Anomalous dimensions in QFT and CFT

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29 September 2025

DESY theory seminar

Anomalous dimensions = spectrum

Anomalous dimensions of composite operators determine scaling and running in general QFT

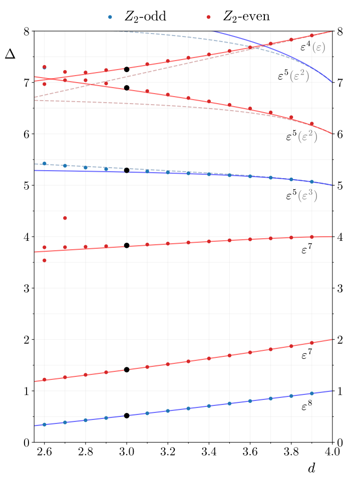
$$\mathcal{O}_{\text{ren}} = Z^{-1}\mathcal{O}_{\text{bare}}, \qquad \gamma = \frac{d}{d\ln\mu}\ln Z$$

$Z = 1 + c$ extracted from

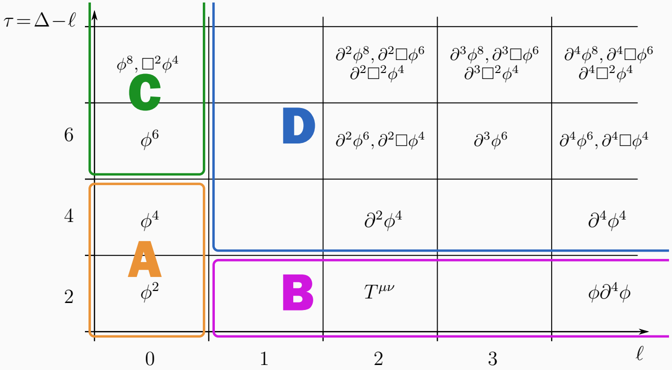
$$\Gamma^{(n)}_{\mathcal{O}} \sim \langle \mathcal{O}_{\text{ren}}(x)\phi(x_1)\cdots\phi(x_n)\rangle \stackrel{!}{=} \text{finite}$$

Take-home messages:

1. $\gamma_{\mathcal{O}}$ determine the spectrum of a CFT
   $\Delta_{\mathcal{O}} = [\mathcal{O}] + \gamma_{\mathcal{O}}$.
2. In CFT, "all" operators are important
3. Analyticity in spin is natural in CFT
4. EFT systematics is suitable for CFT



Plan/chart

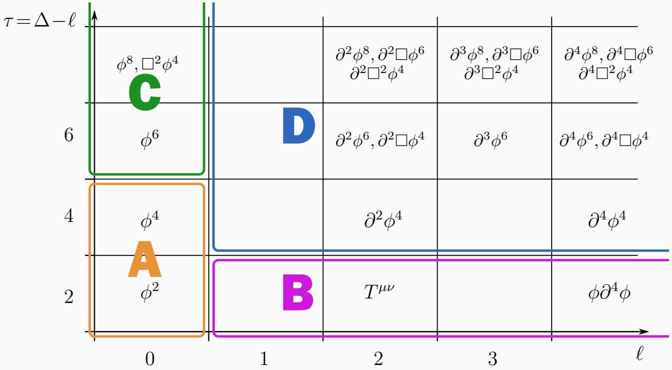


Part 1. Anomalous dimensions in CFT

Part 2. Leading twist and analyticity in spin (region B)

Part 3. EFT systematics for CFT (regions C and D)

Plan/chart



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Perturbative CFT

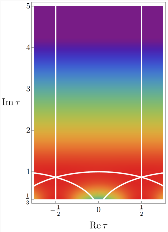
Gague theory beta function

$$\beta(g) = -b_0 g^3 + b_1 g^5 + \ldots$$

$b_0 = \frac{11}{3} N_c - \frac{2}{3} N_f\,, \quad b_1 = -\frac{34}{3} N_c + \frac{10}{3} N_c N_f + \frac{N_c^2-1}{2N_c} N_f$

$\Rightarrow$ fixed-points in $4d$

- $b_0 = b_1 = \ldots = 0$. Superconformal gauge theories. E.g. $\mathcal{N}=4$ SYM: marginal coupling
- $\beta(g_*) = 0$, $g_*^2 = b_0/b_1 \ll 1$ Conformal window/Banks–Zaks fixed-point



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Scalar theory $(\lambda_{abcd}\phi^a\phi^b\phi^c\phi^d)$ beta function $d = 4 - \varepsilon$

$$\beta_{abcd}(\lambda) = -\varepsilon\lambda_{abcd} + (\lambda_{abef}\lambda_{efcd} + \lambda_{acef}\lambda_{efbd} + \lambda_{adef}\lambda_{efbc}) + \ldots$$

$\Rightarrow$ fixed-points in $4-\varepsilon$ dimensions

- Single scalar (Ising CFT) $\beta(\lambda) = -\varepsilon\lambda + 3\lambda^2 + \ldots$, $\lambda_* = \varepsilon/3 + \ldots$.
- $(\vec{\phi}^2)^2$ ($O(n)$ CFT)
- Hypercubic, ...

Anomalous dimensions in QFT and CFT

Why compute anomalous dimensions?

