Unstable Tops and Effective Theory

Andrew Papanastasiou*

22nd November 2011, DESY, Hamburg

*In collaboration with Adrian Signer (IPPP), Paul Mellor (IPPP) and Pietro Falgari (Utrecht)

Outline

Motivation

- Abundance of Top-Quarks produced at the Tevatron and LHC

  $\rightarrow$ opportunity for precision top-studies in next few years

- Top-Quarks considered to be sensitive to New Physics at the EWSB scale

Tantalising hint(?): Tevatron (CDF) $\mathcal{A}_{FB}(m_{t\bar{t}} > 450 \text{ GeV})$ 3.4σ away from SM

- $\Gamma_t \gg \Lambda_{\text{QCD}} \rightarrow$ tops decay before hadronizing

  $\rightarrow$ treat top as unstable for best description: theory challenge

Motivation: Single-Top and $t\bar{t}$

Single Top:

- Background to Higgs searches (e.g. $H \to b\bar{b}$),
- Sensitive to CKM element V_{tb} and structure of tWb-vertex

$t\bar{t}$:

- Important background to Higgs decays (e.g. $H \to WW$) and New Physics signals (e.g. cascade decays in SUSY)
- Mass measurement (constrains SM Higgs mass, BSM parameters)
- Forward-Backward / Charge asymmetry

Experiments have began to challenge current SM-QCD calculations of both processes

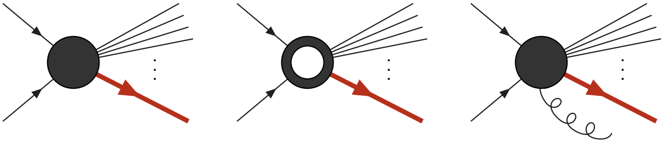
$\to$ better understanding & description of heavy particles at colliders...

Motivation: Effective Theory

Effective Theory
- simplifies calculations ✓
- work with degrees of freedom relevant to energy scale(s) of problem ✓
- powerful machinery, e.g. RG-evolution ✓

Question: can Effective Theory methods simplify NLO calculations for arbitrary observables at colliders?

Stable Top Production and Decay



- top spins summed over

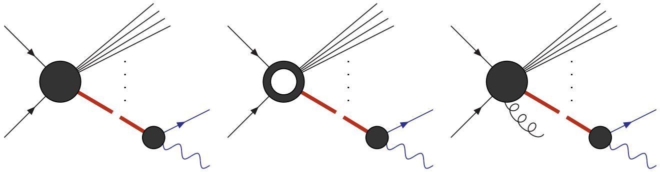
✓ decent approximation for inclusive observables

✗ cuts on final states not possible

✗ no off-shell effects

Single Top: [G. Bordes & B. v Eijk, '95][B.W. Harris et al. '02][S. Willenbrock et al. '97, 04]
$t\bar{t}$: [P. Nason, S. Dawson, R. K. Ellis '89][W. Beenakker et. al. '91]

On-Shell Top Production and Decay



- decay of tops included via (improved) Narrow Width Approximation
- non-factorizable corrections not included

✓ spin correlations and cuts on final states

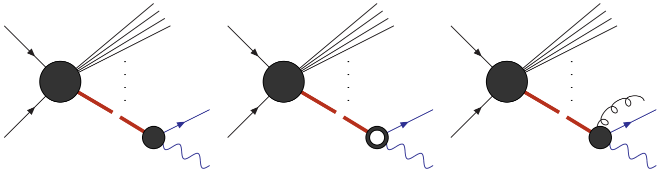
✗ no off-shell effects

OK, for inclusive observables (σ etc) as effects of non-factorizable corrections are small, $\mathcal{O}(\frac{\Gamma_t}{M_t})$ for these [V. S. Fadin et. al. '94][K. Melnikov, O. I. Yakovlev '94]

Need off-shell effects for accuracy of $\delta M_t \lesssim \Gamma_t$

Single Top: [J.M. Campbell et al. '04, '05][Q. H. Hao et al. '05, '10]
$t\bar{t}$: [W. Bernreuther et. al. '01 & '04],[K. Melnikov, M. Schulze '09]

On-Shell Top Production and Decay



- decay of tops included via (improved) Narrow Width Approximation
- non-factorizable corrections not included

