

Run 3 Forward Electron Energy Calibration in ATLAS, with a focus on pileup noise

Giovanni Camia

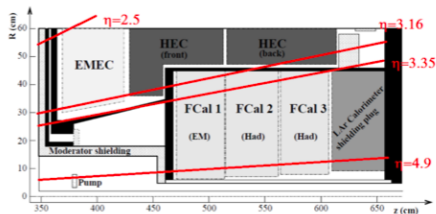
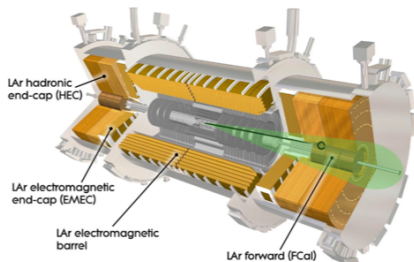
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DESY

September 1, 2025

ATLAS and Forward Region

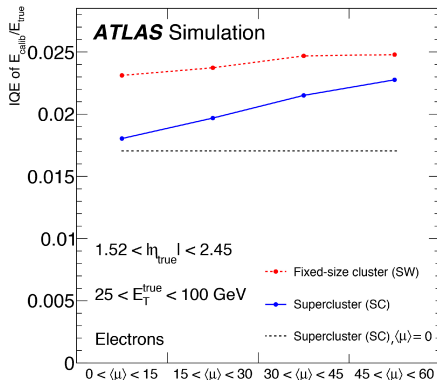
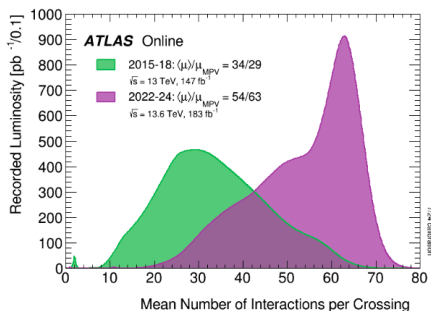
The study is interested in the forward region of the ATLAS detector, which is characterized by a pseudorapidity¹ $\eta > 2.5$. The forward region is split into two different kind of calorimeters: EMEC and FCAL



$$^1\eta = -\ln(\tan(\frac{\theta}{2}))$$

Goal

Investigate the energy resolution of electrons in the forward region from simulation, with a dedicated study of pile-up noise²



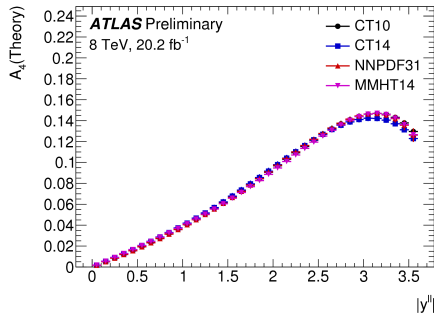
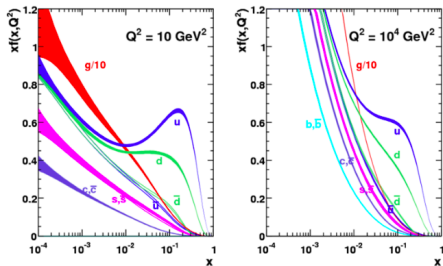
Pile-up refers to the phenomenon where multiple, independent proton-proton collisions occur simultaneously within the same

Forward Electron's Physics

Good calibration on forward electrons allow us to:

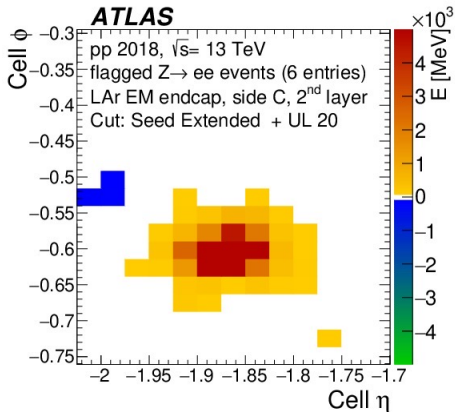
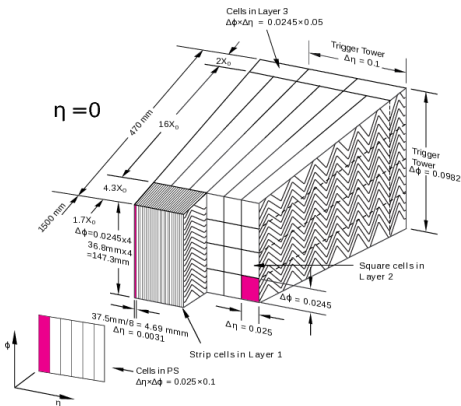
- Study PDFs with low and high x values
- Perform precise measurements of the EW mixing angle: $\sin(\theta_W)$

MSTW 2008 NLO PDFs (68% C.L.)



Energy Reconstruction

Energy is reconstructed by the electromagnetic calorimeters which are divided in cells. The entire e.m. shower is spread across different cells which are then unified in a cluster. $E_{cluster} = E_{reconstructed}$

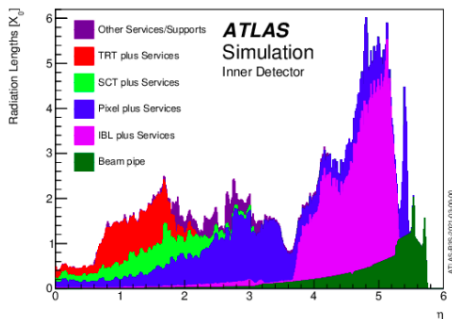
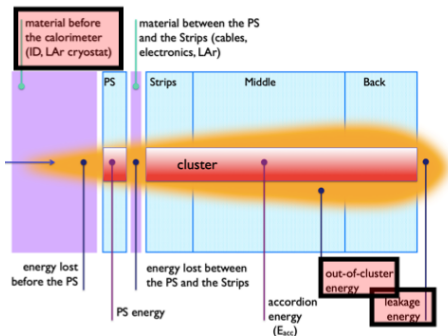


The images refers to the central region of the calorimeters where everything is well understood. The forward region is more of a unknown territory.

Energy Reconstruction

For this project the energy resolution is it going to be estimated by the difference between reconstructed and real energy, so for the relative uncertainty

$$\frac{E_{reconstructed} - E_{truth}}{E_{truth}} = \frac{E_{reconstructed}}{E_{truth}} - 1$$



$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus c \quad (1)$$

- **a**: stochastic term (Poissonian)
- **b**: noise term made up by pile-up and electronics

$$b = \sigma_{pile-up} \oplus \sigma_{electronics} \quad (2)$$

- **c**: constant term due to calibration uncertainty, non uniform detector response, leakage, ...

The resolution receives contribution from a bunch crossing number³-dependent part⁴, term related to the pile-up, and a constant part:

$$\frac{\sigma}{E_t} = \beta\sqrt{\mu} \oplus \alpha \quad (3)$$

$$\left(\frac{\sigma}{E_t}\right)^2 = \frac{\beta^2}{E_t^2} \cdot \mu + \text{constants}$$

To extract the pileup-noise a simple linear fit ($A + Bx$) on the σ^2 vs μ is needed, where the slope(B) contains the pileup noise factor- β

$$\beta \text{ [GeV]} = \sqrt{B} \cdot E_t \quad (4)$$

³ μ
⁴Poissonian

The data used are MC simulated data of a forward electron coming from the Z boson decay.

The work can be divided into steps:

- Create distributions of $\frac{E_{reconstructed}}{E_{truth}}$ for the electron in different bins in p_t , η and μ
- Fit sigma and mean from a Gaussian $\rightarrow \sigma_E$
- Create σ^2 vs μ for each bin
- Extract β using (4) from the linear fit

- **Binning in η_{probe} ($N_\eta = 7$ bins):**

$$\{2.5, 2.7, 2.9, 3.1, 3.1, 3.35, 3.5, 3.5, 3.8, 4.9\}$$

From 3.1 to 3.5 is going to be considered crack region, therefore not shown in plots.

- **Binning in p_t ($N_{p_t} = 4$ bins):**

$$\{20, 30, 40, 50, 100\} \text{ GeV}$$

- **Binning in μ ($N_\mu = 7$ bins):**

$$\{0, 30, 40, 50, 55, 60, 70, 100\}$$

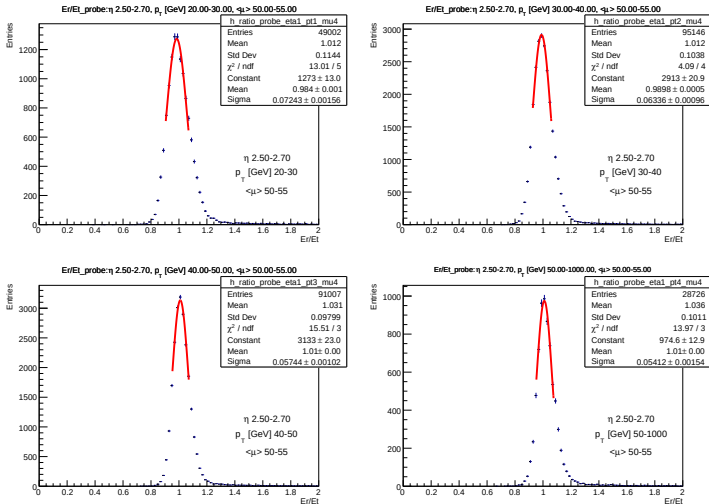


Figure: Example of fitted E_{reco}/E_{truth} distributions for different p_T bins for $2.5 < \eta < 2.7$ and $50 < \mu < 55$

Sigma Trend

Higher $\mu \rightarrow$ higher σ

$$\frac{\sigma}{E_t} = \beta\sqrt{\mu} \oplus \alpha$$

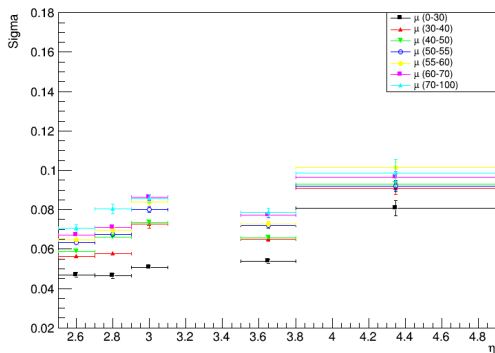
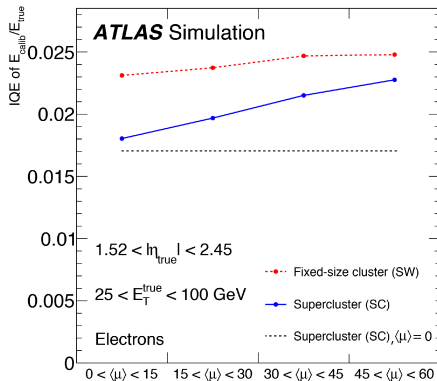
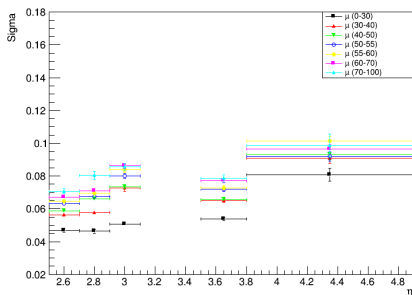


Figure: σ vs η for different μ bins for $30 < p_t[\text{GeV}] < 40$

Differences in η are due to differences in the materials before the calorimeters.

Sigma Difference with Central Case

The resolution in the forward region is much worse than in the central case.



Sigma Trend

Higher $p_t \rightarrow$ smaller σ

$$\frac{\sigma_E}{E} = \frac{a}{\sqrt{E}} \oplus \frac{b}{E} \oplus c$$

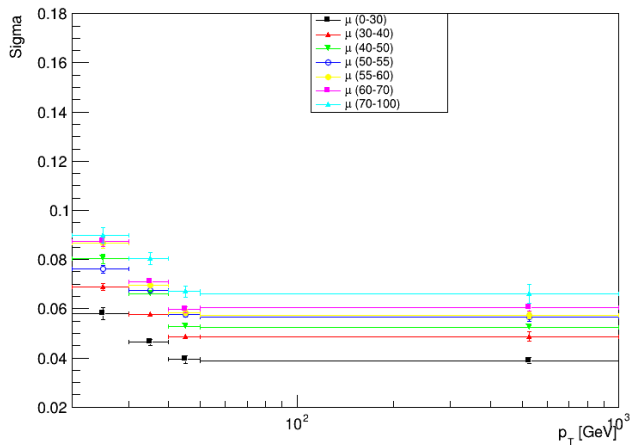


Figure: σ vs p_t for different μ bins for $2.7 < \eta < 2.9$

Linear fit

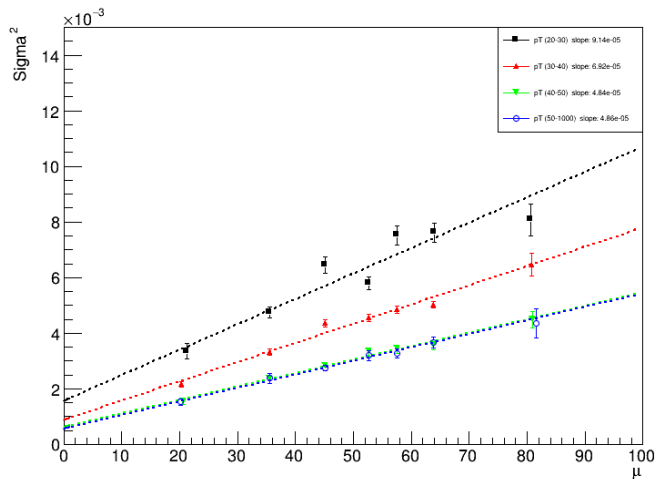


Figure: Linear fit from σ^2 vs μ for different p_t bins for $2.7 < \eta < 2.9$

β Extraction

From the linear fit⁵ is it possible to extract the pile-up noise for each p_t and η bin of the forward electrons⁶:

$$\beta \text{ [GeV]} = \sqrt{B} \cdot E_t$$

$\eta \backslash p_t$ bins	20–30 GeV	30–40 GeV	40–50 GeV	50–1000 GeV
2.50–2.70	0.222 ± 0.010	0.251 ± 0.0073	0.302 ± 0.0073	0.355 ± 0.017
2.70–2.90	0.251 ± 0.010	0.296 ± 0.0067	0.310 ± 0.010	0.424 ± 0.015
2.90–3.10	0.252 ± 0.013	0.368 ± 0.0068	$(5.2 \times 10^{-7}) \pm 1.3 \times 10^5$	$(5.7 \times 10^{-6}) \pm 4.0 \times 10^3$
3.10–3.50	<i>crack region (excluded)</i>			
3.50–3.80	0.184 ± 0.011	0.271 ± 0.0083	0.344 ± 0.028	0.504 ± 0.061
3.80–4.90	0.089 ± 0.029	0.265 ± 0.033	0.352 ± 0.098	$(3.8 \times 10^{-6}) \pm 2.9 \times 10^5$

Table: Values of β [GeV] with uncertainties for different η and p_T bins. The crack region is shaded and excluded.

⁵B is the slope of the linear fit

⁶Previous studies found $\beta \sim 0.1$ GeV for a 50 GeV electron

The pile-up noise in the forward region is much more significant than in the central case.

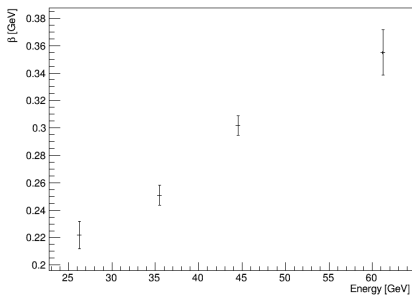


Figure: β vs p_t for $2.5 < \eta < 2.7$

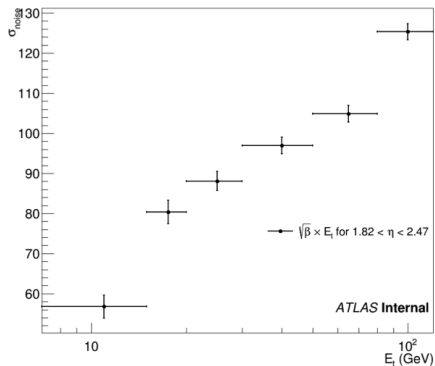


Figure: Results (in MeV) previously obtained for the central region

Conclusions

This work has presented an effective way to estimate the pile-up noise contribution to the energy resolution from simulated electrons in the forward region of the ATLAS calorimeters for the first time.

However, there is still some trends to better understand and problems to overcome, see next slides

Mean⁷ Trend

Higher $\mu \nearrow$ higher mean

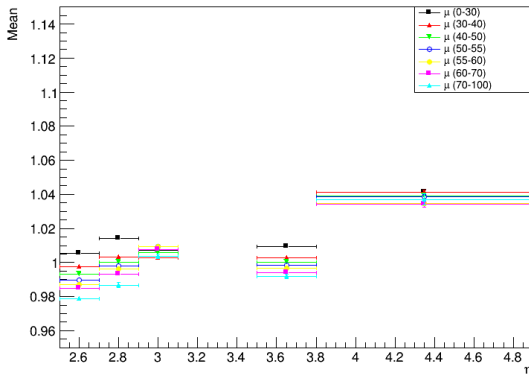


Figure: Mean vs η for different μ bins for $30 < p_t[\text{GeV}] < 40$

⁷Data have not undergone a MVA global calibration, should be the reason for the huge oscillation of the mean

Kinematic Constraint

The analysis is limited by the Z boson decay kinematics.

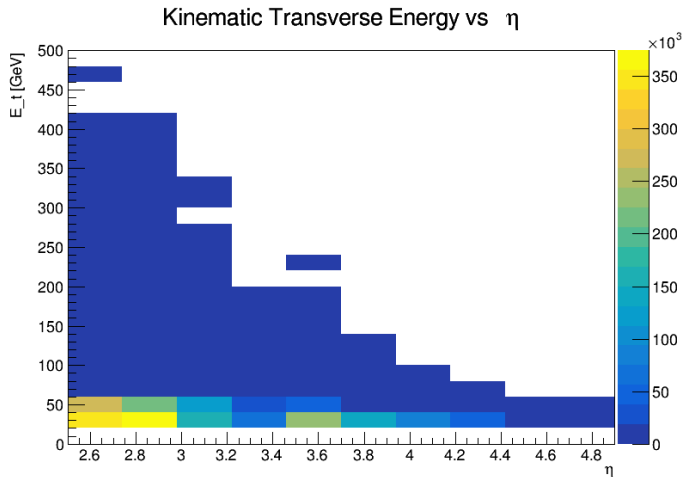


Figure: Number of events for values of E_t and η

Further prospects include:

- Use simulation of single electrons with flat distribution in p_t to avoid statistic problems
- Repeat the study for central electrons

Calorimeter Geometry

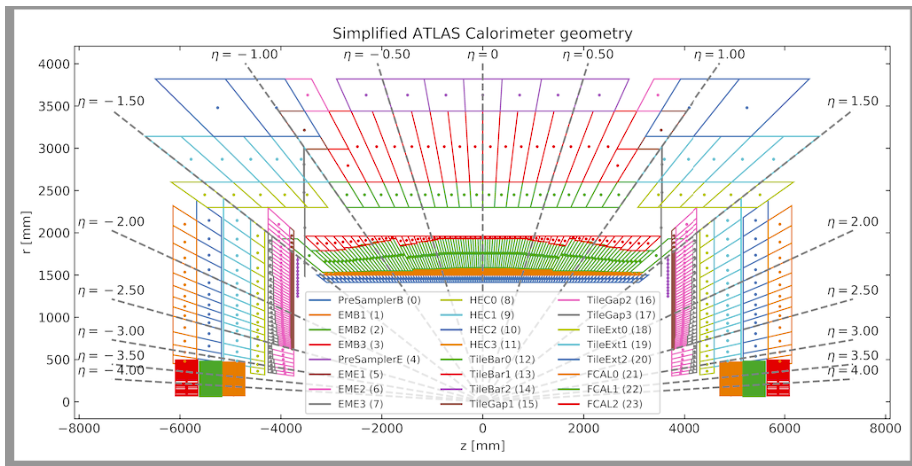


Figure: Full calorimeter disposition

Cookie-Cutting

To have a better linear energy response of the calorimeter cookie-cutting can be applied

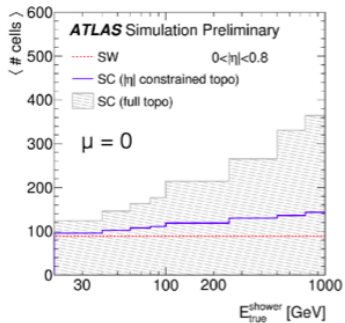
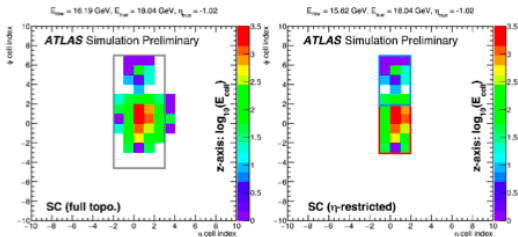
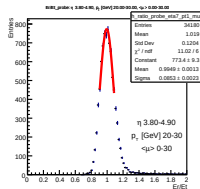
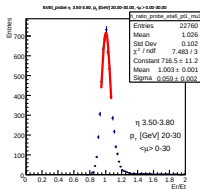
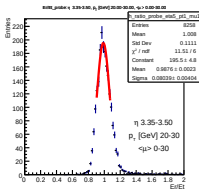
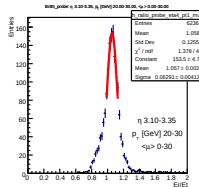
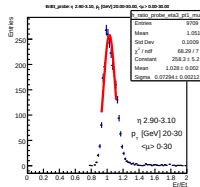
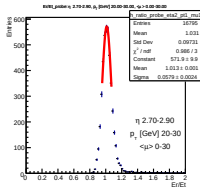
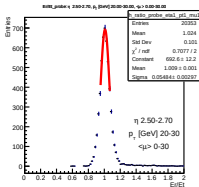
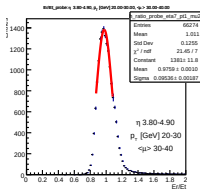
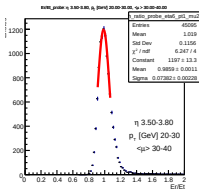
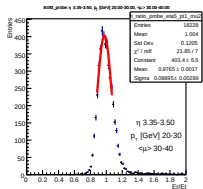
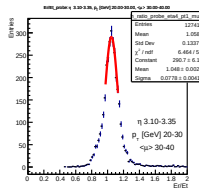
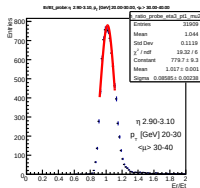
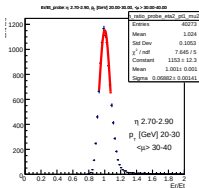
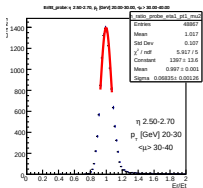


Figure: Cookie-cutting visual representation

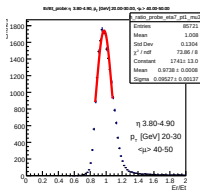
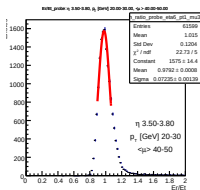
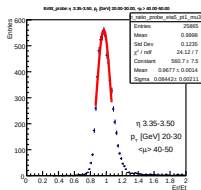
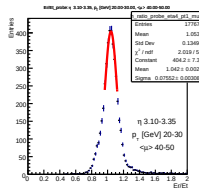
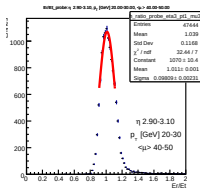
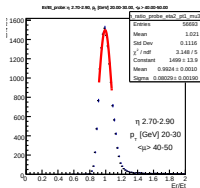
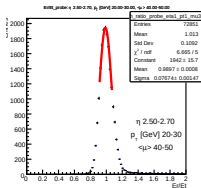
Plot p_t 1, μ 1



