Probing the Higgs Low-Energy Theorem: $h \to \gamma\gamma$ and $h \to Z\gamma$ decays in the BSM Inert Doublet Model

Nataly Debellis

DESY Theory Group

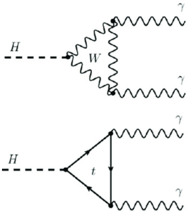
Supervisors: J. Braathen, F. Egle, F. M. Arco Garcia, A. Verduras, G. Weiglein

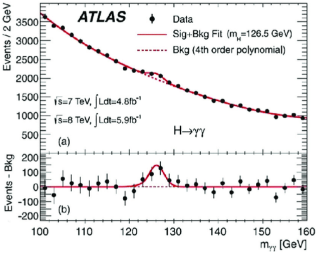
Outline

1. Higgs Boson in the Standard Model and Beyond
2. Inert Doublet Model (IDM)
3. The Low-Energy Higgs Theorem
4. Methods and objectives
5. HLET Application to $h \to \gamma\gamma$
6. HLET Application to $h \to Z\gamma$
7. Conclusions

Higgs Boson in the Standard Model and Beyond

The 2012 discovery of the Higgs boson was a landmark event in Particle Physics.





Any deviation from the SM prediction of the Higgs boson decay channels would be a strong indication of new particles, pointing to **Physics Beyond the Standard Model**.

The Inert Doublet Model (IDM)

The IDM extends the Standard Model by adding a second scalar doublet. These two doublets are distinguished by an exact $\mathbb{Z}_2$ symmetry.

Standard Model Doublet (H_1):

- $H_1 \xrightarrow{\mathbb{Z}_2} H_1$.
- Acquires a VEV, breaking electroweak symmetry.
- Gives mass to SM particles.

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h^0 + iG^0) \end{pmatrix}$$

Inert Doublet (H_2):

- $H_2 \xrightarrow{\mathbb{Z}_2} -H_2$.
- Does not acquire a VEV.
- The lightest scalar particle is stable.

$$H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H^0 + iA^0) \end{pmatrix}$$

The Tree-Level Higgs Potential

The tree-level potential $V(H_1, H_2)$ in the Inert Doublet Model is crucial for defining the masses and interactions of the scalar sector.

$$V(H_1, H_2) = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \lambda_1|H_1|^4 + \lambda_2|H_2|^4 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4|H_1^\dagger H_2|^2 + \frac{1}{2}\lambda_5[(H_1^\dagger H_2)^2 + h.c.] \tag{1}$$

- **Mass Terms (μ_1^2, μ_2^2)**
- **Self-Interaction Terms (λ_1, λ_2)**
- **Mixed Interaction Terms ($\lambda_3, \lambda_4, \lambda_5$)**

The Scalar Masses at Tree Level

Following electroweak symmetry breaking, the scalar sector gives rise to five physical Higgs bosons [1]:

$$M_h^2 = \lambda_1 v^2 \tag{2}$$

$$M_H^2 = \mu_2^2 + \frac{1}{2}\lambda_{345} v^2 \tag{3}$$

$$M_A^2 = \mu_2^2 + \frac{1}{2}\bar{\lambda}_{345} v^2 \tag{4}$$

$$M_{H^\pm}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2 \tag{5}$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$ and $\bar{\lambda}_{345} \equiv \lambda_3 + \lambda_4 - \lambda_5$.

The lightest inert boson is a prime candidate for **dark matter** due to the $\mathbb{Z}_2$ symmetry preventing it from decaying into Standard Model fermions.

The Low-Energy Higgs Theorem

The Higgs low-energy theorem is a powerful tool that relates the amplitudes of two processes which differ by the insertion of a **Higgs-boson leg with zero external momentum** [2, 3].

$$\mathcal{A}_h = \frac{\partial \mathcal{A}}{\partial h} \quad (6)$$

- **First Assumption:** $p_h^\mu \to 0$
  - From the translational invariance, it follows that: $[p_\mu, h] = i\partial_\mu h = 0 \to$ **constant field** h
  - Redefinition of all the masses of the theory that are acquired through the Higgs mechanism: $m_i \to m_i \left(1 + \frac{h}{v}\right)$.