In QFT/EFT: Running of coupling constants $\sum_i C_i \mathcal{O}_i(x)$

$$\mu \frac{d}{d\mu} C_i = \gamma_{ij} C_j \quad \Rightarrow \qquad C(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{-\gamma/(2b_0)} C(\mu_0)$$

In QFT/CFT – consistent definition of composite operators

$$\phi(x)\phi(0) \sim x^{\gamma_{\phi^2} - 2\gamma_\phi} : \phi^2(0) : + \ldots$$

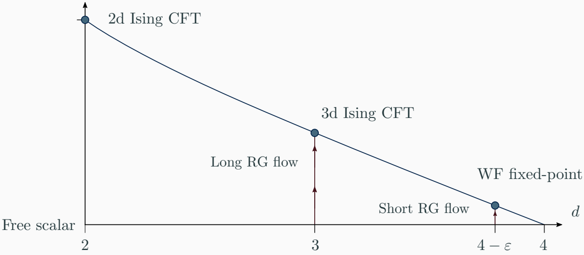
In CFT: OPE in terms of full scaling dimension $\Delta_{\mathcal{O}} = [\mathcal{O}] + \gamma_{\mathcal{O}}$

$$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k x^{\Delta_k - \Delta_i - \Delta_j} \lambda_{ijk} \mathcal{O}_k(0) + \ldots$$

CFT-data: $\{\Delta_i, \lambda_{ijk}\}$ defines the CFT

Ising CFT from $d = 4 - \varepsilon$ to $d = 2$

Universal definition: IR fixed-point in $\lambda\phi^4$ theory $\beta(\lambda) = -\varepsilon\lambda + 3\lambda^2 + \ldots$



Anomalous dimensions $\gamma(\lambda_*) = \gamma(\varepsilon) \leftrightarrow \Delta(d)$ (spectrum continuity) **[Hogervorst et al: 1512.00013]**

OPE $\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_k x^{\Delta_k - \Delta_i - \Delta_j} \lambda_{ijk} \mathcal{O}_k(0) + \ldots$ across d

- $d = 4 - \varepsilon$ (series): $\phi(x)\phi(0) = 1 + x^{\frac{\varepsilon}{3}}(\sqrt{2} + \ldots)\frac{\phi^2}{\sqrt{2}}(0)$
- $d = 3$ (numerics): $\sigma(x)\sigma(0) = 1 + x^{1.4126 - 2\cdot 0.5181} 1.0518\epsilon(0) + \ldots$
- $d = 2$ (exact): $\sigma(x)\sigma(0) = 1 + x^{1 - 2\frac{1}{8}} \frac{1}{2}\epsilon(0) + \ldots$

Ising CFT from $d = 4 - \varepsilon$ to $d = 2$

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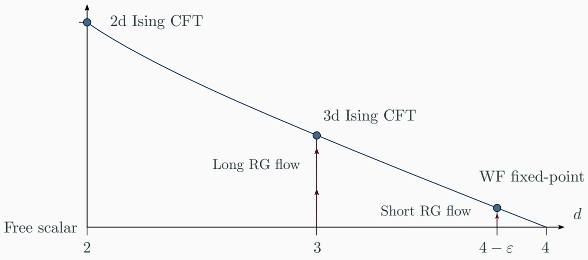

$$\Delta_\phi = 1 - \frac{\varepsilon}{2} + \frac{\varepsilon^2}{108} + \dots$$

$$\Delta_{\phi^2} = 2 - \varepsilon + \frac{\varepsilon}{3} + \dots$$

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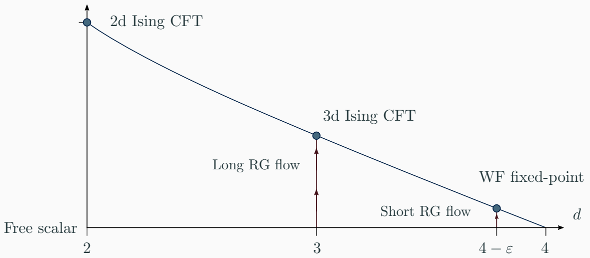

$$\Delta_\sigma = 0.518148806(24)$$

$$\Delta_\epsilon = 1.41262528(29)$$

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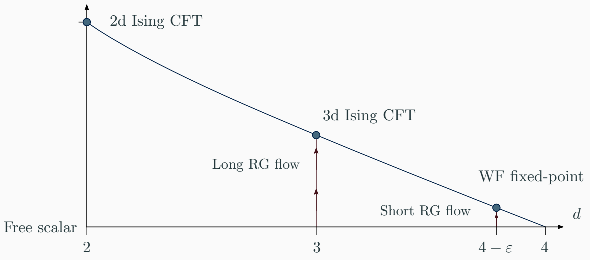

$\Delta_\sigma = \frac{1}{8}$

$\Delta_\epsilon = 1$

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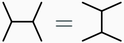
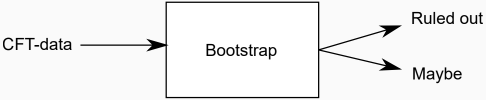
Conformal bootstrap

- CFT four-point function $\langle\sigma\sigma\sigma\sigma\rangle = \mathcal{G}(u,v) = \sum_{\mathcal{O}} \lambda^2_{\sigma\sigma\mathcal{O}} g_{\Delta_{\mathcal{O}},\ell_{\mathcal{O}}}(u,v)$

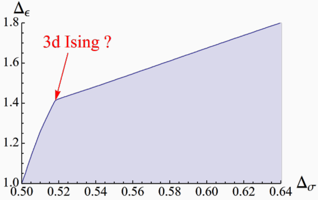
  $g_{\Delta,\ell}(u,v)$ conformal blocks
- Crossing symmetry $\mathcal{G}(u,v) \sim \left(\frac{u}{v}\right)^{\Delta_\sigma} \mathcal{G}(v,u)$