✓ spin correlations and cuts on final states

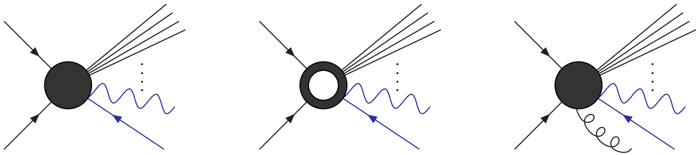
✗ no off-shell effects

OK, for inclusive observables (σ etc) as effects of non-factorizable corrections are small, $\mathcal{O}(\frac{\Gamma_t}{M_t})$ for these [V. S. Fadin et. al. '94][K. Melnikov, O. I. Yakovlev '94]

Need off-shell effects for accuracy of $\delta M_t \lesssim \Gamma_t$

Single Top: [J.M. Campbell et al. '04, '05][Q. H. Hao et al. '05, '10]
$t\bar{t}$: [W. Bernreuther et. al. '01 & '04],[K. Melnikov, M. Schulze '09]

Off-shell Top Production and Decay



- non-factorizable corrections included
- all background diagrams included

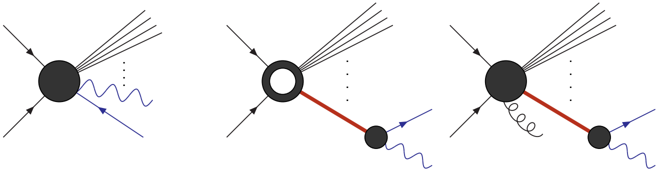
✓ off-shell effects

✓ spin-correlations and cuts on final states

✗ complicated/difficult calculation

Single Top (s-channel): [R. Pittau '96]
$t\bar{t}$: [A. Denner et. al. '10][Bevilacqua at. al. '10]

Resonant Top Production and Decay



- non-factorizable corrections included
- (relevant) background diagrams included

✓ off-shell effects included

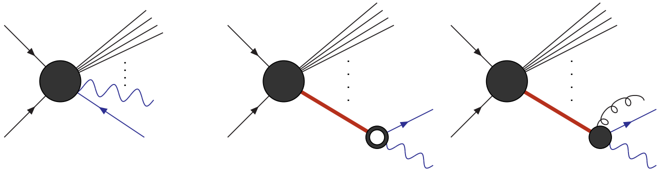
✓ spin-correlations and cuts on final states

✓ simpler calculation

✗ not valid outside resonant region $p_t^2 \sim M_t^2$

Single Top: [P. Falgari et al. '10, '11]
$t\bar{t}$: [P. Falgari, A.P., A. Signer - In Progress]

Resonant Top Production and Decay



- non-factorizable corrections included
- (relevant) background diagrams included

✓ off-shell effects included

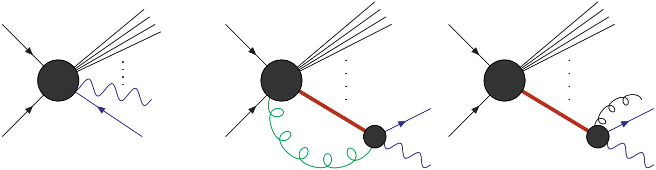
✓ spin-correlations and cuts on final states

✓ simpler calculation

✗ not valid outside resonant region $p_t^2 \sim M_t^2$

Single Top: [P. Falgari et al. '10, '11]
$t\bar{t}$: [P. Falgari, A.P., A. Signer - In Progress]

Resonant Top Production and Decay



- non-factorizable corrections included
- (relevant) background diagrams included

✓ off-shell effects included

✓ spin-correlations and cuts on final states

✓ simpler calculation

✗ not valid outside resonant region $p_t^2 \sim M_t^2$

Single Top: [P. Falgari et al. '10, '11]
$t\bar{t}$: [P. Falgari, A.P., A. Signer - In Progress]

Calculation Simplified

Examine t-channel single top

$$h_1(q,\bar{q})\ h_2(b) \xrightarrow{t} W^+\ b\ + X\ \rightarrow l^+\ \nu_l\ b\ + X$$

(decay of W $\rightarrow$ leptons included via (improved narrow width approximation))

Condition: $p_t^2 = (p_{W^+} + J_b)^2 \sim M_t^2$

Exploit widely separated scales: $p_t^2 - M_t^2\ \sim\ M_t\Gamma_t \ll M_t^2$

$\rightarrow$ Effective Theory inspired approach

[A.P. Chapovsky, V.A. Khoze, A. Signer, W.J. Stirling '01]

