

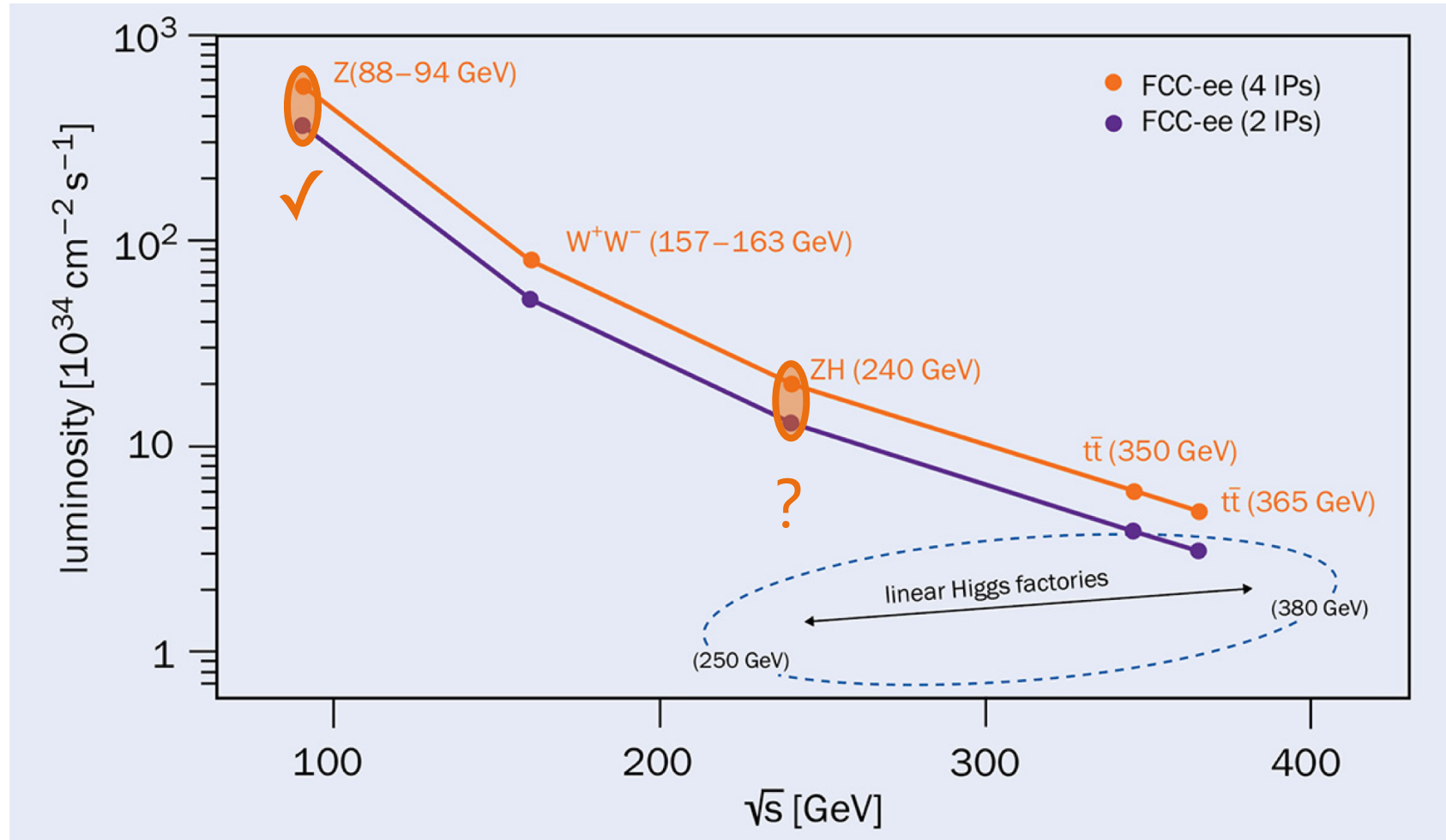
What can we see with the FCC-ee?

Sensitivity to ALPs at FCC-ee ZH run at a center of mass energy $\sqrt{s} = 240$ GeV

30.06.25

Center of mass energy vs. luminosity

Interaction points of the FCC-ee



Processes at ZH (240 GeV)

On Tuesday, we discussed:

- 1) $e^+e^- \rightarrow aZ, a \rightarrow \gamma\gamma$
- 2) $e^+e^- \rightarrow aZZ, a \rightarrow \gamma\gamma$
- 3) $e^+e^- \rightarrow aZH, a \rightarrow \gamma\gamma$

For reference, I will be using m_a (ALP mass) set to 10 GeV and $c_{\gamma\gamma}$ (coupling strength) set to 1.6.

This corresponds to first row of Table 1 in Elnura's thesis.

Keeping everything the same except for process (see above) and beam energy. Only ran 1 event to start.

m_a [GeV]	$c_{\gamma\gamma}$	σ [pb]
10	1.6	6.764
0.3	1.6	7.014
1.0	1.0	2.739
10.0	0.4	0.423
0.3	0.4	0.438