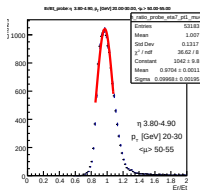
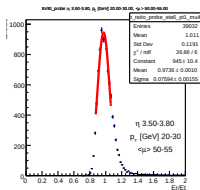
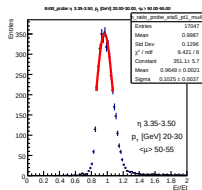
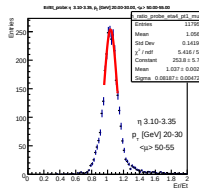
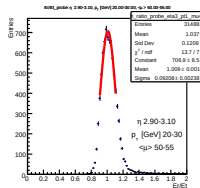
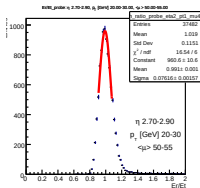
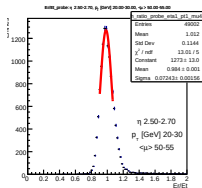
Plot p_t 1, μ 2



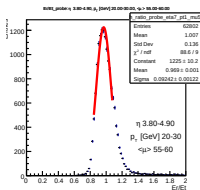
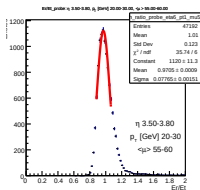
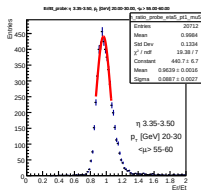
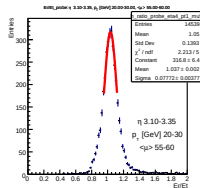
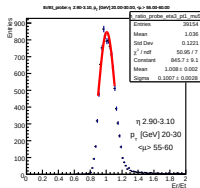
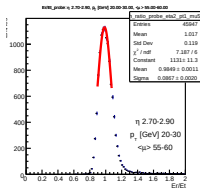
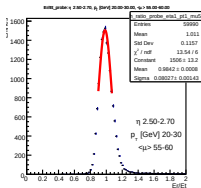
Plot p_t 1, μ 3



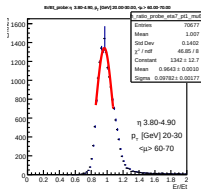
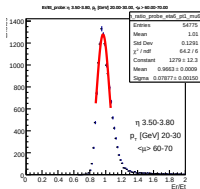
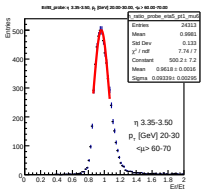
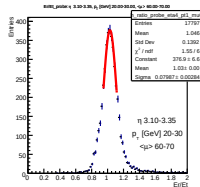
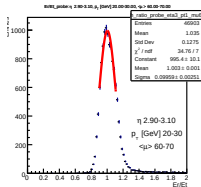
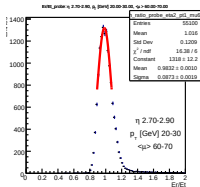
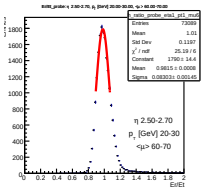
Plot p_t 1, μ 4



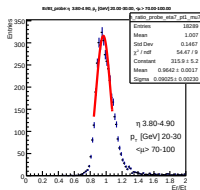
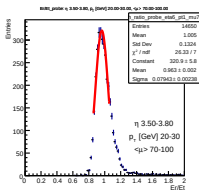
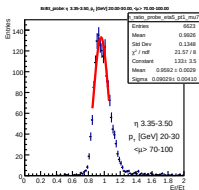
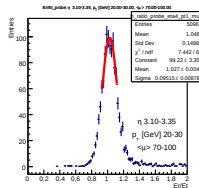
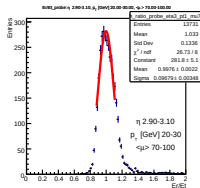
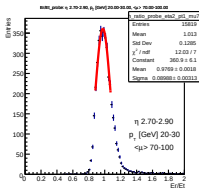
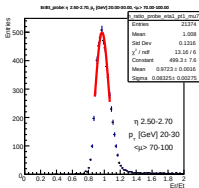
Plot p_t 1, μ 5



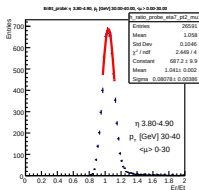
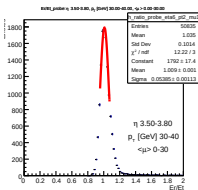
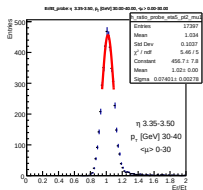
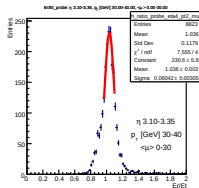
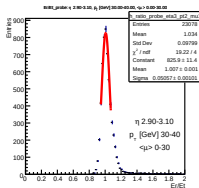
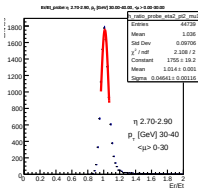
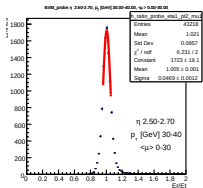
Plot p_t 1, μ 6



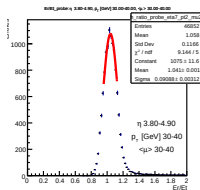
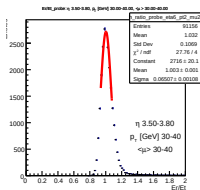
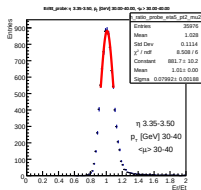
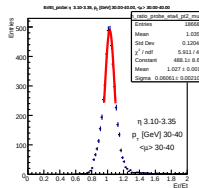
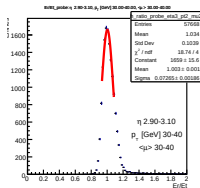
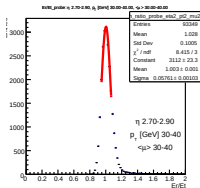
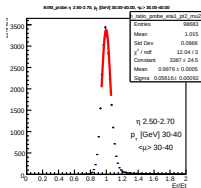
Plot p_t 1, μ 7



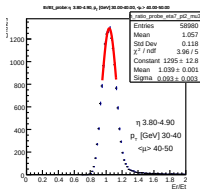
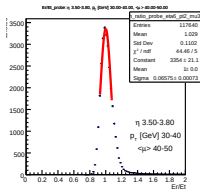
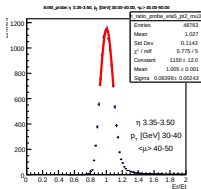
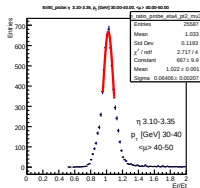
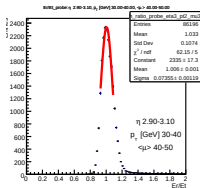
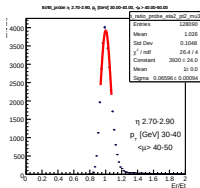
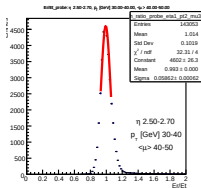
Plot p_t 2, μ 1



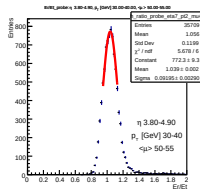
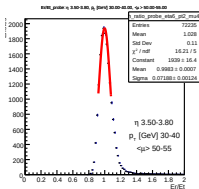
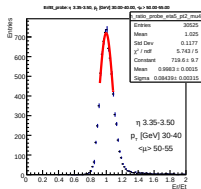
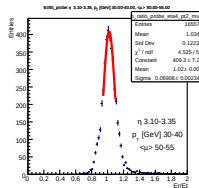
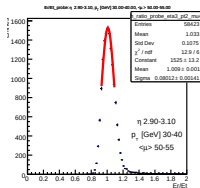
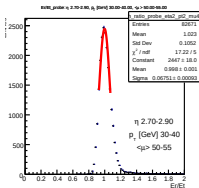
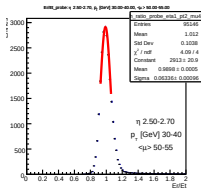
Plot p_t 2, μ 2



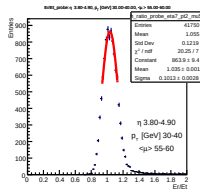
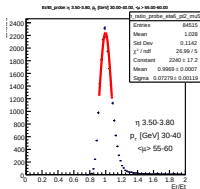
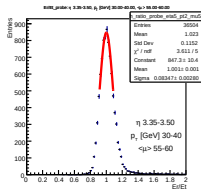
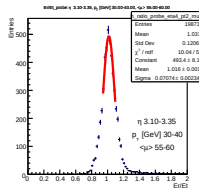
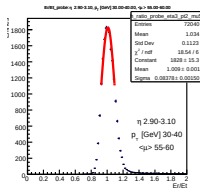
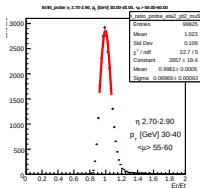
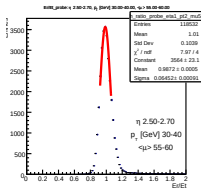
Plot p_t 2, μ 3



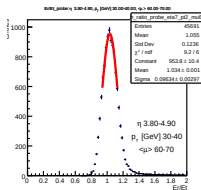
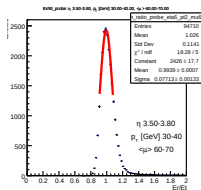
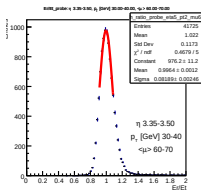
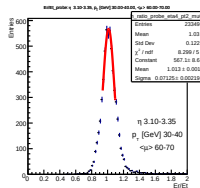
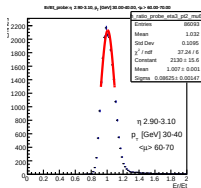
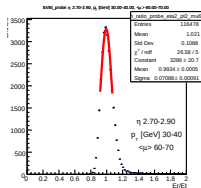
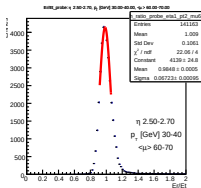
Plot p_t 2, μ 4



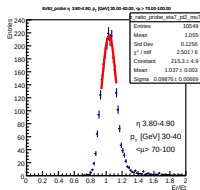
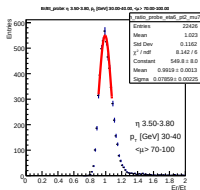
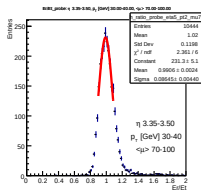
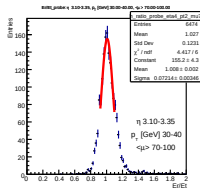
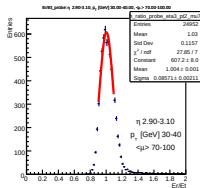
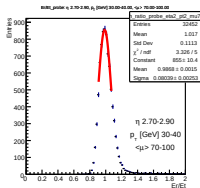
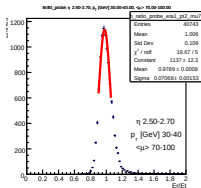
Plot p_t 2, μ 5



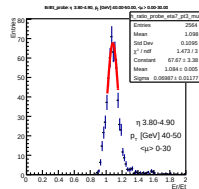
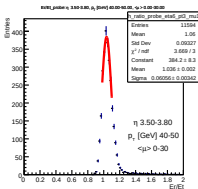
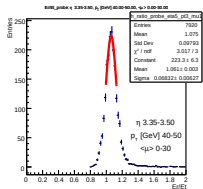
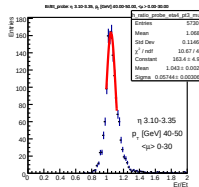
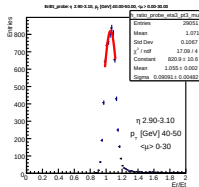
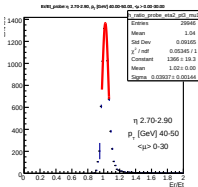
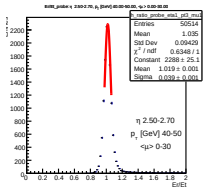
Plot p_t 2, μ 6



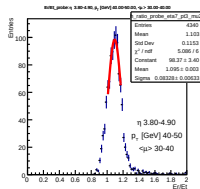
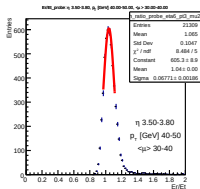
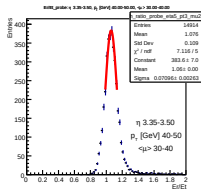
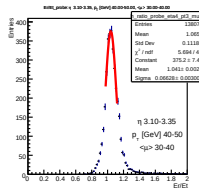
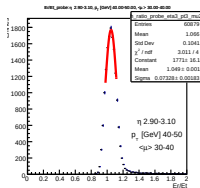
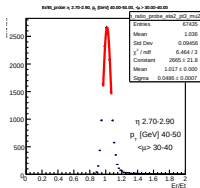
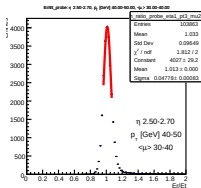
Plot p_t 2, μ 7



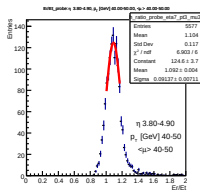
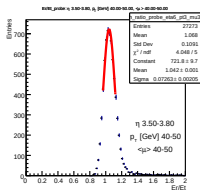
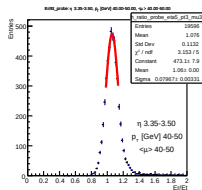
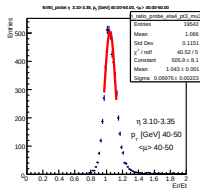
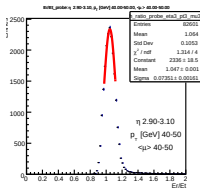
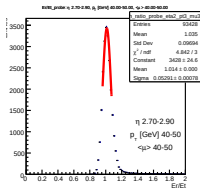
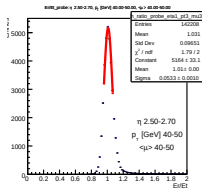
Plot p_t 3, μ 1



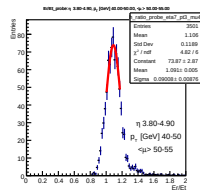
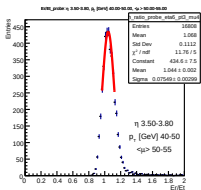
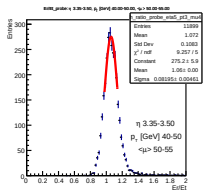
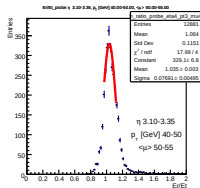
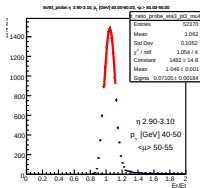
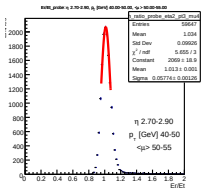
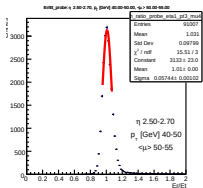
Plot p_t 3, μ 2



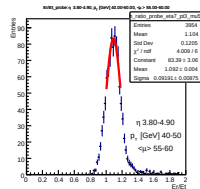
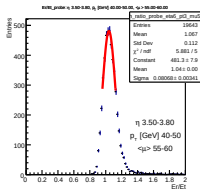
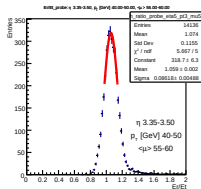
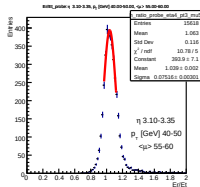
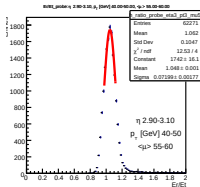
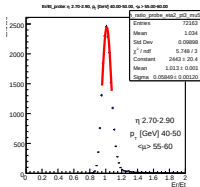
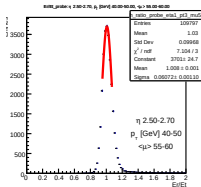
Plot p_t 3, μ 3



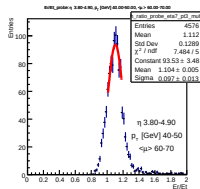
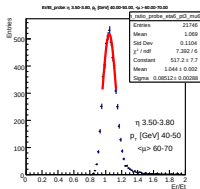
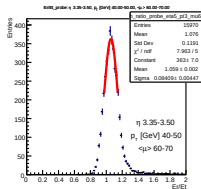
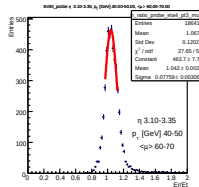
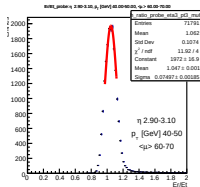
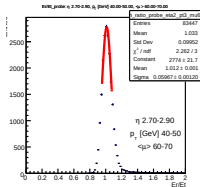
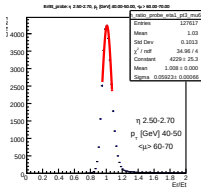
Plot p_t 3, μ 4



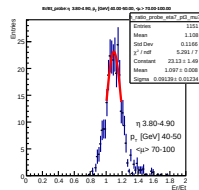
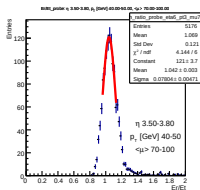
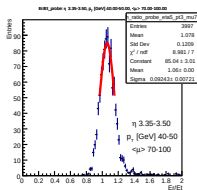
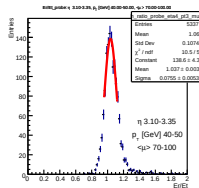
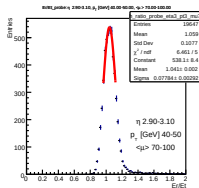
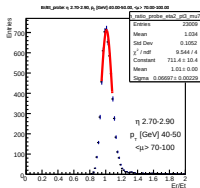
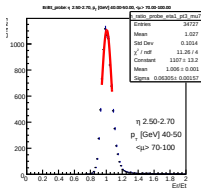
Plot p_t 3, μ 5



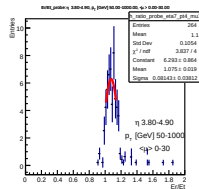
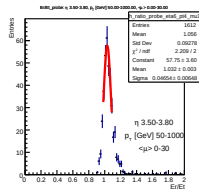
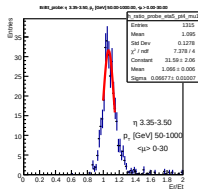
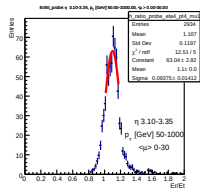
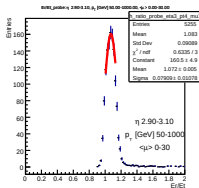
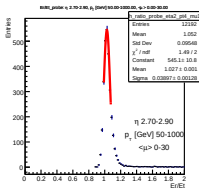
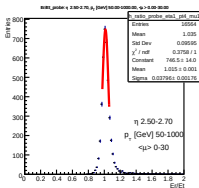
Plot p_t 3, μ 6



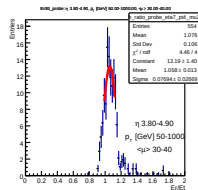
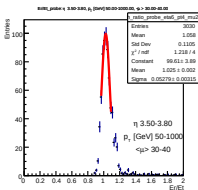
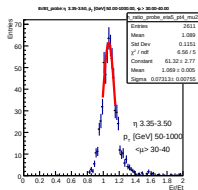
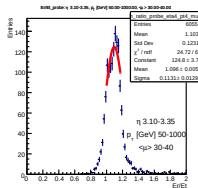
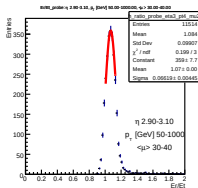
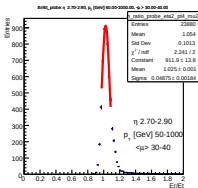
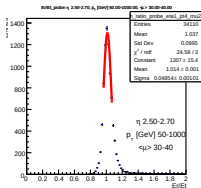
Plot p_t 3, μ 7



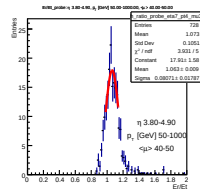
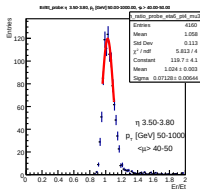
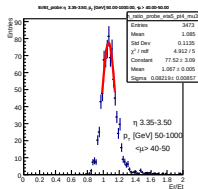
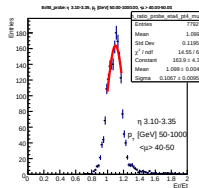
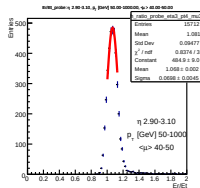
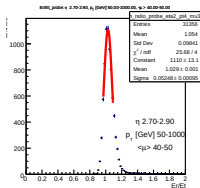
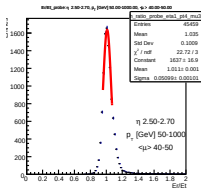
Plot p_t 4, μ 1



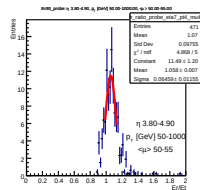
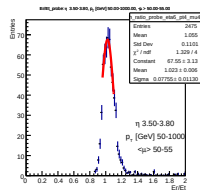
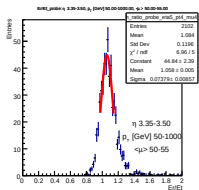
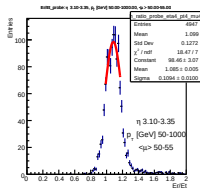
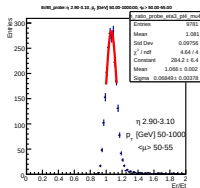
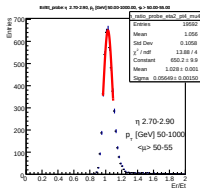
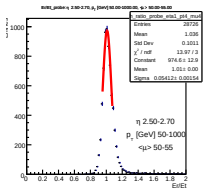
Plot p_t 4, μ 2



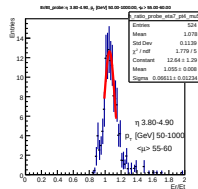
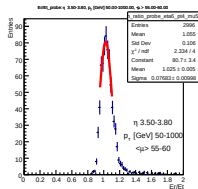
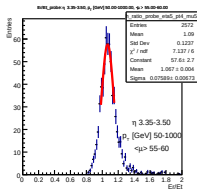
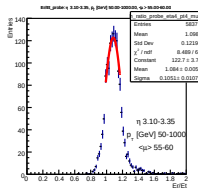
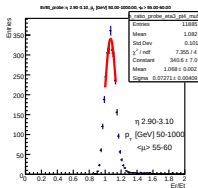
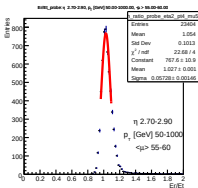
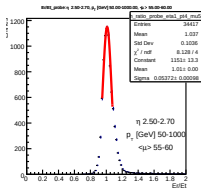
Plot p_t 4, μ 3



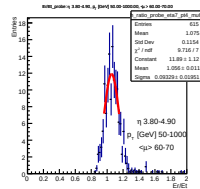
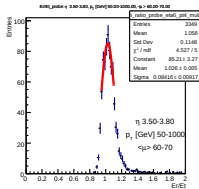
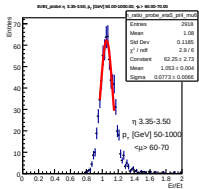
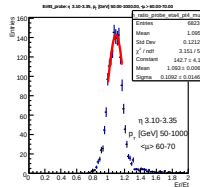
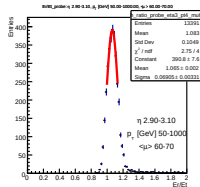
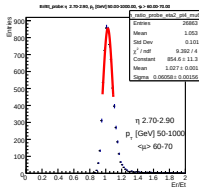
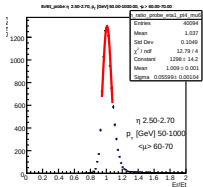
Plot p_t 4, μ 4