[M. Beneke, A.P. Chapovsky, A. Signer, G. Zanderighi '04]

$\rightarrow$ Expand full amplitudes in the small parameters α_s and $\frac{(p_W+J_b)^2-M_t^2}{M_t^2}$

Tree-Level Expansion

Full tree-level amplitude takes the form:

$$\mathcal{A}^{\text{tree}} = \frac{\mathcal{R}(p_i)}{(p_t^2 - M_t^2)} + \mathcal{N}(p_i).$$

Pole Expansion: [A. Aeppli, G. J. v Oldenborgh, D. Wyler '94]

Expand about the complex pole of the propagator: $\mu_t^2 = M_t^2 - iM_t\Gamma_t$

$$\mathcal{A}^{\text{tree}} = \frac{\mathcal{R}(p_i, p_t^2 = \mu_t^2)}{\Delta_t}(1 + \delta\mathcal{R}_t)$$

$$+ \frac{\partial\mathcal{R}}{\partial p_t^2}(p_i, p_t^2 = \mu_t^2) + \mathcal{N}(p_i, p_t^2 = \mu_t^2) + \ldots.$$

$\Delta_t = \frac{p_t^2 - \mu_t^2}{M_t^2} \sim \frac{M_t\Gamma_t}{M_t^2} \sim \alpha_{\text{ew}}$

$(1 + \delta\mathcal{R}_t)$ is the residue of the full top quark propagator at the pole $p_t^2 = \mu_t^2$.

Power Counting

Want to combine expansion in Δ_t with standard expansion in α_s and α_{ew}.

Introduce scalings: $\alpha_s^2 \sim \alpha_{ew} \sim \Delta_t \sim \delta$

$\rightarrow$ expand our full amplitude in δ (up to whatever order we please)

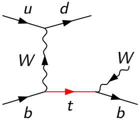
Important: Expansion of Amplitude / Matrix Element is strictly gauge invariant at every order in δ.

To illustrate:
look at tree-level diagrams and assign a power of δ to each diagram

(Note: this will correspond to the leading scaling in δ for each diagram.)

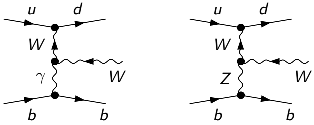
Power Counting

Resonant



$$\sim g_{\rm ew}^3 \frac{1}{\Delta_t} \sim \frac{\delta^{3/2}}{\delta} \sim \delta^{\frac{1}{2}}$$

Non-Resonant (examples)



$$\sim g_{\rm ew}^3 \sim \delta^{\frac{3}{2}}$$

Notation: in expansion of amplitude we have terms $A_s^{(n,m)}$ where
$n, m =$ powers of $g_{\rm ew}$ and g_s respectively that multiply $A_s^{(n,m)}$
$s =$ order to which resummed propagator appears in amplitude.

Write tree-level amplitude as

$$\mathcal{A}^{\text{tree}} = \delta_{31}\delta_{42}\,(\,\underbrace{g_{\text{ew}}^3 A_{-1}^{(3,0)}}_{\delta^{\frac{1}{2}}} + \underbrace{g_{\text{ew}}^3 A_{0}^{(3,0)}}_{\delta^{\frac{3}{2}}} + \ldots\,) + \underbrace{g_{\text{ew}} g_s^2 A^{(1,2)}}_{\delta^1} + \ldots$$

Squaring

$$\mathcal{M}^{\text{tree}} = \underbrace{g_{\text{ew}}^6 |A_{-1}^{(3,0)}|^2}_{\delta^1} + \underbrace{g_{\text{ew}}^6\, 2\,\text{Re}\left(A_{-1}^{(3,0)}\left[A_{0}^{(3,0)}\right]^*\right)}_{\delta^2} + \underbrace{g_{\text{ew}}^2 g_s^4 |A^{(1,2)}|^2}_{\delta^2} + \ldots$$

So, 'LO' contributions $\sim \delta^1$

Going beyond 'LO' consistently requires us to include loop and real emision diagrams.