$$e^+e^- \rightarrow aZ, a \rightarrow \gamma\gamma$$

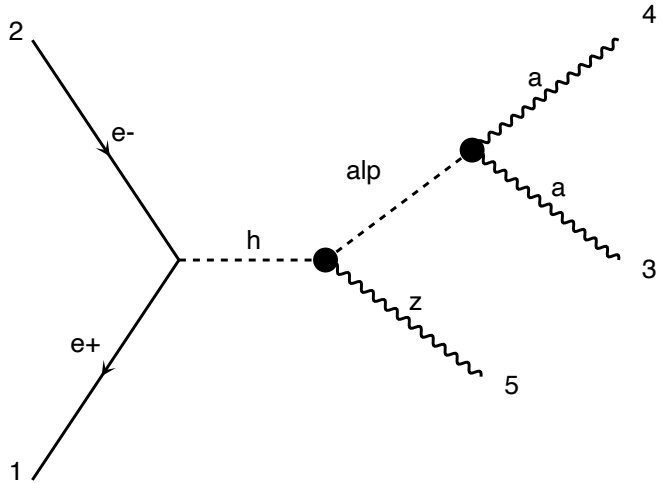


diagram 1

NP=2, QCD=0, QED=3

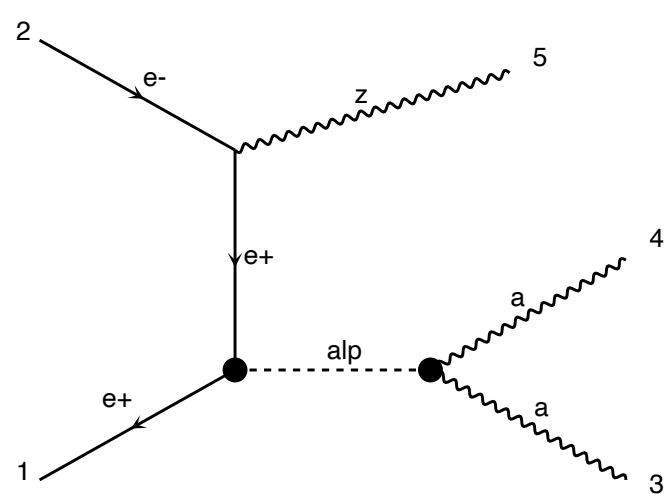


diagram 2

NP=2, QCD=0, QED=3

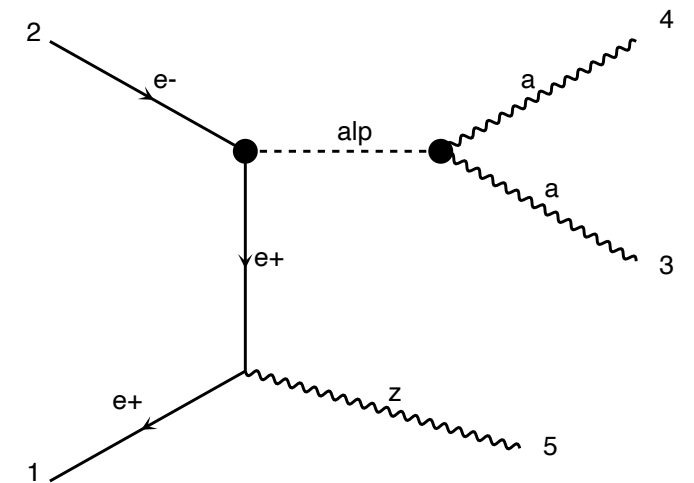


diagram 3

NP=2, QCD=0, QED=3

$$e^+e^- \rightarrow aZZ, a \rightarrow \gamma\gamma$$

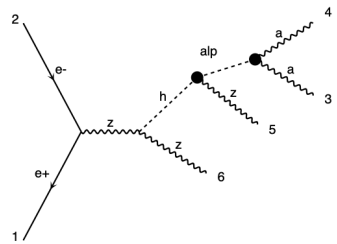


diagram 1 NP=2, QCD=0, QED=4

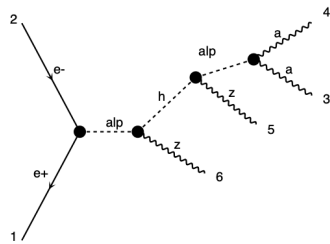


diagram 2 NP=4, QCD=0, QED=2

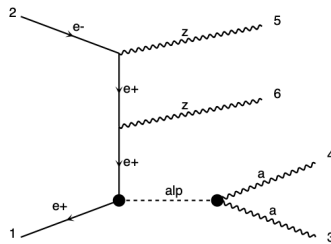


diagram 7 NP=2, QCD=0, QED=4

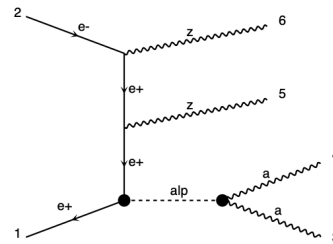


diagram 8 NP=2, QCD=0, QED=4

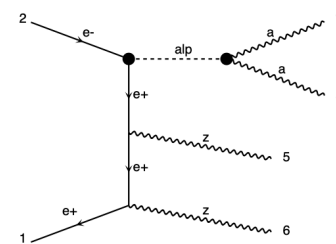


diagram 13 NP=2, QCD=0, QED=4

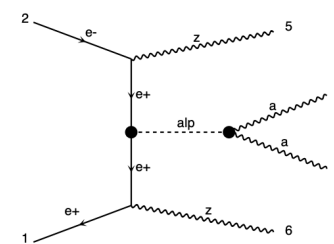


diagram 14 NP=2, QCD=0, QED=4

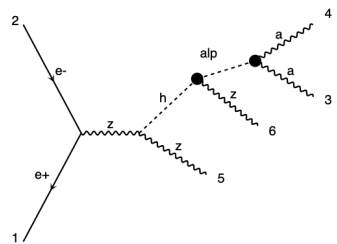


diagram 3 NP=2, QCD=0, QED=4

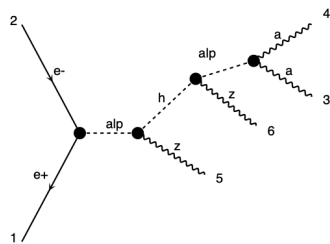


diagram 4 NP=4, QCD=0, QED=2

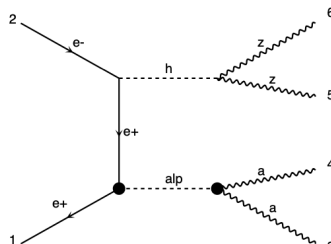


diagram 9 NP=2, QCD=0, QED=4

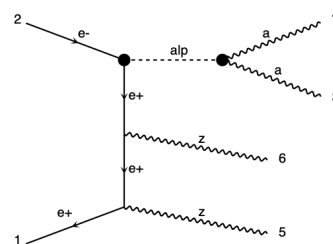


diagram 10 NP=2, QCD=0, QED=4

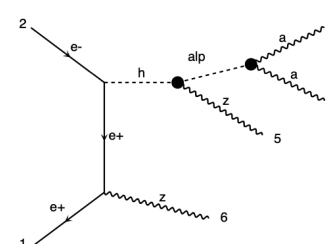


diagram 15 NP=2, QCD=0, QED=4

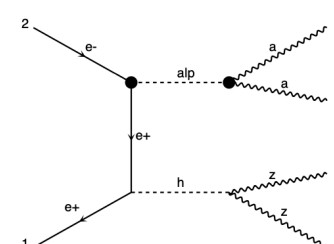


diagram 16 NP=2, QCD=0, QED=4

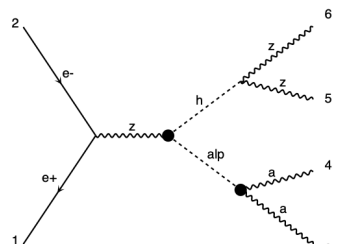


diagram 5 NP=2, QCD=0, QED=4

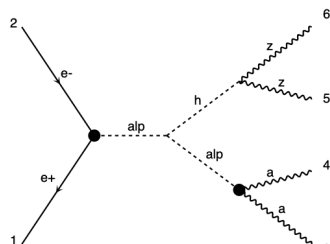


diagram 6 NP=2, QCD=0, QED=4

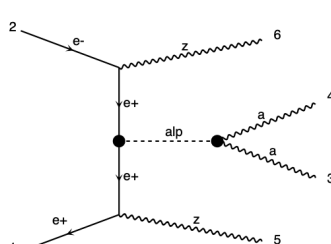


diagram 11 NP=2, QCD=0, QED=4

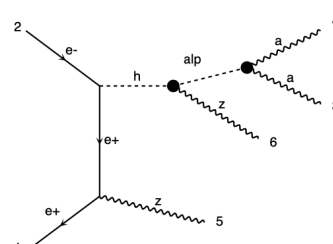


diagram 12 NP=2, QCD=0, QED=4

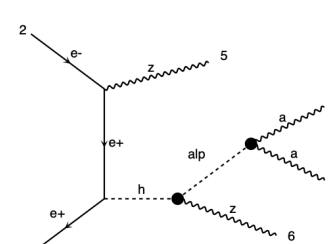


diagram 17 NP=2, QCD=0, QED=4

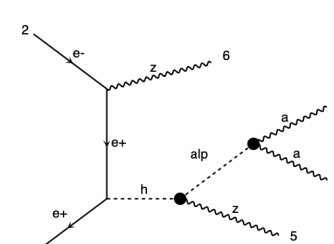
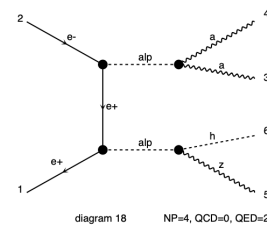
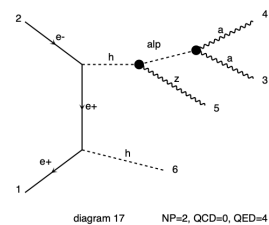
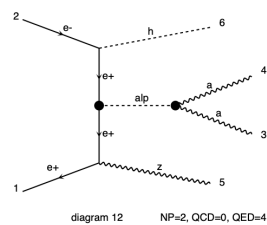
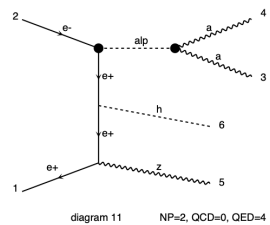
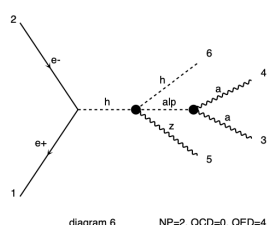
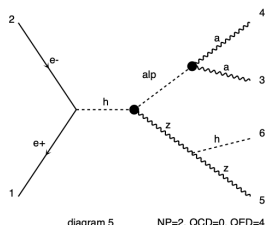
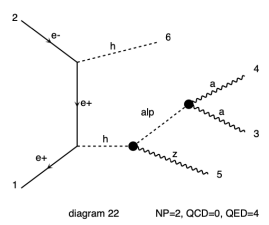
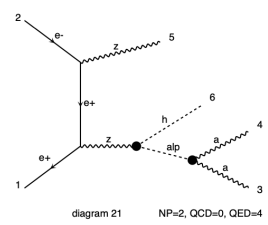
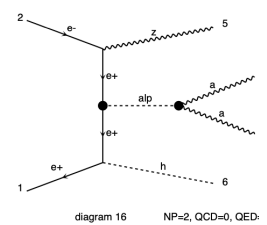
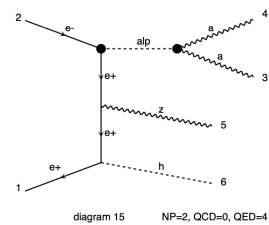
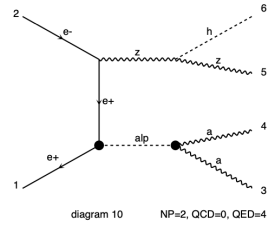
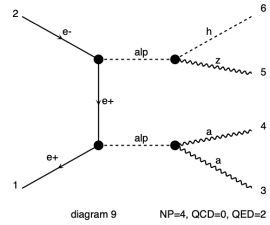
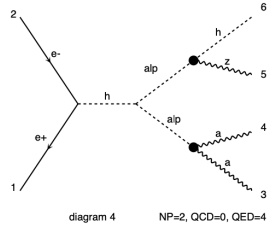
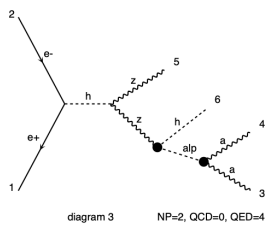
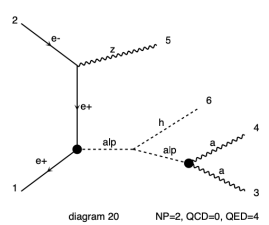
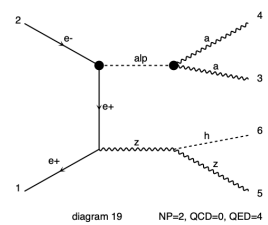
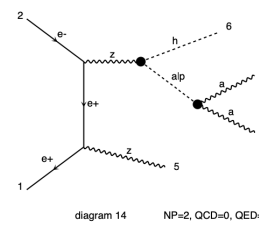
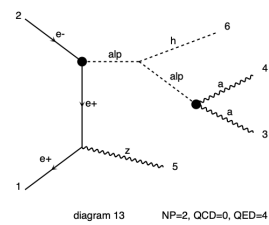
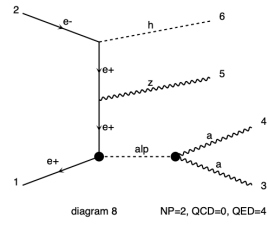
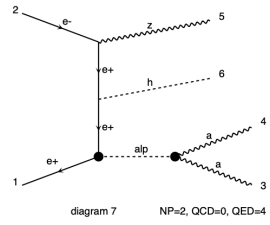
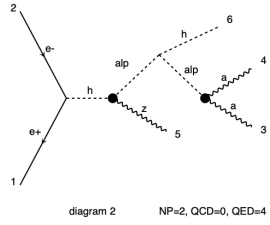
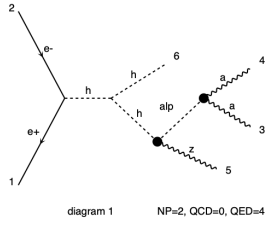


diagram 18 NP=2, QCD=0, QED=4

$$e^+e^- \rightarrow aZH, a \rightarrow \gamma\gamma$$



Couplings

With only the $c_{\gamma\gamma}$ coupling, the processes on the previous three slides give zero cross section.
Maybe we should look into the other couplings a little more closely...

ALP Lagrangian

$$\begin{aligned}
 \mathcal{L}_{\text{eff}} = & \overbrace{\frac{1}{2} (\partial_\mu a)(\partial^\mu a)}^{\text{kinetic energy}} - \overbrace{\frac{m_a^2}{2} a^2}^{\text{mass}} + \overbrace{\sum_f \frac{c_{ff}}{2} \frac{\partial^\mu a}{\Lambda} \bar{f} \gamma_\mu \gamma_5 f}^{\text{fermions}} \\
 & + \underbrace{g_s^2 C_{GG} \frac{a}{\Lambda} G_{\mu\nu}^A \tilde{G}^{\mu\nu,A}}_{\text{SU(3)}_c; \text{ gluons}} + \underbrace{g^2 C_{WW} \frac{a}{\Lambda} W_{\mu\nu}^A \tilde{W}^{\mu\nu,A}}_{\text{SU(2)}_L; \text{ W gauge bosons}} + \underbrace{g'^2 C_{BB} \frac{a}{\Lambda} B_{\mu\nu} \tilde{B}^{\mu\nu}}_{\text{U(1)}_Y; \text{ hypercharge gauge bosons}},
 \end{aligned} \tag{1}$$

[Georgi, Kaplan, Randall (1986)]

ALP Lagrangian continued

In the broken phase of the electroweak symmetry, the ALP couples to the photon and the Z boson as

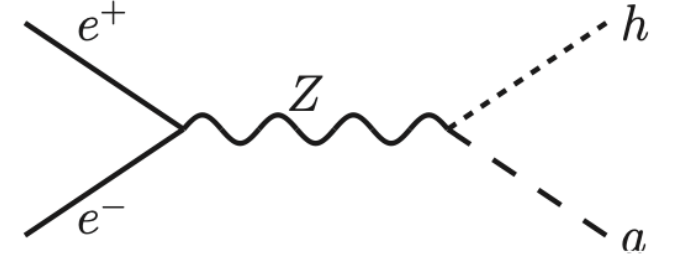
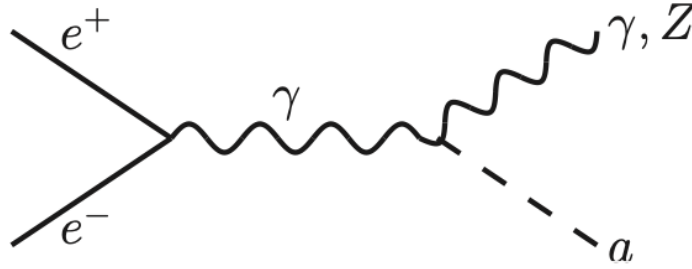
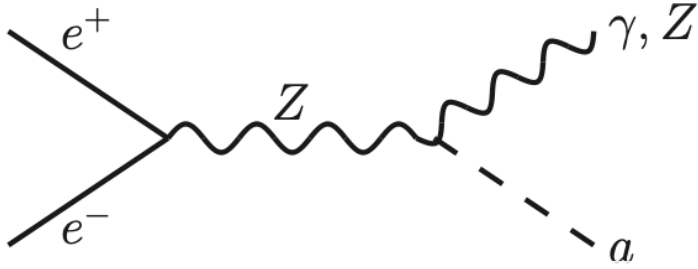
$$\mathcal{L}_{\text{eff}} \ni e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{2e^2}{s_w c_w} C_{\gamma Z} \frac{a}{\Lambda} F_{\mu\nu} \tilde{Z}^{\mu\nu} + \frac{e^2}{s_w^2 c_w^2} C_{ZZ} \frac{a}{\Lambda} Z_{\mu\nu} \tilde{Z}^{\mu\nu}. \quad (2)$$

The relevant Wilson coefficients are given by

$$C_{\gamma\gamma} = C_{WW} + C_{BB}, \quad C_{\gamma Z} = c_w^2 C_{WW} - s_w^2 C_{BB}, \quad C_{ZZ} = c_w^4 C_{WW} + s_w^4 C_{BB}, \quad (3)$$

where s_w and c_w are the sine and cosine of the weak mixing angle, respectively. The exotic decay $Z \rightarrow \gamma a$ is governed by the Wilson coefficient $C_{\gamma Z}$.

Differential cross sections



$$\frac{d\sigma(e^+e^- \rightarrow \gamma a)}{d\Omega} = 2\pi\alpha\alpha^2(s) \frac{s^2}{\Lambda^2} \left(1 - \frac{m_a^2}{s}\right)^3 (1 + \cos^2 \theta) (|V_\gamma(s)|^2 + |A_\gamma(s)|^2), \quad (16)$$

$$\frac{d\sigma(e^+e^- \rightarrow Z a)}{d\Omega} = 2\pi\alpha\alpha^2(s) \frac{s^2}{\Lambda^2} \lambda^{\frac{3}{2}}(x_a, x_Z) (1 + \cos^2 \theta) (|V_Z(s)|^2 + |A_Z(s)|^2), \quad (17)$$

$$\frac{d\sigma(e^+e^- \rightarrow h a)}{d\Omega} = \frac{\alpha}{128\pi c_w^2 s_w^2} \frac{|C_{Zh}^{\text{eff}}|^2}{\Lambda^2} \frac{s m_Z^2}{(s - m_Z^2)^2} \lambda^{\frac{3}{2}}(x_a, x_h) \sin^2 \theta (g_V^2 + g_A^2), \quad (18)$$

$$V_\gamma(s) = \frac{C_{\gamma\gamma}^{\text{eff}}}{s} + \frac{g_V}{2c_w^2 s_w^2} \frac{C_{\gamma Z}^{\text{eff}}}{s - m_Z^2 + im_Z \Gamma_Z}, \quad A_\gamma(s) = \frac{g_A}{2c_w^2 s_w^2} \frac{C_{\gamma Z}^{\text{eff}}}{s - m_Z^2 + im_Z \Gamma_Z}, \quad (19)$$

$$V_Z(s) = \frac{1}{c_w s_w} \frac{C_{\gamma Z}^{\text{eff}}}{s} + \frac{g_V}{2c_w^3 s_w^3} \frac{C_{ZZ}^{\text{eff}}}{s - m_Z^2 + im_Z \Gamma_Z}, \quad A_Z(s) = \frac{g_A}{2c_w^3 s_w^3} \frac{C_{ZZ}^{\text{eff}}}{s - m_Z^2 + im_Z \Gamma_Z}, \quad (20)$$

Exotic decay rates

$$\Gamma(Z \rightarrow \gamma a) = \frac{\alpha \alpha(m_Z) m_Z^3}{96\pi^3 s_w^2 c_w^2 f^2} |c_{\gamma Z}|^2 \left(1 - \frac{m_a^2}{m_Z^2}\right)^3, \quad (10)$$

$$\Gamma(h \rightarrow Za) = \frac{m_h^3 v^2}{64\pi f^6} |c_{Zh}|^2 \lambda^{3/2} \left(\frac{m_Z^2}{m_h^2}, \frac{m_a^2}{m_h^2}\right), \quad (11)$$

$$\Gamma(h \rightarrow aa) = \frac{m_h^3 v^2}{32\pi f^4} |c_{ah}|^2 \left(1 - \frac{2m_a^2}{m_h^2}\right)^2 \sqrt{1 - \frac{4m_a^2}{m_h^2}}. \quad (12)$$

Couplings

```
# set to electron beams (0 for ele, 1 for proton)
set lpp1 0
set lpp2 0
set ebeam1 45.594
set ebeam2 45.594
# set ALP mass
set Ma ALPMASS
# set ALP couplings
set cWW = 0.0
set CYY = ALPCOUPLING
set cGG = 0.
set cuu = 0.
set cdd = 0.
set ccc = 0.
set css = 0.
set ctt = 0.
set cbb = 0.
set cee = 0.
set cmumu = 0.
set ctautau = 0.
set cah = 0.
set cZh5 = 0.
```

Couplings

Playing around with the couplings for:

1) $e^+e^- \rightarrow aZ, a \rightarrow \gamma\gamma$

- Set all couplings to zero except CYY and cah (0.01) → Survey return zero cross section
- Set all couplings to zero except CYY and **cZh5** (0.01) → 4.768e-11 +- 1.058e-13 pb

How does cross section depend on cZh5?

Coupling	Cross Section
0.001	4.783e-13 +- 9.372e-16 pb
0.01	4.749e-11 +- 1.097e-13 pb
0.1	4.767e-09 +- 1.054e-11 pb
0.5	1.19e-07 +- 2.743e-10 pb
1.0	4.781e-07 +- 1.557e-09 pb

ALP Lagrangian continued

Interactions with the Higgs boson, ϕ , appear only at dimension-6 and higher,

$$\mathcal{L}_{\text{eff}}^{D \geq 6} = \underbrace{\frac{C_{ah}}{\Lambda^2} (\partial_\mu a)(\partial^\mu a) \phi^\dagger \phi}_{h \rightarrow aa} + \underbrace{\frac{C_{Zh}}{\Lambda^3} (\partial^\mu a) (\phi^\dagger i D_\mu \phi + \text{h.c.}) \phi^\dagger \phi}_{h \rightarrow Za} + \dots, \quad (5)$$

where the first operator mediates the decay $h \rightarrow aa$, while the second one is responsible for $h \rightarrow Za$. Note that a possible dimension-5 operator coupling the ALP to the Higgs current vanishes by the equations of motion. However, in theories where a heavy new particle acquires most of its mass through electroweak symmetry breaking, the non-polynomial dimension-5 operator

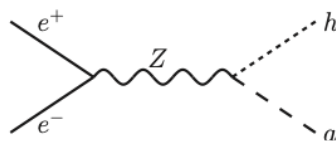
$$\frac{C_{Zh}^{(5)}}{\Lambda} (\partial^\mu a) (\phi^\dagger i D_\mu \phi + \text{h.c.}) \ln \frac{\phi^\dagger \phi}{\mu^2} \quad (6)$$

can be present [25, 28, 66, 67]. In our analysis we allow for the presence of such an operator.