- **Second Assumption:** $m_h \ll m_{\mathbf{loop}}$
  By integrating out heavy degrees of freedom, we get an **Effective Lagrangian** for a generic decay $h \to XX$:

$$\mathcal{L}^h_{\text{eff}} \supset -\frac{1}{4} C_{hXX}\, h\, X_{\mu\nu} X^{\mu\nu} \quad (7)$$

in which the coupling is defined as:

$$C_{hXX} = \frac{\partial}{\partial h} \Pi_{XX}(p^2 = 0)\Big|_{h=0} \quad (8)$$

and Π_{XX} is the gauge boson's vacuum polarization.

The effective coupling, C_{hXX}, is given by the HLET relation:

$$C_{hXX} = \frac{\partial \Pi_{XX}}{\partial h} = \frac{\partial \Pi_{XX}}{\partial m_{\text{loop}}} \frac{\partial m_{\text{loop}}}{\partial h} \quad (9)$$

This is a computational shortcut, as the derivative with respect to the loop mass (m_{loop}) is equivalent to the derivative with respect to the Higgs field (h) itself.

Methods and objectives

My work explores **one-loop-induced Higgs boson decays** [4].

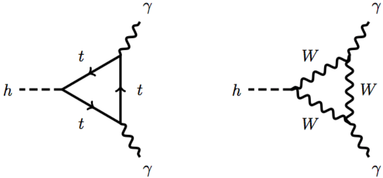


Figure: Top loop and W boson loop contributions

- Application of the **Background Field Method** [5, 6] and t'Hooft-Feynman gauge fixing ($\xi_Q = 1$) for the quantum fields.
- Comparison between the SM results, both in the HLET approximation and the full loop calculation, and the IDM ones.
- Study of the **decoupling regime** as a function of the λ_3 coupling and the $H^\pm$ mass.

HLET Application to $h \to \gamma\gamma$

The **effective operator** [1] obtained with the HLET is:

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{4} C_{h\gamma\gamma} h F_{\mu\nu} F^{\mu\nu} \tag{10}$$

and the coupling $C_{h\gamma\gamma}$ reads:

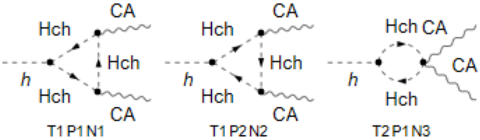
$$C_{h\gamma\gamma} = \left. \frac{\partial}{\partial h} \Pi_{\gamma\gamma}(p^2 = 0) \right|_{h=0} \tag{11}$$

- The vacuum polarization function $\Pi_{\gamma\gamma}(p^2)$ is defined from the **photon self-energy tensor**:

$$\Sigma^{\mu\nu}_{\gamma\gamma}(p^2) = (p^2 g^{\mu\nu} - p^\mu p^\nu) \Pi_{\gamma\gamma}(p^2). \tag{12}$$

Comparison between SM and IDM decay widths

The IDM Feynman diagrams for $h \to \gamma\gamma$ include additional contributions from the charged Higgs boson H^+:

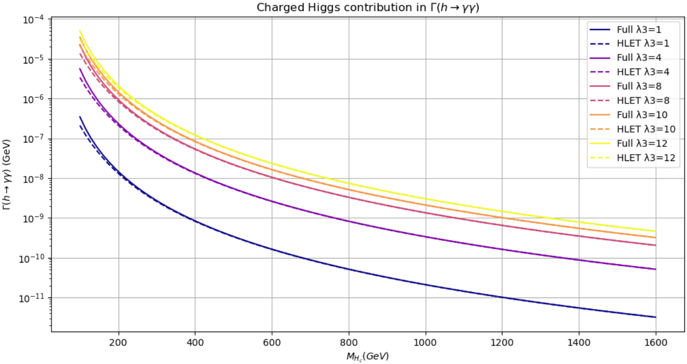


- The HLET one-loop main contributions in the decay width [7] are given by the following terms:

$$\Gamma(h \to \gamma\gamma) = \frac{\sqrt{2}\alpha_{\text{em}}^2 G_F m_h^3}{16\pi^3} \left| I_t^{(1)} + I_W^{(1)} + I_{H^\pm}^{(1)} \right|^2 \qquad (13)$$