Aim: compute Matrix Element to $\mathcal{O}(\delta^{\frac{3}{2}})$ in counting

$\rightarrow \mathcal{O}(\alpha_s)$-corrections to resonant contributions

$$\mathcal{M}^{\text{NLO}} \sim g_{\text{ew}}^6 g_s^2\, 2\,\text{Re}\left(A_{-1}^{(3,2)}\left[A_{-1}^{(3,0)}\right]^*\right) \sim \delta^{\frac{3}{2}}$$

Define: 'NLO' $\sim \delta^{\frac{3}{2}}$

Loop diagrams: Method of Regions

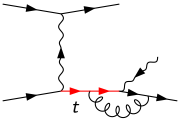
Wish to compute the contributions from virtual diagrams which scale as $\sim \delta^1$.

Technically this is achieved by computing the integrals via the Method of Regions [M. Beneke, V. A. Smirnov '98]

This involves splitting the integrals into 'hard' and 'soft' parts:

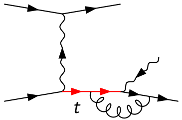
- expand the integrands in the 2 cases of the loop momentum scaling as $q^\mu \sim 1$ and $q^\mu \sim \delta \ll 1$
- only keep contributions which scale as δ^1

Method of Regions: Hard-Soft separation: Example



$$\sim g_s^2 g_{\rm ew}^3 \cdots \int d^4 q \, \frac{1}{q^2} \, \frac{\not{p}_b - \not{q}}{(p_b - q)^2} \, \gamma^\nu \, \frac{\not{p}_t - \not{q} + M_t}{(p_t - q)^2 - M_t^2} \, \frac{1}{\Delta_t} \cdots$$

Method of Regions: Hard-Soft separation: Example



$$\sim g_s^2 g_{\rm ew}^3 \ldots \int d^4q \, \frac{1}{q^2} \, \frac{\not{p}_b - \not{q}}{(p_b - q)^2} \, \gamma^\nu \, \frac{\not{p}_t - \not{q} + M_t}{(p_t - q)^2 - M_t^2} \, \frac{1}{\Delta_t} \ldots$$

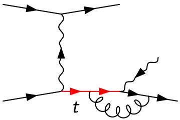
Hard: $q^\mu \sim 1, \quad d^4q \sim 1, \quad \Delta_t \sim \delta$

E.g. $\quad (p_t - q)^2 - M_t^2 \; = \; q^2 - 2q.p_t + \Delta_t \; \rightarrow \; q^2 - 2q.p_t$

$$\rightarrow \; \sim g_s^2 \;\; g_{\rm ew}^3 \ldots \int d^4q \, \frac{1}{q^2} \, \frac{\not{p}_b - \not{q}}{q^2 - 2q.p_b} \, \gamma^\nu \, \frac{\not{p}_t - \not{q} + M_t}{q^2 - 2q.p_t} \, \frac{1}{\Delta_t} \ldots$$

$\delta^{\frac{1}{2}} \quad \delta^{\frac{3}{2}} \qquad 1 \quad 1 \qquad 1 \qquad\qquad 1 \qquad \frac{1}{\delta} \qquad \sim \delta$ keep!

Method of Regions: Hard-Soft separation: Example



$$\sim g_s^2 g_{\text{ew}}^3 \ldots \int d^4q \, \frac{1}{q^2} \, \frac{\not{p}_b - \not{q}}{(p_b - q)^2} \, \gamma^\nu \, \frac{\not{p}_t - \not{q} + M_t}{(p_t - q)^2 - M_t^2} \, \frac{1}{\Delta_t} \ldots$$

Soft: $q^\mu \sim \delta, \quad d^4q \sim \delta^4, \quad \Delta_t \sim \delta$

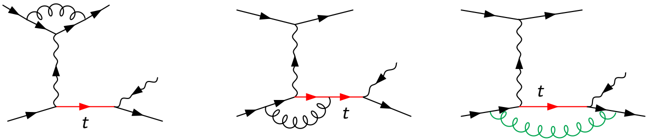
E.g. $(p_t - q)^2 - M_t^2 \; = \; q^2 - 2q.p_t + \Delta_t \; \to \; -2q.p_t + \Delta_t$

$$\to \; \sim g_s^2 \;\; g_{\text{ew}}^3 \ldots \int d^4q \, \frac{1}{q^2} \, \frac{\not{p}_b}{-2q.p_b} \, \gamma^\nu \, \frac{\not{p}_t + M_t}{-2q.p_t + \Delta_t} \, \frac{1}{\Delta_t} \ldots$$

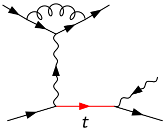
$$\delta^{\frac{1}{2}} \;\; \delta^{\frac{3}{2}} \qquad \delta^4 \quad \frac{1}{\delta^2} \quad \frac{1}{\delta} \qquad \frac{1}{\delta} \quad \frac{1}{\delta} \qquad \sim \delta$$

keep!