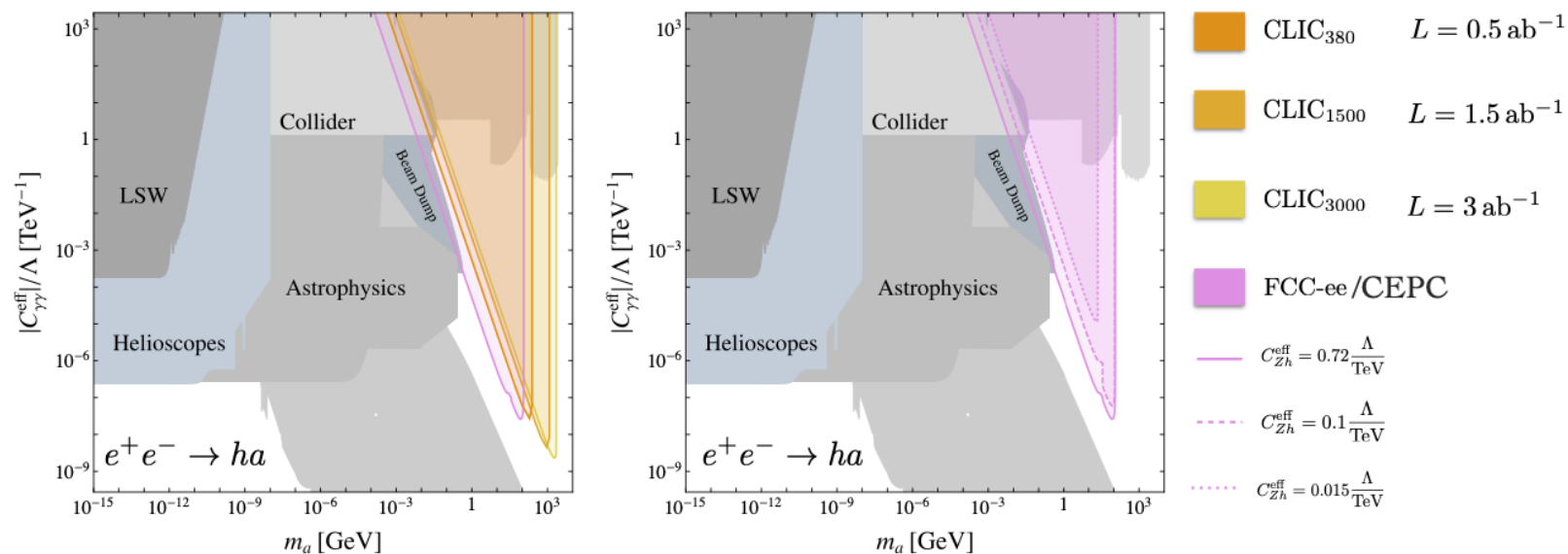
Sensitivity

https://indico.mit.edu/event/876/contributions/2864/attachments/1086/1793/Thamm_ALPs.pdf

- ALP associated production with a H



- ALP decay into photons



02.07.25

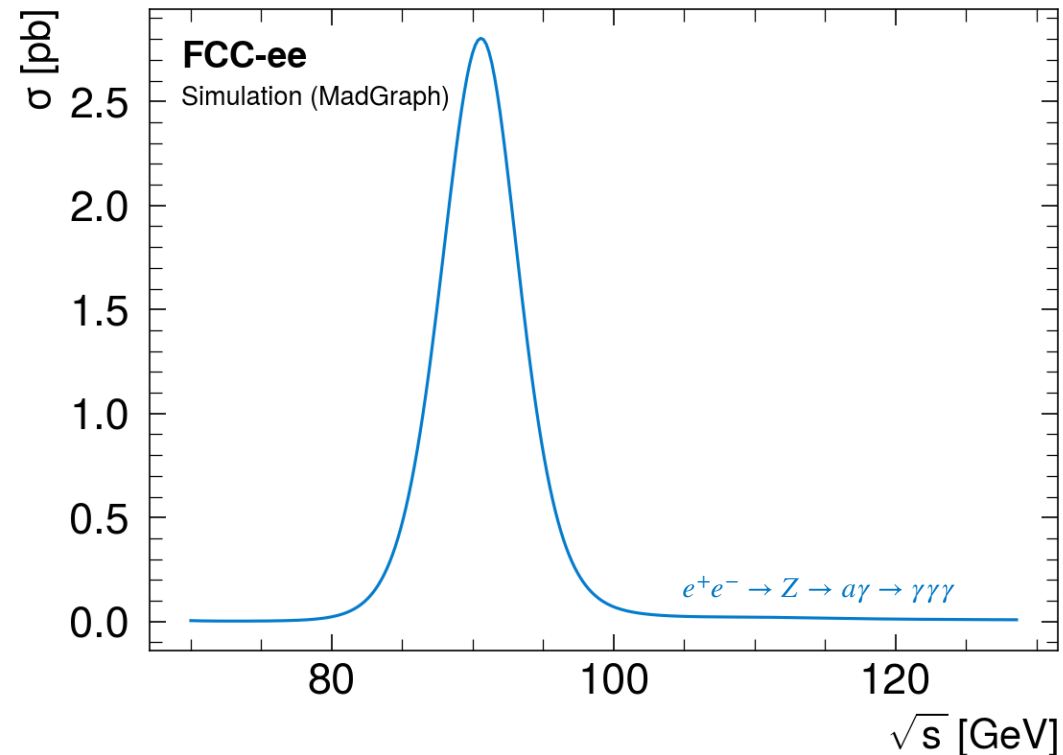
Zero cross section $e^+e^- \rightarrow Z \rightarrow a\gamma \rightarrow \gamma\gamma\gamma$ at higher centre-of-mass energy?

Reran the same process Elnura generated, but at the Z-Higgs associated production center of mass energy (240 GeV). To keep things consistent, only kept $c_{\gamma\gamma}$ coupling. Using m_a of 1.0 GeV and $c_{\gamma\gamma}$ of 1.0

Got zero cross section → **What is happening?**

Seems to work at 91.188 GeV, but not 240 GeV. Decided to start at 240 GeV and keep lowering centre-of-mass energy until there was a non-zero cross section.

$\sqrt{s}/2$	Cross Section
35	$0.004201 \pm 1.205\text{e-}05$ pb
40	$0.0225 \pm 6.359\text{e-}05$ pb
45.594	2.732 ± 0.0074 pb
50	0.07077 ± 0.0001945 pb
55	$0.02093 \pm 5.927\text{e-}05$ pb
60	$0.01154 \pm 2.86\text{e-}05$ pb
62.5	$0.009352 \pm 2.504\text{e-}05$ pb
63.75	$0.008561 \pm 2.382\text{e-}05$ pb
64.0625	$0.008402 \pm 3.167\text{e-}05$ pb
64.21875	$0.008339 \pm 2.23\text{e-}05$ pb
64.296875	$0.008304 \pm 2.27\text{e-}05$ pb
64.3	$0.008293 \pm 2.237\text{e-}05$ pb
64.31	Survey return zero cross section.



Breit-Wigner distribution cutoff

The Z boson decay $Z \rightarrow \gamma a$ is on shell and the process $e^+e^- \rightarrow Z \rightarrow a\gamma$ is resonant on the Z pole

According to Olivier Mattelaer,

"The run_card parameter **bwcutoff** defines what is considered to be on-shell s-channel resonances: The resonance is considered to be on-shell if the invariant mass of an s-channel resonance is within $M \pm \text{bwcutoff} \cdot \Gamma$, and will then be included as a comment particle in the LHE event file (with status code 2). The value of bwcutoff does not affect the cross section of any processes, except if

1) you use the decay chain formalism: $[a b \rightarrow c d, c \rightarrow e f, d \rightarrow g h]$ Decay chain processes are strictly valid only in the narrow width limit, and we have therefore chosen to allow only production of on-shell particles. This means that the cross section will get smaller as bwcutoff gets smaller, since more of the tails of the Breit Wigner distributions are cut off."

Check.

Our allowed energy range around resonance is: $M_Z - \text{bw cutoff} \times \Gamma_Z < \sqrt{s} < M_Z + \text{bw cutoff} \times \Gamma_Z$

Given:

$$M_Z = 91.1880 \text{ GeV}$$

$$\Gamma_Z = 2.4955 \text{ GeV}$$

$$\text{bw cutoff} = 15$$



from run_card.dat

```
#####  
# BW cutoff (M+/-bw cutoff*Gamma) ! Define on/off-shell for "$" and decay  
#####  
15.0 = bw cutoff ! (M+/-bw cutoff*Gamma)  
#####
```

Putting this together:

$$53.7555 \text{ GeV} < \sqrt{s} < 128.6205 \text{ GeV}$$

MadGraph gives a zero cross section somewhere between 128.6 GeV and 128.62 GeV, so this is consistent!

Note that this only applies when you use decay chain formalism or explicitly forbid s-channel particles to be on shell.

Though we aren't particularly interested in this process at the ZH run, this is still something to be aware of, as it may show up again in calculations of cross sections for other processes.

DESY.

```
#####
def remove_empty_events(self, Gdir):
    """return Gdir strip from the one providing empty events.lhe files."""

    reasons = collections.defaultdict(list)
    Gdirs = Gdir[:]
    for G in Gdirs[:]:
        try:
            size = os.path.getsize(pjoin(G, 'events.lhe'))
        except Exception as error:
            size = 0
        if size < 10:
            Gdirs.remove(G)
            try:
                log = misc.BackRead(pjoin(G, 'log.txt'))
            except Exception as error:
                log = misc.BackRead(pjoin(G, 'run1_app.log'))

            found = -1
            for line in log:
                if 'Deleting file events.lhe in line:'
                    found = 0
                elif 'Impossible BW configuration in line:'
                    reasons['bwconfig'].append(G)
                    break
                elif found < -150:
                    reasons['not found'].append(G)
                    Gdirs.append(G)
                    break
                elif found < 0:
                    found -= 1
                elif 'Loosen cuts or increase max_events' in line:
                    reasons['cuts'].append(G)
                    break
                elif 'all returned zero' in line:
                    reasons['zero'].append(G)
                    break
                elif found > 5:
                    reasons['unknown'].append(G)
                    break
                else:
                    found += 1

    if len(reasons):
        logger.debug('Reasons for empty events.lhe:')
        if len(reasons['unknown']):
            logger.debug(' - unknown: %s' % len(reasons['unknown']))
            logger.log(10, 'DETAIL: ' + ', '.join(['/'.join(G.rsplit(os.sep)[-2:]) for G in reasons['unknown'][:10]]))
        if len(reasons['not found']):
            logger.debug(' - not found in log: %s' % len(reasons['not found']))
            logger.log(10, 'DETAIL: ' + ', '.join(['/'.join(G.rsplit(os.sep)[-2:]) for G in reasons['not found'][:10]]))
        if len(reasons['zero']):
            logger.debug(' - zero amplitudes: %s' % len(reasons['zero']))
            logger.log(10, 'DETAIL: ' + ', '.join(['/'.join(G.rsplit(os.sep)[-2:]) for G in reasons['zero'][:10]]))
        if len(reasons['bwconfig']):
            critical_bwconfig = set()
            for G in reasons['bwconfig']:
                base = G.rsplit('.', 1)[0]
                if any(G2.startswith(base) for G2 in Gdirs):
                    continue
            else:
                critical_bwconfig.add(os.sep.join(base.rsplit(os.sep)[-2:]))
        for G in critical_bwconfig:
            logger.warning('Gdirectory %s has no events.lhe file. (no BW config found %s times)' % (G, 0))

        logger.debug(' - impossible BW configuration: %s' % len(reasons['bwconfig']))
        logger.debug(' - channel with no possible BW configuration: %s' % len(critical_bwconfig))
```

As a quick fix, I removed the "%s times" portion from the string. I'm not sure if the Gdirectory is meant to be matched with a specific count, but dropping the second placeholder resolved the crash on my end.

```
if len(reasons):
    logger.debug('Reasons for empty events.lhe:')
    if len(reasons['unknown']):
        logger.debug(' - unknown: %s' % len(reasons['unknown']))
        logger.log(10, 'DETAIL: ' + ',.join(['/'.join(G.rsplit(os.sep)[-2:])) for G in reasons['unknown']][:10]))
    if len(reasons['not found']):
        logger.debug(' - not found in log: %s' % len(reasons['not found']))
        logger.log(10, 'DETAIL: ' + ',.join(['/'.join(G.rsplit(os.sep)[-2:])) for G in reasons['not found']][:10]))
    if len(reasons['zero']):
        logger.debug(' - zero amplitudes: %s' % len(reasons['zero']))
        logger.log(10, 'DETAIL: ' + ',.join(['/'.join(G.rsplit(os.sep)[-2:])) for G in reasons['zero']][:10]))
    if len(reasons['bwconfig']):
        critical_bwconfig = set()
        for G in reasons['bwconfig']:
            base = G.rsplit('.', 1)[0]
            if any(G2.startswith(base) for G2 in Gdirs):
                continue
            else:
                critical_bwconfig.add(os.sep.join(base.rsplit(os.sep)[-2:]))
        for G in critical_bwconfig:
            logger.warning('Gdirectory %s has no events.lhe file. (no BW config found %s times)' % G)

    logger.debug(' - impossible BW configuration: %s' % len(reasons['bwconfig']))
    logger.debug(' - channel with no possible BW configuration: %s' % len(critical_bwconfig))
```

Going through each process systematically

What are the associated cross sections for the following processes?

1) $e^+e^- \rightarrow aH, a \rightarrow \gamma\gamma$

2) $e^+e^- \rightarrow aZ, a \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow a\gamma, a \rightarrow \gamma\gamma$

3) $e^+e^- \rightarrow aZZ, a \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow a\gamma\gamma, a \rightarrow \gamma\gamma$

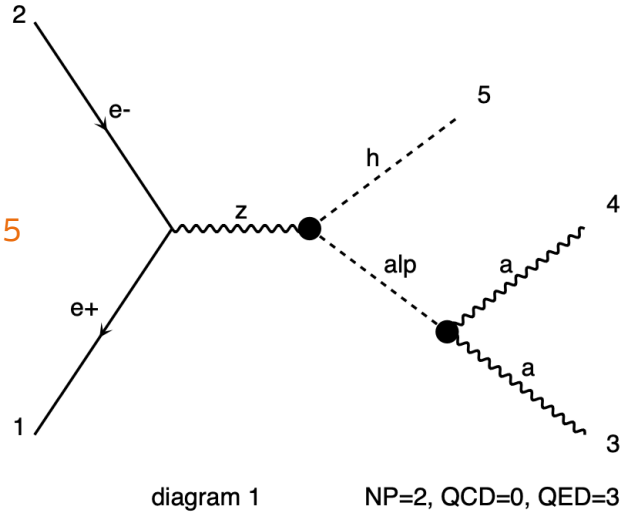
4) $e^+e^- \rightarrow aZH, a \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow a\gamma H, a \rightarrow \gamma\gamma$

First, look at all possible diagrams and determine relevant couplings

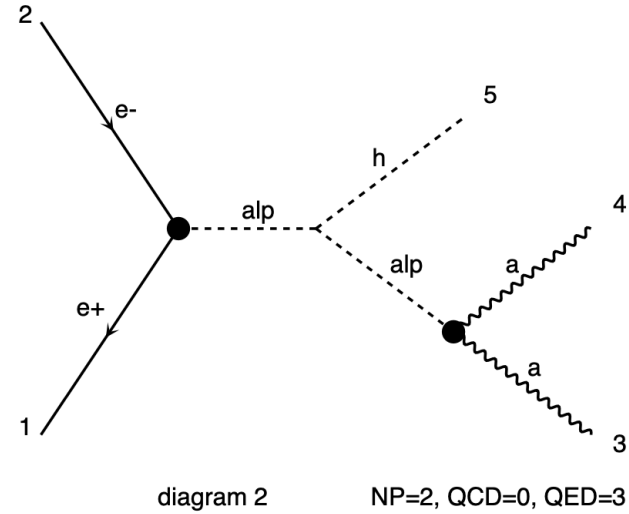
MadGraph generates Feynman diagrams before calculating cross sections and using couplings

