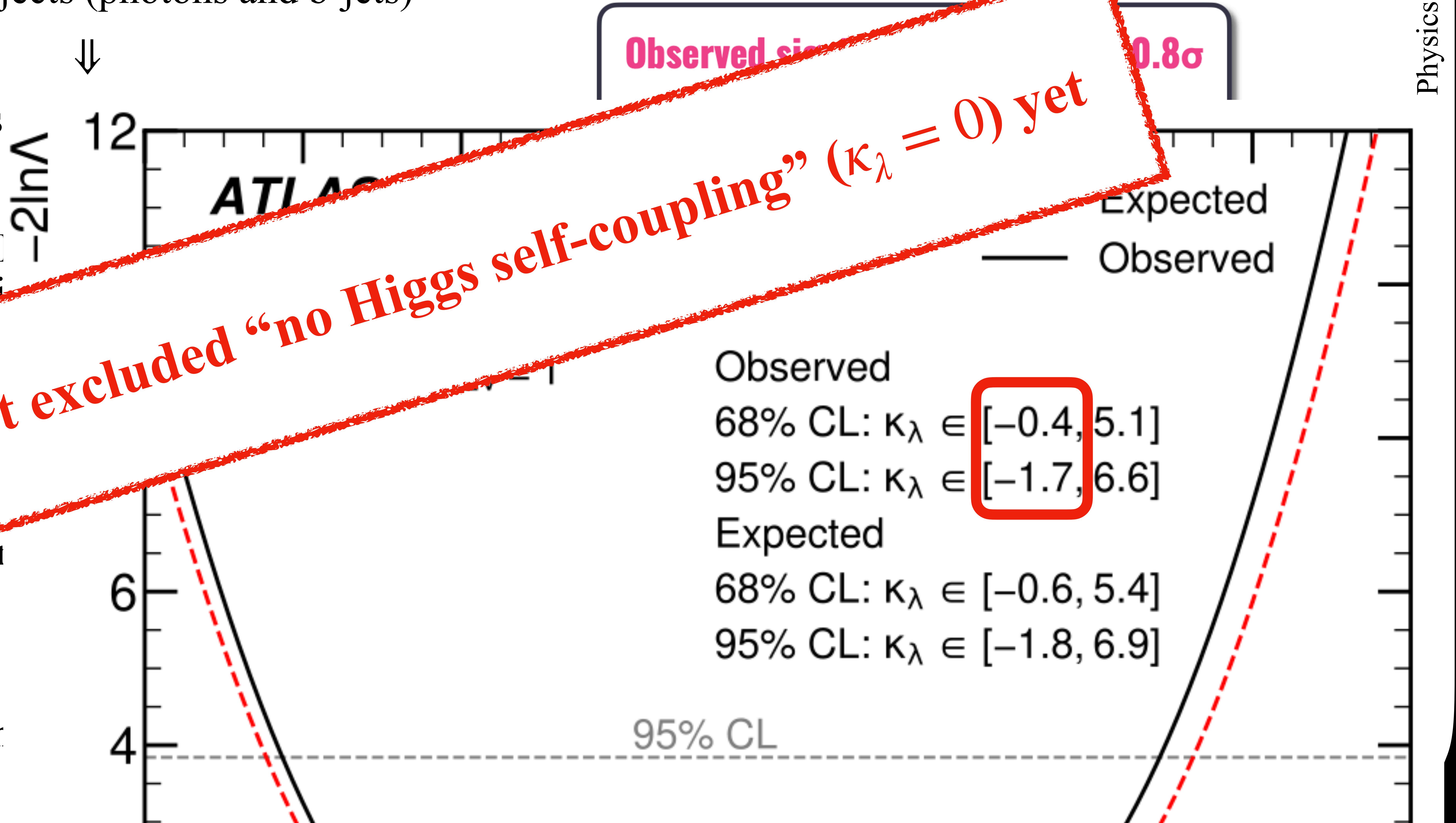
cor

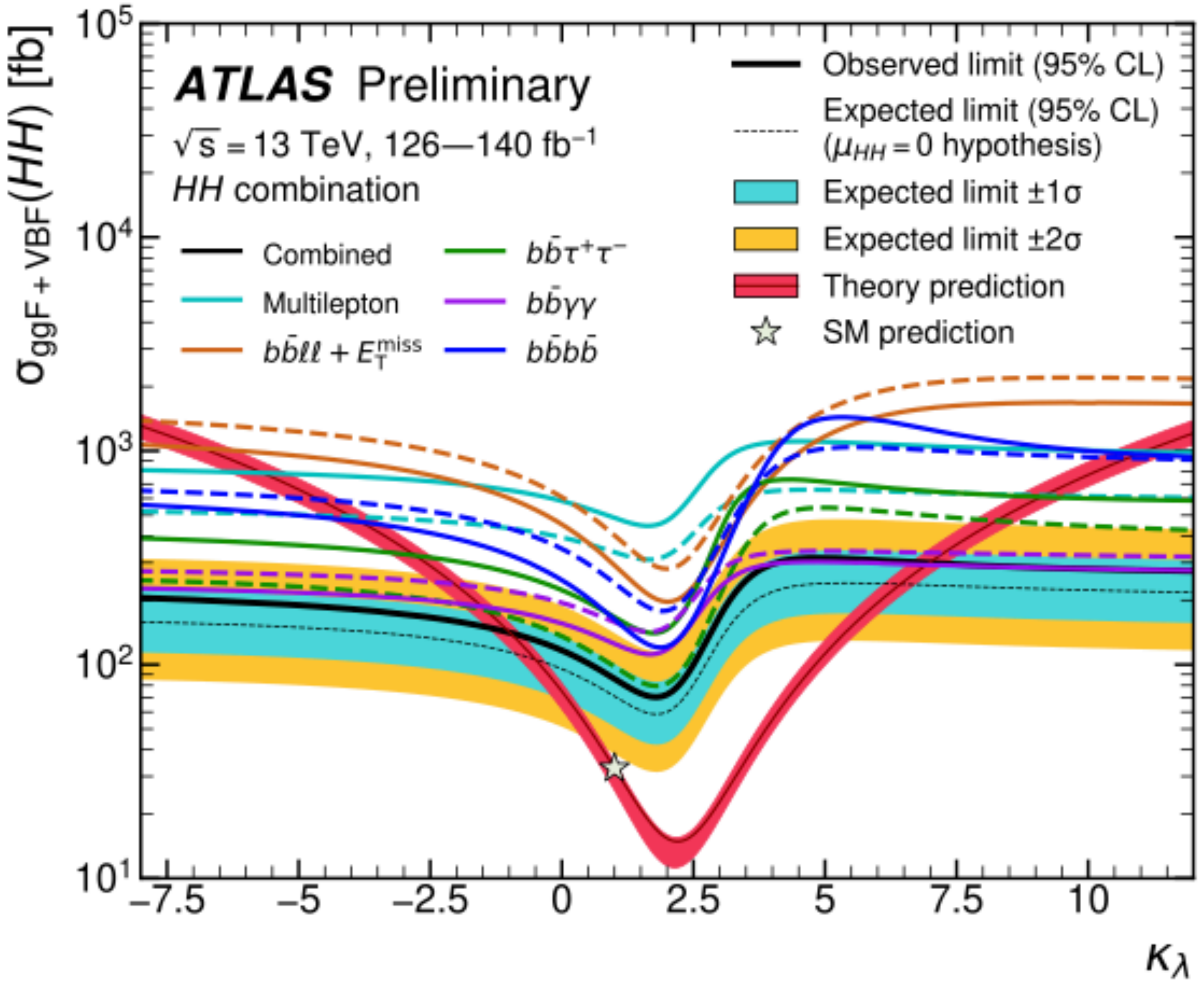


We cannot excluded “no Higgs self-coupling” ($\kappa_\lambda = 0$) yet

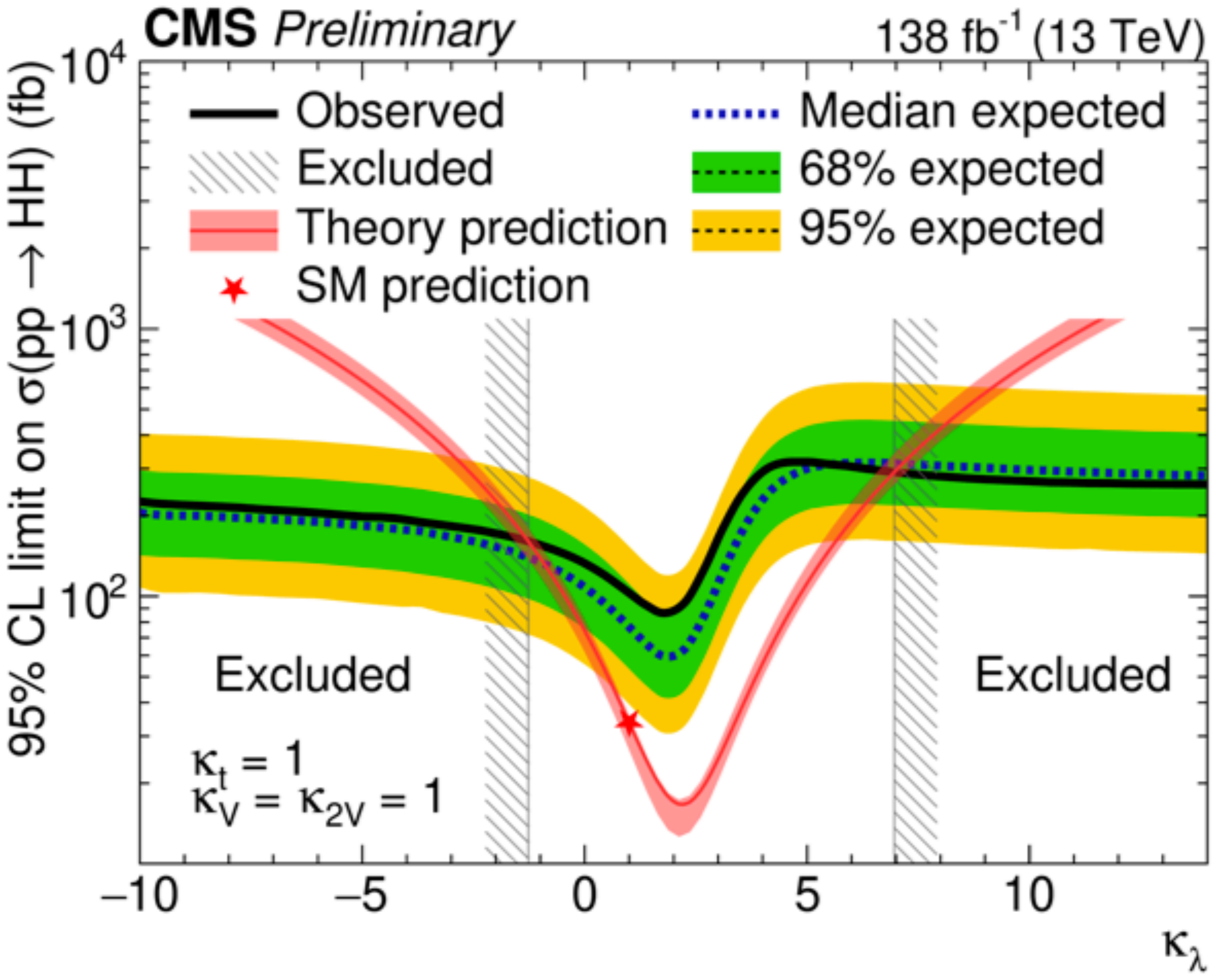
Physics Workshop

Newest di-Higgs constraints

ATLAS Run 2



CMS Run 2



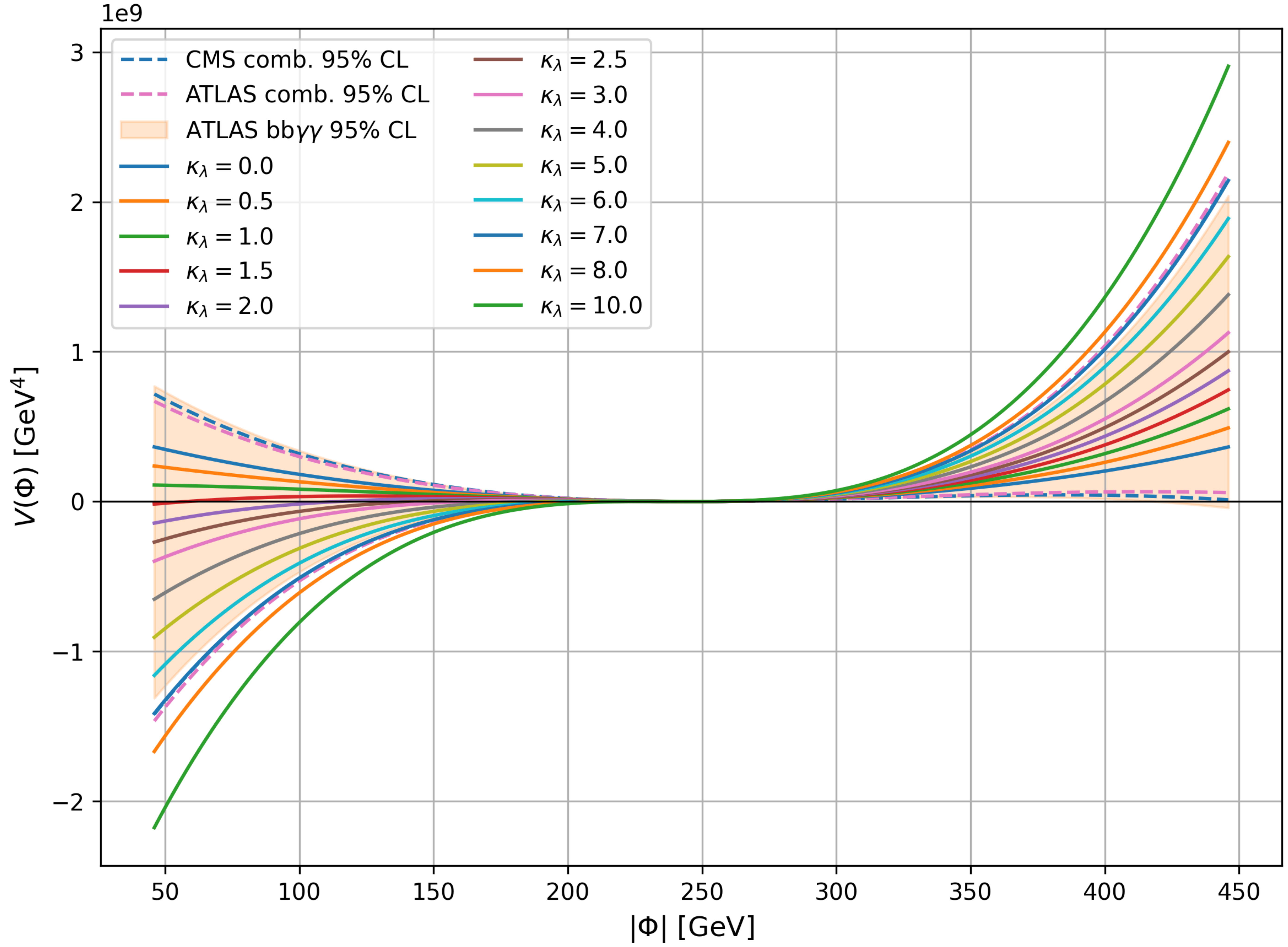
Combined Run 2: $-1.2 < \kappa_\lambda < 7.2$ at 95% C.L.

$-1.4 < \kappa_\lambda < 7.0$ at 95% C.L.

New $bb\gamma\gamma$ analysis: $-1.6 < \kappa_\lambda < 6.6$ at 95% C.L.

Upper bound on λ of currently about **7 × (SM value)**

Translated to the potential shape



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SM = Standard Model
BSM = Beyond Standard Model

Are the current limits interesting?

Standard model

higher-order contributions to λ, mostly from top loop:

~ 0.07×SM

BSM models (UV-complete)

mass splitting between BSM Higgs states yields large enhancement of λ:

~ several × SM

Effective field theories

BSM effects parameterized as higher-dimensional operators, large enhancement of λ possible:

~ several × SM

⇒ **the current limits are already interesting for theory**

⇒ κ_λ **can be modified up to** ~ 7 × SM **from BSM**

Future limitations and show stoppers?

Current limitation: STATISTICS

$bb\gamma\gamma$ systematic uncertainties

Uncertainty source	$\Delta\sigma$ [%]
Photon Energy Resolution (PER)	± 0.42
Photon Energy Scale (PES)	± 3.5
Jet	$< \pm 0.1$
E_T^{miss}	$< \pm 0.1$
Muon	$< \pm 0.1$
Photon Efficiency	± 0.27
Flavor Tagging	$< \pm 0.1$
QCD Scale + m_{top}, PDF+α_S	± 7.1
Branching ratio	± 0.32
Parton showering model	$< \pm 0.1$
Heavy-flavour content	± 0.98
Pileup	$< \pm 0.1$
Luminosity	$< \pm 0.1$
Background model (spurious signal)	$< \pm 0.1$
All systematics	± 13

preliminary

largest systematic uncertainty

J. Davies, Higgs Pairs 2025

Theoretical uncertainties

The theoretical uncertainties on the di-Higgs cross-section are directly related to the uncertainty on κ_λ

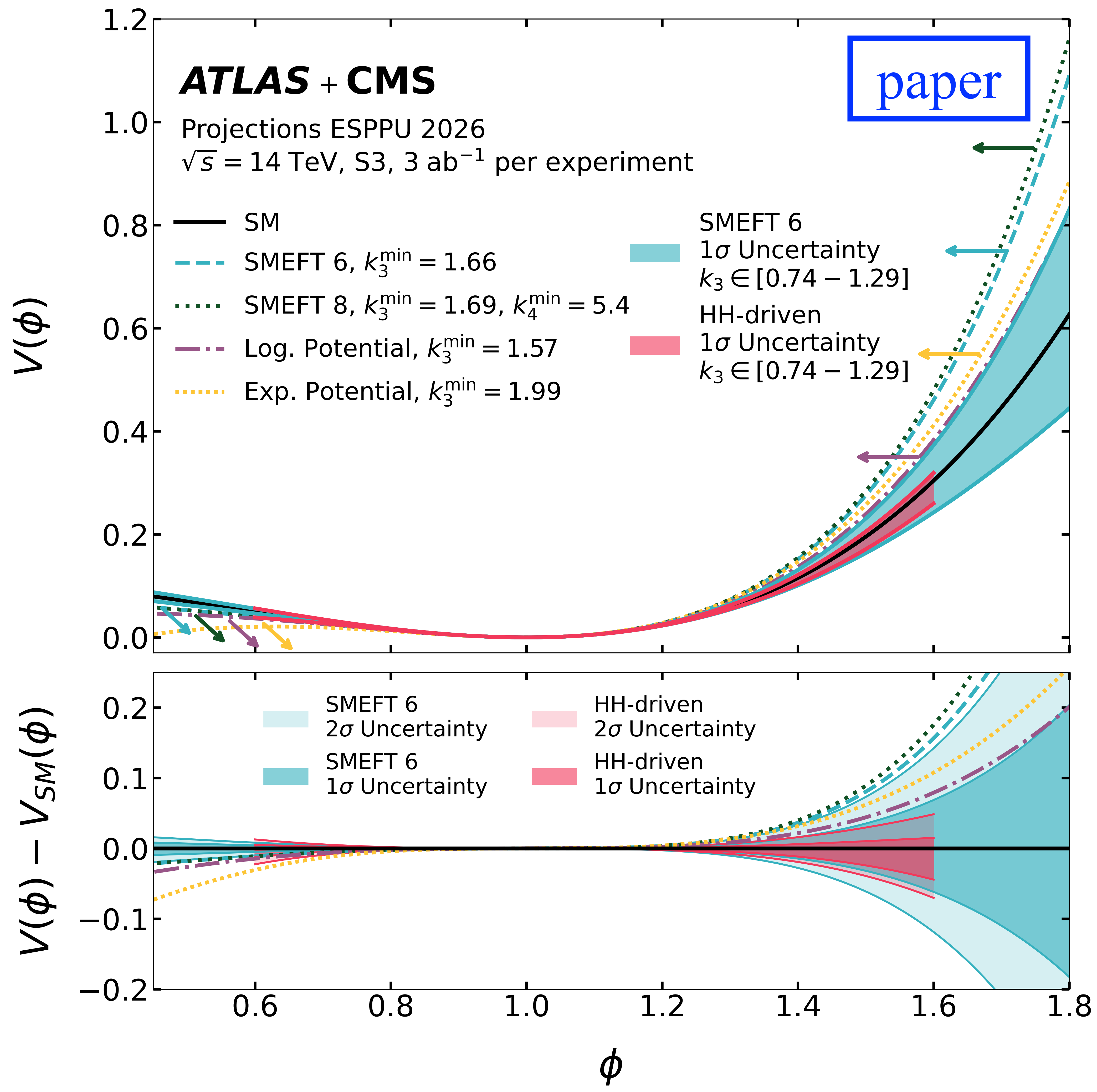
$\sqrt{s}$	13 TeV	13.6 TeV	14 TeV
$\sigma_{\text{NNLO FTapprox}}$ [fb]	30.77	34.13	36.37
PDF+α_s	$\pm$2.3%	$\pm$2.3%	$\pm$2.2%
ren. scale	+2.2%/-5.0%	+2.1%/-4.9%	+2.1%/-4.9%
m_t	+4%/-18%	+4%/-18%	+4%/-18%

worsens scale

- top mass scheme
- renormalization and factorization scales
- parton distribution functions (PDFs)
- strong coupling (α_S)

HL-LHC = High Luminosity LHC

A look at the future: HL-LHC



Arrows indicate where four BSM scenarios predict a strong first-order phase transition

This means that at **HL-LHC** with 3 ab^{-1} the 95% CL red band excludes as good as all possible strong-FOPT scenarios across the four alternative hypotheses:

in specific models we can either find hints of a First Order Phase Transition or exclude one!

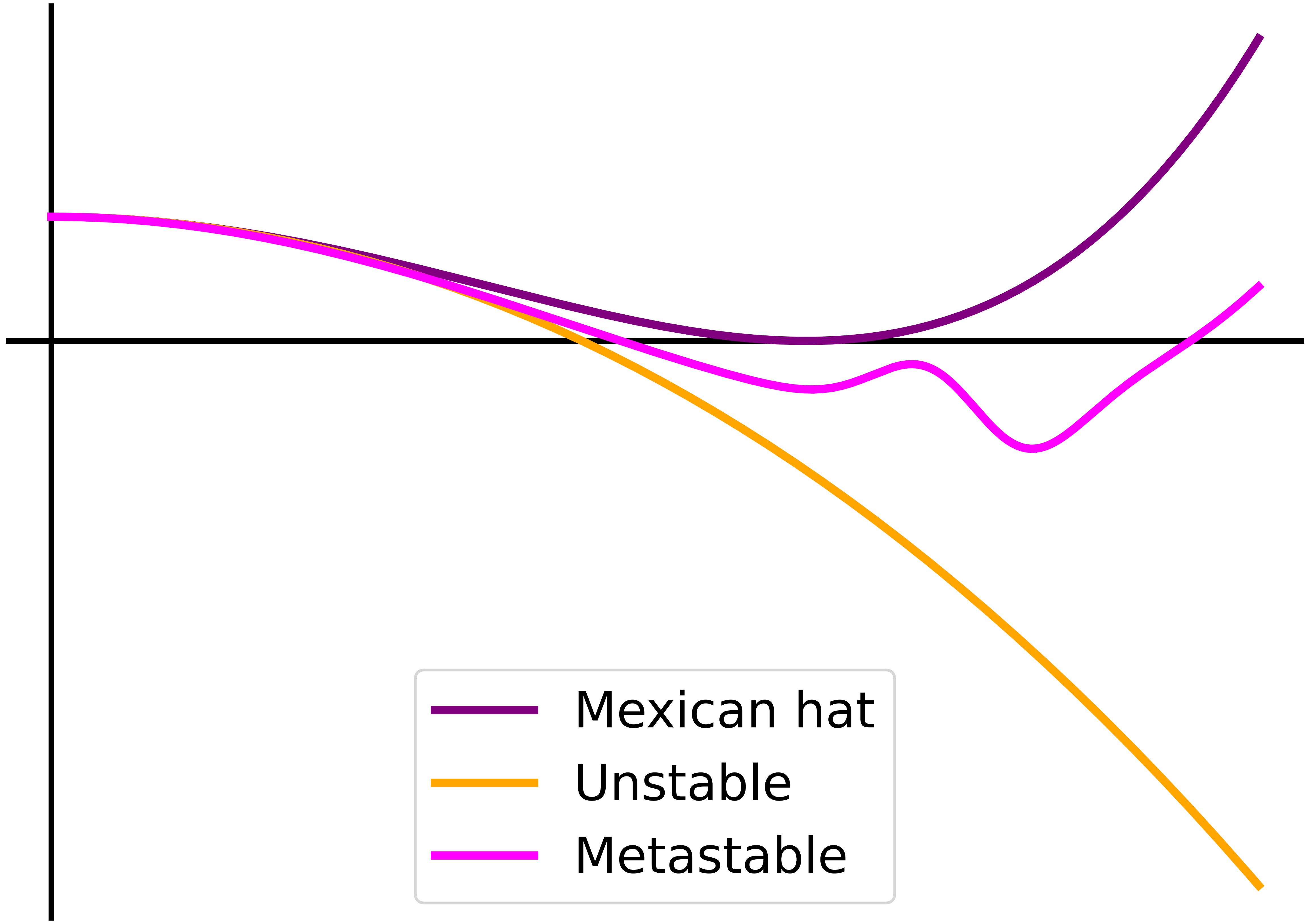
Cédrine Hügli (DESY), Astroparticle and Particle Physics Workshop

Fate of the universe

Plot for illustration purpose

Stable, metastable and unstable

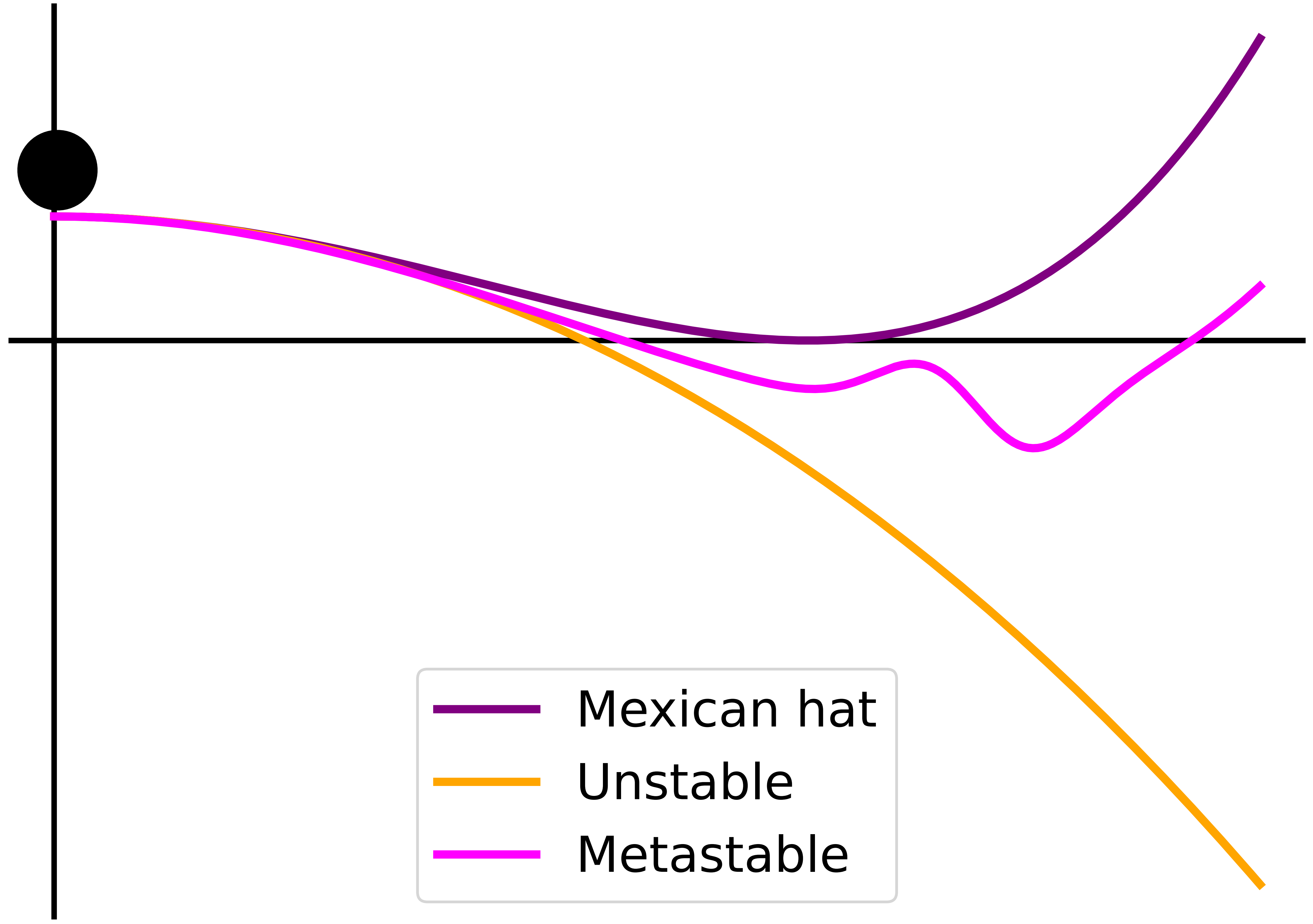
How could a Higgs potential look like?