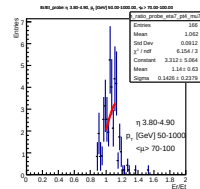
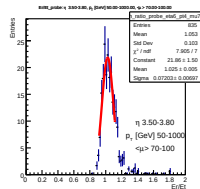
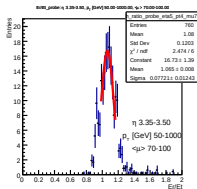
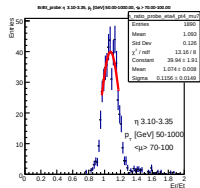
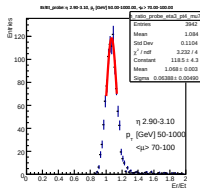
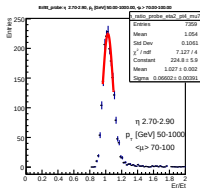
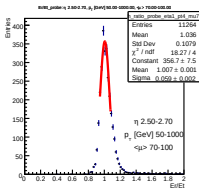
Plot p_t 4, μ 5



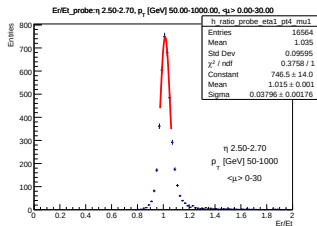
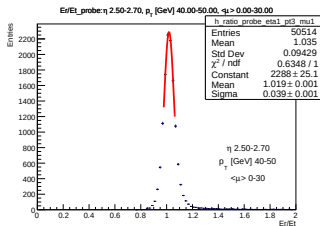
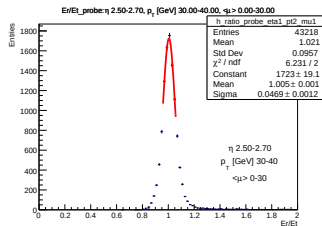
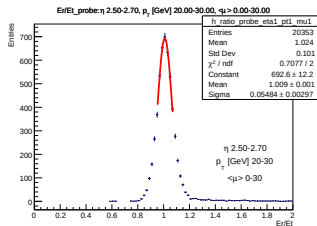
Plot p_t 4, μ 6



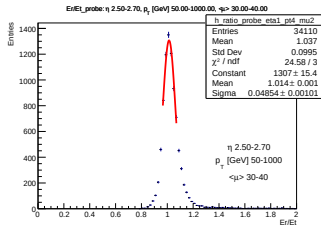
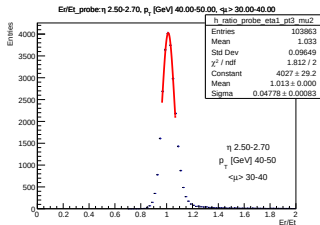
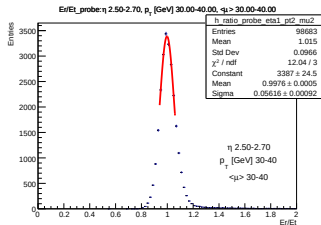
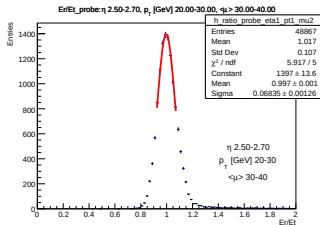
Plot p_t 4, μ 7



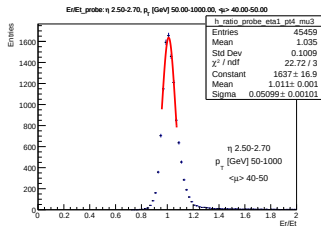
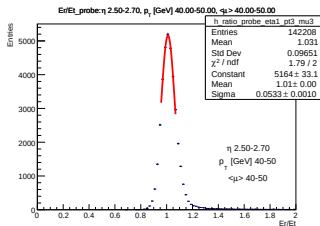
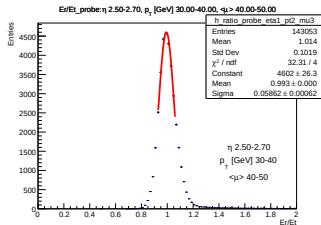
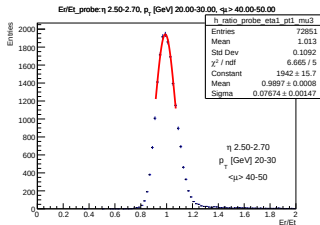
Plot η 1, μ 1



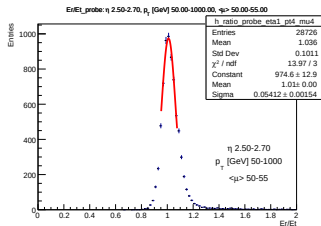
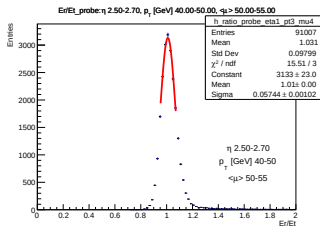
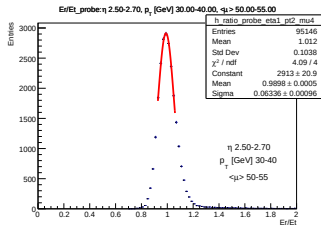
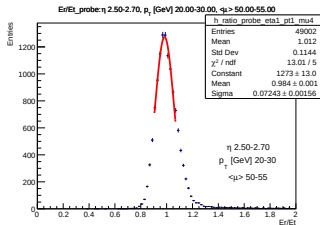
Plot η 1, μ 2



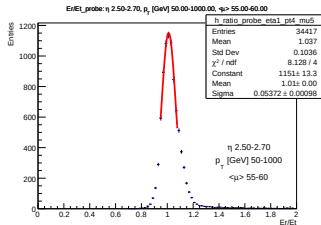
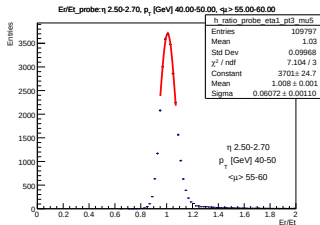
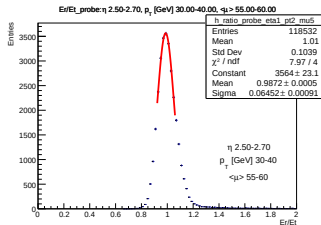
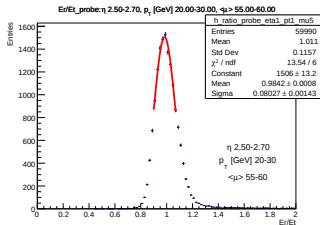
Plot η 1, μ 3



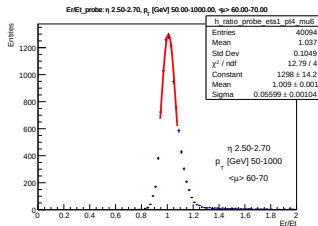
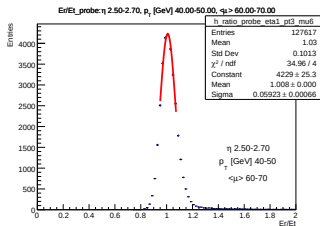
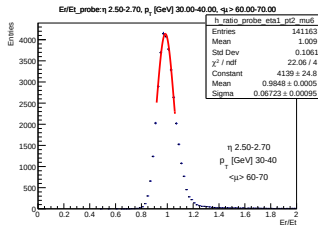
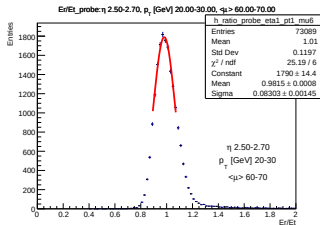
Plot η 1, μ 4



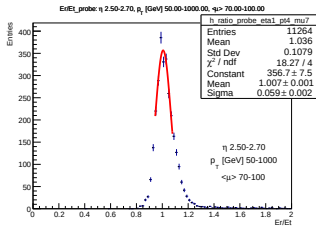
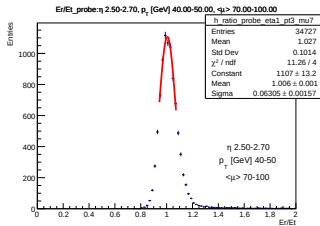
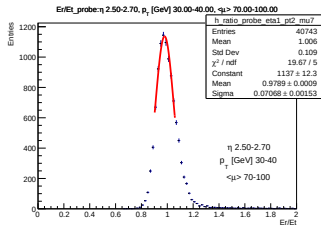
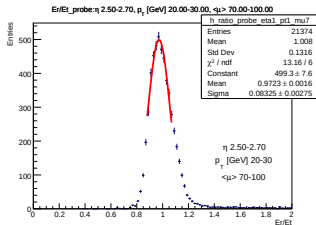
Plot η 1, μ 5



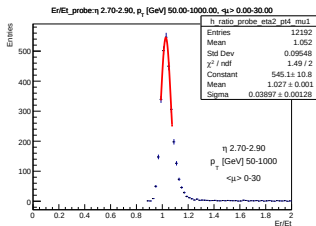
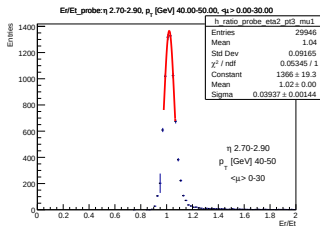
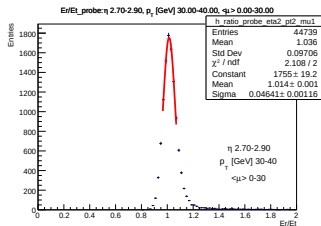
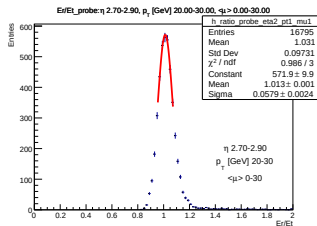
Plot η 1, μ 6



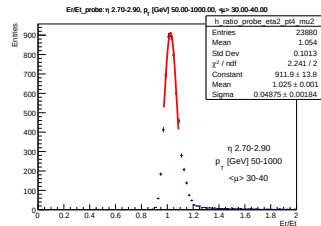
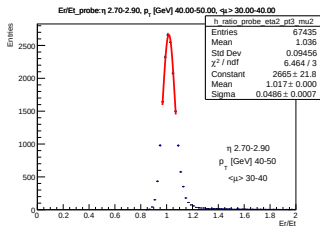
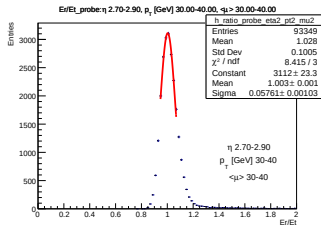
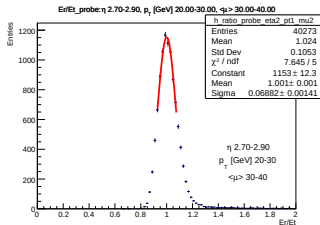
Plot η 1, μ 7



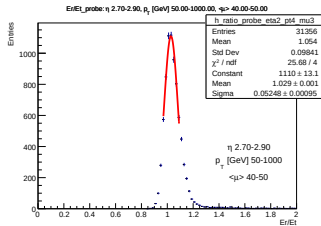
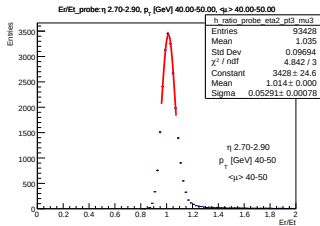
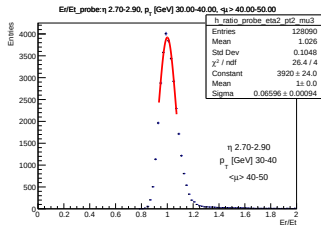
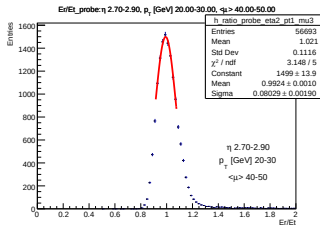
Plot η 2, μ 1



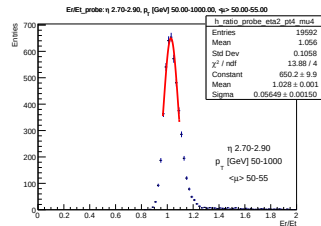
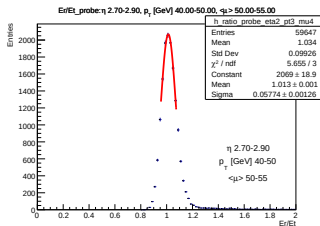
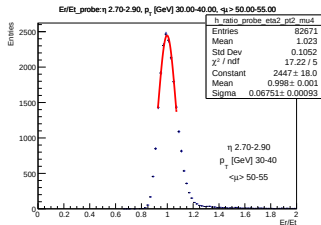
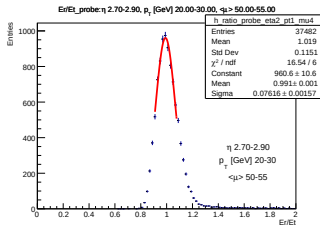
Plot η 2, μ 2



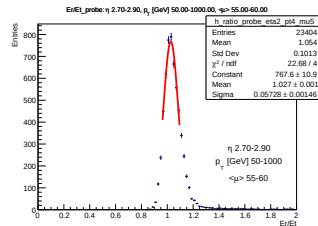
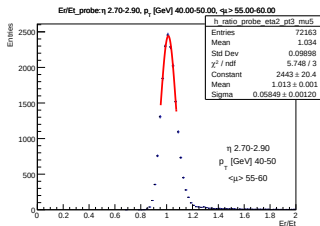
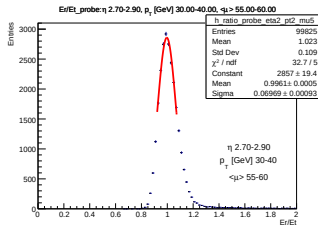
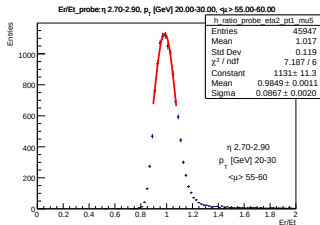
Plot η 2, μ 3



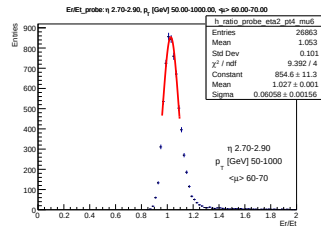
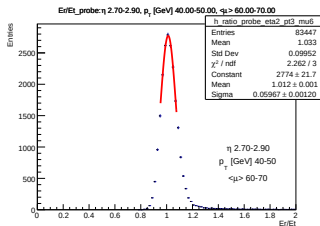
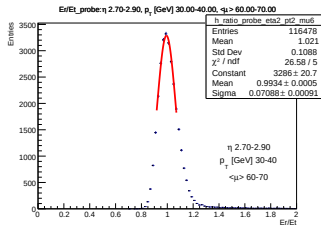
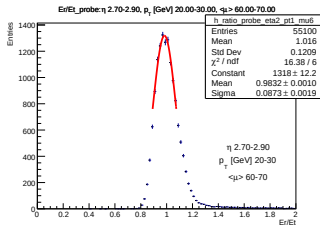
Plot η 2, μ 4



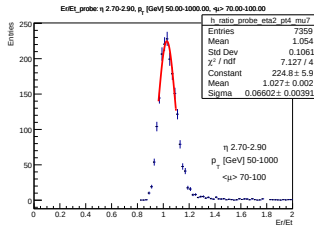
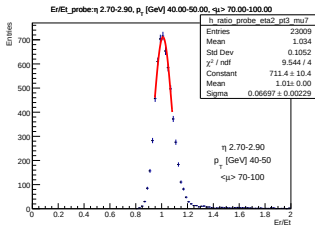
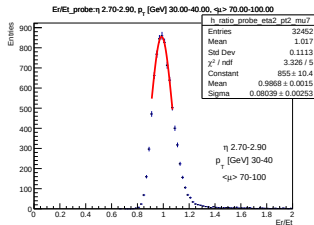
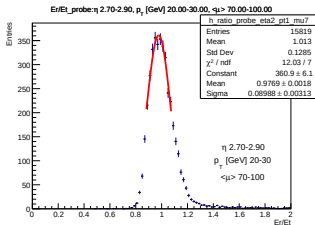
Plot η 2, μ 5



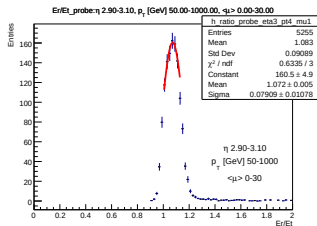
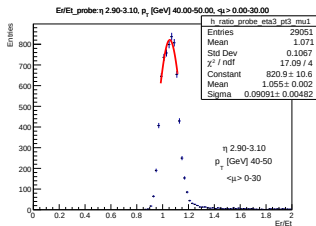
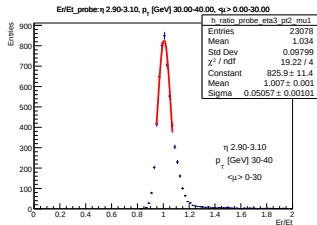
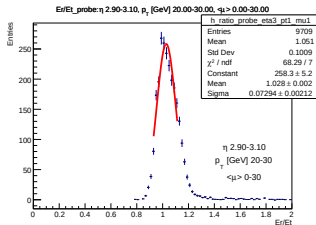
Plot η 2, μ 6



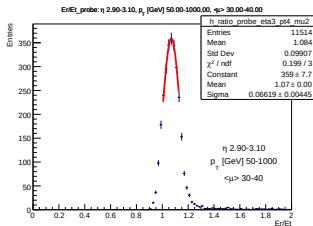
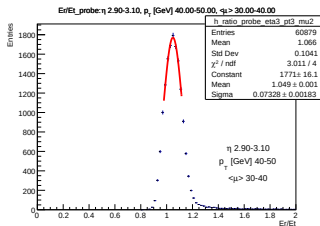
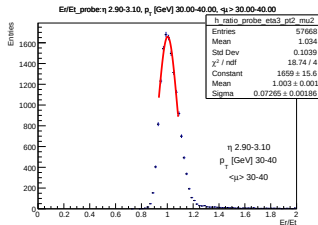
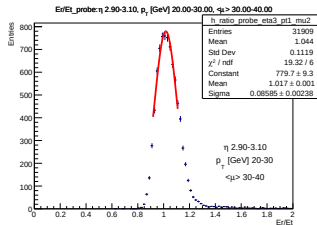
Plot η 2, μ 7



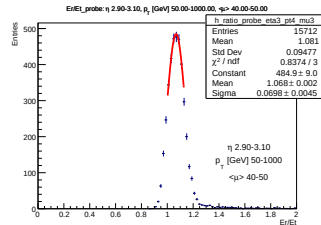
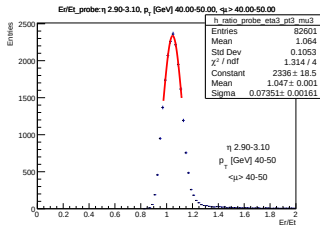
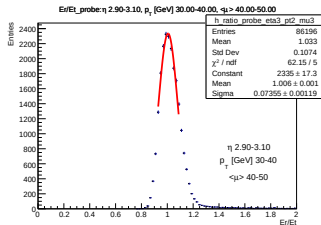
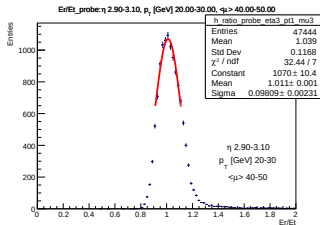
Plot η 3, μ 1



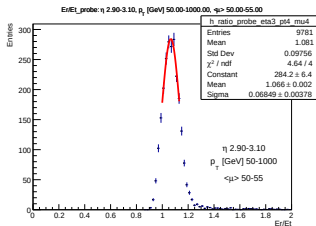
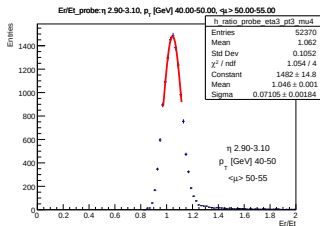
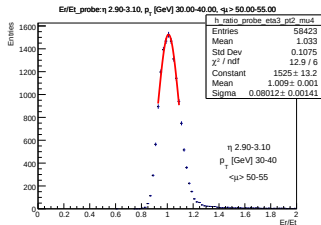
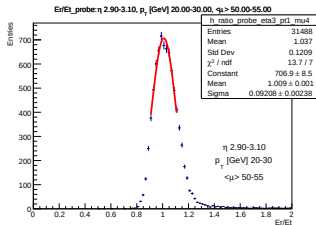
Plot η 3, μ 2



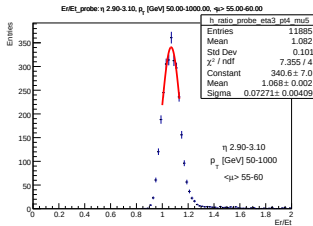
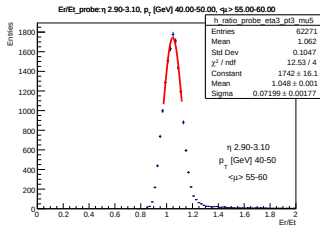
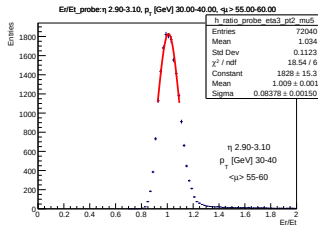
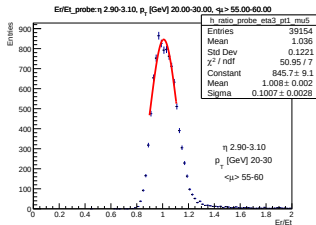
Plot η 3, μ 3



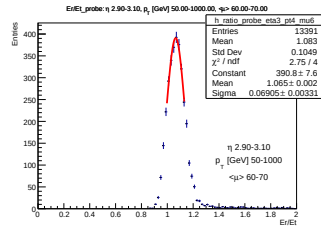
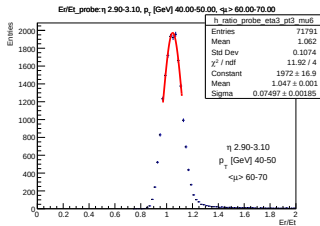
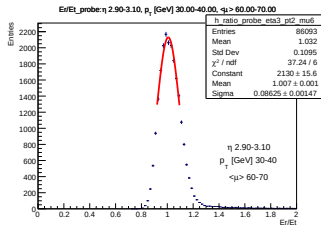
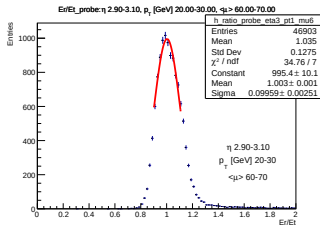
Plot η 3, μ 4



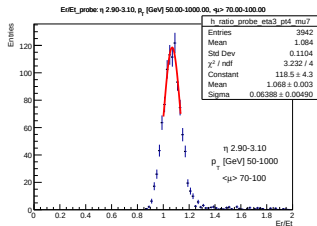
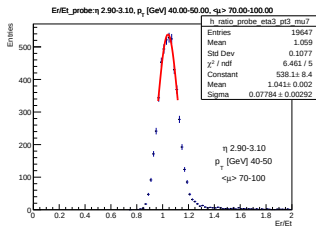
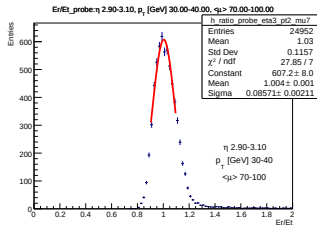
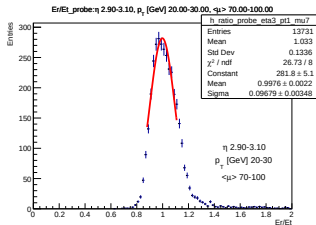
Plot η 3, μ 5



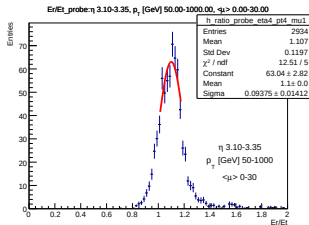
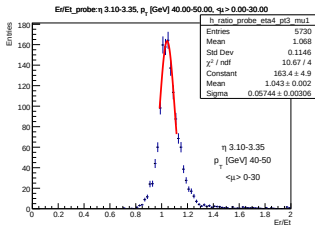
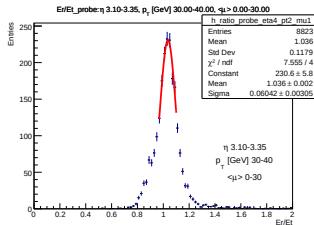
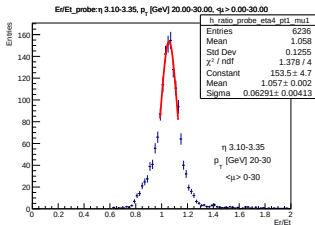
Plot η 3, μ 6