1) $e^+e^- \rightarrow aH, a \rightarrow \gamma\gamma$

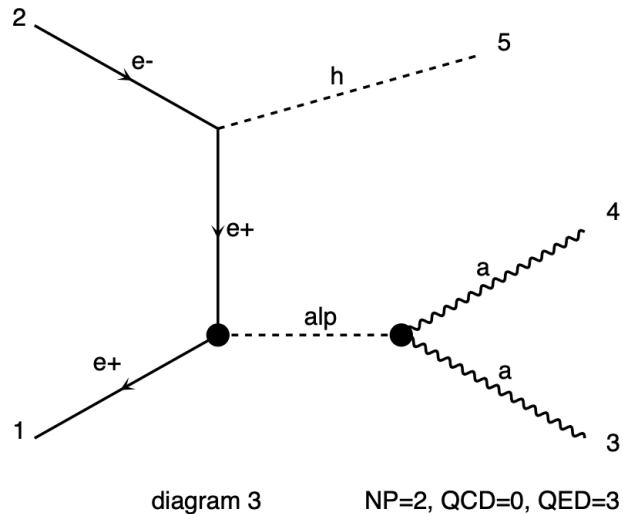
coupling(s): CYY, cZh5



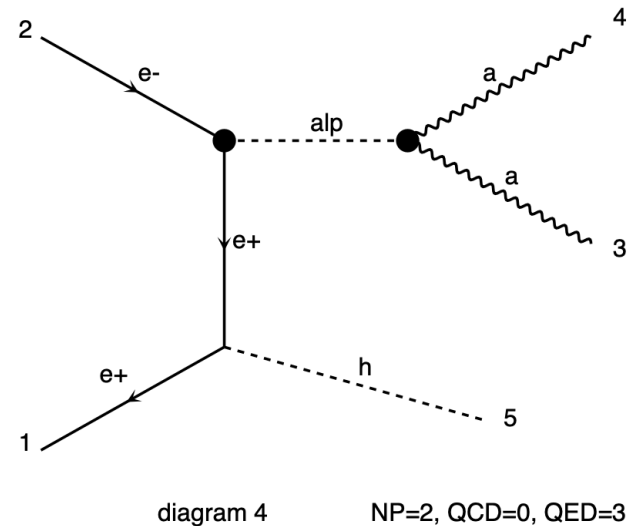
coupling(s): CYY, cah



coupling(s): CYY, cee



coupling(s): CYY, cee



2a) $e^+e^- \rightarrow aZ, a \rightarrow \gamma\gamma$

coupling(s): CYY, cZh5

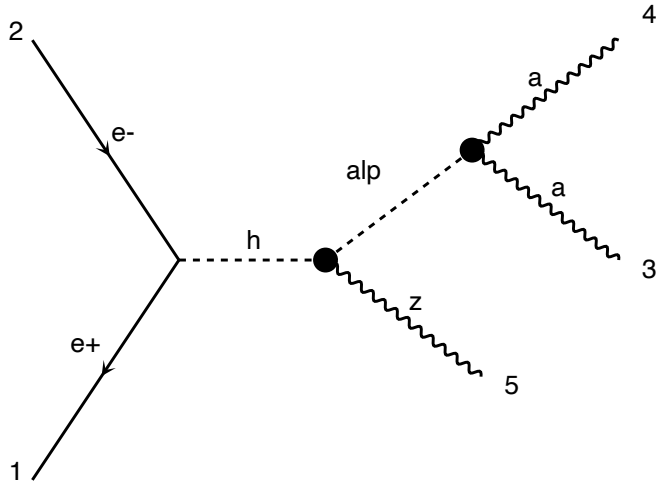


diagram 1

NP=2, QCD=0, QED=3

coupling(s): CYY, cee

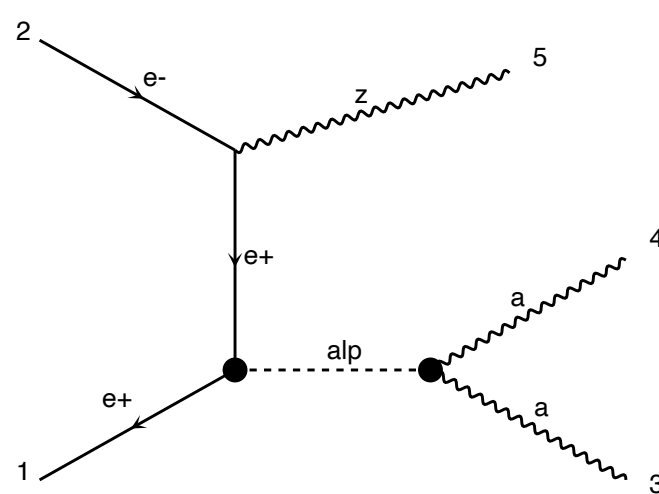


diagram 2

NP=2, QCD=0, QED=3

coupling(s): CYY, cee

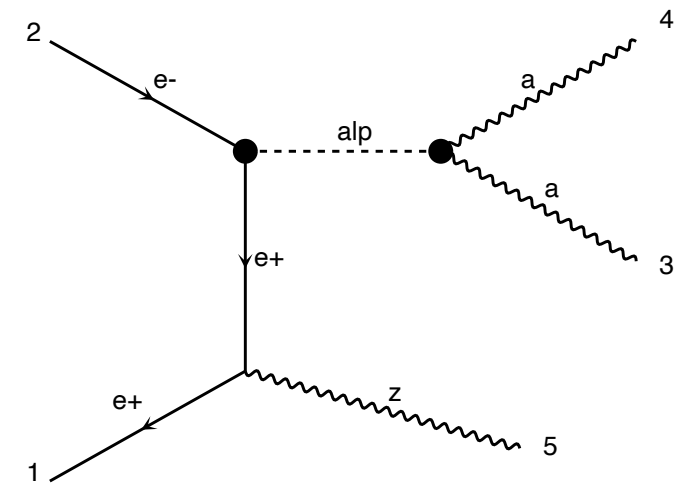


diagram 3

NP=2, QCD=0, QED=3

2b) $e^+e^- \rightarrow a\gamma, a \rightarrow \gamma\gamma$

coupling(s): CYY, cee

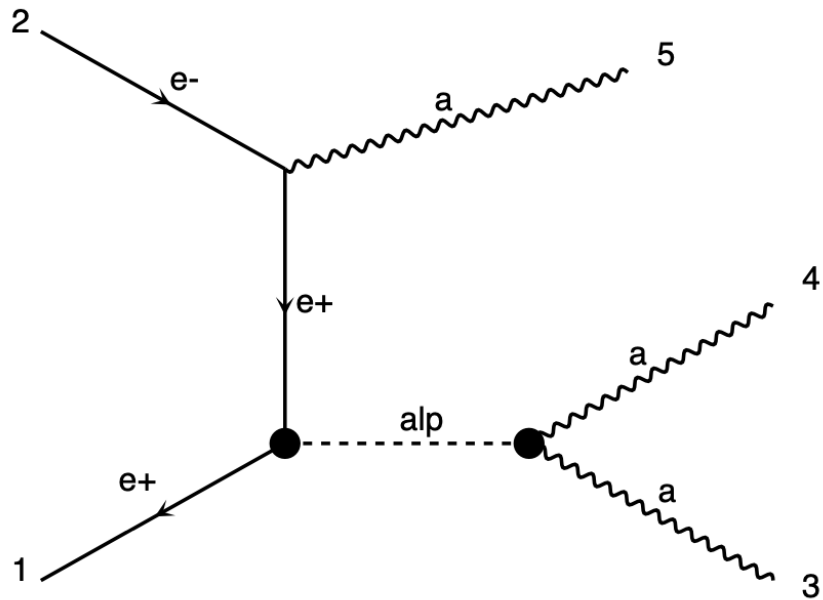


diagram 1

NP=2, QCD=0, QED=3

coupling(s): CYY, cee

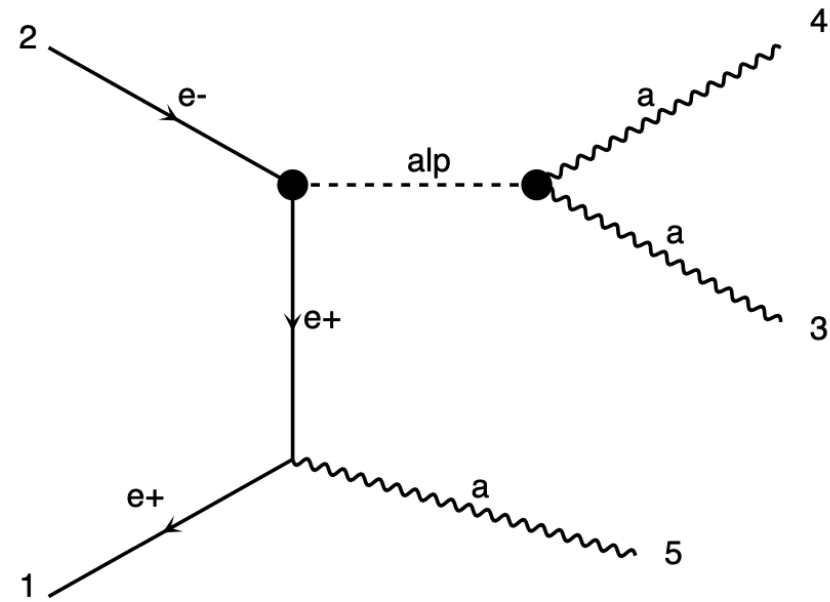


diagram 2

NP=2, QCD=0, QED=3

3a) $e^+e^- \rightarrow aZZ, a \rightarrow \gamma\gamma$

coupling(s): CYY, cZh5

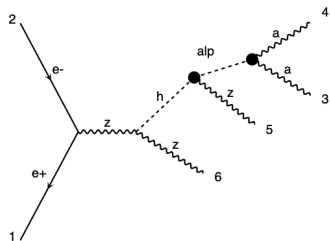


diagram 1 NP=2, QCD=0, QED=4

coupling(s): CYY, cee, cZh5

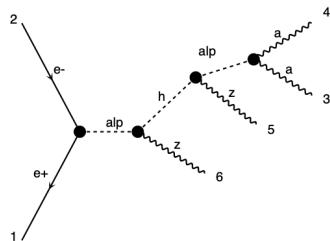


diagram 2 NP=4, QCD=0, QED=2

coupling(s): CYY, cee

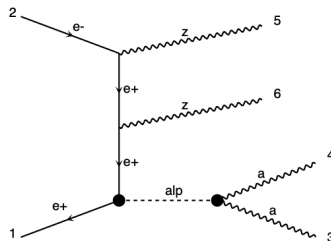


diagram 7 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

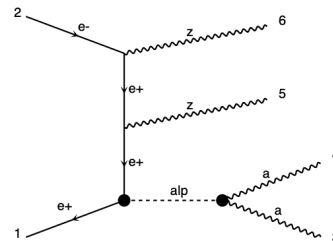


diagram 8 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

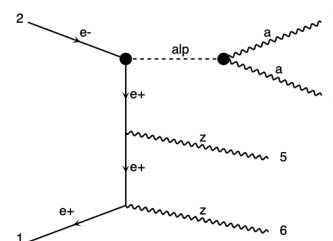


diagram 13 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

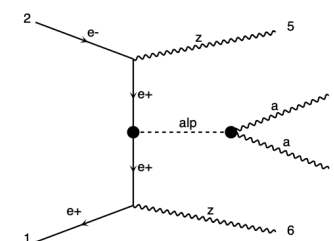


diagram 14 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

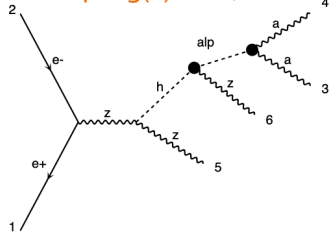


diagram 3 NP=2, QCD=0, QED=4

coupling(s): CYY, cee, cZh5

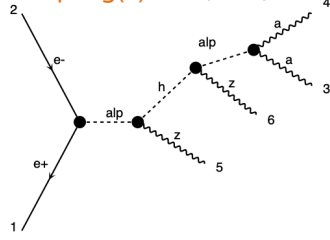


diagram 4 NP=4, QCD=0, QED=2

coupling(s): CYY, cee

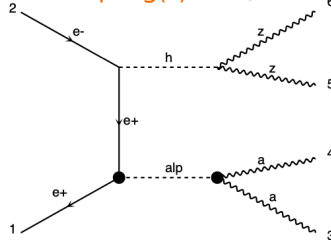


diagram 9 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

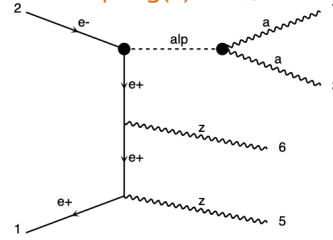


diagram 10 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

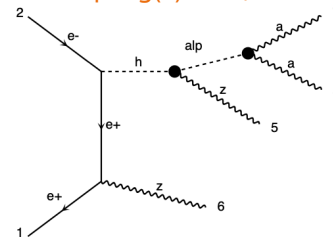


diagram 15 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

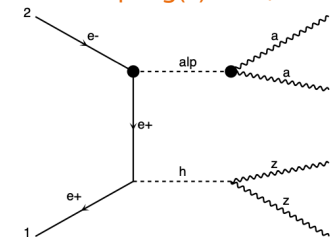


diagram 16 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

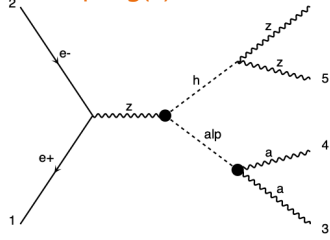


diagram 5 NP=2, QCD=0, QED=4

coupling(s): CYY, cee, cah

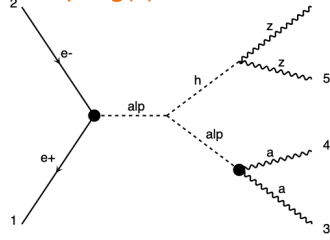


diagram 6 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

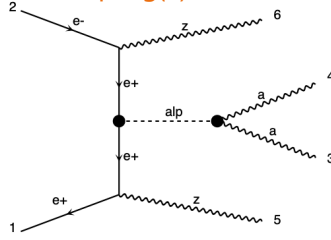


diagram 11 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

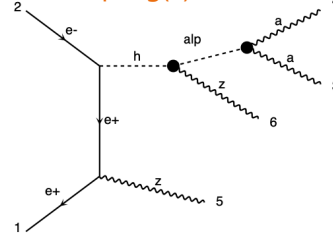


diagram 12 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

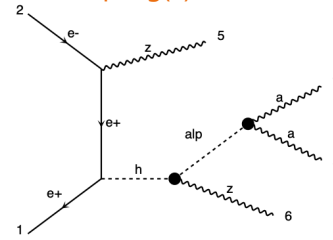


diagram 17 NP=2, QCD=0, QED=4

coupling(s): CYY, cZh5

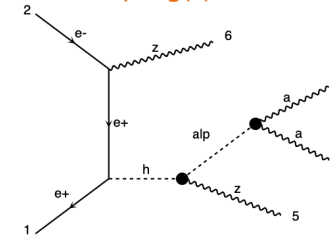


diagram 18 NP=2, QCD=0, QED=4

3b) $e^+e^- \rightarrow a\gamma\gamma, a \rightarrow \gamma\gamma$

