Rare Kaon and Hyperon Decays from Lattice QCD

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Astroparticle and Particle Physics Workshop

- Direct experimental results are described very well by the Standard Model of Particle Physics
- SM doesn't explain Matter-Antimatter asymmetry, Dark Matter, Neutrino masses, and is incompatible with General Relativity

Where is new physics?

- Search for BSM physics is the interplay between experiment and theory
- Two main types of searches: Precision and rare processes

Search for New Physics

Precision searches

- Look for tiny deviations from SM
- Need very high experimental precision
  $\rightarrow$ many events
  $\rightarrow$ highly probable channels
- Need very robust SM predictions
  $\rightarrow$ QCD often makes this hard
- Examples: Muon anomalous magnetic moment, $K^+ \rightarrow \mu^+\nu$ and $\pi^+ \rightarrow \mu^+\nu$ decays, etc.

Can we find the pea under the mattresses?

Search for New Physics

Rare/Forbidden processes

- Look for relatively large deviations from tiny SM background
- Remove SM background entirely by breaking SM symmetries:
  e.g. $K^+ \to \pi^- \mu^+ \mu^+$, $K \to \mu^+ e^-$, etc.
- SM suppression by vanishing at tree-level:
  e.g. Flavour Changing Neutral Currents

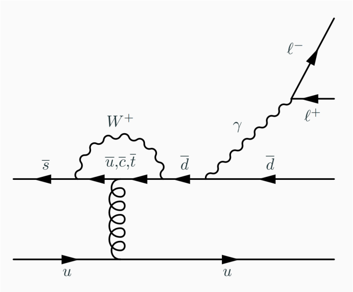
Many interesting FCNCs

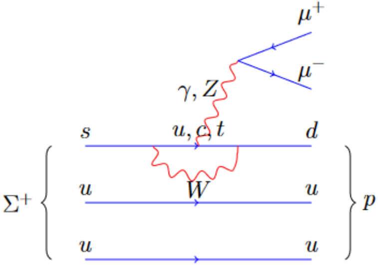
- $K_{L,S} \to \ell^+ \ell^-$
- $K \to \pi \ell^+ \ell^-$
- $K \to \pi \nu \bar{\nu}$
- $\Sigma^+ \to p \ell^+ \ell^-$
- $\Sigma^+ \to p \nu \bar{\nu}$
- $D \to \pi \ell^+ \ell^-$
- $\Lambda_c \to p \ell^+ \ell^-$
- $B \to K^{(*)} \ell^+ \ell^-$
- etc.

Can we find the pea hidden in the grass?

Rare K and Σ Decays

- Focus on $K^{+/S} \to \pi^{+/0}\ell^+\ell^-$ and $\Sigma^+ \to p\ell^+\ell^-$
- Dominated by intermediate virtual photon $\gamma^* \to \ell^+\ell^-$
- Complicated by low energy QCD $\to$ Non-perturbative methods





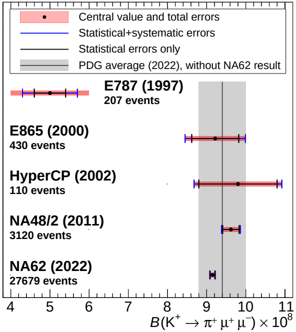
(a)

- Future ambition to also compute $D \to \pi\ell^+\ell^-$

Rare K Decay Experiment

There are several different channels in $K \to \pi\,\ell^+\ell^-$:

- $K^+ \to \pi^+\mu^+\mu^-$ most precise: latest result from NA62
  $\mathcal{B} = (9.15 \pm 0.08) \times 10^{-8}$
- $K^+ \to \pi^+e^+e^-$ measured by NA48
  $\mathcal{B} = (3.11 \pm 0.12) \times 10^{-7}$
- $K_S^0 \to \pi^+\mu^+\mu^-$ and $K_S^0 \to \pi^+e^+e^-$ observed at NA48.
  Much less precise
  $\mathcal{B} = (2.9^{+1.5}_{-1.2} \pm 0.2) \times 10^{-9}$ and $(5.8^{+2.9}_{-2.4}) \times 10^{-9}$



[hep-ex 2209.05076]

- $K_L^0 \to \pi^0\ell^+\ell^-$ not observed. Bounds set by KTeV at Fermilab
  $\mathcal{B} < 3.8 \times 10^{-10}$ and $< 2.8 \times 10^{-10}$

Rare K Decay Phenomenology

$K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$

- $\mathcal{B} = C_{1\gamma}\, a^2_{+,S}$
- Dominated by CP conserving intermediate $\gamma^* \to \ell^+\ell^-$
- Non-perturbative physics determined by parameters a_+ and a_S
- Experiment $a_+^{\text{exp}} = -0.575(13)$, theory $a_+^{\text{SM}} = -1.59(8)$