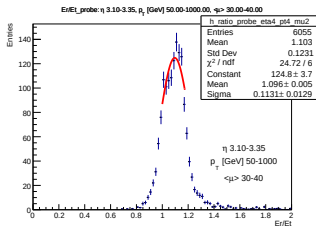
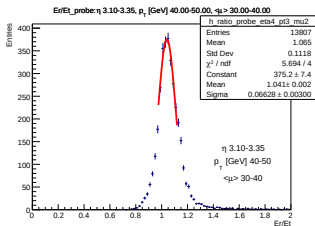
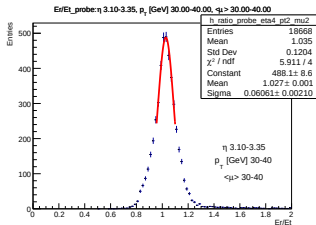
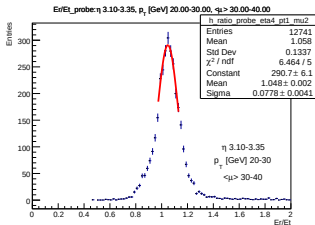
Plot η 3, μ 7



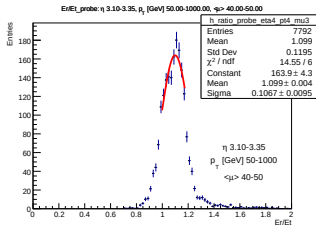
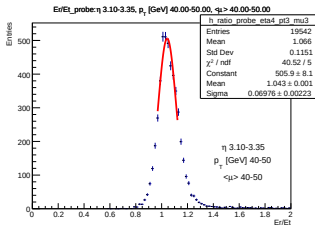
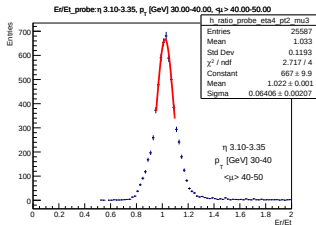
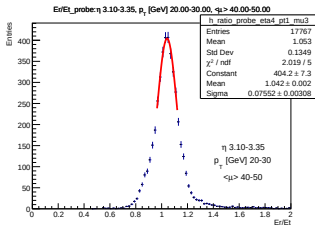
Plot η 4, μ 1



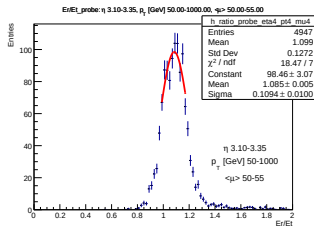
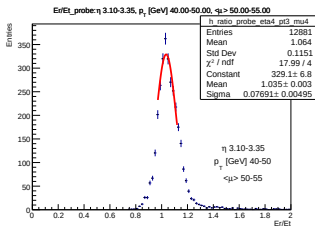
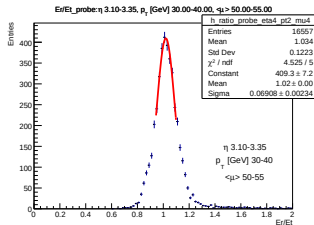
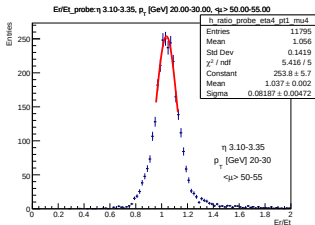
Plot η 4, μ 2



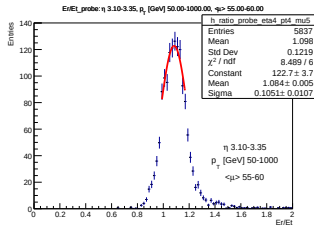
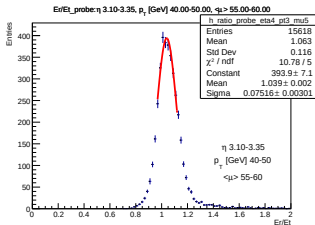
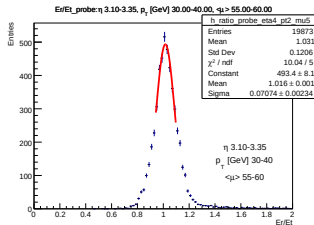
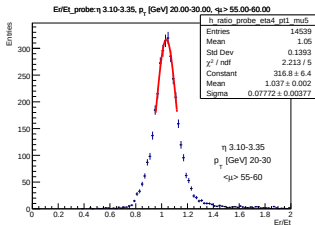
Plot η 4, μ 3



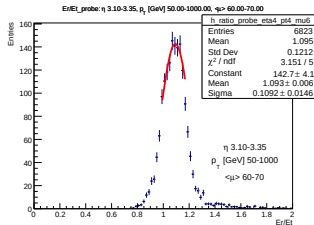
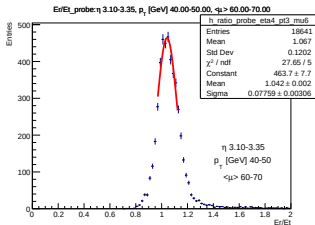
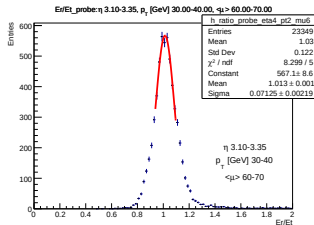
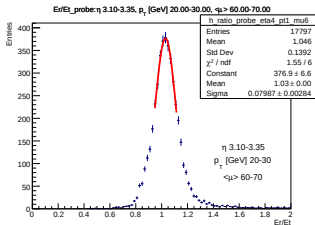
Plot η 4, μ 4



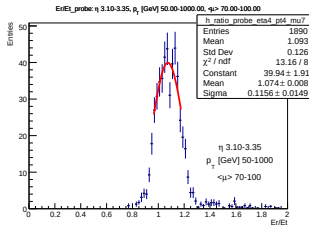
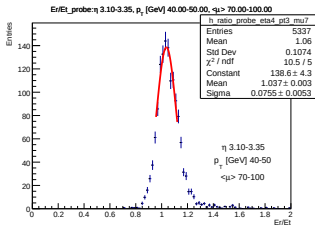
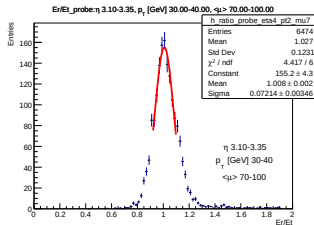
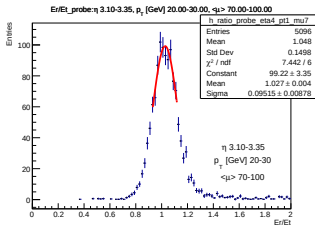
Plot η 4, μ 5



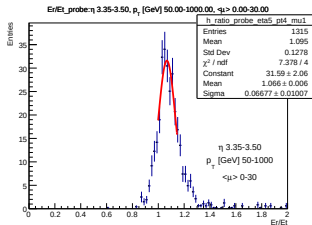
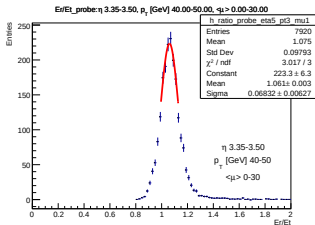
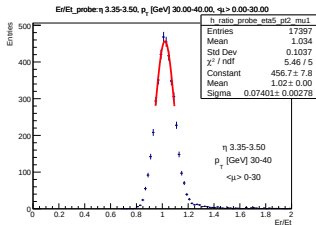
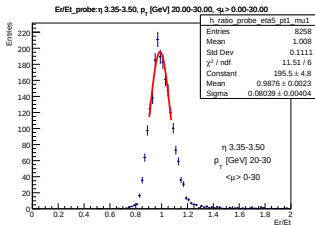
Plot η 4, μ 6



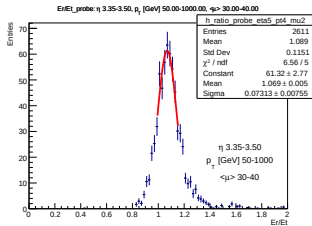
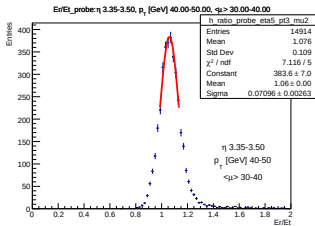
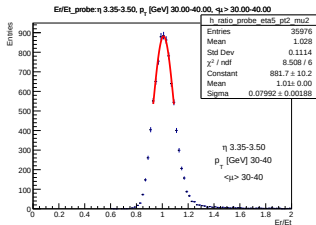
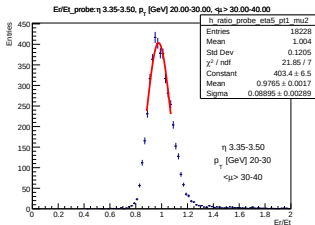
Plot η 4, μ 7



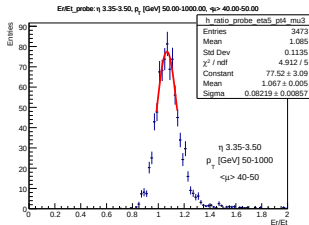
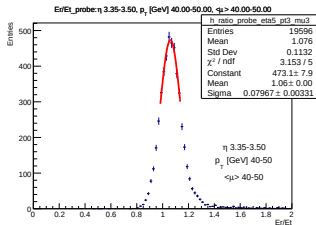
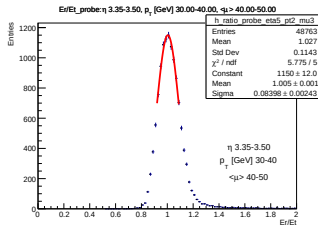
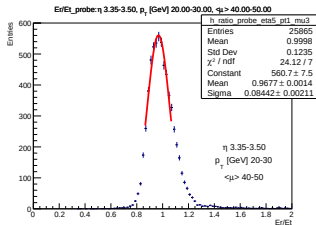
Plot η 5, μ 1



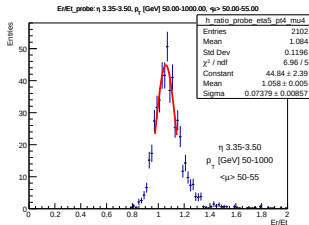
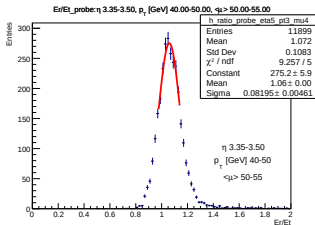
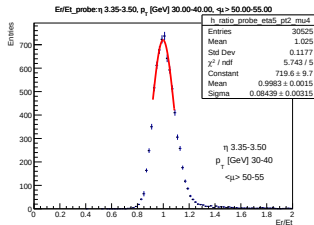
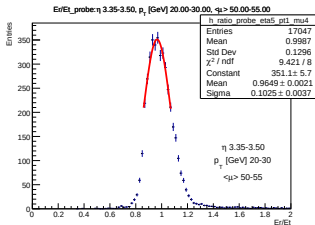
Plot η 5, μ 2



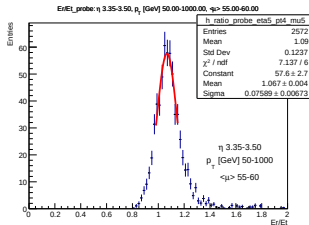
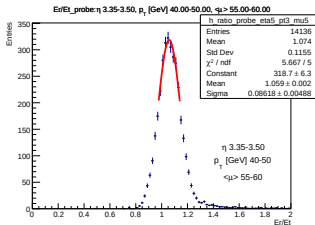
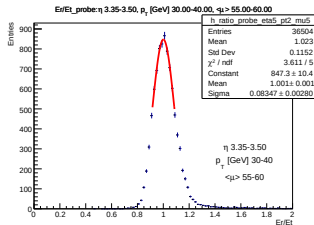
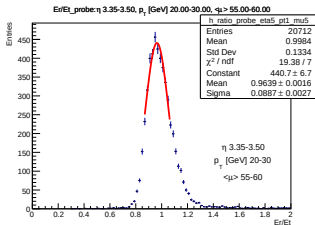
Plot η 5, μ 3



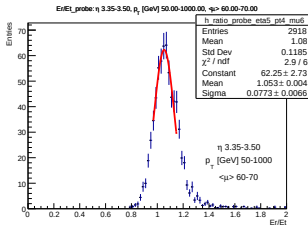
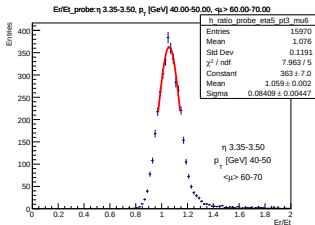
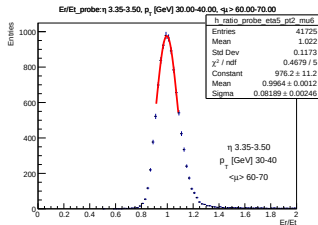
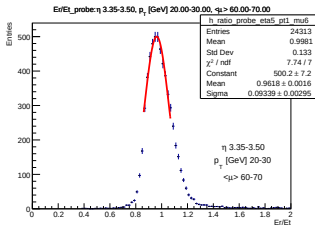
Plot η 5, μ 4



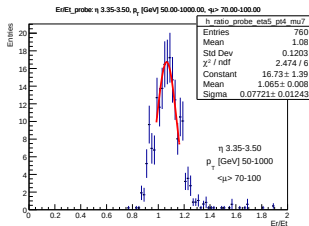
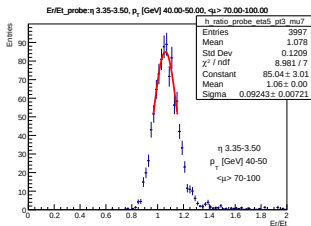
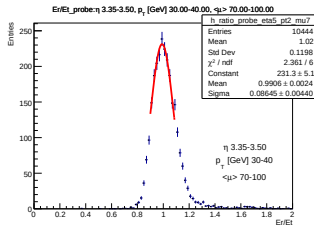
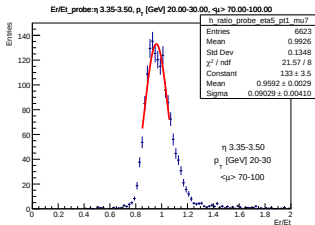
Plot η 5, μ 5



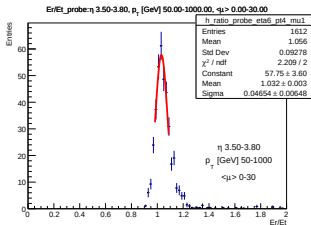
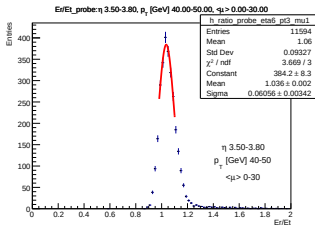
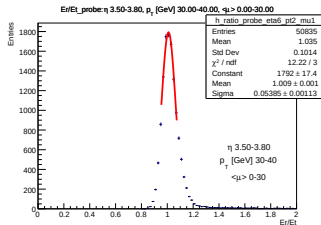
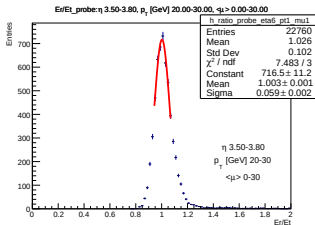
Plot η 5, μ 6



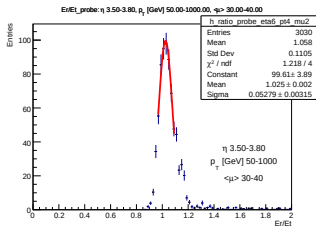
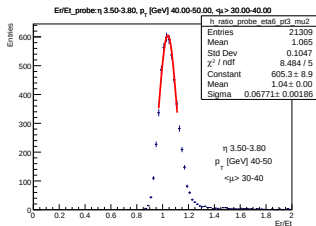
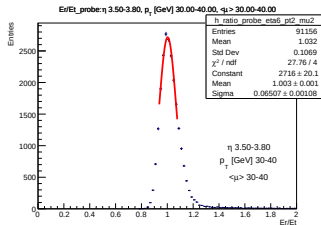
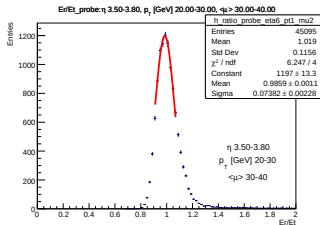
Plot η 5, μ 7



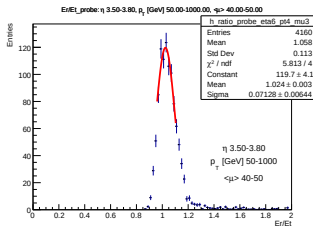
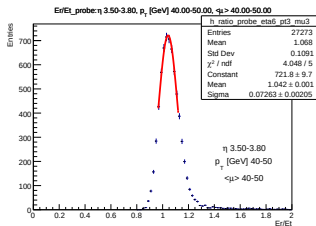
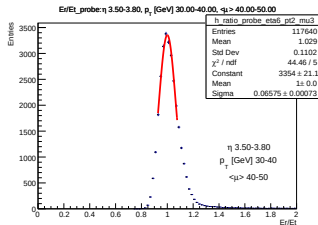
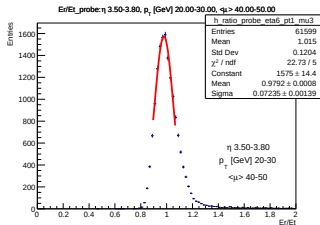
Plot η 6, μ 1



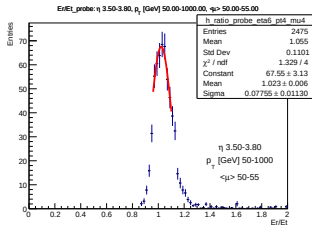
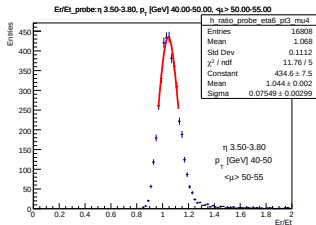
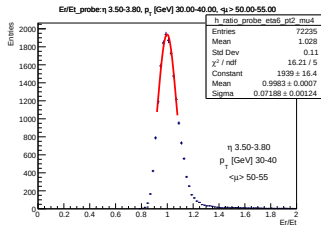
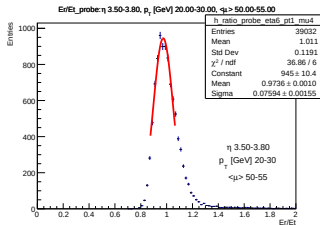
Plot η 6, μ 2



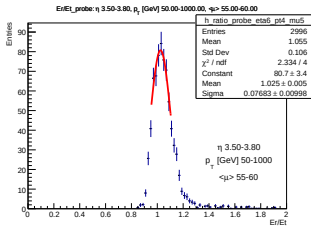
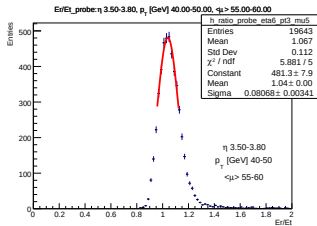
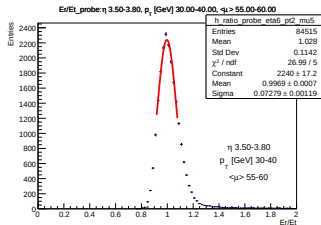
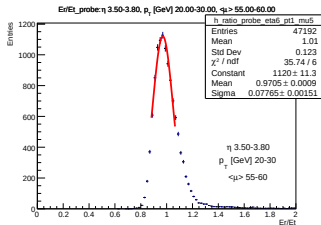
Plot η 6, μ 3



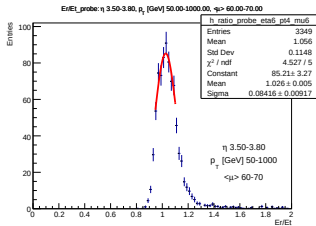
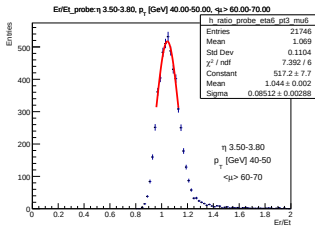
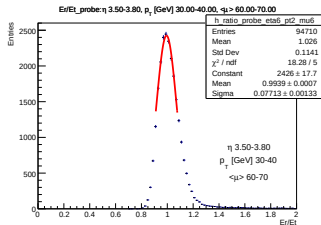
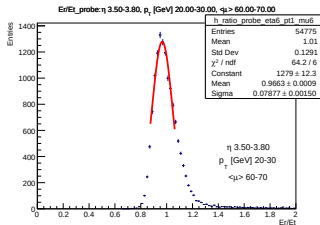
Plot η 6, μ 4



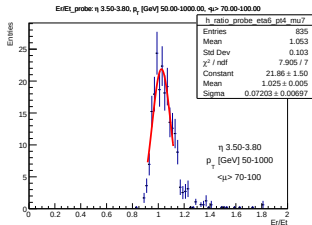
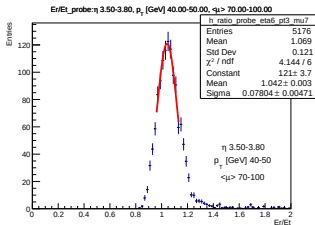
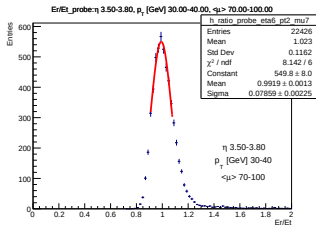
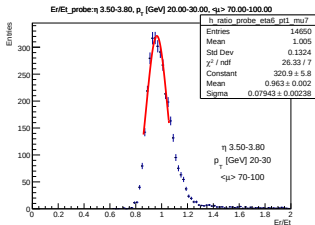
Plot η 6, μ 5



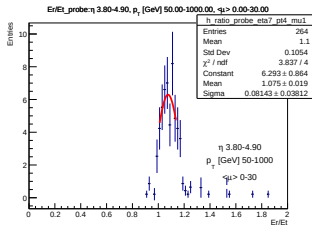
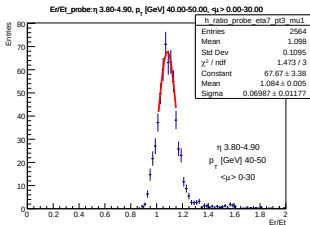
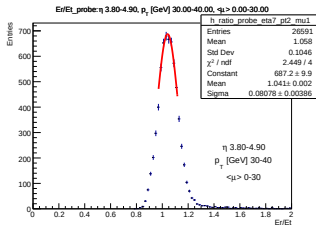
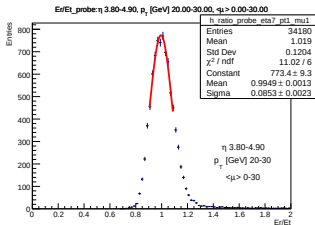
Plot η 6, μ 6



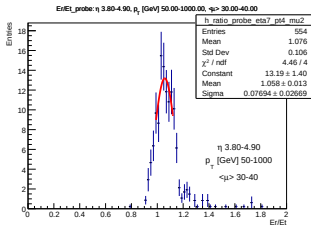
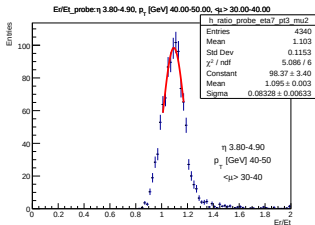
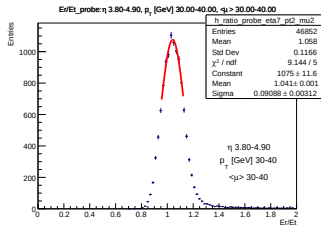
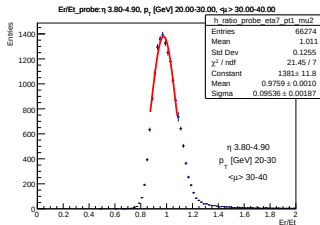
Plot η 6, μ 7