Plot for illustration purpose

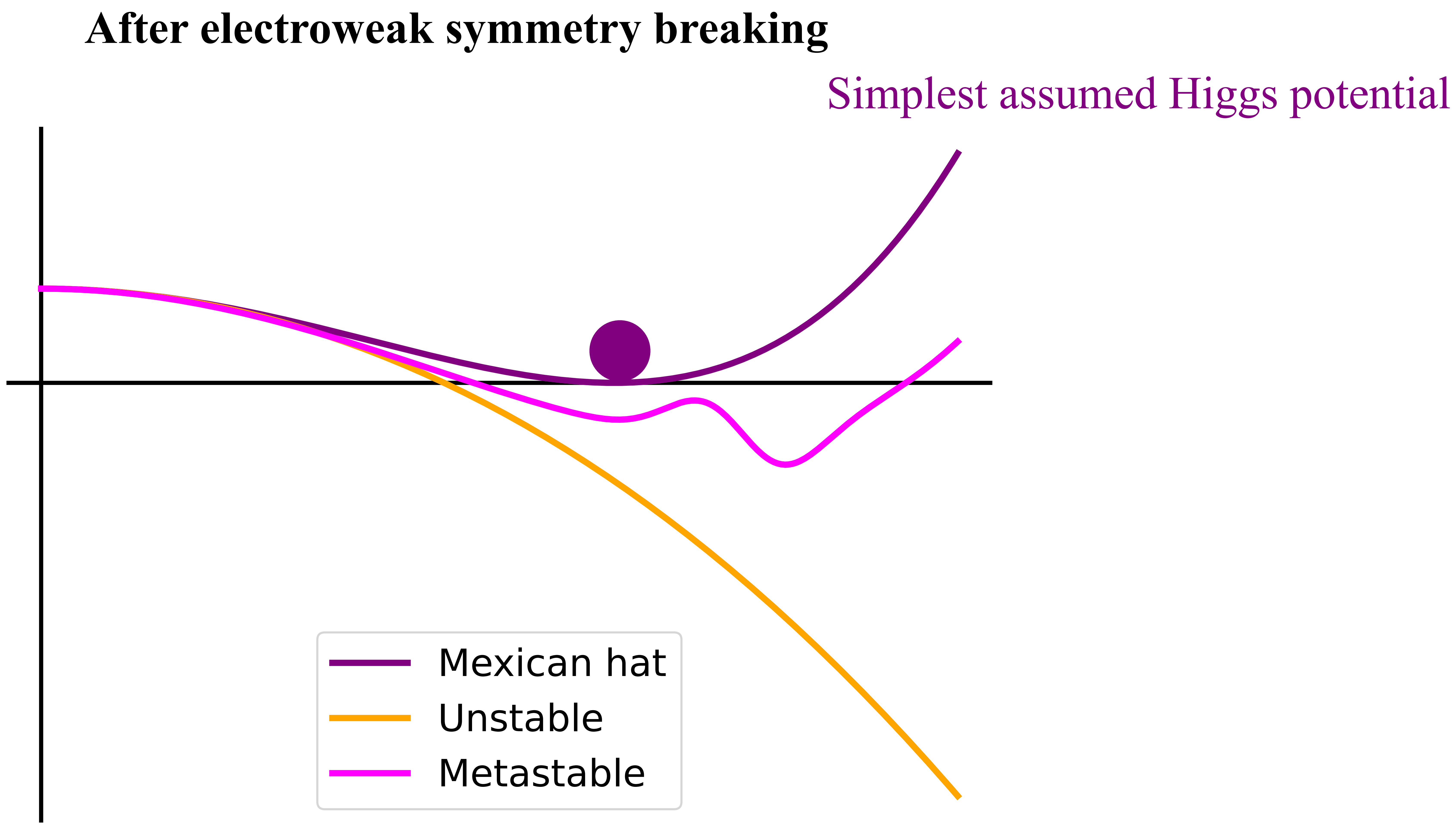
Stable, metastable and unstable

Before electroweak symmetry breaking



Plot for illustration purpose

Stable, metastable and unstable



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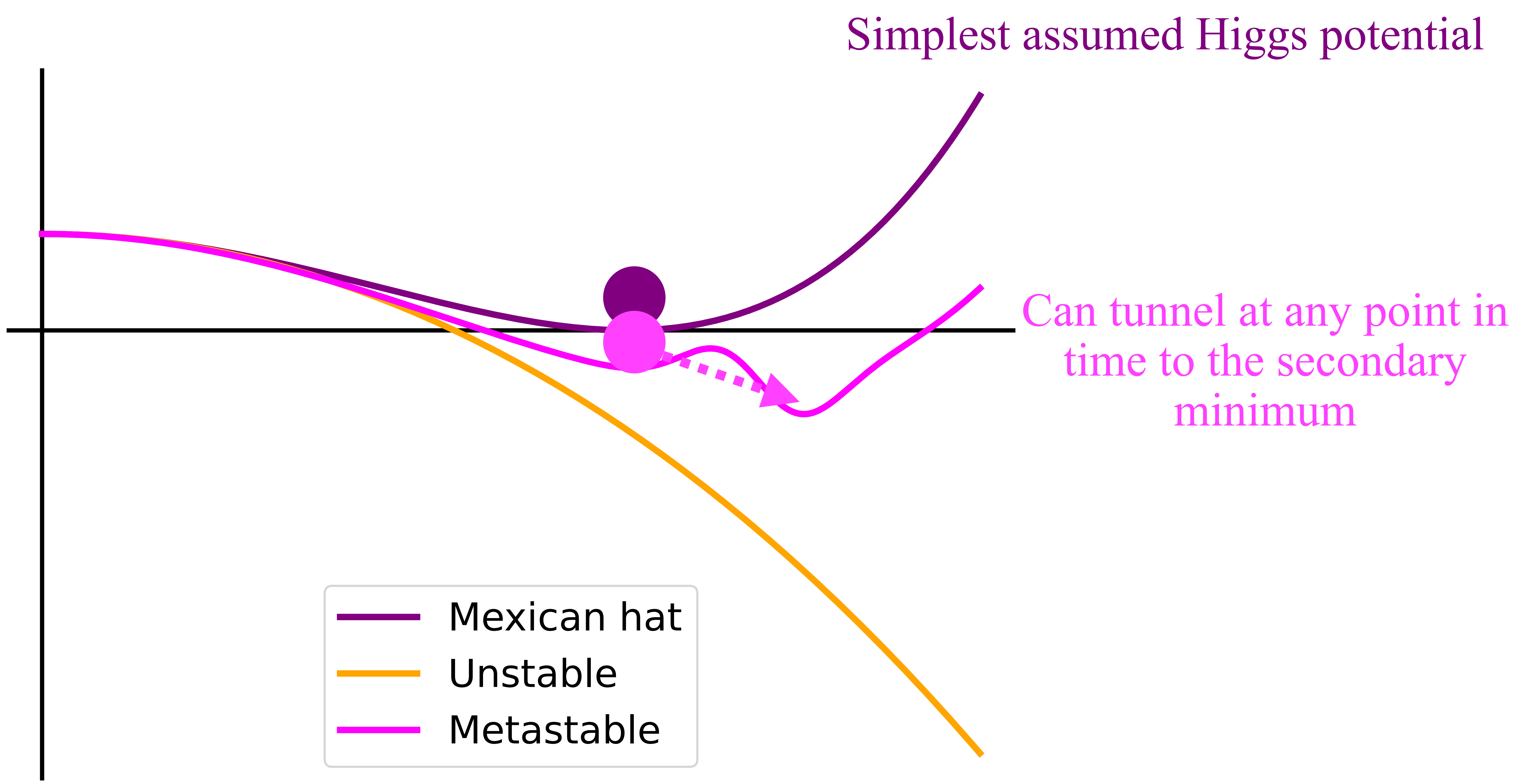
After electroweak symmetry breaking



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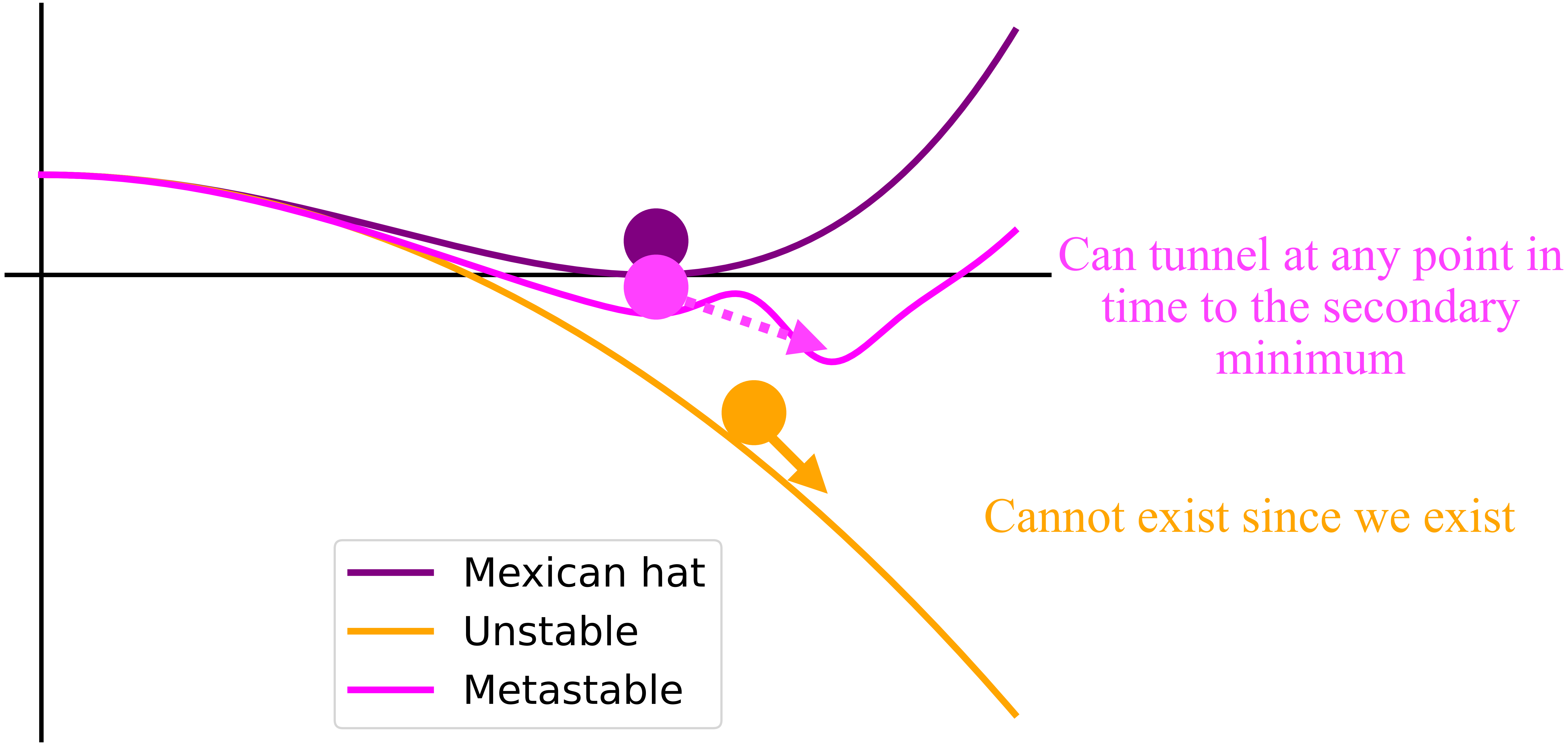
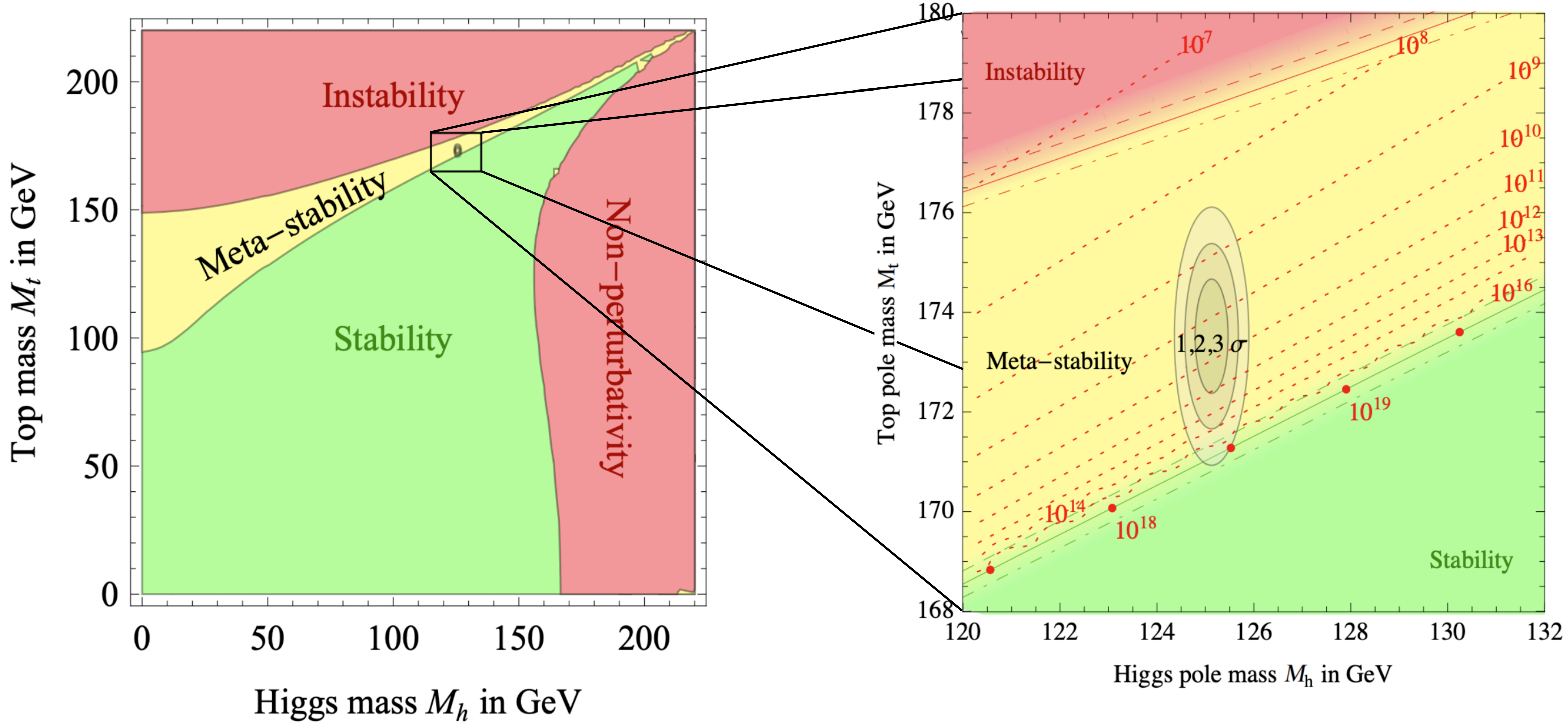


arXiv:1307.3536

Stability of the universe and Higgs self-coupling

Stability of the universe ↔ Higgs and top masses



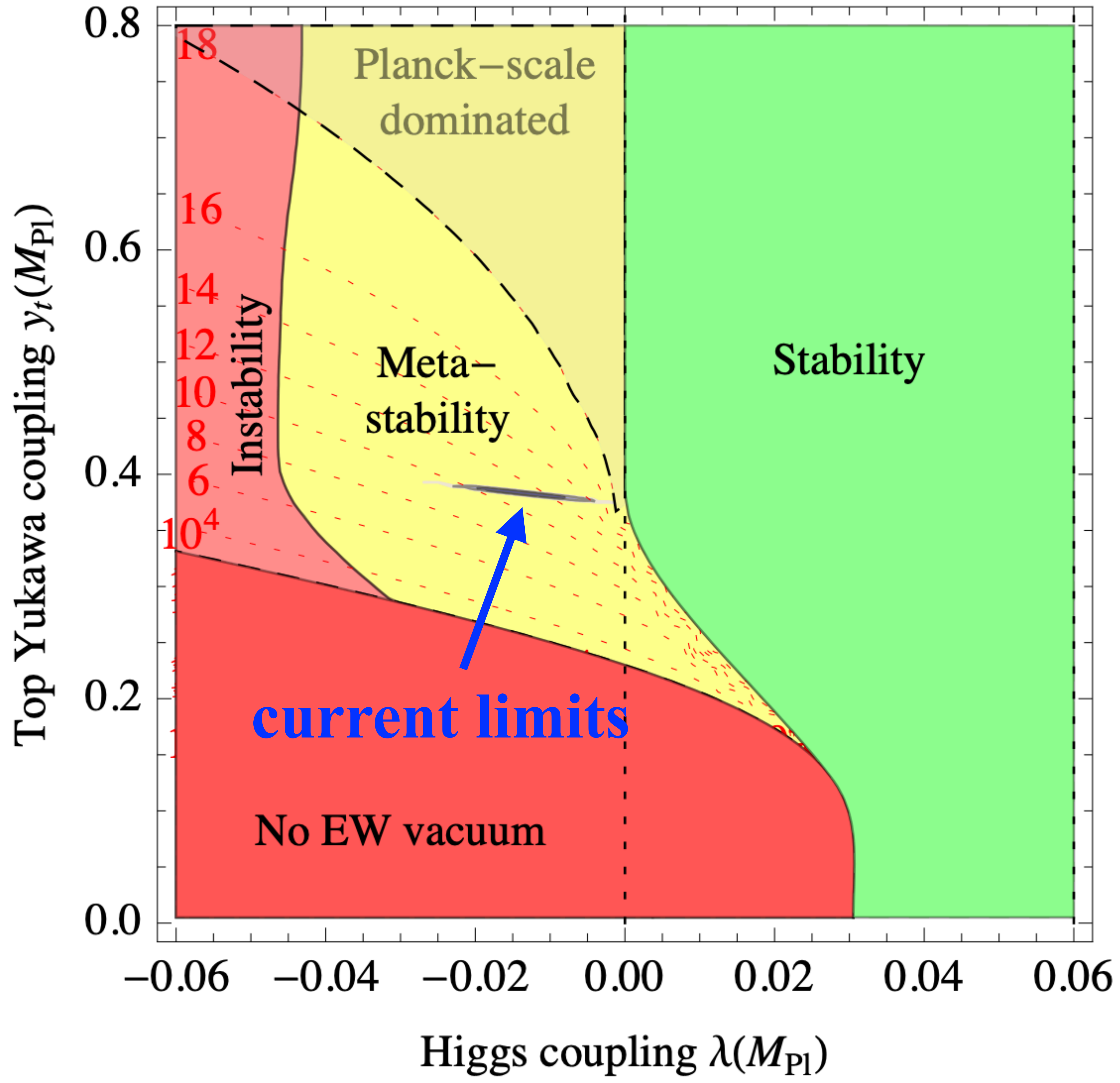
arXiv:1307.3536

Stability of the universe and Higgs self-coupling

Stability of the universe ↔ Higgs and top masses ↔ Top Yukawa coupling and Higgs coupling

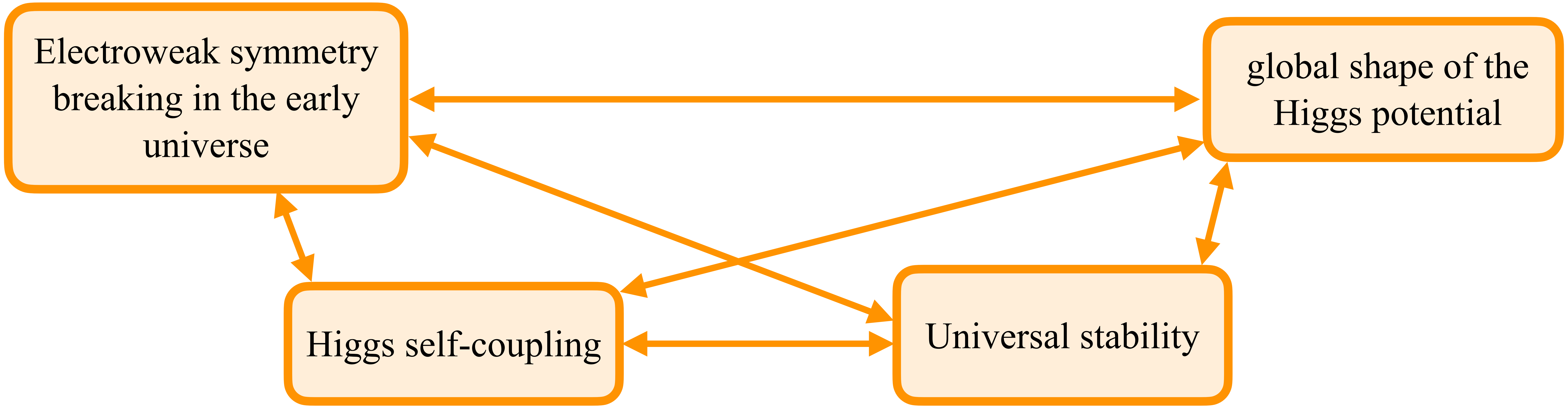
$$V = const. + m_H^2 |H|^2 + \lambda |H|^4$$

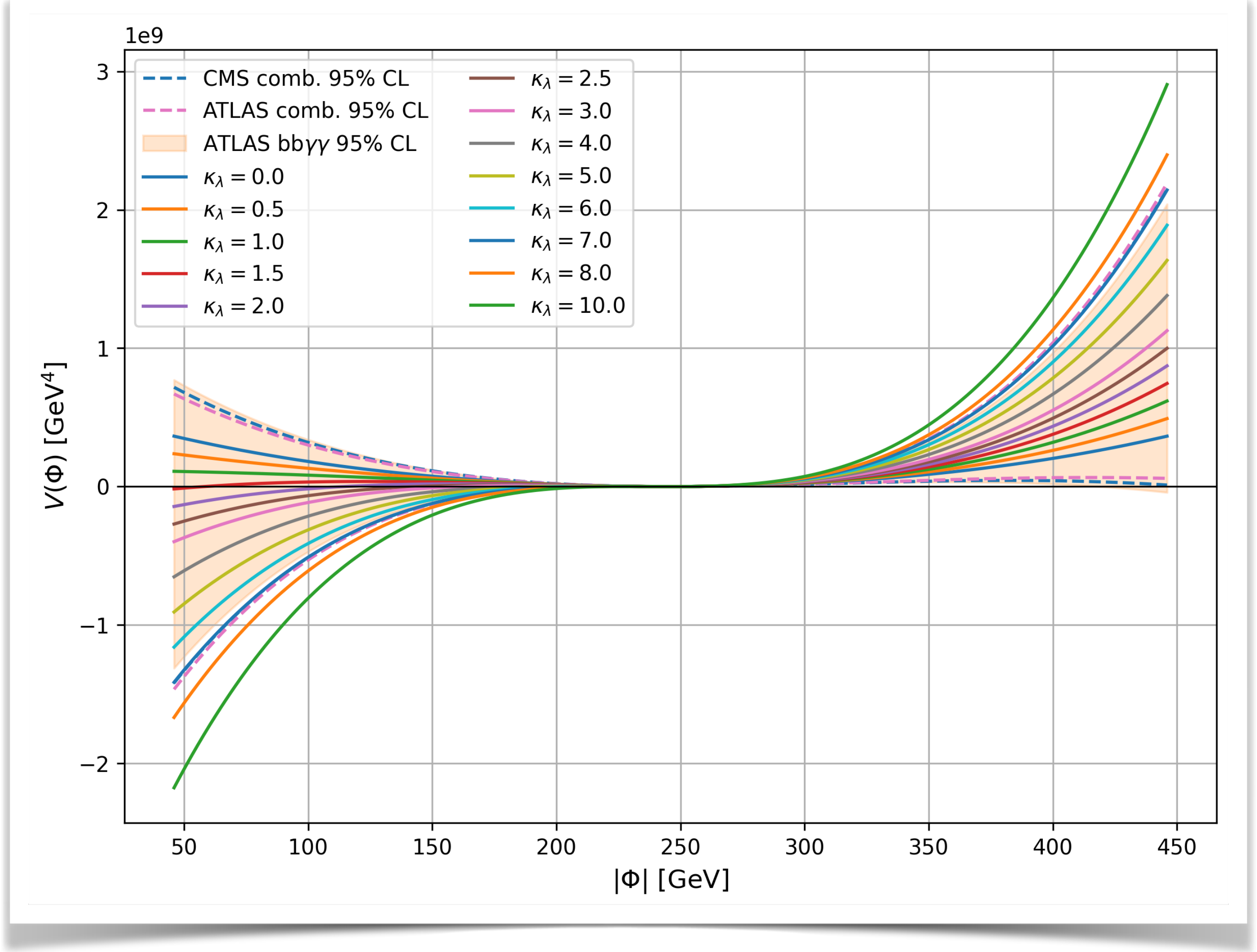
Small variations of any of these three parameters with respect to their measured values could have devastating consequences for our life-friendly universe.

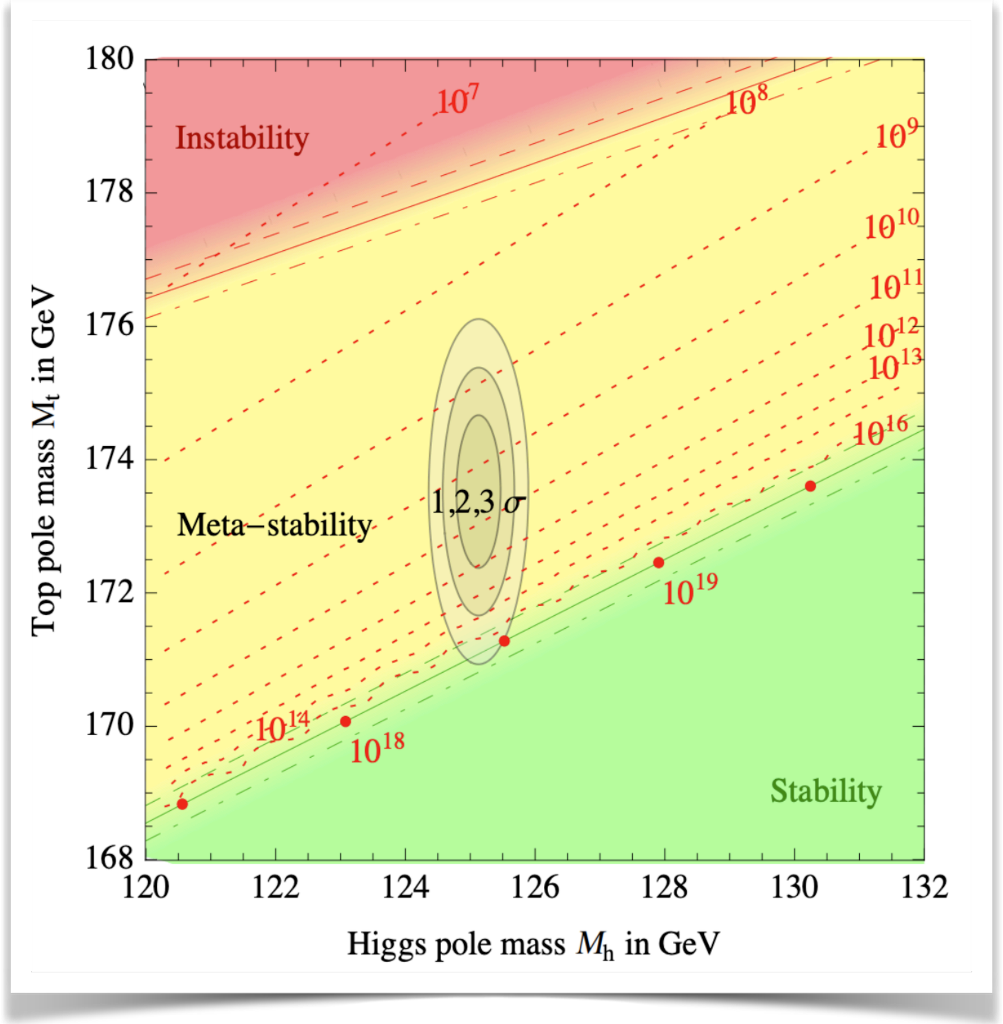


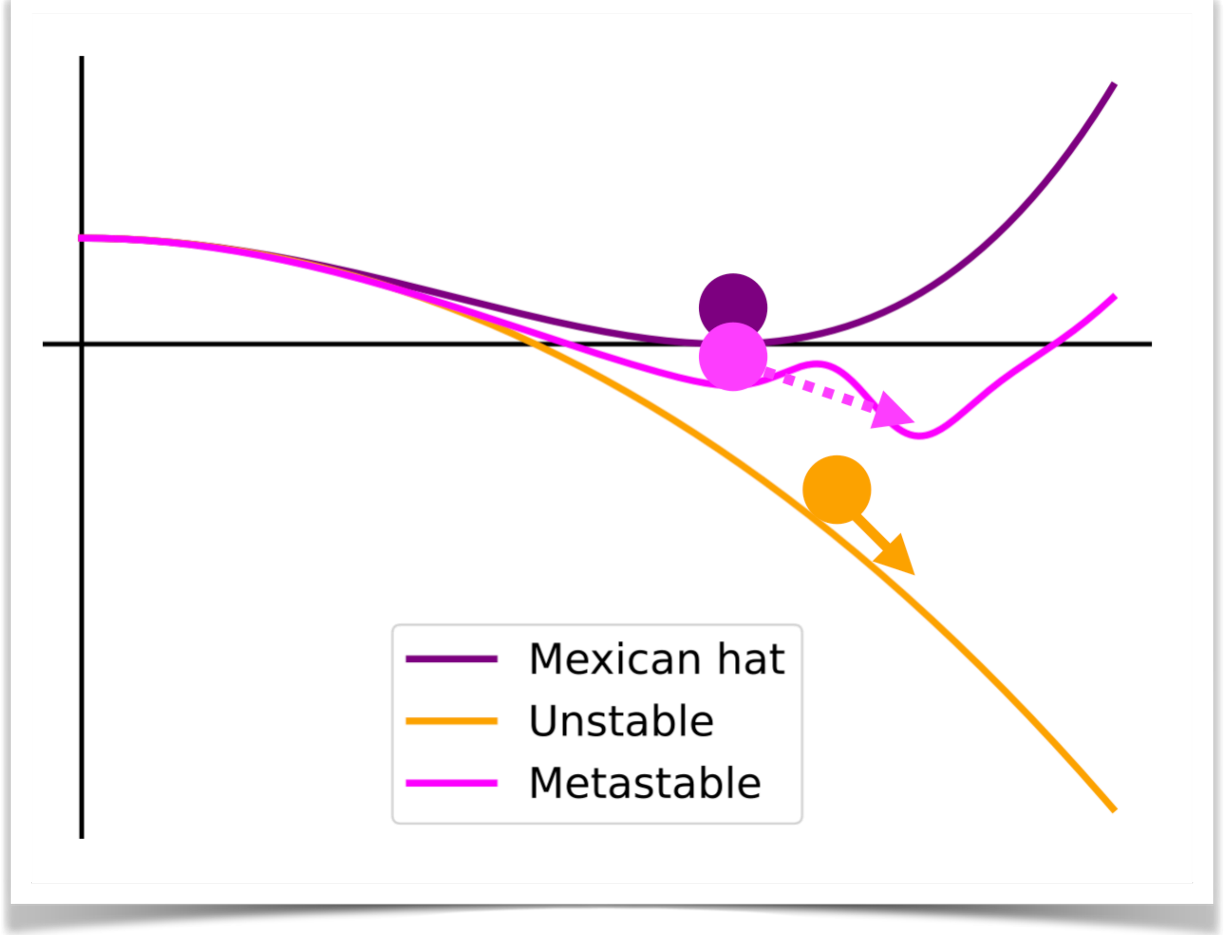
Conclusion

- Electroweak symmetry breaking: what phase transition? We don't know yet
- The current limits are already useful to exclude some Beyond the Standard Model models
- The limiting factor to the Higgs boson self coupling discovery is the statistics that should be achieved at HL-LHC, at that point the limiting factor will be the theory prediction precision
- Current knowledge tells us we could in theory disappear instantly due to the universe being metastable

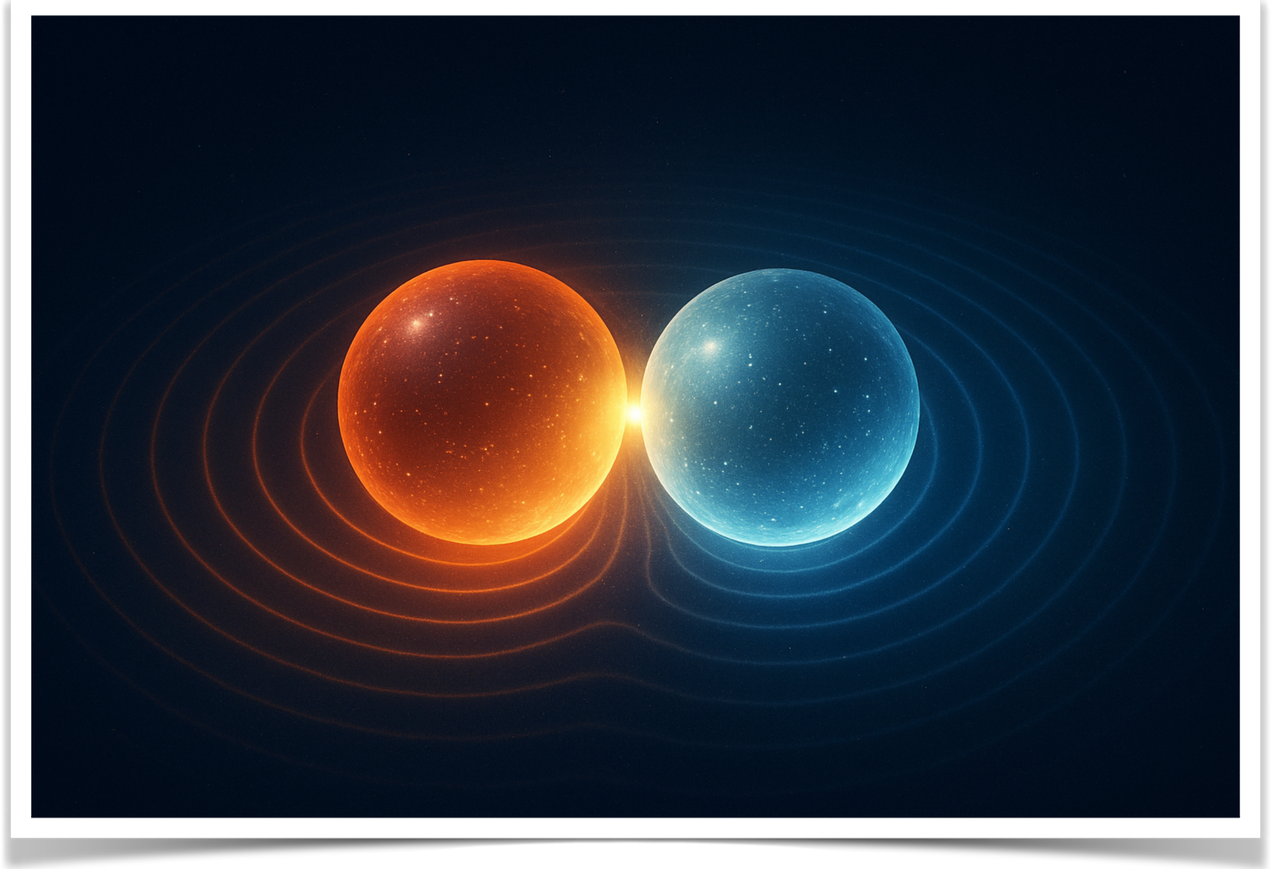


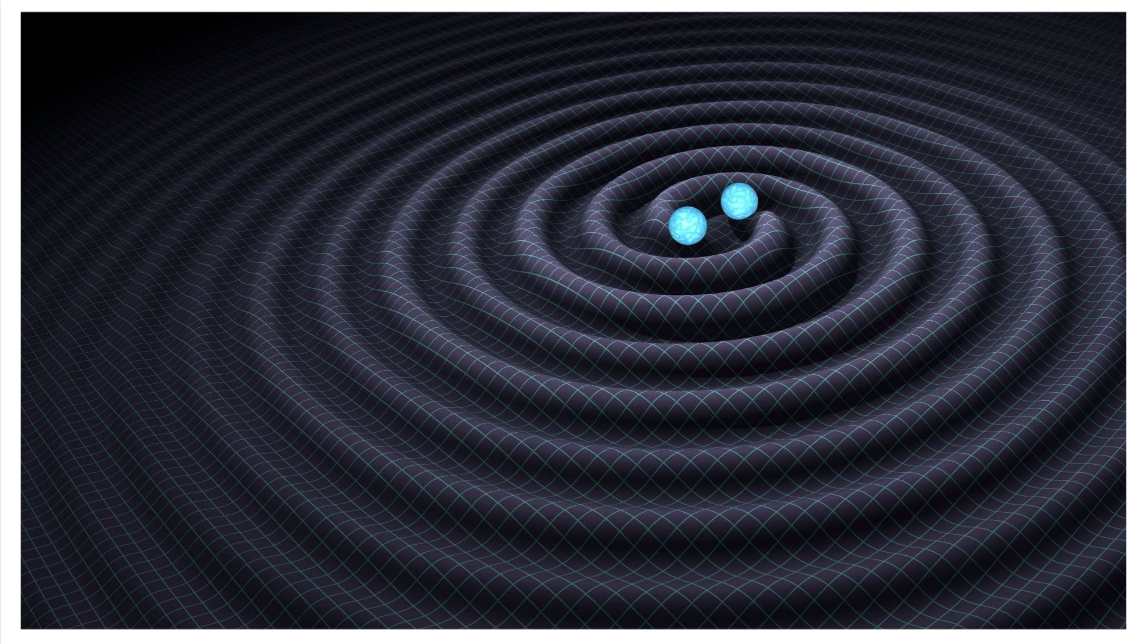






Thank you for listening!
Questions?