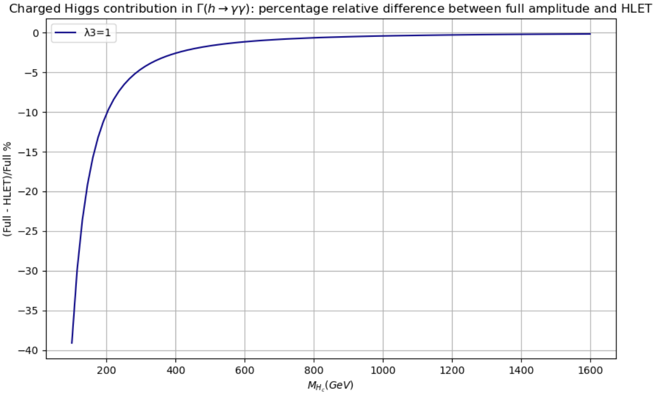
where the IDM adds: $I_{H^\pm}^{(1)} = -\frac{1}{12}\left(1 - \frac{\mu_2^2}{m_{H^\pm}^2}\right) = -\frac{1}{24}\left(\frac{\lambda_3 v^2}{m_{H^\pm}^2}\right)$

$H^{\pm}$ contributions in the decay width



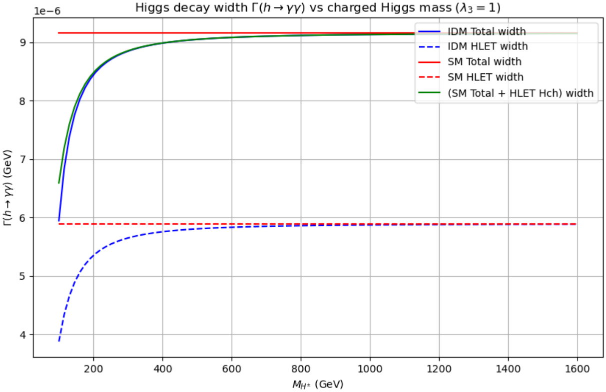
- Suppression of the $H^{\pm}$ loop contribution in the high mass limit in line with the **decoupling theorem**;
- Enhancement of the contribution for higher values of the λ_3 coupling.

$H^{\pm}$ contributions: relative difference between full amplitude and HLET approximation



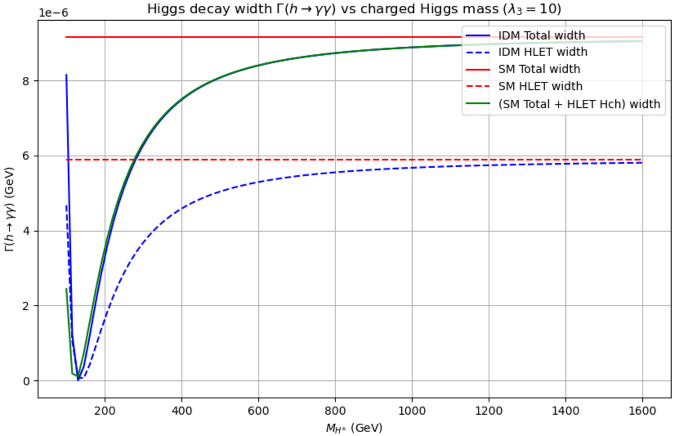
- The HLET approximation becomes accurate ($< 1\%$) for $M_{H^{\pm}} > 600 GeV$.

$\lambda_3 = 1$



- The HLET **underestimates** the decay widths both for SM and IDM;
- The IDM width converges to the SM prediction for $M_{H^\pm} > 500 - 600 GeV$;
- The mixed approach (green line SM + HLET for $H^\pm$) is accurate in the decoupling limit of $M_{H^\pm} \to \infty$.

