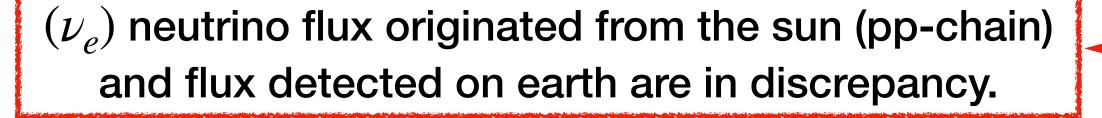


# A timeline of neutrino physics discoveries



Prediction of an undetected particle  $(\bar{\nu}_{\rho})$ in  $\beta$ -decay

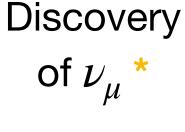




**Nobel Prize** 

Discovery of  $\nu_e^*$ 

Prediction of neutrino oscillations



solar neutrino puzzle \*

Discovery of  $u_{ au}$ 

(1998/2001)Clear observation of neutrino oscillations \*







2000

1930

1956

1957



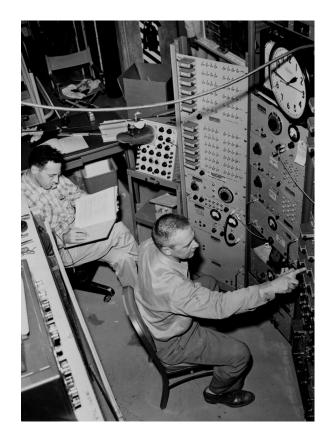
1960'

**DONUT Experiment** 

2015



W. Pauli



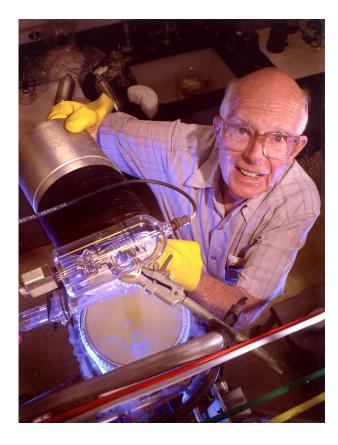
F. Reines and C. Cowan \*



B. Pontecorvo (Z. Maki, M. Nakagawa, S. Sakata)



L. Lederman \*



Homestake Experiment \*



A. McDonald \* **SNO** Experiment



T. Kajita \* Super-Kamiokande



#### Neutrino oscillations in the vacuum

$$|\nu_{\alpha}\rangle = \sum_{i} U_{\alpha i}^{*} |\nu_{i}\rangle$$

$$|\nu_{i}(t)\rangle = e^{-i(E_{i}t - \mathbf{p}_{i} \cdot \mathbf{x})} |\nu_{i}\rangle$$



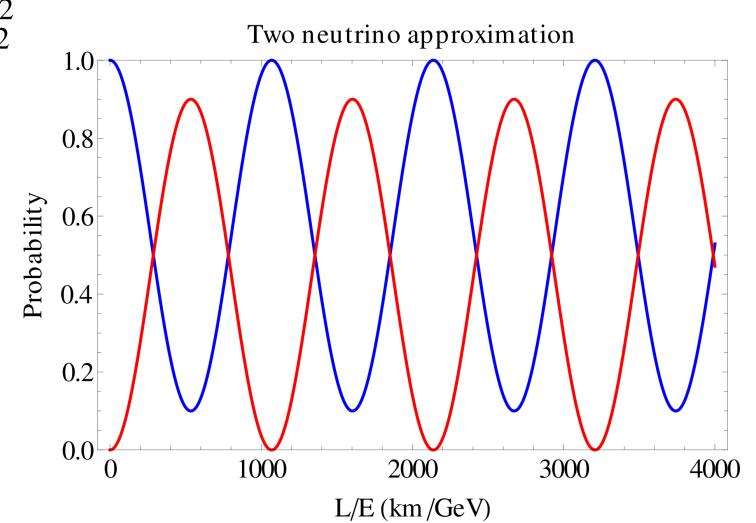
#### 2-flavour case

Ultrarelativistic limit  $t \approx L = |\mathbf{x}|$ 

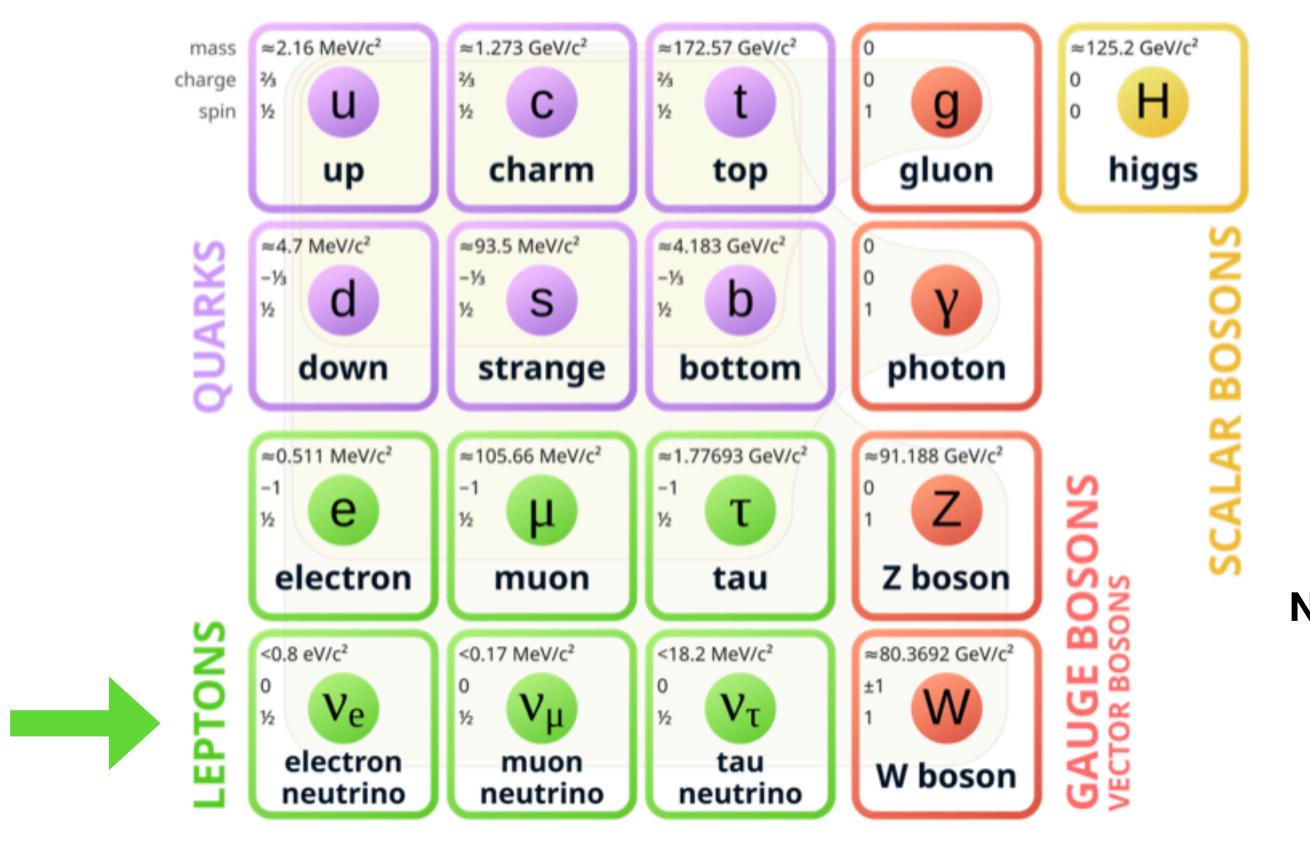
$$P_{\alpha \to \beta} = |\langle \nu_{\beta}(L) | \nu_{\alpha}(0) \rangle|^2 = \sin^2 \left(\frac{\Delta m^2 L}{4E}\right) \sin^2 \theta$$

$$\Delta m^2 = m_1^2 - m_2^2$$

Neutrinos cannot be massless!