Cédrine Hügli (DESY), Astroparticle and Particle Physics Workshop

BACKUP

Backup content

- Top mass scheme
- Gravitational Waves
- An introduction to EFT
- Heavy top limit
- Theoretical uncertainties: PDF issues
- Plot: LO interference
- Plot: Stability - mass
- Plank scale
- Plot: Stability - coupling
- Kinematic fit detailes

Top mass scheme

1. The Problem: What is the Top Quark Mass?

- Top quark mass is a fundamental parameter in the Standard Model.
- Different ways to define "mass" lead to slightly different numerical values.
- The uncertainty isn't just experimental — it's **theoretical**, depending on the scheme used.

2. Mass Schemes

Scheme	Definition	Key Feature	Uncertainty Source
Pole Mass	Mass of the quark as an on-shell particle (propagator pole)	Intuitive, linked to physical mass	Ambiguous due to infrared effects (~Λ_QCD ≈ 200 MeV)
MS-bar ($\overline{\mathrm{MS}}$)	Short-distance mass, defined in dimensional regularization	Scale-dependent, avoids long-distance ambiguity	Needs scale choice; less direct physical interpretation
1S / PS / Kinetic	Masses tied to bound-state properties or low-energy observables	Reduces long-distance QCD effects	Small scheme conversion uncertainties (~100 MeV)

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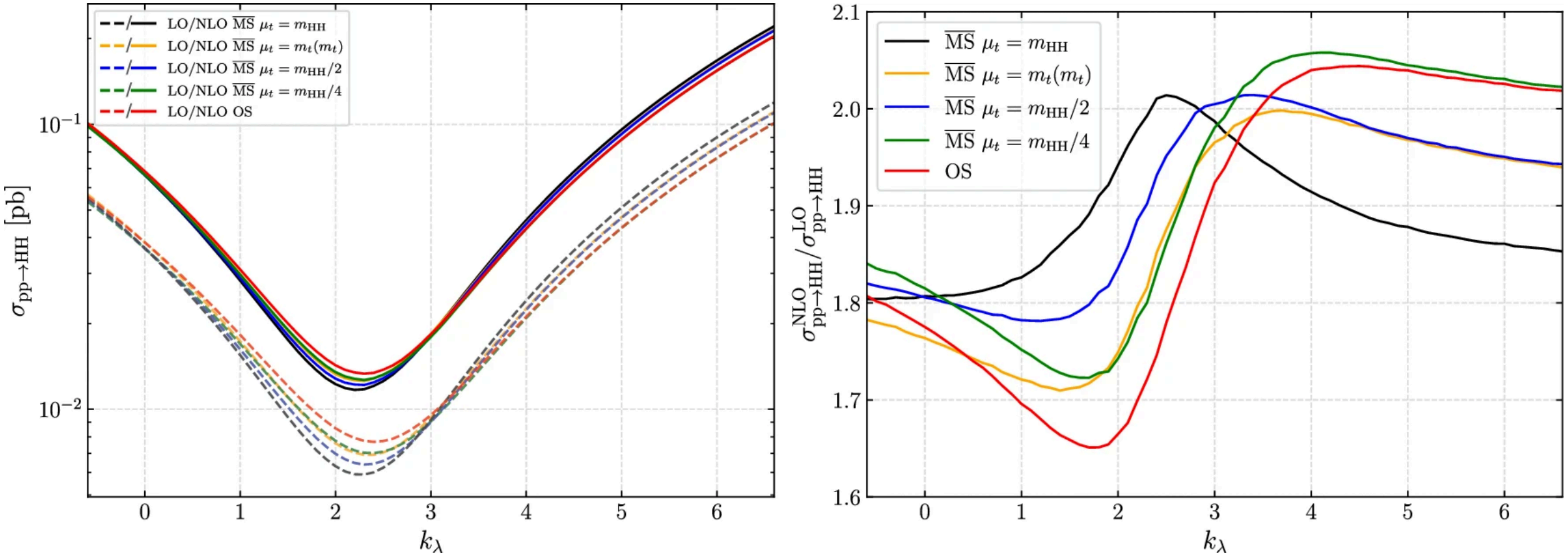
Top mass scheme and κ_λ

Fig. 1

From: Higgs boson pair production at NLO in the POWHEG approach and the top quark mass uncertainties



Left: the total inclusive cross sections at $\sqrt{s} = 13.6$ TeV for different choices of the top mass renormalization scheme, at LO (dashed) and NLO (solid), as a function κ_λ. Right: the corresponding K-factors

Bagnaschi et al., Eur. Phys. J. C 83, 1054 (2023)

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Top mass scheme and m_{HH} distribution

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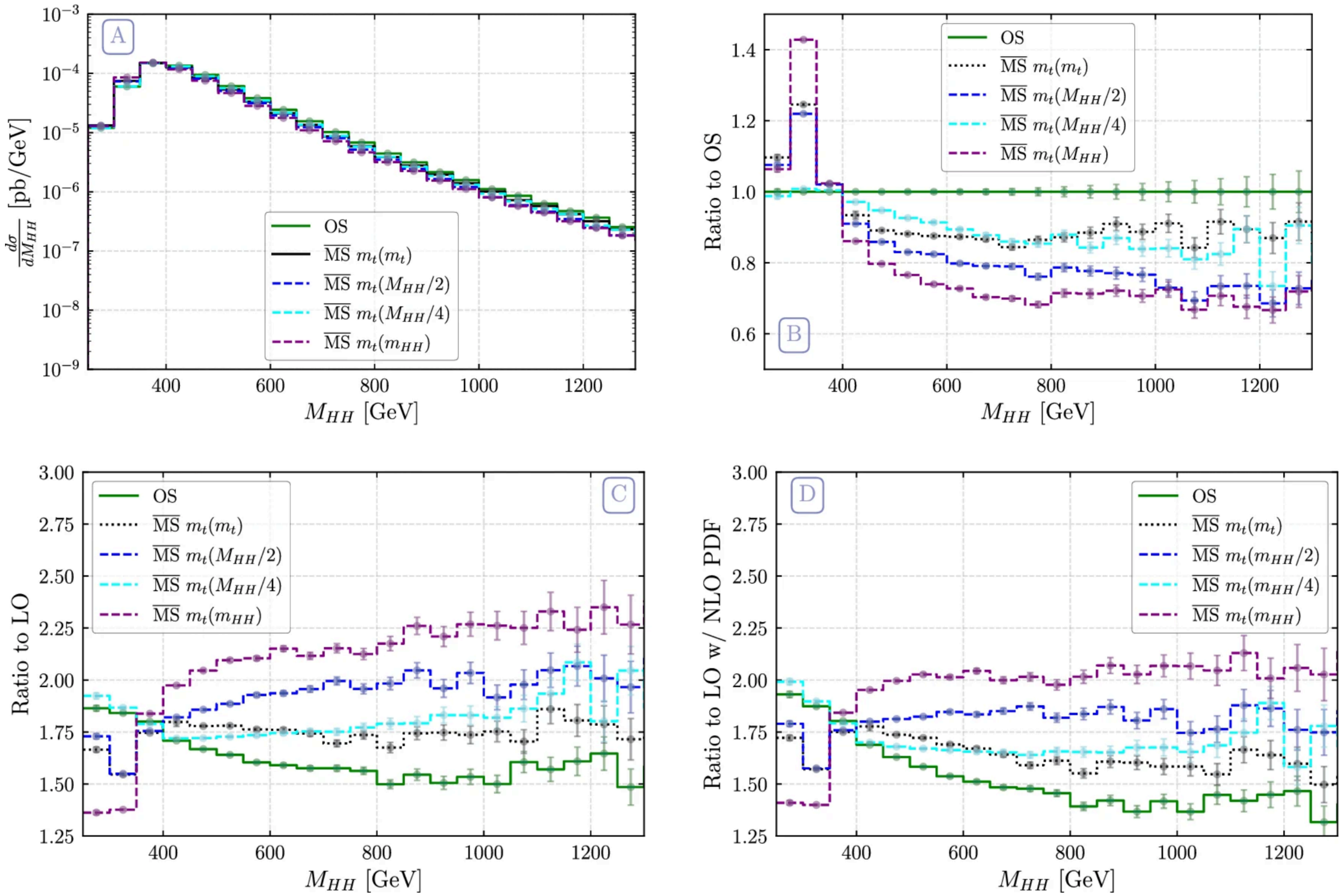

The invariant mass distribution of the two-Higgs system for different choices of the top-mass renormalization scheme: (**A**) absolute distributions at NLO+PS; (**B**) ratio between the $\overline{\text{MS}}$ predictions and the OS one; (**C**) ratio between the distributions computed at NLO+PS and their LO counterpart (K-factors); (**D**) same as **C** but with the LO distributions computed with NLO PDFs

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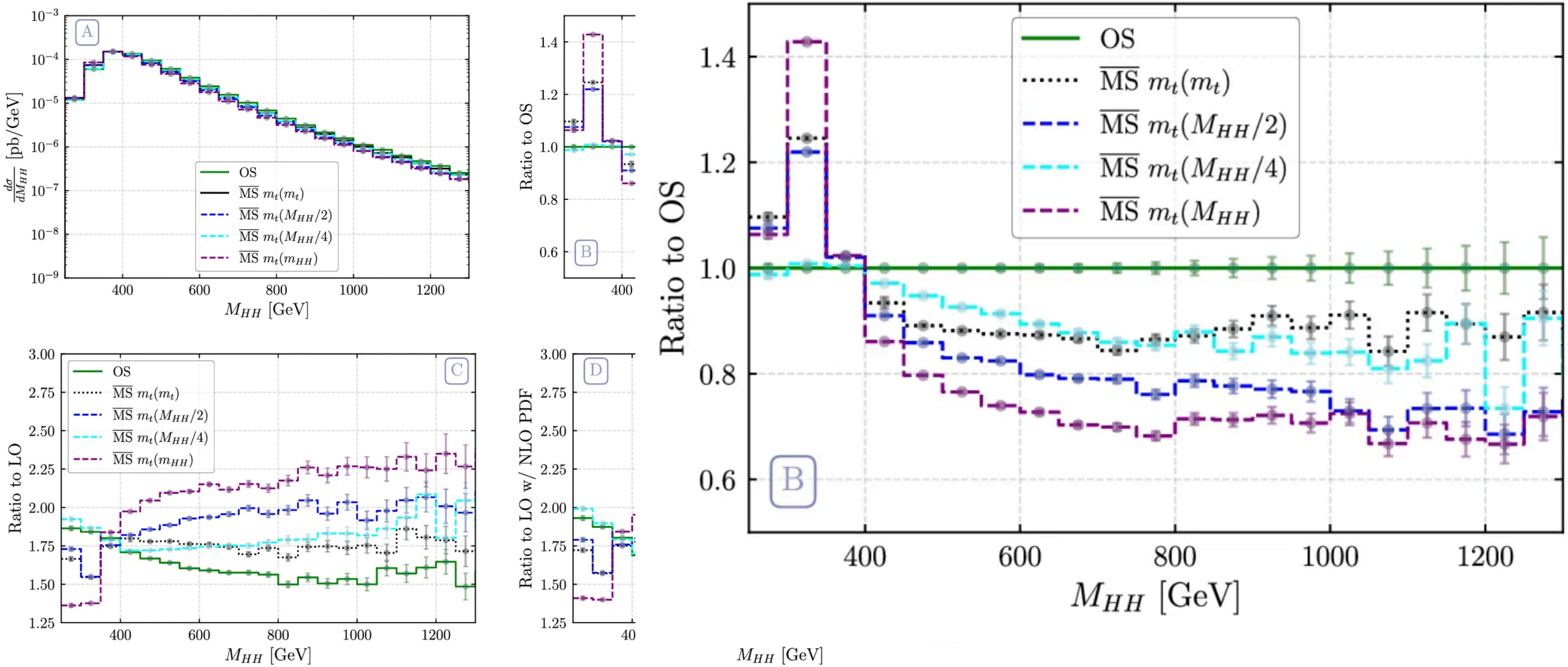

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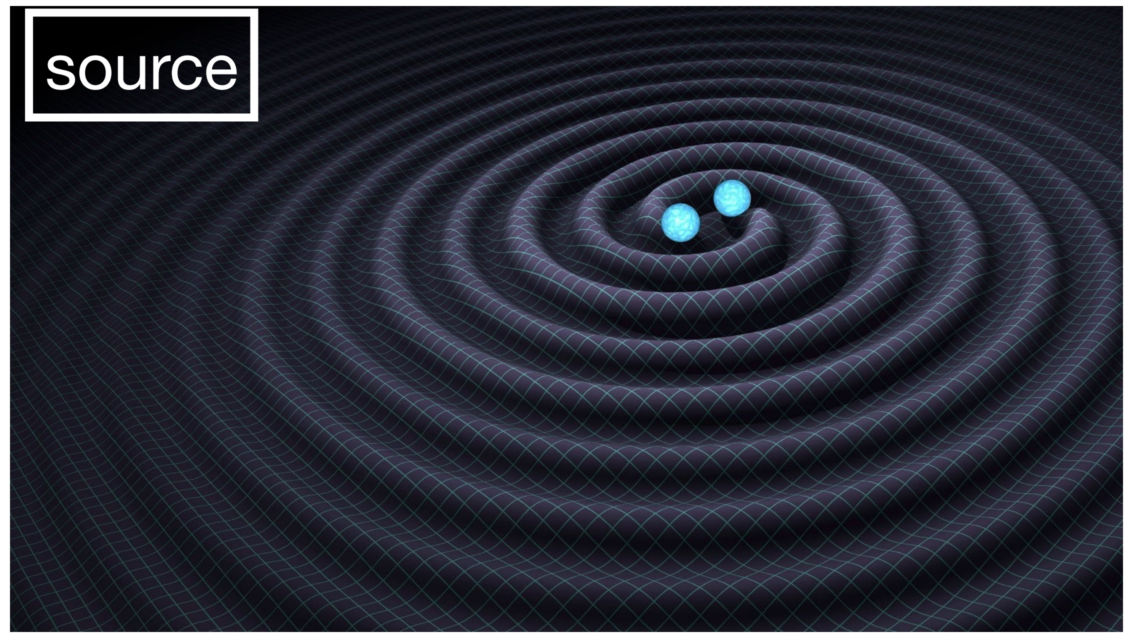
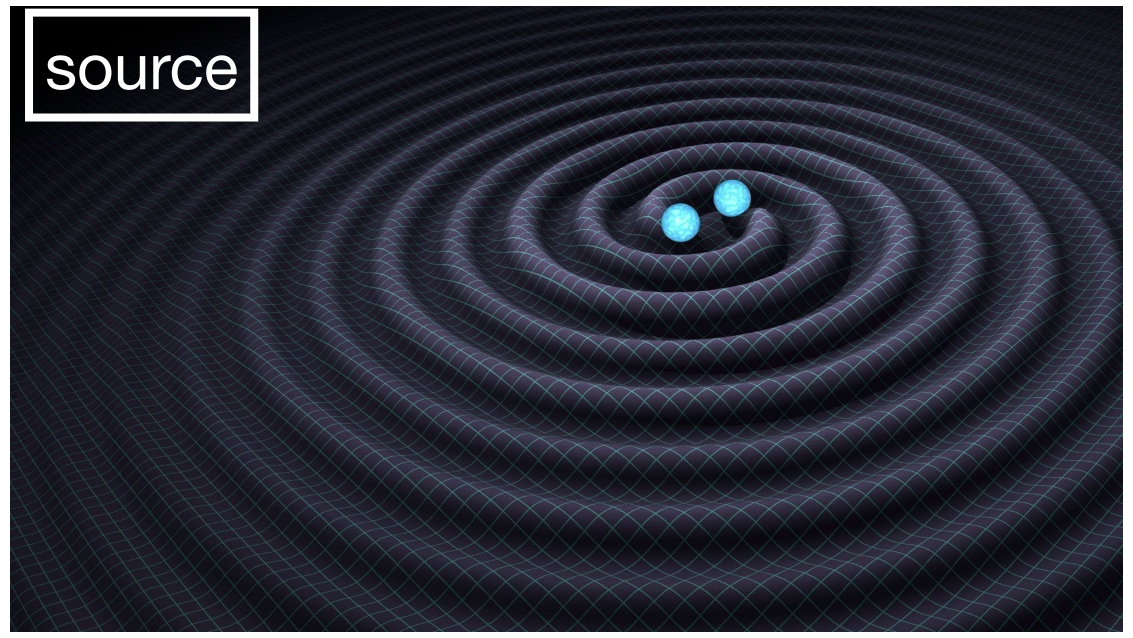
Gravitational Waves (GWs)



- are ripples in spacetime
- produced by violent cosmic events (e.g. black hole merges)
- can be detected with experiments like LIGO and Virgo

Gravitational Waves (GWs)



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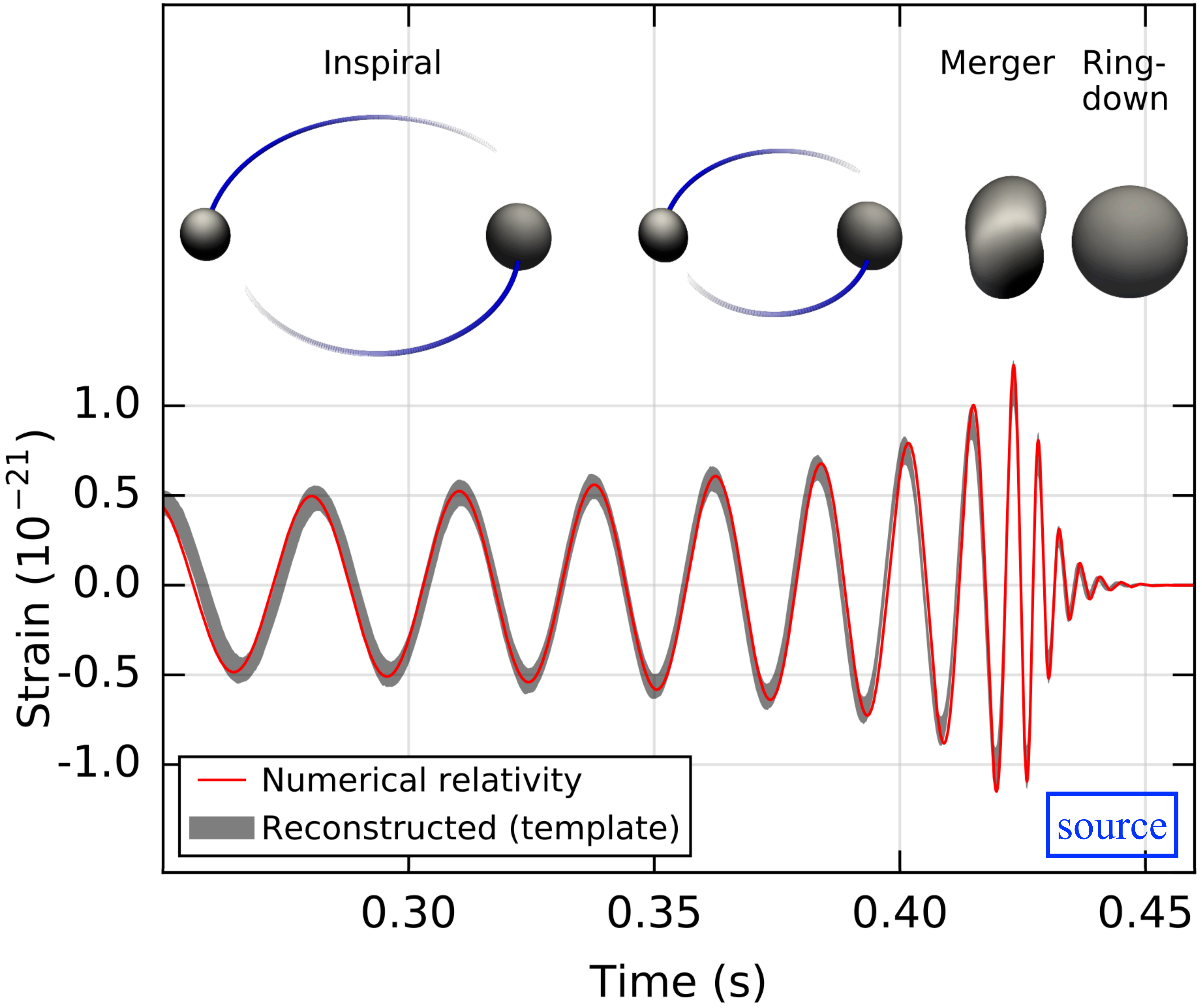
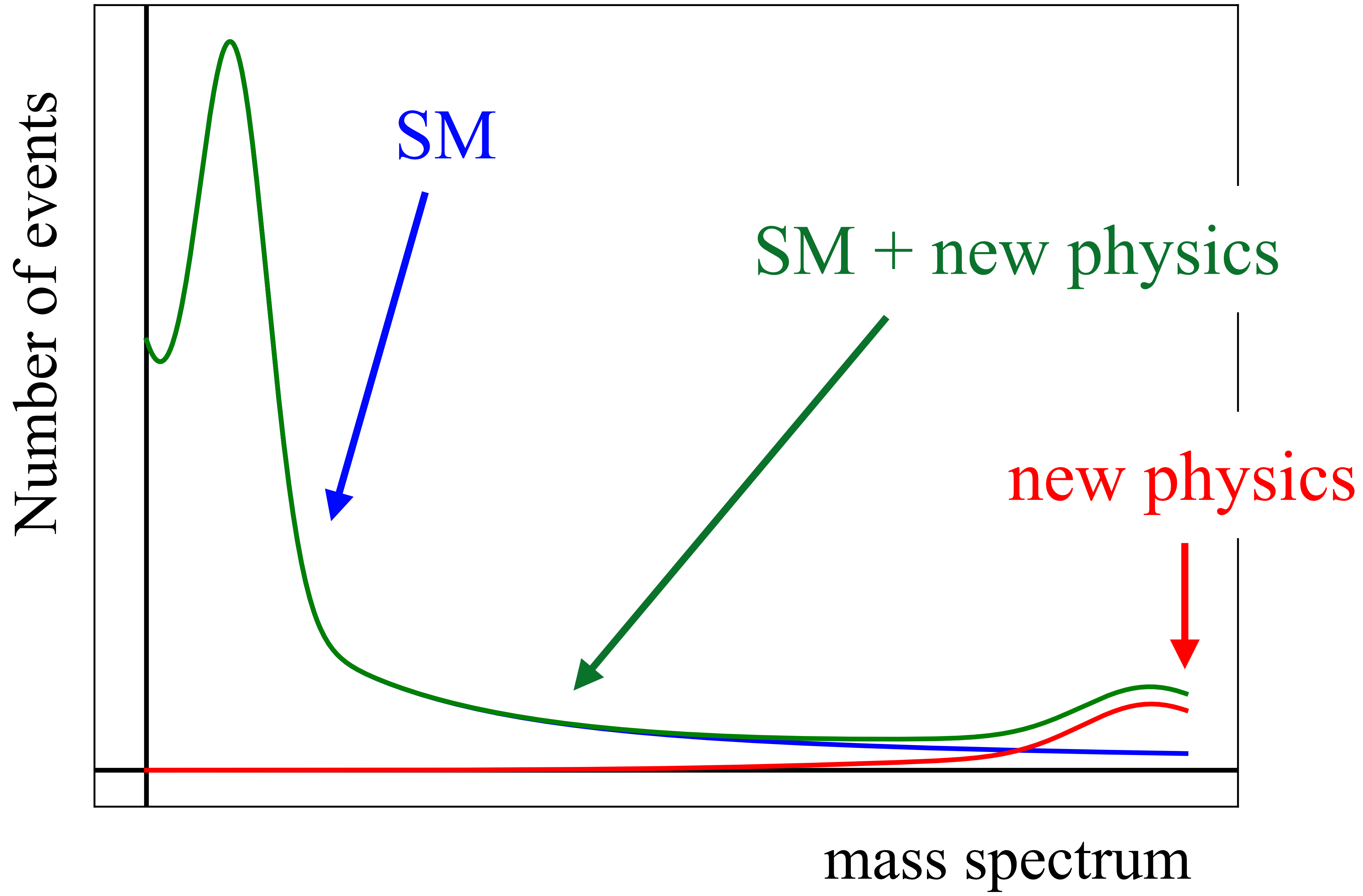


What do gravitational waves have to do with the Higgs bosons?

Effective field theory (EFT) - Idea



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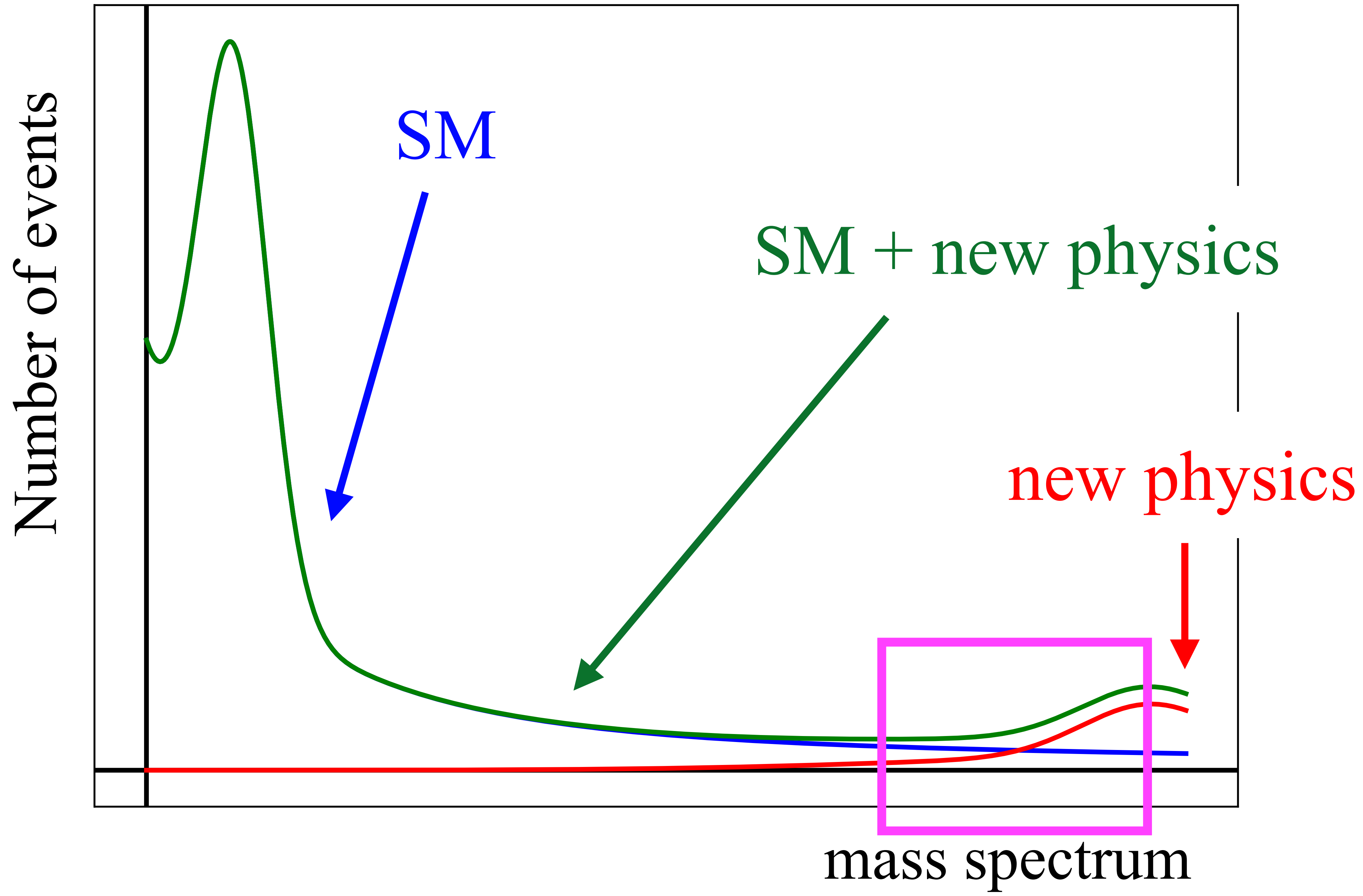

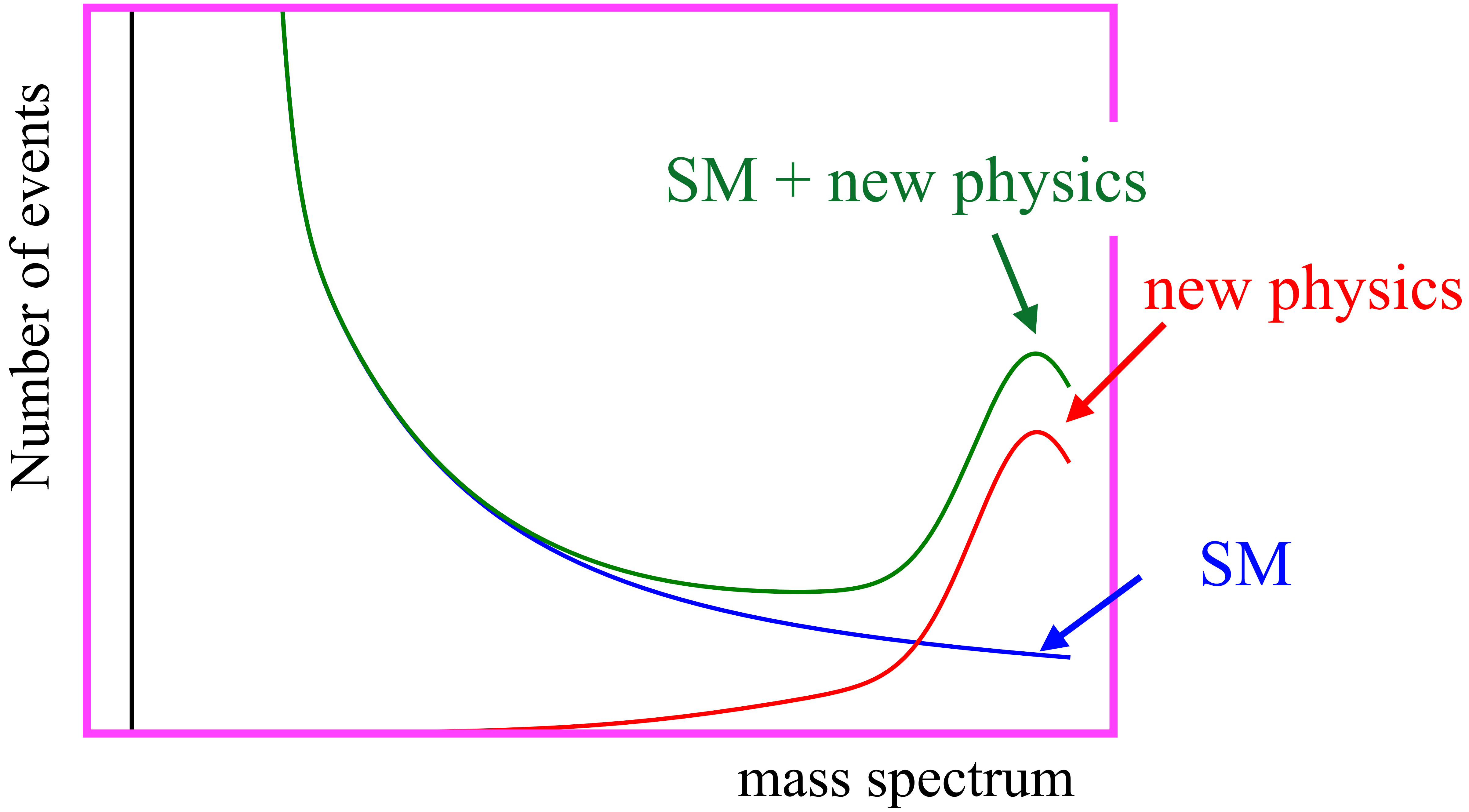

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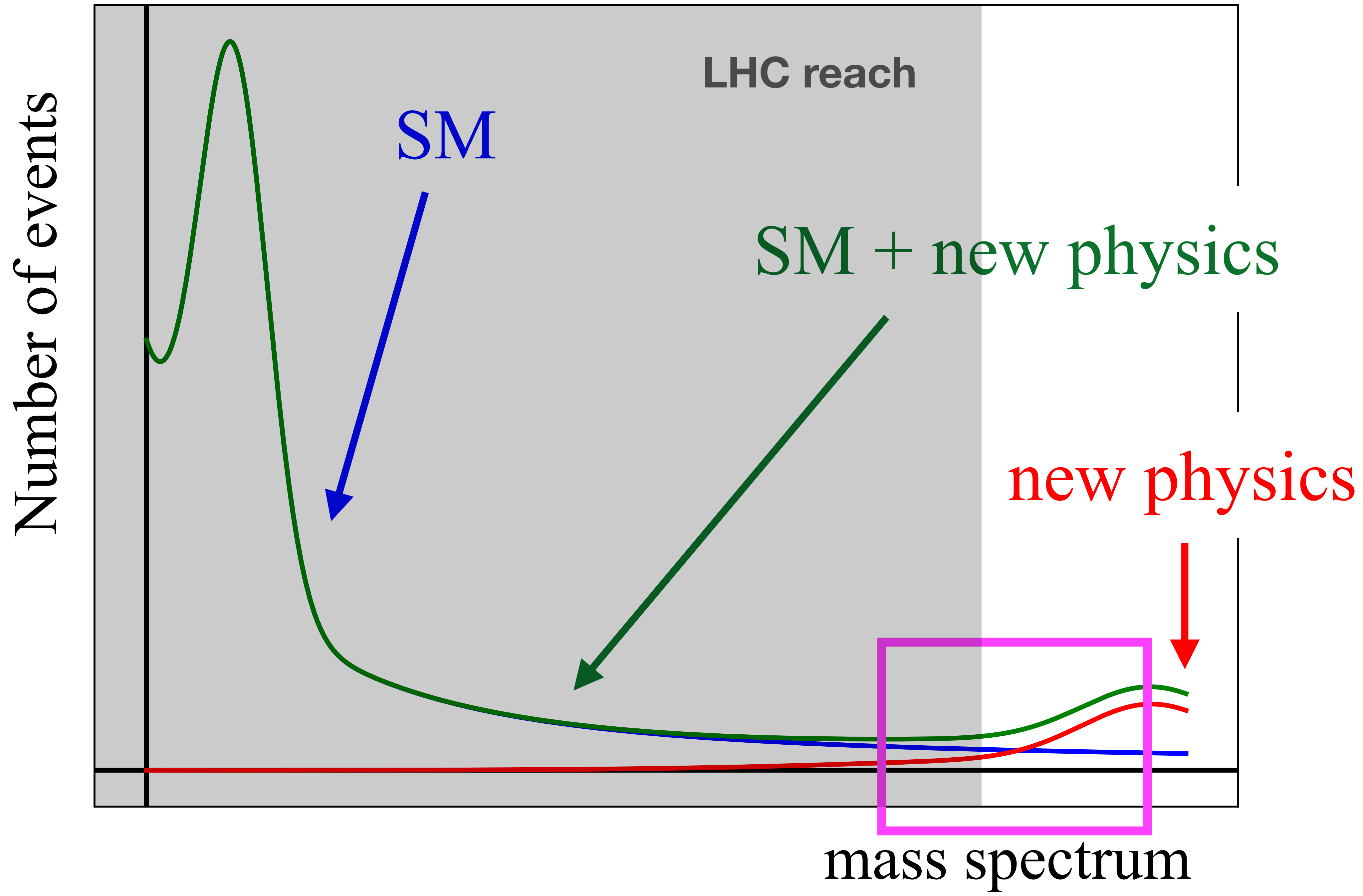