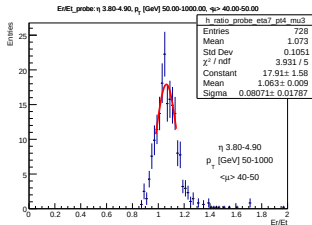
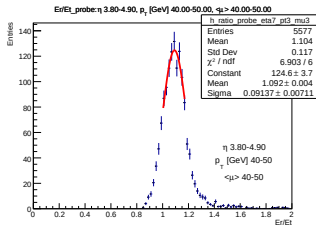
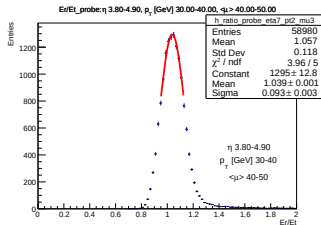
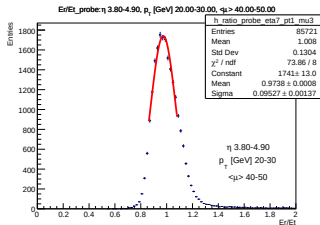
Plot η 7, μ 1



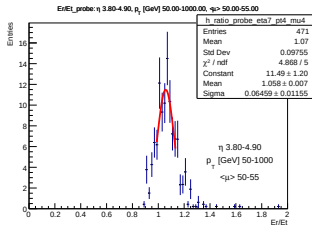
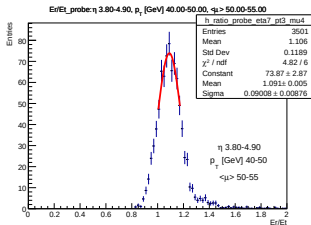
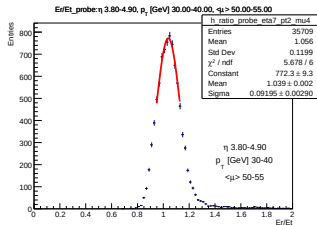
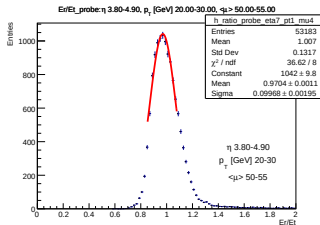
Plot η 7, μ 2



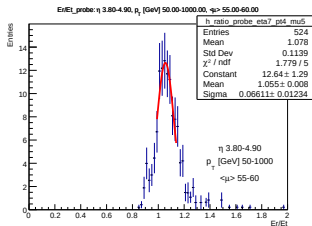
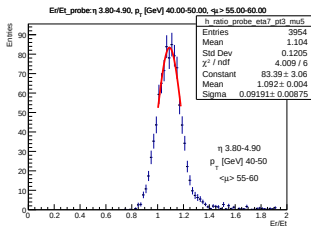
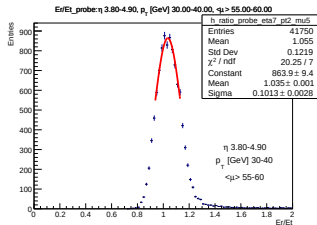
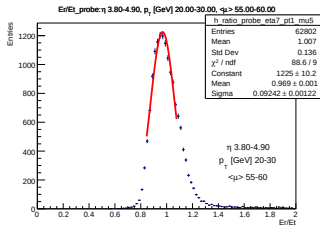
Plot η 7, μ 3



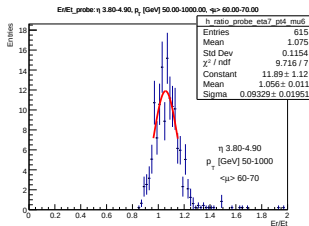
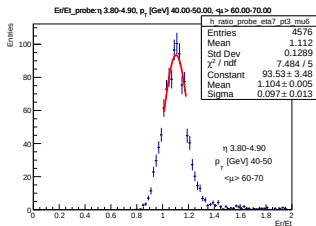
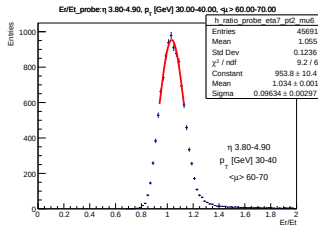
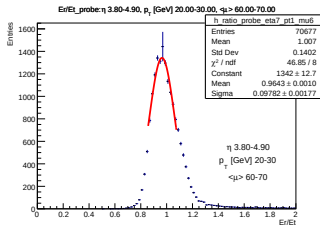
Plot η 7, μ 4



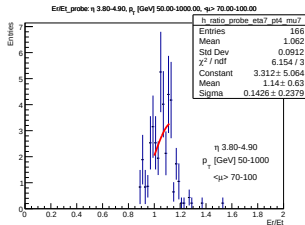
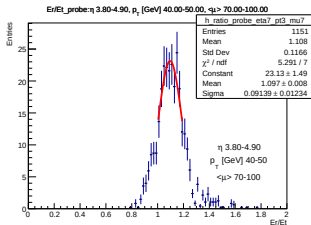
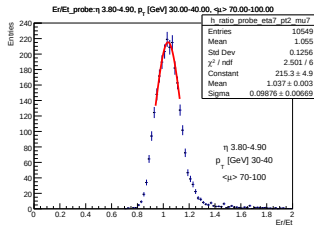
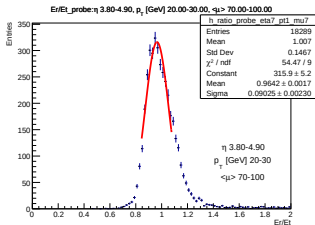
Plot η 7, μ 5



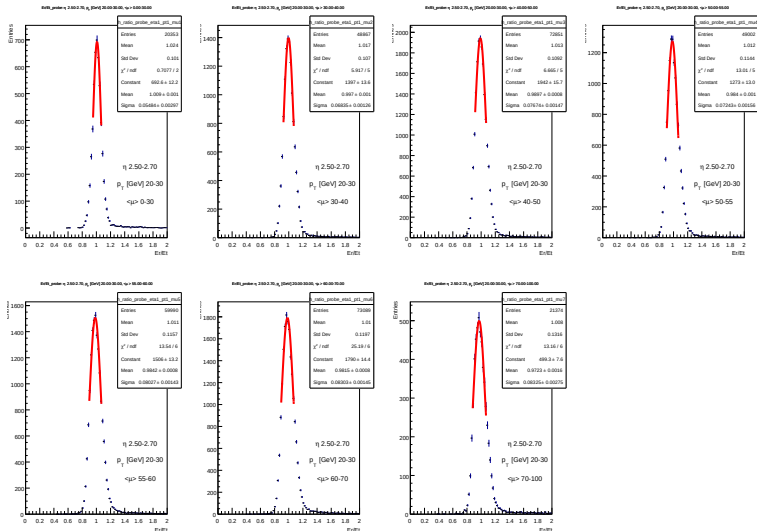
Plot η 7, μ 6



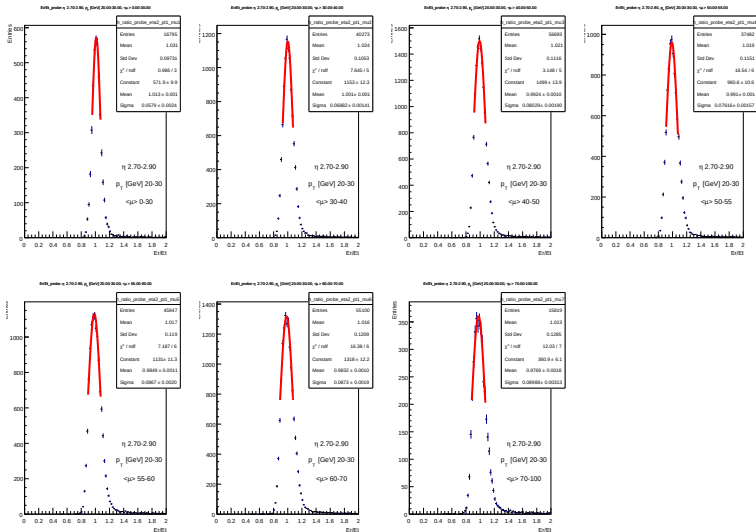
Plot η 7, μ 7



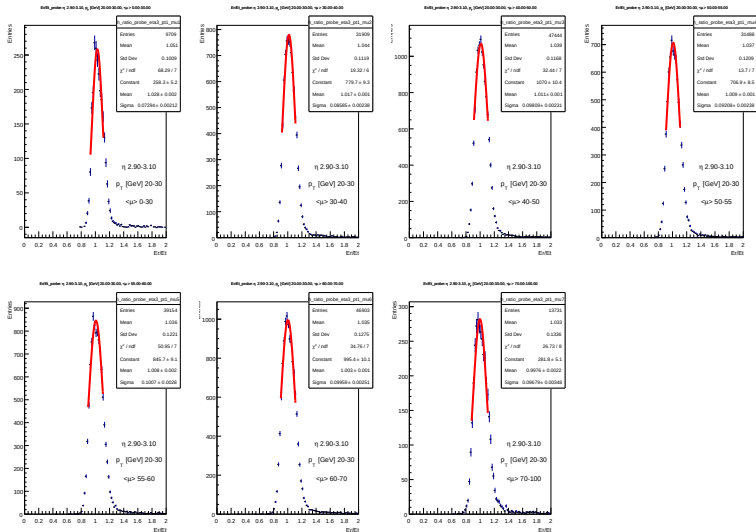
Plot p_t 1, η 1



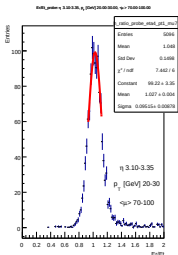
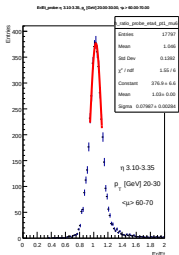
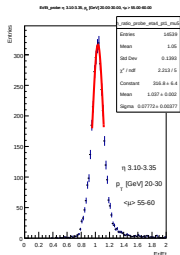
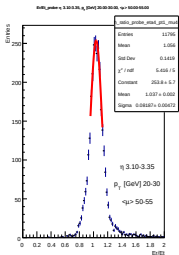
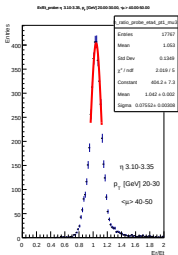
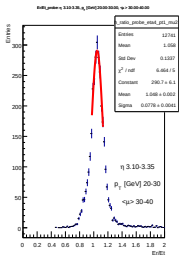
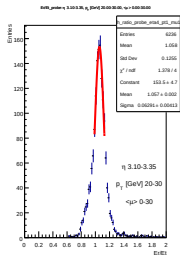
Plot p_t 1, η 2



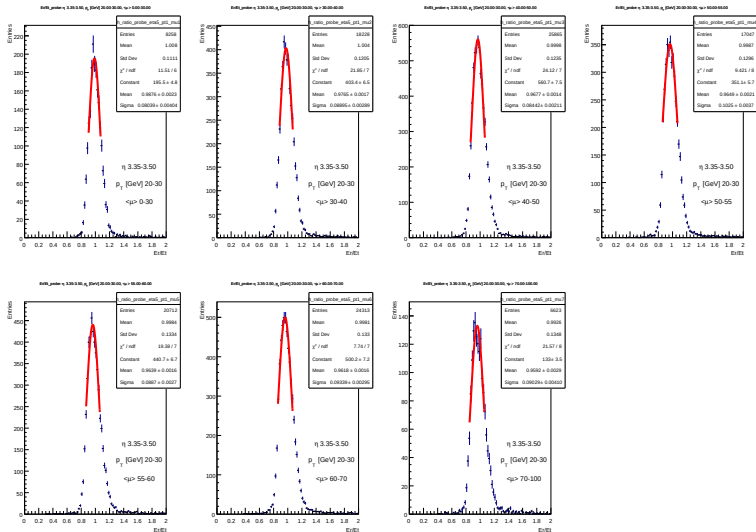
Plot p_t 1, η 3



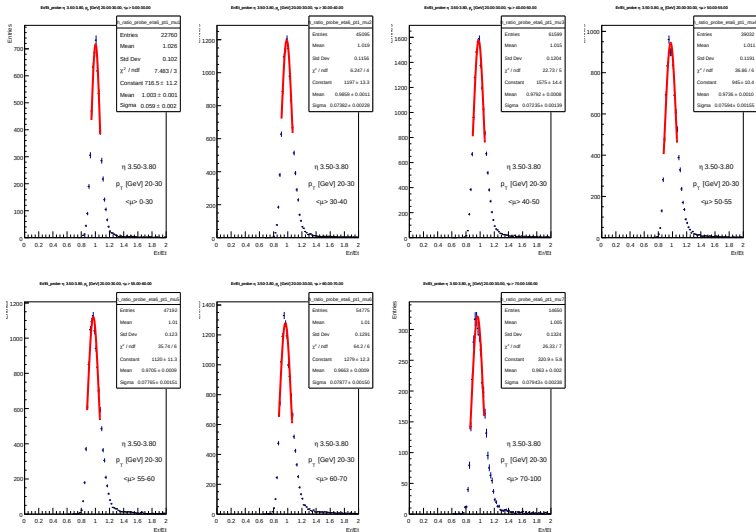
Plot p_t 1, η 4



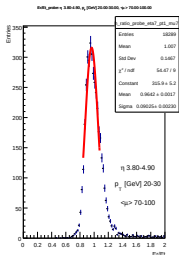
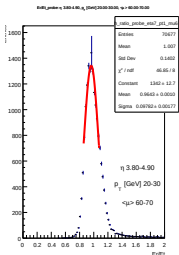
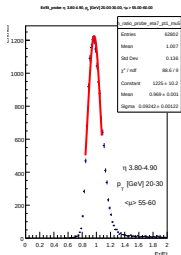
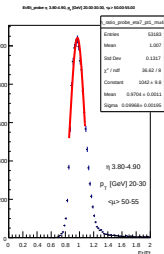
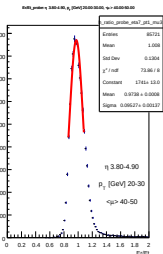
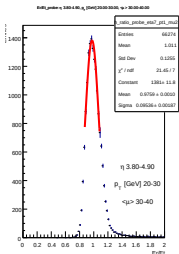
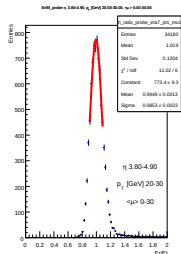
Plot p_t 1, η 5



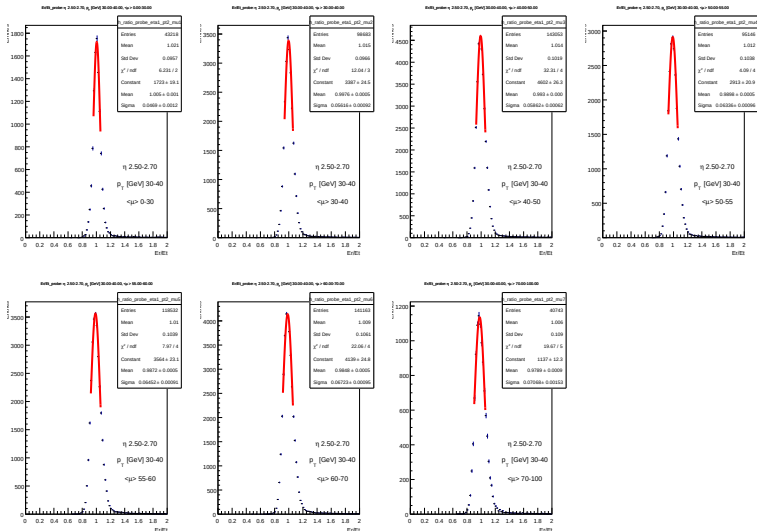
Plot p_t 1, η 6



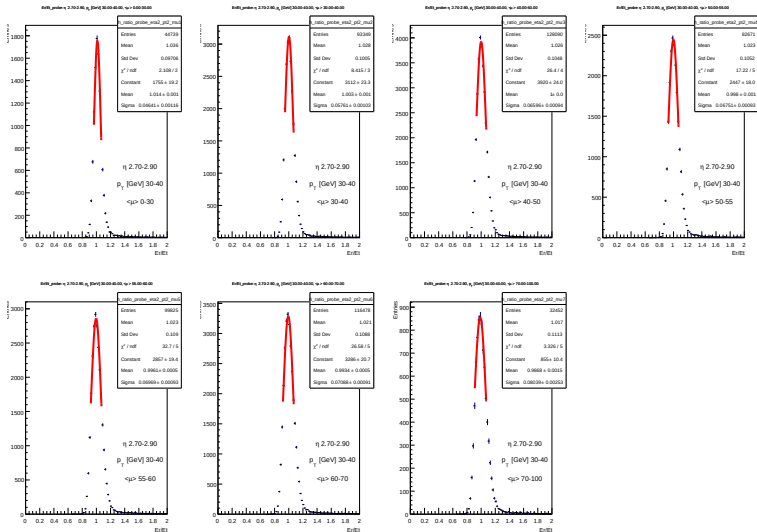
Plot p_t 1, η 7



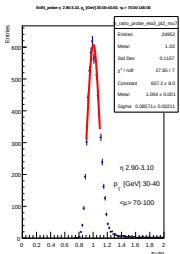
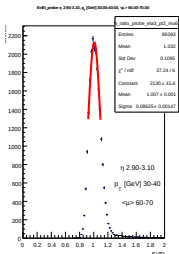
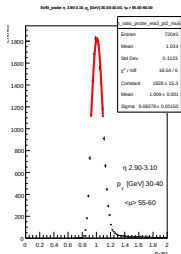
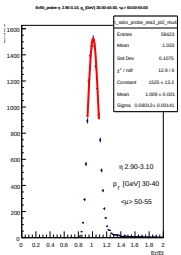
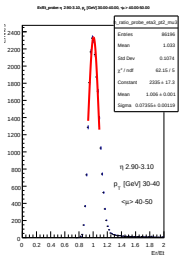
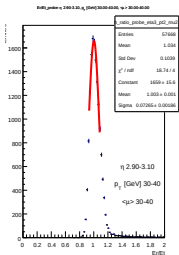
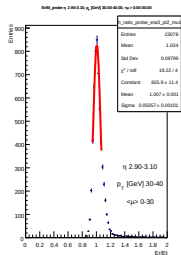
Plot p_t 2, η 1



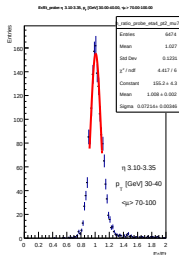
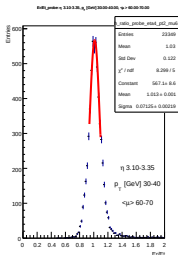
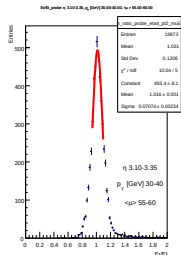
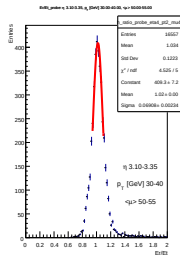
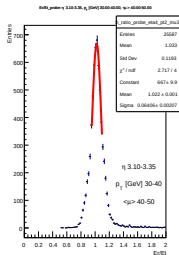
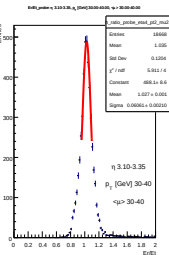
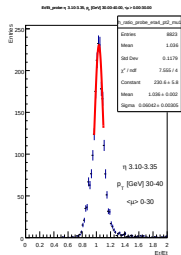
Plot p_t 2, η 2



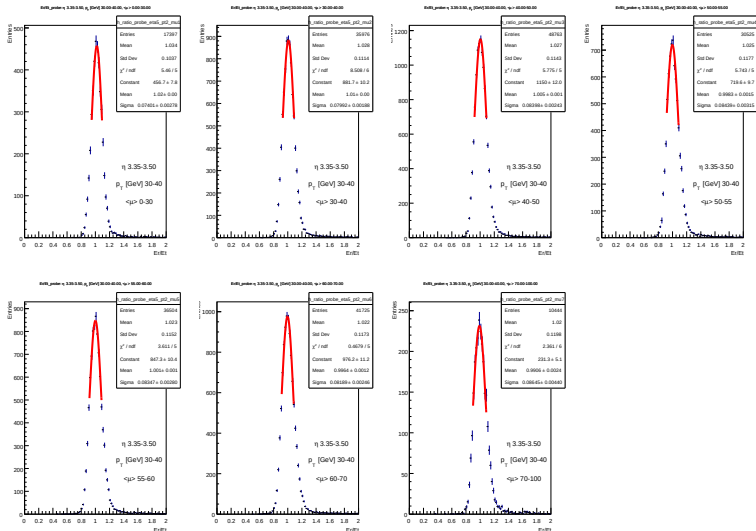
Plot p_t 2, η 3



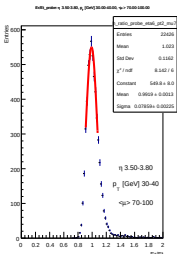
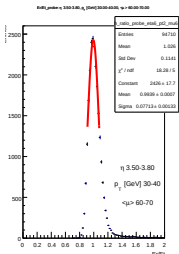
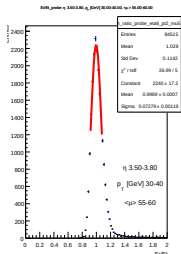
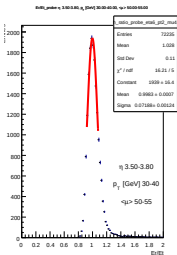
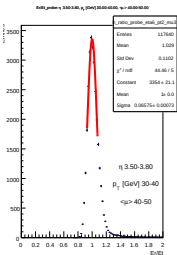
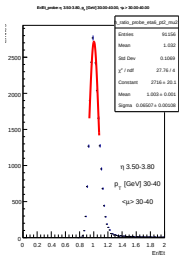
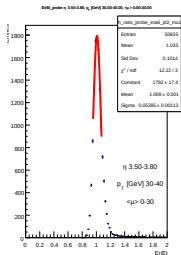
Plot p_t 2, η 4