- Unitarity $\Delta \geqslant \Delta_{u.b.}$ and $\lambda_{ijk} \in \mathbb{R}$

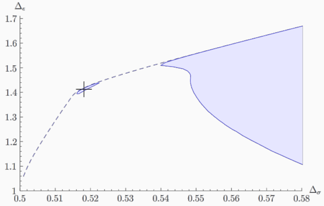


Single correlator



[El-Showk et al 2012: 1203.6064]

Mixed correlators

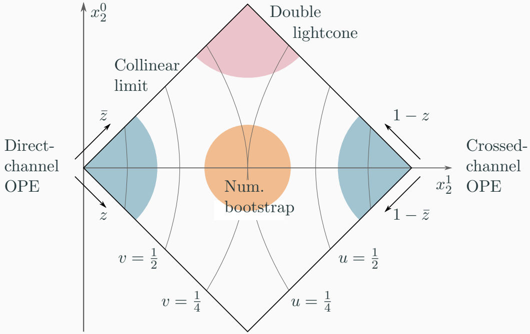


[Kos et al 2014: 1406.4858]

Kinematics limits of the four-point function

$\mathcal{G}(u,v) \sim \langle \sigma(x_1)\sigma(x_2)\sigma(x_3)\sigma(x_4)\rangle$ at $x_1^\mu = 0$, $x_4^\mu = \infty$, $x_3^\mu = (1,0,0,0)$

$u = z\bar{z}$, $v = (1-z)(1-\bar{z})$,

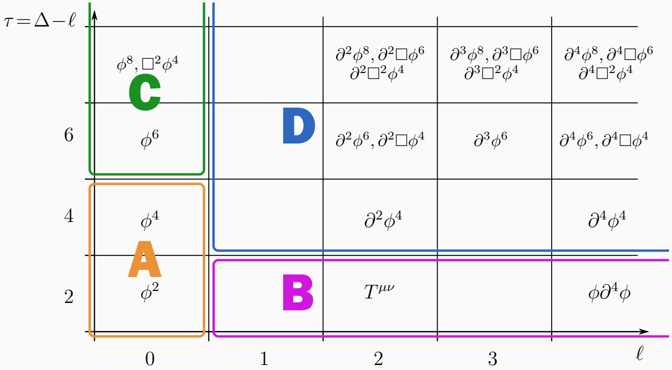


OPE limit $x^\mu \to 0$ – dominance of low-Δ

Lightcone/collinear limit $x^2 \to 0$, x^μ finite – dominance of low-τ

Crossing-symmetric point – x^μ finite – mild dominance of low-Δ operators

Plan/chart



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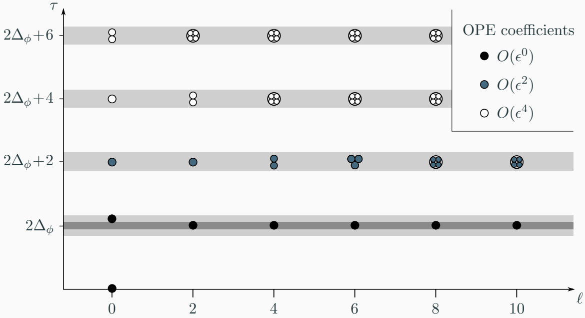
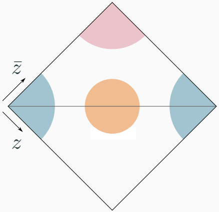
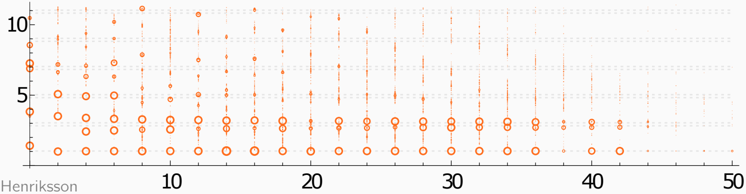
Ising spectroscopy (1)

Operators in the $\phi \times \phi$ OPE [JH: 2008.12600]





Operators in the $\sigma \times \sigma$ OPE [Simmons-Duffin: 1612.08471]



Twist-2 operators

Gauge theory: $\mathcal{O}_q = \bar{\psi}\gamma^{\mu_1} D^{\mu_2} \cdots D^{\mu_\ell} \psi - \text{traces}$, $\qquad \tau = \Delta - \ell$

$$\mathcal{O}_g = G^{\alpha\mu_1} D^{\mu_2} \cdots D^{\mu_{\ell-1}} G^{\mu_\ell}{}_\alpha - \text{traces}$$

$\ell = \text{spin} = \text{Mellin moment}$, $\qquad \gamma_{i,j,\ell} = -\int_0^1 x^{\ell-1} P_{i\leftarrow j}(x)$, P splitting function

Scalar theory: $\mathcal{O} = \phi \partial^{\mu_1} \cdots \partial^{\mu_\ell} \phi$-traces

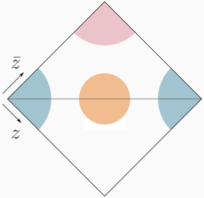
Anomalous dimensions ($\Delta = 2 + \ell - \varepsilon + \gamma_\ell$) [Derkachov, Gracey, Manashov: hep-ph/9705268]

$$\gamma_\ell = \frac{\varepsilon^2}{54}\left(1 - \frac{6}{\ell(\ell+1)}\right) + \frac{\varepsilon^3}{5832}\left(\frac{109\ell^4 + 218\ell^3 + 373\ell^2 - 384\ell - 324}{\ell^2(\ell+1)^2} - \frac{432 S_1(\ell)}{\ell(\ell+1)}\right) + \ldots$$