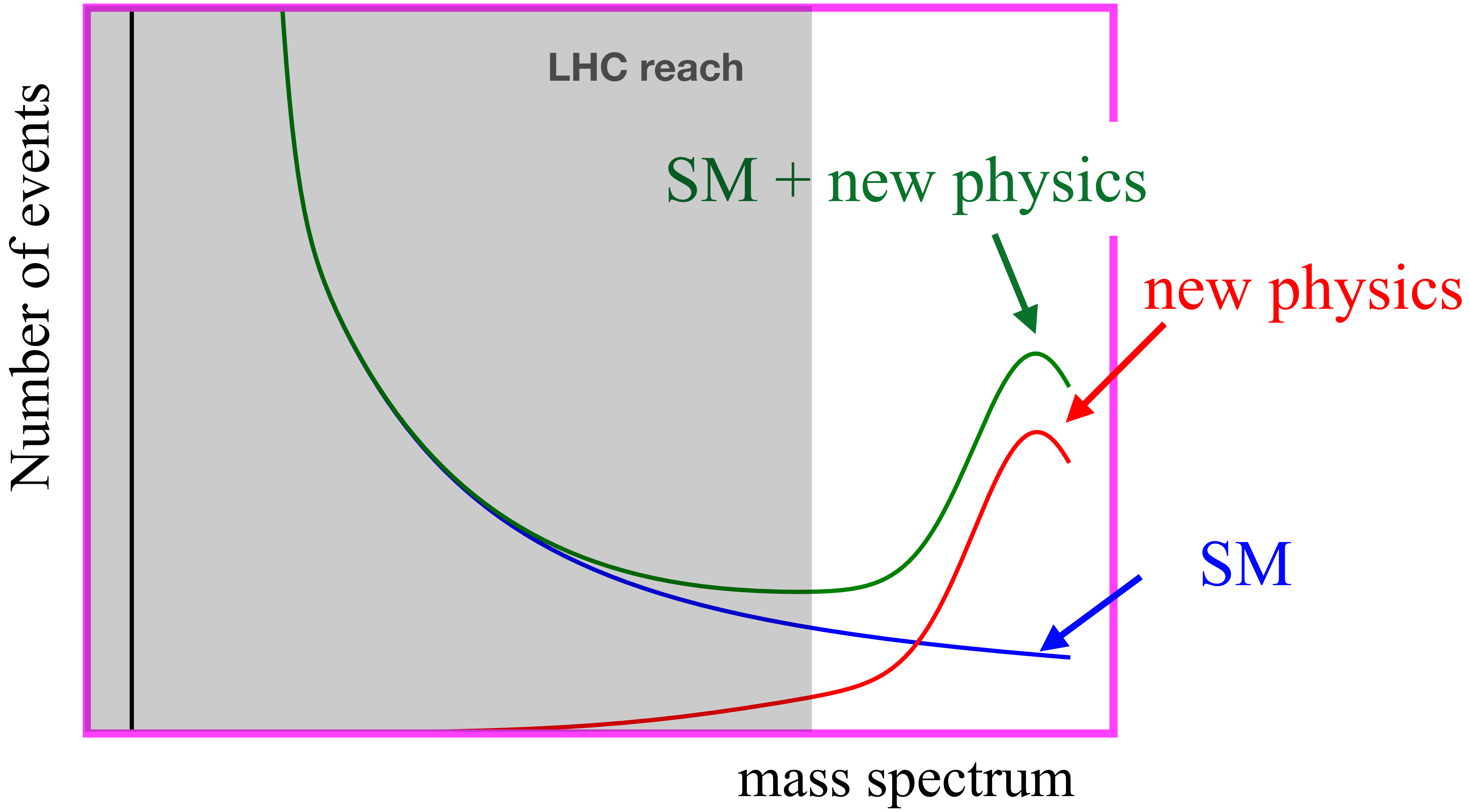

Effective field theory (EFT) - SMEFT lagrangian

Extend the Standard Model (SM) Lagrangian with higher orders:

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda}\mathscr{L}_5 + \frac{1}{\Lambda^2}\mathscr{L}_6 + \ldots$$

Scale of new physics

with $\mathscr{L}_i = \sum_j C_j O_j^i$

In many BSM models, Λ is interpreted as the mass of the new particle

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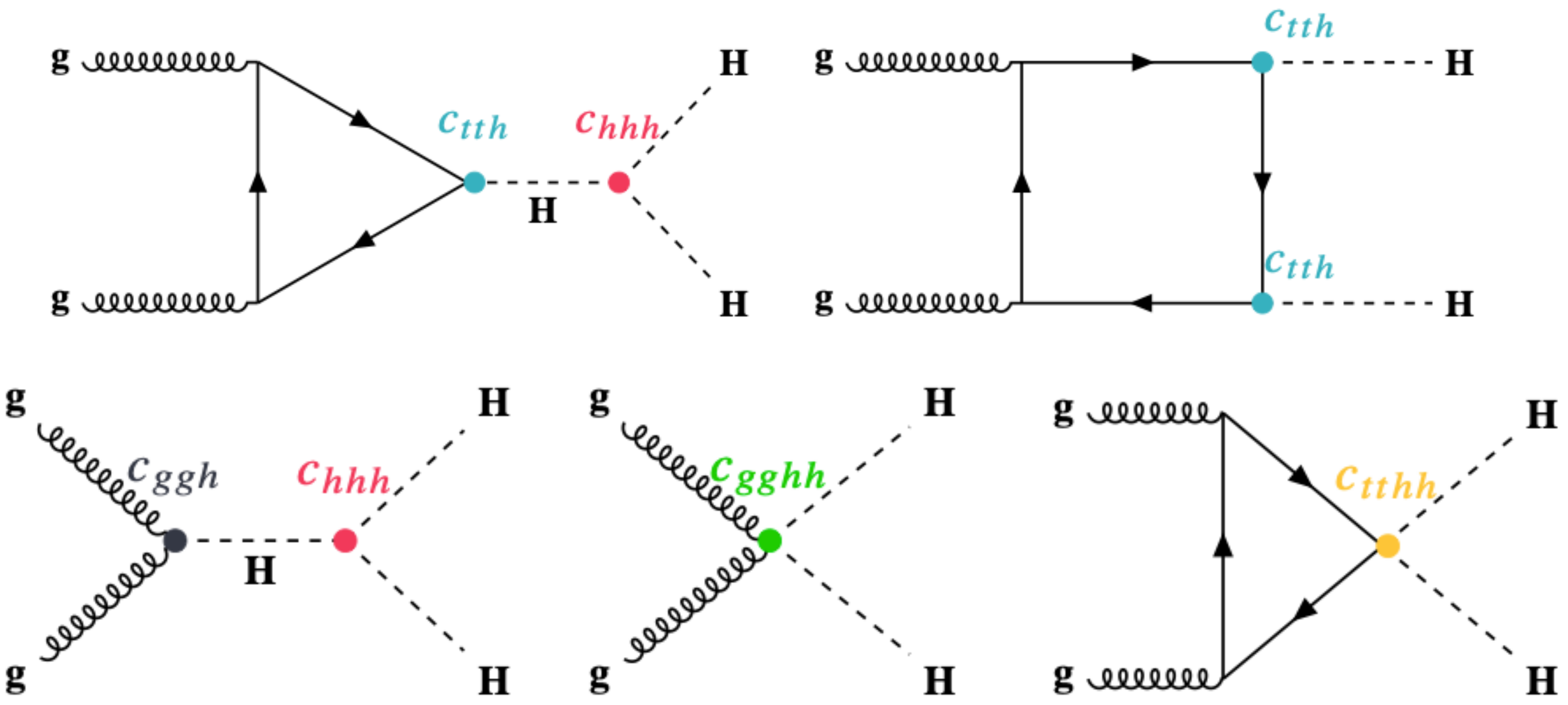
Operators

Wilson coefficients

Wilson coefficients

The Wilson coefficients are our free parameters in the EFT theories

They can be imaginged as follows

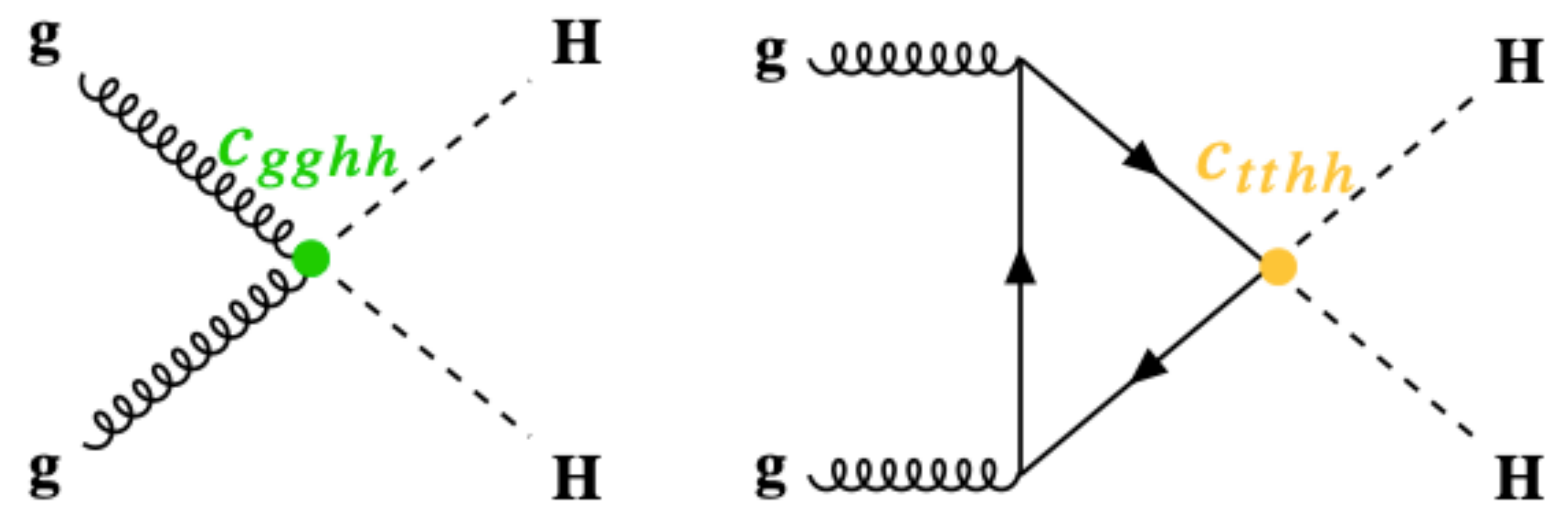


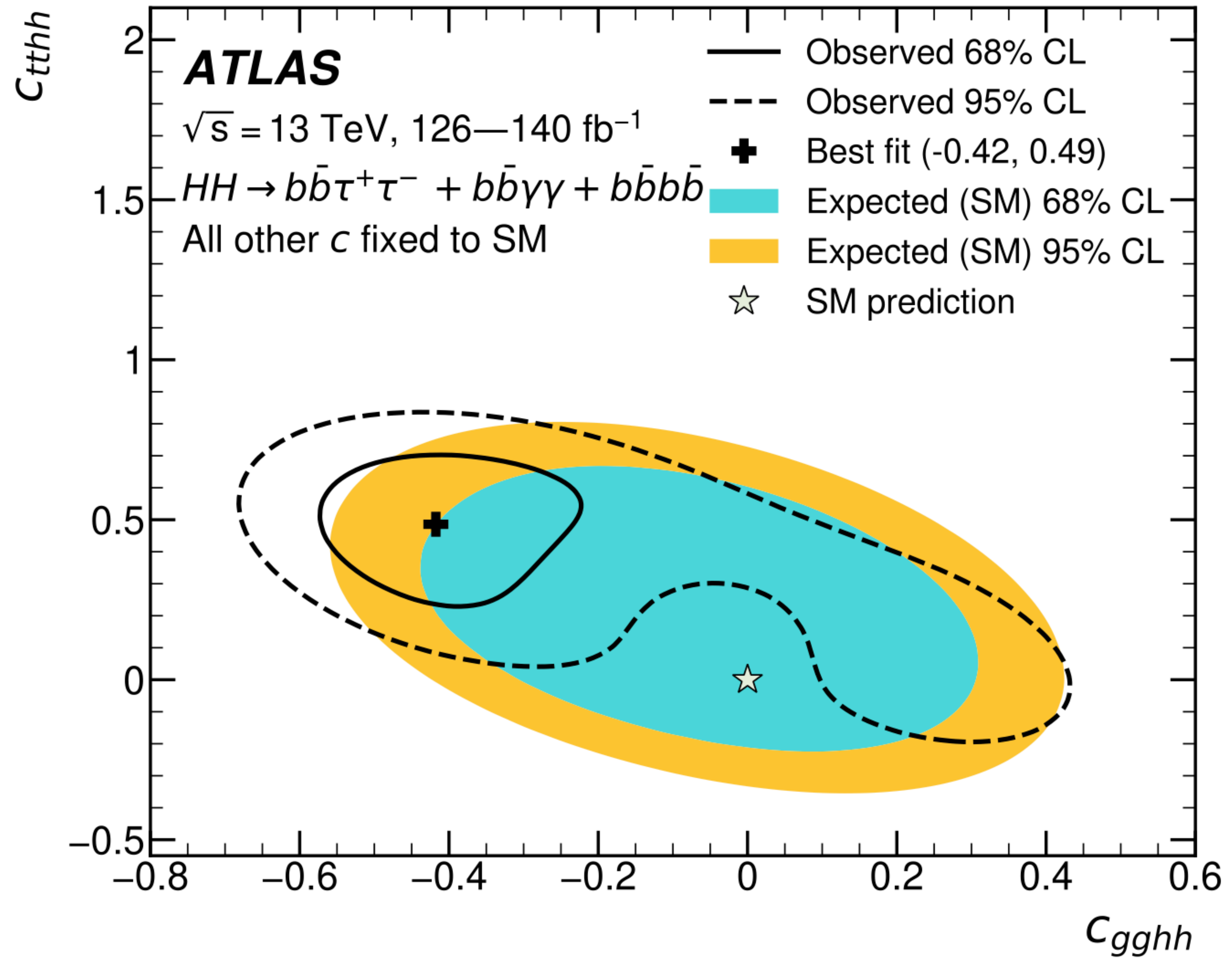
Standard Model is refound with the Wilson coefficients at zero.

So any significant deviation from zero is interesting.

paper

Wilson coefficients - constraints





Observed 95% C.L. limits:

$$-0.38 < c_{gghh} < 0.49$$

$$-0.19 < c_{tthh} < 0.70$$

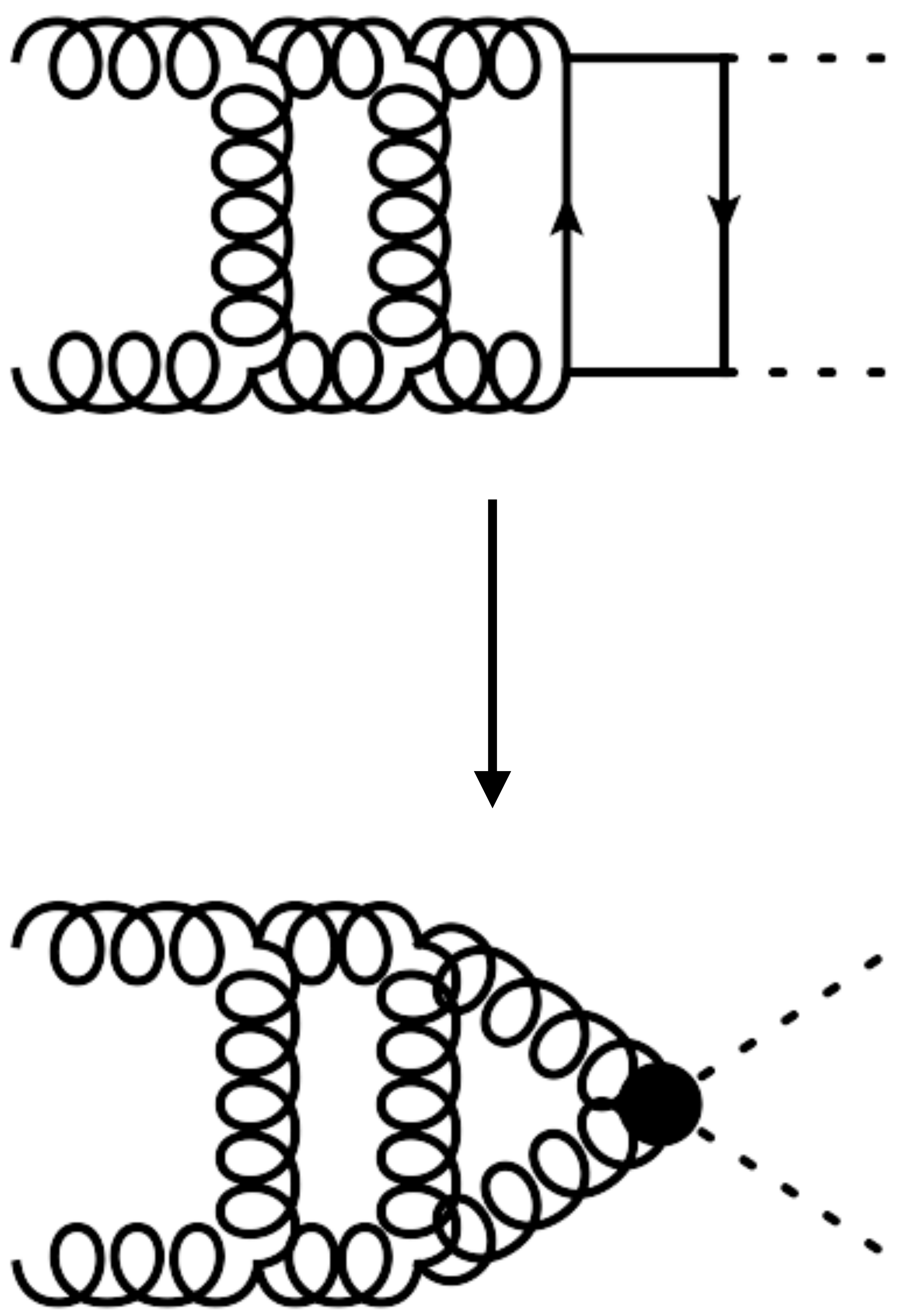
These represent the most stringent constraints to date on c_{gghh} and c_{tthh}

The results are compatible with the SM predictions

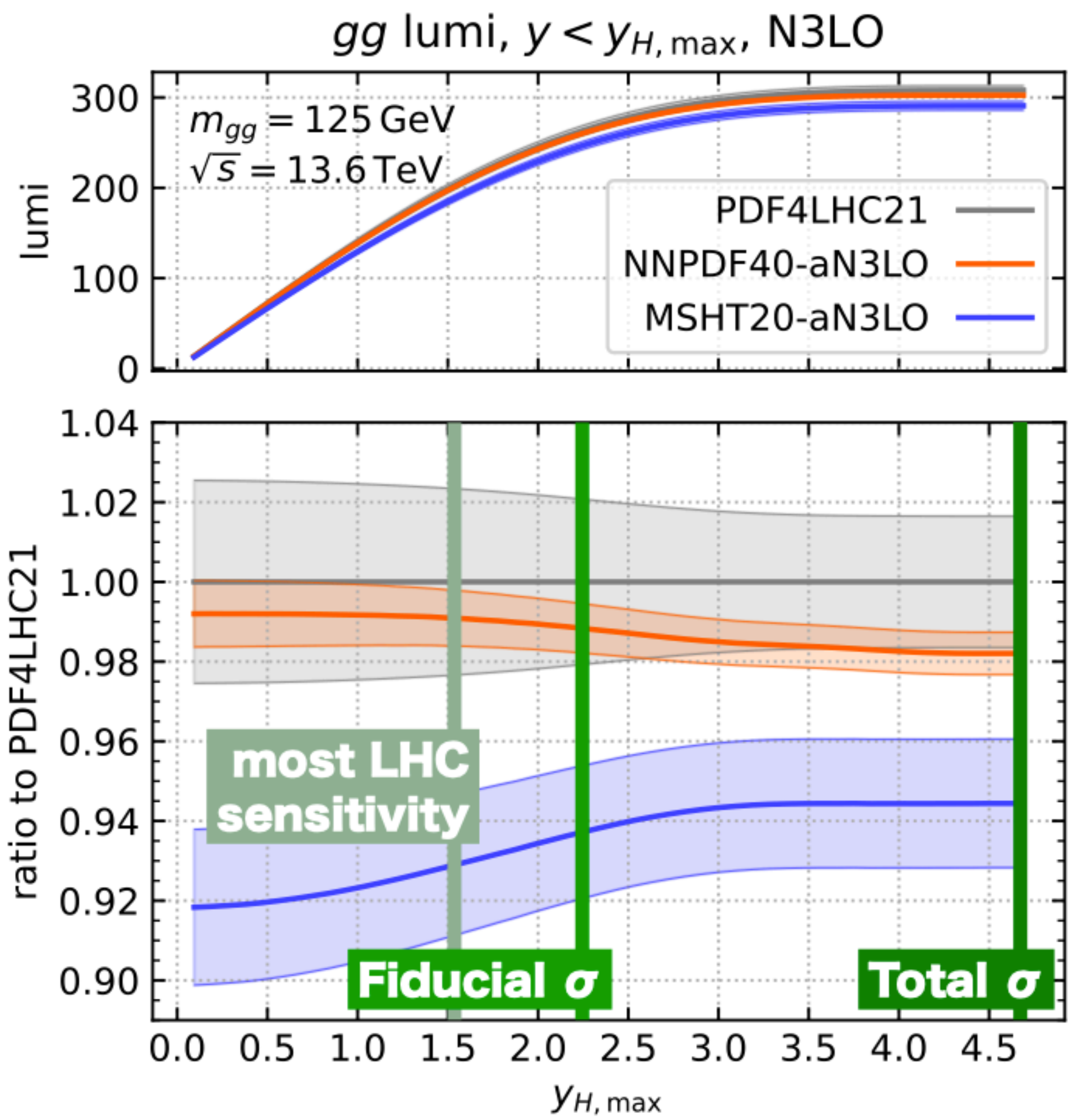
No new physics found yet

Heavy Top Limit

This is the approximation made to compute it more simply



Theroretical uncertainties - example: PDF issues



At NNLO: 1-2% difference between PDF4LHC

At N3LO: up to ~ 6%

Problematic for a measurement aiming to ~1% precision

Gavin Salam

arXiv:1910.00012

LO diagram interference

The effect of the trilinear Higgs self-coupling in the LO total cross section amounts to a reduction of about 50% with respect to the box-only contribution, due to the large destructive interference.

The QCD corrections increase the total cross section by about a factor of two with respect to the LO prediction,

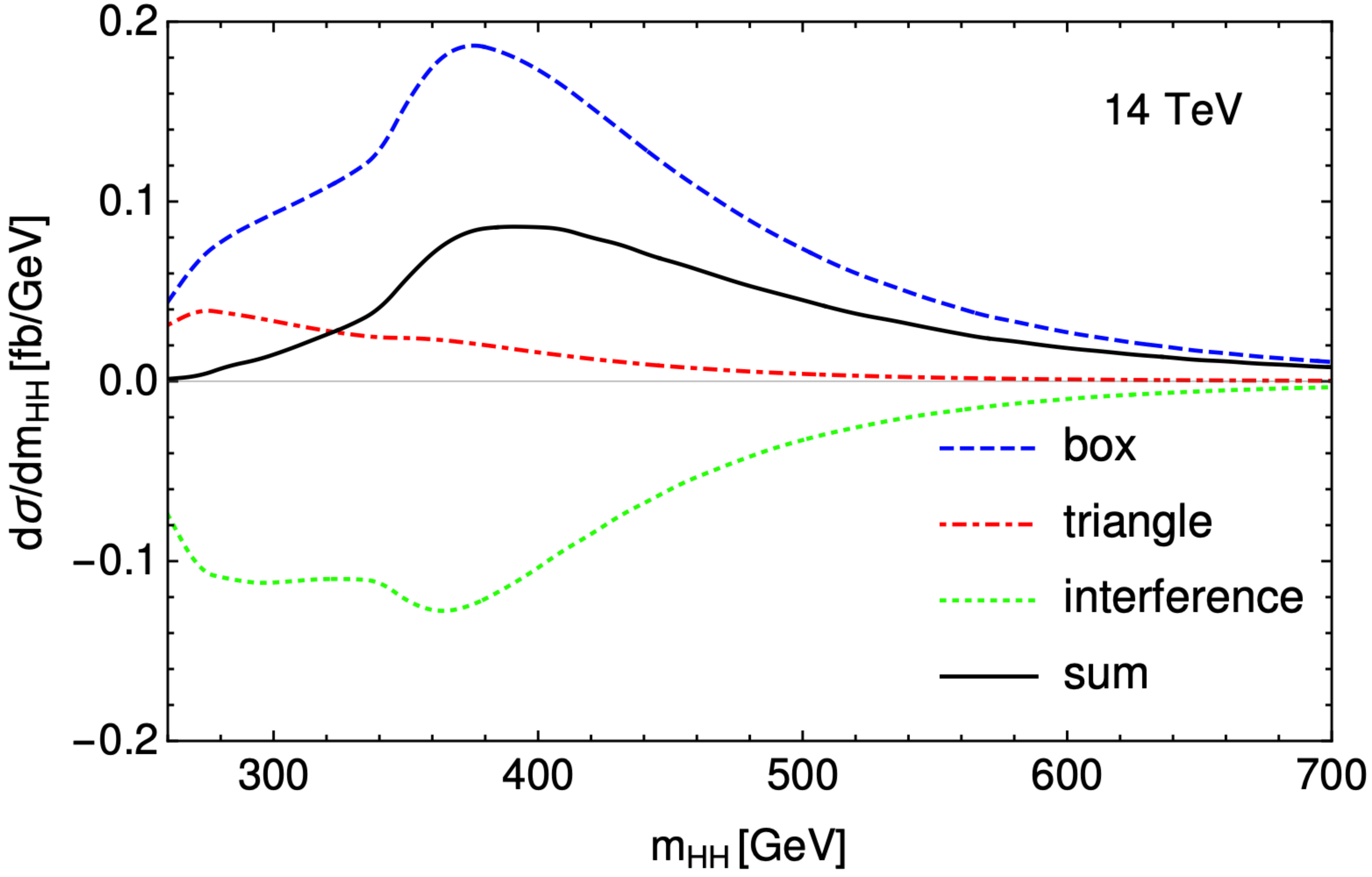
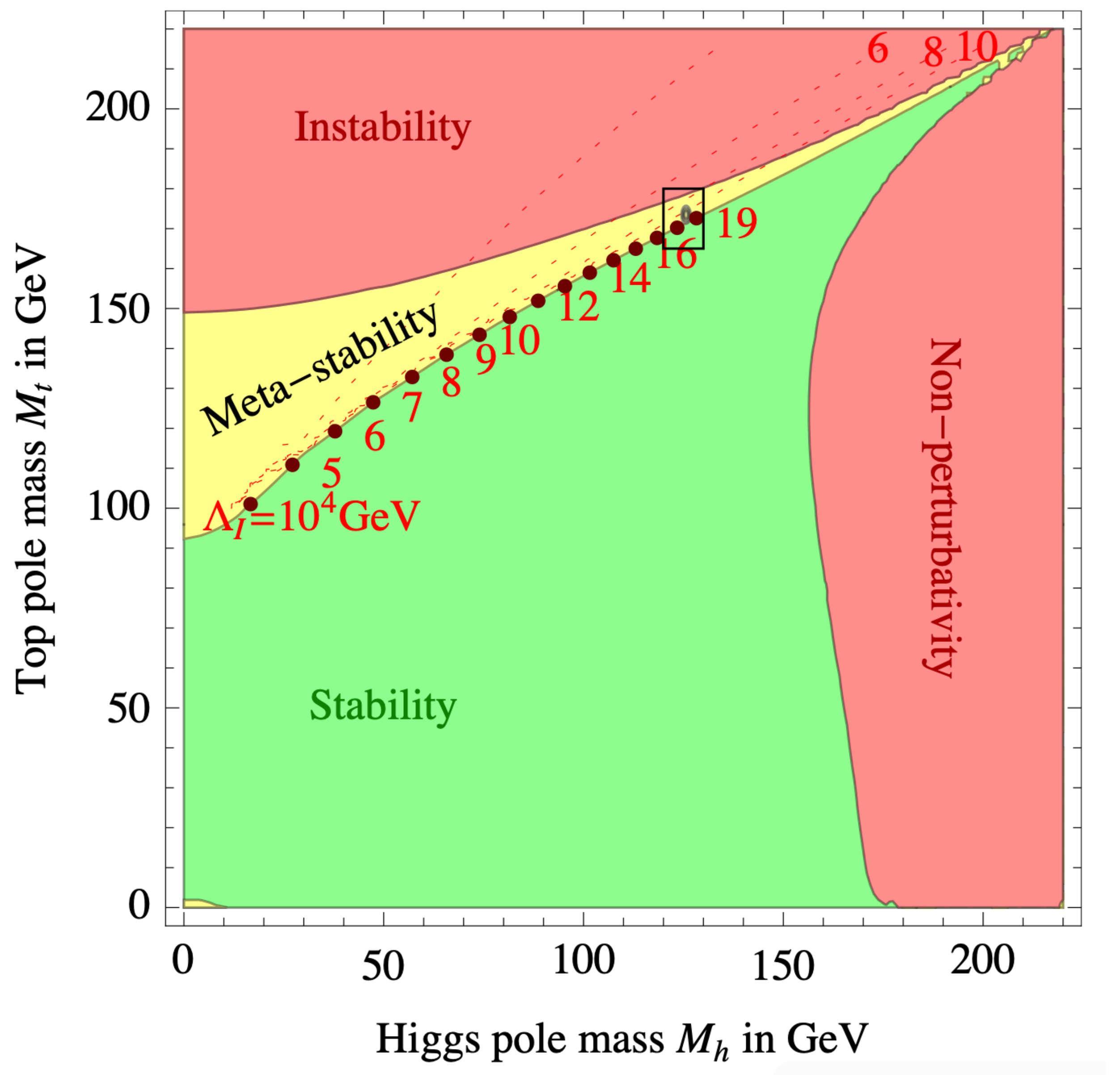
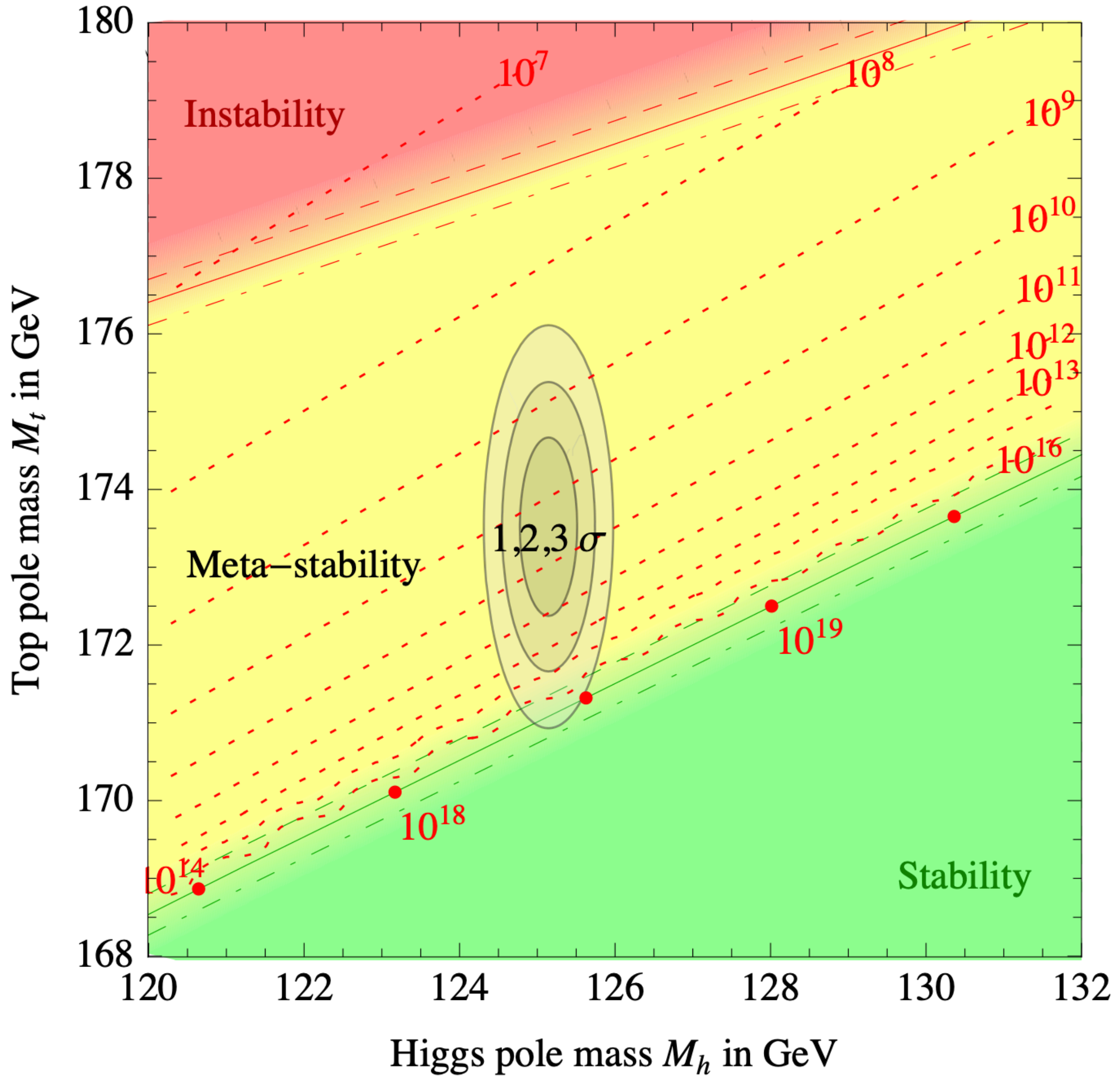


Figure 1.3: Higgs pair invariant mass distribution at leading order for the different contributions to the gluon fusion production mechanism and their interference.