#### Neutrino oscillations in the vacuum



 $|\nu_{\alpha}\rangle = \sum_{i} U_{\alpha i}^{*} |\nu_{i}\rangle$   $|\nu_{i}(t)\rangle = e^{-i(E_{i}t - \mathbf{p}_{i} \cdot \mathbf{x})} |\nu_{i}\rangle$ 



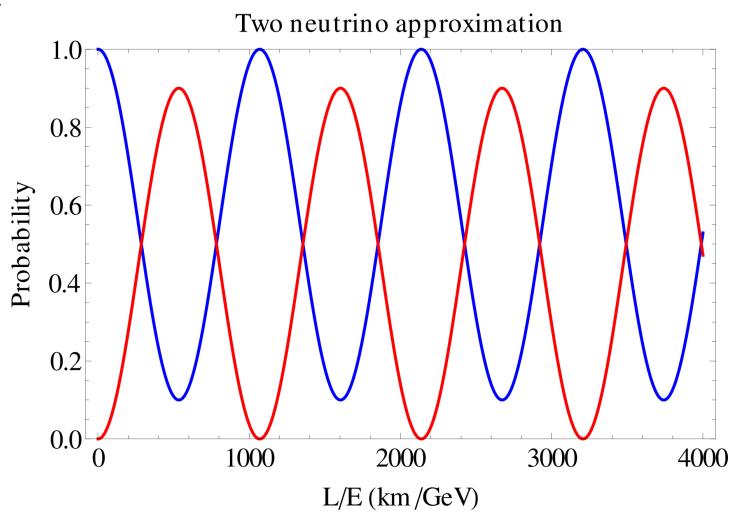
2-flavour case

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$$P_{\alpha \to \beta} = |\langle \nu_{\beta}(L) | \nu_{\alpha}(0) \rangle|^{2} = \sin^{2}\left(\frac{\Delta m^{2}L}{4E}\right) \sin^{2}\theta$$

$$\Delta m^{2} = m_{1}^{2} - m_{2}^{2}$$
Two neutrino approximation

**Neutrinos cannot** be massless!



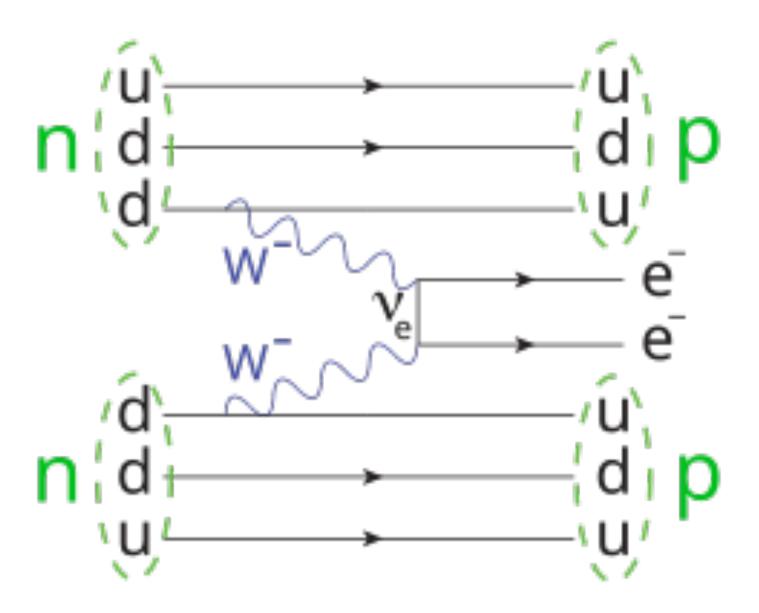
Need to revisit SM to account for massive neutrinos



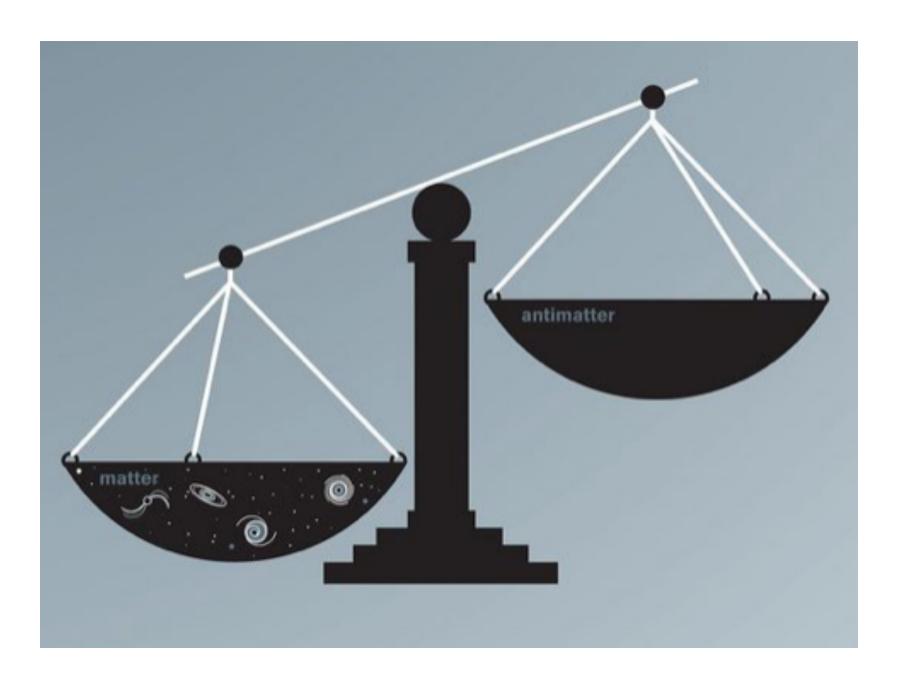
# Some open questions

♦ How do neutrinos acquire their masses? Fine tuning of Yukawa coupling? See-saw mechanism?

 $\spadesuit$  Are they Majorana particles? ( $\nu_{\ell}=\bar{\nu}_{\ell}$ ) addressed by  $0\nu\beta\beta$  experiments



lacktriangle Do  $ar{
u}_{\ell}$  oscillate differently than  $u_{\ell}$ ? CP violation Spharelons Matter-Antimatter Asymmetry



(Sakharov conditions)
[Talk by G. Zaharijas 06/10]