$S_1(\ell) = \sum_{k=1}^{\ell} \frac{1}{k} = \psi(\ell+1) + \gamma_E$ harmonic number

- γ_ℓ analytic in ℓ
- Reciprocity. In CFT [Alday, Bissi, Łukowski: 1502.07707]: $\gamma_\ell = \hat{\gamma}(\frac{\tau}{2} + \ell + \frac{1}{2}\gamma_\ell)$
  $\hat{\gamma}(\bar{h})$ symmetric under $\bar{h} \leftrightarrow 1 - \bar{h}$

Twist-2 operators



QCD: Duality between γ_ℓ and splitting functions $P(x)$

$\gamma_{i,j,\ell} = -\int_0^1 x^{\ell-1} P_{i\leftarrow j}(x)$, P splitting function

CFT: Duality between γ_ℓ and lightcone coordinate $\bar{z}$

Conformal blocks simplify in lightcone limit $z \to 0$

$$g_{\Delta,\ell}(z,\bar{z}) \sim z^{\frac{\Delta-\ell}{2}} k_{\frac{\Delta+\ell}{2}}(\bar{z}), \qquad k_{\bar{h}} = \bar{z}^{\bar{h}}\, {}_2F_1(\bar{h},\bar{h};2\bar{h};\bar{z})$$

Insert $\Delta = 2 + \ell + \gamma_\ell$

$$\sum_\ell \lambda^2_{\phi\phi\mathcal{O}_\ell} g_{\Delta,\ell}(z,\bar{z}) \sim \frac{z\ln z}{2} \sum_\ell \lambda^2_{\phi\phi\mathcal{O}_\ell} \gamma_\ell k_{\bar{h}}(\bar{z}) + \text{non-}\ln z \text{ terms}$$

Using $(\lambda^{(0)}_{\phi\phi\mathcal{O}_\ell})^2 = \frac{2\Gamma(\ell+1)^2}{\Gamma(2\ell+1)}$, $z \to 0$ limit generates sums of the form

$$F(\bar{z}) = \sum_{\bar{h}} \frac{2\Gamma(\bar{h})^2}{\Gamma(2\bar{h}-1)} \hat{\gamma}(\bar{h}) k_{\bar{h}}(\bar{z}) \qquad\qquad \bar{h} = \ell + 1$$

Twist-2 operators

The $z \to 0$ limit generates sums of the form ($\ell = \bar{h} - 1$)

$$\mathcal{G}(z,\bar{z})|_{\ln z} \sim \sum_{\ell} z^{\frac{\Delta-\ell}{2}} k_{\frac{\Delta+\ell}{2}}(\bar{z})|_{\ln z} \sim \sum_{\bar{h}} \frac{2\Gamma(\bar{h})^2}{\Gamma(2\bar{h}-1)} \hat{\gamma}(\bar{h}) k_{\bar{h}}(\bar{z}).$$

It is sometimes possible to compute these sums (see e.g. [JH: 2008.12600])!

$$\hat{\gamma}(\bar{h}) = A + \frac{B}{\bar{h}(\bar{h}-1)} + C S_1(\bar{h}-1) + \frac{D S_1(\bar{h}-1)}{\bar{h}(\bar{h}-1)}$$

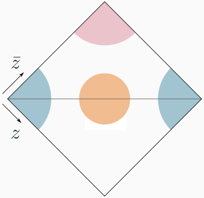
$\leftrightarrow$

$$\mathcal{G}(z,\bar{z})|_{\ln z} = \frac{A}{1-\bar{z}} + \frac{B}{2}\ln^2(1-\bar{z}) - \frac{C}{2}\frac{\ln(1-\bar{z})}{1-\bar{z}} - \frac{D}{12}\ln^3(1-\bar{z}) + \text{reg.}$$

Each coefficient of A, B, C, D has a different $\bar{z} \to 1$ limit

Sums of leading twist operators probe singularities as $z \to 0$, $\bar{z} \to 1$ (double lightcone limit)

Duality between twist-2 anomalous dimensions and singularities of four-point function in double lightcone limit



Twist-2 operators

Duality between twist-2 anomalous dimensions and singularities of four-point function in double lightcone limit. Sum $\leftrightarrow$ inversion fromula

$$\sum_{\bar{h}} \frac{2\Gamma(\bar{h})^2}{\Gamma(2\bar{h}-1)} \hat{\gamma}(\bar{h}) k_{\bar{h}}(\bar{z}) = F(\bar{z}) \leftrightarrow \hat{\gamma}(\bar{h}) = \frac{\Gamma(\bar{h})^2}{2\pi^2\Gamma(2\bar{h})} \int_0^1 \frac{d\bar{z}}{\bar{z}^2} k_{\bar{h}}(\bar{z}) \mathrm{dDisc}[F(\bar{z})]$$

Example $F(\bar{z}) = \frac{B}{2} \ln^2(1-\bar{z})$

$$\mathrm{dDisc}[F(\bar{z})] = 4\pi^2 \frac{B}{2}, \qquad \hat{\gamma}(\bar{h}) = \frac{\Gamma(\bar{h})^2 B}{\Gamma(2\bar{h})} \int_0^1 \frac{d\bar{z}}{\bar{z}^2} k_{\bar{h}}(\bar{z}) = \frac{B}{\bar{h}(\bar{h}-1)}$$