coupling(s): CYY, cee

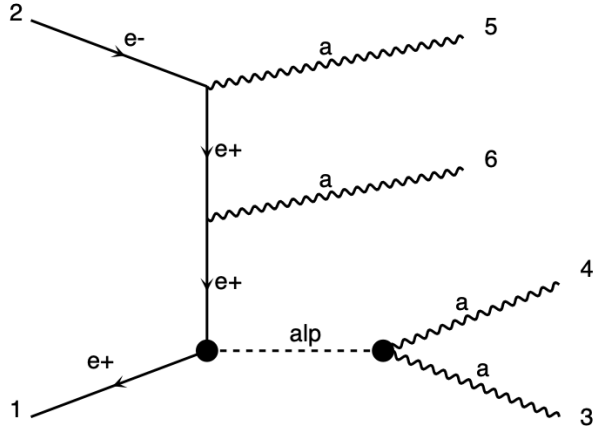


diagram 1 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

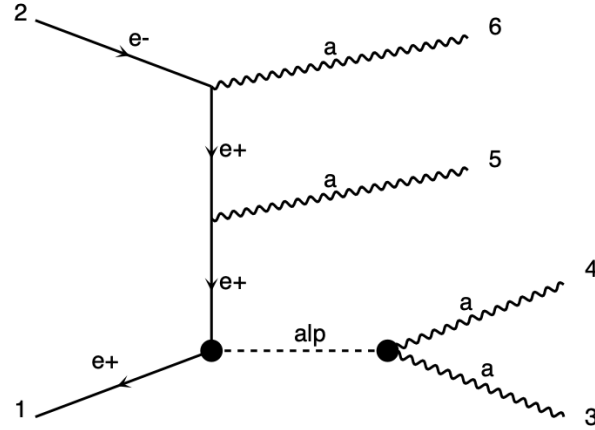


diagram 2 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

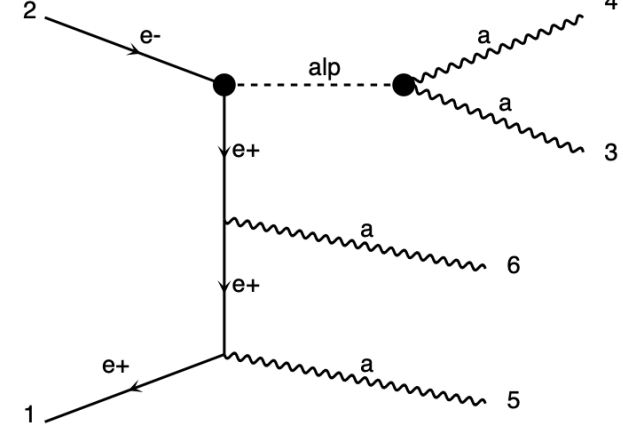


diagram 3 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

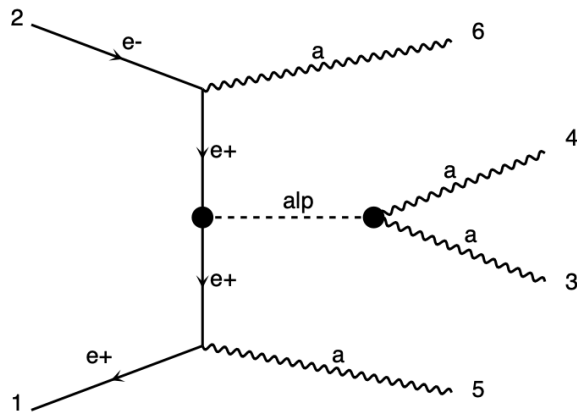


diagram 4 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

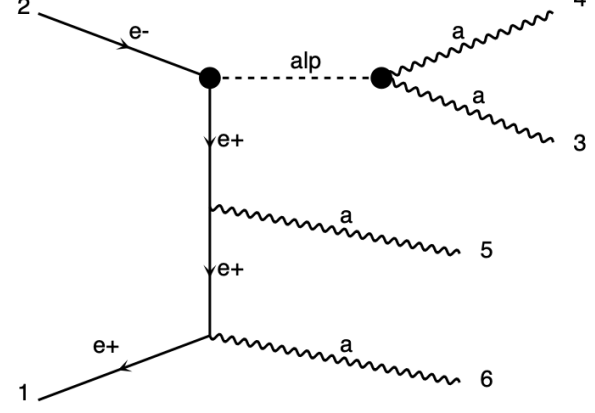


diagram 5 NP=2, QCD=0, QED=4

coupling(s): CYY, cee

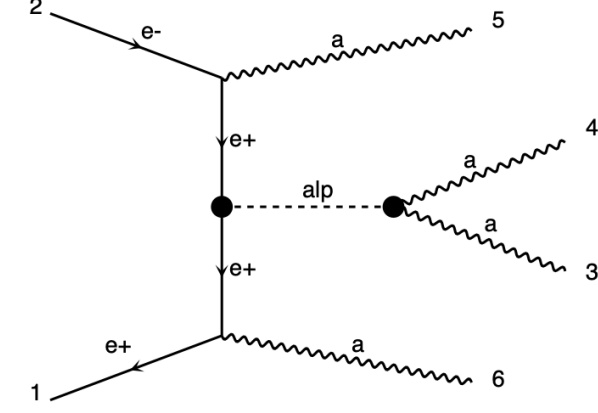
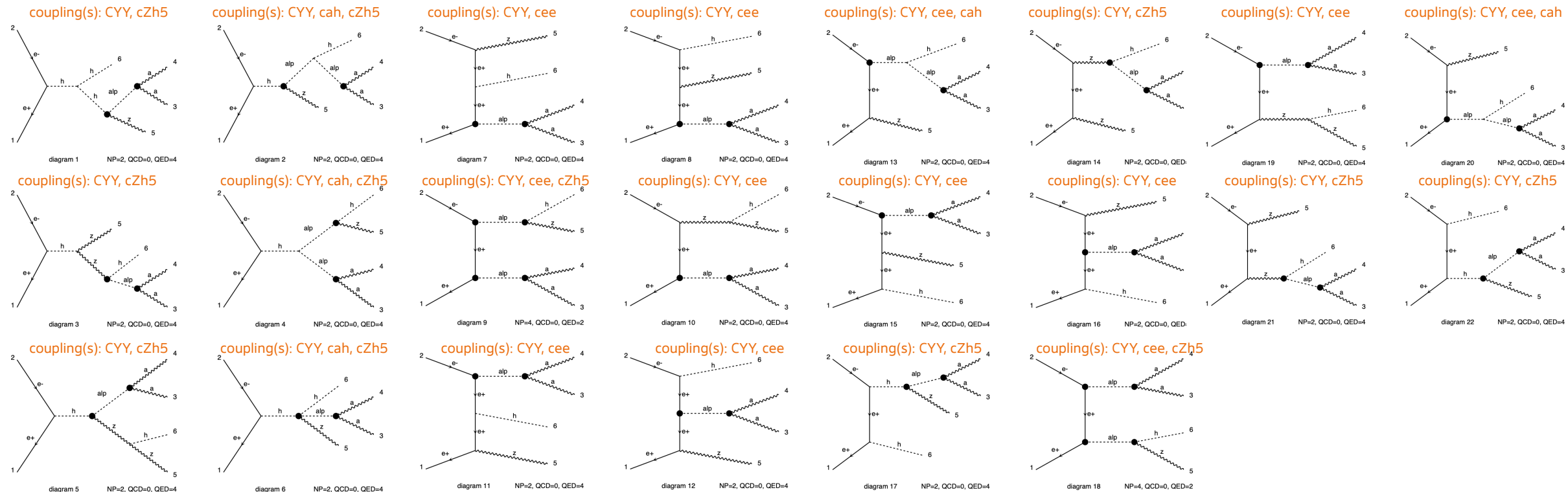
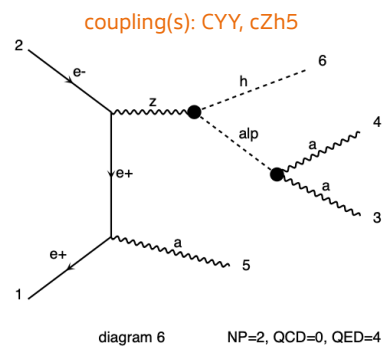
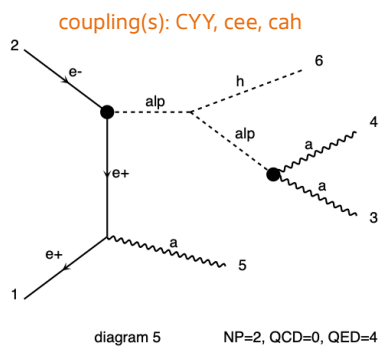
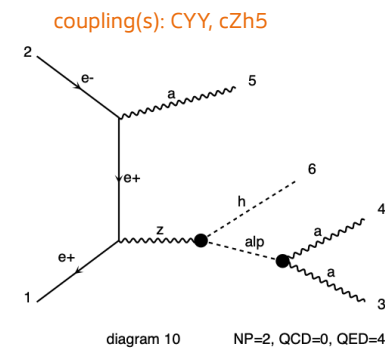
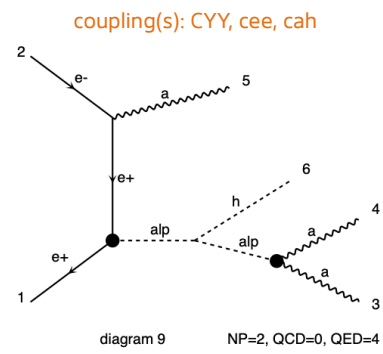
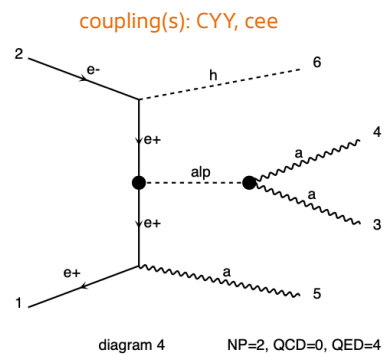
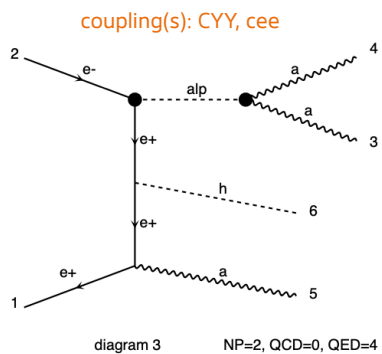
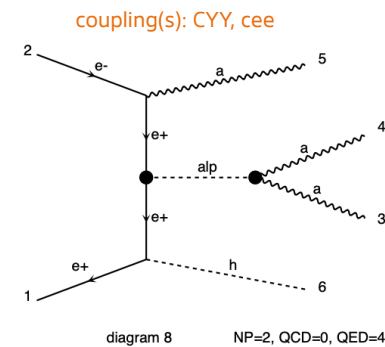
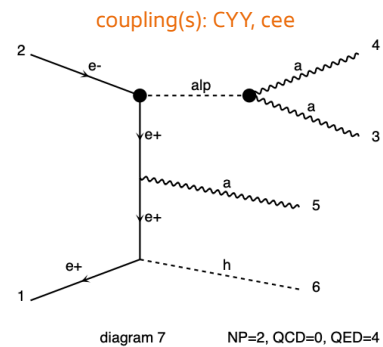
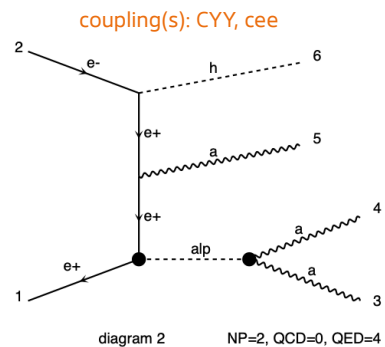
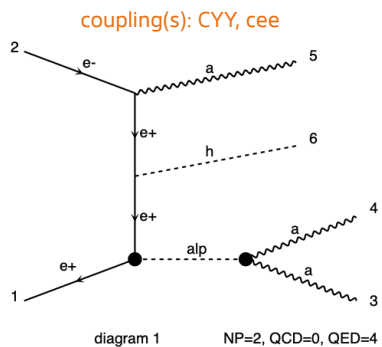


diagram 6 NP=2, QCD=0, QED=4

4a) $e^+e^- \rightarrow aZH, a \rightarrow \gamma\gamma$



4b) $e^+e^- \rightarrow a\gamma H, a \rightarrow \gamma\gamma$



Cross sections

Code in the table corresponds to couplings:

Code **ABCD**:

A: coupling to photons **CYY**

B: coupling to electrons **cee**

C: coupling to ALP and Higgs **cah**

D: coupling to Z and Higgs **cZh5**

Example: 1001

CYY set to 1.

cee set to 0.

cah set to 0.

cZh5 set to 1.

If coupling is enabled, it is set to 1.0

CYY is always enabled since we specify decay of ALP to two photons

Mass of ALP also set to 1.0 GeV for all processes

Process	Code	Cross Section
$e^+e^- \rightarrow aH, a \rightarrow \gamma\gamma$	1001	165.6 ± 0.6018 pb
	1010	Survey return zero cross section.
	1100	$2.208\text{e-}07 \pm 2.196\text{e-}09$ pb
$e^+e^- \rightarrow aZ, a \rightarrow \gamma\gamma$	1001	$4.805\text{e-}07 \pm 1.045\text{e-}09$ pb
	1100	$6.243\text{e-}07 \pm 2.044\text{e-}09$ pb
$e^+e^- \rightarrow a\gamma, a \rightarrow \gamma\gamma$	1100	$6.301\text{e-}07 \pm 1.438\text{e-}09$ pb
$e^+e^- \rightarrow aZZ, a \rightarrow \gamma\gamma$	1001	5697 ± 12.04 pb
	1100	$5.762\text{e-}11 \pm 4.292\text{e-}13$ pb
	1101	4247 ± 9.412 pb
	1110	$1.042\text{e-}05 \pm 1.069\text{e-}07$ pb
$e^+e^- \rightarrow a\gamma\gamma, a \rightarrow \gamma\gamma$	1100	$2.747\text{e-}08 \pm 6.527\text{e-}10$ pb
$e^+e^- \rightarrow aZH, a \rightarrow \gamma\gamma$	1001	$0.003187 \pm 2.133\text{e-}05$ pb
	1011	$0.003182 \pm 1.456\text{e-}05$ pb
	1100	$1.674\text{e-}12 \pm 6.996\text{e-}15$ pb
	1101	$0.002328 \pm 2.152\text{e-}05$ pb
	1110	$1.005\text{e-}07 \pm 4.229\text{e-}10$ pb
	1001	3.502 ± 0.02914 pb
$e^+e^- \rightarrow a\gamma H, a \rightarrow \gamma\gamma$	1100	$4.326\text{e-}09 \pm 3.305\text{e-}11$ pb
	1110	$0.003682 \pm 4.148\text{e-}05$ pb