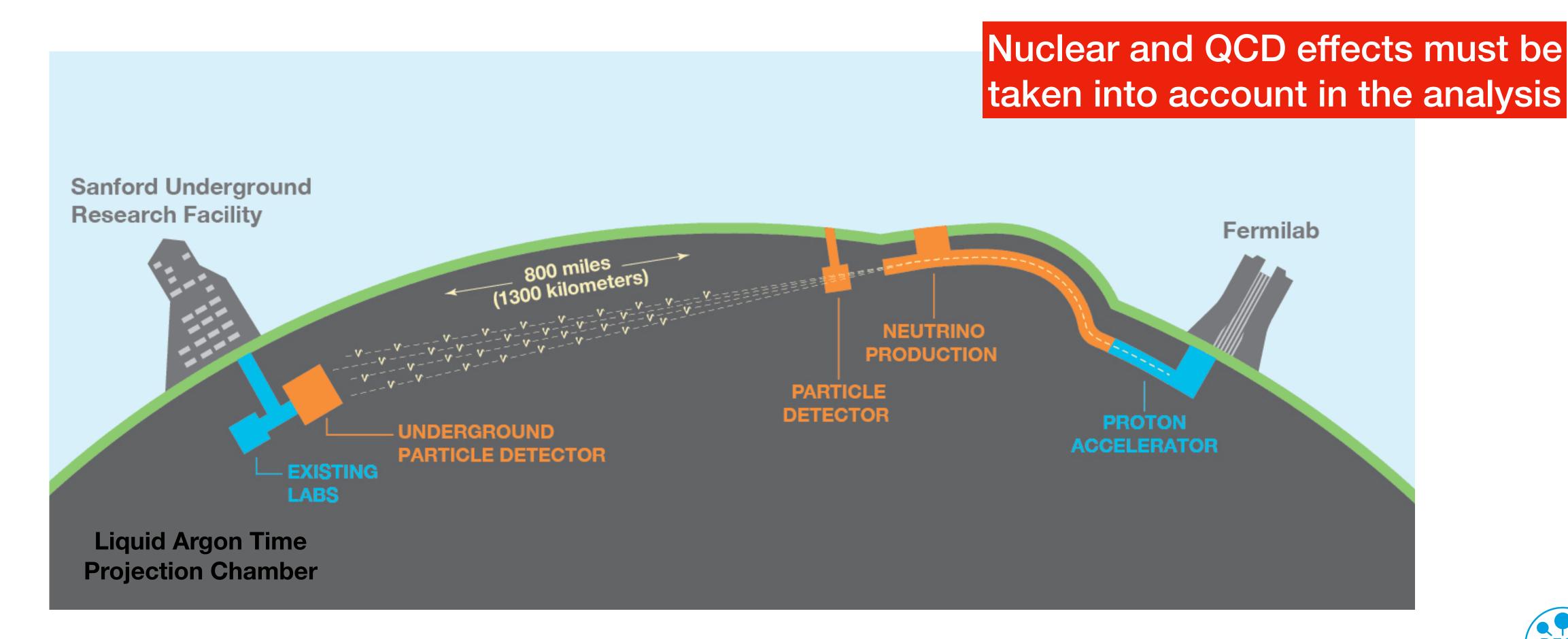


## Next-generation neutrino oscillation experiments: DUNE

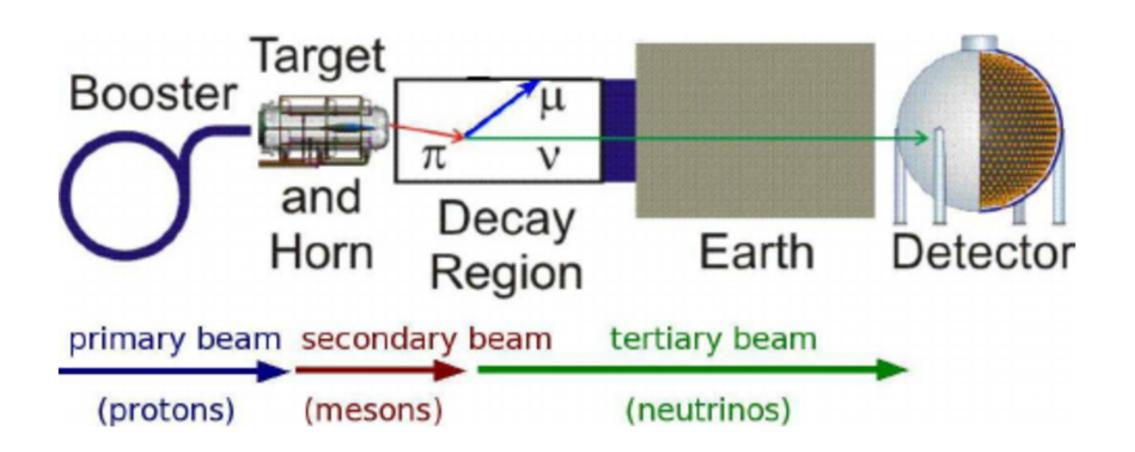
35 countries, ~200 institutions involved

> \$3B investment

Next-generation experiments aim to improve the accuracy of the neutrino mixing parameters and observe any asymmetry in how neutrinos and antineutrinos oscillate



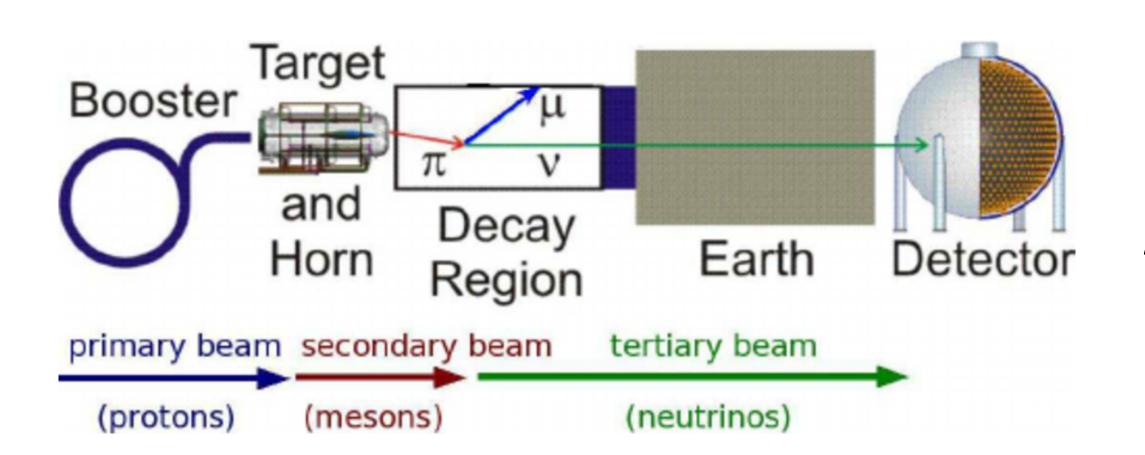
# Neutrino-nucleon interactions in neutrino oscillation experiments: MiniBooNE



 $u_u$  flux is artificially produced and then detected after a travel length L

$$\nu_{\mu} \; n \stackrel{\mathcal{J}^-}{\longrightarrow} \mu^- p \qquad \frac{d\sigma^{(\nu N)}}{dQ^2} \propto |\langle N|\mathcal{J}^-|N\rangle|^2 \qquad \mathcal{J}^- \text{ is the weak current}$$

## Neutrino-nucleon interactions in neutrino oscillation experiments: MiniBooNE



 $u_{\mu}$  flux is artificially produced and then detected after a travel length L

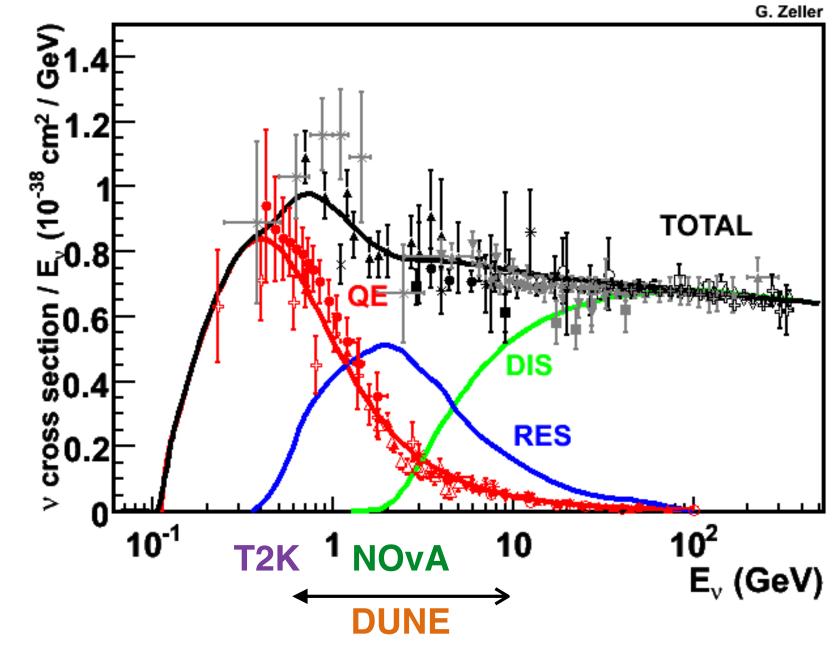
$$\nu_{\mu} \; n \xrightarrow{\mathcal{J}^{-}} \mu^{-} p \qquad \frac{d\sigma^{(\nu N)}}{dQ^{2}} \propto |\langle N|\mathcal{J}^{-}|N\rangle|^{2} \qquad \mathcal{J}^{-} \; \text{is the weak current}$$

#### Main challenges

- Nuclear model to factorise neutrino-nuclei scattering into neutrino-nucleon scattering (known from theory);
- ullet  $Q^2$  in a range where excited nucleons are produced!

USQCD Collaboration [arXiv:2203.09030, 2207.07641]

Required also knowledge of  $\langle N^* | \mathcal{J}^- | N \rangle$ ,  $\langle \Delta | \mathcal{J}^- | N \rangle$  and  $\langle N\pi | \mathcal{J}^- | N \rangle$ 