Stability of the universe - mass plot





The uncertainty from α_s and from theoretical errors are indicated by the dashed lines and the colour shading along the borders.

Also shown are contour lines of the instability scale Λ_I

a network of contour lines labeled by the instability scale

Λ_I: the energy where λ_eff(μ)=0

From this result we conclude that vacuum stability of the SM up to the Planck scale is excluded at 2.8σ (99.8% C.L. one-sided). Since the main source of uncertainty in eq. (64) comes from M_t, any refinement in the measurement of the top mass is of great importance for the question of EW vacuum stability.

2. Quantum corrections: from $V(\phi)$ to $V_{\text{eff}}(\phi)$

In quantum field theory (QFT), particles interact via loops — virtual fluctuations that shift the energy levels.
So the true energy density for a given field configuration isn't just the classical $V(\phi)$, but the **effective potential** $V_{\text{eff}}(\phi)$, which includes **loop corrections**.

Mathematically:

$$V_{\text{eff}}(\phi) = V_{\text{tree}}(\phi) + \Delta V_{\text{1-loop}}(\phi) + \Delta V_{\text{2-loop}}(\phi) + \cdots$$

Each ΔV term accounts for higher-order quantum effects (Feynman diagrams with loops).

3. Why the effective potential matters

- The **vacuum state** of the universe is the field configuration that *minimizes* $V_{\text{eff}}(\phi)$.
- Quantum corrections can **shift or even create new minima** at large field values.
  For example, due to the heavy top quark, the Higgs quartic coupling $\lambda(\mu)$ can run negative at high energy scales.
- This means the SM potential may develop a **second, deeper minimum** at large ϕ, making our electroweak vacuum only **metastable**.

That's what Buttazzo et al. study: how quantum corrections (encoded in V_{eff}) change the vacuum structure.

4. The role of the renormalization group (RG)

The effective potential depends on the **renormalization scale** μ.

To make physical predictions stable under changes in μ, we use **renormalization group equations (RGEs)** to evolve the couplings $(\lambda, g, y_t, \ldots)$ with energy:

$$\frac{d\lambda}{d\ln\mu} = \beta_\lambda(\lambda, g, y_t, \ldots)$$

The **RG-improved effective potential** is then:

$$V_{\text{eff}}(h) \simeq \frac{1}{4}\lambda_{\text{eff}}(h)\, h^4$$

for large Higgs field values $h \gg v$.

This form shows that if $\lambda_{\text{eff}}(h) < 0$ at some scale h, the potential turns downward — indicating **vacuum instability**.

2. Quantum corrections make λ run with energy

Quantum loops (especially involving the **top quark**, W, Z, and the Higgs itself) modify how the parameters behave at different energy scales.

So λ becomes **scale-dependent** via **Renormalization Group Equations (RGEs)**:

$$\frac{d\lambda}{d\ln\mu} = \beta_\lambda(\lambda, g_i, y_t, \ldots)$$

Here:

- μ is the renormalization scale (the energy at which you probe the theory),
- g_i are the gauge couplings,
- y_t is the top Yukawa coupling.

This "running" of $\lambda(\mu)$ is computed from the SM's β-functions — typically up to **3-loop precision** in modern analyses like Buttazzo et al. (2013).

$$V_{\text{eff}}(h) = \lambda_{\text{eff}}(h)\frac{h^4}{4} \qquad h \gg v$$

6. How the instability scale links to lifetime qualitatively

Instability scale Λ_I	Behavior of λ_{eff}	Vacuum lifetime
$\Lambda_I > M_{\text{Pl}}$	No zero crossing → stable	Infinite (no decay)
$\Lambda_I \sim 10^{17}\,\text{GeV}$	Crosses just below Planck	Extremely long (> 10^{50}× age of universe)
$\Lambda_I \sim 10^{10-12}\,\text{GeV}$	Crosses moderately early	Still enormous lifetime (metastable)
$\Lambda_I \lesssim 10^{6}\,\text{GeV}$	Crosses low, λ very negative	Lifetime shorter than universe (unstable)

So the **lower** the instability scale, the **faster** the potential decay — because λ_eff turns negative sooner and more strongly.

⚙️ 2. "Perturbative" vs. "Non-perturbative"

In the SM, we compute the running of couplings (λ, g_i, y_t, ...) using **perturbation theory** — i.e. we expand observables in powers of small couplings:

$$\text{Observable} = \text{tree level} + c_1\, g^2 + c_2\, g^4 + \cdots$$

This works only when *all* the relevant couplings are small (< 1) so that higher-order terms are suppressed.

If a coupling grows too large, the perturbative expansion stops converging — the theory becomes **non-perturbative**, and our usual RGEs and loop calculations no longer give trustworthy results.

🔺 3. What causes that in the Higgs sector

For very large Higgs masses M_h, the **quartic coupling** λ must also be large, because:

$$\lambda(v) = \frac{m_h^2}{2v^2}$$

So if M_h is, say, 500 GeV, then $\lambda(v) \sim 0.5$; if $M_h = 1\mathrm{TeV}$, $\lambda(v) \sim 8$ — enormous.

Then, as we run $\lambda(\mu)$ to higher scales using the RG equations, it increases even more and can **diverge** at a finite energy scale — the so-called **Landau pole**.

That means:

- The perturbative calculation breaks down.
- We lose predictive control before reaching the Planck scale.
- The SM cannot be extrapolated reliably there.

📈 5. The big picture

Region	Behavior of λₑff(μ)	Interpretation
Stable	λₑff > 0, small, up to Planck	SM vacuum absolutely stable
Metastable	λₑff crosses 0 slightly, stays small	Long-lived false vacuum
Unstable	λₑff < 0, sizable	Vacuum would have decayed
Non-perturbative	λₑff ≫ 1 or diverges (Landau pole)	SM loses predictivity; need new physics

🪐 4. What it tells us about our universe

Using the measured values
$M_h \simeq 125$ GeV, $M_t \simeq 173$ GeV, and $\alpha_s(M_Z)$,
the authors find:

$$\Lambda_I \approx 10^{10\text{–}11}\,\mathrm{GeV},$$

and the vacuum lifetime $\tau \gg t_{\mathrm{Universe}}$.

So **our universe sits in the metastable region**, extremely close to the boundary with absolute stability — what they call *near-criticality*.

Plank scale

Quantity	Definition	Numerical Value
Planck mass	$\sqrt{\hbar c/G}$	$\sim 2.2 \times 10^{-8}\ \mathrm{kg}$
Planck energy	$M_{\mathrm{Pl}}c^2$	$\sim 1.22 \times 10^{19}\ \mathrm{GeV}$
Planck length	$\sqrt{\hbar G/c^3}$	$\sim 1.616 \times 10^{-35}\ \mathrm{m}$
Planck time	l_{Pl}/c	$\sim 5.39 \times 10^{-44}\ \mathrm{s}$

- **Connection:**

$$E_{\mathrm{Pl}} \sim M_{\mathrm{Pl}} \sim \frac{1}{l_{\mathrm{Pl}}} \sim \frac{1}{t_{\mathrm{Pl}}}$$

1. What they did in Figure 3

In **Figure 3**, the phase diagram is drawn in terms of the **low-energy, measured parameters**:

- Higgs mass M_h
- Top quark mass M_t

These are **pole masses** measured near the electroweak scale (~ 100 GeV).

- From these masses, you **infer the low-energy Higgs quartic coupling** $\lambda(v)$ and **top Yukawa** $y_t(v)$.
- Then you use **RGEs** to run them up to high energies ($\mu \sim 10^{10-19}$ GeV) to see if the vacuum is stable, metastable, or unstable.

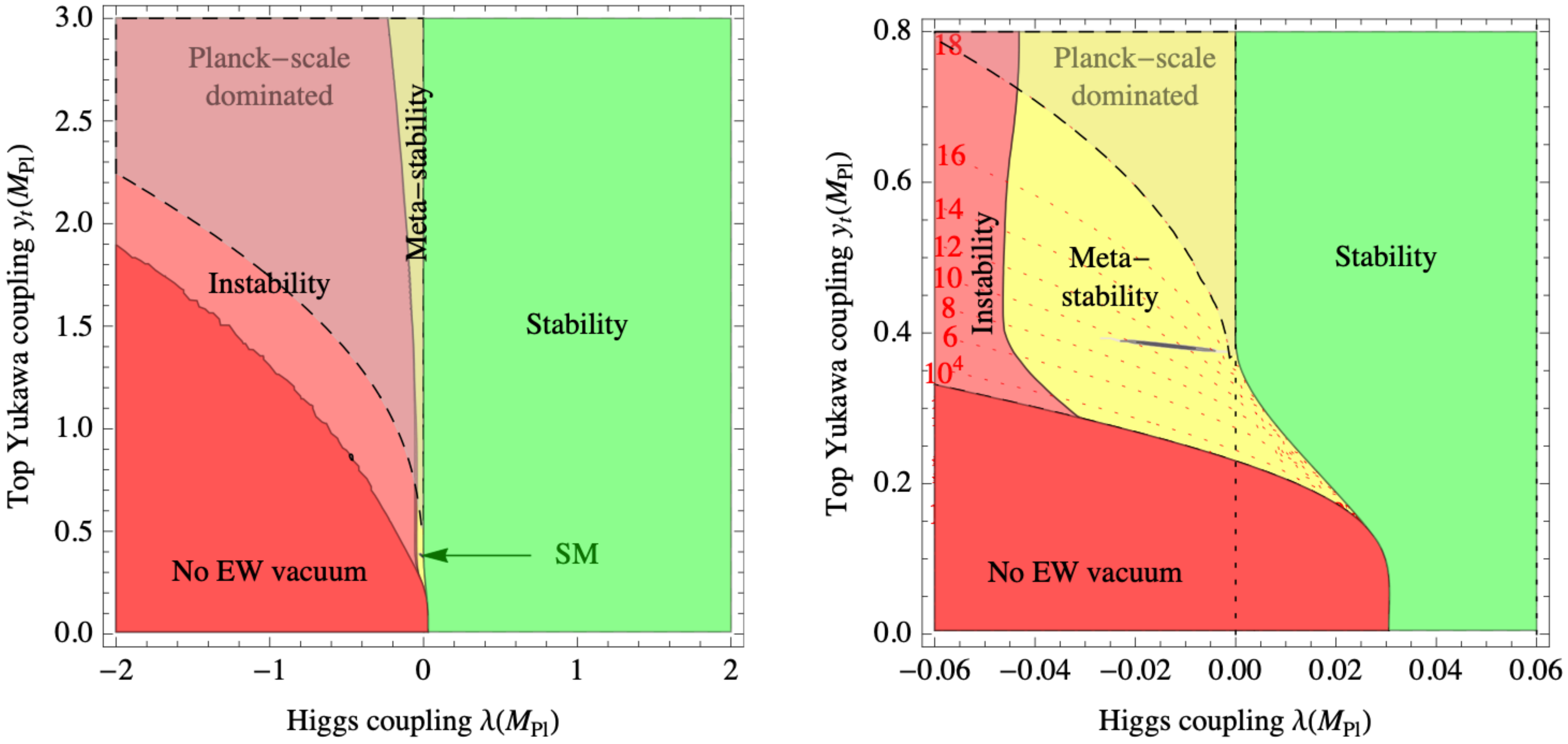
2. The idea behind Figure 4

In Figure 4, the authors **reparametrize the phase diagram** using **the values of the couplings themselves at a high-energy scale**:

$$\lambda(\mu_{\text{high}}), \quad y_t(\mu_{\text{high}})$$

- Typically, $\mu_{\text{high}} \sim 10^{17-18}$ GeV, just below the Planck scale.
- This is **where the potential is near its critical behavior** — i.e., near the scale where λ_{eff} could cross zero.

Stability - coupling plot



The area denoted as 'no EW vacuum' corresponds to a situation in which λ is negative at the weak scale, and therefore the usual Higgs vacuum does not exist.

In the region denoted as 'Planck-scale dominated' the instability scale ΛI is larger than 10^18 GeV.

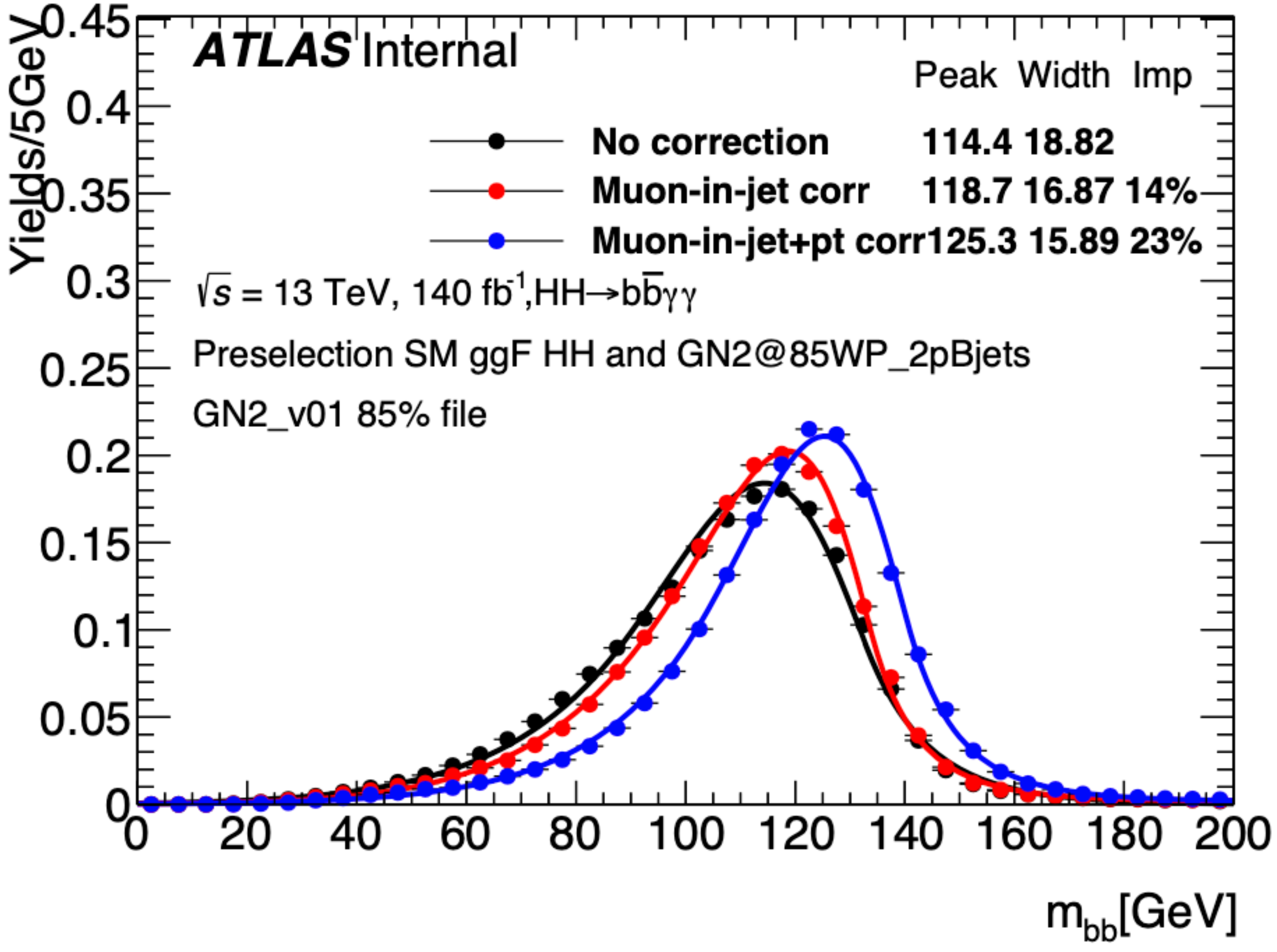
In this situation we expect that both the Higgs potential and the tunnelling rate receive large gravitational corrections and any assessment about vacuum stability becomes unreliable.

BJetCalibration correction

Improve the m_{bb} resolution by correction for

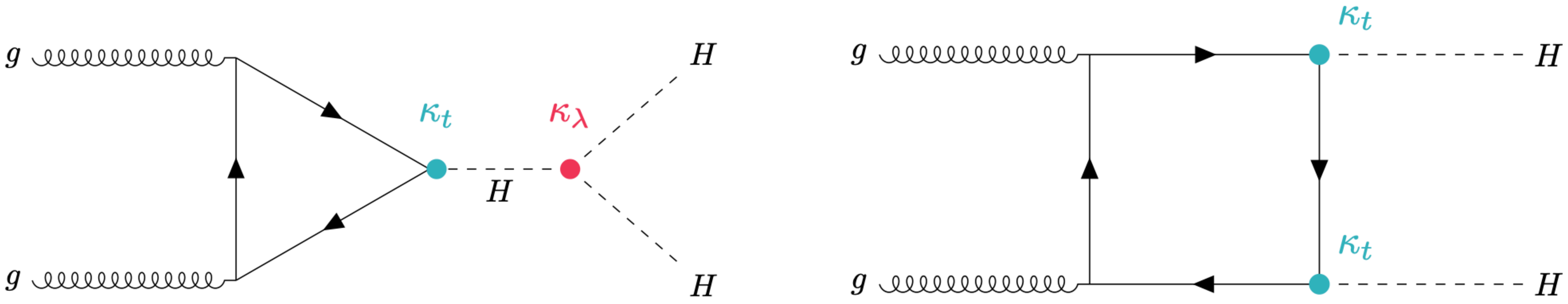
- muons escaping the cone
- neutrinos
- other out-of-cone effects

Improve by ~ 23% the m_{bb} resolution



κ_λ and κ_{2V}

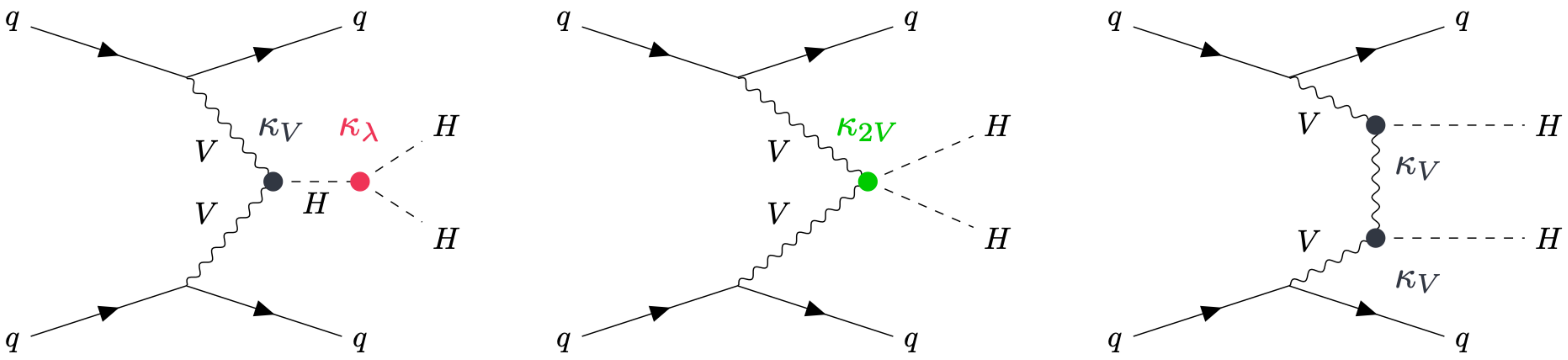
ggF production



(a) Trilinear coupling

(b) Box diagram

VBF production



(a) Trilinear coupling

(b) $HHVV$ vertex

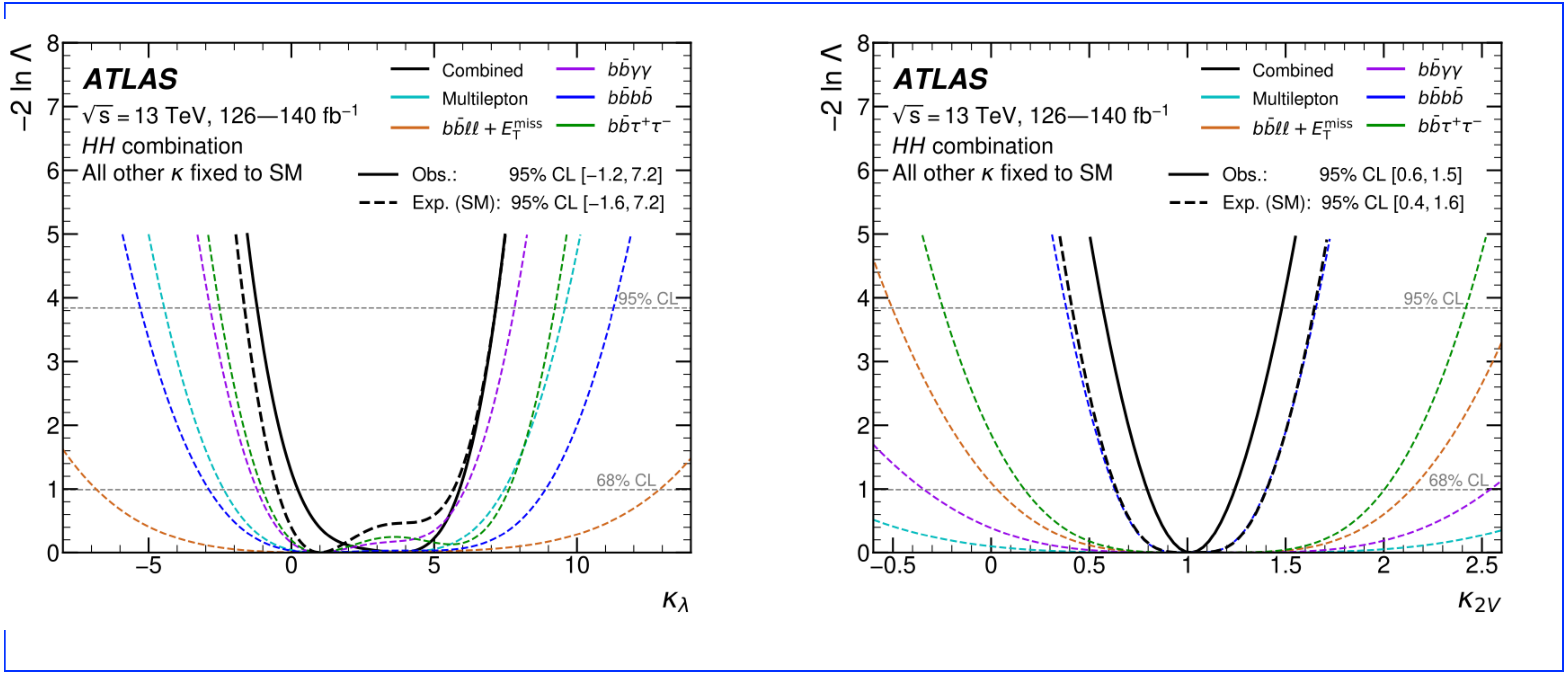
(c) VVH Production mode

Systematic uncertainties

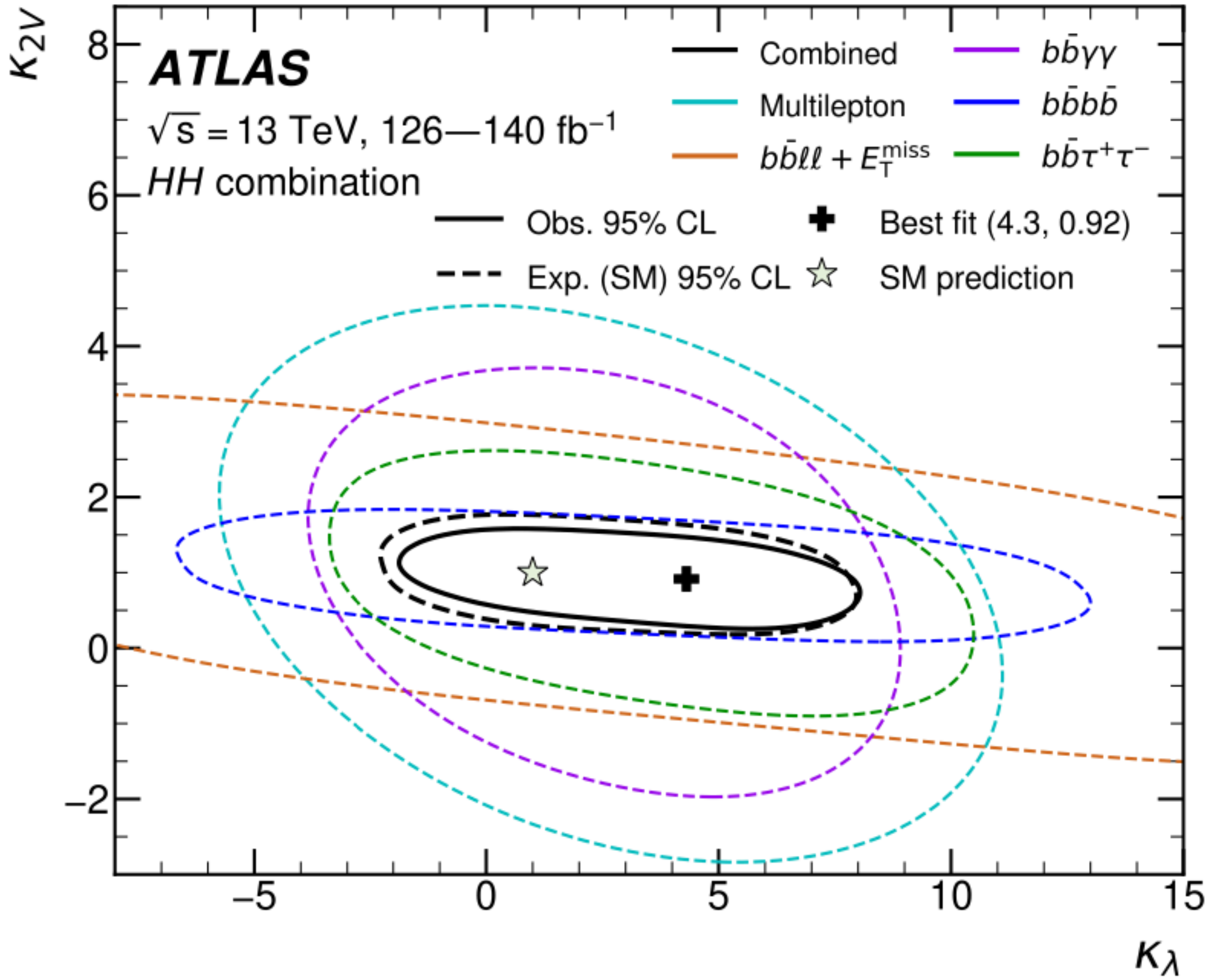
Systematic uncertainties affect the shape and normalisation of the diphoton invariant mass distributions of the Higgs boson pair signal and single Higgs boson backgrounds. Nevertheless, due to the limited number of events and the small signal-to-background ratio, the impact of the systematic uncertainties is small compared with that of the statistical uncertainties.