Cross section $e^+e^- \rightarrow aH, a \rightarrow \gamma\gamma$

165.6 ± 0.6018 pb

1001: CYY, cZh5

Cross section if cZh5 set to limit
from paper (0.72): 86.02 pb

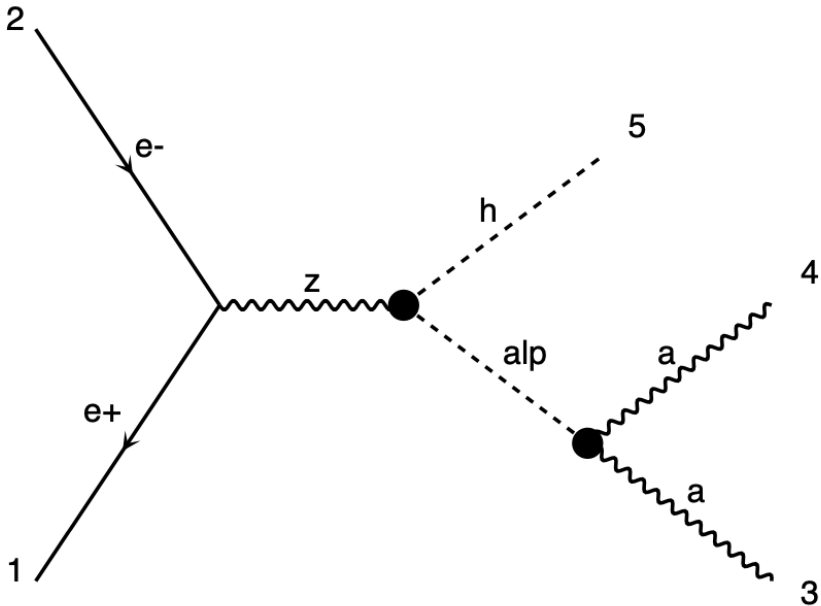


diagram 1 NP=2, QCD=0, QED=3

s= 165.61 ± 0.602 (pb)

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1.7	165.6	0.602	7.009	391.0	2.36
G4	0	0	0.0	1.0	0
G3	0	0	0.0	1.0	0
G2.8	0	0	0.0	0.0	0
G2.7	0	0	0.0	1.0	0
G1.8	0	0	0.0	0.0	0

Cross section $e^+e^- \rightarrow aZZ, a \rightarrow \gamma\gamma$

5697 ± 12.04 pb

1001: CYY, cZh5

$s = 5697.1 \pm 12$ (pb)

Cross section if cZh5 set to limit
from paper (0.72): 2939 pb

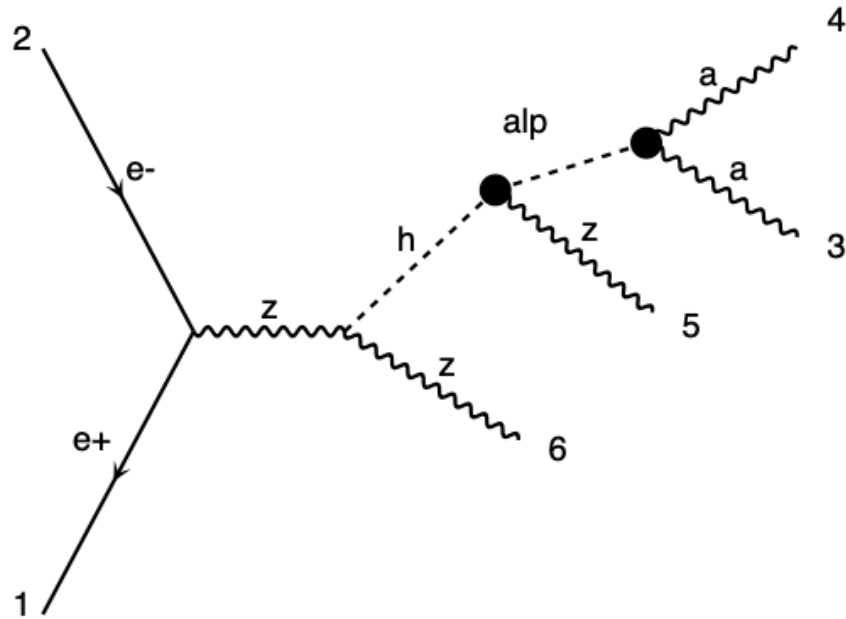


diagram 1

NP=2, QCD=0, QED=4

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1	5697	12	7.009	384.0	0.0674
G5.25	0.007277	5.65e-05	7.009	403.0	5.54e+04
G17	8.197e-17	9.7e-19	7.009	421.0	5.14e+18
G12	8.138e-17	8.88e-19	7.009	392.0	4.82e+18
G16.02	0	0	0.0	1.0	0
G11	0	0	0.0	1.0	0
G10	0	0	0.0	1.0	0
G9.06	0	0	0.0	1.0	0
G7	0	0	0.0	1.0	0
G6.26	0	0	0.0	0.0	0
G6.25	0	0	0.0	1.0	0
G5.26	0	0	0.0	0.0	0
G2.26	0	0	0.0	0.0	0
G2.25	0	0	0.0	1.0	0
G2.23	0	0	0.0	0.0	0
G2.22	0	0	0.0	1.0	0

Cross section $e^+e^- \rightarrow aZZ, a \rightarrow \gamma\gamma$

4247 ± 9.412 pb

1101: CYY, cee, cZh5

$s = 4246.7 \pm 9.41$ (pb)

Cross section if cZh5 set to limit
from paper (0.72): 2207 pb

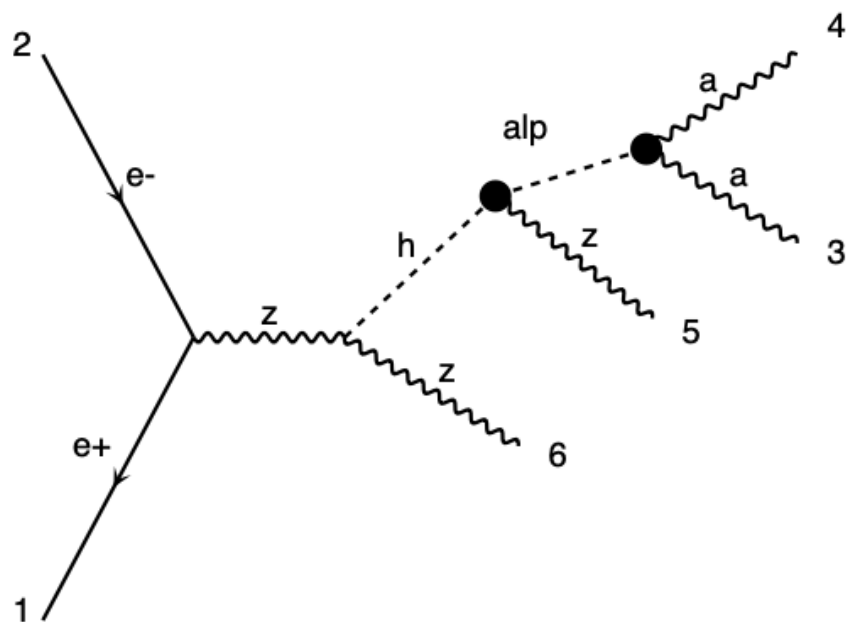


diagram 1

NP=2, QCD=0, QED=4

Graph	Cross-Section ↓	Error	Events (K)	Unwgt	Luminosity
G1	4246	9.41	7.009	386.0	0.0909
G2.22	0.8363	0.00278	7.009	400.0	478
G10	0.1002	0.00129	7.009	380.0	3.79e+03
G7	0.0997	0.00143	7.009	390.0	3.91e+03
G2.25	0.05683	0.002	15.009	391.0	6.88e+03
G11	0.01657	0.000189	7.009	425.0	2.57e+04
G5.25	0.004842	3.57e-05	7.009	388.0	8.01e+04
G12	5.062e-07	2.73e-09	7.009	386.0	7.63e+08
G17	5.042e-07	2.59e-09	7.009	384.0	7.62e+08
G9.06	9.707e-12	1e-13	7.009	390.0	4.02e+13
G16.02	9.68e-12	1.75e-13	7.009	398.0	4.11e+13
G6.26	0	0	0.0	0.0	0
G6.25	0	0	0.0	1.0	0
G5.26	0	0	0.0	0.0	0
G2.26	0	0	0.0	0.0	0
G2.23	0	0	0.0	0.0	0

08.07.25

ALPnlo

recently obtained from Bauer by Abu Dhabi colleagues, no known problem, except lack of phase space factors in decays into fermions

```
set gPU 0
set gPd 0
set gPI 0
set cah 0
set cZh5 0
set cWW 0.
set cBB 1.
set cGG 0
set Lambda 1000.
set mh 125
set MALP 1.
set WALP auto
```

ALP

obtained from Thamm in 2019 only works if mh is put high

```
set gPU 0
set gPd 0
set gPI 0
set cah 0
set cZh5 0
set cAA 1.
set cZA -0.223
set cGG 0
set Lambda 1000.
set mh 1000
set MALP 1.
set WALP auto
```

ALP_NLO_UFO

used for Snowmass report, available in LLP git, seems to yield sensible results only for CWW=0

```
set Ma 1.  
set cWW = 0.0  
set CYY = 1.  
set cGG = 0.  
set cuu = 0.  
set cdd = 0.  
set ccc = 0.  
set css = 0.  
set ctt = 0.  
set cbb = 0.  
set cee = 0.  
set cmumu = 0.  
set ctautau = 0.  
set cah = 0.  
set cZh5 = 0.  
set falp = 6.33  
set WALP auto
```

ALP_linear_UFO

Brivio et al.

From Giacomo:

In order to match the input parameters for this UFO (ALP_linear_UFO) to the ones for the Bauer et al. UFO (ALP), the following formulas are useful:

$$\begin{aligned} \text{CBtil} &= \text{CBB} * 4\pi\alpha/cw2 = \text{CBB} * 0.128215343 \\ \text{CWtil} &= \text{CWW} * 4\pi\alpha/sw2 = \text{CWW} * 0.420418893 \end{aligned}$$

$$\begin{aligned} \text{CBB} &= -\text{CZA} + \text{CAA} * (1 - sw2) \\ \text{CWW} &= \text{CZA} + \text{CAA} * sw2 \end{aligned}$$

where the input parameters for the W/gamma sector in card are

Mimasu: CBtil, CWtil
Thamm: CAA, CZA

The correspondance of other parameters is

Mimasu	Thamm
-----	-----
fa	Lambda
Ma	MALP
Wax (*)	WALP

(*) In the original version of the UFO the width of the ALP was hardcoded to zero. I modified the UFO files in order to get it parametric.

```
set pta 0.
set ptl 0.
set ptj 0.
set ptb 0.
set etaa 2.6
set draa 0.0
set ea 0.0
set CGtil 0.
set CWtil 0.
set CBtil 0.1282155343
set CaPhi 0
set fa 1000.
set Ma 1
set Wax auto
```

Cross section calculation

Based on formula 16 of Bauer et al. arXiv:1808.10323

$$\frac{d\sigma(e^+e^- \rightarrow \gamma a)}{d\Omega} = 2\pi\alpha\alpha^2(s)\frac{s^2}{\Lambda^2} \left(1 - \frac{m_a^2}{s}\right)^3 (1 + \cos^2 \theta) (|V_\gamma(s)|^2 + |A_\gamma(s)|^2), \quad (16)$$

```
1  import math
2  #-----
3  # Calculate production cross-section for e+e-->\gamma a
4  # based on formula 16 of Bauer et al. arXiv:1808.10323
5  # GP 06/07/25
6  #-----
7  # electroweak constants
8  #-----
9  XSZpole=58500 # pb L0 XS at sqrts=91.2 GeV
10 MZ =91.1876 # GeV
11 MZ2=MZ**2
12 GamZ=2.4952 # GeV
13 aEW1=127.9
14 aEW1Z=128.952
15 Gf=0.0000116637 # GeV-2
16 aEW=1/aEW1
17 aEWZ=1/aEW1Z
18 Mw=math.sqrt(MZ**2/2. + math.sqrt(MZ**4/4. - (aEW*math.pi*MZ**2)/(Gf*math.sqrt(2))))
19 sw2=1 - Mw**2/MZ**2
20 cw2=1 - sw2
21 gv=2*sw2-0.5
22 ga=-0.5
23 #-----
```

```
24 # e+e-->\gamma a cross-section calculation
25 def calc_xs_ga(sqs,ma,cgg,ggz,Lam):
26     s=sqs**2
27     # vector and axial coupling, formula 19
28     Vg=cgg/s+gv*cgz/2/sw2/cw2/complex(s-MZ2,MZ*GamZ)
29     Vg2=(Vg*Vg.conjugate()).real
30     Ag=ga*cgz/2/sw2/cw2/complex(s-MZ2,MZ*GamZ)
31     Ag2=(Ag*Ag.conjugate()).real
32     # formula 16
33     const=2*math.pi*aEW*aEWZ**2*s**2/Lam**2*(1-ma**2/s)**3*(Vg2+Ag2)
34     # integral of phase space integrate on (1+cos^2\theta)
35     phsp=2*math.pi*((1+1/3)-(-1-1/3))
36     # convert using hc^2 into picobarns
37     xs=const*phsp*0.389379*1e9
38     return xs
39 #-----
40 # width of Z into gamma alp (GeV), formula 12
41 def gam_galp(ma,cgz,Lam):
42     gamalp=0
43     if(ma<MZ):
44         gamalp=8*math.pi*aEW*aEWZ*MZ**3*cgz**2*(1-ma**2/MZ**2)**3/3/sw2/cw2/Lam**2
45     return gamalp
46 #-----
47 # transform from cww, cbb to cgg, cgz, czz, formula 3
48 def calc_coeff(cbb,cww):
49     cgg=cww+cbb
50     cgz=cw2*cww-sw2*cbb
51     czz=cw2**2*cww+sw2**2*cbb
52     return cgg,cgz,czz
```

Cross sections

Test each model and compare cross sections

Process: $e^+ e^- \rightarrow a$, ($a > a \text{ ALP}$, ($\text{ALP} > a a$))

Particle decaying to itself so potentially some issues with infinite recursion?