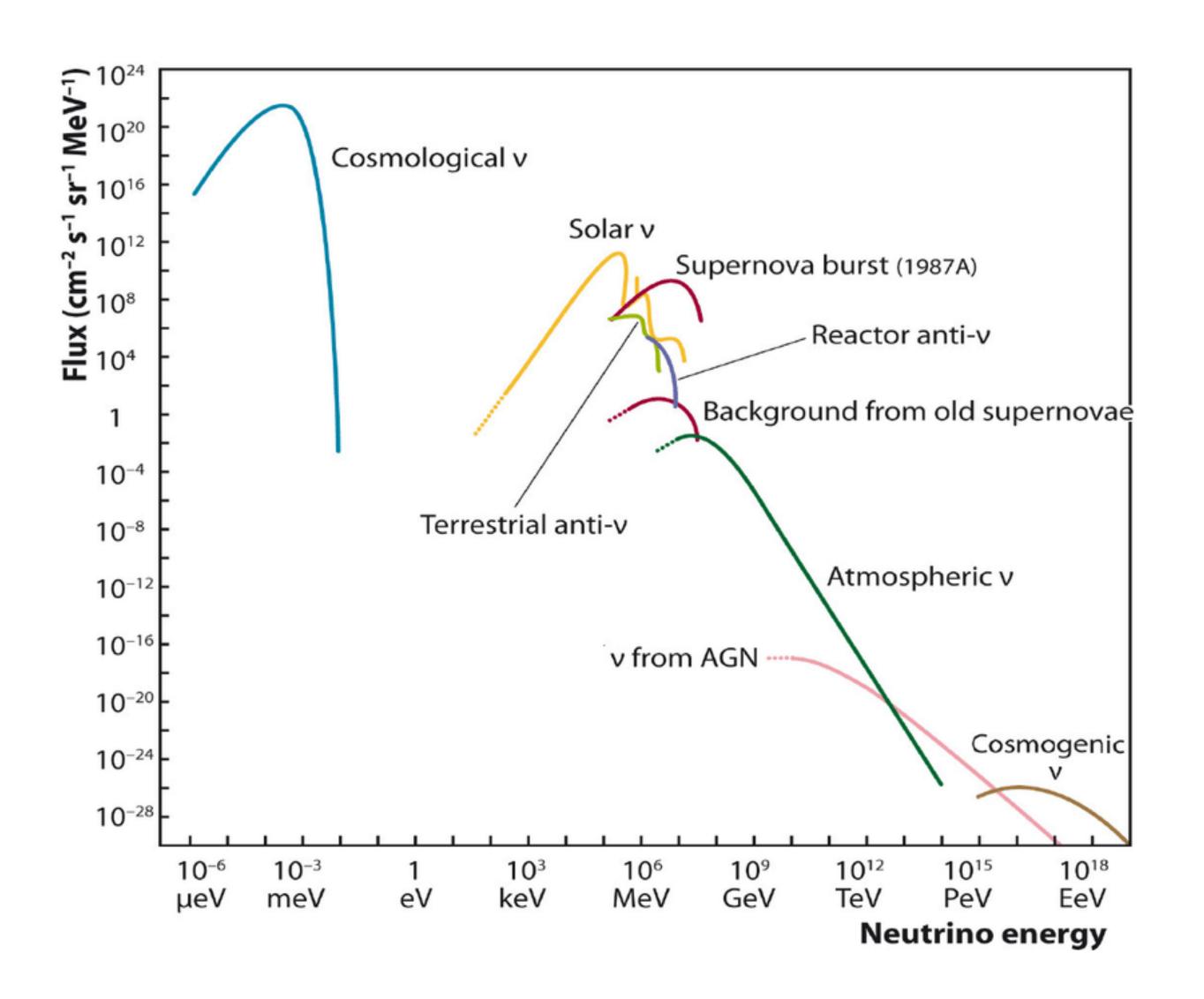
[J. A. Formaggio, G. Zeller, RMP.84.1307]

I am the first to investigate  $\langle N\pi | \mathcal{J}^- | N \rangle$  with LQCD

[**LB**, et al., *Phys.Rev.D* 107 (2023) 5, L051505, *arXiv:2110.11908*]



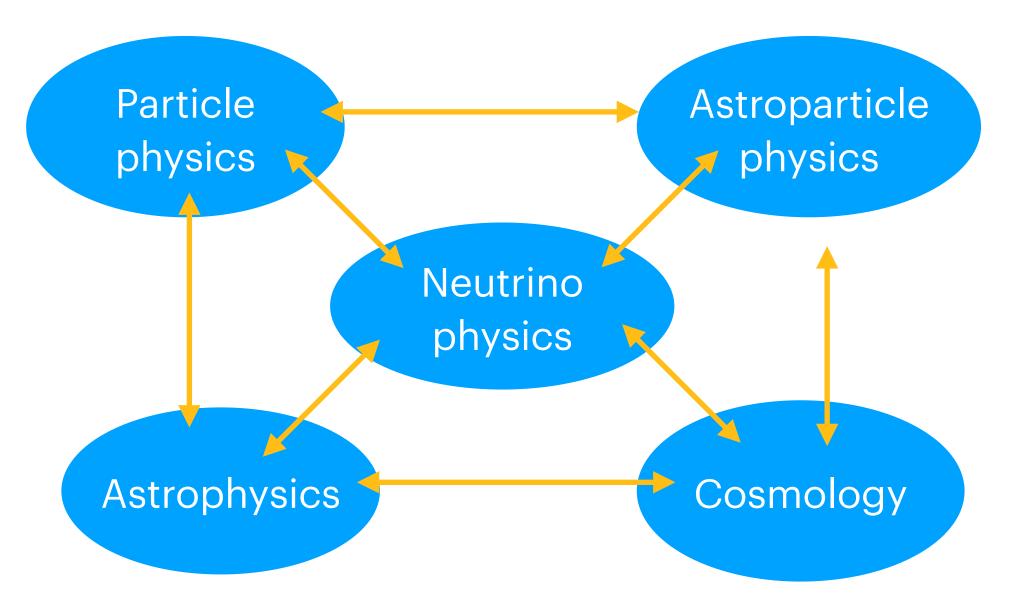
#### Neutrino flux from different sources



# Caveat

Similar nonperturbative effects may occur in other analysis involving "low-energy" neutrinos

Neutrino physics: a truly multidisciplinary field





Defined as 
$$\sigma_{fN} = m_f \langle N | \bar{q}_f q_f | N \rangle$$
 they enter in:

• Nucleon mass decomposition:  $m_N = \langle \ \frac{p}{2g} F^2 + \sum_{f=u,d,s,c,b,t} (1+\gamma_m) \ m_f \ \bar{q}_f q_f \ \rangle_N$ 

Defined as 
$$\sigma_{\!f\!N} = m_{\!f} \langle N \, | \, \bar{q}_f q_f \, | \, N \rangle$$

they enter in:

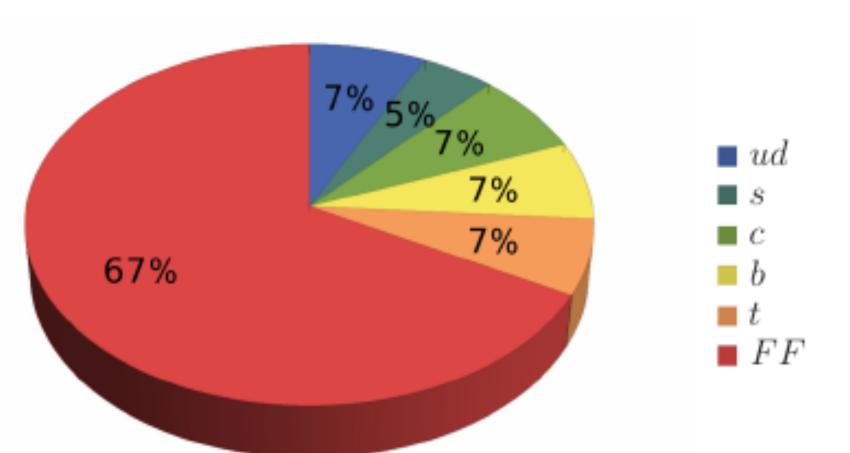
 $m_N = \left\langle \frac{\beta}{2g} F^2 + \sum_{f=u,d,s,c,b,t} (1 + \gamma_m) m_f \bar{q}_f q_f \right\rangle_N$ 

• Nucleon mass decomposition:

"Trace Anomaly" contribution due to gluon dynamics

(67% of the total nucleon mass)

[M. Hoferichter, J. Ruiz de Elvira Encyclopedia of Particle Physics]



Nucleon mass decomposition

"Quark mass" contribution (33% of the total nucleon mass)

Fun Fact In the chiral limit ( $m_f \rightarrow 0$ ) Nucleons are still heavy



Defined as 
$$\sigma_{\!f\!N} = m_{\!f} \langle N \, | \, \bar{q}_f q_f \, | \, N \rangle$$

they enter in:

• Nucleon mass decomposition:

$$m_N = \langle \frac{\beta}{2g} F^2 + \sum_{f=u,d,s,c,b,t} (1 + \gamma_m) m_f \bar{q}_f q_f \rangle_N$$

• BSM scenarios: WIMP, SUSY, ...