Systematic uncertainty source	Relative impact [%]
Experimental	
Photon energy resolution	0.4
Photon energy scale	0.1
Flavour tagging	0.1
Theoretical	
Factorisation and renormalisation scale	4.8
$\mathcal{B}(H \to \gamma\gamma, b\bar{b})$	0.2
Parton showering model	0.2
Heavy-flavour content	0.1
Background model (spurious signal)	0.1

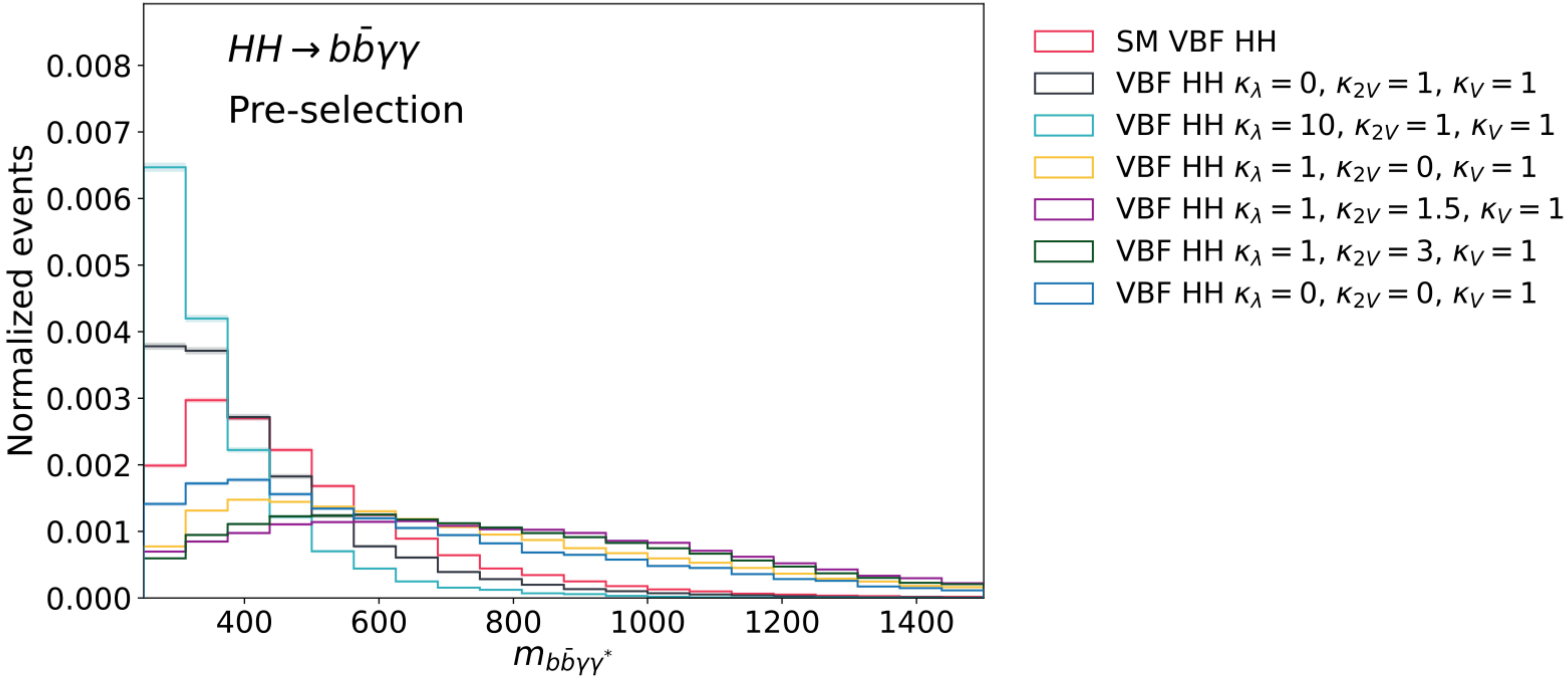
Complementarity of bbyy and 4b



Complementarity of bbyy and 4b



VBF parameter dependency

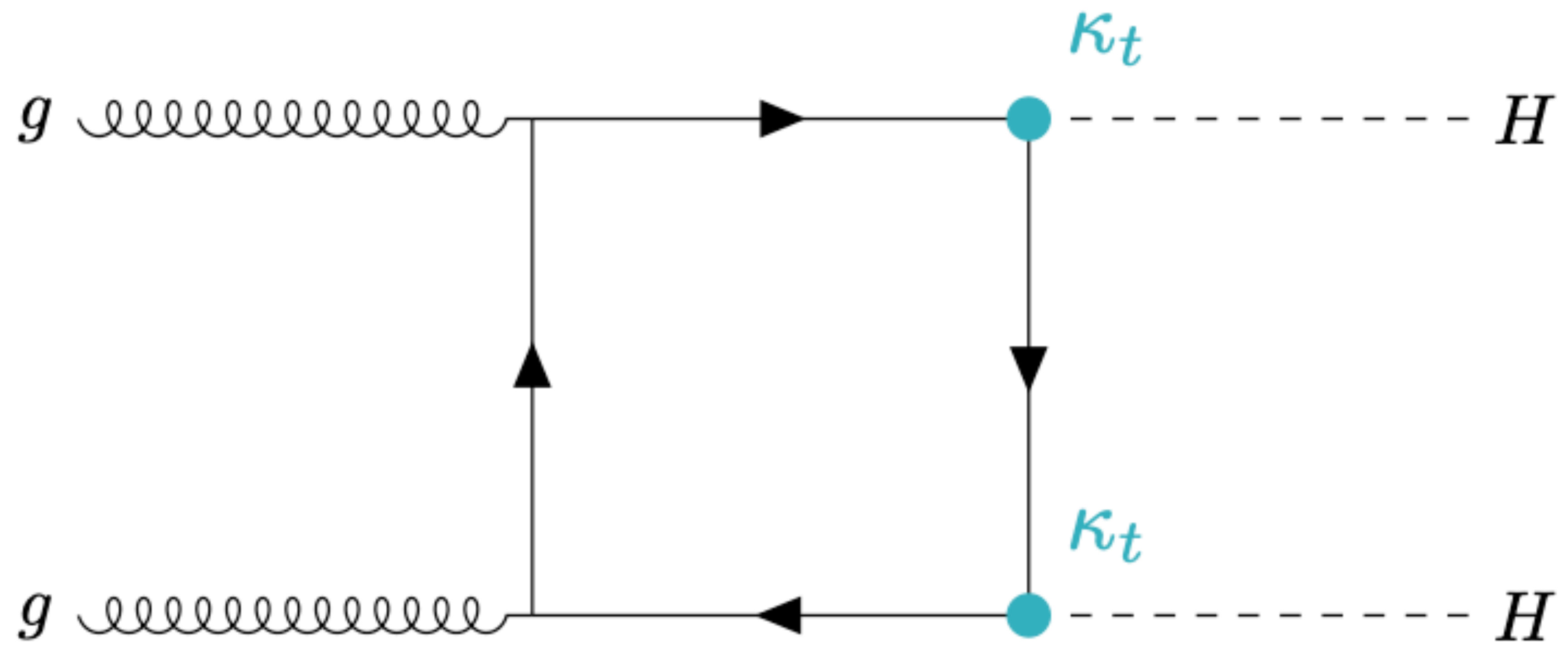


Parametrisations

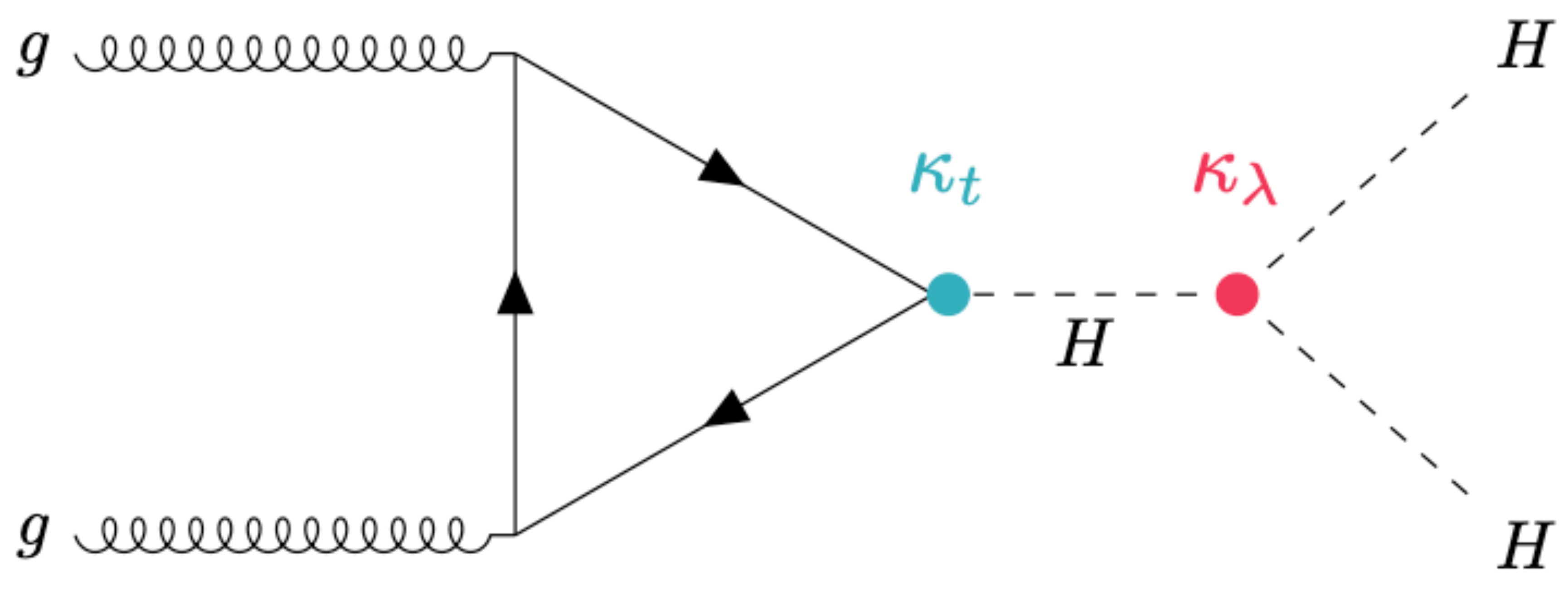
$$\frac{d\sigma_{ggF}}{d\Phi}(\kappa_\lambda) = \kappa_\lambda^2 \cdot \left[\frac{1}{380} \cdot \left(19 \cdot \frac{d\sigma_{ggF}}{d\Phi}(0) - 20 \cdot \frac{d\sigma_{ggF}}{d\Phi}(1) + \frac{d\sigma_{ggF}}{d\Phi}(20)\right)\right]$$
$$+ \kappa_\lambda \cdot \left[\frac{1}{380} \cdot \left(-399 \cdot \frac{d\sigma_{ggF}}{d\Phi}(0) + 400 \cdot \frac{d\sigma_{ggF}}{d\Phi}(1) - \frac{d\sigma_{ggF}}{d\Phi}(20)\right)\right]$$
$$+ \frac{d\sigma_{ggF}}{d\Phi}(0)$$

$$\frac{d\sigma_{VBF}}{d\Phi}(\kappa_\lambda, \kappa_{2V}, \kappa_V) = \left(\kappa_{2V}^2 - \frac{373\kappa_{2V}\kappa_\lambda\kappa_V}{594} - \frac{1150\kappa_{2V}\kappa_V^2}{297} - \frac{\kappa_\lambda^2\kappa_V^2}{9} + \frac{1033\kappa_\lambda\kappa_V^3}{594} + \frac{853\kappa_V^4}{297}\right) \cdot \frac{d\sigma_{VBF}}{d\Phi}(1, 1, 1)$$
$$+ \left(\frac{2\kappa_{2V}\kappa_\lambda\kappa_V}{3} - \frac{2\kappa_{2V}\kappa_V^2}{3} + \frac{\kappa_\lambda^2\kappa_V^2}{10} - \frac{53\kappa_\lambda\kappa_V^3}{30} + \frac{5\kappa_V^4}{3}\right) \cdot \frac{d\sigma_{VBF}}{d\Phi}(0, 1, 1)$$
$$+ \left(\frac{\kappa_{2V}\kappa_\lambda\kappa_V}{27} - \frac{\kappa_{2V}\kappa_V^2}{27} + \frac{\kappa_\lambda^2\kappa_V^2}{90} - \frac{13\kappa_\lambda\kappa_V^3}{270} + \frac{\kappa_V^4}{27}\right) \cdot \frac{d\sigma_{VBF}}{d\Phi}(10, 1, 1)$$
$$+ \left(-\frac{4\kappa_{2V}^2}{3} - \frac{4\kappa_{2V}\kappa_\lambda\kappa_V}{33} + \frac{60\kappa_{2V}\kappa_V^2}{11} + \frac{4\kappa_\lambda\kappa_V^3}{33} - \frac{136\kappa_V^4}{33}\right) \cdot \frac{d\sigma_{VBF}}{d\Phi}(1, 1.5, 1)$$
$$+ \left(\frac{\kappa_{2V}^2}{3} + \frac{5\kappa_{2V}\kappa_\lambda\kappa_V}{66} - \frac{10\kappa_{2V}\kappa_V^2}{11} - \frac{5\kappa_\lambda\kappa_V^3}{66} + \frac{19\kappa_V^4}{33}\right) \cdot \frac{d\sigma_{VBF}}{d\Phi}(1, 3, 1)$$
$$+ \left(-\frac{16\kappa_{2V}\kappa_\lambda\kappa_V}{33} + \frac{16\kappa_{2V}\kappa_V^2}{33} + \frac{16\kappa_\lambda\kappa_V^3}{33} - \frac{16\kappa_V^4}{33}\right) \cdot \frac{d\sigma_{VBF}}{d\Phi}(-5, 1, 0.5)$$

The non-resonant $bb\gamma\gamma$ channel



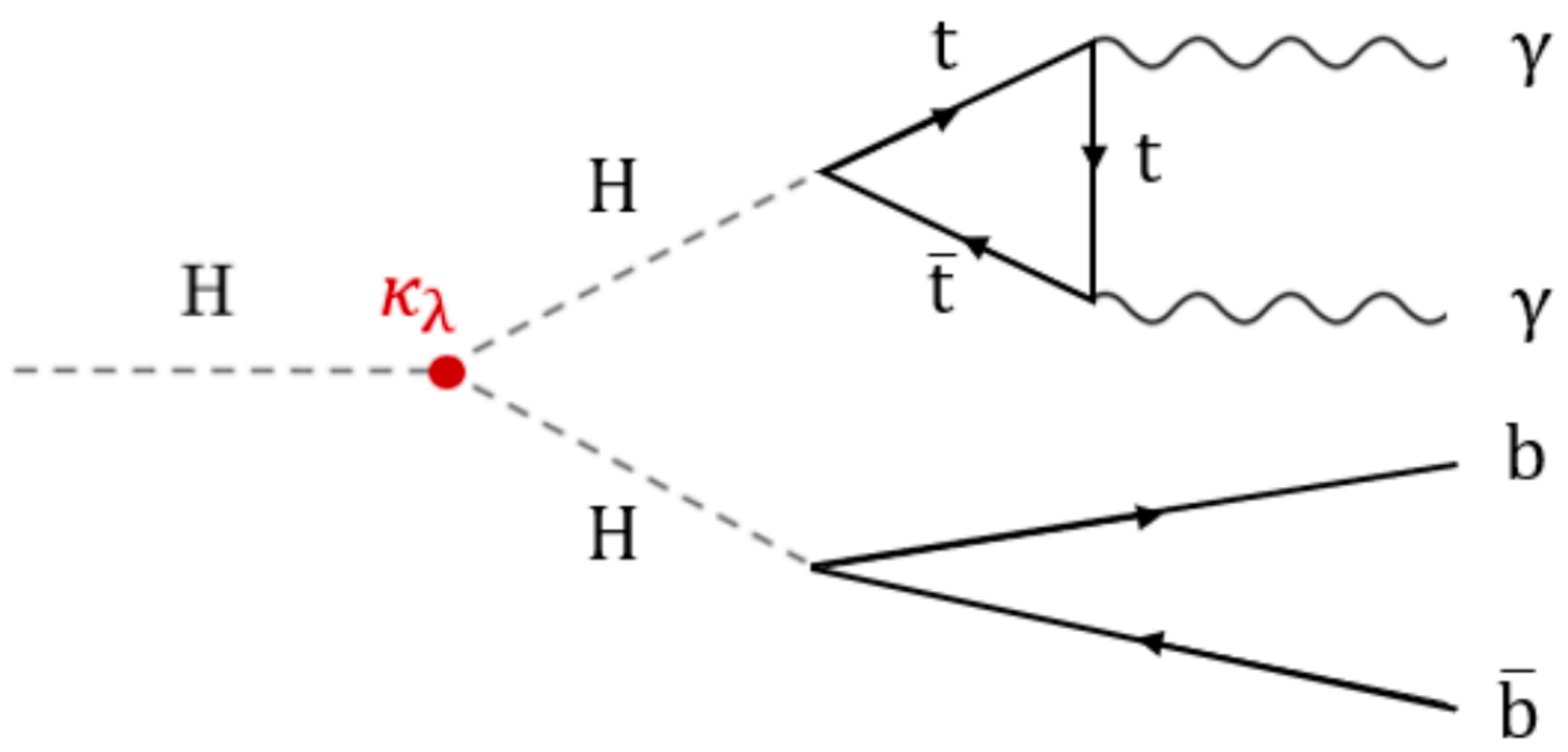
(a) Box Diagram

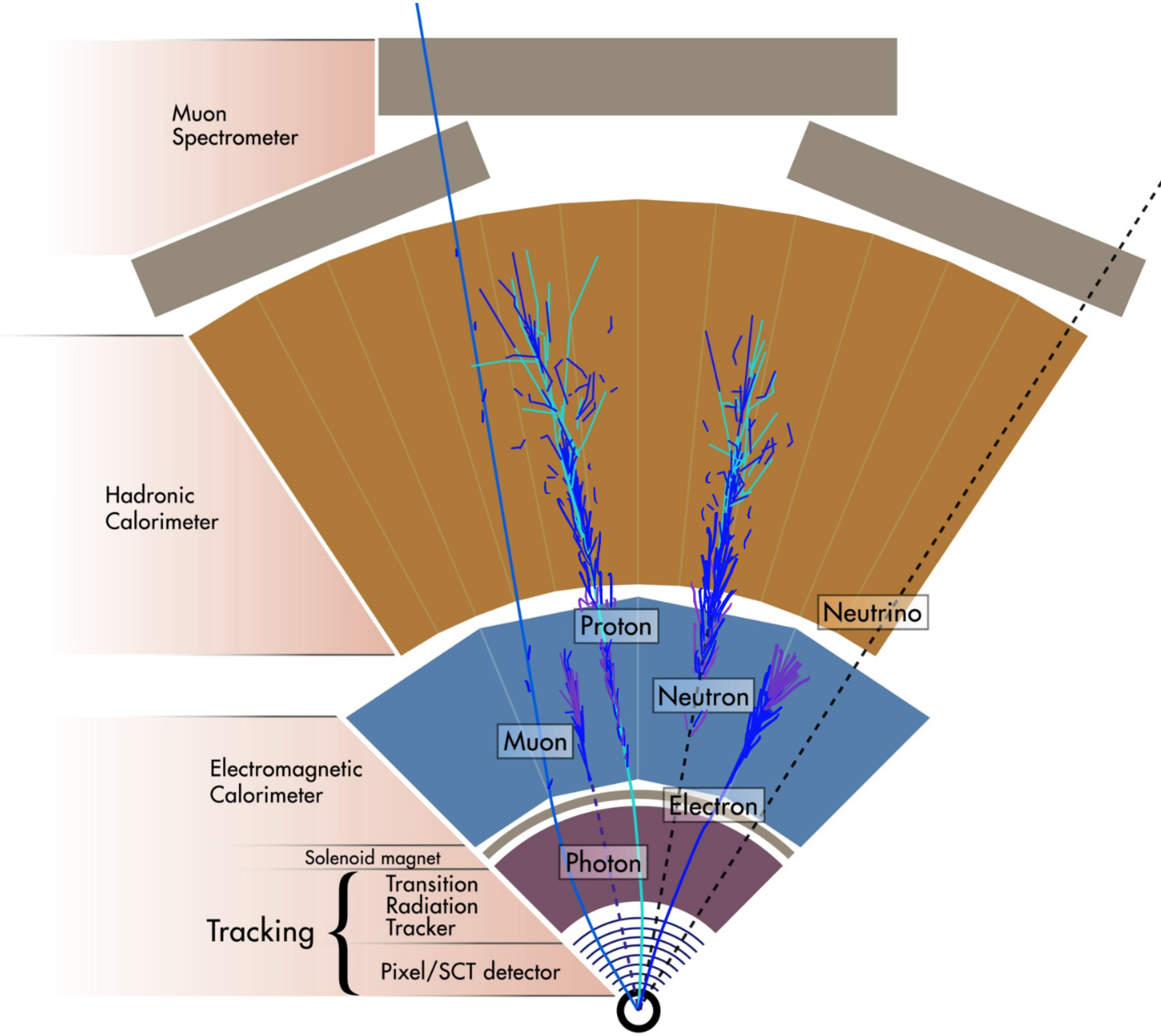


(b) Trilinear Coupling

DiHiggs production with one Higgs decaying to two photons and one to two b-quarks

The non-resonant $bb\gamma\gamma$ channel



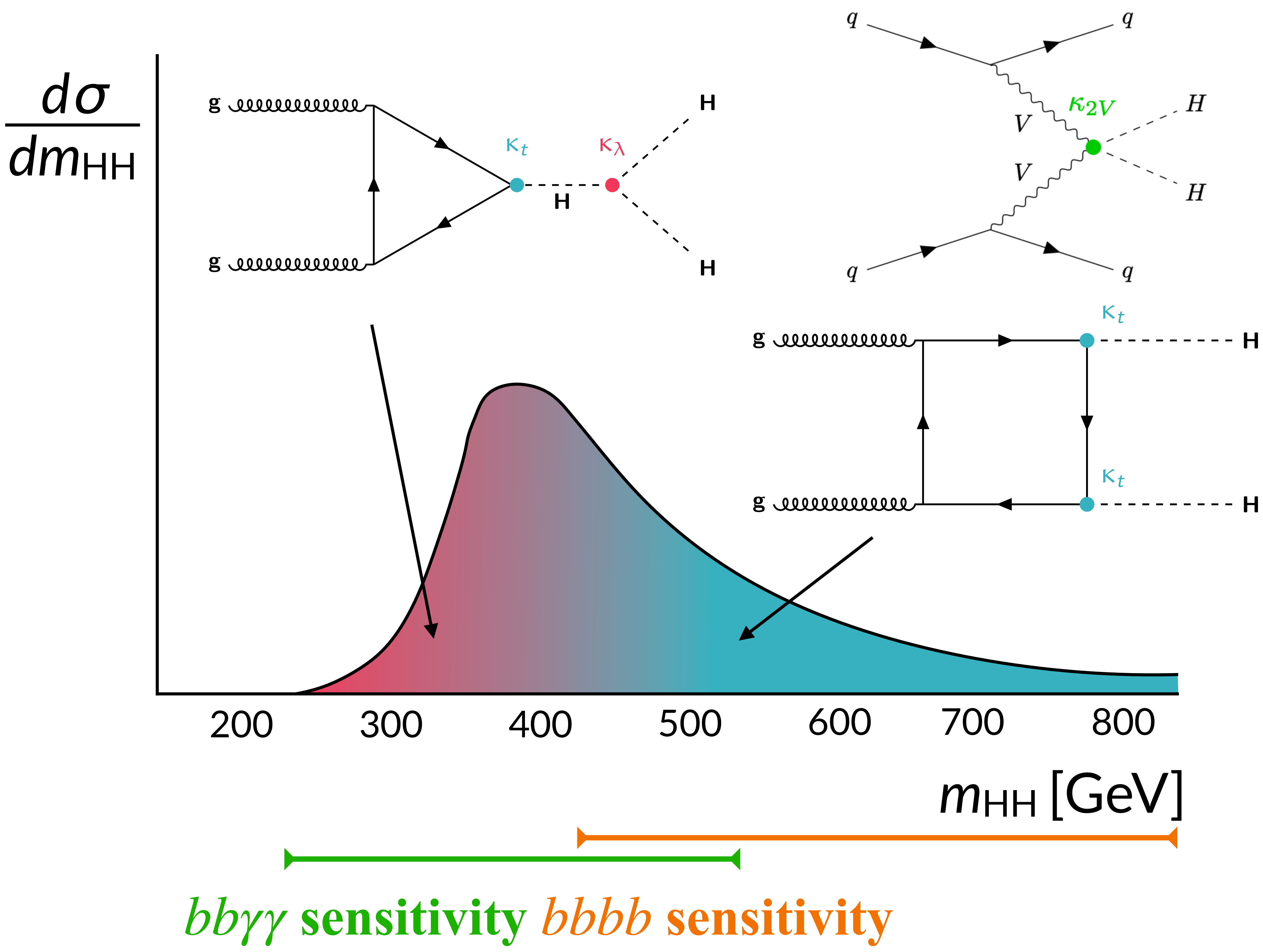


Final state particles observed in the ATLAS detector: photons and *b*-jets

⇒ photon reconstruction and calibration

⇒ *b*-jet reconstruction and calibration

Motivation to use the $bb\gamma\gamma$ channel



The low m_{HH} region is essential to constrain the trilinear coupling

The diphoton signature has the advantages:

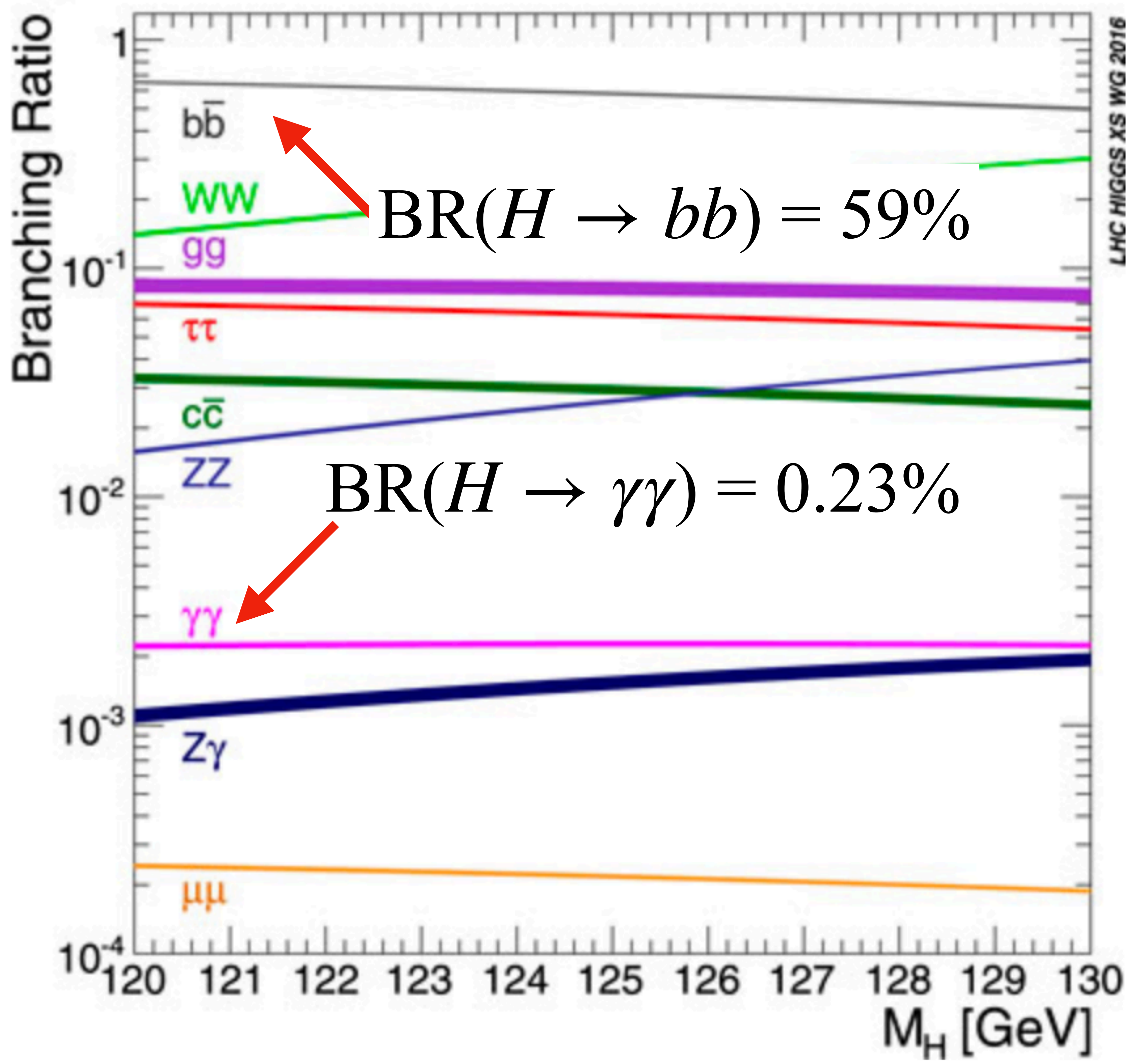
- efficient photon reconstruction
- low energy resolution
- diphoton trigger is able to get the Higgs at rest

This ensures good sensitivity to low m_{HH}

We would love to do HH to $\gamma\gamma\gamma\gamma$

However,….

Higgs decay modes



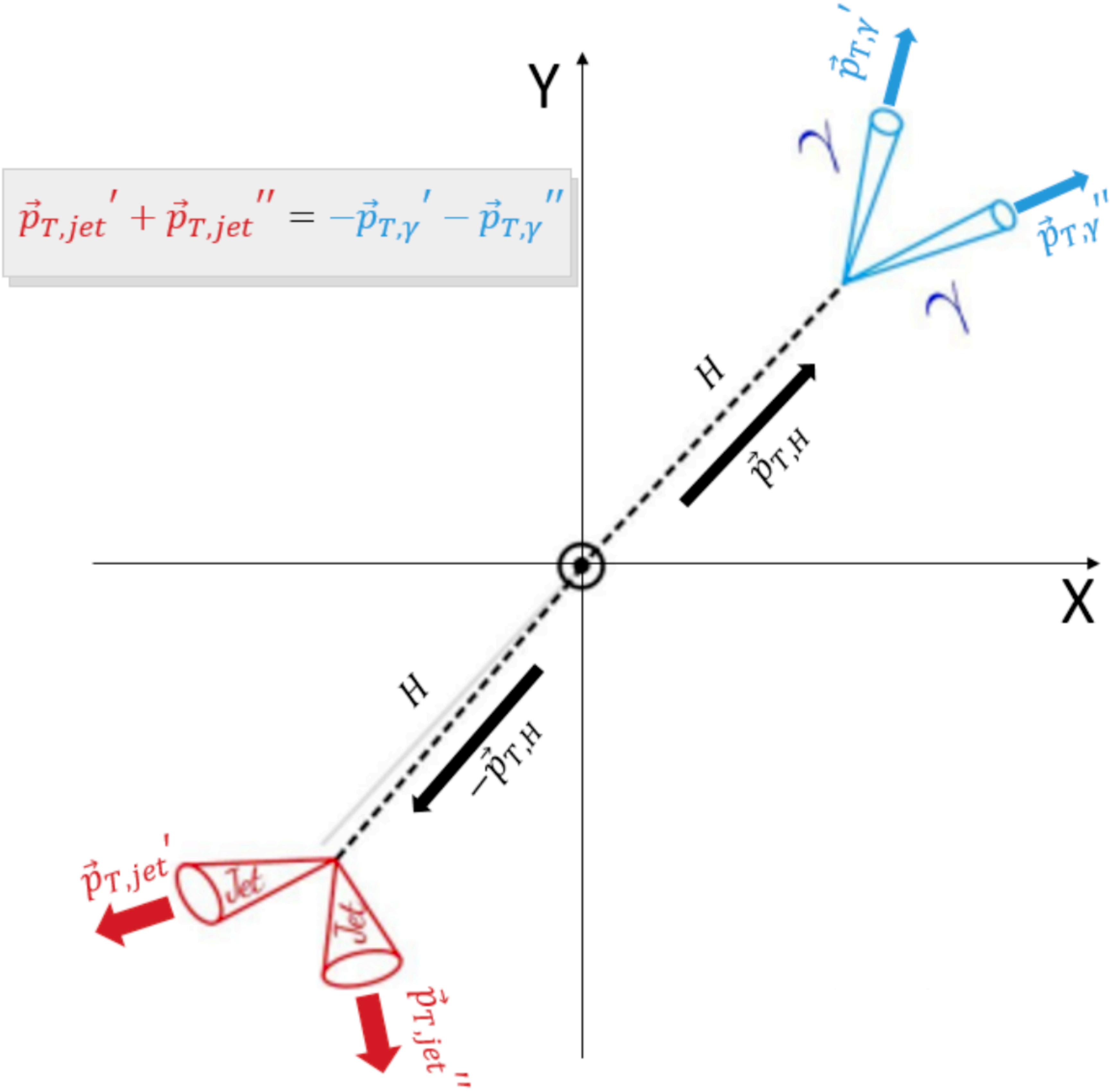
	bb	WW	ττ	ZZ	γγ
bb	34%				
WW	25%	4.6%			
ττ	7.3%	2.7%	0.39%		
ZZ	3.1%	1.1%	0.33%	0.069%	
γγ	$\boxed{0.26\%}$	0.10%	0.028%	0.012%	$\boxed{0.0005\%}$