$\lambda_3 = 10$


Higgs decay width Γ(h → γγ) vs charged Higgs mass (λ3 = 10)
1e−6
Γ(h → γγ) (GeV)
0
2
4
6
8
200
400
600
800
1000
1200
1400
1600
MH± (GeV)
IDM Total width
IDM HLET width
SM Total width
SM HLET width
(SM Total + HLET Hch) width

HLET Application to $h \to Z\gamma$

- **Effective Lagrangian for $Z\gamma$:**

$$\mathcal{L}_{\text{eff}} \supset -\frac{1}{4} F^{0\mu\nu} Z^0_{0\mu\nu} \Pi^0_{Z\gamma}(0), \tag{14}$$

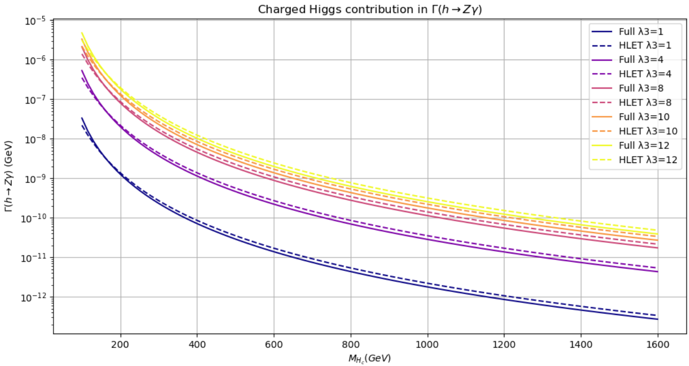
where $\Pi^0_{Z\gamma}(0)$ is the $Z\gamma$ self-energy at zero external momentum. [2]

- **Decay width:**

$$\Gamma(h \to Z\gamma) = \frac{\sqrt{2}\alpha_{\text{em}}^2 G_F m_h^3}{128\pi^3} \left(1 - \frac{m_z^2}{m_h^2}\right)^3 \left| J_t^{(1)} + J_W^{(1)} + J_{H^\pm}^{(1)} \right|^2 \tag{15}$$

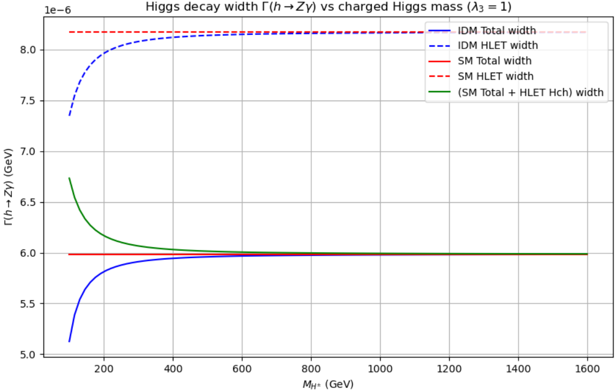
where $J_t^{(1)}, J_W^{(1)}, J_{H^\pm}^{(1)}$ are respectively the top loop, W boson and $H^\pm$ amplitude contributions.

$H^{\pm}$ contributions in the decay width



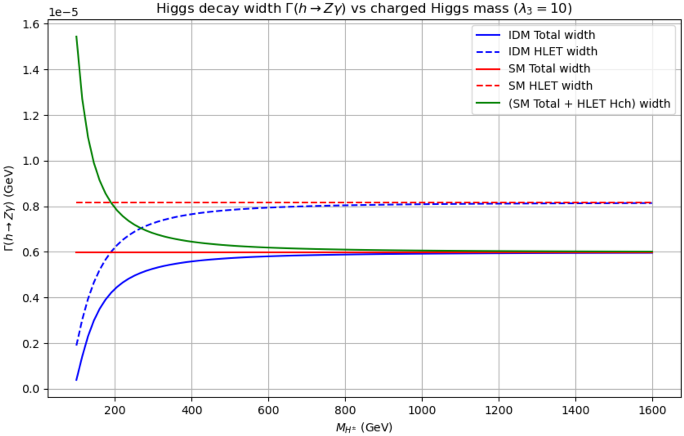
- Suppression of the $H^{\pm}$ loop contribution in the high mass limit in line with the **decoupling theorem**;
- Enhancement of the contribution for higher values of the λ_3 coupling.