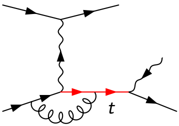
Virtuals Simplified



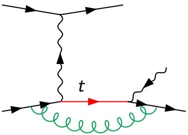
Virtuals Simplified



$H \sim \delta$ ✓
$S \to 0$ (tadpole)

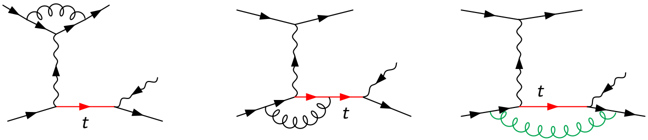


$H \sim \delta$ ✓
$S \sim \delta$ ✓



$H \sim \delta^2$ ✗
$S \sim \delta$ ✓

Virtuals Simplified



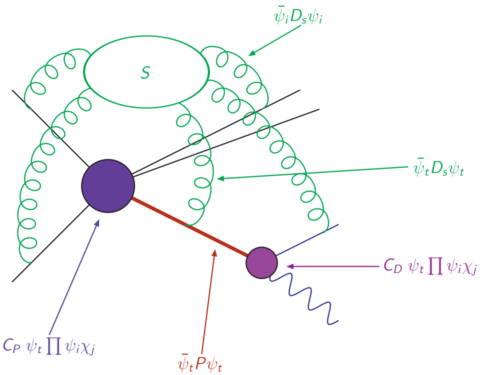
$H \sim \delta$ ✓	$H \sim \delta$ ✓	$H \sim \delta^2$ ✗
$S \to 0$ (tadpole)	$S \sim \delta$ ✓	$S \sim \delta$ ✓

Note: Expansion provides a gauge invariant separation of:

- Factorizable Corrections $\leftrightarrow$ 'Hard corrections', i.e. corrections to on-shell production/decay
- Non-Factorizable Corrections $\leftrightarrow$ 'Soft corrections', i.e. corrections connecting production and decay

[A.P. Chapovsky, V. Khoze, A. Signer, W. J. Stirling '01]

Effective Theory View

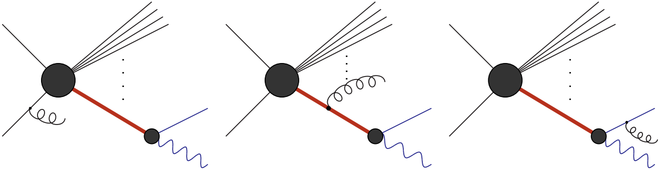


'Hard Contributions' contained in matching coefficients, C_P and C_D

Matching onto full theory done on-shell, $p_t^2 = \mu_t^2$

$\bar{\psi}_t \not{D}_s \psi_t$ encodes dynamical degrees of freedom (soft gluons, cf SCET)

Real



(Conceptually) Harder than virtual corrections: don't know what to expand in:

$$(p_W + p_b)^2 - M_t^2 \sim \delta \text{ or } (p_W + p_b + p_g)^2 - M_t^2 \sim \delta.$$

Effective Theory: gluon momentum can introduce a new scale to the problem, which can spoil the power-counting/expansion.

Subtraction

$$\sigma_{\rm NLO} = \int d\Phi_n \, |M_n^{\rm born}|^2 + \int d\Phi_n \, |M_n^{\rm virt}|^2 + \int d\Phi_{n+1} \, |M_{n+1}^{\rm real}|^2$$

Usually write:

$$\int d\Phi_{n+1} \, |M_{n+1}^{\rm real}|^2 = \int d\Phi_{n+1} \, \left(|M_{n+1}^{\rm real}|^2 - |M_{n+1}^{\rm c.t.}|^2\right)_{\epsilon=0} + \int d\Phi_{n+1} \, |M_{n+1}^{\rm c.t.}|^2$$

where $\int d\Phi_1 \, |M_{n+1}^{\rm c.t.}|^2$ cancels the epsilon poles in $|M_n^{\rm virt.}|^2$.