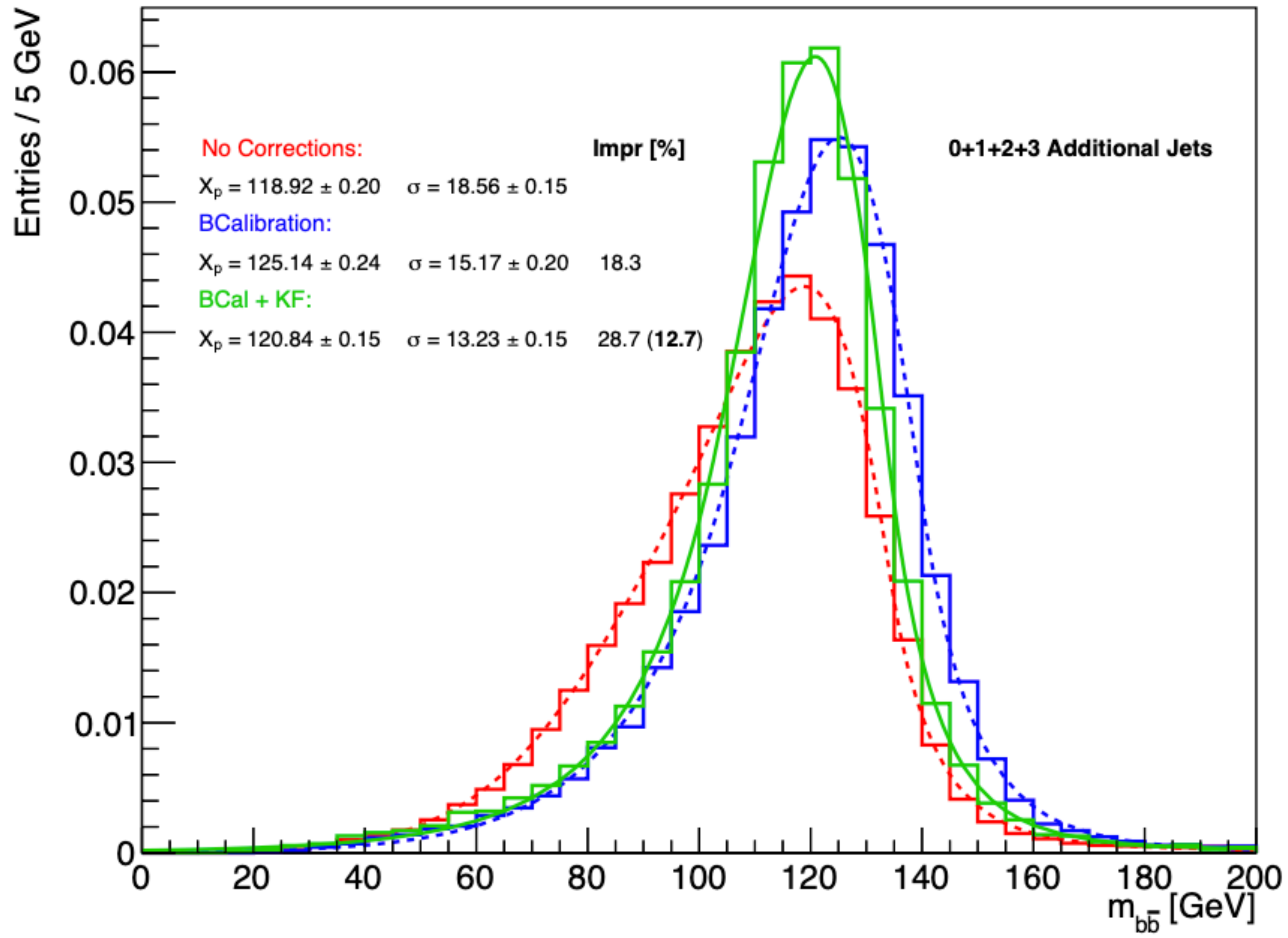
$H \to \gamma\gamma$ has a very low branching ratio

At the moment DiHiggs searches are limited by statistics $\Rightarrow bb\gamma\gamma$

$\Rightarrow bb\gamma\gamma$ is thus great to access low m_{HH} region and still have a reasonable statistics

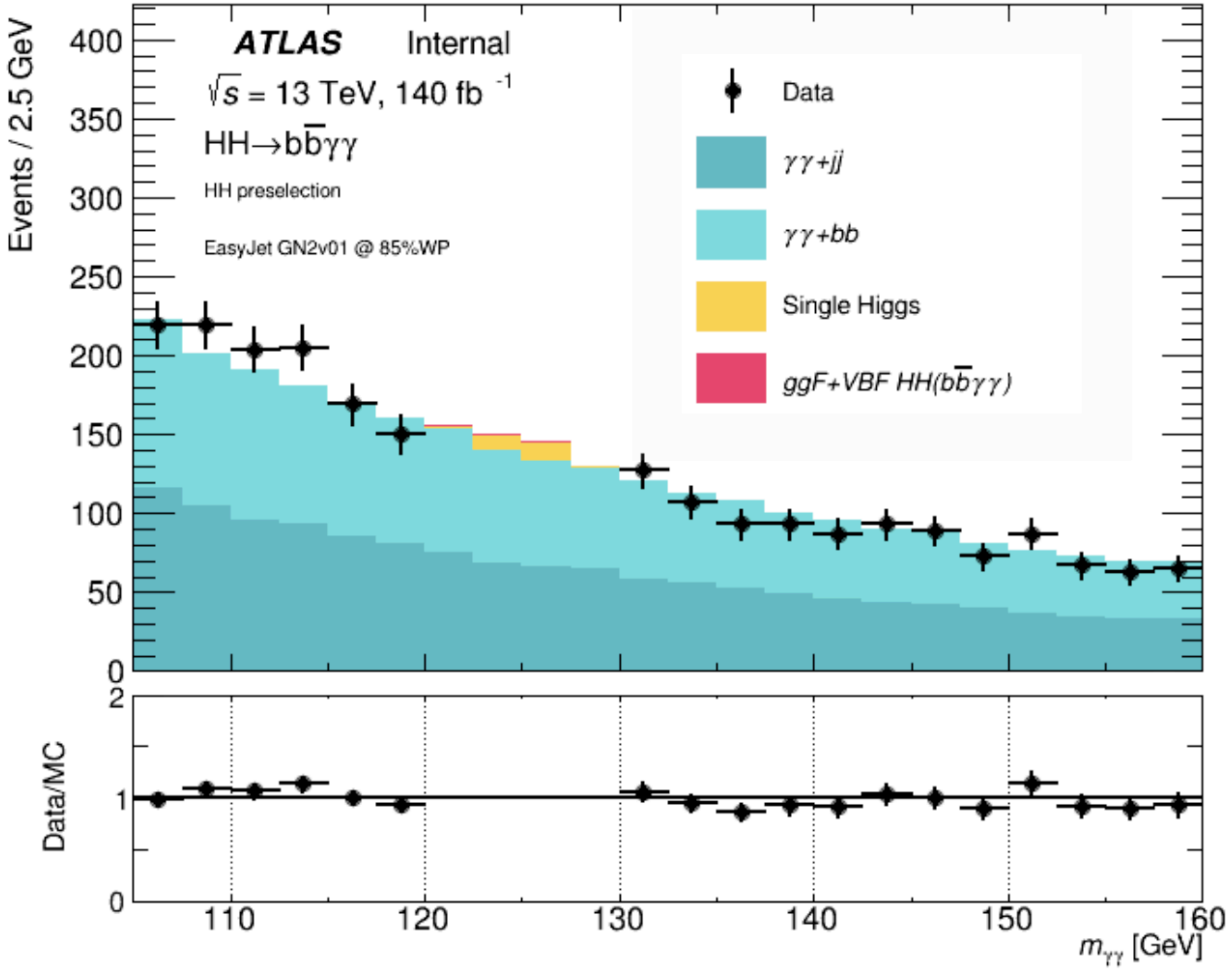
Kinematic fit





Improve the m_{bb} resolution by an additional 16% compared to the BCalibration correction

$m_{\gamma\gamma}$ distribution



$m_{\gamma\gamma}$ is the distribution we want to fit to extract κ_λ

The main background is $\gamma\gamma$+jets

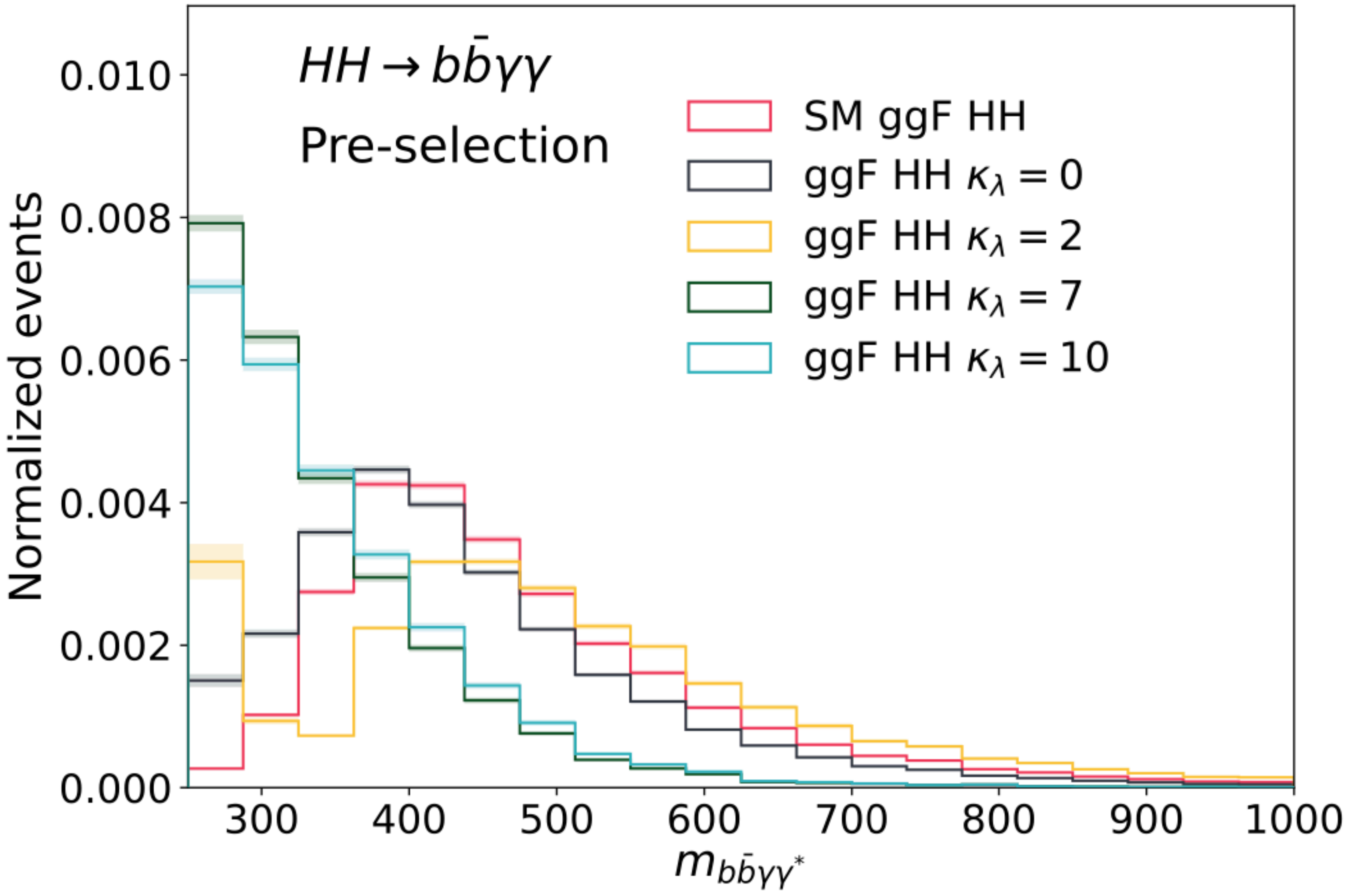
	SM ggF HH	SM VBF HH
$\sqrt{s}$ = 13 TeV	30.77 fb	1.687 fb
$\sqrt{s}$ = 13.6 TeV	34.13 fb	1.874 fb

Boosted Decision Tree (BDT) training

$$m^*_{bbyy} = m_{bbyy} - m_{bb} - m_{yy} + 250\ \text{GeV}$$

HM (high mass) region = SM-like region: $m^*_{bbyy} \geq 350$ GeV

LH (low mass) region = BSM-like region: $m^*_{bbyy} < 350$ GeV



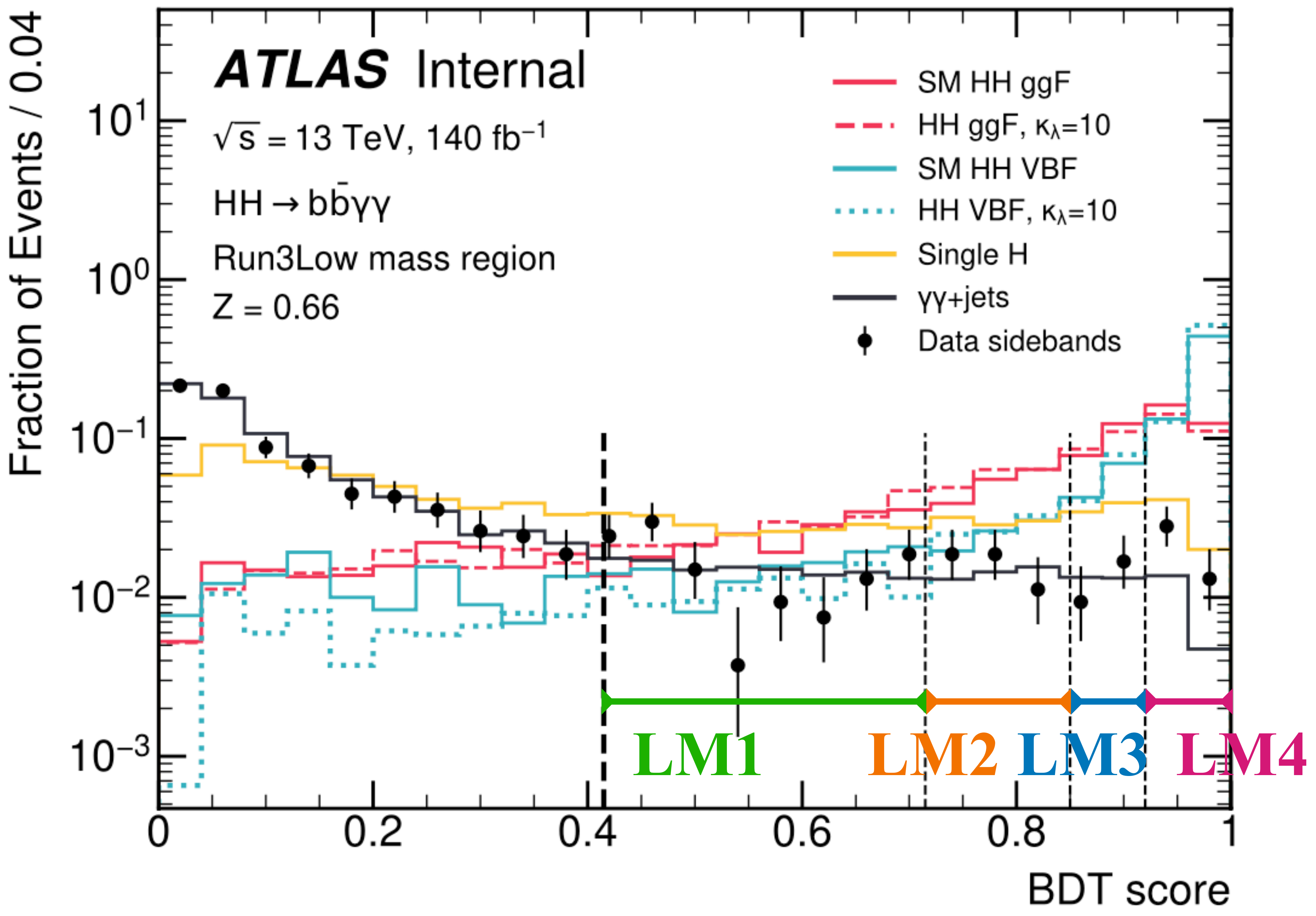
m^*_{bbyy} is preferred over simply m_{bbyy} since it improves the signal mass resolution due to the cancellation of detector effects

A BDT is trained for each region to distinguish signal from backgrounds

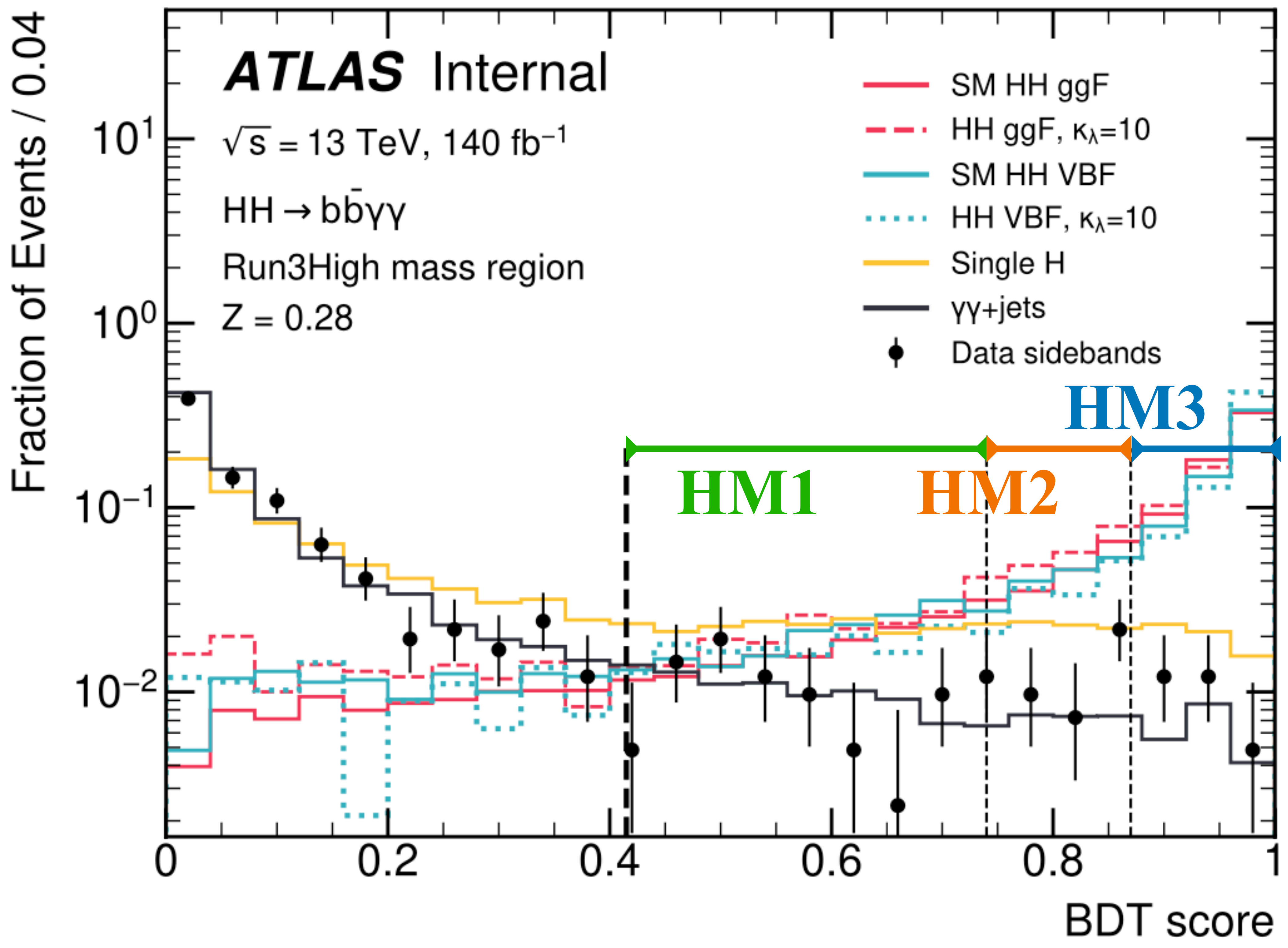
Event categories

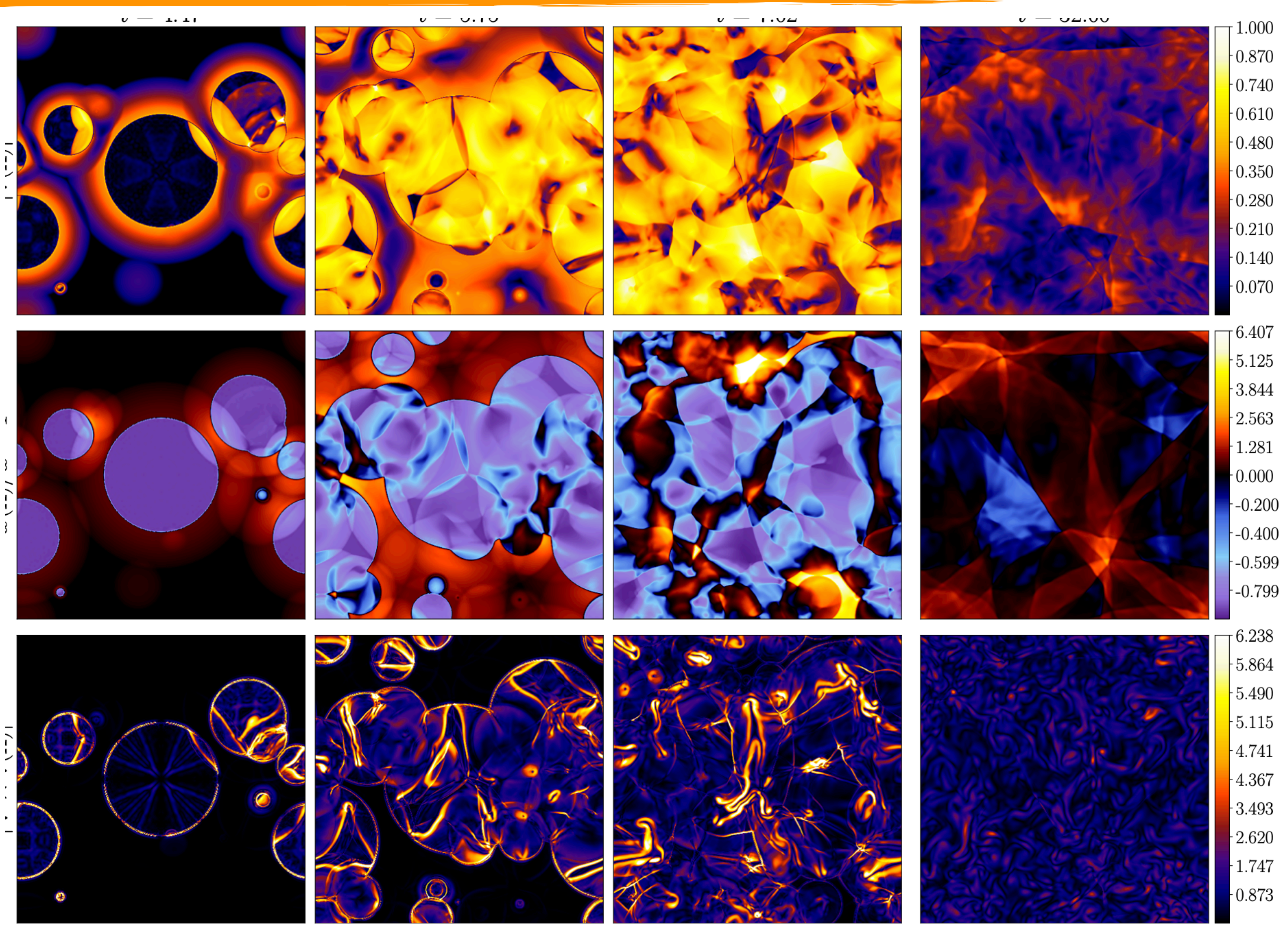
3 categories for HM region and 4 for LM region are defined based on the BDT scores