Many dark-matter candidates (e.g.  $\chi$ ) interact with nucleons via the exchange of a scalar particle (e.g. H)

$$\mathcal{L}_{\text{eff}} \supset \sum_{q} \frac{g_{\chi} g_{q}}{m_{h}^{2}} (\bar{\chi}\chi)(\bar{q}q)$$

WIMP- nucleon scalar cross section

$$\sigma^{SI} \propto |f_N|^2$$
  $f_N \propto \sigma_{fN}$ 

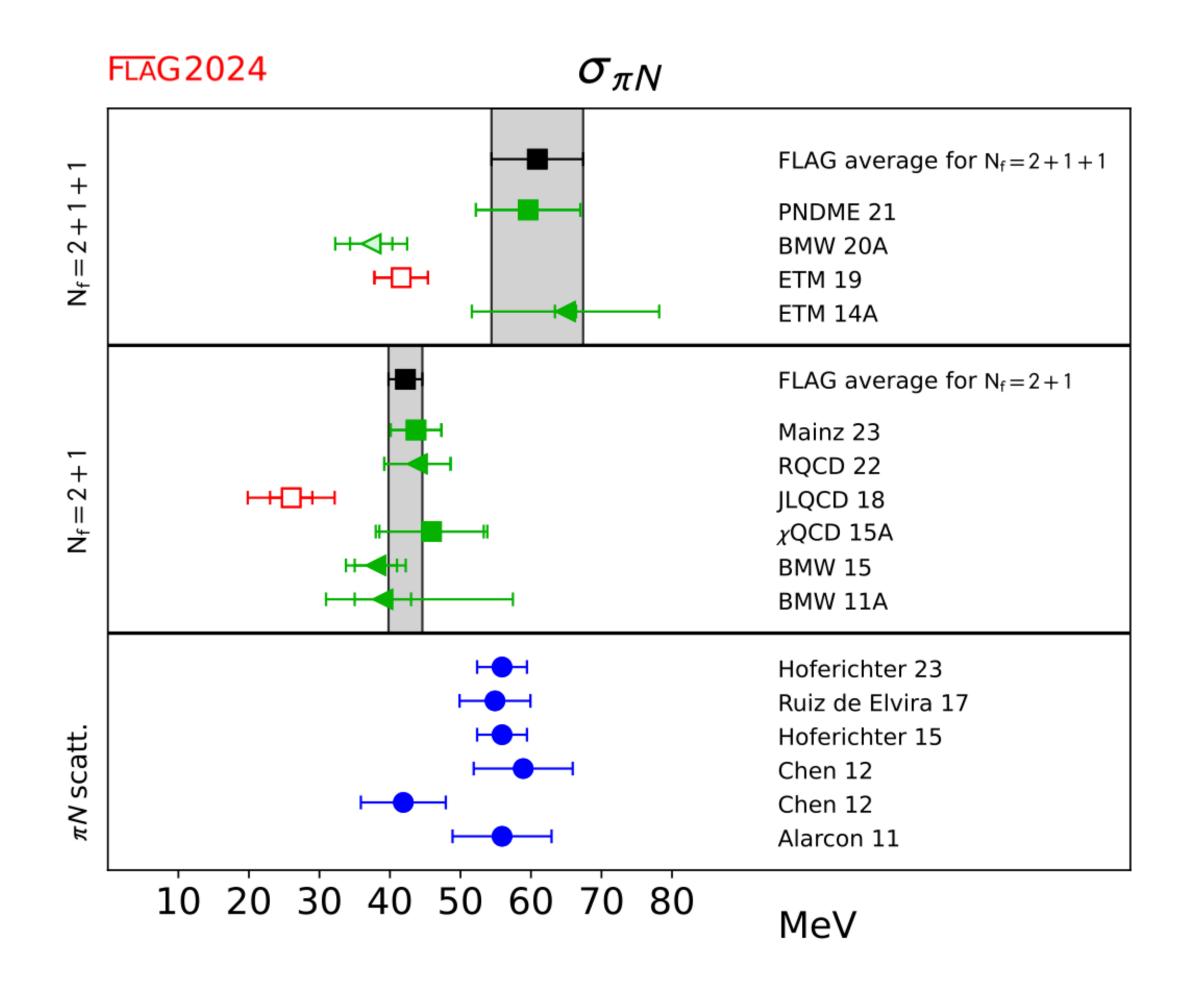
particle physics

Flux 
$$(\gamma, \nu)$$
 =  $\frac{1}{4\pi} \frac{(\sigma_{\text{ann}} \nu)}{2 m_{\chi}^2} \chi \sum_{i} BR_{i} \frac{dN_{\gamma}^{i}}{dE_{\gamma}} \chi$ 

cosmology

[Talk by G. Zaharijas 06/10]





$$\sigma_{fN} = m_f \langle N | \bar{q}_f q_f | N \rangle$$

$$\sigma_{\pi N} = \sigma_{uN} + \sigma_{dN}$$

Possible sources of uncontrolled systematic effects (lattice):

- Finite volume → Infinite Volume, continuum limit
- Physical quark mass extrapolation (arXiv:2508.11435)
   (Need good EFT)
- Uncontrolled excited state effects

[**LB**, et al., Phys.Rev.D 111 (2025) 3, L031505] [**LB**, arXiv:2508.09006 (Letter to PRD)]

Need to correctly account for excited state effects for reliable extraction of matrix elements.



# Lattice QCD Hadron Structure Calculations: Basics

$$\langle \mathcal{O}'_{N}(t) \mathcal{J}(\tau) \bar{\mathcal{O}}_{N}(0) \rangle = \int [dU][dq][d\bar{q}] \frac{e^{-S_{QCD}}}{\mathcal{Z}_{QCD}} \mathcal{O}'_{N}(t) \mathcal{J}(\tau) \bar{\mathcal{O}}_{N}(0)$$

(Spectral decomposition) = 
$$\langle N' | \mathcal{J} | N \rangle A'_N A_N e^{-E'_N(t-\tau)} e^{-E_N \tau}$$

$$+ \sum_{i,f \neq N} \langle i | \mathcal{J} | f \rangle A'_f A_i e^{-E'_f(t-\tau)} e^{-E_i \tau}$$

High-dimensional integral estimated via Monte Carlo sampling using HPC on supercomputers

 $\langle N' | \mathcal{J} | N \rangle$  can be extracted

For 
$$t \gg \tau \gg 0$$
  $(E'_f, E_i > E'_N, E_N)$ 

Excited states are suppressed.



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 $\langle N' | \mathcal{J} | N \rangle$  can be extracted

For 
$$t \gg \tau \gg 0$$
  $(E'_f, E_i > E'_N, E_N)$ 

Excited states are suppressed.

At large  $t, \tau$ , the statistical error grows (Lepage-Parisi) Analysis is thus restricted to short  $t, \tau$ 

Contribution from excited states is large And must be carefully accounted for.



# Lattice QCD Hadron Structure Calculations: Variational method (GEVP)

Instead of using a single nucleon operator  $O_N$   $O_N|0\rangle = c_N|N\rangle + c_{N\pi}|N\pi\rangle + \dots$   $\langle O_N'(t) \mathcal{J}(\tau) \bar{O}_N(0)\rangle$  construct a basis of operators with nucleon quantum numbers:  $O_i = O_N, O_{N\pi}, \dots$   $\langle O_i'(t) \mathcal{J}(\tau) \bar{O}_j(0)\rangle$  diagonalise the system, and construct improved operators  $\Omega_N$  s.t.

$$\langle \Omega'_{N}(t) \mathcal{J}(\tau) \bar{\Omega}_{N}(0) \rangle = \langle N' | \mathcal{J} | N \rangle A'_{N} A_{N} e^{-E'_{N}(t-\tau)} e^{-E_{N}\tau}$$

The variational approach allows us to extract also transition matrix elements like

$$\langle N\pi | \mathcal{J} | N \rangle$$
,  $\langle \Delta(1232) | \mathcal{J} | N \rangle$ ,  $\langle N^*(1440) | \mathcal{J} | N \rangle$ , ...

GEVP commonly used to study spectroscopy: [J. Green talk 06/07, A. Stump poster]



# Lattice QCD Hadron Structure Calculations: Variational method (GEVP)

Applied successfully for different nucleon structure quantities:

(Pilot studies at  $m_{\pi} = 430 \text{ MeV}$ )

$$O_N$$
,  $O_{N\pi}$ 

$$\langle N | \mathcal{A}_{\mu} | N \rangle$$
,  $\langle N \pi | \mathcal{A}_{\mu} | N \rangle$ 

$$O_N$$
,  $O_{N\sigma}$ 

$$\langle N | \mathcal{S} | N \rangle$$
,

$$O_N$$
,  $O_{N\rho}$ 

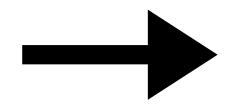
$$\langle N | \mathcal{V}_{\mu} | N \rangle$$

• [ETMc, PRD 110 (2024) 9, 094514]

Found a pattern of current-enhanced excited states

• [LB, arXiv:2508.09006 (letter to PRD)]

and a formalism to explain such current-enhanced excited states with broad applications



Plenary talk at Lattice Conference 2025, TIFR, Mumbai, 2-8 November



# Lattice QCD Hadron Structure Calculations: Plans for the future

Reliable physical point extrapolation of

- $\sigma_{\pi N}$  with  $O_N$ ,  $O_{N\sigma}$  ~3 years (very soon results at 220 MeV)
- $\langle N | \mathcal{J}_W | N \rangle$ ,  $\langle N\pi | \mathcal{J}_W | N \rangle$ ,  $\langle \Delta(1232) | \mathcal{J}_W | N \rangle$  with  $O_N$ ,  $O_{N\pi}$ ,  $O_{\Delta}$  5 7 years

Working in collaboration with researchers from U. Regensburg

Fermilab, Berkeley, Los Alamos

Many US institutions are very interested in this research ( $\rightarrow$ DUNE, DM experiments)

Currently writing competitive grant applications for continuing this work.