Now write:

$$\int d\Phi_{n+1} \, |M_{n+1}^{\rm real}|^2 = \int d\Phi_{n+1} \, \left(|M_{n+1}^{\rm real}|^2 - |M_{n+1}^{\rm c.t.}|^2\right)_{\epsilon=0} + \int d\Phi_{n+1} \, |M_{n+1}^{\rm c.t.,\ exp}|^2$$

where $\int d\Phi_1 \, |M_{n+1}^{\rm c.t.,\ exp}|^2$ cancels the epsilon poles in $|M_n^{\rm virt.,\ exp}|^2$.

Drawbacks to this method:

Ideally, would like 'hard/soft' virtual singularities to be cancelled by 'hard/soft' real singularities

Want to understand better how to separate factorizable/non-factorizable real contributions,

i.e. arrange real contributions into same ET structure as virtuals.

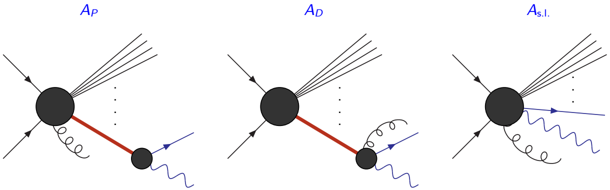
$\rightarrow$ could then consistently use a hard ($\mu_h = M_t$) and a soft scale ($\mu_s = \Gamma_t$) for $\alpha_s(\mu)$ for the 'hard' and 'soft' contributions
(previously used common (hard) scale for both sets of contributions)

$\stackrel{?}{\rightarrow}$ could lead to an enhancement of the soft/non-factorizable corrections (??).

want to achieve resummation in μ_s/μ_h

Production/Decay Split

Solution: split full amplitude for real corrections into a production part, A_P and a decay part, A_D.



$A_P \sim \frac{1}{\Delta_t}$ and approximates full real amplitude when $(p_W + p_b)^2 \sim M_t^2$

$A_D \sim \frac{1}{\Delta_{tg}}$ and approximates full real amplitude when $(p_W + p_b + p_g)^2 \sim M_t^2$

$\longrightarrow A_P + A_D + A_{\text{s.l.}}$ approximates full amplitude always

Pole Cancellation

$$|M_{n+1}^{\text{real}}|^2 \simeq |A_P + A_D + A_{\text{s.l.}}|^2$$

$|A_P|^2$ and $|A_D|^2$ correspond to factorizable corrections

$2\,\text{Re}\,(A_P\,[A_D]^*)$ correspond to non-factorizable corrections

Other terms, $2\,\text{Re}\,\left(A_{\text{s.l.}}\,\left[A_{P/D}\right]^*\right)$, $|A_{\text{s.l.}}|^2$ are subleading.

Now pole cancellation works as expected and desired:

$\int d\Phi_g\ |A_{P/D}|^2$ results in $\frac{1}{\epsilon^2}$ and $\frac{1}{\epsilon}$ poles

$\rightarrow$ cancelled by hard virtual contributions.

$\int d\Phi_g\ 2\,\text{Re}\,(A_P\,[A_D]^*)$ results in a standard $\frac{1}{\epsilon}$ soft-pole

$\rightarrow$ cancelled by $\frac{1}{\epsilon}$-pole of the soft virtual contribution.

Results: Process Definition

General Setup : 7 TeV LHC

Use MSTW2008 PDF set, with $\alpha_s(M_Z) = 0.12018$,

$$M_t = 172.0 \text{ GeV} \quad M_W = 80.4 \text{ GeV}$$

$$\Gamma_t = 1.328 \text{ GeV} \quad \Gamma_W = 2.14 \text{ GeV}$$

$$\mu_F = \mu_R = M_t$$

Cuts: (example: any are possible) $k_\perp$-algorithm, $D_{\text{jet}} = 0.7$

$p_T(J_b) > 20$ GeV	$\not{E}_T > 20$ GeV
$p_T(\text{hardest}J_l) > 20$ GeV	$\eta(J_b) < 2.5$
$p_T(e) > 20$ GeV	$\eta(\text{hardest}J_l) < 2.0$
$120 < m_{\text{inv}} < 200$ GeV	$\eta(e) < 2.5$