Low Mass region



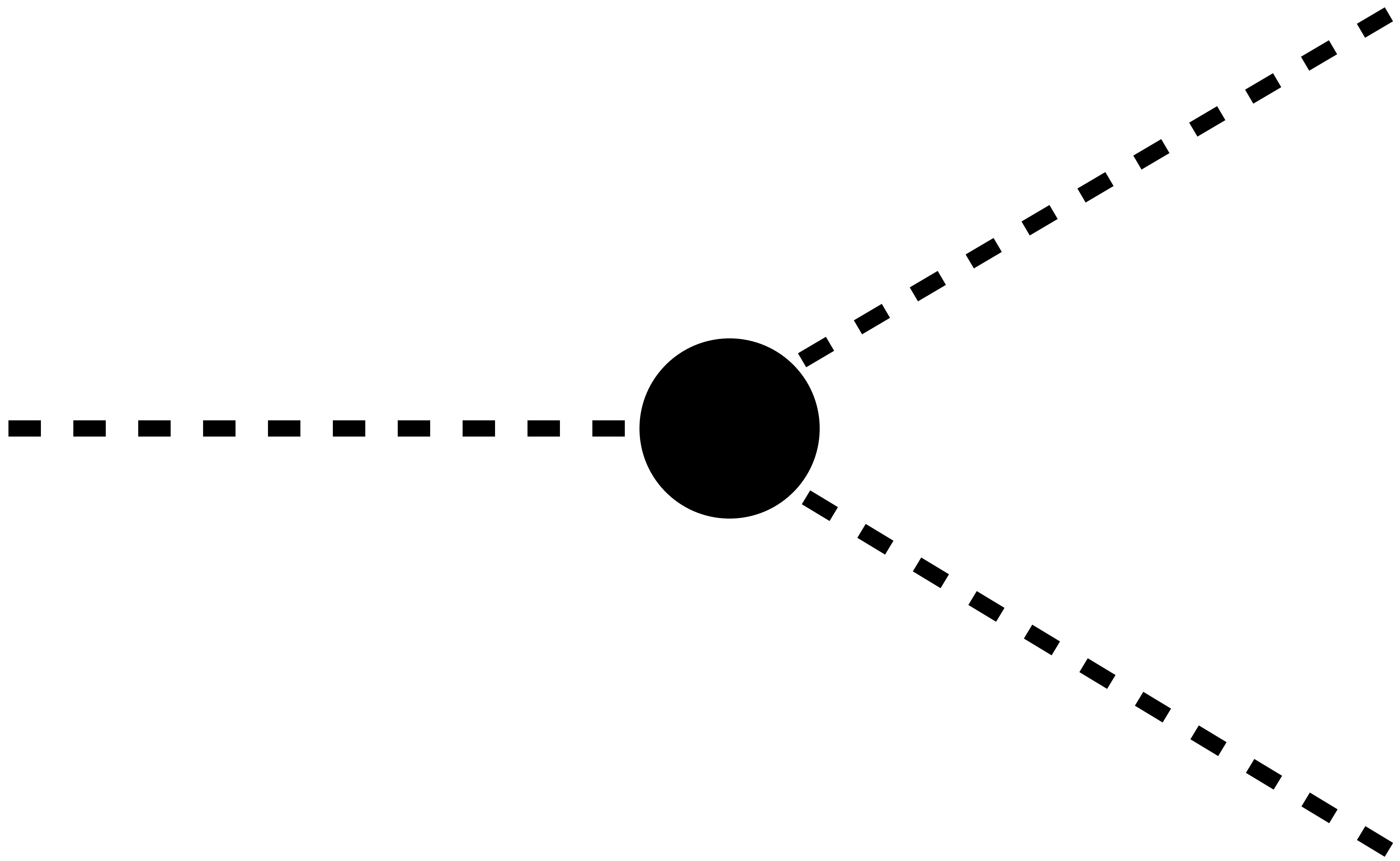
High Mass region

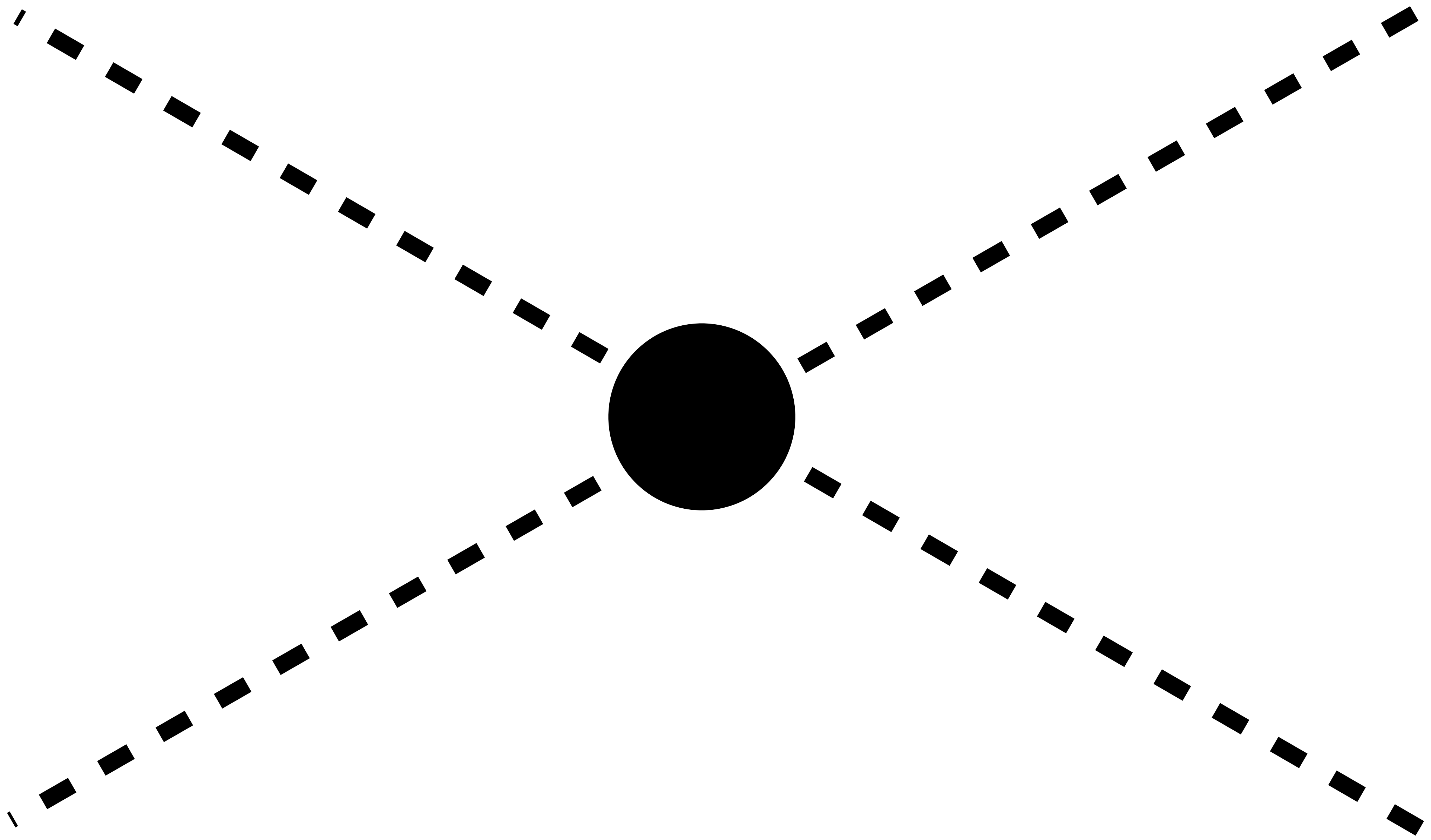




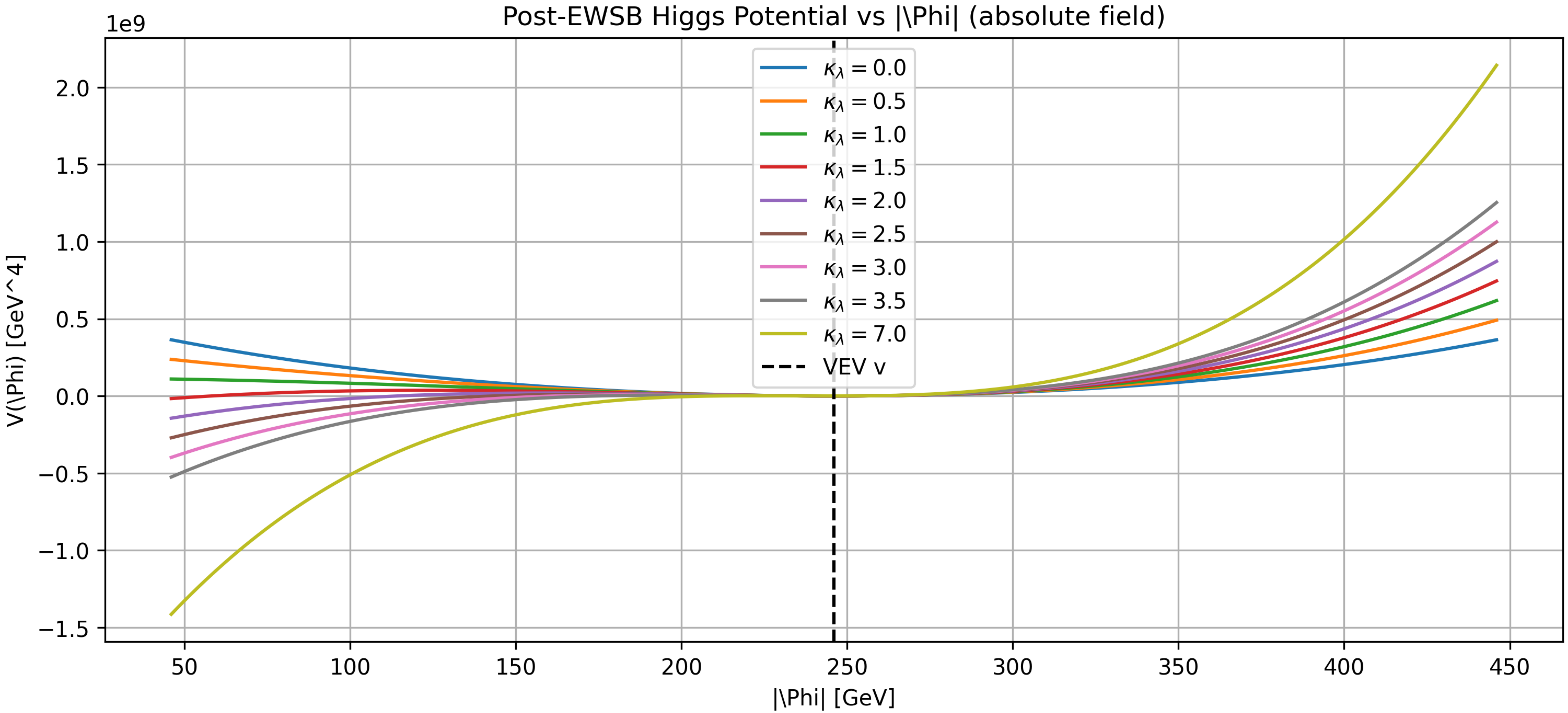
https://arxiv.org/pdf/2409.03651

To delete

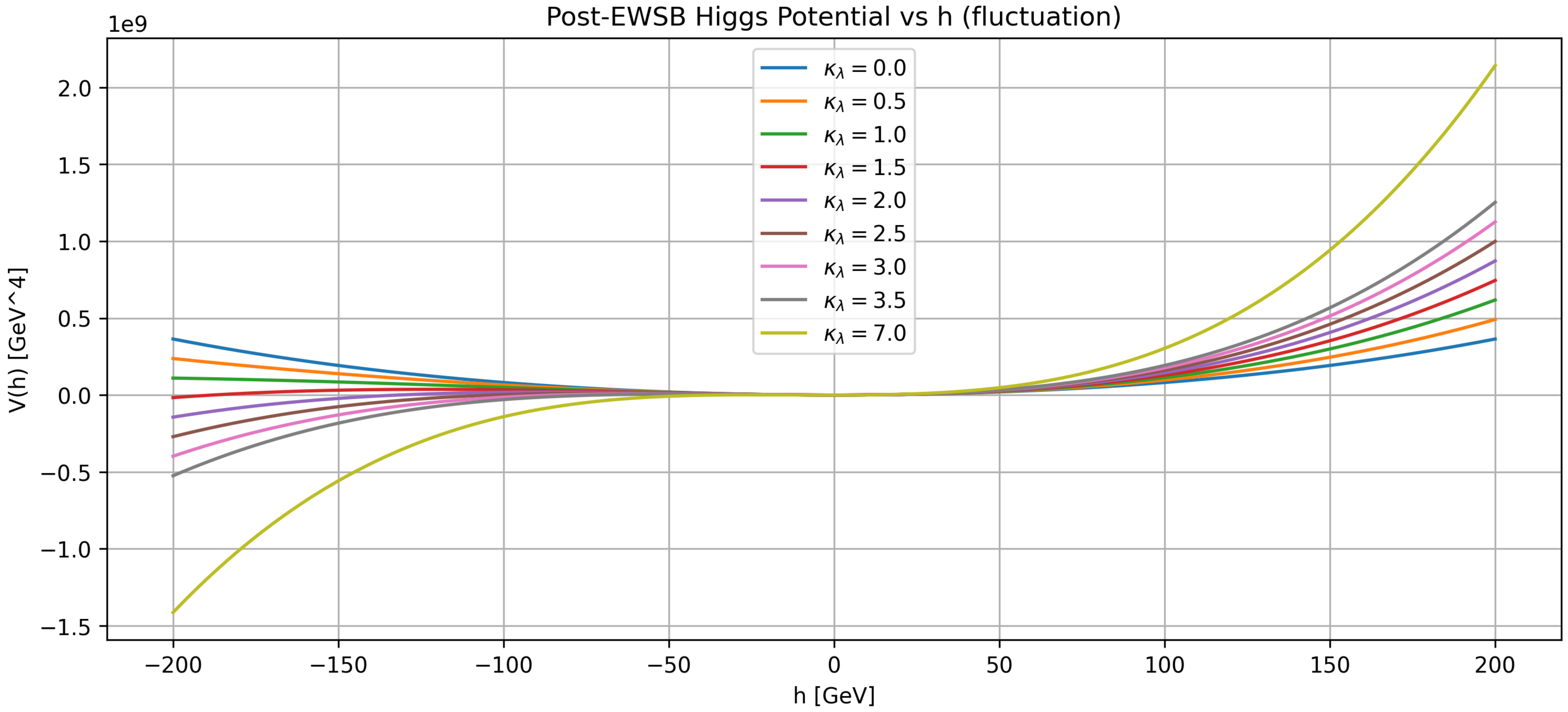


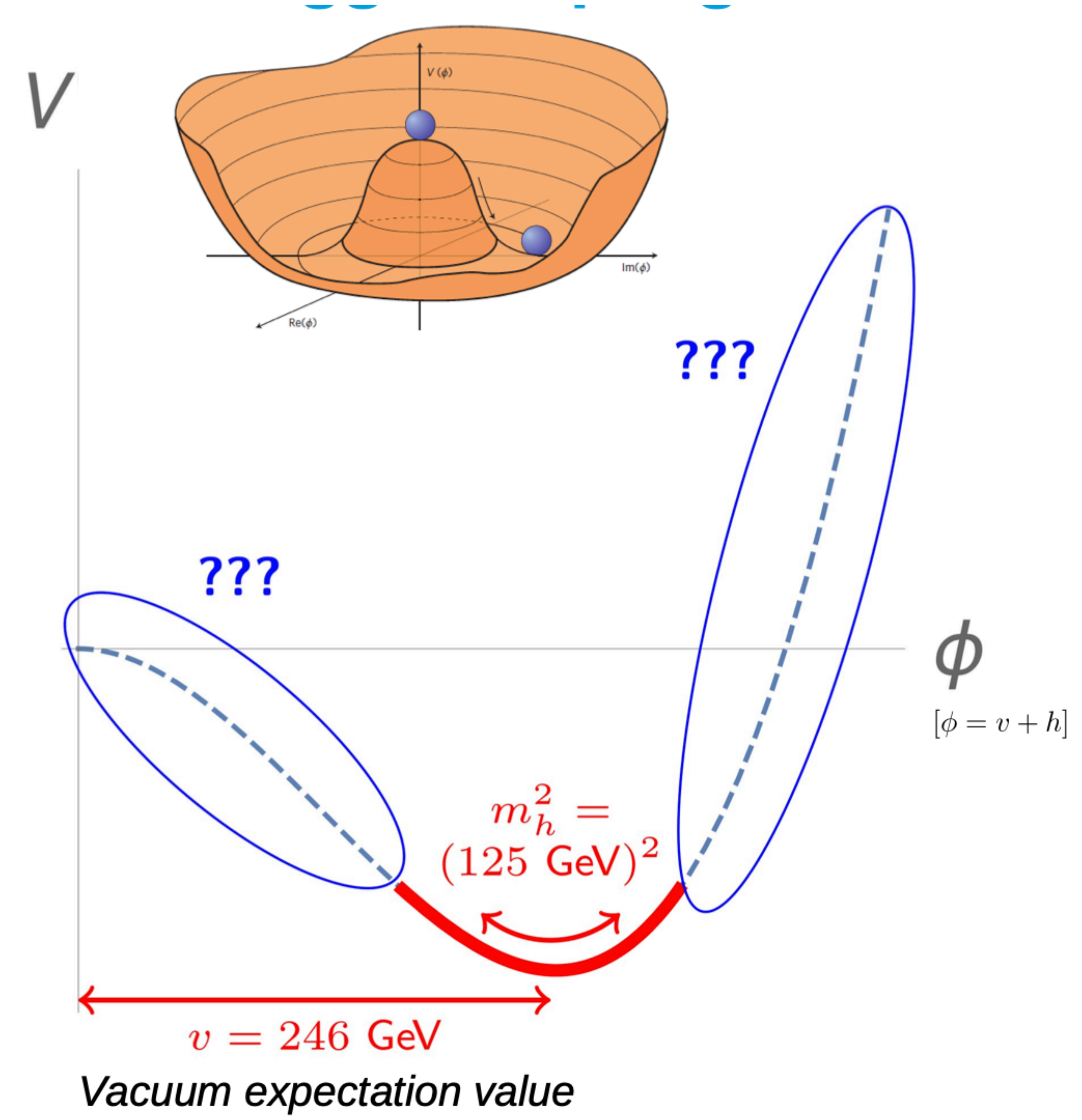


Higgs potential shape and kl

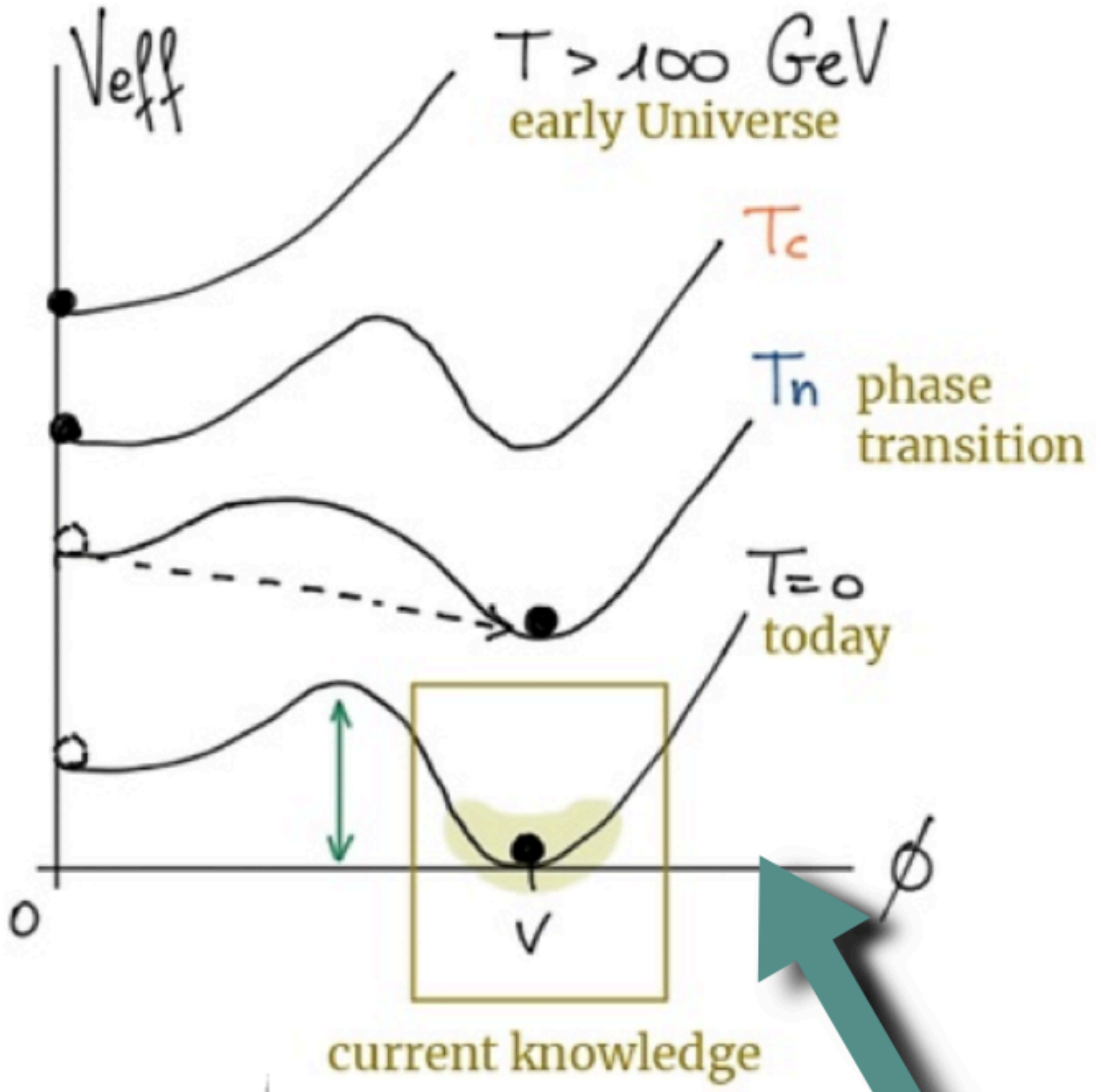


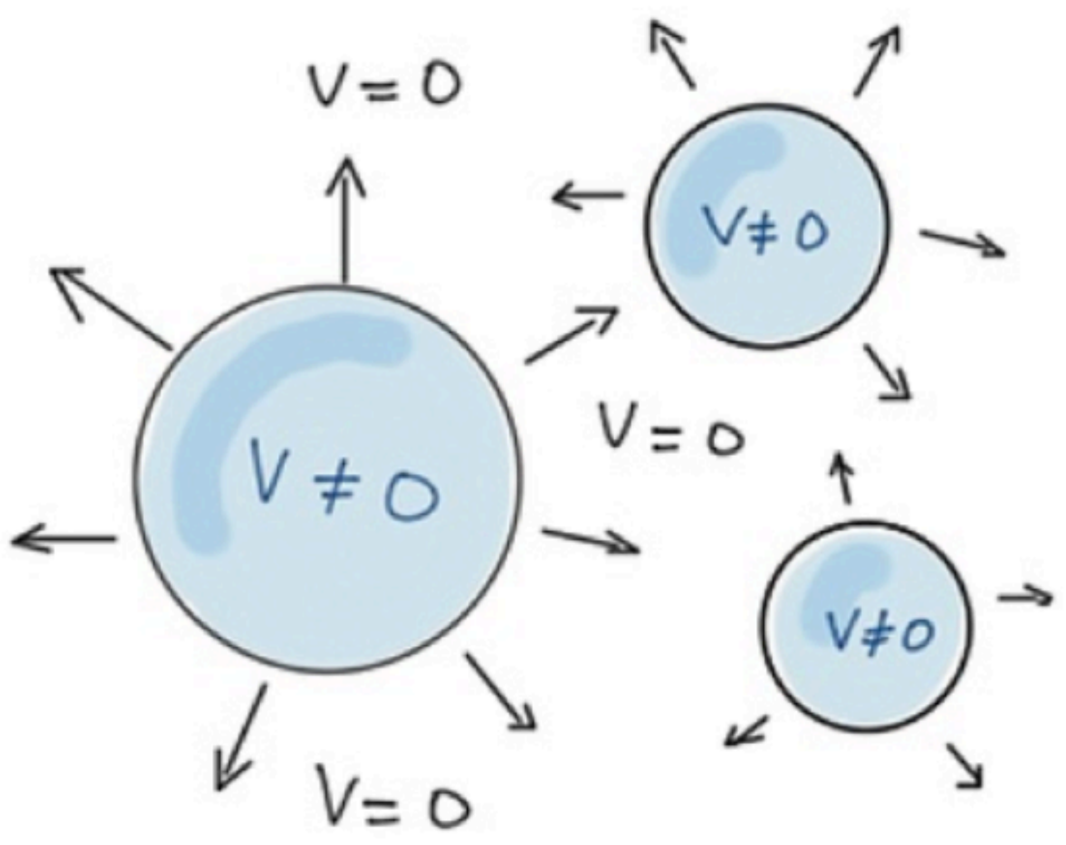
Higgs potential shape and kl




V
???
???
ϕ
$[\phi = v + h]$
$m_h^2 = (125\ \text{GeV})^2$
$v = 246$ GeV
Vacuum expectation value

Understanding the beginning of Universe





A violent EW symmetry breaking

First Order Phase Transition

(FOPT)

boiling universe

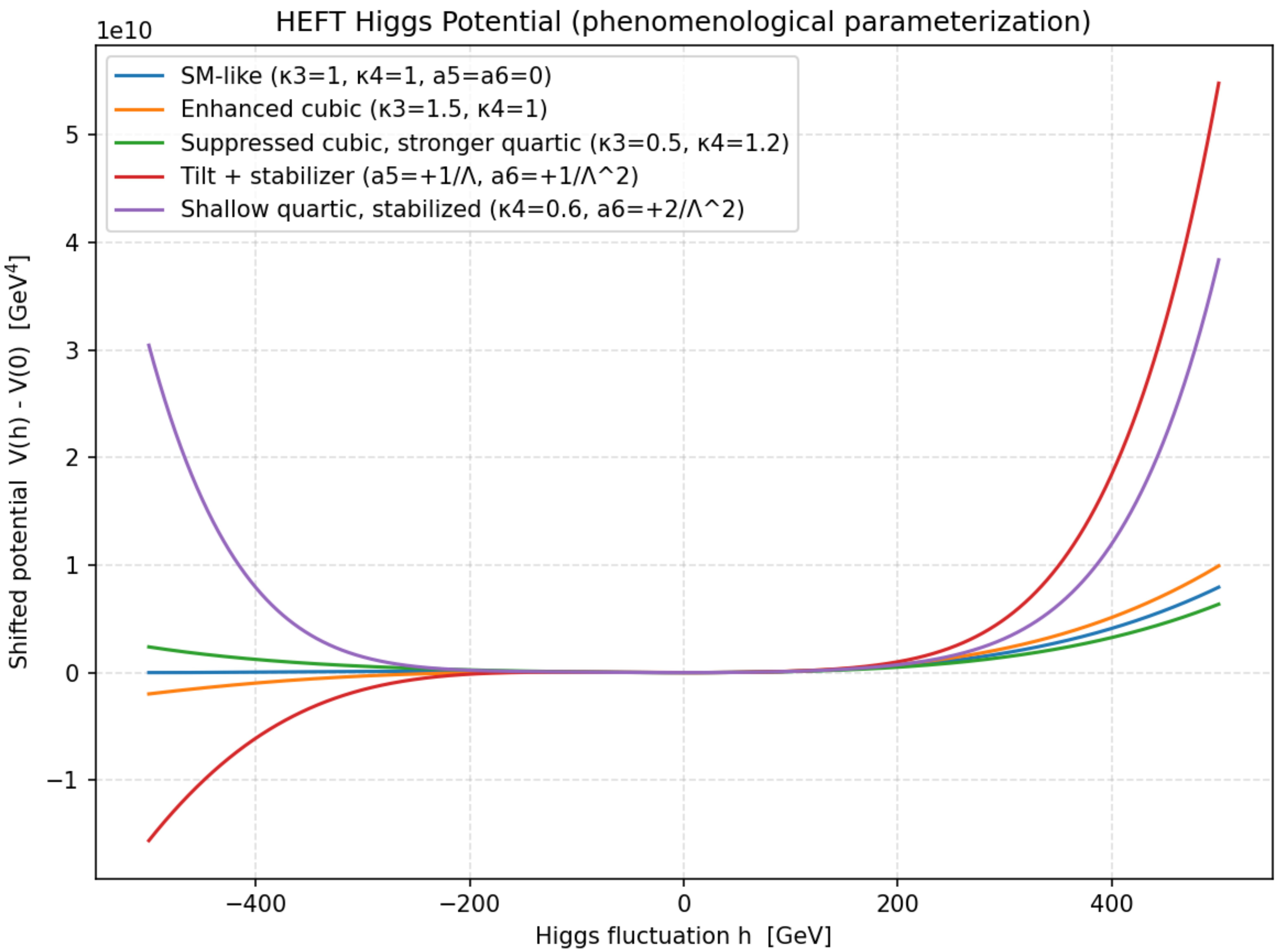
Why the Universe is made of matter?

The Higgs potential shape is crucial

Ph

ONLY a Strong FOPT can explain a boiling universe

17



HEFT Higgs Potential (phenomenological parameterization)
1e10
SM-like (κ3=1, κ4=1, a5=a6=0)
Enhanced cubic (κ3=1.5, κ4=1)
Suppressed cubic, stronger quartic (κ3=0.5, κ4=1.2)
Tilt + stabilizer (a5=+1/Λ, a6=+1/Λ^2)
Shallow quartic, stabilized (κ4=0.6, a6=+2/Λ^2)
Shifted potential V(h) - V(0) [GeV⁴]
5
4
3
2
1
0
−1
−400
−200
0
200
400
Higgs fluctuation h [GeV]

Primordial GWs frequency ranges?

Primordial gravitational wave frequencies span a broad range, with current and future observatories targeting specific bands: Pulsar Timing Arrays (PTAs) probe the nanohertz (nHz) to microhertz (μHz) range, space-based interferometers like LISA target the millihertz (mHz) to 1 Hz range, and ground-based detectors like the Einstein Telescope (ET) and Cosmic Explorer (CE) aim for the 1 Hz to 10 kHz band. Scientists also look for even higher frequencies, from kilohertz to 100 gigahertz (GHz), though these frequencies are primarily associated with sources like cosmic strings that have left no imprint on current detectors

- The strength and nature of the electroweak phase transition depend partly on the Higgs potential — and thus on the **Higgs self-coupling**.
- The mexican hat SM is a smooth transition from 0 to VEV thus not first order
- The higgs selcoupling gives us information about the shape of the higgs potential and thus conseguently on how the Higgs did “roll” into that minima
- for a first-order phase transition the frequency range would depend on the temperature of the universe when it happened
- A first order phase transition higgs potential would have a secondary minima that can be reached suddenly through tunneling for example
- First connection between the very small and the very big? (SM and gravity?)

HL-LHC results will stay for years to come and some will be reference until Fcc-hh

[De Blas et al., 2020]

kappa-0	HL-LHC	LHeC	HE-LHC S2	HE-LHC S2′	ILC 250	ILC 500	ILC 1000	CLIC 380	CLIC 15000	CLIC 3000	CEPC	FCC-ee 240	FCC-ee 365	FCC-ee/eh/hh
κ_W [%]	1.7	0.75	1.4	0.98	1.8	0.29	0.24	0.86	0.16	0.11	1.3	1.3	0.43	0.14
κ_Z [%]	1.5	1.2	1.3	0.9	0.29	0.23	0.22	0.5	0.26	0.23	0.14	0.20	0.17	0.12
κ_g [%]	2.3	3.6	1.9	1.2	2.3	0.97	0.66	2.5	1.3	0.9	1.5	1.7	1.0	0.49
κ_γ [%]	1.9	7.6	1.6	1.2	6.7	3.4	1.9	98⋆	5.0	2.2	3.7	4.7	3.9	0.29
$\kappa_{Z\gamma}$ [%]	10.	–	5.7	3.8	99⋆	86⋆	85⋆	120⋆	15	6.9	8.2	81⋆	75⋆	0.69
κ_c [%]	–	4.1	–	–	2.5	1.3	0.9	4.3	1.8	1.4	2.2	1.8	1.3	0.95
κ_t [%]	3.3	–	2.8	1.7	–	6.9	1.6	–	–	2.7	–	–	–	1.0
κ_b [%]	3.6	2.1	3.2	2.3	1.8	0.58	0.48	1.9	0.46	0.37	1.2	1.3	0.67	0.43
κ_μ [%]	4.6	–	2.5	1.7	15	9.4	6.2	320⋆	13	5.8	8.9	10	8.9	0.41
κ_τ [%]	1.9	3.3	1.5	1.1	1.9	0.70	0.57	3.0	1.3	0.88	1.3	1.4	0.73	0.44

k_γ, k_Z, $k_{Z\gamma}$ will be the most precise measurements for a long time

Higgs potential shape and kl

2. Where a secondary minimum comes from

To have a second minimum at large field values, you need the **full Higgs potential** at large h, including:

- Higher-order terms beyond h^4
- Radiative corrections (Coleman–Weinberg potential)
- Possible new physics contributions

For example:

- In the SM, RG running of $\lambda(\mu)$ can make it negative at high scale → metastability
- In BSM, extra terms can create another local minimum

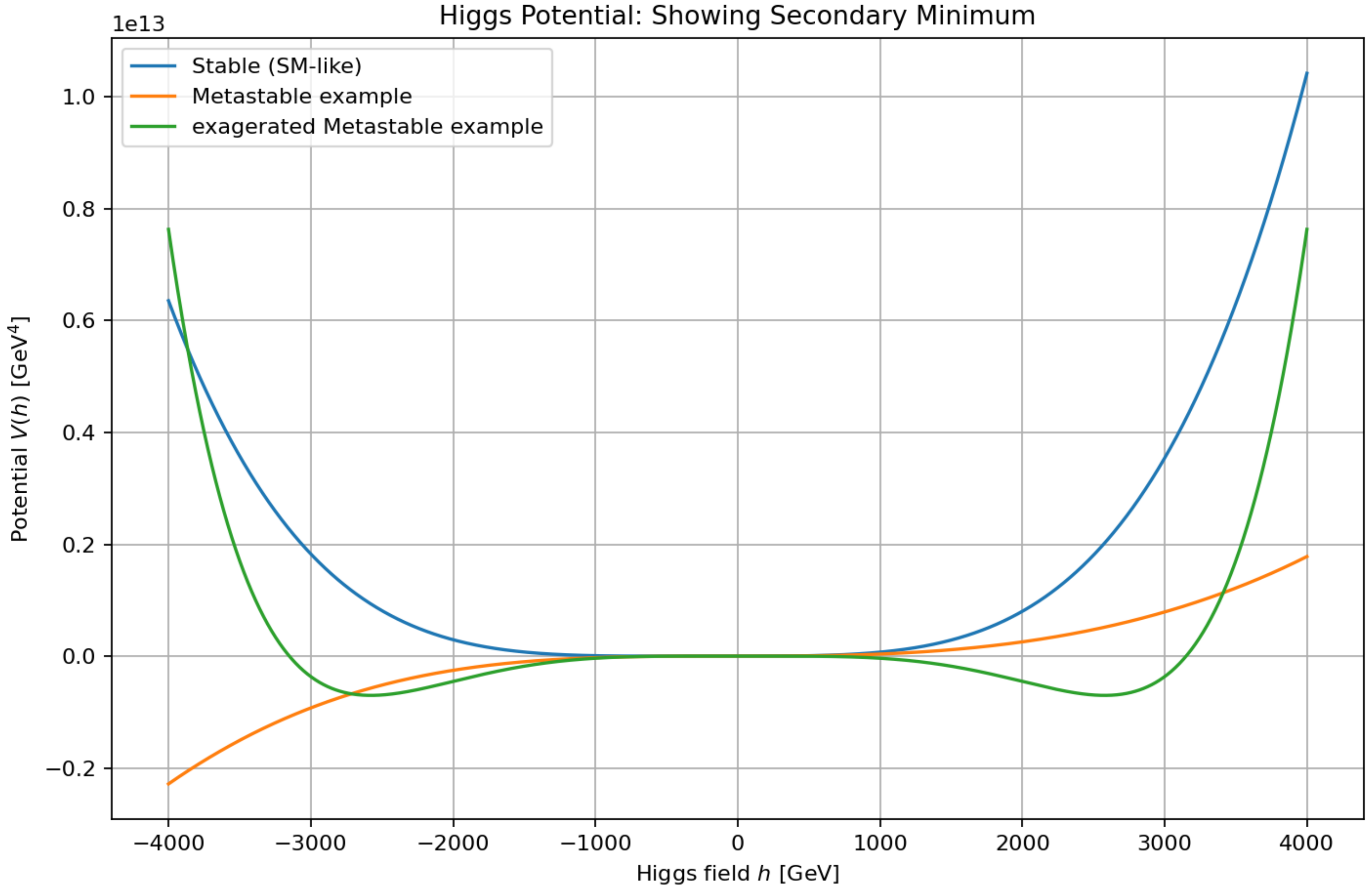
3. Link between κ_λ and secondary minima

Here's the subtlety:

- Measuring κ_λ tells you **about the third derivative of $V(h)$ at $h = v$**:

$$\kappa_\lambda \propto \frac{1}{v} V'''(h)\big|_{h=0}$$

- A secondary minimum, however, depends on the *integrated shape* of $V(h)$ far from $h = v$.
- A wildly different κ_λ **can** be a sign that the quartic (and higher) terms are also different, which might allow for a second minimum — but κ_λ alone doesn't prove or rule it out.



Higgs Potential: Showing Secondary Minimum
1e13
Stable (SM-like)
Metastable example
exagerated Metastable example
Potential V(h) [GeV^4]
Higgs field h [GeV]
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−4000
−3000
−2000
−1000
0
1000
2000
3000
4000

Effective field theory (EFT) - Warsaw basis

if you start from “all operators of dimension 6,” you get a very redundant set

Why do we need to chose a basis? Because there is no *unique* way to remove redundancies

We need to chose a basis.

A basis is the minimal set of independent operators (parameters) for the most general classification of BSM effects.

We pick by convention the Warsaw basis

The Warsaw basis (Grzadkowski et al. 2010, *JHEP* 10 (2010) 085) is the most widely used complete and nonredundant set of dimension-6 SMEFT operators in the linear realization of EWSB

deforming SM correlations

SMEFT SU(2)xU(1)/U(1)

"SM is a good IR reference point"

- clear power counting
- comparably few (bosonic) parameters e.g. [Gröber et al. `15]
- tight correlations across Higgs multiplicities
- non-linear elw vacuum: non-trivial technical implications…

HEFT SU(2)xSU(2)/SU(2)

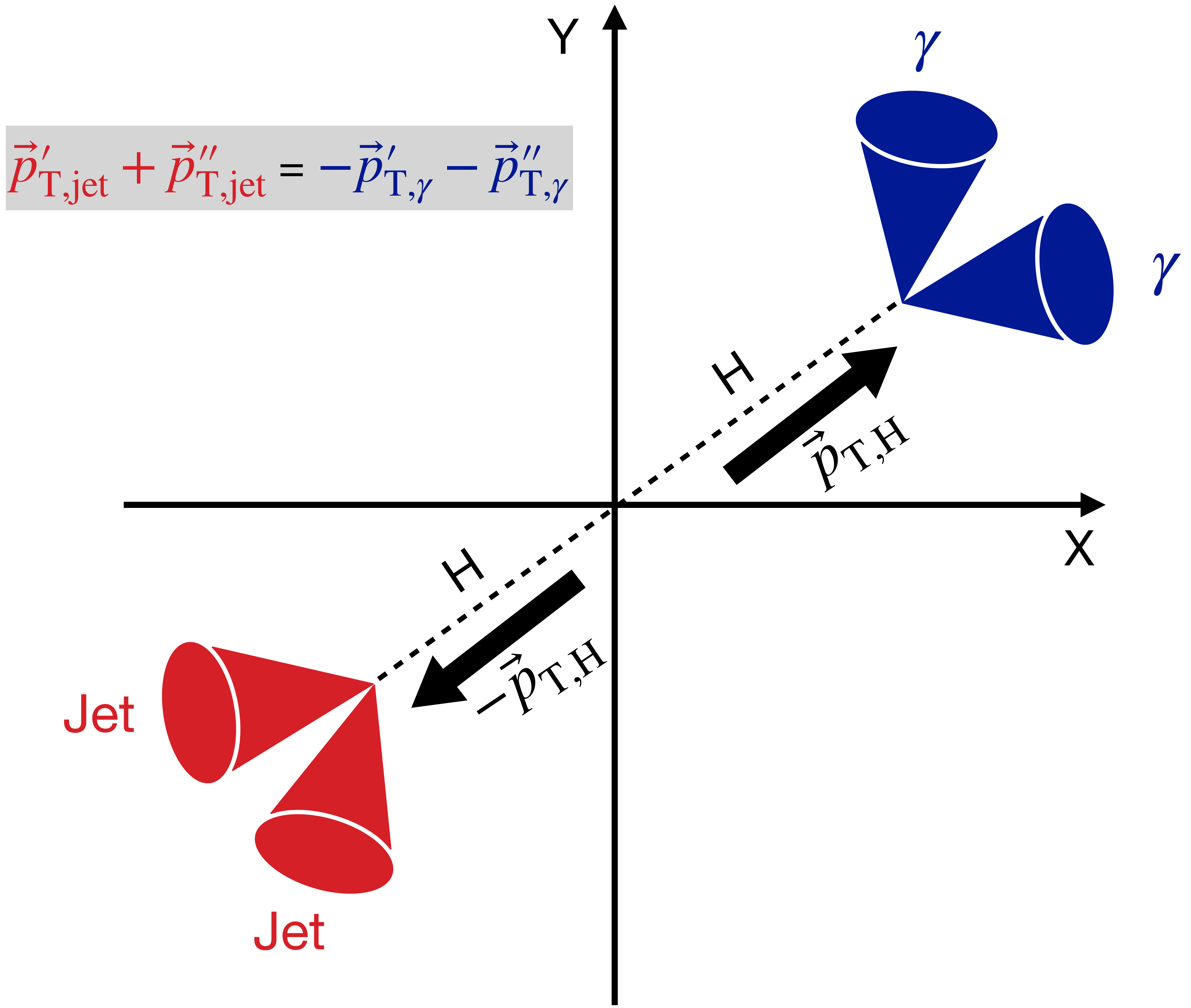
"SM EWSB perhaps too limiting"

- power counting debatable
- large number of parameters
- data informs Higgs multiplicity correlations
- elw vacuum coarse-grained: technical simplifications…

Advantages and disadvantages of EFT

- Good for model independent searches +
- Can be matched to UV complete theories +
- Allows to scan a wide range of BSM +
- Depending on the scenario, number of willson coefficients to test can be huge –
- Choices need to be made how to scan in these cases and also deal with degeneracies e.g. Eigenvector decompositions –
- Based on this, all results include baked in assumptions such as other coefficients fixed to SM case –

→ Looking for some (benchmark) UV complete models directly?



$\vec{p}'_{\mathrm{T,jet}} + \vec{p}''_{\mathrm{T,jet}} = -\vec{p}'_{\mathrm{T},\gamma} - \vec{p}''_{\mathrm{T},\gamma}$
Y
X
γ
γ
H
$\vec{p}_{\mathrm{T,H}}$
H
$-\vec{p}_{\mathrm{T,H}}$
Jet
Jet