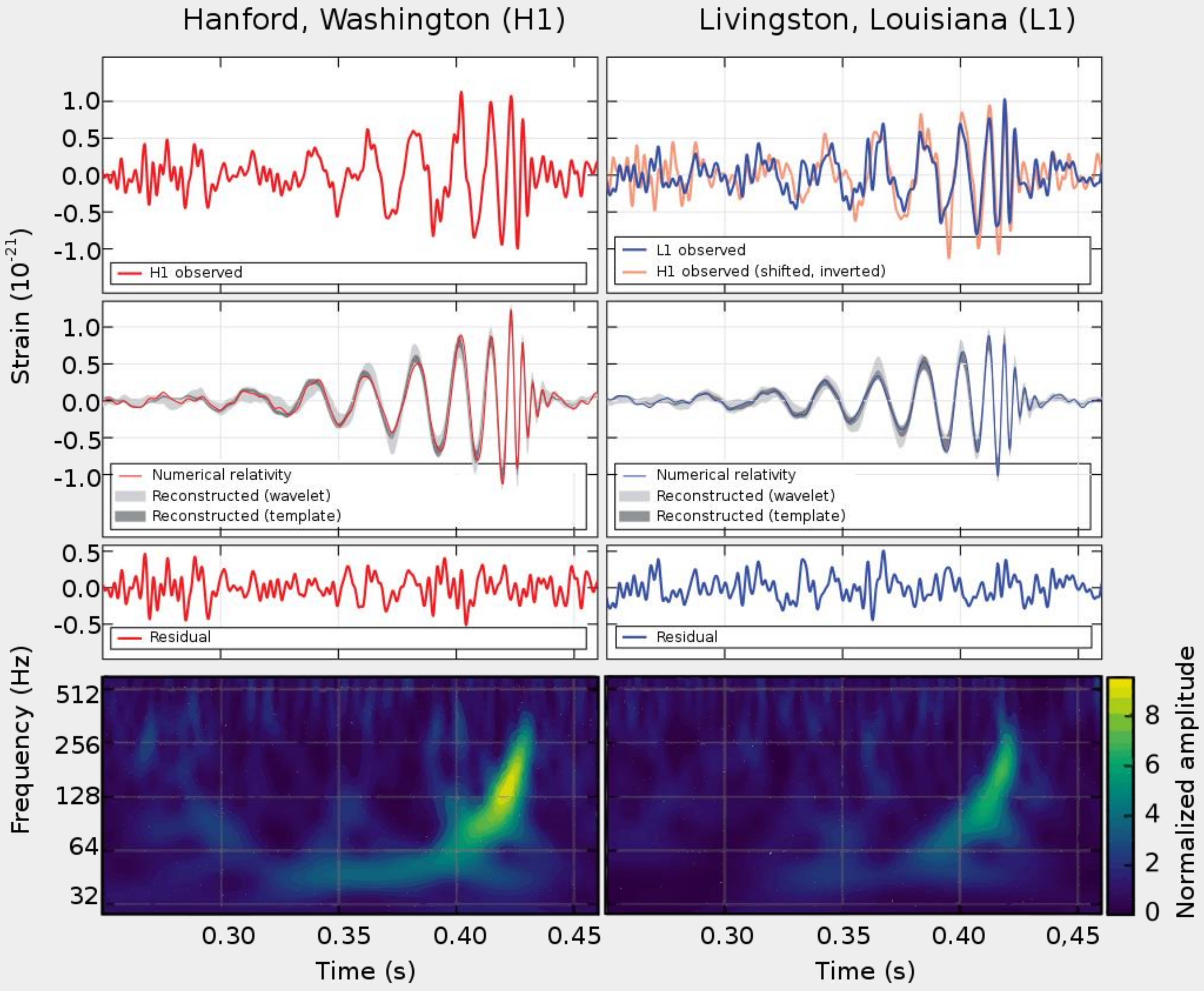
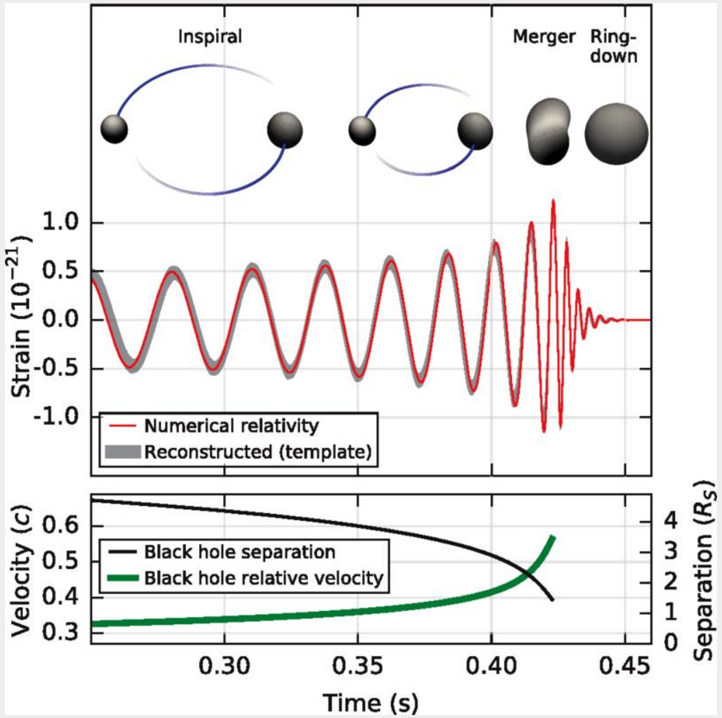
Bound-state binary dynamics from particle physics tools