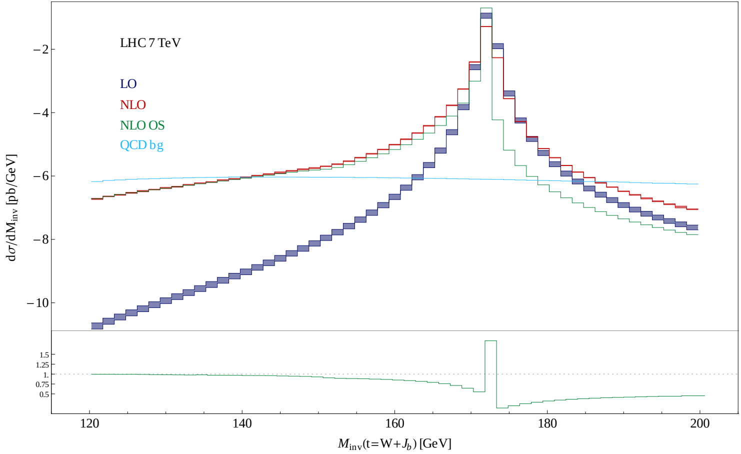
Inclusive Cross-Section

Process: $pp \to J_b\, J_l\, e^+\, \not{E}_T + X$

	ET	**NWA**
LO [fb]	1137.0(2)	1152.3(1)
ub [fb]	-66.8(2)	-69.5(1)
ug [fb]	68.9(1)	69.0(1)
gb [fb]	-52.63(1)	-53.3(1)
NLO [fb]	1086.5(2)	1098.5(2)

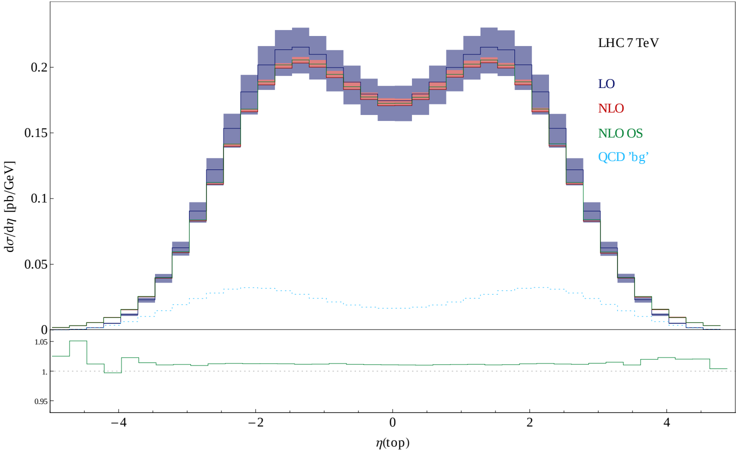
As expected, off-shell effects amount to a 1-2% correction to the on-shell NLO inclusive cross-section.

Example distribution: $M_{\rm inv}$(top)



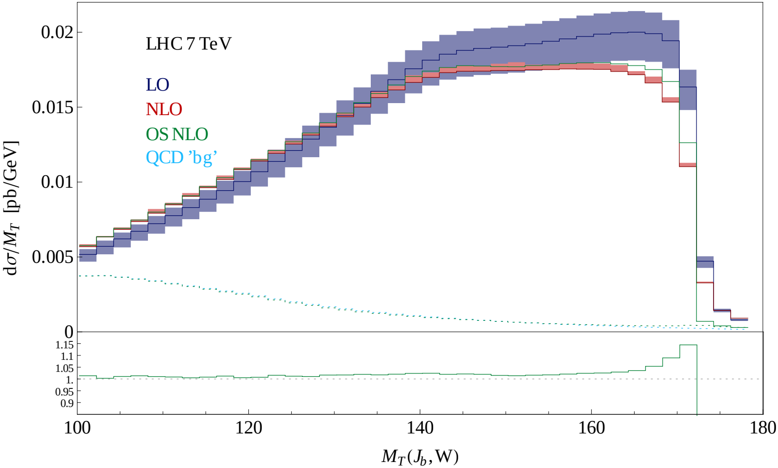
Off-shell effects important

Example distribution: η(top)



Off-shell effects small

Example distribution: M_T(top)

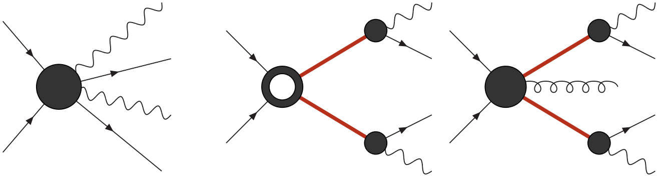


Off-shell effects important!!