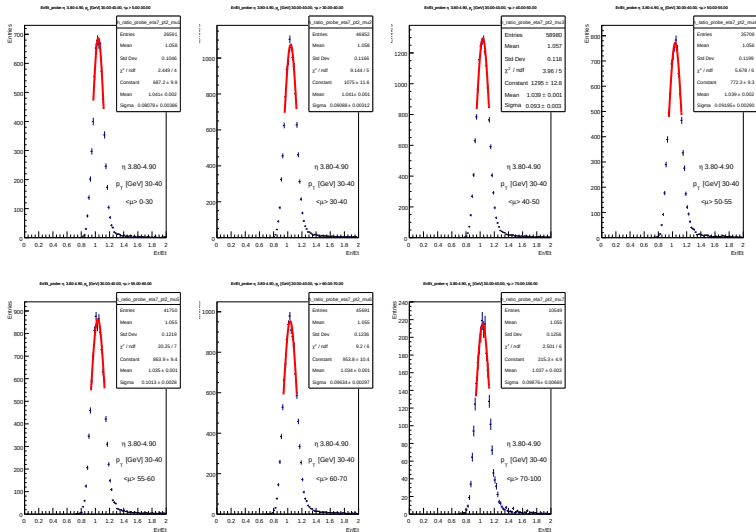
Plot p_t 2, η 5



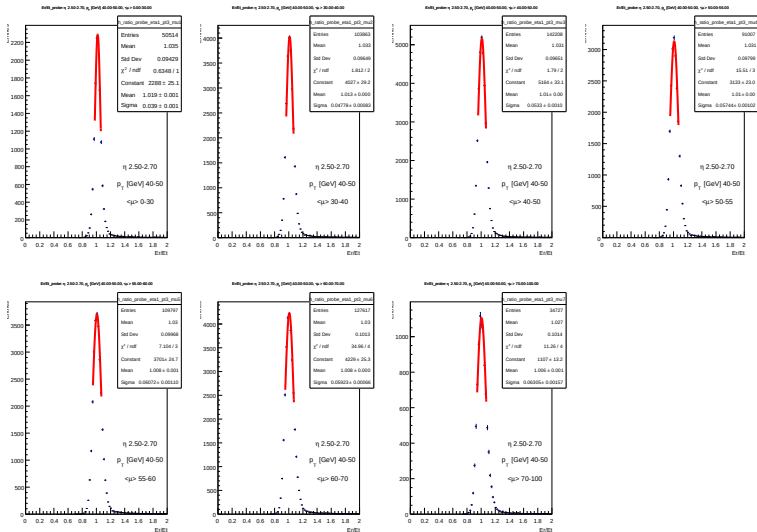
Plot p_t 2, η 6



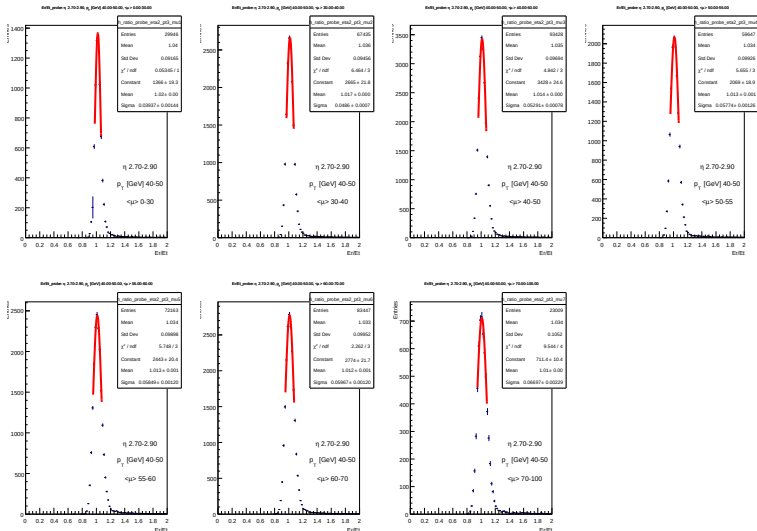
Plot p_t 2, η 7



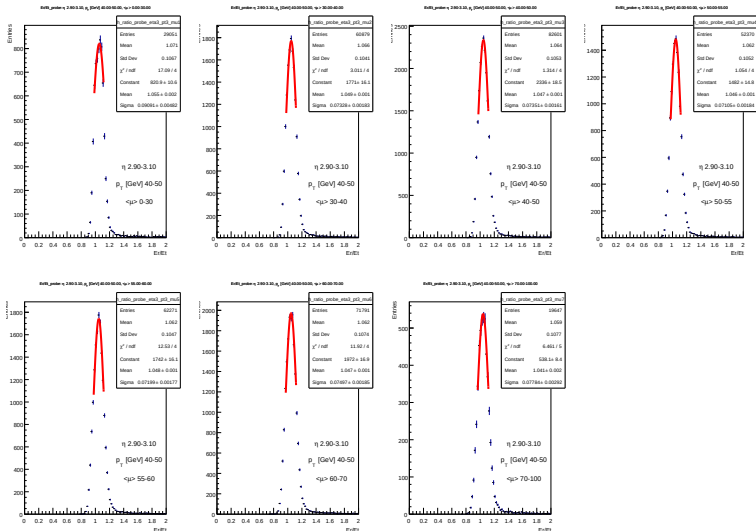
Plot p_t 3, η 1



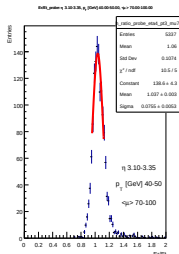
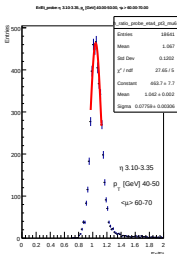
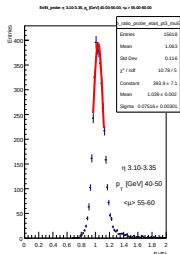
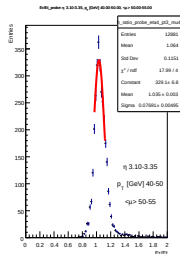
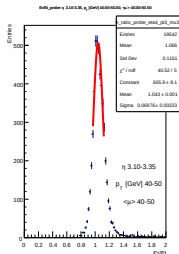
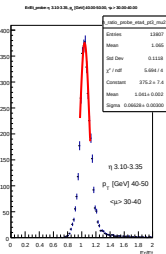
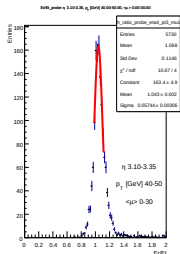
Plot p_t 3, η 2



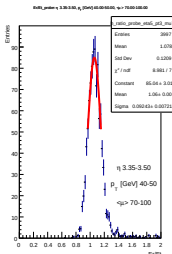
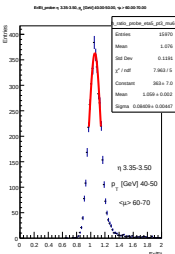
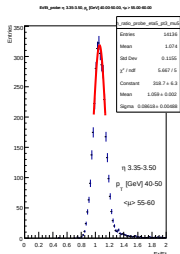
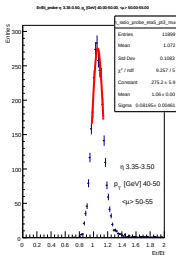
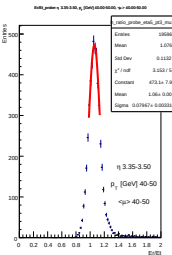
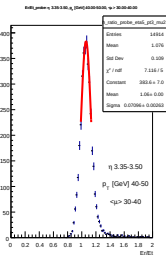
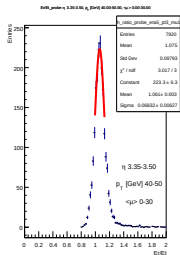
Plot p_t 3, η 3



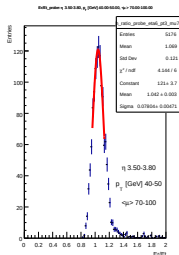
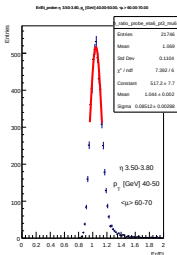
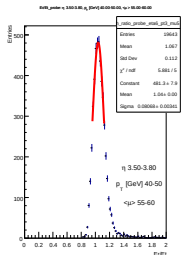
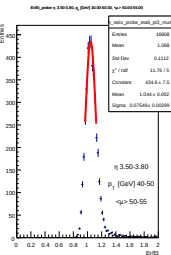
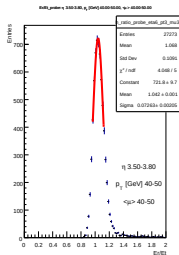
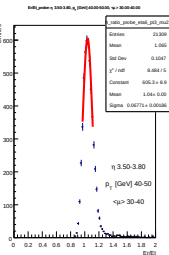
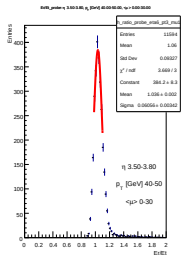
Plot p_t 3, η 4



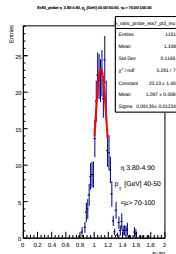
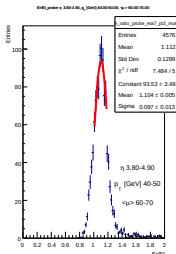
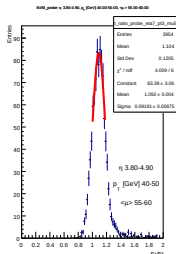
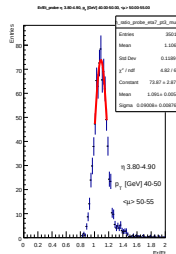
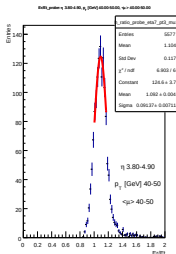
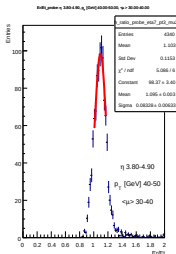
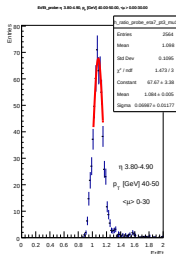
Plot p_t 3, η 5



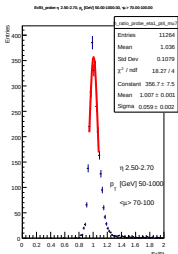
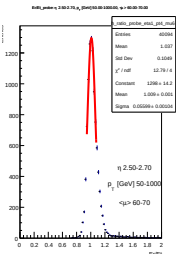
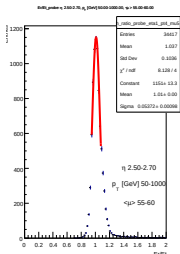
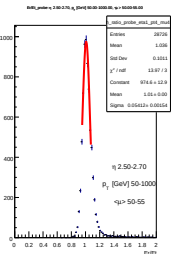
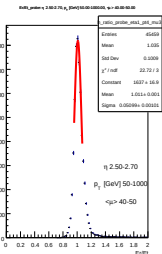
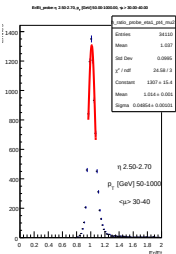
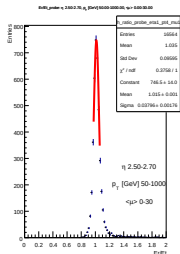
Plot p_t 3, η 6



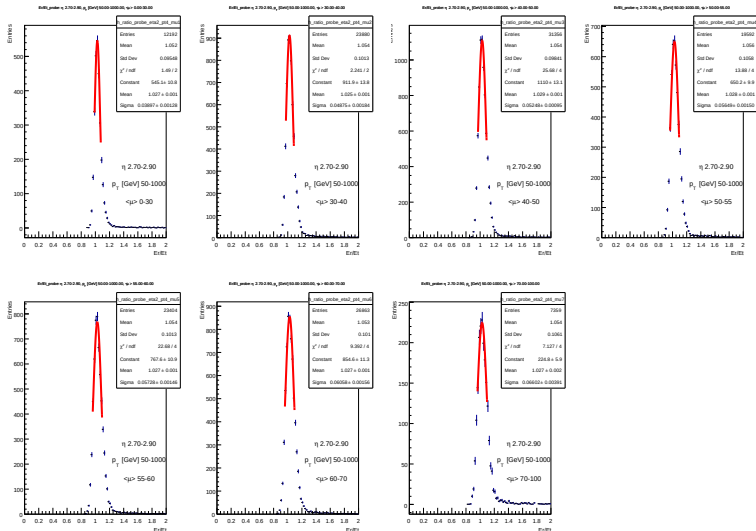
Plot p_t 3, η 7



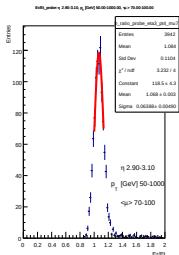
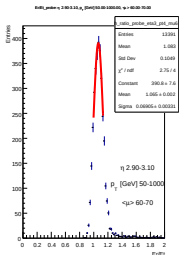
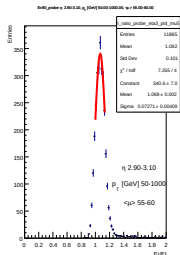
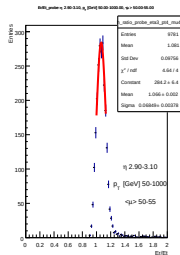
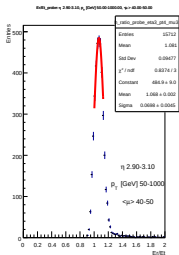
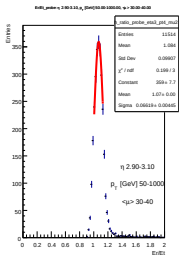
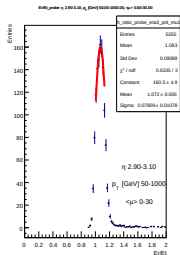
Plot p_t 4, η 1



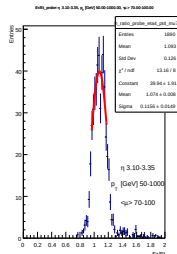
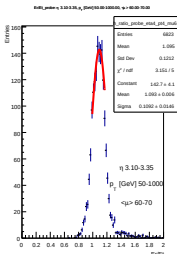
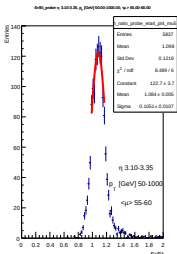
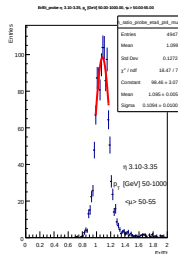
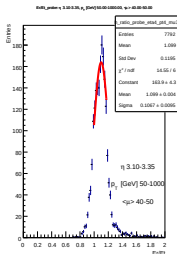
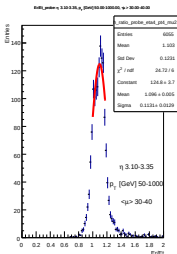
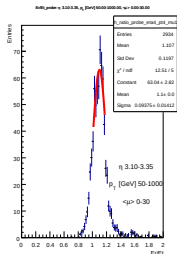
Plot p_t 4, η 2



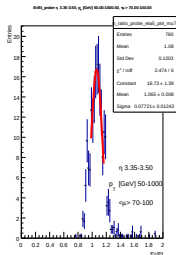
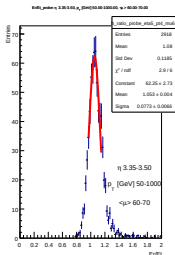
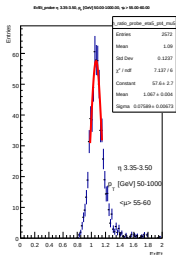
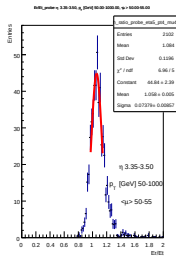
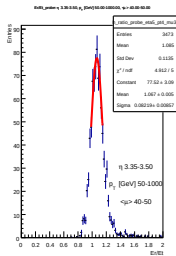
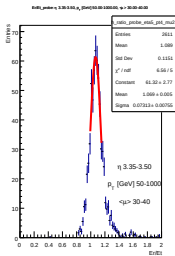
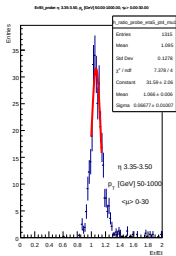
Plot p_t 4, η 3



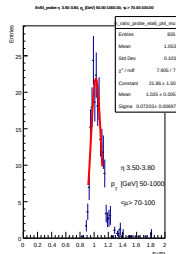
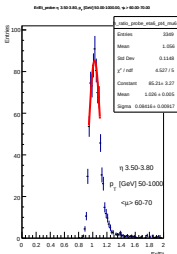
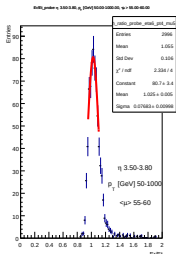
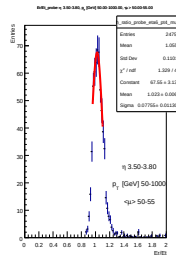
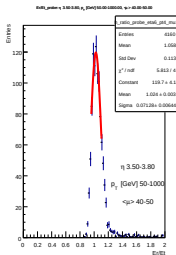
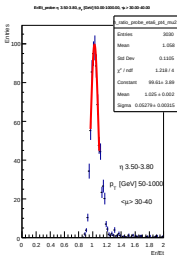
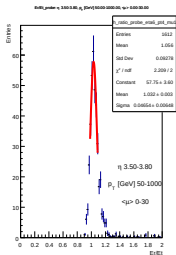
Plot p_t 4, η 4



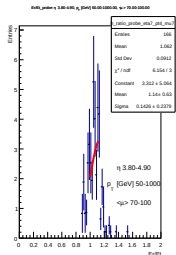
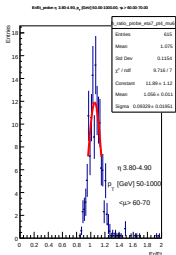
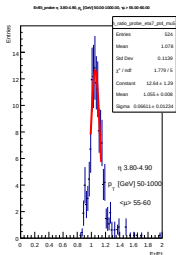
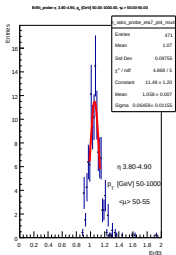
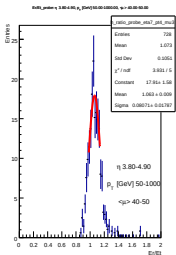
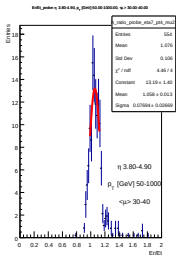
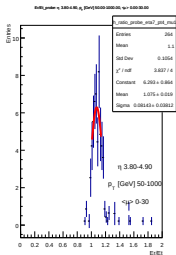
Plot p_t 4, η 5