Christoph Dlapa

European Research Council
Established by the European Commission

First detected gravitational wave

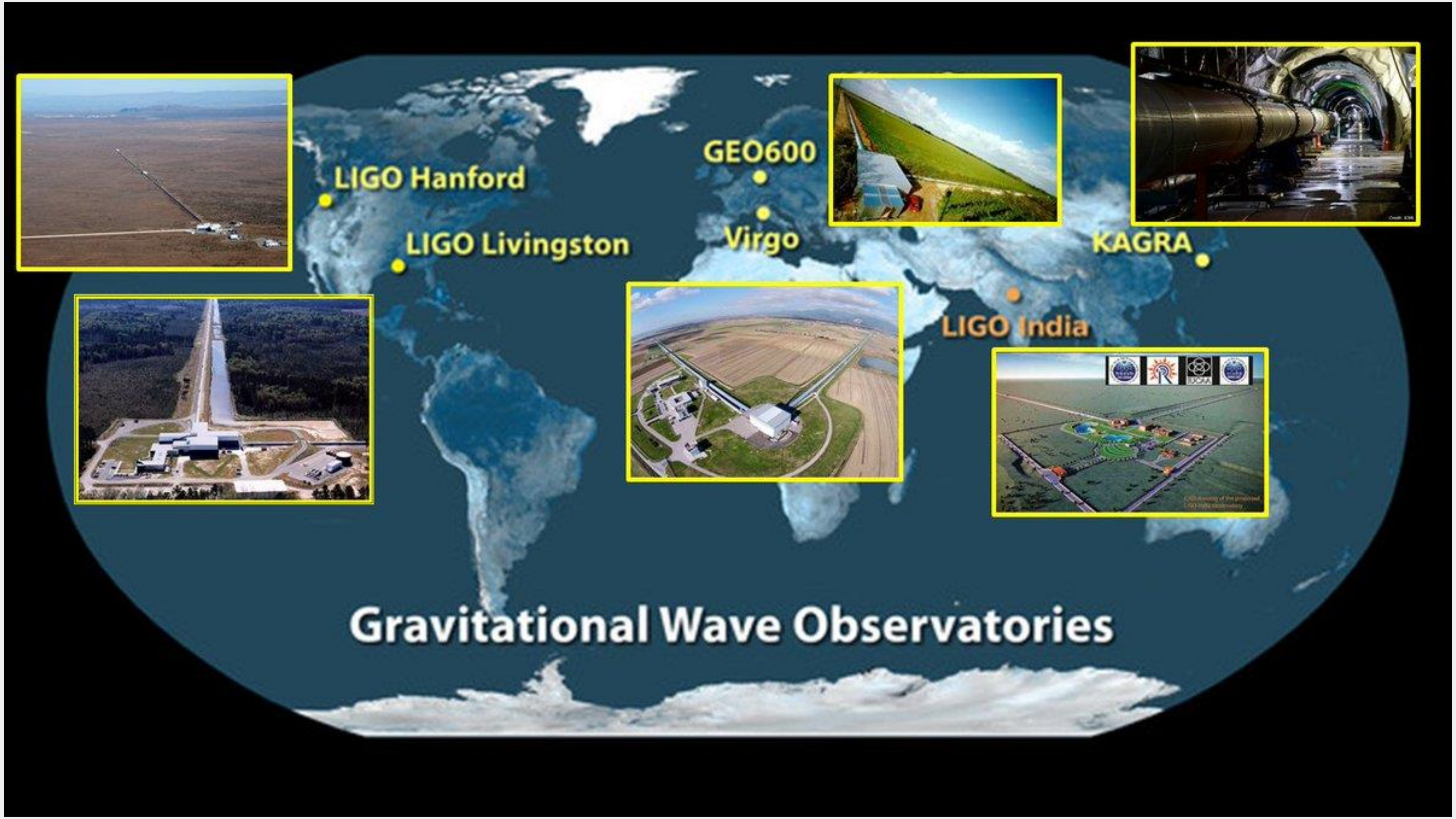




[LIGO]

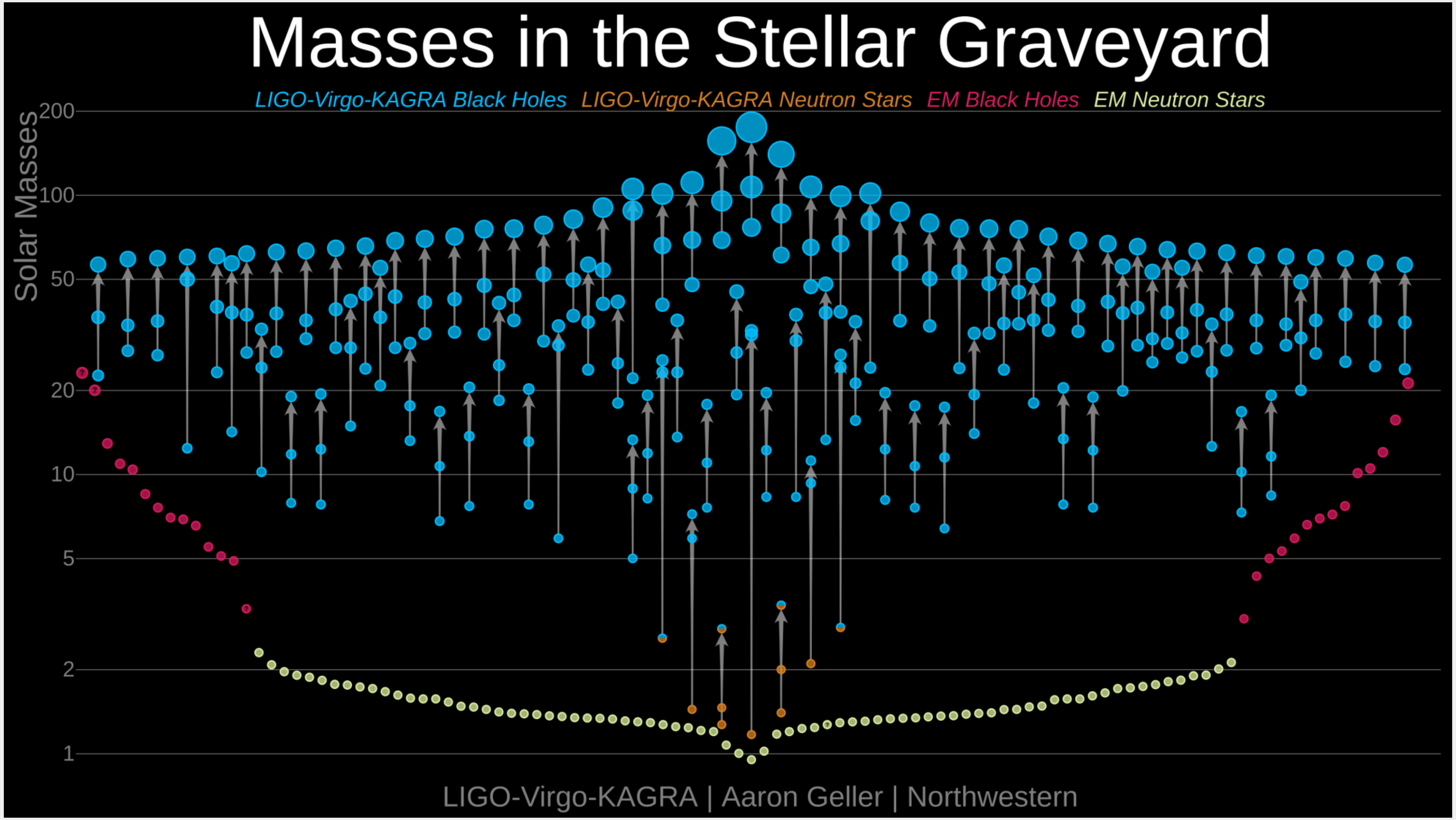
2017 Nobel Prize in Physics:
Weiss, Barish and Thorne

Global Network of 2G detectors



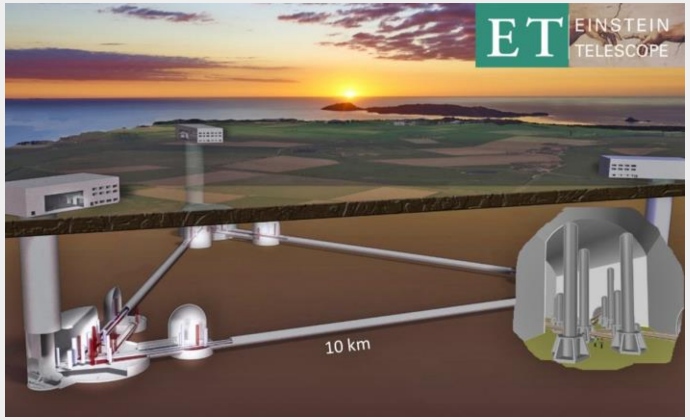
[LIGO]