$\sqrt{s}$, m_a , c_{VV}	Model	Cross Section
91.188, 1, 1	ALPnlo	0.01919 +- 5.113e-05 pb
	ALP	0.01921 +- 5.29e-05 pb
	ALP_NLO_UFO	0.01914 +- 5.157e-05 pb
	ALP_linear_UFO	0.01919 +- 4.968e-05 pb
240, 1, 1	ALPnlo	0.01915 +- 6.019e-05 pb
	ALP	0.01919 +- 5.259e-05 pb
	ALP_NLO_UFO	0.01916 +- 7.818e-05 pb
	ALP_linear_UFO	0.01918 +- 5.112e-05 pb

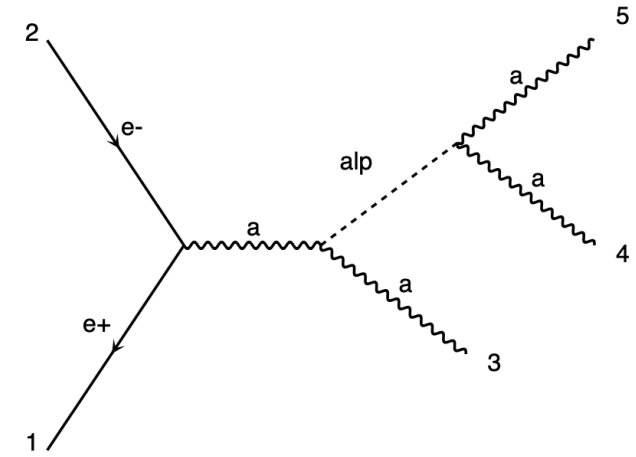


diagram 1

QCD=0, QED=7

Direct calculation (crossalp.py)

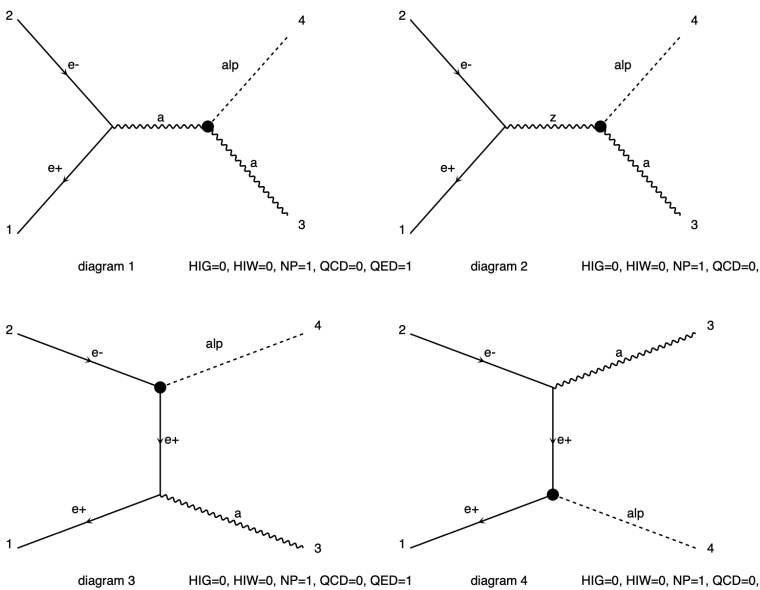
Inputs: $m_a = 1$ GeV; $\Lambda = 1000$ GeV; $\sqrt{s} = 91.188$ GeV
 $c_{WW} = 0$ $c_{BB} = 1$
 $c_{gg} = 1$ $c_{gz} = -0.2337$
 xs from formula **2.769765571070452 pb**

Inputs: $m_a = 1$ GeV; $\Lambda = 1000$ GeV; $\sqrt{s} = 240$ GeV
 $c_{WW} = 0$ $c_{BB} = 1$
 $c_{gg} = 1$ $c_{gz} = -0.2337$
 xs from formula **0.02304513132546023 pb**

Cross sections

Test each model and compare cross sections
Process: $e^+ e^- \rightarrow a \text{ ALP}$

$\sqrt{s}, m_a, c_{\gamma\gamma}$	Model	Cross Section
91.188, 1, 1	ALPnlo	Zero cross section
	ALP	2.469 +- 0.006559 pb
	ALP_NLO_UFO	Zero cross section
	ALP_linear_UFO	2.778 +- 0.006595 pb
240, 1, 1	ALPnlo	Zero cross section
	ALP	0.02082 +- 4.045e-05 pb
	ALP_NLO_UFO	Zero cross section
	ALP_linear_UFO	0.02115 +- 4.063e-05 pb



Direct calculation (crossalp.py)

Inputs: $m_a = 1 \text{ GeV}$; $\Lambda = 1000 \text{ GeV}$; $\sqrt{s} = 91.188 \text{ GeV}$
 $c_{WW} = 0$ $c_{bb} = 1$
 $c_{gg} = 1$ $c_{gz} = -0.2337$
xs from formula **2.769765571070452 pb**

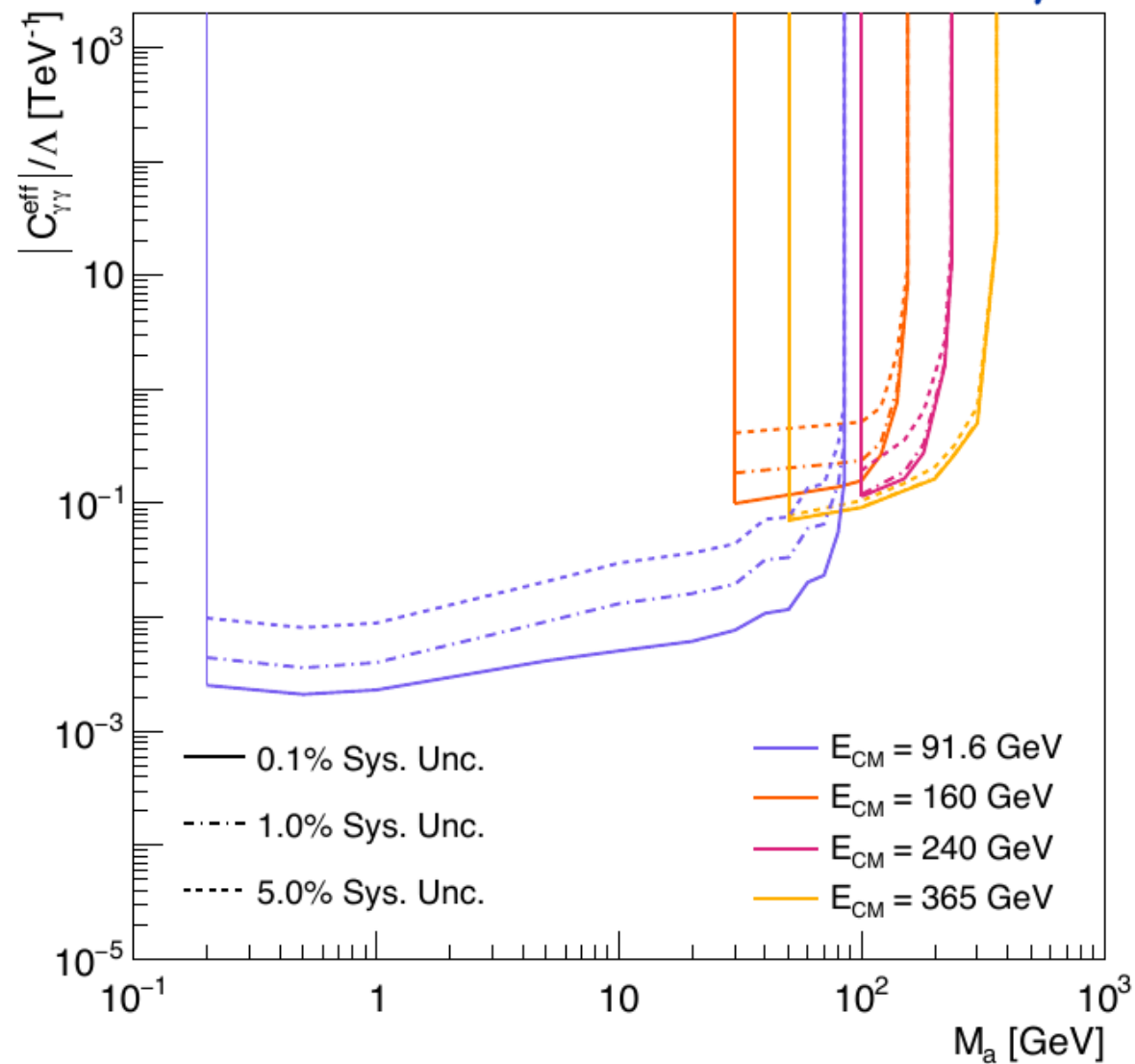
Inputs: $m_a = 1 \text{ GeV}$; $\Lambda = 1000 \text{ GeV}$; $\sqrt{s} = 240 \text{ GeV}$
 $c_{WW} = 0$ $c_{bb} = 1$
 $c_{gg} = 1$ $c_{gz} = -0.2337$
xs from formula **0.02304513132546023 pb**

Use Brivio et al. model? Unfortunately the coupling of the ALP to Higgs and fermions has not been mapped yet

Preliminary plot

Plot made by Giacomo's student

FCCee IDEA: $e^+e^- \rightarrow \gamma a$



Cross sections

Now testing ALP_linear UFO model

Process: $e^+ e^- \rightarrow a \, ax, ax \rightarrow a \, a$

In the broken phase of the electroweak symmetry, the ALP couples to the photon and the Z boson as

$$\mathcal{L}_{\text{eff}} \ni e^2 C_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{2e^2}{s_w c_w} C_{\gamma Z} \frac{a}{\Lambda} F_{\mu\nu} \tilde{Z}^{\mu\nu} + \frac{e^2}{s_w^2 c_w^2} C_{ZZ} \frac{a}{\Lambda} Z_{\mu\nu} \tilde{Z}^{\mu\nu}. \quad (2)$$

The relevant Wilson coefficients are given by

$$C_{\gamma\gamma} = C_{WW} + C_{BB}, \quad C_{\gamma Z} = c_w^2 C_{WW} - s_w^2 C_{BB}, \quad C_{ZZ} = c_w^4 C_{WW} + s_w^4 C_{BB}, \quad (3)$$

Parameters	Cross Section
$\sqrt{s} = 240, C_{BB} = 1, C_{WW} = 0$	0.02068 +- 5.766e-05 pb
$\sqrt{s} = 240, C_{BB} = 0.5, C_{WW} = 0.5$	0.02345 +- 7.016e-05 pb
$\sqrt{s} = 240, C_{BB} = 0.25, C_{WW} = 0.75$	0.03436 +- 7.736e-05 pb
$\sqrt{s} = 240, C_{BB} = 0.75, C_{WW} = 0.25$	0.01892 +- 7.23e-05 pb

Couplings

ALP production in association with a photon, Z or Higgs boson

For $e^+e^- \rightarrow \gamma a \rightarrow 3\gamma$ and $e^+e^- \rightarrow Z a \rightarrow Z\gamma\gamma$, the process only depends on the photon coupling $|C_{\gamma\gamma}^{\text{eff}}|/\Lambda$ once a specific relation between C_{WW} and C_{BB} is assumed, see (3). The projected reach can therefore be compared to the limits in Figure 4. If the FCC-ee will operate at different values of the center-of-mass energy, it is in principle possible to measure the two coefficients $C_{\gamma Z}^{\text{eff}}$ and $C_{\gamma\gamma}^{\text{eff}}$ independently, as pointed out in [25]. Also, for the proposed Z -pole run of the FCC-ee, the process $e^+e^- \rightarrow \gamma a \rightarrow 3\gamma$ would correspond to on-shell decay of the Z boson to an ALP, $Z \rightarrow \gamma a$, which will be discussed below.

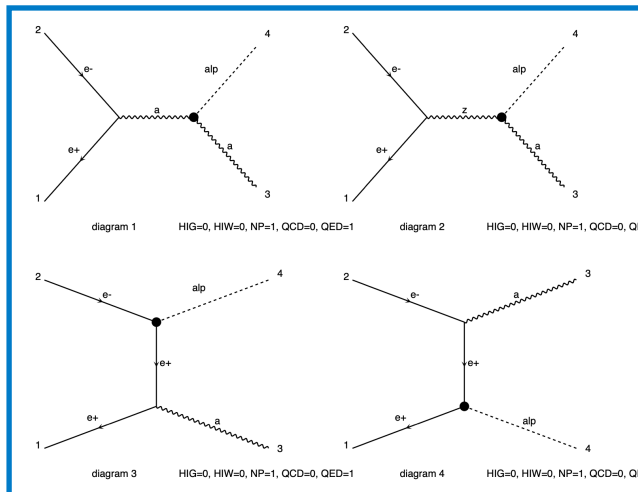
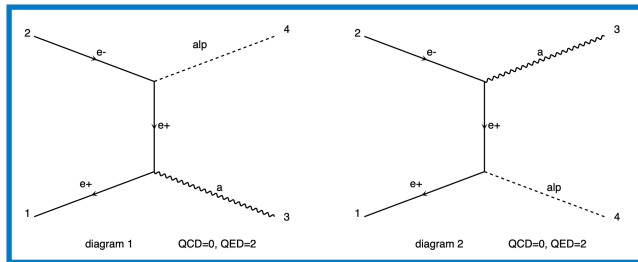
are loop suppressed. If all Wilson coefficients are of the same magnitude and the ALP is lighter than the pion (or if it does not couple to colored particles at all), the dominant decay mode is $a \rightarrow \gamma\gamma$. The leptonic modes $a \rightarrow \ell^+\ell^-$ are only significant near the thresholds $m_a \gtrsim 2m_\ell$, where they can be dominant. If the ALP–boson couplings are loop suppressed,

09.07.25/14.07.25

Testing ALP models

$$e^+e^- \rightarrow a\gamma$$

$\sqrt{s} = 91.188$, $m_a = 1$ GeV, $C_{YY} = 1$, $C_{WW} = 0$, $C_{BB} = 1$, $n_{\text{events}} = 1$



Model	Cross Section
ALPnlo	Survey returns zero cross section.
ALP_NLO_UFO	Survey returns zero cross section.
ALP	2.475 +- 0.007508 pb
ALP_linear_UFO	2.747 +- 0.006761 pb

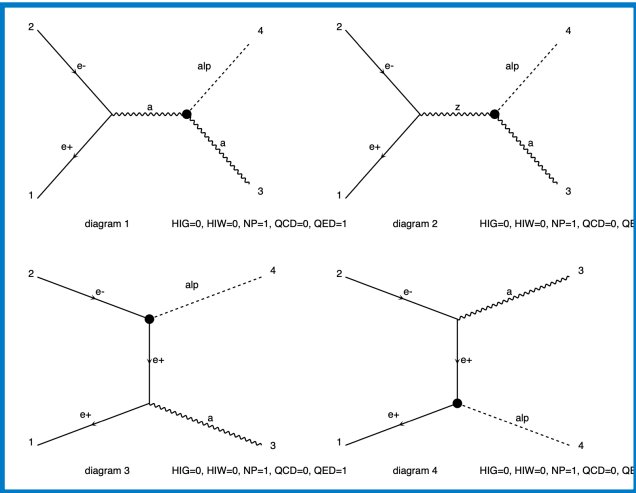
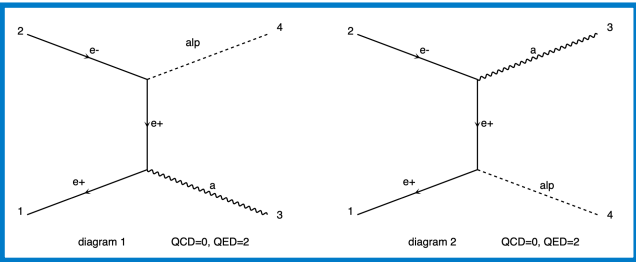
Not sure what is going on here... ALPnlo and ALP_NLO_UFO don't have ALP couplings to Z/gamma or corresponding vertices? Pretty sure MadGraph generates diagrams before incorporating these model parameters though... could be wrong though

Testing ALP models

$$e^+e^- \rightarrow a\gamma$$

Same process as previous slide, just lower centre-of-mass energy (below Z pole) so expect cross section to go down

$$\sqrt{s} = 45, m_a = 1 \text{ GeV}, C_{YY} = 1, C_{WW} = 0, C_{BB} = 1, \text{nevents} = 1$$



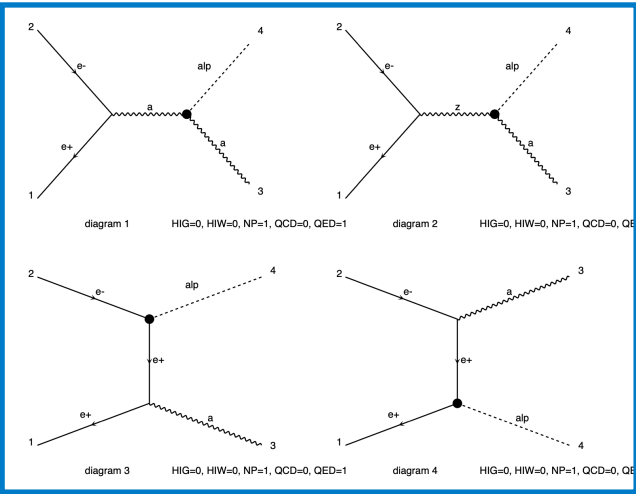
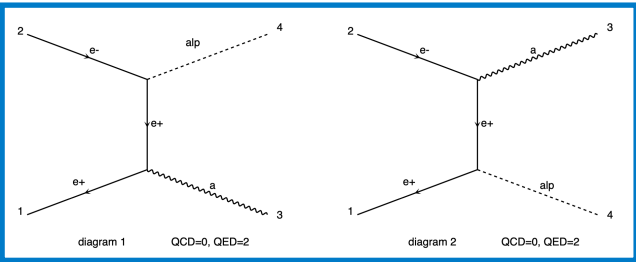
Model	Cross Section
ALPnlo	Survey returns zero cross section.
ALP_NLO_UFO	Survey returns zero cross section.
ALP	0.01705 +- 4.64e-05 pb
ALP_linear_UFO	0.01969 +- 4.192e-05 pb