Resonant Top-Pair Production and Decay

Study di-lepton channel:

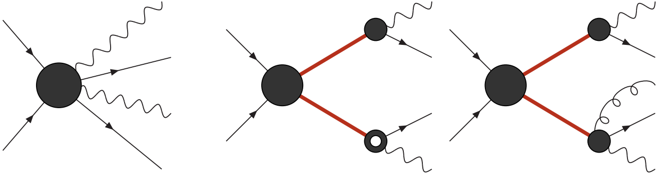
$$h_1(q,g)\; h_2(\bar{q},g) \xrightarrow{t\bar{t}} W^+\, b\, W^-\, \bar{b}\; + X \to l^+\, \nu_l\, b\, l^-\, \bar{\nu}_l\, \bar{b}\; + X$$



Resonant Top-Pair Production and Decay

Study di-lepton channel:

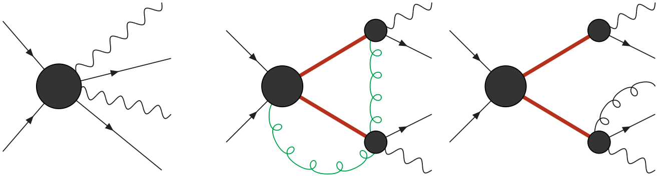
$$h_1(q,g)\; h_2(\bar{q},g) \xrightarrow{t\bar{t}} W^+\, b\, W^-\, \bar{b}\; + X \rightarrow l^+\, \nu_l\, b\, l^-\, \bar{\nu}_l\, \bar{b}\; + X$$



Resonant Top-Pair Production and Decay

Study di-lepton channel:

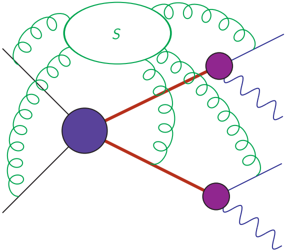
$$h_1(q,g)\ h_2(\bar{q},g) \xrightarrow{t\bar{t}} W^+\ b\ W^-\ \bar{b}\ + X \rightarrow l^+\ \nu_l\ b\ l^-\ \bar{\nu}_l\ \bar{b}\ + X$$



Resonant Top-Pair Production and Decay

Study di-lepton channel:

$$h_1(q,g)\ h_2(\bar{q},g) \xrightarrow{t\bar{t}} W^+\ b\ W^-\ \bar{b}\ + X \rightarrow l^+\ \nu_l\ b\ l^-\ \bar{\nu}_l\ \bar{b}\ + X$$



Similar Effective Theory structure: production, decay, soft gluons etc...

Status

Hard Virtual Corrections: [S. Badger, R. Sattler, V. Yundin '10]
Soft Virtual Corrections: with Pietro Falgari (new triangle, box and pentagon)

Real Corrections: Conceptual issues sorted out & tested in Single Top calculation $\longrightarrow$ follow same strategy for $t\bar{t}$

$\longrightarrow$ Implementation into two independent Monte Carlos is already underway...

A full study of off-shell effects will be carried out & we will be making comparisons both to:

Fully Off-Shell Results: [A. Denner et. al. '10][Bevilacqua at. al. '10]
(Spin-Correlated) On-Shell Results: [W. Bernreuther et. al. '01 & '04],[K. Melnikov, M. Schulze '09]

Conclusions

- Various approaches to unstable particle production and decay
- E.T. inspired approach $\rightarrow$ gauge invariant expansion of amplitude in resonant region
- Structure to field-theory amplitudes

Not only applicable to Tops, but also to processes involving New Heavy Particles (e.g. SUSY particles...)

- Valid for any observable / arbitrary cuts on final states

Method is systematically improvable: know what we have to compute if we want to go to the next order in δ ('NNLO').

- Hard/Soft separation of virtual and real corrections $\leftrightarrow$ gauge invariant separation of factorizable / non-factorizable corrections

Outlook

Next few months...

- Finish $t\bar{t}$ implementation; study off-shell effects in distributions, mass determination

Extensions to Effective Theory Approach

- Further disentangle hard/soft and collinear separation of real corrections and cancellation of associated singularities with hard/soft/collinear virtual corrections
- Progress towards resummation of large logs $\sim \log(\frac{\mu_s}{\mu_h})$ for exclusive observables

(Making baby steps first with Single Top)

More Processes

- Add in hadronic decays of W's
- Straight-forward to include anomalous couplings of tops [J.A. Aguilar-Saavedra '10][C. Zhang, S. Willenbrock '11]
- Off-shell effects in SUSY decays...??

Thanks for listening!