$K_L \to \pi^0\ell^+\ell^-$

- $\mathcal{B} = (C_{2\gamma} + C_{\text{dir}} + C_{\text{ind}} a_S^2 \pm C_{\text{int}} a_S)$
- $C_{2\gamma}$: CP conserving $\gamma^*\gamma^* \to \ell^+\ell^-$
- C_{dir} : Direct CP violation (short distance)
- C_{ind} : Indirect CP violating ($K^0 - \bar{K}^0$ mixing $\times\, K_S \to \pi^0\ell^+\ell^-$)
- C_{int} : Interference between direct and indirect CP violation

Need to know sign of a_S to disentangle interference vs new physics

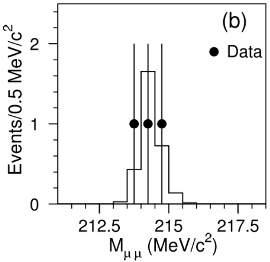
Rare Σ Decay Experiment

First observed by HyperCP: [hep-ex/0501014]

- 3 events seen

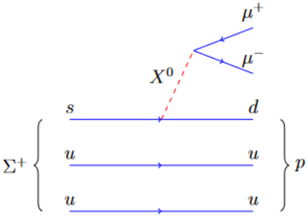
$$\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{\mathrm{HCP}} = 8.6^{+6.6}_{-5.4} \pm 5.5 \times 10^{-8}$$

- HyperCP anomaly: possible new particle $\Sigma^+ \to pX^0 \to pX^0 \to \mu^+\mu^-$ with $m_{X^0} \simeq 214$ MeV?



Lots of attention from the BSM theory community

- X^0 scalar or pseudoscalar?
- Sgoldstino?
- Light Higgs?
- Secluded $U(1)$?

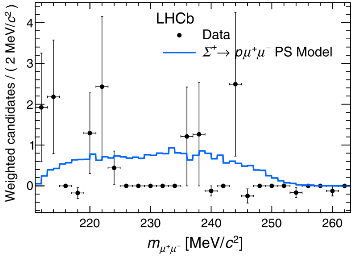


(c)

Later measured at LHCb: [hep-ex/1712.08606]

- $\simeq$ 10 events

$$\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{\text{LHCb}} = 2.2^{+1.8}_{-1.3} \times 10^{-8}$$

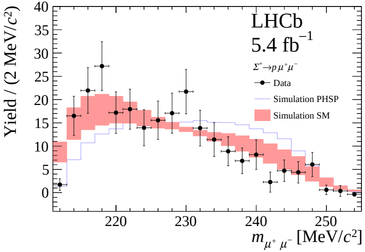


Recent observation at LHCb: [hep-ex/2504.06096]

- $\simeq$ 240 events

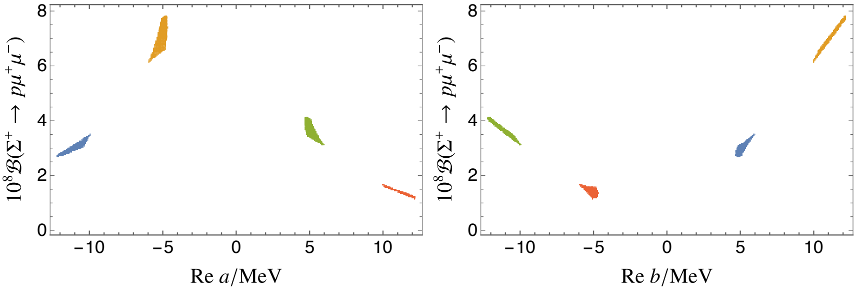
$$\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-)_{\text{LHCb}} = 1.09 \pm 0.17 \times 10^{-8}$$

- Currently working on additional measurements
  - + angular observables
  - + CP violation
  - + e^+e^- mode



No evidence of the HyperCP anomaly

- Spin degree of freedom gives access to extra BSM operators
- SM prediction uses combination of experimental input ($\Sigma^+ \to p\gamma$), Vector Meson Dominance, Baryon χ_{PT}
- 4-fold ambiguity from experimental input



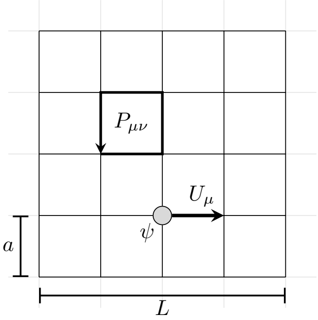
[hep-ph 2404.15268]

Need ab initio non-perturbative methods → Lattice QCD

Lattice QCD

Need to regulate infinite, continuous QCD problem to put it onto a computer

- Discretize space-time
- Restrict to finite size lattice
- Define discretisation of quarks ψ and gluons U_μ

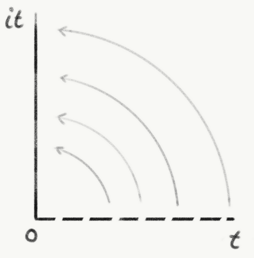


Compute observables on the lattice $\mathcal{O}(a, L)$

Recover QCD by taking the infinite volume and continuum limits

$$\mathcal{O}_{\mathrm{QCD}} = \lim_{a \to 0} \lim_{L \to \infty} \mathcal{O}(a, L)$$

Also cheaper to compute with unphysical pion mass
Then take $m_\pi \to m_\pi^{\mathrm{phys}}$ limit