Dictionary: {reciprocity-resp. harmonic sums, $(\ell(\ell+1))^{-1}$} $\leftrightarrow$ {Polylogs}

Duality only sensitive to limit $\bar{z} \to 1 \leftrightarrow$ tail of sum. But in examples expressions are valid for any $\ell \geqslant 2$

$$\gamma_\ell = \frac{\varepsilon^2}{54}\left(1 - \frac{6}{\ell(\ell+1)}\right) + \frac{\varepsilon^3}{5832}\left(\frac{109\ell^4+218\ell^3+373\ell^2-384\ell-324}{\ell^2(\ell+1)^2} - \frac{432 S_1(\ell)}{\ell(\ell+1)}\right) + \ldots$$

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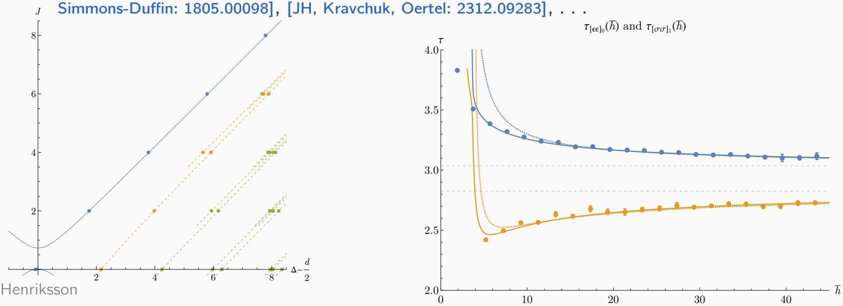
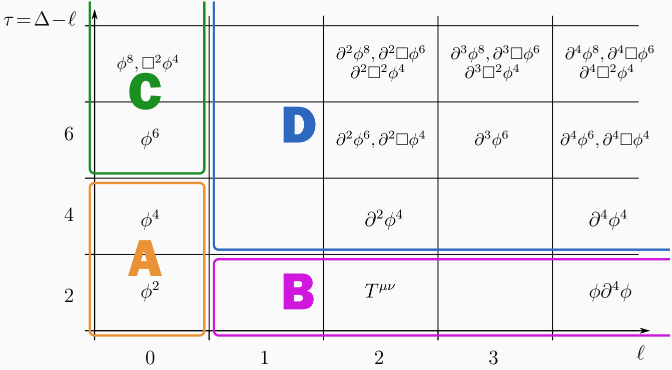
- Lightcone = large spin [Fitzpatrick et al: 1212.3616], [Komargodski, Zhiboedov: 1212.4103]
- Lorentzian **inversion formula** [Caron-Huot: 1703.00278]

$$C(\Delta,\ell)=\kappa_{\Delta,\ell}\int_0^1 dz d\bar{z} K_{\Delta,\ell}(z,\bar{z})\mathrm{dDisc}[\mathcal{G}(z,\bar{z})] \quad \overset{z\to 0}{\Rightarrow} \quad \hat{\gamma}(\bar{h}) \leftrightarrow F(\bar{z})$$

- Large spin perturbation theory [Alday: 1611.01500]: **compute CFT-data**
  – Analytic $(\mathbf{1}+\phi^2)|_{\text{crossed}} \overset{\sim}{\to} \frac{1}{1-\bar{z}}+\ln^2(1-\bar{z}) \overset{\sim}{\to} \frac{\varepsilon^2}{54}\left(1-\frac{6}{\ell(\ell+1)}\right)$ [Alday, JH, Van Loon: 1712.02314]. γ_ℓ to $O(\varepsilon^4)$.
  – Numeric [Simmons-Duffin: 1612.08471]
- Lorentzian inversion formula gives data for **lightray operators** [Kravchuk, Simmons-Duffin: 1805.00098], [JH, Kravchuk, Oertel: 2312.09283], . . .



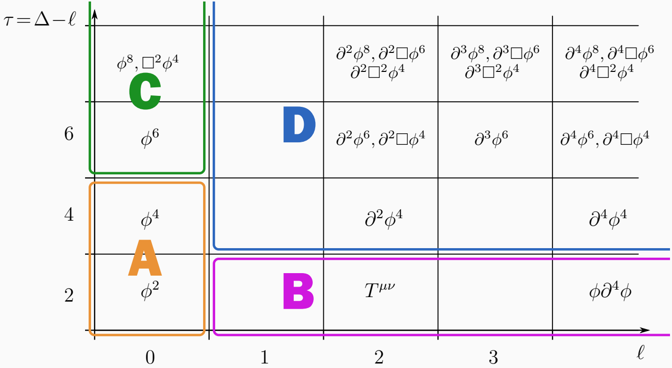
Plan/chart



Status:

- A. Impressive results $O(\varepsilon^7)$ [Schnetz: 2212.03663]
- B. Impressive results $O(\varepsilon^4)$ [Derkachov, Gracey, Manashov: hep-ph/9705268] & Lorentzian inversion formula [Alday, JH, Van Loon: 1712.02314]
- C. (?) $\rightarrow$ Impressive results [Cao, Herzog, Melia, Roosmale Nepveu: 2105.12742]
- D. Embarrassing, but some systematics at 1-loop [Kehrein, Wegner: hep-th/9405123]. Want a) General field indices, b) spinning operators