More and more lattice groups will start using variational approach for hadron structure.



#### Conclusions.

#### What is the relevance of your research within your research field?

Very relevant. It has important implications in hadron structure calculations.

### What is the relevance of your research for other fields?

Understanding the low-energy structure of nucleons is of high relevance for particle physics, astroparticle physics (DM searches, e.g. Lux-zeplin, xenonnt, ...) and cosmology (matter-antimatter).

### What methods are you using or developing that could be useful for others?

HPC on supercomputers, Monte Carlo sampling and variational analysis for faster ground state convergence



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HPC on supercomputers, Monte Carlo sampling and variational analysis

for faster ground state convergence

Thank you!



Backup slides

# Imprints of baryon asymmetry in neutrino physics

Spharelons 



## Symmetry:

Neutrinos and antineutrinos change flavors at the same rate.

Neutrino oscillations



Antineutrino



## Asymmetry:

Neutrinos and antineutrinos change flavors at different rates.

Neutrino oscillations





Antineutrino oscillations





# Neutrinoless double beta-decay

$$(A, Z) \to (A, Z + 2) + 2e^{-}$$

$$\Gamma_{\beta\beta}^{0\nu} = \frac{1}{T_{\beta\beta}^{0\nu}} \propto |M^{0\nu}|^2 \langle m_{\beta\beta} \rangle^2$$

$$\langle m_{\beta\beta} \rangle = \sum_{i} U_{ei}^2 m_i$$

Isotope ÷	Experiment +	lifetime $T^{0\nu}_{\beta\beta}$ [years] $\Rightarrow$
$^{48}\mathrm{Ca}$	ELEGANT-VI	$>1.4\cdot 10^{22}$
$^{76}{ m Ge}$	Heidelberg-Moscow <sup>[14]</sup>	$> 1.9 \cdot 10^{25}$ [14]
$^{76}{ m Ge}$	GERDA	$> 1.8 \cdot 10^{26}$ [15]
$^{82}\mathrm{Se}$	NEMO-3	$>1.0\cdot 10^{23}$
$^{82}\mathrm{Se}$	CUPID-0	$>4.6\cdot 10^{24}$ [16]
$^{96}{ m Zr}$	NEMO-3	$>9.2\cdot 10^{21}$
$^{100}\mathrm{Mo}$	NEMO-3	$>2.1\cdot 10^{25}$
$^{116}\mathrm{Cd}$	Solotvina	$>1.7\cdot 10^{23}$
$^{130}\mathrm{Te}$	CUORE	$>2.2\cdot 10^{25}$
$^{136}\mathrm{Xe}$	EXO	$> 3.5 \cdot 10^{25}$ [17]
$^{136}\mathrm{Xe}$	KamLAND-Zen	$> 1.07 \cdot 10^{26}$ [18]
$^{150}\mathrm{Nd}$	NEMO-3	$>2.1\cdot 10^{25}$

 $M^{0
u}$  is the nuclear matrix element

 $U_{ei}$  is a PMNS matrix element

#### **LQCD** studies

#### **Short & Long-distance**

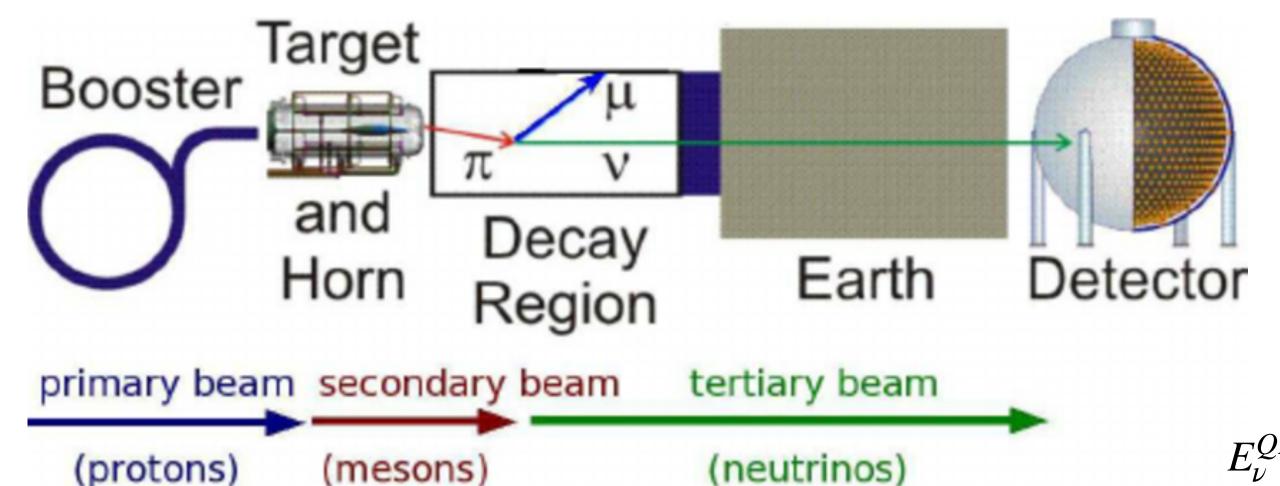
$$\pi^- \rightarrow \pi^+ e^- e^-$$

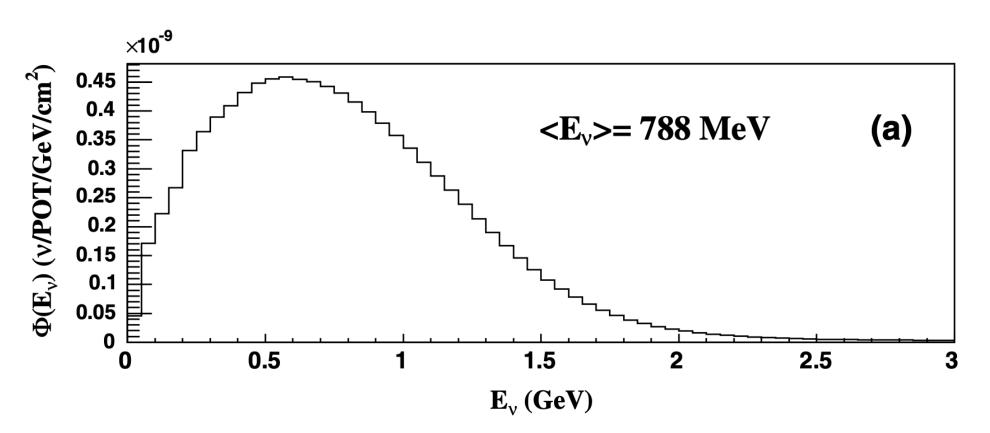
MIT, *Phys.Rev.D* 107 (2023) 9, 094501

NPLQCD Collab., arXiv:2004.07404

very rare process

# MiniBooNE experiment





Predicted flux at the detector

$$E_{\nu}^{QE} = \frac{2M_{N}E_{\mu} - M_{N}^{2} + m_{\mu}^{2} + M_{p}^{2}}{2[M_{N} - E_{\mu} + \cos\theta_{\mu}\sqrt{E_{\mu}^{2} - m_{\mu}^{2}}]}$$

$$M_N = M_n - E_b$$

(Carbon binding energy)

Booster: proton beam is accelerated

**Detector:** spherical steel tank filled with 818 tons of mineral oil  $(CH_2)$ 

Target: proton beam scatters off a target, producing a meson beam

Mesons' decay: (muon) neutrino beam is produced

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

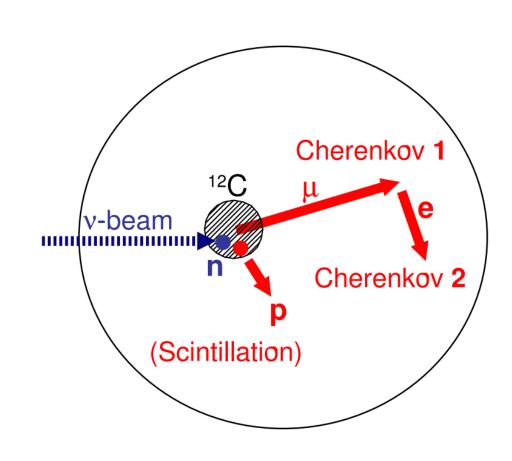
**Earth:** neutrino beam travels a length  $L \approx 450 \mathrm{m}$ 