Detections since then

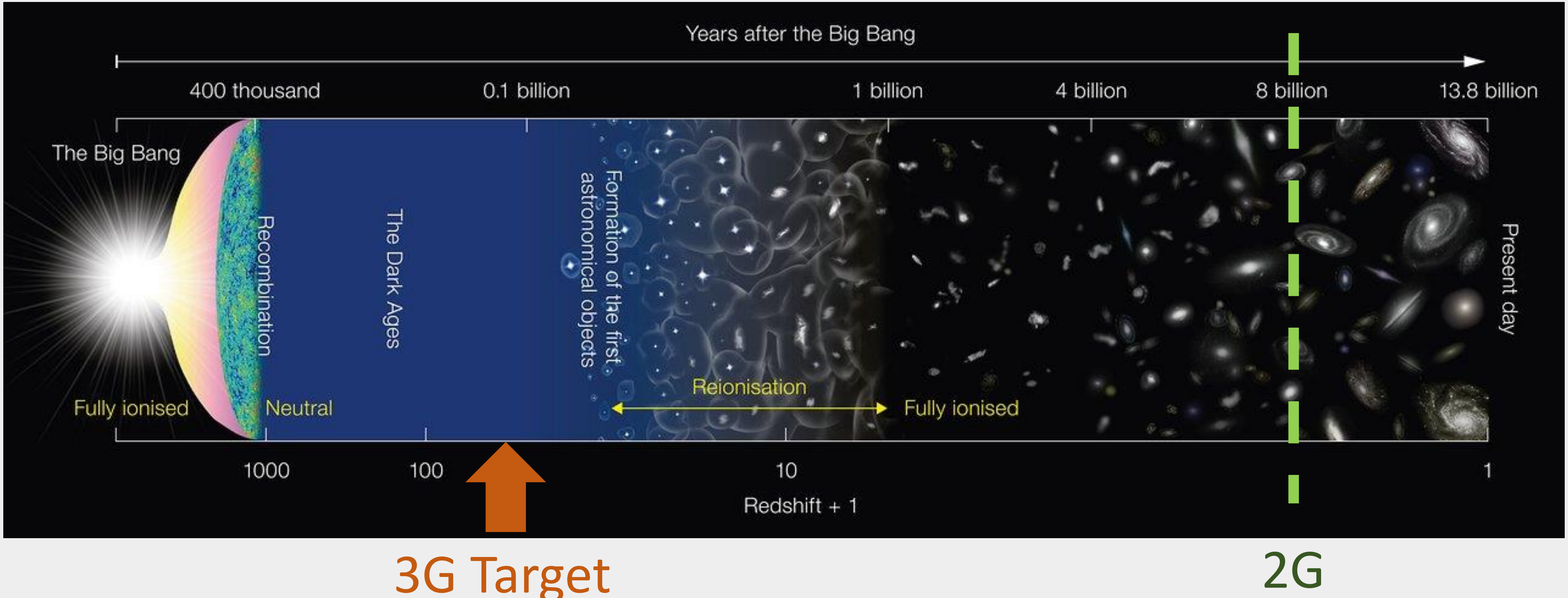


Proposed third-generation detectors

- Einstein Telescope (ET)
  - Belgium-Netherlands-Germany border or Lusatia
  - Sardinia (Italy)
  - underground, triangular/2L shape, 10 km
- Cosmic Explorer (CE)
  - US
  - above-ground, L-shaped, 40 km + 20 km
- Laser Interferometer Space Antenna (LISA)
  - space-based (ESA), heliocentric orbit
  - triangular shape, 2.5 million km



Probing the Universe



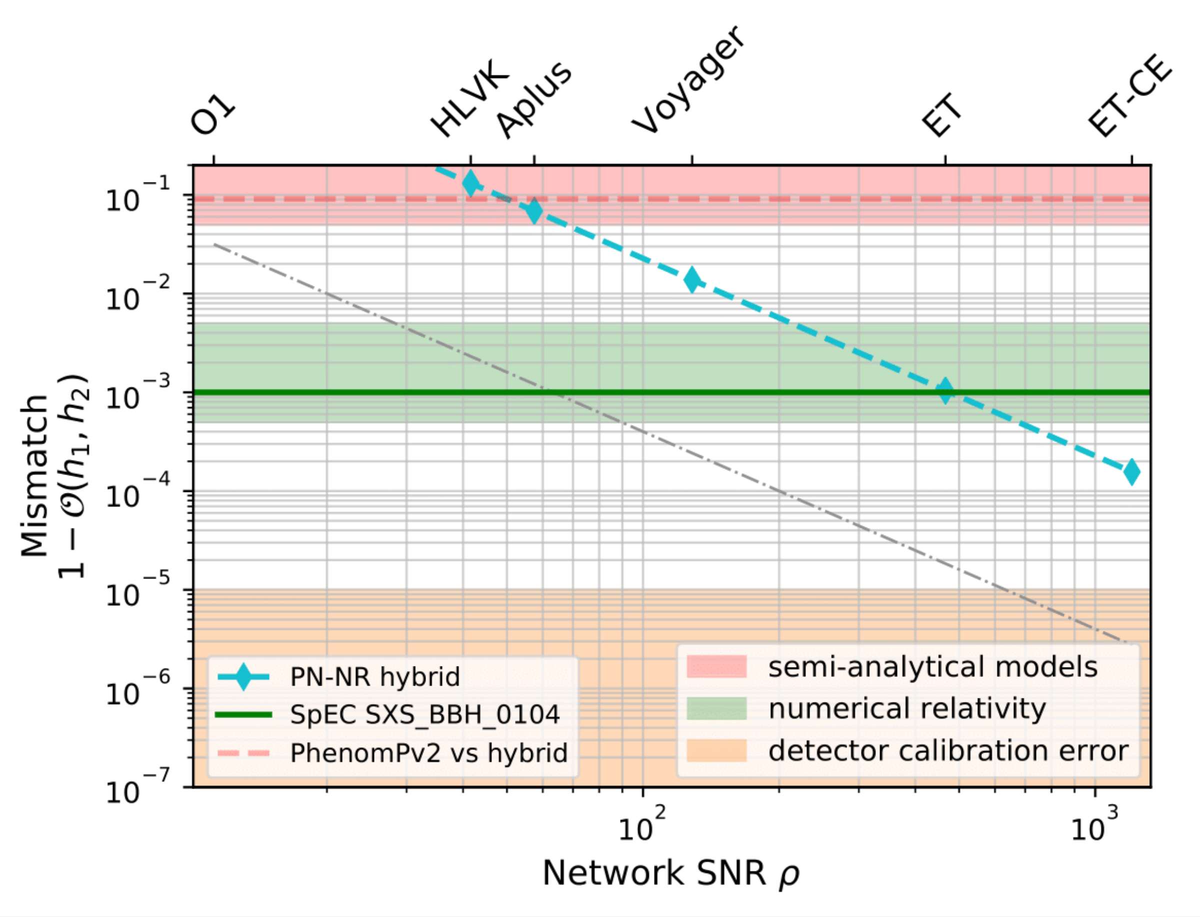
[ALMA collaboration]

Prospects

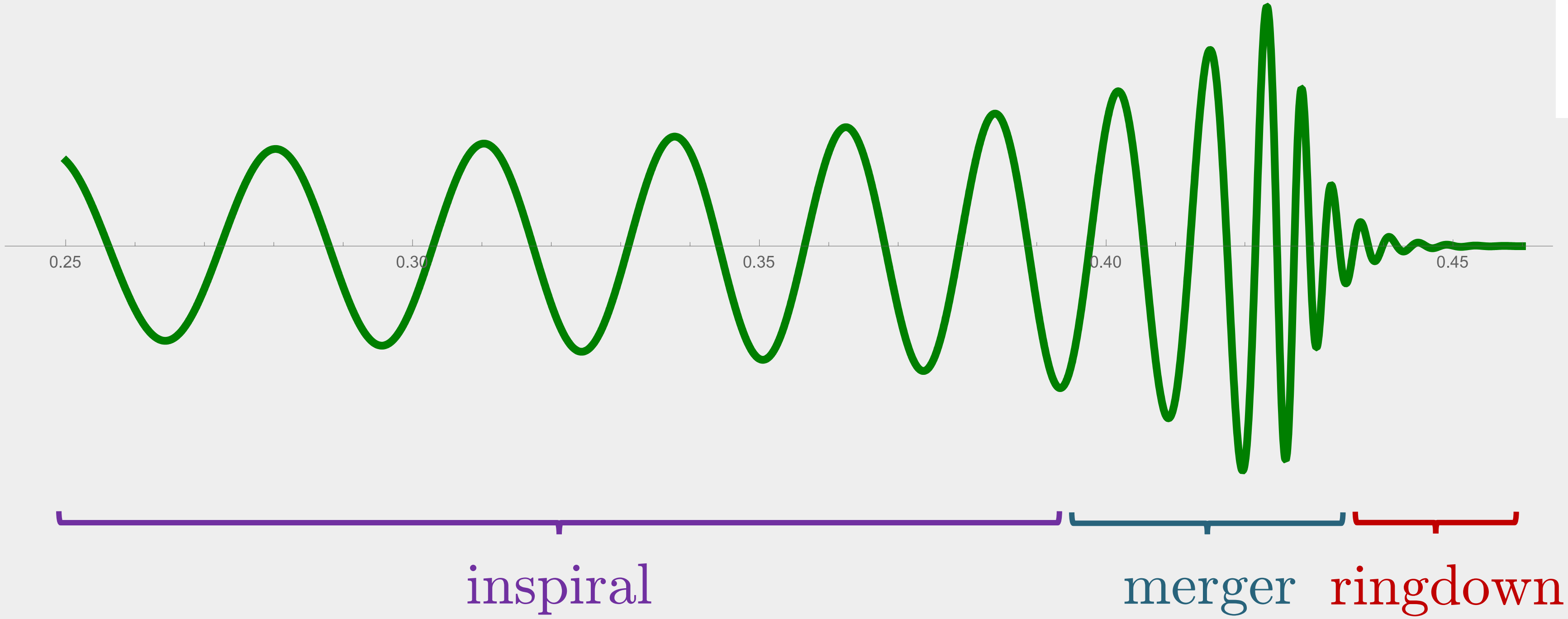
- Black hole properties
- Neutron star properties
- New astrophysical sources
- Dark matter (e.g. accreting on compact objects)
- Testing Einstein gravity (modified GW propagation)