Plan/chart



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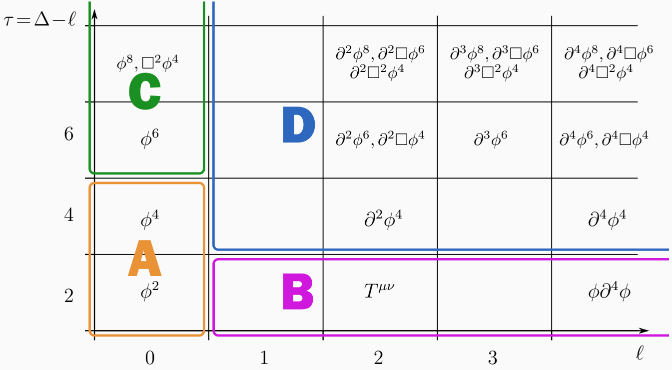
- C. (?) $\rightarrow$ Impressive results [Cao, Herzog, Melia, Roosmale Nepveu: 2105.12742]
- D. Embarrassing, but some systematics at 1-loop [Kehrein, Wegner: hep-th/9405123]. Want a) General field indices, b) spinning operators

Plan/chart



Status:

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Higher-dimensional operators

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a - \frac{1}{24}\lambda^{abcd}\phi^a\phi^b\phi^c\phi^d + \mathcal{L}_{\text{spin-0}} + u_\mu\mathcal{L}^\mu_{\text{spin-1}} + v_{(\mu\nu)}\mathcal{L}^{(\mu\nu)}_{\text{spin-2}} + w_{[\mu\nu]}\mathcal{L}^{[\mu\nu]}_{\text{spin-2}}$$

$$\mathcal{L}_{\text{spin-0}} = \frac{c^{(5,0)}_{(abcde)}}{120}\phi^a\phi^b\phi^c\phi^d\phi^e + \frac{c^{(6,0)}_{(abcdef)}}{720}\phi^a\phi^b\phi^c\phi^d\phi^e\phi^f - \frac{c^{(6,0)}_{((ab),(cd))}}{4}\phi^a\phi^b\partial_\mu\phi^c\partial^\mu\phi^d$$

$$\mathcal{L}^\mu_{\text{spin-1}} = c^{(3,1)}_{[ab]}\phi^a\partial^\mu\phi^b + \frac{c^{(4,1)}_{(ab)c}}{2}\phi^a\phi^b\partial^\mu\phi^c + \frac{c^{(5,1)}_{(abc)d}}{6}\phi^a\phi^b\phi^c\partial^\mu\phi^d + \frac{c^{(6,1)}_{(abcd)e}}{24}\phi^a\phi^b\phi^c\phi^d\partial_\mu\phi^e ,$$

$$\mathcal{L}^{(\mu\nu)}_{\text{spin-2}} = c^{(4,2)}_{(ab)}\left(\phi^a\partial^\mu\partial^\nu\phi^b - 2\,\partial^\mu\phi^a\partial^\nu\phi^b\right) + 2\,c^{(5,2)}_{a(bc)}\left(\phi^a\phi^b\partial^\mu\partial^\nu\phi^c - 2\,\phi^a\partial^\mu\phi^b\partial^\nu\phi^c\right) + c^{(6,2)}_{(ab)(cd)}\left(\phi^a\phi^b\phi^c\partial^\mu\partial^\nu\phi^d - 2\,\phi^a\phi^b\partial^\mu\phi^c\partial^\nu\phi^d\right)$$

$$\mathcal{L}^{[\mu\nu]}_{\text{spin-2}} = -c^{(5,\{1,1\})}_{[abc]}\phi^a\partial^\mu\phi^b\partial^\nu\phi^c - c^{(6,\{1,1\})}_{a[bcd]}\phi^a\phi^b\partial^\mu\phi^c\partial^\nu\phi^d$$

- Step 1: Derivation of general results. Generation of Feynman diagrams and evaluation using R^* method [JH, Herzog, Kousvos, Roosmale Nepveu 2507.12518]
  https://github.com/jasperrn/EFT-RGE
- Step 2: Extraction for specific theories. Project onto tensor structures compatible with global symmetry G [JH, Kousvos, Roosmale Nepveu WIP]

Ising spectroscpy

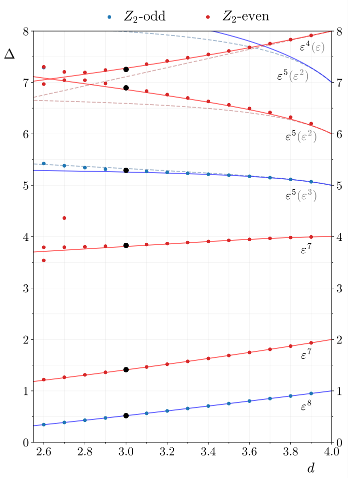
Bootstrap:

- 3d spectrum [Simmons-Duffin: 1612.08471]
- Level repulsion [JH, Kousvos, Reehorst: 2207.10118]
- Recent precision island [Chang et al: 2411.15300]

New perturbative estimates:

TABLE I. Our results at $\Delta \leq 6$ in the Ising CFT, compared with numerical bootstrap results in 3d with statistical and rigorous error intervals respectively.