Testing ALP models

$$e^+e^- \rightarrow a\gamma$$

Same process again, but now higher centre-of-mass energy (ZH run)

$$\sqrt{s} = \mathbf{240}, m_a = 1 \text{ GeV}, C_{YY} = 1, C_{WW} = 0, C_{BB} = 1, \text{nevents} = 1$$



Model	Cross Section
ALPnlo	Survey returns zero cross section.
ALP_NLO_UFO	Survey returns zero cross section.
ALP	0.02089 +- 4.58e-05 pb
ALP_linear_UFO	0.02106 +- 4.315e-05 pb

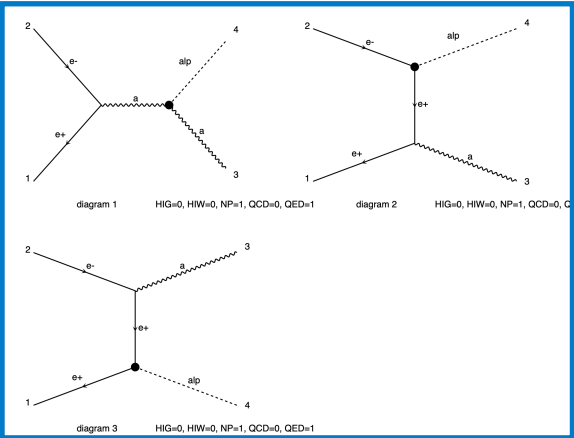
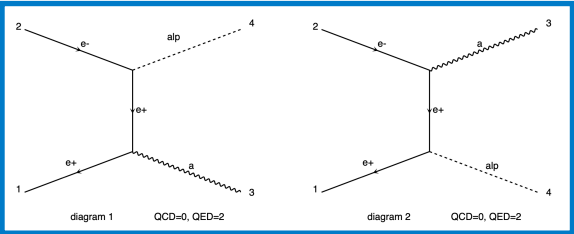
Testing ALP models

What if we forbid the Z boson in the s-channel?

No longer see enhancement at Z pole since no Z production; also see slightly increasing cross section with greater CME

$$e^+e^- \rightarrow a\gamma \text{ \textit{\$ \$} } Z$$

$m_a = 1 \text{ GeV}, C_{YY} = 1, C_{WW} = 0, C_{BB} = 1, \text{nevents} = 1$

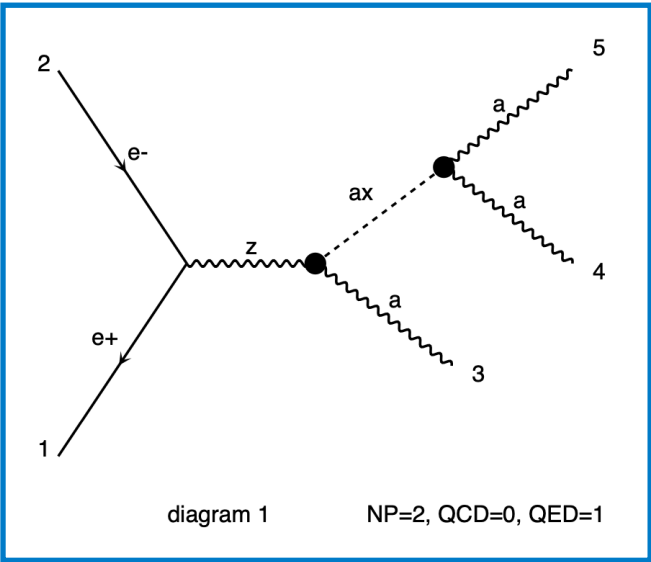


$\sqrt{s}$	Model	Cross Section
45 GeV	ALPnlo	Survey returns zero cross section.
	ALP_NLO_UFO	Survey returns zero cross section.
	ALP	0.01664 +- 4.565e-05 pb
	ALP_linear_UFO	0.01915 +- 4.287e-05 pb
91.188 GeV	ALPnlo	Survey returns zero cross section.
	ALP_NLO_UFO	Survey returns zero cross section.
	ALP	0.01886 +- 5.176e-05 pb
	ALP_linear_UFO	0.0192 +- 5.974e-05 pb
240 GeV	ALPnlo	Survey returns zero cross section.
	ALP_NLO_UFO	Survey returns zero cross section.
	ALP	0.01927 +- 4.198e-05 pb
	ALP_linear_UFO	0.0192 +- 4.777e-05 pb

Testing ALP models

$$e^+e^- \rightarrow Z, (Z > a\gamma, (a > \gamma\gamma))$$
 Running process Elnura used

$$\sqrt{s} = 91.188, m_a = 1 \text{ GeV}, C_{YY} = 1, C_{WW} = 0, C_{BB} = 1, \text{nevents} = 1$$



Model	Cross Section
ALPnlo	2.728 +- 0.007485 pb
ALP_NLO_UFO	2.734 +- 0.006922 pb
ALP	2.492 +- 0.00776 pb
ALP_linear_UFO	2.747 +- 0.006959 pb

This works for all models and gives the same diagram. Doesn't work for any of the models at 240 GeV due to bw cutoff

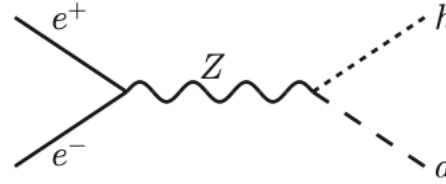
Model testing

Out of the four models (ALPnlo, ALP_NLO_UFO, ALP, and ALP_linear_UFO), only two seem to calculate cross sections. According to Giacomo, this depends on whether or not they have the NP vertex implemented.

Model testing

ALP model supposedly only works when Higgs mass is set high, but seems to work fine with $m_h = 125$ GeV

However, we should check



$$\frac{d\sigma(e^+e^- \rightarrow ha)}{d\Omega} = \frac{\alpha}{128\pi c_w^2 s_w^2} \frac{|C_{Zh}^{\text{eff}}|^2}{\Lambda^2} \frac{s m_Z^2}{(s - m_Z^2)^2} \lambda^{\frac{3}{2}}(x_a, x_h) \sin^2 \theta (g_V^2 + g_A^2)$$

Run MadGraph and compare the cross section from the output

```

1 import math
2
3 alpha = 1 / 137
4 MZ = 91.1876
5 MZ2 = MZ**2
6 sw2 = 0.2312208188
7 cw2 = 1 - sw2
8 sw = math.sqrt(sw2)
9 cw = math.sqrt(cw2)
10 gv = 2 * sw2 * 0.5
11 ga = 0.5
12
13 def kallen_lambda(a, b):
14     return (1 - a - b)**2 - 4 * a * b
15
16 def lambda_3_half(xa, xh):
17     lam = kallen_lambda(xa, xh)
18     return lam**1.5 if lam > 0 else 0
19
20 def sigma_total_ha(sqr, ma, mh, Ceff_Zh, Lam):
21     s = sqr**2
22     xa = MZ**2 / s
23     xh = mh**2 / s
24
25     lam_factor = lambda_3_half(xa, xh)
26     couplings = gv**2 + ga**2
27
28     pre = alpha / (128 * math.pi * cw2 * sw2)
29     form_factor = abs(Ceff_Zh)**2 / Lam**2
30     prop = s * MZ2 / (s - MZ2)**2
31
32     solid_angle_integral = (8 * math.pi) / 3
33     sigma = pre * form_factor * prop * lam_factor * couplings * solid_angle_integral
34
35     sigma_pb = sigma * 0.389379e9
36     return sigma_pb
37
38 sqr = 240.0
39 ma = 1.0
40 mh = 125.0
41 Lam = 1000.0
42 Ceff_Zh = 0.72
43
44 sigma_pb = sigma_total_ha(sqr, ma, mh, Ceff_Zh, Lam)
45 print("Total [GeV] to ha) = (sigma_pb, 4f) pb")

```

Setting:

$\sqrt{s} = 240$ GeV

$m_a = 1$ GeV

$m_H = 125$ GeV

$\Lambda = 1000$

$C_{Zh} = 0.72$

Cross sections:

From eq. 18: 0.0034 pb

From MadGraph: 0.003531 pb

Cross sections (ALP)

Code in the table corresponds to couplings:

Code **ABC**:

A: coupling to photons **CYY**

B: coupling to ALP and Higgs **cah**

C: coupling to Z and Higgs **cZh5**

Example: 101

CYY set to 1.

cah set to 0.

cZh5 set to 1.

If coupling is enabled, it is set to 1.0

CYY is always enabled since we specify decay of ALP to two photons

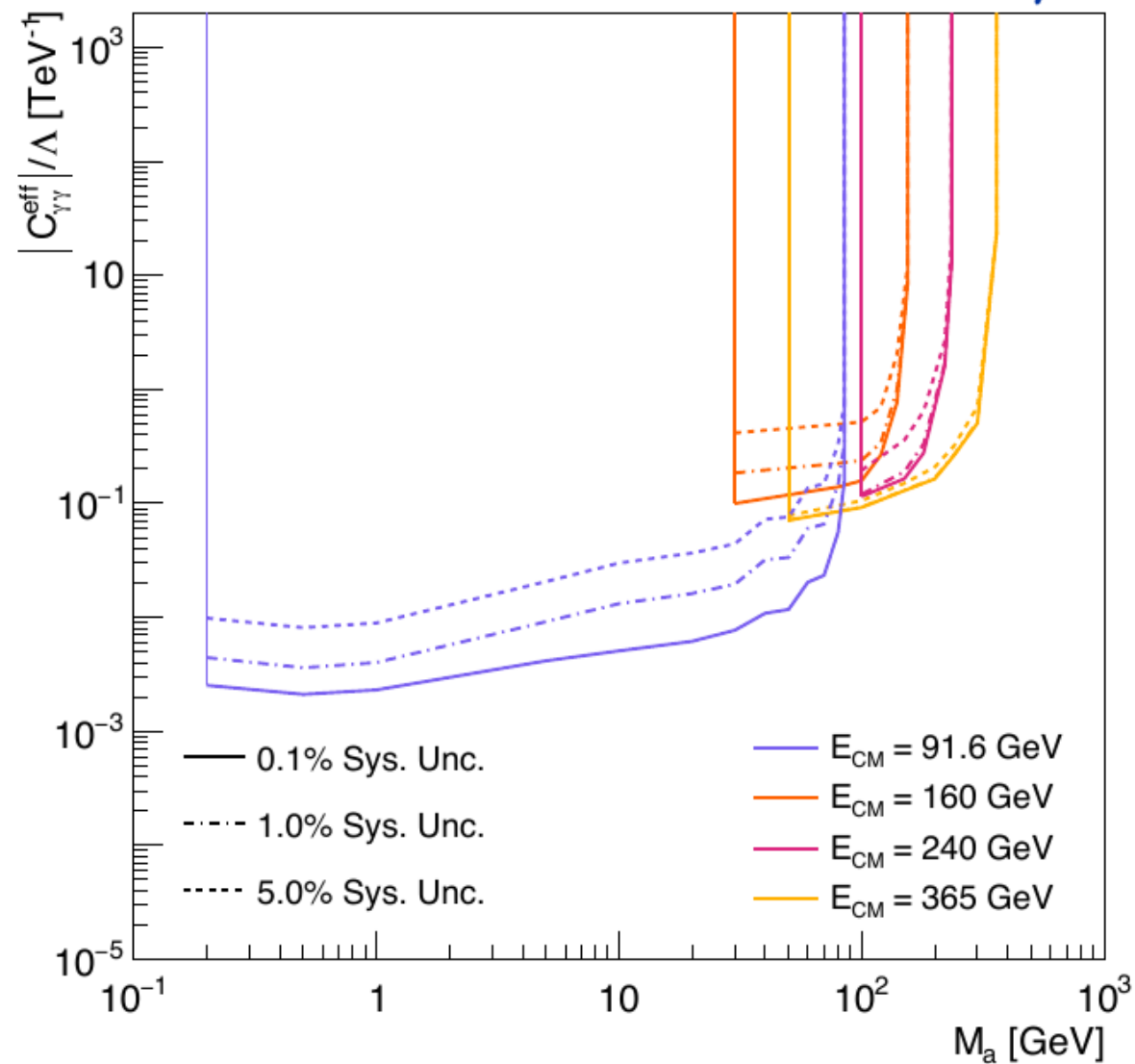
Mass of ALP also set to 1.0 GeV for all processes

Process	Code	Cross Section
$e^+e^- \rightarrow aH, a \rightarrow \gamma\gamma$	100	Survey return zero cross section.
	101	$0.006644 \pm 3.084\text{e-}05$ pb
	110	Survey return zero cross section.
$e^+e^- \rightarrow aZ, a \rightarrow \gamma\gamma$	100	$0.003806 \pm 8.293\text{e-}06$ pb
	101	$0.003787 \pm 8.963\text{e-}06$ pb
$e^+e^- \rightarrow a\gamma, a \rightarrow \gamma\gamma$	100	$0.02046 \pm 6.906\text{e-}05$ pb
	100	$4.72\text{e-}07 \pm 2.865\text{e-}09$ pb
$e^+e^- \rightarrow aZZ, a \rightarrow \gamma\gamma$	101	0.1155 ± 0.0002531 pb
	110	$4.696\text{e-}07 \pm 2.241\text{e-}09$ pb
$e^+e^- \rightarrow a\gamma\gamma, a \rightarrow \gamma\gamma$	100	$0.000944 \pm 1.7\text{e-}05$ pb
	100	$3.162\text{e-}09 \pm 1.583\text{e-}11$ pb
	101	$1.238\text{e-}07 \pm 7.137\text{e-}10$ pb
$e^+e^- \rightarrow aZH, a \rightarrow \gamma\gamma$	110	$3.132\text{e-}09 \pm 1.147\text{e-}11$ pb
	111	$1.226\text{e-}07 \pm 9.248\text{e-}10$ pb
$e^+e^- \rightarrow a\gamma H, a \rightarrow \gamma\gamma$	100	$9.309\text{e-}06 \pm 4.065\text{e-}08$ pb
	101	$0.0001516 \pm 1.693\text{e-}06$ pb
	110	$9.265\text{e-}06 \pm 4.223\text{e-}08$ pb

Preliminary plot

Plot made by Giacomo's student

FCCee IDEA: $e^+e^- \rightarrow \gamma a$



Cross Sections

$e^+e^- \rightarrow aH, a \rightarrow \gamma\gamma, H \rightarrow \gamma\gamma$: $5.77\text{e-}06 \pm 2.461\text{e-}08$ pb

$e^+e^- \rightarrow a\gamma, a \rightarrow \gamma\gamma$: $0.02032 \pm 6.076\text{e-}05$ pb