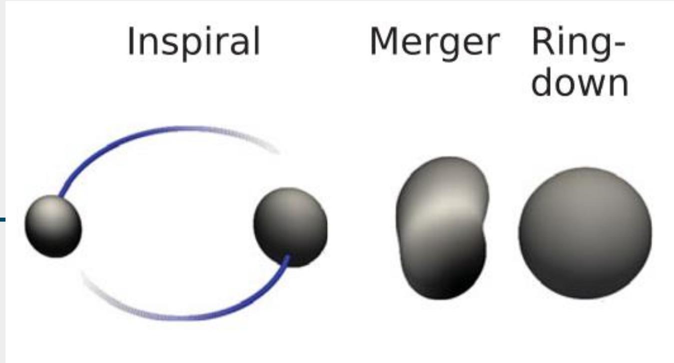
3G Detectors: Higher accuracy needed

[Pürrer, Haster, '20]



Gravitational two-body problem





- Inspiral: analytic models for parameter estimation studies (PES)
- Merger: numerical relativity (NR)
- Ringdown: BH perturbation theory

Effective one-body (EOB) theory for inspiral-merger-ringdown waveforms

PN vs PM

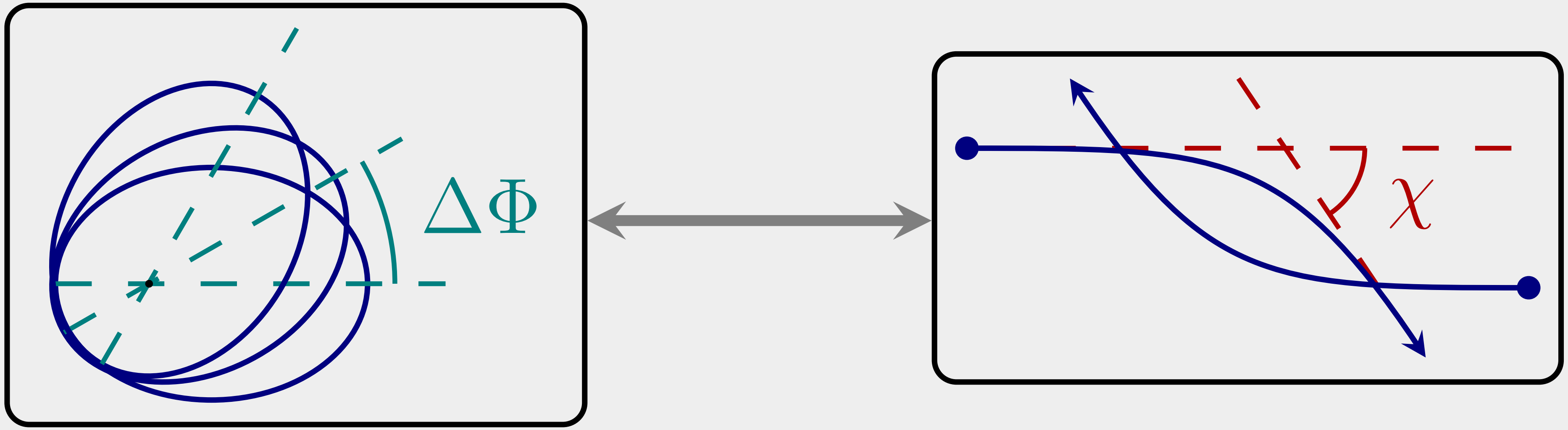
- Post-Newtonian (PN): $(v^2 \ll 1)$
  - non-relativistic expansion
  - integrals are non-relativistic (3d)
  - is simultaneously an expansion in the coupling

PN vs PM

- Post-Newtonian (PN): $(v^2 \ll 1)$
  - non-relativistic expansion
  - integrals are non-relativistic (3d)
  - is simultaneously an expansion in the coupling
- Post-Minkowskian (PM): $G \ll 1$
  - expansion in coupling
  - ➢ natural expansion for QFT tools
  - incorporates all orders in velocities (more general)
  - results are naturally for scattering events

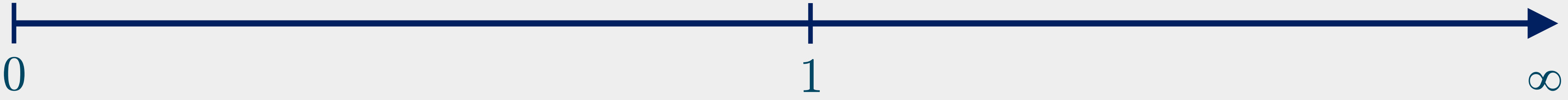
Bound vs Unbound motion

[Kälin, Porto, '20]



$\mathcal{E} > 0$ analytic continuation in binding energy $\mathcal{E} < 0$

Eccentricity e



PM-EFT

[Kälin, Porto, 20'; Kälin, Neef, Porto, 21']

$$S_{\text{pp}} = -\sum_{a=1,2} \frac{m_a}{2} \int \mathrm{d}\tau \, g_{\mu\nu} v_a^\mu v_a^\nu, \qquad S_{\text{EH}} = -2M_{\text{Pl}}^2 \int \mathrm{d}^4x \, \sqrt{-g} \, R$$

- EFT approach:

$$e^{iS_{\text{eff}}[x_a]} = \int \mathcal{D}h_{\mu\nu} e^{iS_{\text{pp}}[x_a,h] + iS_{\text{EH}}[h]}, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

  - Post-Minkowskian expansion:

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  - Post-Minkowskian expansion:

$$S_{\rm eff} = \sum_{n=0}^{\infty} \int \mathrm{d}\tau_1 G^n \mathcal{L}_n[x_1(\tau_1), x_2(\tau_2)]$$

  - Total momentum change through EOM:

$$0 = \sum_{n=0}^{\infty} \left(\frac{\partial \mathcal{L}_n}{\partial x_1^\nu} - \frac{\mathrm{d}}{\mathrm{d}\tau_1} \frac{\partial \mathcal{L}_n}{\partial v_1^\nu} \right)$$

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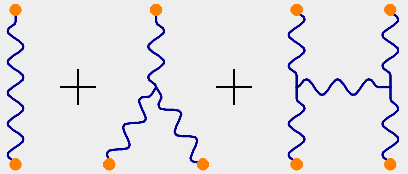
$\Delta p^\mu =$  $+ \dots$

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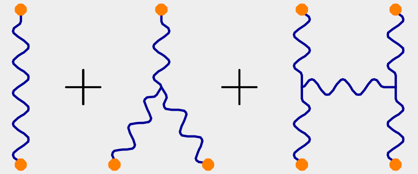
$\Delta p^\mu =$  $+ \ldots$

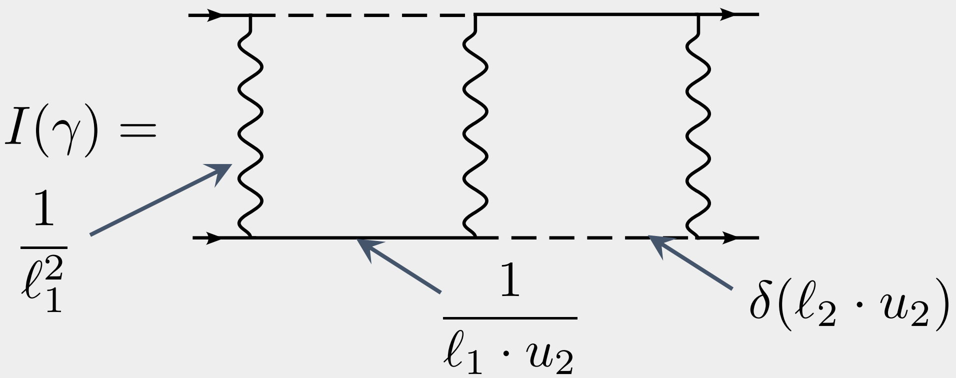
Other approaches:

- Scattering amplitudes [Cheung, Rothstein, Solon; Bern, Parra-Martinez, Roiban, Ruf, Shen, Zeng, ...]
- WL-QFT [Jakobsen, Mogull, Plefka, Steinhoff, '21]
- Eikonal [Di Vecchia, Heissenberg, Russo, Veneziano]
- ...