- Wick rotation $t \to it$: Euclidean spacetime

$$\int \mathcal{D}\psi \ldots e^{iS_M} \to \int \mathcal{D}\psi \ldots e^{-S_E}$$

- Use Monte Carlo Markov Chain to compute path integral
- Propagation in time e^{iEt} becomes decaying (or growing) e^{-Et}

Need to get back to Minkowski quantity:

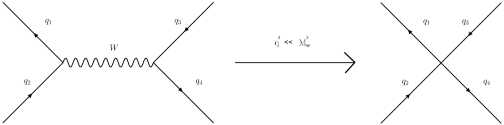
- Trivial for many quantities: e.g. hadrons masses
- Non-trivial for non-local matrix elements like rare decays (problematic growing exponentials)
- K and Σ decays can be done manually
- Heavier decays like $D \to \pi \ell^+ \ell^-$ need alternative methods: e.g. numerical inverse Laplace transform $\to$ Can other fields help?

Electroweak Interactions on the Lattice

Can we simulate the full SM on the lattice? **No!**

- Practical problem: Would need $L \gg m_\pi^{-1}$ and $a \ll \sqrt{G_F}$
  $(L/a)^4 \gg 10^{13}$ lattice sites. Currently limits $128^3 \times 256 \sim 5 \times 10^8$
- Theoretical problem: EW is a chiral gauge theory
  Fermion discretisations generally break chiral symmetry

Instead we work in Low-Energy Effective Field Theory
$\rightarrow$ integrate out heavy degrees of freedom

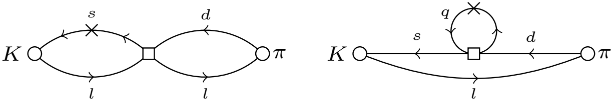


+ ...

Compute matrix elements of these operators in pure QCD

Amplitude to calculate (very similar for Σ decay)

$$\mathcal{A}_\mu = \int d^4x \, \langle\pi| \, T\{H_W(x) J_\mu(0)\} \, |K\rangle$$
$$= i\int_0^\infty d\omega \frac{1}{2\omega} \frac{\langle\pi| J_\mu |\omega\rangle \langle\omega| H_W |K\rangle}{\omega - E_\pi} + i\int_0^\infty d\omega \frac{1}{2\omega} \frac{\langle\pi| H_W |\omega\rangle \langle\omega| J_\mu |K\rangle}{\omega - E_K}$$



etc.

Quark loops diagrams are problematic

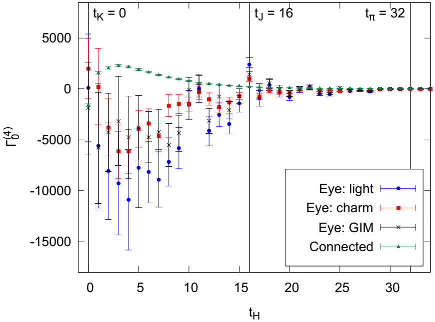
- Need to take the trace of gigantic ($\sim 10^9 \times 10^9$) matrix $\text{Tr}\left[D^{-1}\right]$
- Done stochastically: recent developments have had massive improvement

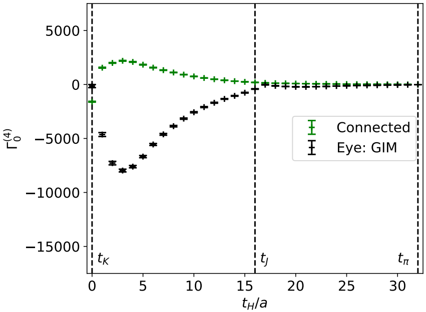
Baryons also have a signal-to-noise problem $\rightarrow$ even harder than kaon decay

Rare K and Σ Calculations

	Method	Exploratory	Physical	Improvement
$K \to \pi\ell^+\ell^-$	2015	2016	2022 [RH]	Analysis ongoing [RH]
$\Sigma \to \pi\ell^+\ell^-$	2022 [RH]	2025 [RH]		

- $K \to \pi\ell^+\ell^-$ stochastic loop noise blows up as $m_\pi \to m_\pi^{\text{phys}}$
- Unexpectedly loop noise dominates in $\Sigma \to \pi\ell^+\ell^-$ at $m_\pi = 340$ Mev
- $\Sigma \to \pi\ell^+\ell^-$ large cancellation between two time orderings
- Improved stochastic loop estimator





Conclusions/Outlook

Big question to answer:

- **Where is all the new physics?**

Rare FCNC decays have potential to answer this

- Current theory methods struggle to make robust predictions in some areas
- Need first principles non-perturbative calculations $\rightarrow$ Lattice QCD

Lattice still has challenges to overcome

- Stochastic loop noise $\rightarrow$ even better estimators?
- Unexpected cancellations $\rightarrow$ can we reformulate the problem
- Baryon signal-to-noise problem $\rightarrow$ would have massive benefit to many lattice calculations
- Developments towards numerically undoing Wick rotation / inverse Laplace transform $\rightarrow$ lattice has made progress in recent years

Does your field have any solutions?