Operator	ϕ^5	$\phi^2\partial_\mu\partial_\nu\phi$	$\phi^3\partial_\mu\partial_\nu\phi$
Previous loop order	ε^3 [64]	ε^2 [65]	ε^1 [66]
New loop order	ε^5	ε^5	ε^5
Padé approximant	5.257395	4.162978	5.465027
Statistical error [67]	5.2906(11)	4.180305(18)	5.50915(44)
Rigorous error [68]	5.262(89)	—	5.499(17)



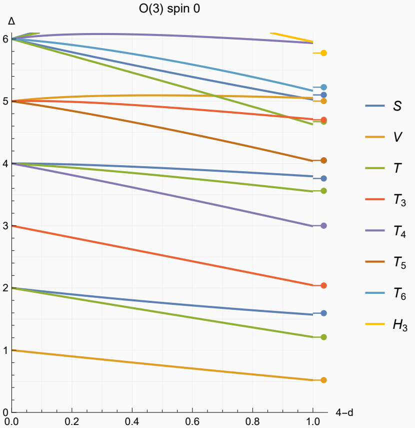
O(3) spectroscopy

Bootstrap:

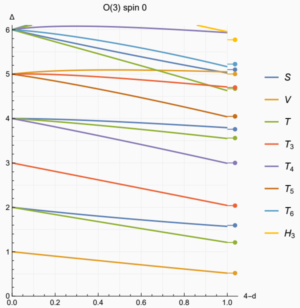
- Precision island [Chester et al: 2011.14647]
- Spectrum [Chester et al unpublished]

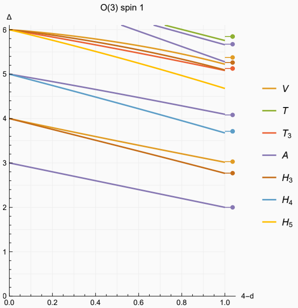
New perturbative estimates:

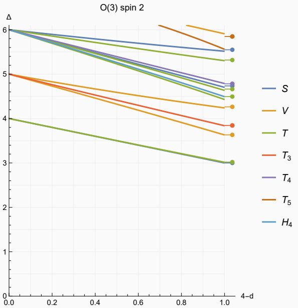
R	ℓ	$\mathcal{O}$	order	Padé	Bootstrap	MC
V	0	ϕ	(ε^8)	.5188246	.518942(51)	.518920(25) [72]
T	0	ϕ^2	(ε^6)	1.210809	1.20954(32)	1.2094(3) [73]
S	0	ϕ^2	(ε^7)	1.571279	1.59489(59)	1.5948(2) [72]
A	1	$\partial\phi^2$	exact	2		
T_3	0	ϕ^3	(ε^6)	2.042931	2.03867(23)	2.0385(3) [74]
H_3	1	$\partial\phi^3$	$\varepsilon \to \varepsilon^5$	2.766426	2.77025(22)	2.67 [71]
T_4	0	ϕ^4	(ε^6)	2.991664	$<$ 2.99056	2.9857(9) [74]
S	2	$\partial^2\phi^2$	exact	3		
T	2	$\partial^2\phi^2$	$\varepsilon^4 \to \varepsilon^5$	3.015701	3.013(18)	
V	1	$\partial\phi^3$	$\varepsilon^2 \to \varepsilon^5$	3.015815	3.03120(32)	
T	0	ϕ^4	(ε^6)	3.550026	3.561(13)	
V	2	$\partial^2\phi^3$	$\varepsilon^2 \to \varepsilon^5$	3.630425	3.633(4)	
H_4	1	$\partial\phi^4$	$\varepsilon \to \varepsilon^5$	3.676439	3.713(9)	
A_3	1^-	$\partial^2\phi^3$	$\varepsilon \to \varepsilon^5$	3.787955	NA	
S	0	ϕ^4	(ε^7)	3.793620	3.7667(10)	3.759(2) [72]
T_3	2	$\partial^2\phi^3$	$\varepsilon^2 \to \varepsilon^5$	3.837674	3.841(19)	



O(3) spectroscopy







Hypercubic CFT

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + [\lambda_{4,1}(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) + \lambda_{4,2}\,\delta_{abcd}]\,\phi^a\phi^b\phi^c\phi^d$$

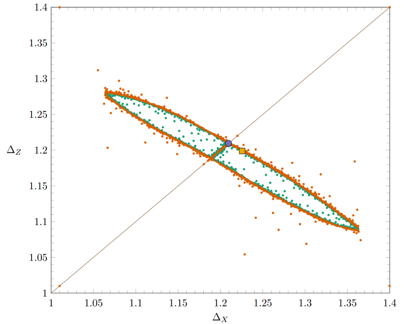
$\lambda_{4,2}\,\delta_{abcd}$ breaks $O(n)$ symmetry to $\mathbb{Z}_2^n \rtimes S_n$ (symmetry group of hypercube)

Perturbative spectrum:

- One-loop: [Antipin, Bersini : 1903.04950] [Bednyakov, JH, Kousvos: 2304.06755]
- Six-loop for ϕ^k, $k \leqslant 4$ [ibid]

Bootstrap:

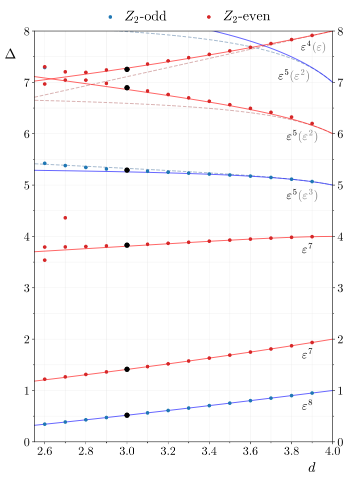
- Earlier work [Stergiou: 1801.07127] [Kousvos, Stergiou: 1810.10015,1911.00522]
- Recent [Kousvos, Stergiou: 2507.05338] Gap assumptions motivated by our new perturbative estimates