Analytic computation methods

[CD, Kälin, Liu, Porto, 23']

- Feynman integral computation



$D = 4 - 2\epsilon$

  - Integration-by-parts reduction
  - Method of differential equations (+ elliptic) $\partial_x \vec{f} = A(x,\epsilon)\vec{f}$

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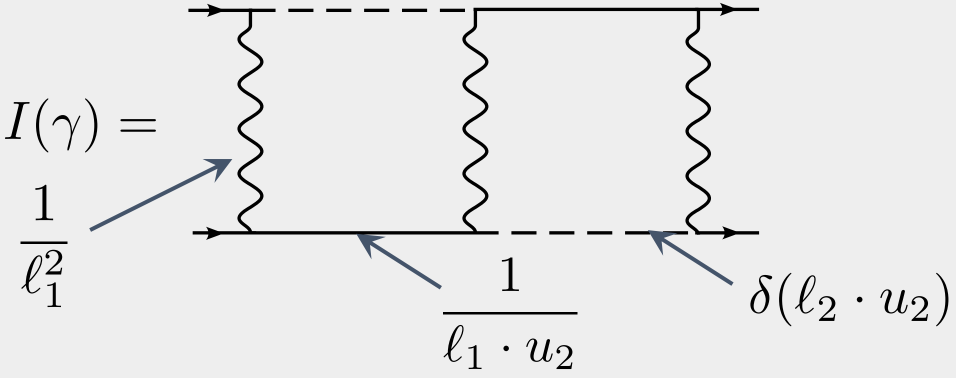
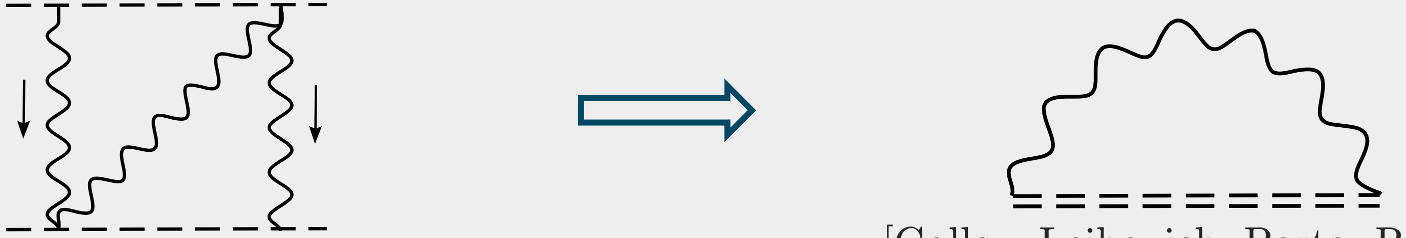


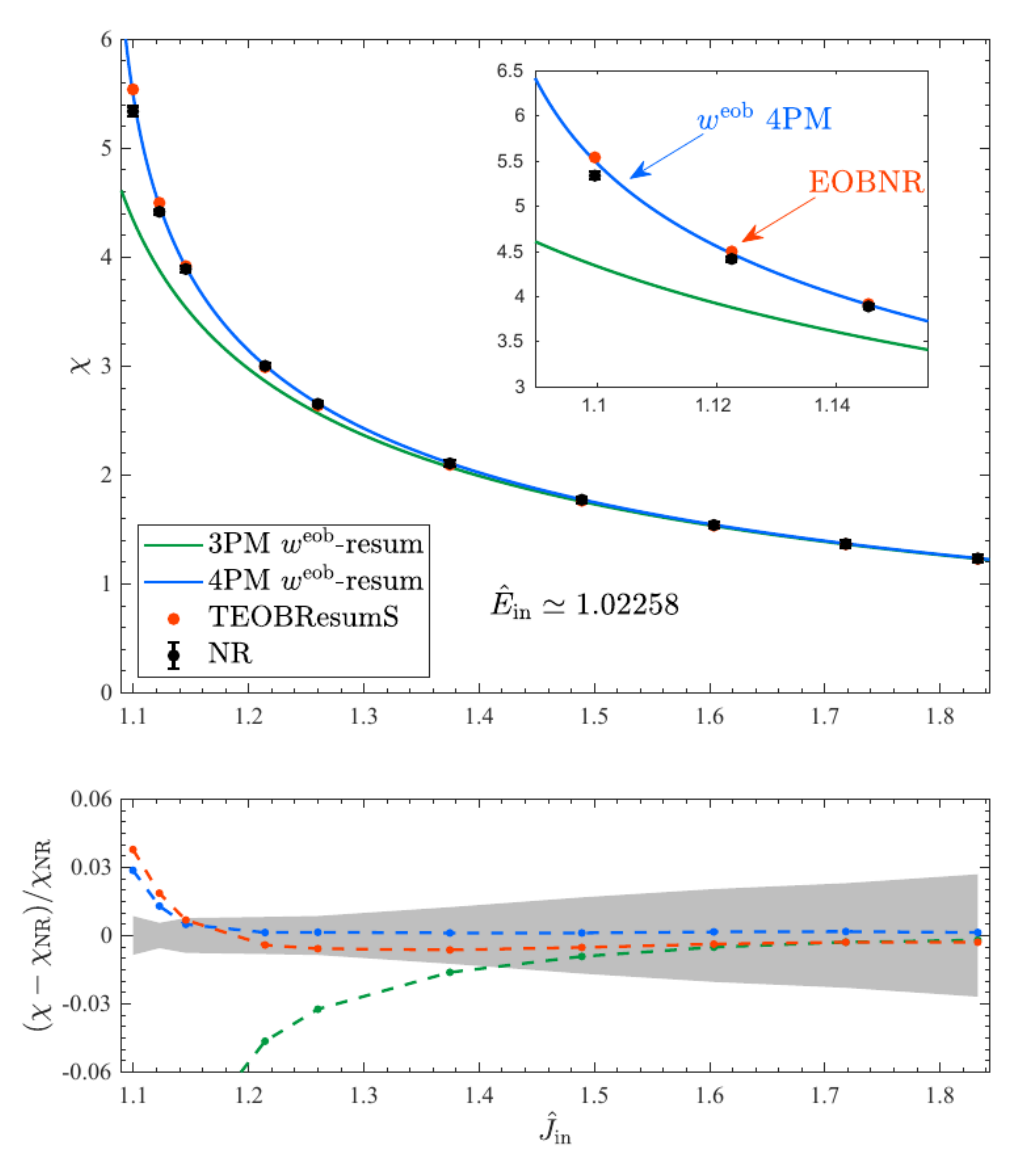
$D = 4 - 2\epsilon$

  - Integration-by-parts reduction
  - Method of differential equations (+ elliptic) $\partial_x \vec{f} = A(x, \epsilon)\vec{f}$
  - Boundary conditions from PN $(v^2 \ll 1)$



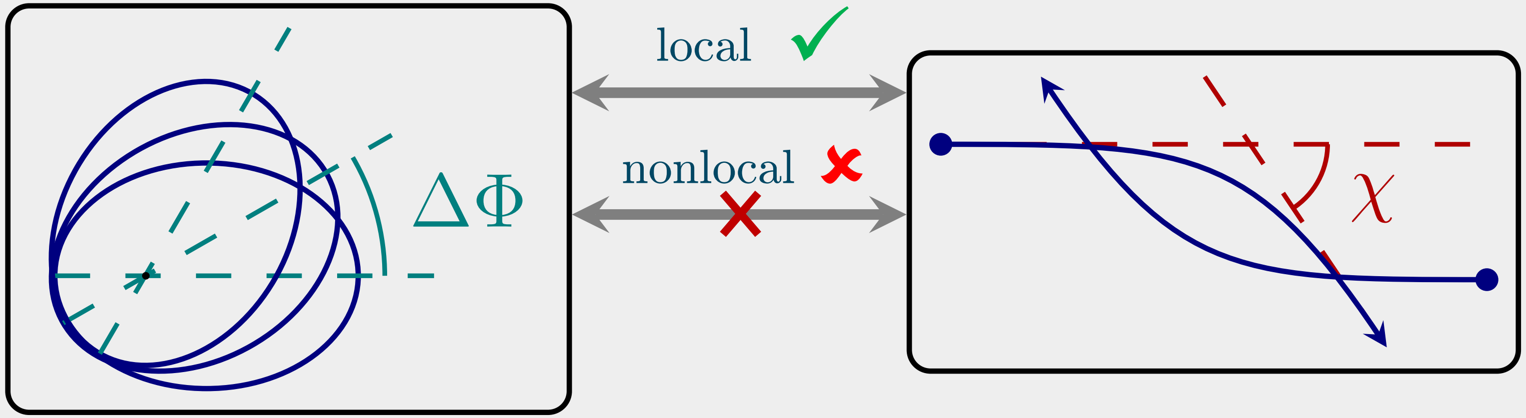
[Galley, Leibovich, Porto, Ross, '15]