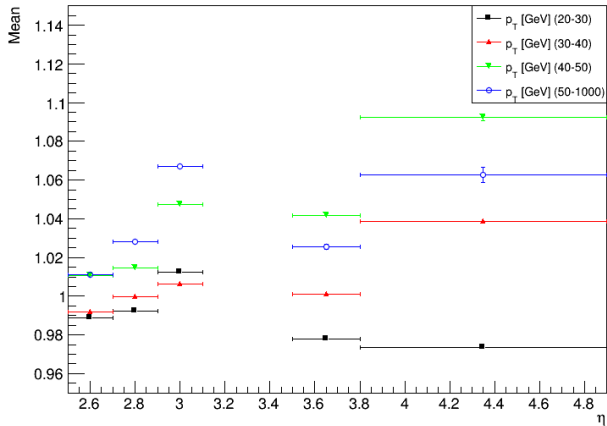
Plot p_t 4, η 6



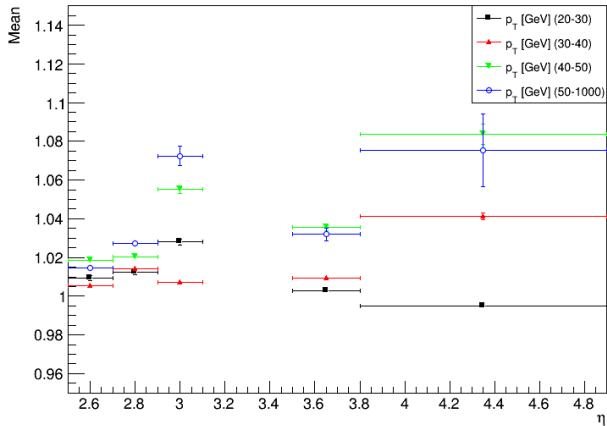
Plot p_t 4, η 7



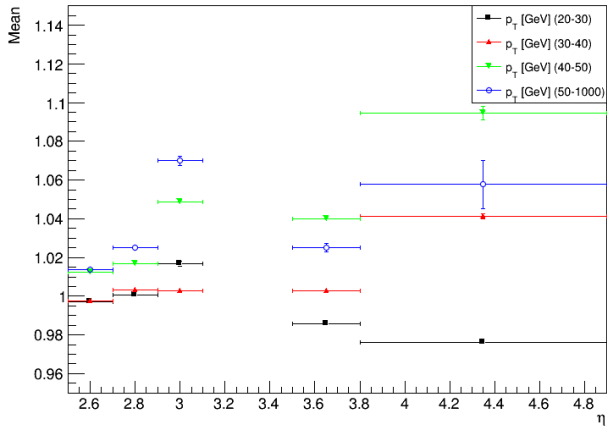
Mean μ_0



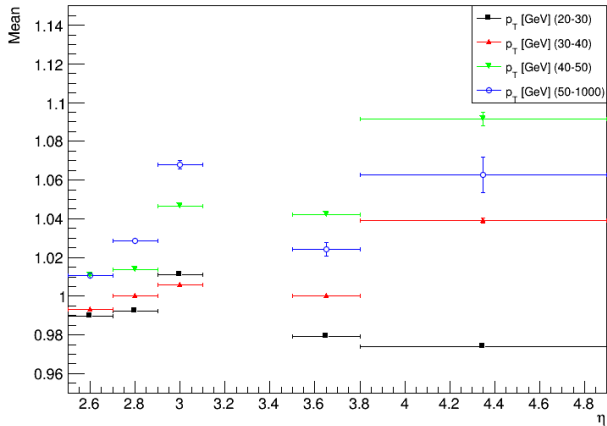
Mean μ 1



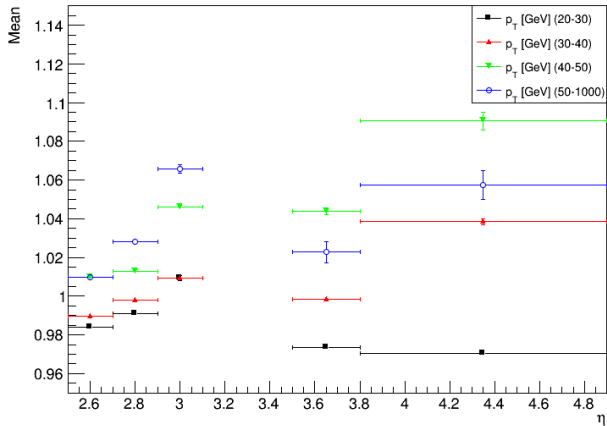
Mean μ 2



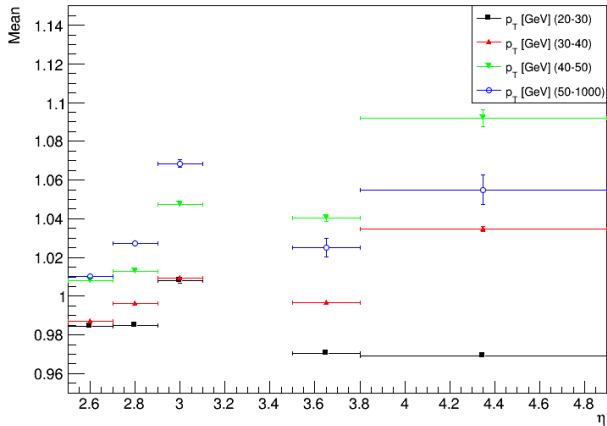
Mean μ 3



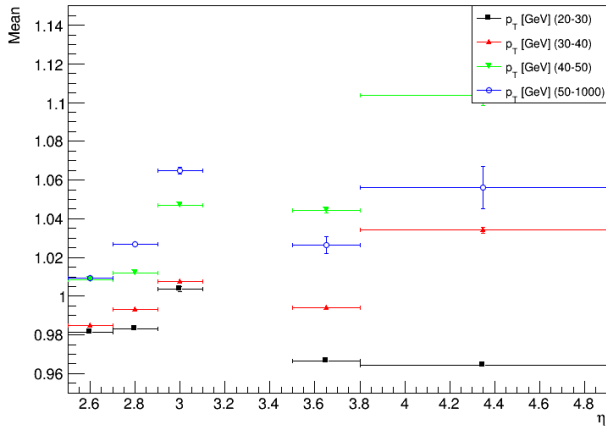
Mean μ 4



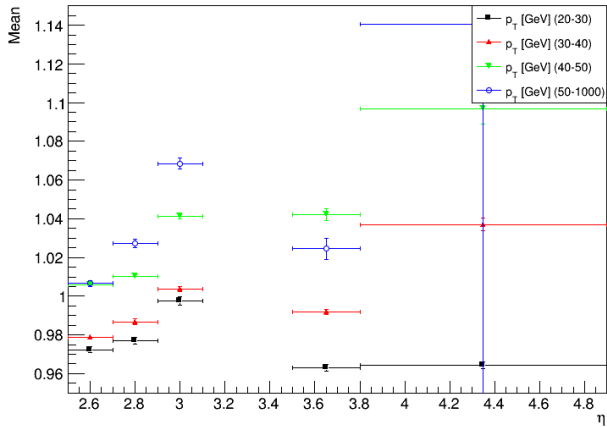
Mean μ 5



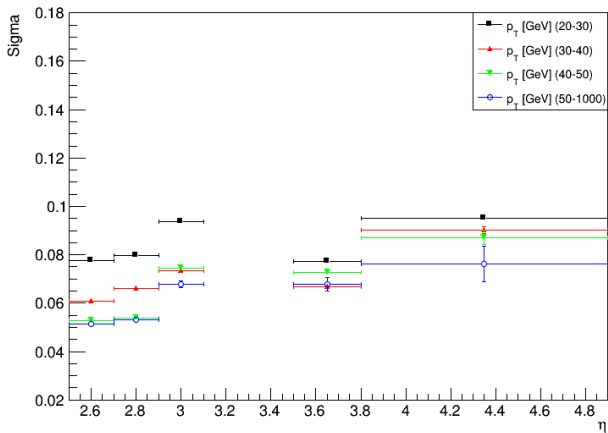
Mean μ 6



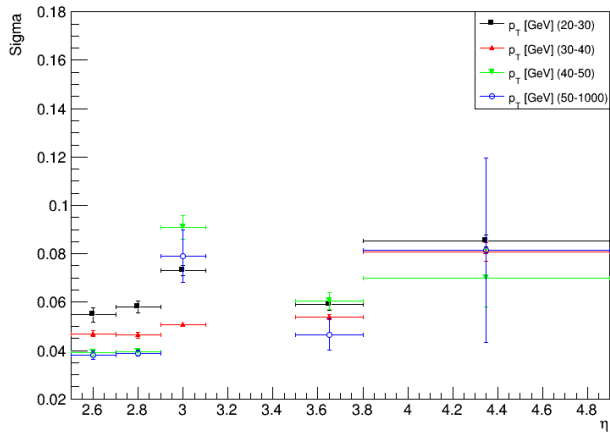
Mean μ 7



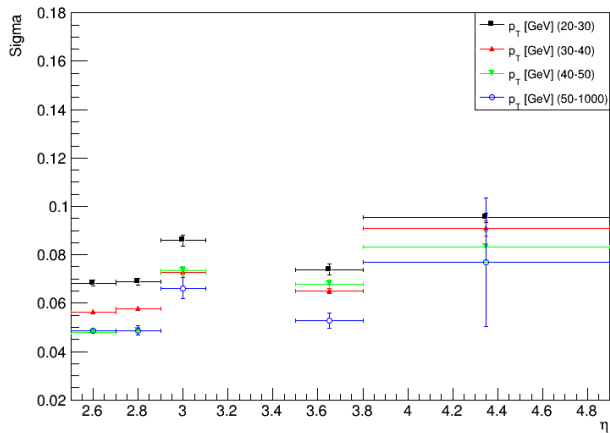
Sigma μ 0



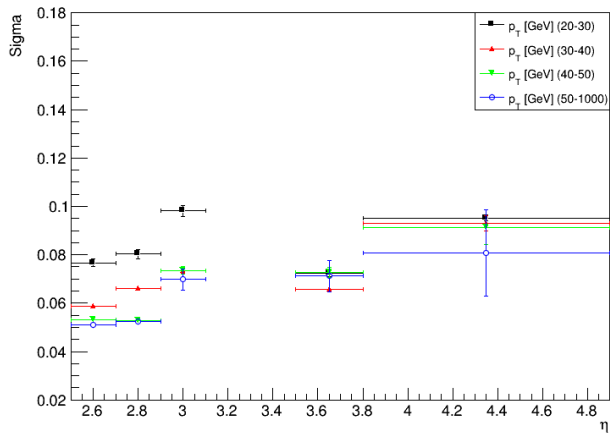
Sigma μ 1



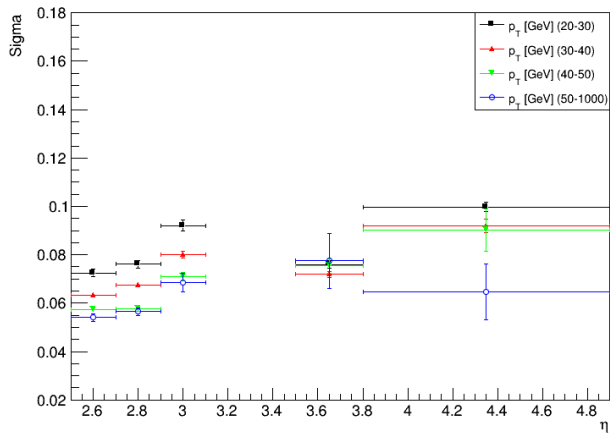
Sigma μ 2



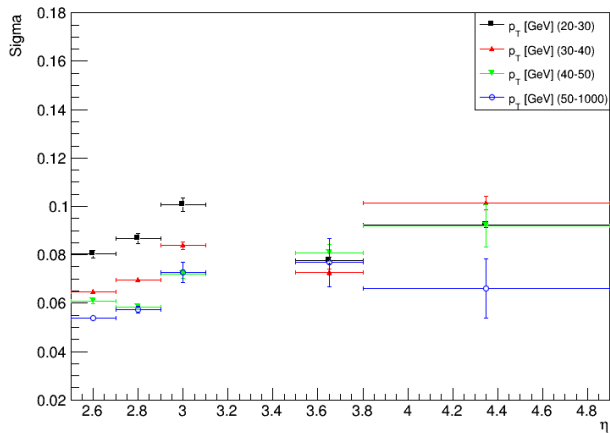
Sigma μ 3



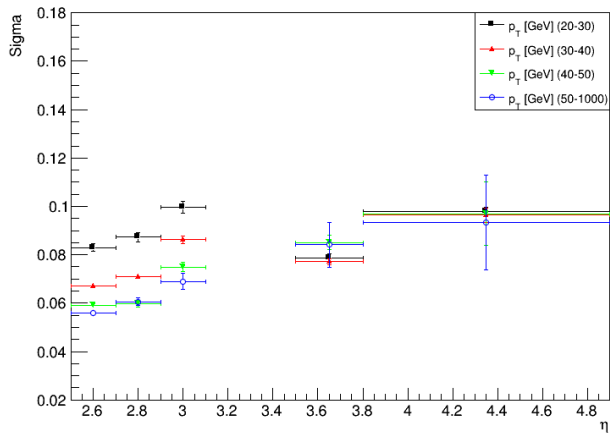
Sigma μ 4



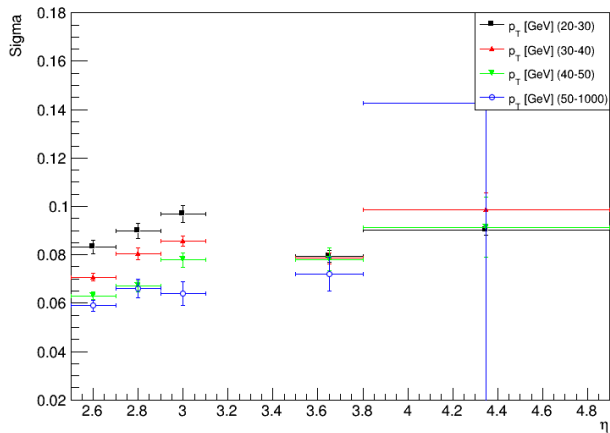
Sigma μ 5



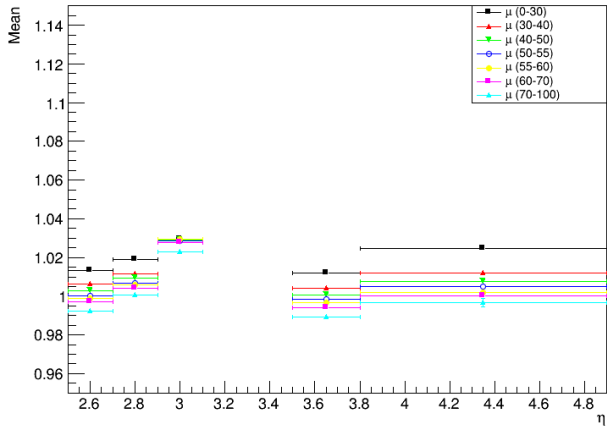
Sigma μ 6



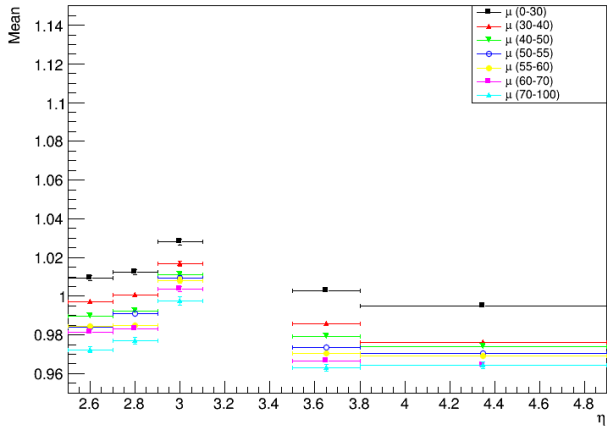
Sigma μ 7



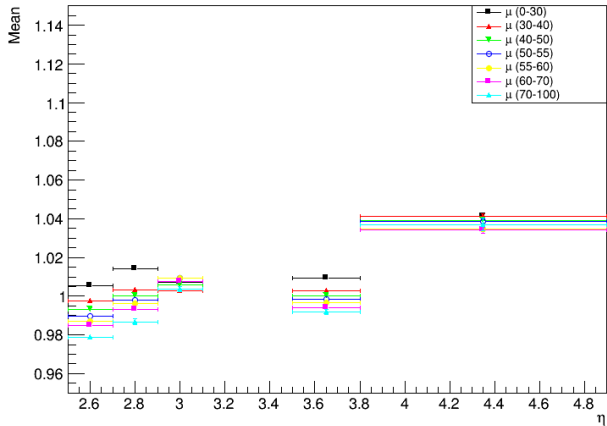
Mean pt 0



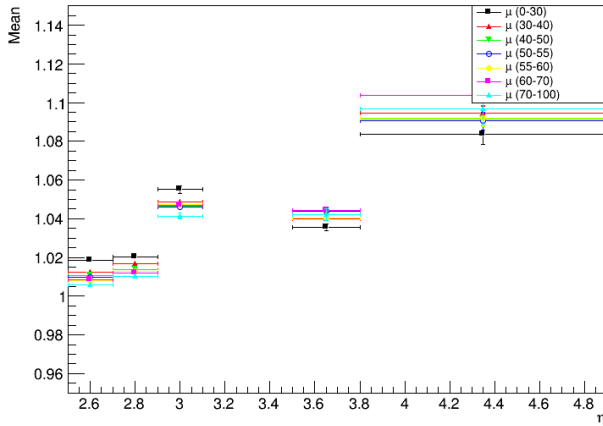
Mean pt 1



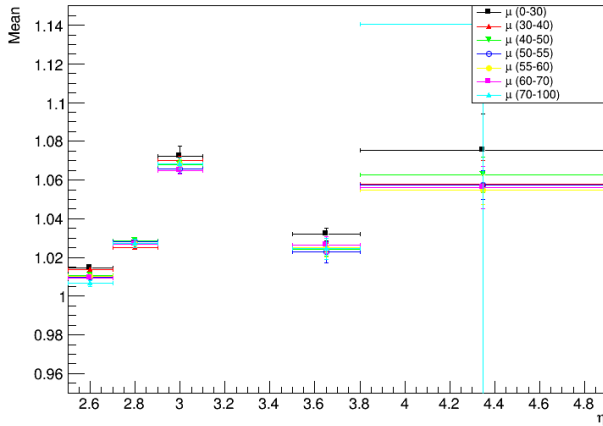
Mean pt 2



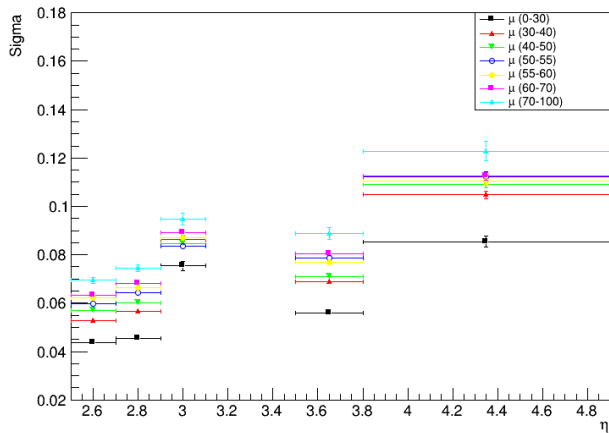
Mean pt 3



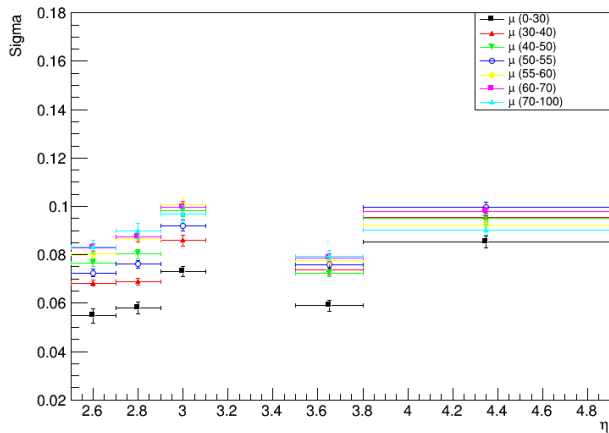
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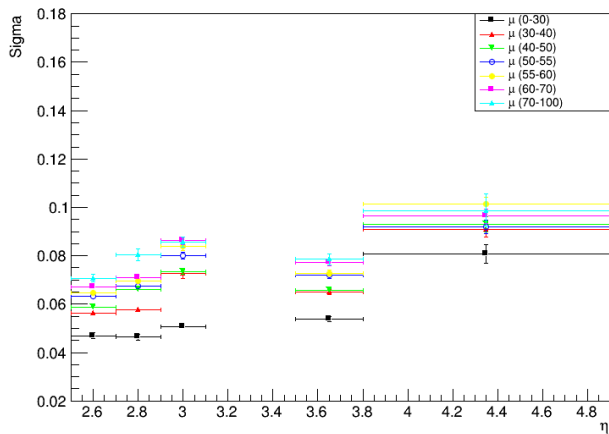
Sigma pt 0