**Detector:** muons are produced and accelerated in matter

$$\nu_{\mu} + n \rightarrow \mu^{-} + p$$

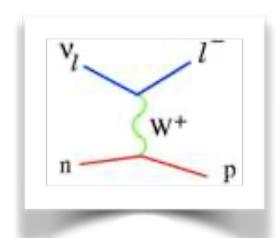
**Detector:** muons are produced and accelerated in matter 
$$\nu_{\mu} + n \rightarrow \mu^{-} + p$$

-> Cherenkov light (refractive index)  $n = \frac{c}{v} = 1.47$  -> Cherenkov threshold:  $\beta = \frac{v}{c} > 0.68$ 

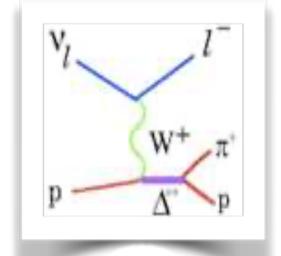


CCQE interaction at the MiniBoone detector

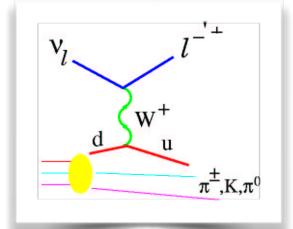
#### Quasi-elastic scattering (QE)

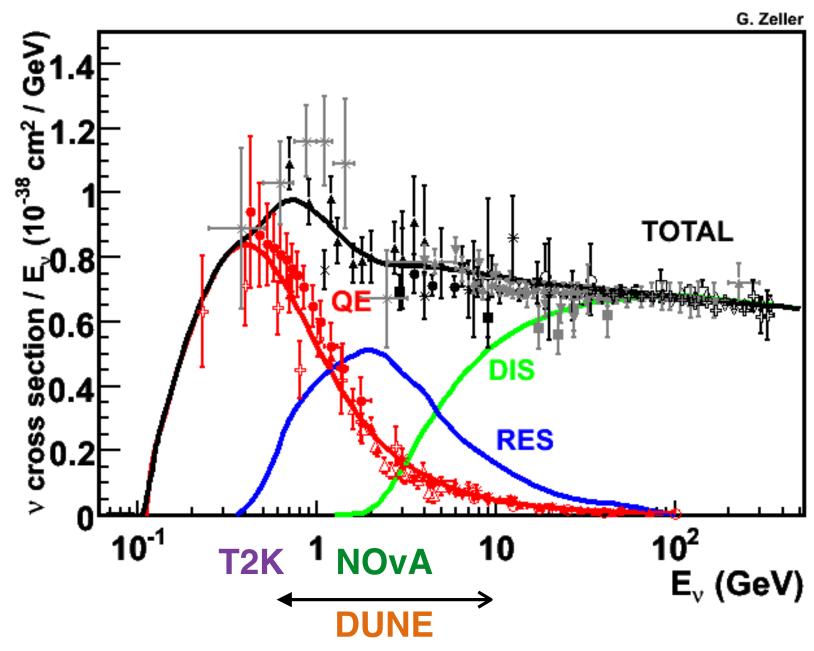


#### Resonance production (RES)



#### Deep Inelastic scattering (DIS)





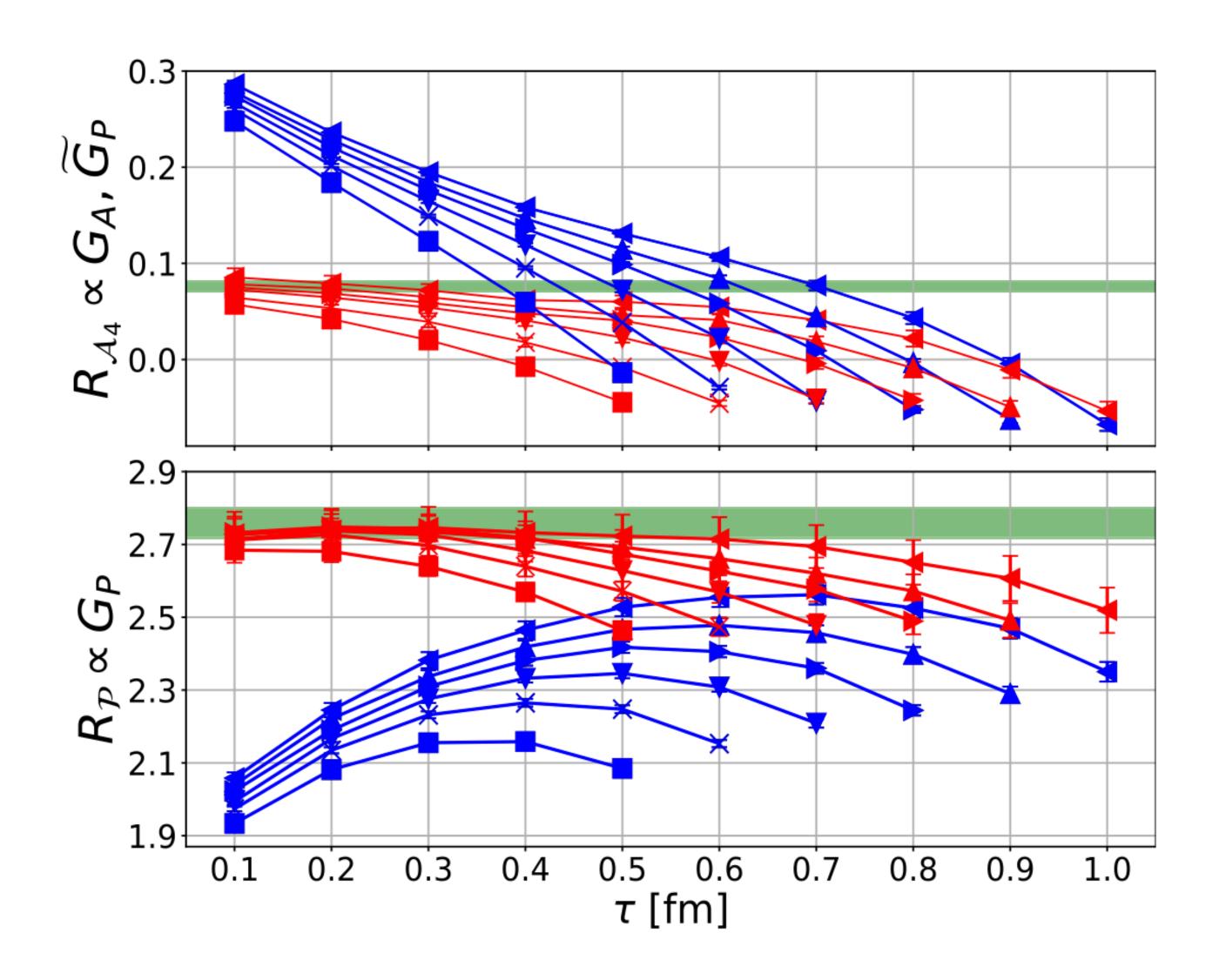
J.A. Formaggio, G. Zeller, Reviews of Modern Physics, 84 (2012)

[arXiv:2203.09030]

neutrino process	abbreviation	reaction	fraction (%)
CC quasielastic	CCQE	$ u_{\mu} + n \rightarrow \mu^{-} + p $	39
NC elastic	NCE	$ u_{\mu} + p(n)  ightarrow  u_{\mu} + p(n)$	16
CC $1\pi^+$ production	$CC1\pi^+$	$ u_{\mu}+p(n) ightarrow\mu^{-}+\pi^{+}+p(n)$	25
$CC 1\pi^0$ production	$\mathrm{CC}1\pi^0$	$ u_{\mu}+n ightarrow\mu^{-}+\pi^{0}+p$	4
NC $1\pi^{\pm}$ production	$\mathrm{NC}1\pi^{\pm}$	$ u_{\mu} + p(n) \rightarrow \nu_{\mu} + \pi^{+}(\pi^{-}) + n(p) $	4
NC $1\pi^0$ production	$ ext{NC}1\pi^0$	$ u_{\mu}+p(n) ightarrow u_{\mu}+\pi^{0}+p(n)$	8
multi pion production, DIS, etc.	other	$\nu_{\mu} + p(n) \to \mu^{-} + N\pi^{\pm} + X$ , etc.	4