PM radiative state-of-the-art accuracy



[Damour, Rettegno, '22]

Nonlocal-in-time contributions



[CD, Kälin, Liu, Porto, 24']

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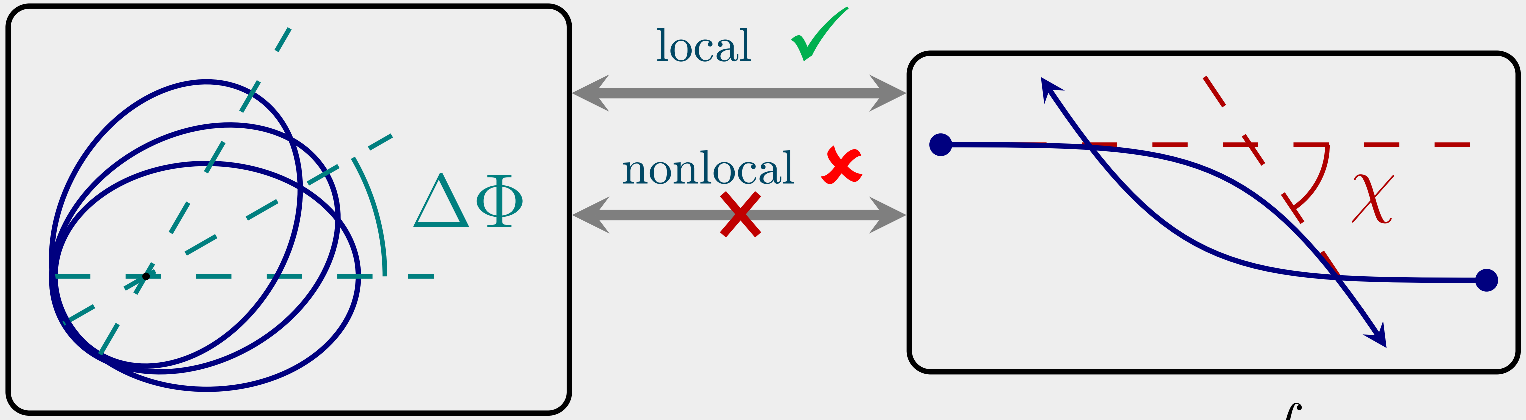
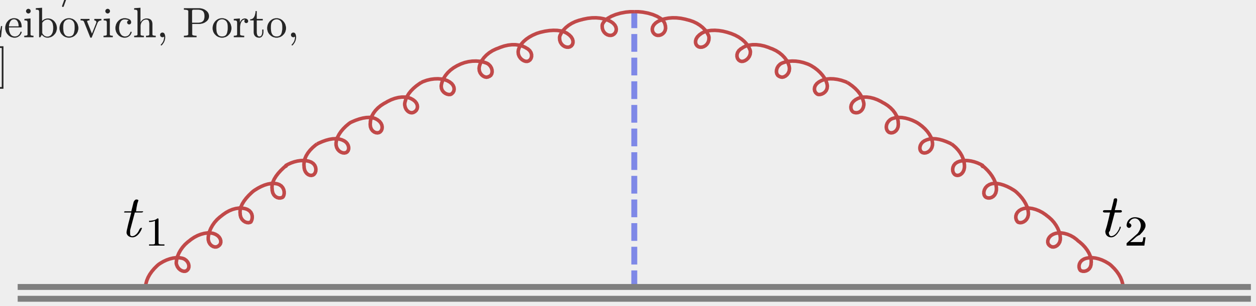


[Cho, Kälin, Porto, '22 / Damour, Jaranowski, Schäfer, '14 / Galley, Leibovich, Porto, Ross, '15]



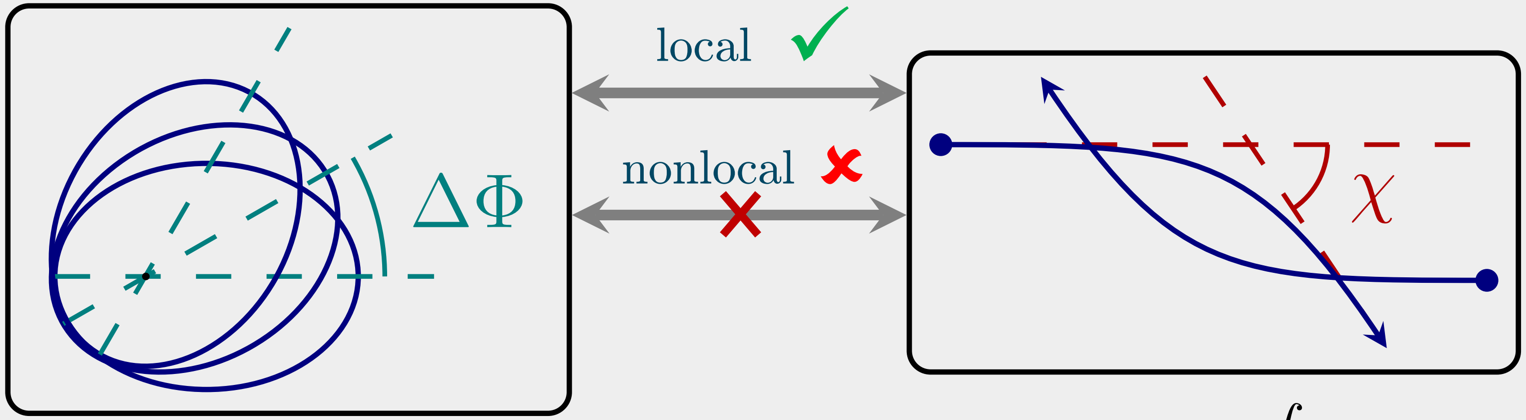
$$\mathcal{S}_r^{\mathrm{nloc}} \sim GE \int \mathcal{F}(t_1, t_2) \frac{\mathrm{d}t_1 \mathrm{d}t_2}{|t_1 - t_2|},$$

$$\sim GE \int \frac{\mathrm{d}E}{\mathrm{d}\omega} \log(\omega^2)\, \mathrm{d}\omega$$

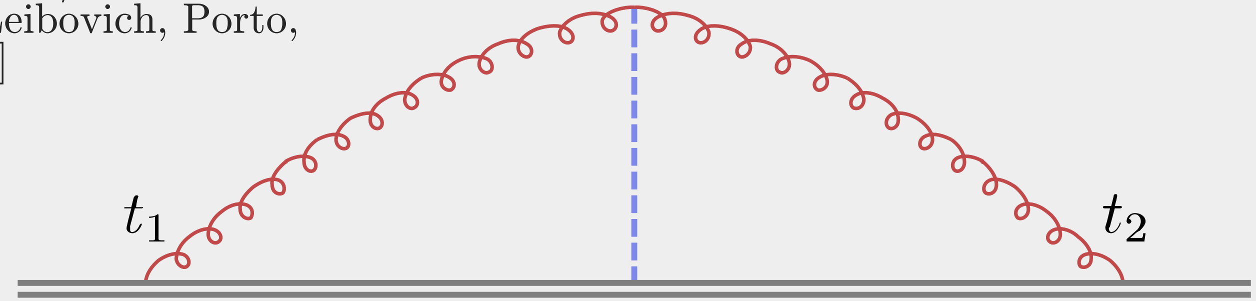
$\chi \sim \partial_j \mathcal{S}_r$

[CD, Kälin, Liu, Porto, 24']