[Kousvos, Stergiou: 2507.05338]

Conclusions

- Anomalous dimension provide the spectrum of a CFT
- Spinning anomalous dimensions can be computed by the Lorentzian inversion formula from singuarities in the four-point function
- “All” operators, including complicated composite ones, are essential to make the CFT self-consistent (bootstrap)
- EFT techiques for higher-dimensional operators can be directly applied to CFT



Outlook

Recent EFT work

- *Anomalous Dimension of a General Effective Gauge Theory* **[Aebischer, Bresciani, Selimovic: 2502.14030]**
- *Renormalization of general Effective Field Theories* **[Fonseca, Olgoso, Santiago: 2501.13185]**

$$\mathscr{L}_{d\leq 4} = -\frac{1}{4}(a_{KF})_{AB}F^A_{\mu\nu}F^{B\,\mu\nu} + \frac{1}{2}(a_{K\phi})_{ab}D_\mu\phi_a D^\mu\phi_b + (a_{K\psi})_{ij}\bar{\psi}_i \mathrm{i}\not{D}\psi_j - \frac{1}{2}\left[(m_\psi)_{ij}\psi_i^T C\psi_j + \text{h.c.}\right] \\ -\frac{1}{2}(m^2_\phi)_{ab}\phi_a\phi_b - \eta_a\phi_a - \frac{1}{2}\left[Y_{ija}\psi_i^T C\psi_j + \text{h.c.}\right]\phi_a - \frac{\kappa_{abc}}{3!}\phi_a\phi_b\phi_c - \frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d, \qquad (5)$$

- *One-loop renormalization group equations in generic effective field theories* **[Misiak, Nałęcz: 2501.17134]**

Potential CFT applications

4d CFTs: Banks–Zaks fixed-point, $\mathcal{N} = 1$ susy, $\mathcal{N} = 2$ susy (e.g. conf. SQCD)

3d CFTs: Gross–Neveu–Yukawa models, 3d conf. gauge theories

Multicritical models: $d = d_c - \varepsilon$. $\quad d_c(\phi^k) = \frac{2k}{k-2} = 6, 4, \frac{10}{3}, 3, \ldots$

Back up slides

Inversion dictionary

Table 1: Inversions used in the ε-expansion. $J^2 = \bar{h}(\bar{h}-1)$

$G(\bar{z})$	$A(J)$
$\log^2(1-\bar{z})$	$\frac{4}{J^2}$
$\log^3(1-\bar{z})$	$-\frac{24S_1}{J^2}$
$\log^4(1-\bar{z})$	$\frac{96}{J^2}\left(S_1^2-\zeta_2-S_{-2}\right)$
$\log^2(1-\bar{z})\,\mathrm{Li}_2(1-\bar{z})$	$\frac{4}{J^2}(\zeta_2+2S_{-2})$
$\log^3(1-\bar{z})\,\mathrm{Li}_2(1-\bar{z})$	$\frac{24}{J^2}((S_{-3}-2S_{-2,1})-3(S_{-3}-2S_{1,-2})+3\zeta_2S_1-2S_3)$
$\log^2(1-\bar{z})\,\mathrm{Li}_3(1-\bar{z})$	$\frac{4}{J^2}(-2(S_{-3}-2S_{1,-2})+\zeta_3+2\zeta_2S_1-2S_3)$
$\log^2(1-\bar{z})\,\mathrm{Li}_3\left(\frac{\bar{z}-1}{\bar{z}}\right)$	$\frac{4}{J^2}\left(-2\zeta_3-\frac{1}{J^6}-\frac{2}{J^4}+2S_3\right)$
$\log^2(1-\bar{z})\log\bar{z}$	$-\frac{4}{J^4}$
$\log^2(1-\bar{z})\log^2\bar{z}$	$\frac{8}{J^2}\left(-\zeta_2+\frac{1}{J^4}+\frac{1}{J^2}-2S_{-2}\right)$
$\log^2(1-\bar{z})\log^3\bar{z}$	$\frac{24}{J^2}\left(2(S_{-3}-2S_{1,-2})+\zeta_3-\frac{1}{J^6}-\frac{2}{J^4}+\frac{\zeta_2}{J^2}+\frac{2S_{-2}}{J^2}-2\zeta_2S_1\right)$
$\log^3(1-\bar{z})\log\bar{z}$	$\frac{24}{J^2}\left(-\zeta_2+\frac{S_1}{J^2}-2S_{-2}\right)$
$\log^4(1-\bar{z})\log\bar{z}$	$\frac{48}{J^2}\left(-\frac{2S_1^2}{J^2}-4(S_{-3}-2S_{-2,1})+6(S_{-3}-2S_{1,-2}) +3\zeta_3+\frac{2\zeta_2}{J^2}+\frac{2S_{-2}}{J^2}-6\zeta_2S_1+2S_3\right)$
$\log^2(1-\bar{z})\mathrm{Li}_2(1-\bar{z})\log\bar{z}$	$\frac{4}{J^2}\left(-6(S_{-3}-2S_{1,-2})-\frac{\zeta_2}{J^2}-3\zeta_3-\frac{2S_{-2}}{J^2}+6\zeta_2S_1\right)$

[Alday, JH, Van Loon: 1712.02314]