[PRD.81.092005]

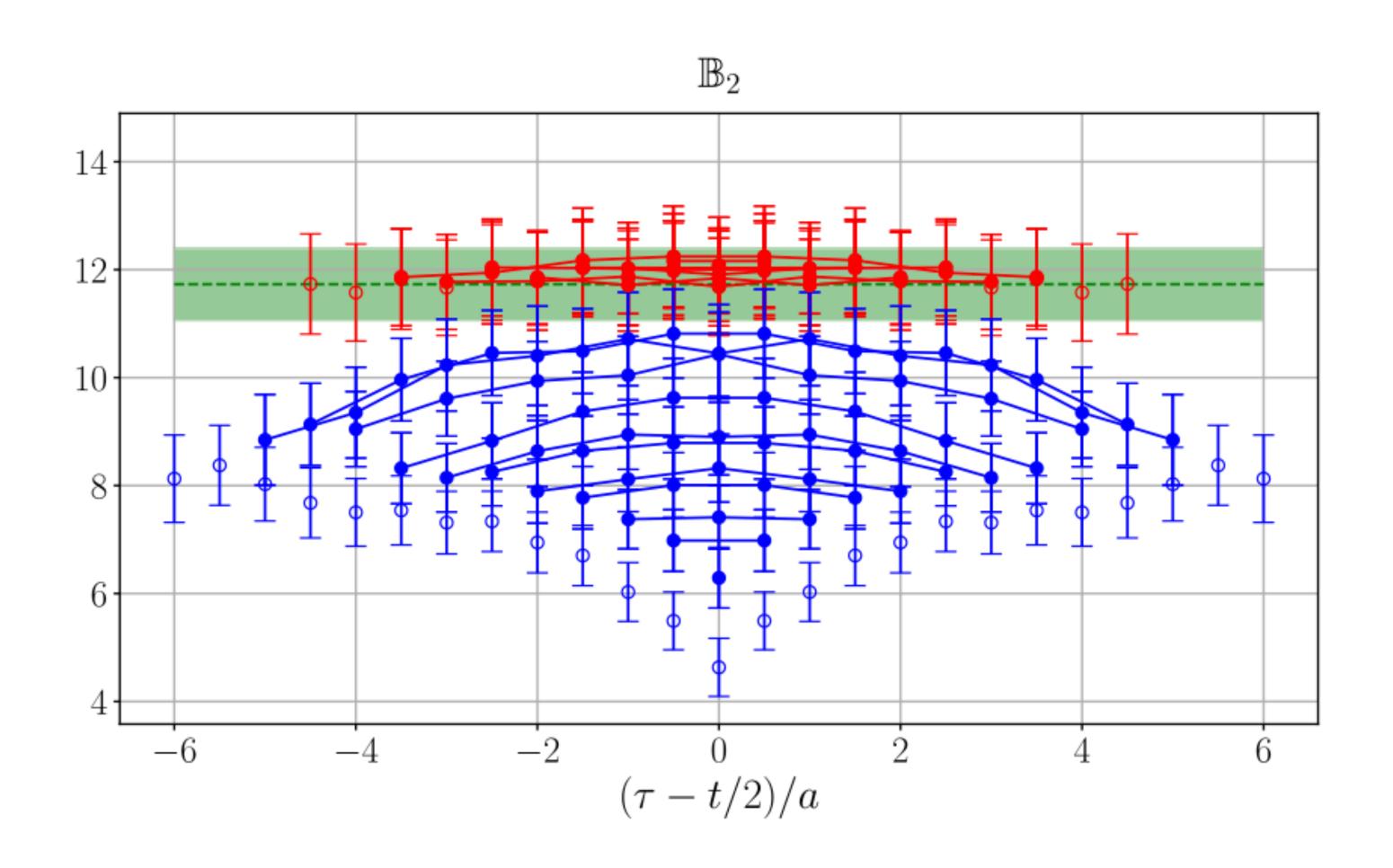
# Examples: Excited states for Nucleon weak matrix elements



Data should be constant in  $\tau$  if no excited states contribute

Red data are with GEVP

# Examples: Excited states for Nucleon scalar matrix elements

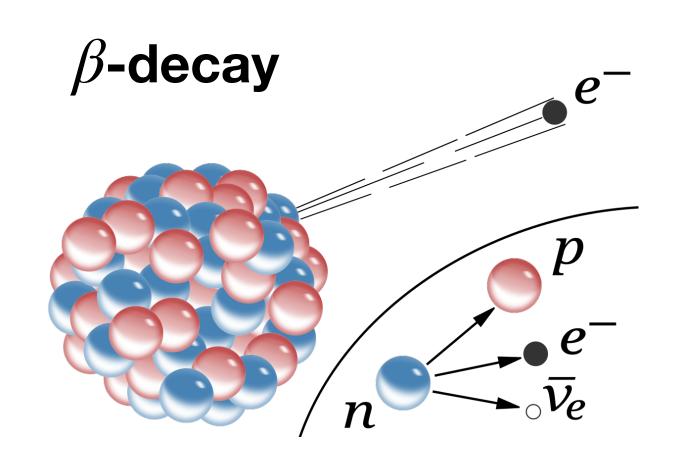


Great improvement after GEVP.

# Weak Nucleon Structure

# Weak Form Factors

$$G_A(Q^2)$$
,  $\widetilde{G}_P(Q^2)$ ,  $F_1^W(Q^2)$ ,  $F_2^W(Q^2)$ 



#### Weak interactions mediated by Vector - Axial currents

Decay Rate

$$\Gamma \propto G_A(Q^2 = 0), F_1^W(Q^2 = 0)$$

## $\nu-N$ scattering (CC)

$$\frac{\mathrm{d}\sigma^{(\nu p,\bar{\nu}p)}}{\mathrm{d}Q^2} = \frac{G_F m^2}{8\pi E_{\nu}} \left[ A(Q^2) \mp B(Q^2) \frac{(s-u)}{m^2} + C(Q^2) \frac{(s-u)^2}{m^4} \right]$$

$$A(Q^2), B(Q^2), C(Q^2) \propto G_A(Q^2), \widetilde{G}_P(Q^2), F_1^W(Q^2), F_2^W(Q^2)$$

"Axial Structure of the Nucleon" [V. Bernard, L. Elouadrhiri, U. Meißner]

# Lorentz decomposition of nucleon matrix elements

$$\langle N(\mathbf{p}') | \mathcal{J}(\mathbf{q}) | N(\mathbf{p}) \rangle = \bar{u}_{\mathbf{p}'} FF[\mathcal{J}] u_{\mathbf{p}}$$

$$\frac{d\sigma^{(\nu N)}}{dQ^2} \propto |\langle N|\mathcal{J}^-|N\rangle|^2$$

$$\mathcal{J} = \bar{q}\Gamma q \qquad \Gamma \in \{\gamma_5, \gamma_\mu, \gamma_\mu \gamma_5\}$$

 $FF[\mathcal{J}]$  = Lorentz decomposition of the nucleon matrix element in terms of nucleon form factors  $G(Q^2)$ 

**Pseudoscalar** 

$$\Gamma = \gamma_5 : \mathcal{J} = \mathscr{P}$$

$$\langle N(\mathbf{p}') | \mathcal{P}(\mathbf{q}) | N(\mathbf{p}) \rangle = \bar{u}_{\mathbf{p}'} \gamma_5 G_P(Q^2) u_{\mathbf{p}}$$

**Axial** 

$$\Gamma = \gamma_{\mu} \gamma_5 : \mathcal{J} = \mathcal{A}_{\mu}$$

$$\left| \langle N(\mathbf{p}') \, | \, \mathcal{A}_{\mu}(\mathbf{q}) \, | \, N(\mathbf{p}) \rangle = \bar{u}_{\mathbf{p}'} \left[ \gamma_{\mu} \gamma_5 G_A(Q^2) + \frac{q_{\mu}}{2m_N} \gamma_5 \widetilde{G}_P(Q^2) \right] u_{\mathbf{p}} \right|$$

$$g_A \equiv G_A(0)$$
  
Axial charge

**Vector** 

$$\Gamma = \gamma_{\mu} : \mathscr{J} = \mathscr{V}_{\mu}$$

$$\langle N(\mathbf{p}') | \mathcal{V}_{\mu}(\mathbf{q}) | N(\mathbf{p}) \rangle = \bar{u}_{\mathbf{p}'} \left[ \gamma_{\mu} F_1(Q^2) + \frac{i \sigma_{\mu\nu} q_{\nu}}{2m_N} F_2(Q^2) \right] u_{\mathbf{p}}$$