Nonlocal-in-time contributions



[Cho, Kälin, Porto, '22 / Damour, Jaranowski, Schäfer, '14 / Galley, Leibovich, Porto, Ross, '15]



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$$\chi \sim \partial_j \mathcal{S}_r$$

$$\Delta p_1 \to \Delta E$$

- Related to previous computation: $\Delta E = \int \frac{\mathrm{d}E}{\mathrm{d}\omega} \mathrm{d}\omega,$ $\Rightarrow$ add log to integrand

[CD, Kälin, Liu, Porto, 24']

Nonlocal-in-time integrals at 4PM

- Nonlocal action: $\mathcal{S}_r^{\text{nonloc}} \sim GE \int \frac{\mathrm{d}E}{\mathrm{d}\omega} \log(\omega^2)\,\mathrm{d}\omega,$ $\omega = k \cdot u_{\text{com}}$

- Feynman integral computation:
  - Integration-by-parts reduction to master integrals
  - Differential equations
  - Boundary

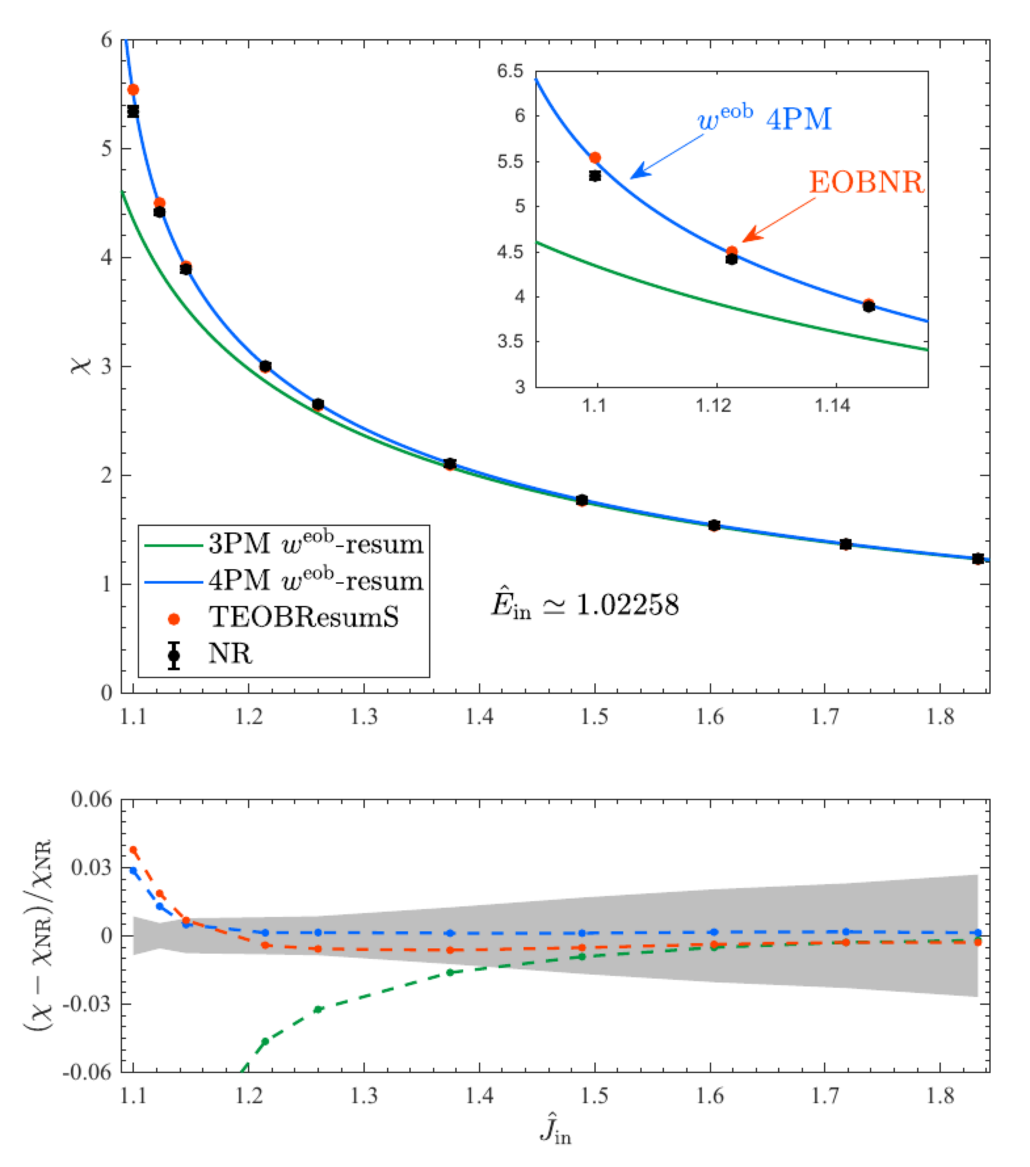
input from mathematics, computer science and particle physics

output to particle physics

- Goal is to compute the bound-case dynamics from this and eventually the waveform

potential impact on cosmology, APP, dark matter, PP, ...

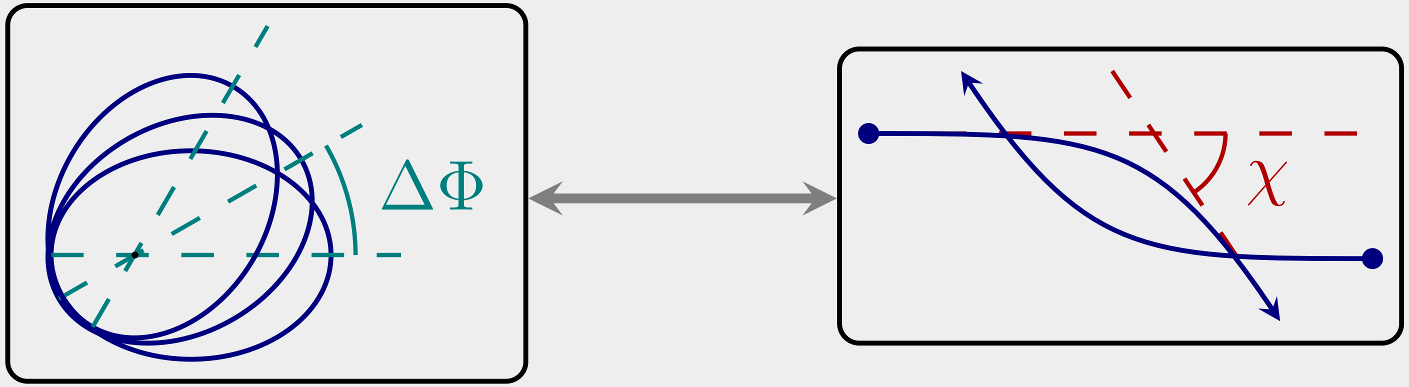
PM radiative state-of-the-art accuracy



[Damour, Rettegno, '22]

Summary

- Theory is not ready for 3G GW observatories
- Gravitational two-body problem
  - Post-Minkowskian approximation
  - Use tools from particle physics
  - Result for scattering shows remarkable agreement with NR
- B2B map:
  - Bound from scattering
  - Nonlocality problematic
- Outlook: Waveform



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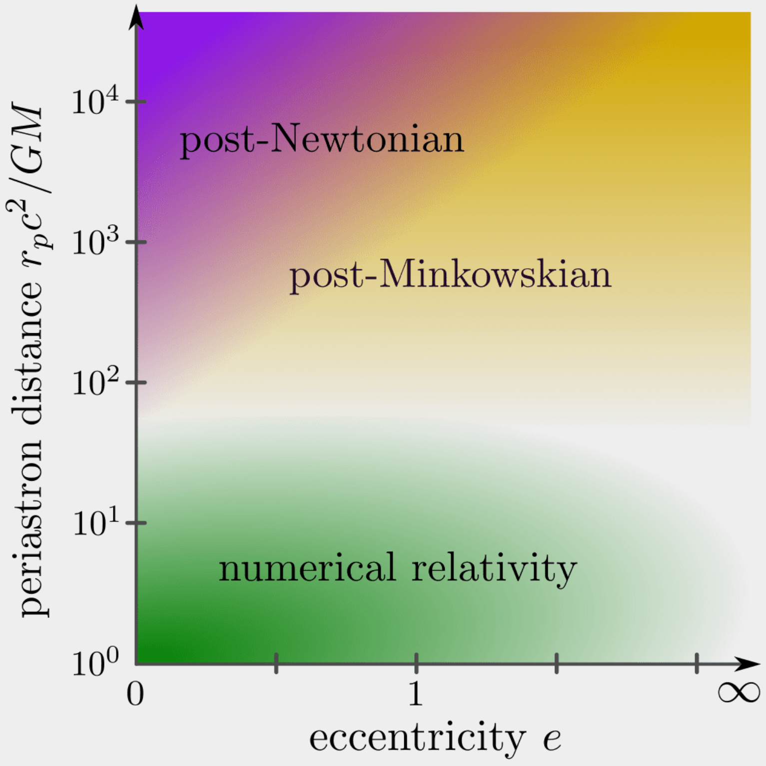
Post-Minkowskian (PM) expansion