$\lambda_3 = 1$



- The HLET here **overestimates** the decay widths both for SM and IDM;
- The IDM width converges to the SM prediction for about $M_{H\pm} > 600 GeV$ as well;
- The mixed approach (green line SM + HLET for $H^{\pm}$) remains accurate in the decoupling limit of $M_{H\pm} \to \infty$.

$\lambda_3 = 10$



Higgs decay width Γ(h → Zγ) vs charged Higgs mass (λ3 = 10)
1e−5
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Γ(h → Zγ) (GeV)
200
400
600
800
1000
1200
1400
1600
MH± (GeV)
IDM Total width
IDM HLET width
SM Total width
SM HLET width
(SM Total + HLET Hch) width

Conclusions

- **Comparison between full SM and HLET results**
  The Low-Energy Theorem fails to accurately describe the W loop contribution because the assumption $m_h \ll m_W$ is invalid, resulting in a ~30% error. As a consequence, the HLET underestimates the $\Gamma(h \to \gamma\gamma)$ decay width and overestimates $\Gamma(h \to Z\gamma)$.

- **IDM contributions**
  The IDM introduces contributions from the charged Higgs boson which, in the decoupling limit ($M_{H^\pm} \to \infty$), approach zero, as expected.

- **HLET high-mass validity in the IDM**
  The HLET approximation (SM Total + HLET $H^\pm$) is shown to be reliable at high values of $M_{H^\pm}$.

Thank you!

Appendix:

HLET One-Loop Corrections for $\Gamma(h \to \gamma\gamma)$

The one-loop corrections to the **Higgs to di-photon decay width** [7] are given by the following terms:

$$\Gamma(h \to \gamma\gamma) = \frac{\sqrt{2}\alpha_{\rm em}^2 G_F m_h^3}{16\pi^3} \left| I_t^{(1)} + I_W^{(1)} + I_{H^\pm}^{(1)} \right|^2 \tag{16}$$

The expressions for the one-loop contributions read:

- $I_{H^\pm}^{(1)} = -\frac{1}{12}\left(1 - \frac{\mu_2^2}{m_{H^\pm}^2}\right) \quad = -\frac{1}{24}\left(\frac{\lambda_3 v^2}{m_{H^\pm}^2}\right)$
- $I_t^{(1)} = -\frac{4}{9}$
- $I_W^{(1)} = \frac{7}{4}$

Appendix: Mathematica calculation steps

Calculation Procedure:

1. **Amplitude generation** using FeynArts for:
   - Top quark loop contribution
   - W-boson loop contribution
   - $H^{\pm}$ loop contribution (IDM)
2. **Algebraic manipulation** with FeynCalc:
   - Projection with projPi$^{\mu\nu}(p)$
   - Tensor reduction via TID
   - Expression in terms of Passarino-Veltman integrals
3. **Dimensional regularization** ($D = 4 - 2\epsilon$):

   UV divergence expansion:
   - $A_0(x) \to \epsilon A_{0e}(x) + A_0^{\text{fin}}(x) + \frac{x}{\epsilon} + \mathcal{O}(\epsilon^2)$
   - $B_0(x) \to \epsilon B_{0e}(x) + B_0^{\text{fin}}(x) + \frac{1}{\epsilon} + \mathcal{O}(\epsilon^2)$
   - $C_0(x) \to \epsilon C_{0e}(x) + C_0^{\text{fin}}(x) + \mathcal{O}(\epsilon^2)$
4. **Finite part evaluation** at $p^2 \to 0$:

   Finite contributions:
   - $A_0^{\text{fin}}(x) \to x\left(1 - \log\left(\frac{x}{Q^2}\right)\right)$
   - $B_0^{\text{fin}}(p^2, x, x) \to \frac{(p^2)^2}{60x^2} + \frac{p^2}{6x} - \log\left(\frac{x}{Q^2}\right) + \mathcal{O}(p^4)$
   - $C_0^{\text{fin}}(0, 0, y, x, x, x) \to -\frac{y}{24x^2} - \frac{1}{2x} + \mathcal{O}(y)$ (valid for y « x)

HLET Implementation for $h \to \gamma\gamma$:

- $\mathcal{A}(h \to \gamma\gamma) = \mathcal{A}_W + \mathcal{A}_t + \mathcal{A}_{H\pm}$

W-boson contribution:

- $\mathcal{A}_W \propto \frac{\partial}{\partial h}\Pi^W_{\gamma\gamma}(h)\Big|_{h=0}$
- $m_W(h) = m_W\left(1 + \frac{h}{v}\right)$
- `D[Pizero$G /. mW -> mW (v + h)/v, h] /. h -> 0`

Top quark contribution:

- $\mathcal{A}_t \propto \frac{\partial}{\partial h}\Pi^t_{\gamma\gamma}(h)\Big|_{h=0}$
- $m_t(h) = m_t\left(1 + \frac{h}{v}\right)$
- `D[Pizero$t /. mt -> mt (v + h)/v, h] /. h -> 0`

Charged Higgs contribution (IDM):

- $\mathcal{A}_{H\pm} \propto \frac{\partial}{\partial h}\Pi^{H\pm}_{\gamma\gamma}(h)\Big|_{h=0}$
- `D[Pizero, h] /. h -> 0`

Appendix:

Background Field Method and Gauge fixing

- Calculation performed using the **Background Field Method** (BFM) [5, 6]
  - Splits fields: $A_\mu = \hat{A}_\mu + a_\mu$ (background + quantum)
  - **Preserves** gauge invariance for background fields
  - **Breaks** gauge symmetry only for quantum fluctuations
- **'t Hooft-Feynman gauge fixing ($\xi_Q = 1$)** for quantum fields
  - Significant simplification of loop algebra

The BFM preserves background gauge symmetry while breaking symmetry only for quantum fluctuations. The choice $\xi_Q = 1$ streamlines calculations without affecting physical results.

Appendix:

Theoretical Constraints on the Higgs Potential

- **Vacuum Stability**
  The scalar potential must be bounded from below.
  - $\lambda_1 > 0$, $\lambda_2 > 0$
  - $\lambda_3 + \lambda_4 + \lambda_5 + \sqrt{\lambda_1 \lambda_2} > 0$

- **Perturbative Unitarity**
  Scattering probabilities of scalar particles must not violate unitarity.

- **Inert Vacuum Condition**
  The model requires that the Standard Model-like vacuum $(\langle H_1 \rangle \neq 0, \langle H_2 \rangle = 0)$ is the global minimum of the potential.
  - $m_{H_2}^2 = \mu_2^2 + \frac{1}{2}\lambda_3 v^2 > 0$

References

[1] M.Aiko, J.Braathen, and S.Kanemura. "Leading two-loop corrections to the Higgs di-photon decay in the Inert Doublet Model". In: (2025).

[2] B.A.Kniehl and M.Spira. "Low-Energy theorems in Higgs Physics". In: (1995).

[3] H.E.Haber S.Dawson. "A Primer on Higgs Boson Low-Energy Theorems". In: (1989).

[4] A.Djouadi. "The Anatomy of Electro–Weak Symmetry Breaking". In: Tome I: The Higgs boson in the Standard Model (2005).

[5] L.F.Abbott. "Introduction to the Background Field Method". In: (1981).

[6] A. Denner, G. Weiglein, and S.Dittmaier. "Application of the Background-Field Method to the electroweak Standard Model". In: (1994).

[7] J.Braathen, M.Gabelmann, and P.Stylianou T.Robens. "Probing the Inert Doublet Model via Vector-Boson Fusion at a Muon Collider". In: (2025).