- Natural pQFT approach $\Rightarrow$ recycle tools $G \ll 1$
- PM incorporates PN $(v^2 \ll 1) \Rightarrow$ similar results for overlap
- PM velocity resummation increases accuracy for scattering-like events
  - large-eccentricity, hyperbolic
  - detection rate is expected to increase in 3G detectors!
  - [Khalil, Buonanno, Steinhoff, Vines, '22; Damour, Rettegno, '23]
  - higher accuracy needed
  - [Pürrer, Haster, '20]

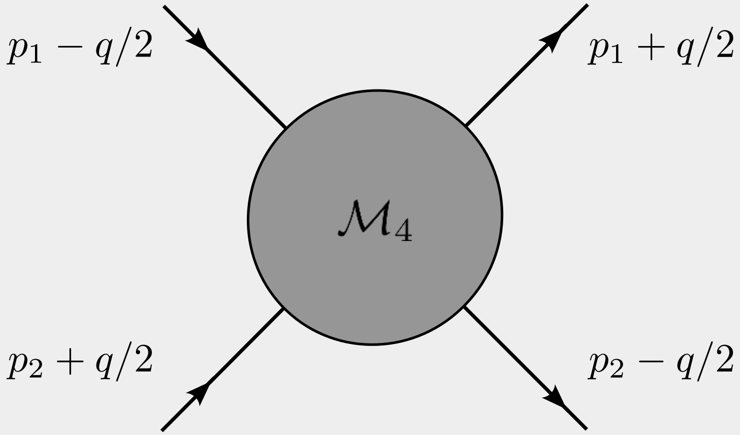


[Khalil, Buonanno, Steinhoff, Vines, '22]

[Bern, Cheung, Roiban, Shen, Solon, Zeng, '20]

The Scattering Amplitudes Approach

$$S_{\mathrm{GR}} = \int \mathrm{d}^D x \sqrt{-g}\left[-\frac{1}{16\pi G}R + \frac{1}{2}\sum_{a=1,2}\left(D^\mu \phi_a D_\mu \phi_a - m_a^2 \phi_a^2\right)\right]$$



$$p_1 \cdot q = p_2 \cdot q = 0$$

$$p_1^\mu = m_1 u_1^\mu, \quad p_2^\mu = m_2 u_2^\mu$$

- Integrand construction:
  - Generalized unitarity $\quad C = \sum M_{(1)}^{\text{tree}} M_{(2)}^{\text{tree}} \ldots M_{(m)}^{\text{tree}}$
  - Double copy $\quad M_{\text{tree}} \sim (A_{\text{tree}})^2$

    Gravity $\quad$ Yang-Mills

- Classical limit: $\quad G \longleftrightarrow$ number of loops

  $\hbar \longleftrightarrow |q| \quad$ leading term in soft expansion!

$$\frac{1}{(\ell + p_i - q/2)^2 - m_1^2} \simeq \frac{1}{2p_1 \cdot \ell} = \frac{1}{m_1\, 2u_1 \cdot \ell}$$

- Potential from EFT matching

Elliptic integrals

- Polylogarithms for most sectors

$$\partial_x \vec{g} = \epsilon\, \tilde{A}(x)\vec{g}, \qquad \tilde{A}(x) = \sum_i M_i a_i(x)$$

constant matrices

$$a_i(x) \in \left\{ \frac{1}{x}, \frac{1}{x-1}, \frac{1}{x+1}, \frac{x}{1+x^2} \right\}$$

new at 4PM

- Algorithms do not work for one sector
  - analyze ϵ^0-part:

$$\partial_x \vec{f} = A(x,\epsilon)\vec{f}$$

$$\partial_\gamma \vec{f} = A(\gamma,\epsilon)\vec{f} \longrightarrow \sqrt{\gamma^2-1}$$

$$T^{(0)}(\gamma)$$

$$A(x,\epsilon) = A^{(0)}(x) + \epsilon A^{(1)}(x) + \mathcal{O}(\epsilon^2), \qquad \partial_x T^{(0)}(x) = A^{(0)}(x) T^{(0)}(x)$$

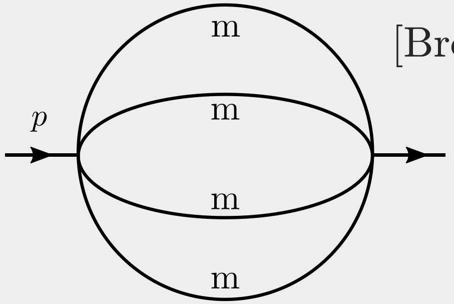
remove with transformation

Elliptic integrals

- Polylogarithms for most sectors $\partial_x \vec{g} = \epsilon\, \tilde{A}(x)\vec{g}, \quad \tilde{A}(x) = \sum_i M_i a_i(x)$

  constant matrices

- Algorithms do not work for one sector
  - Third-order DE: $$\left[\partial_x^3 - \frac{6x}{1-x^2}\partial_x^2 - \frac{1-4x^2+7x^4}{x^2(1-x^2)^2}\partial_x - \frac{1+x^2}{x^3(1-x^2)}\right]\Psi_{1,2,3} = 0$$
  - Solve with Mathematica: $\Psi_1 = x\mathrm{K}^2(1-x^2), \quad \Psi_2 = x\mathrm{K}(1-x^2)\mathrm{K}(x^2), \quad \Psi_3 = x\mathrm{K}^2(x^2)$



[Broedel, Duhr, Dulat, Marzucca, Penante, Tancredi, '19;
Broedel, Duhr, Matthes, '21;
Pögel, Wang, Weinzierl, '22]

$$\mathrm{K}(z) = \int_0^1 \frac{\mathrm{d}t}{\sqrt{(1-t^2)(1-zt^2)}},$$

$$\mathrm{E}(z) = \int_0^1 \frac{1-zt^2}{\sqrt{1-t^2}}\mathrm{d}t$$

ϵ-form using INITIAL

- Elliptic differential equations

68 master integrals:
$\Rightarrow$ constant 68×68 matrices

$$\partial_x \vec{g} = \epsilon\, \tilde{A}(x) \vec{g}, \qquad \tilde{A}(x) = \sum_i M_i a_i(x)$$

$$a_i(x) \in \left\{ \frac{\pi^2}{x(1-x^2)\mathrm{K}^2(1-x^2)}, \frac{1}{1-x}, \frac{1}{x}, \frac{1}{1+x}, \frac{x}{1+x^2}, \frac{\mathrm{K}^2(1-x^2)}{\pi^2 x(1-x^2)}, \frac{\mathrm{K}^2(1-x^2)}{\pi^2(1-x^2)}, \frac{\mathrm{K}^2(1-x^2)}{\pi^2 x}, \frac{\mathrm{K}^2(1-x^2)}{\pi^2}, \frac{(1-x^2)\mathrm{K}^2(1-x^2)}{\pi^2 x}, \frac{\mathrm{K}^4(1-x^2)}{\pi^4 x(1-x^2)}, \frac{\mathrm{K}^4(1-x^2)}{\pi^4 x}, \frac{(1-x^2)\mathrm{K}^4(1-x^2)}{\pi^4 x}, \frac{(1-x^2)^2\mathrm{K}^4(1-x^2)}{\pi^4 x} \right\}$$

  - Differential equations not problematic for us, despite elliptics
  - Canonical form just as in polylogarithmic 3PM case