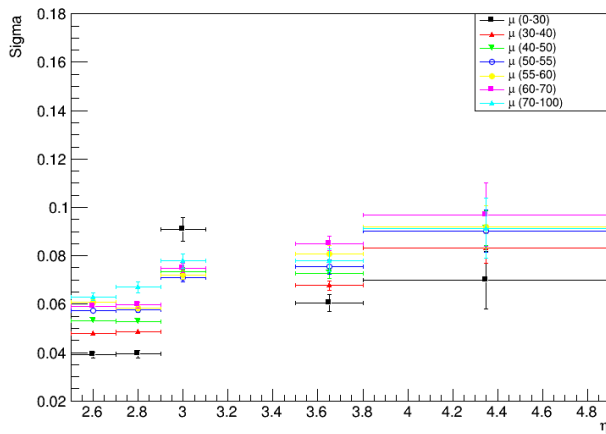
Sigma pt 1



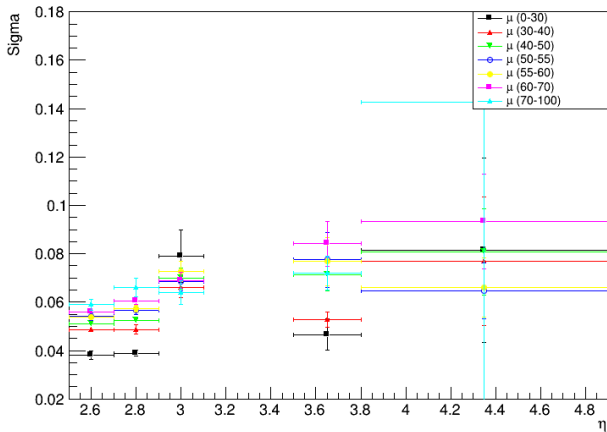
Sigma pt 2



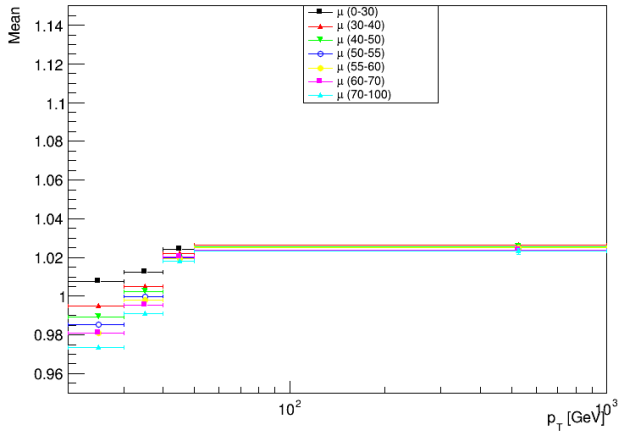
Sigma pt 3



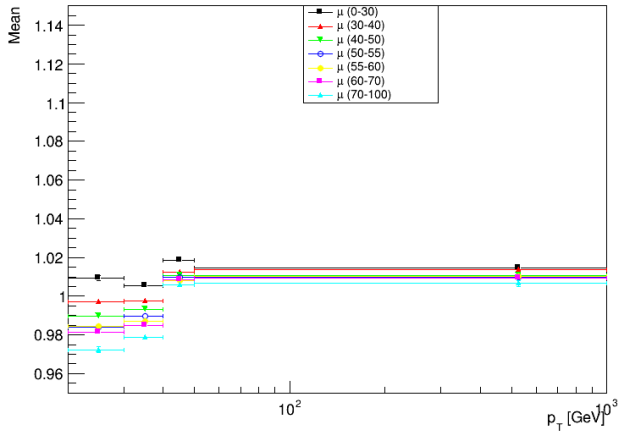
Sigma pt 4



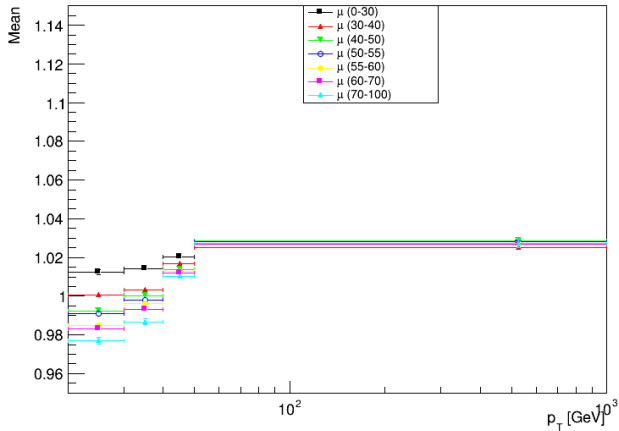
Mean η 0



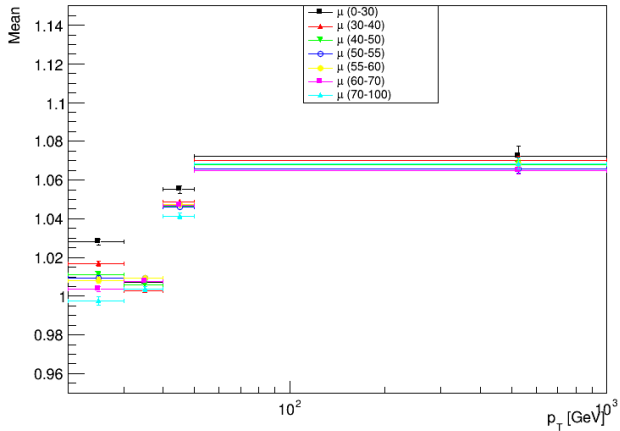
Mean η 1



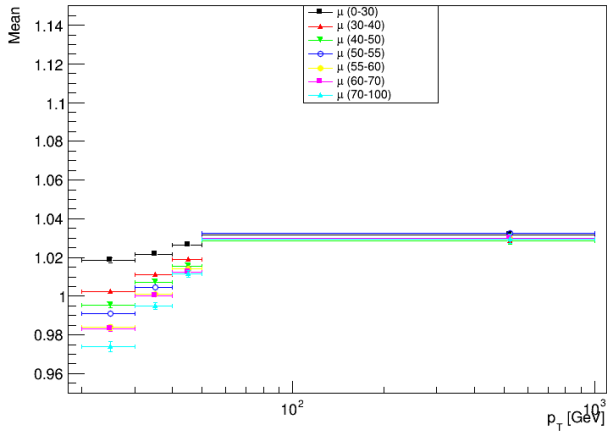
Mean η 2



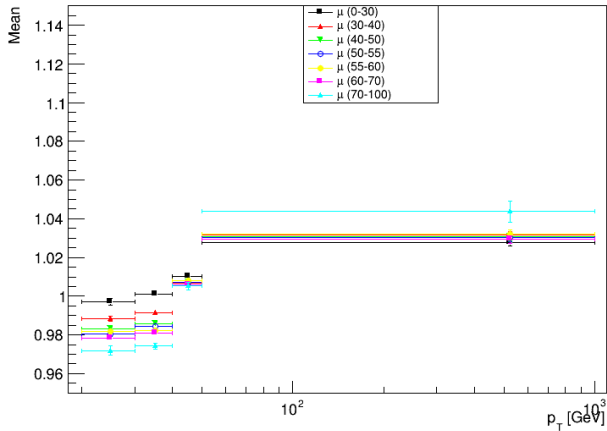
Mean η 3



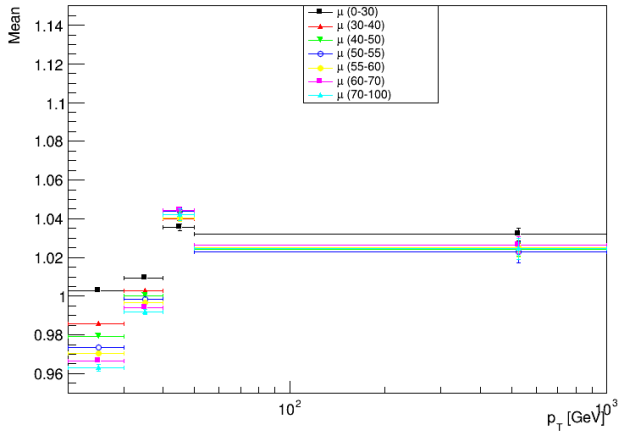
Mean η 4



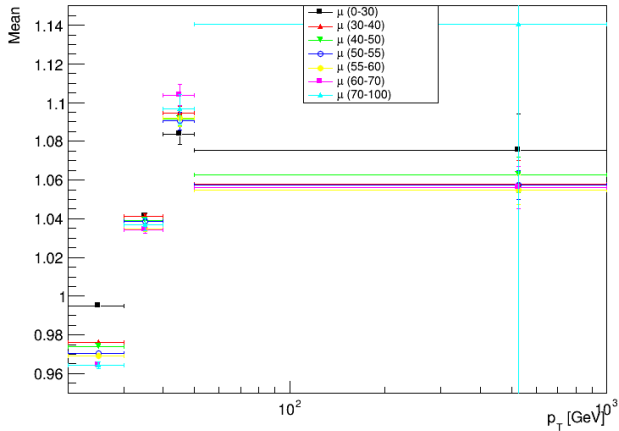
Mean η 5



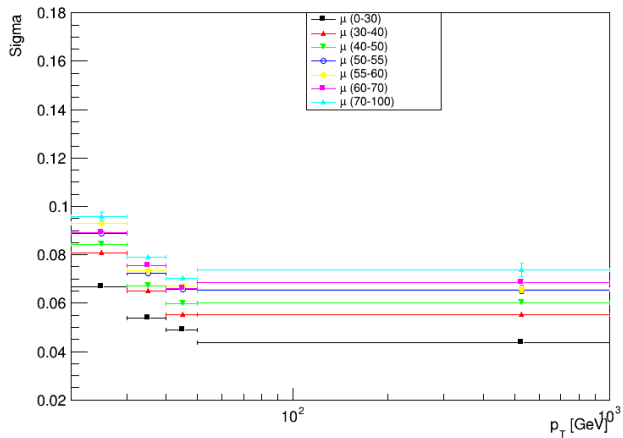
Mean η 6



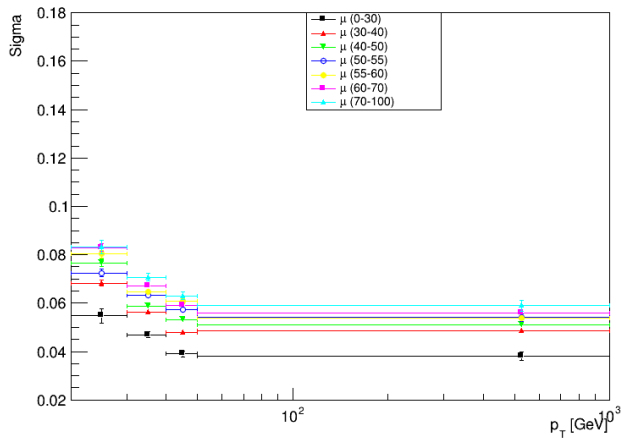
Mean η 7



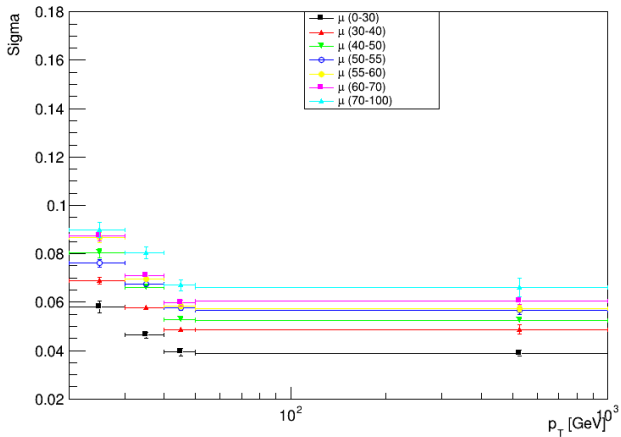
Sigma η 0



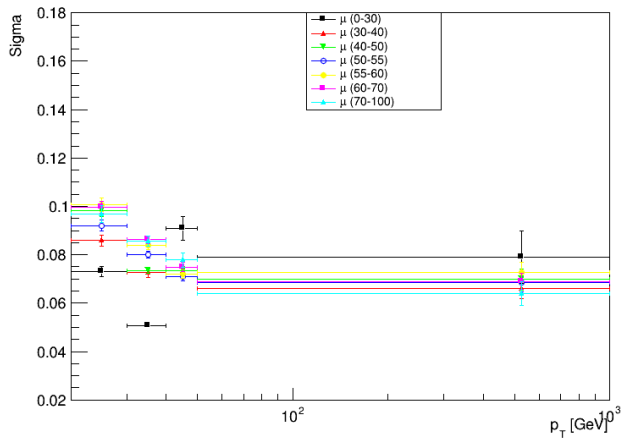
Sigma η 1



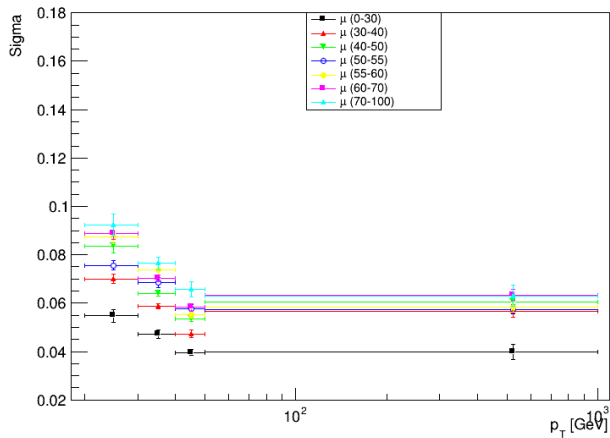
Sigma η 2



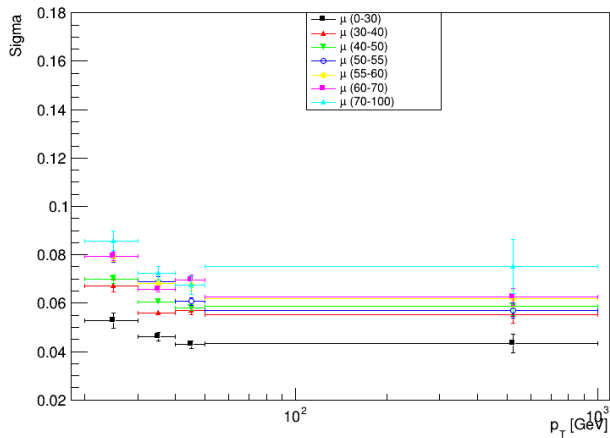
Sigma η 3



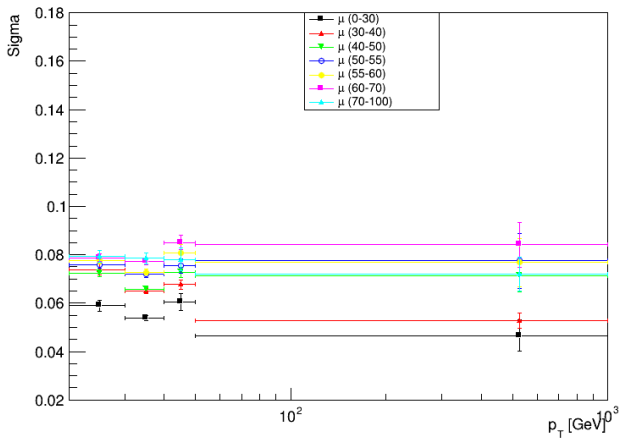
Sigma η 4



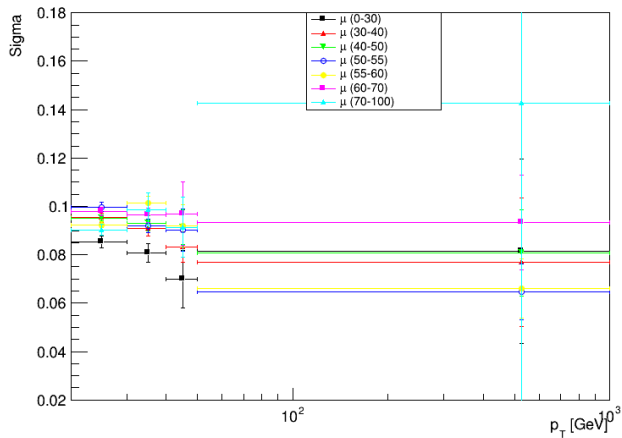
Sigma η 5



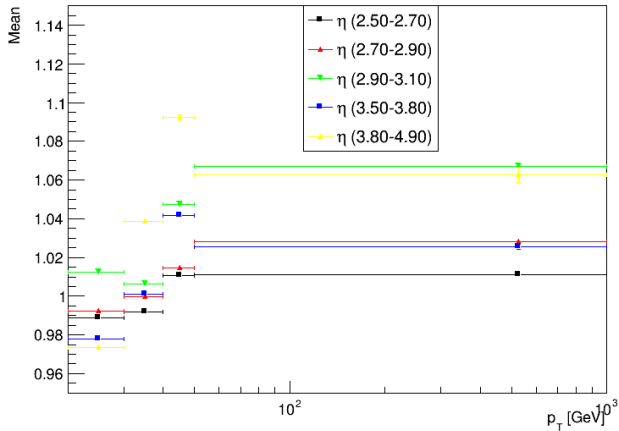
Sigma η 6



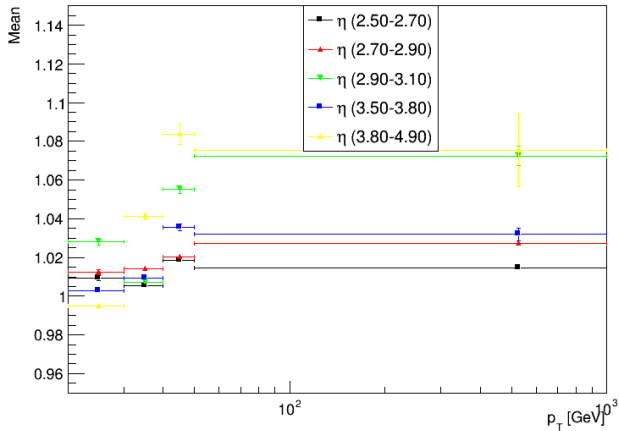
Sigma η 7



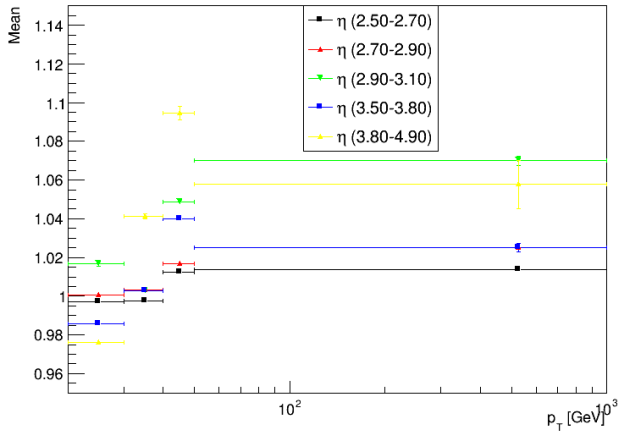
Mean μ_0



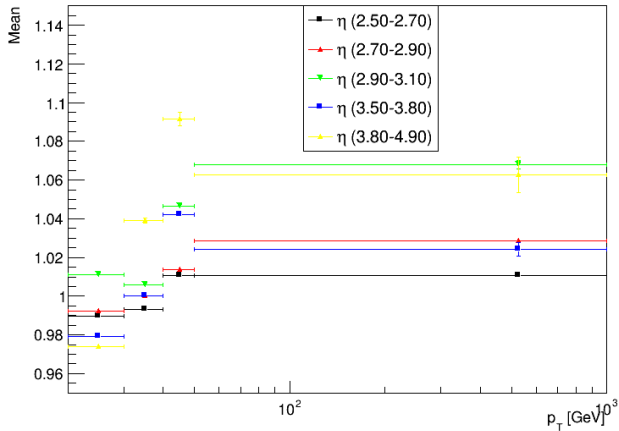
Mean μ 1



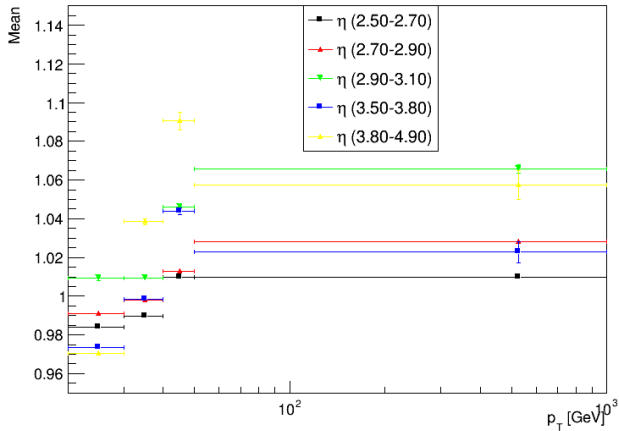
Mean μ 2



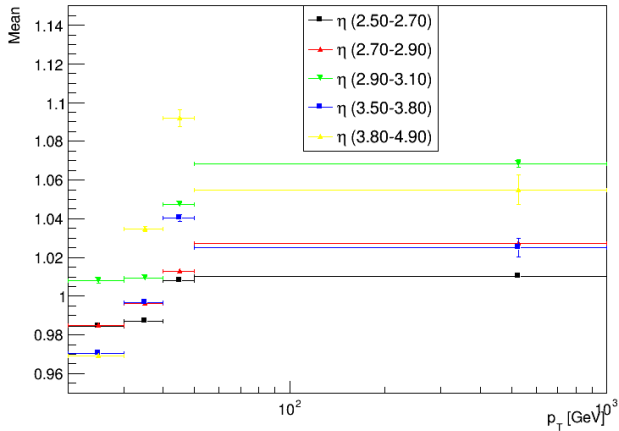
Mean μ 3



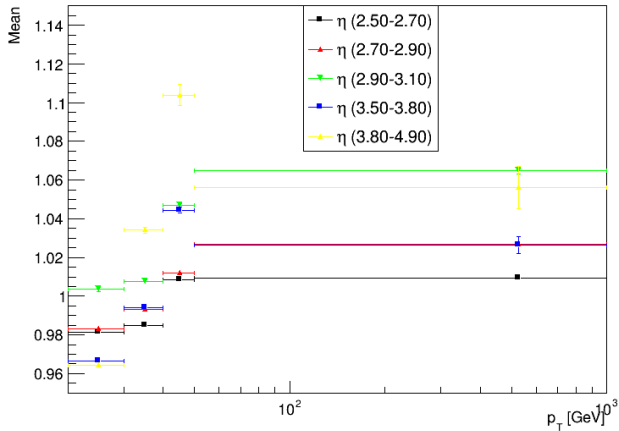
Mean μ_4



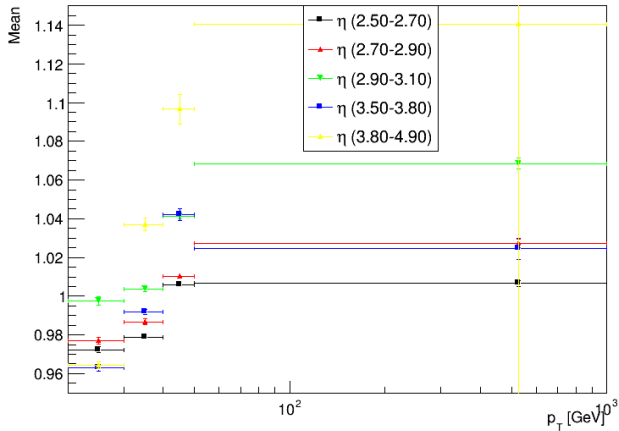
Mean μ 5



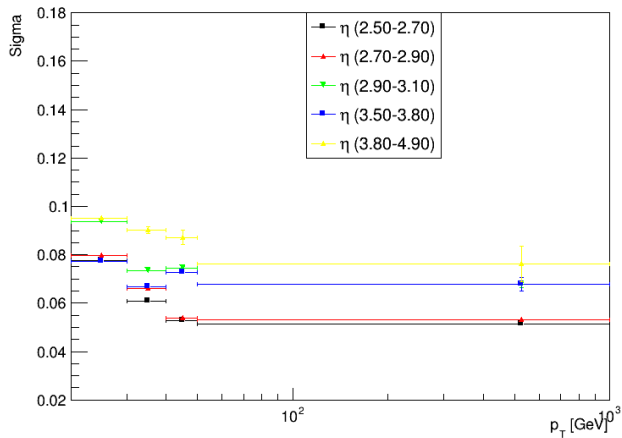
Mean μ 6



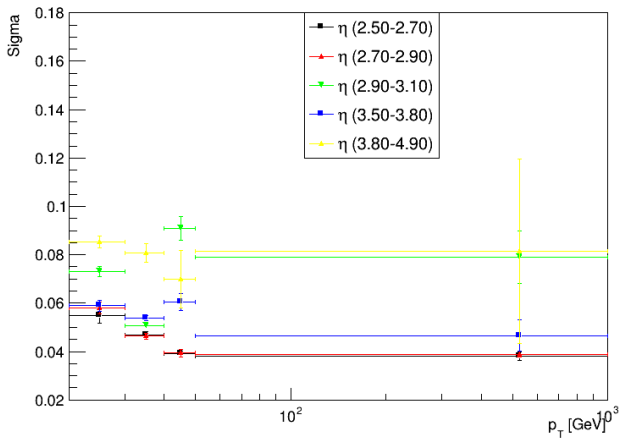
Mean μ 7



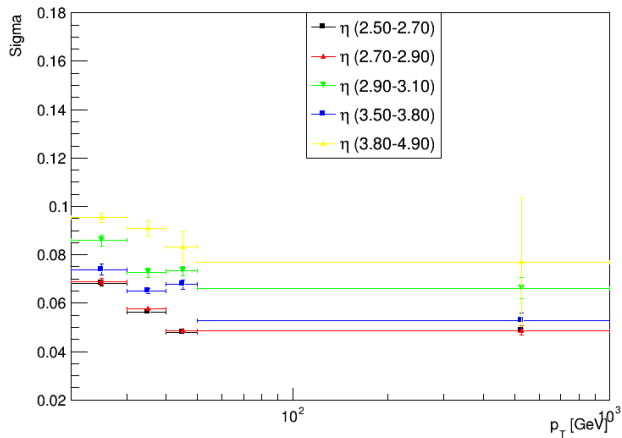
Sigma μ 0



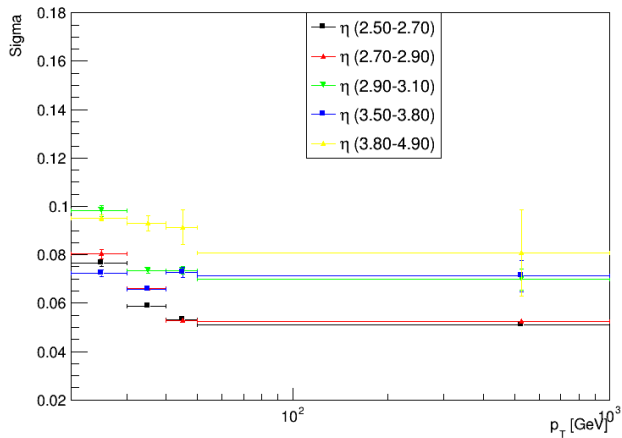
Sigma μ 1



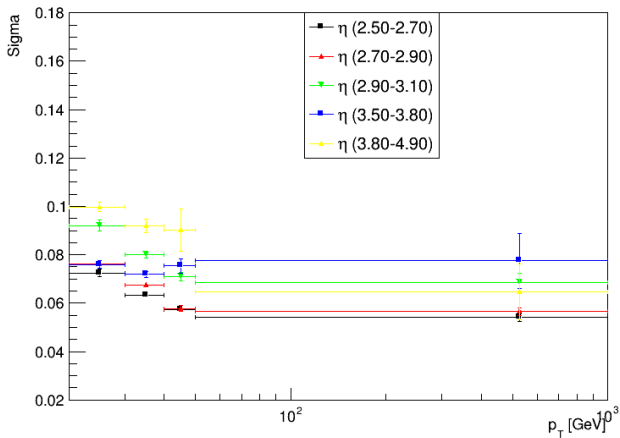
Sigma μ 2



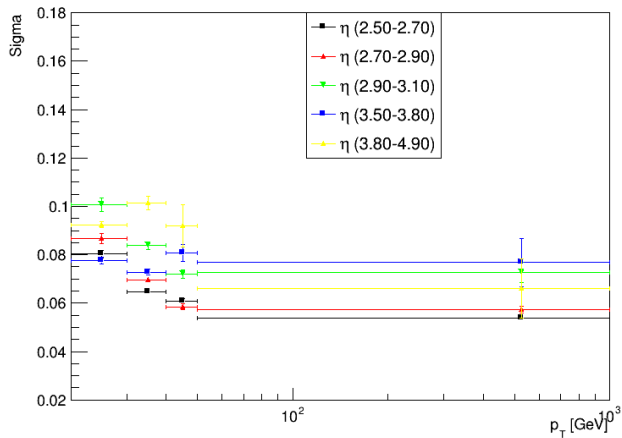
Sigma μ 3



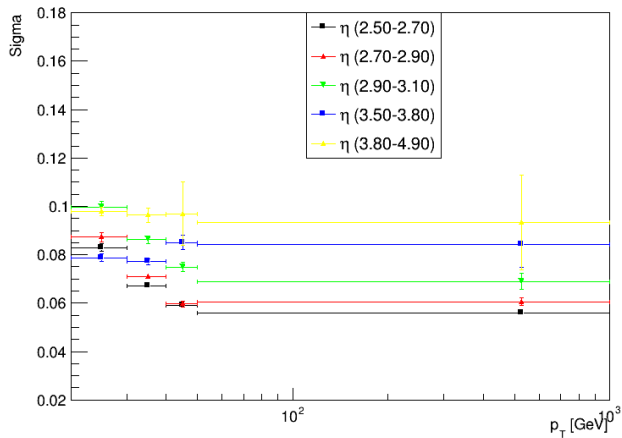
Sigma μ 4



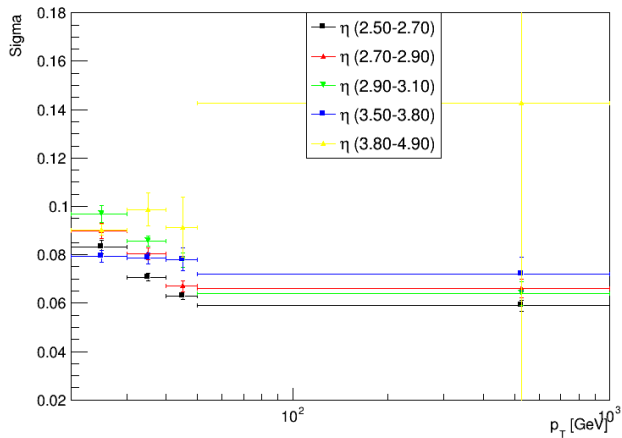
Sigma μ 5



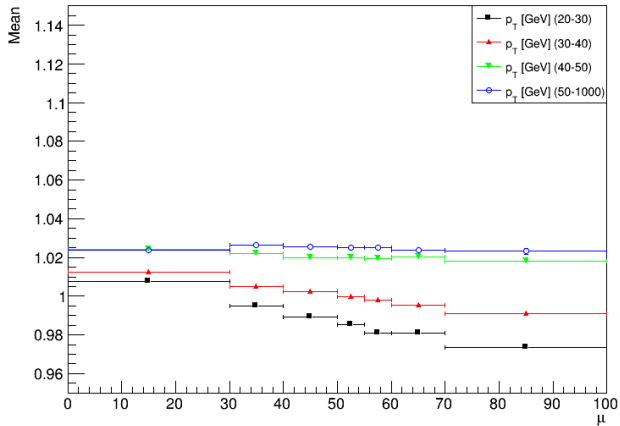
Sigma μ 6



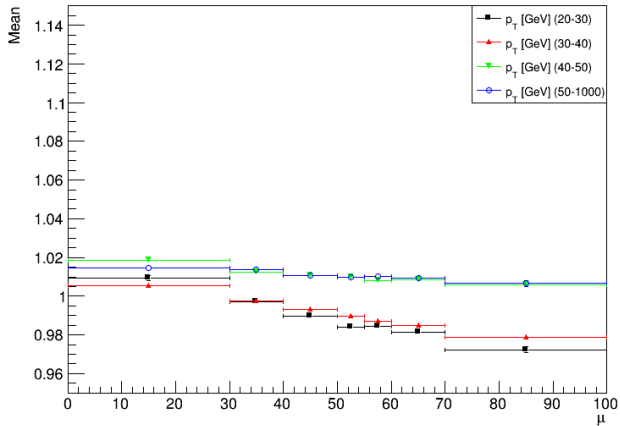
Sigma μ 7



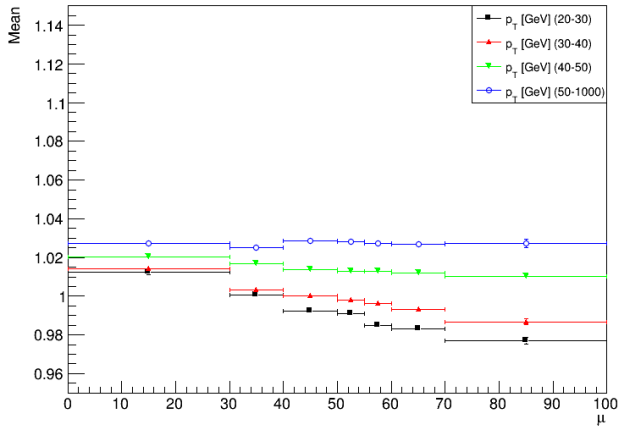
Mean η 0



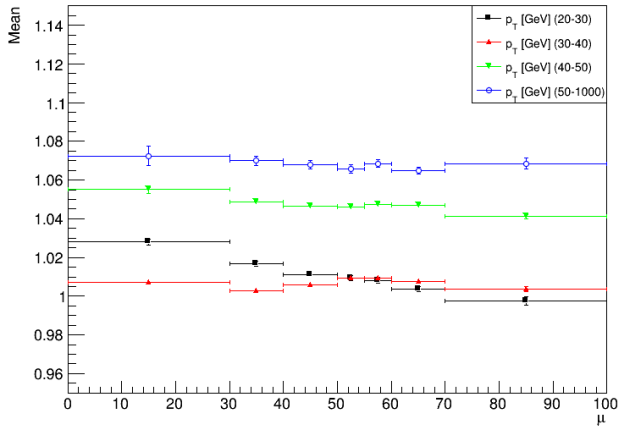
Mean η 1



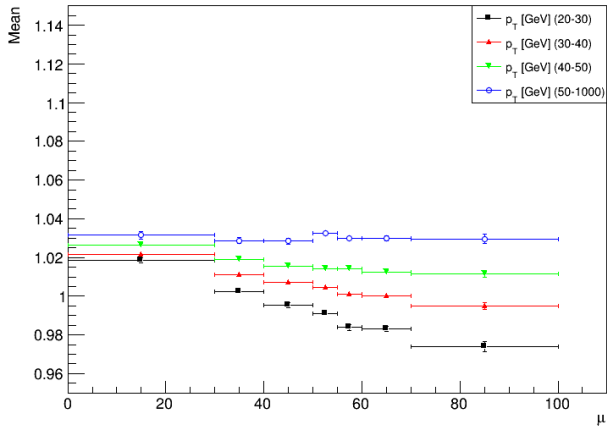
Mean η 2



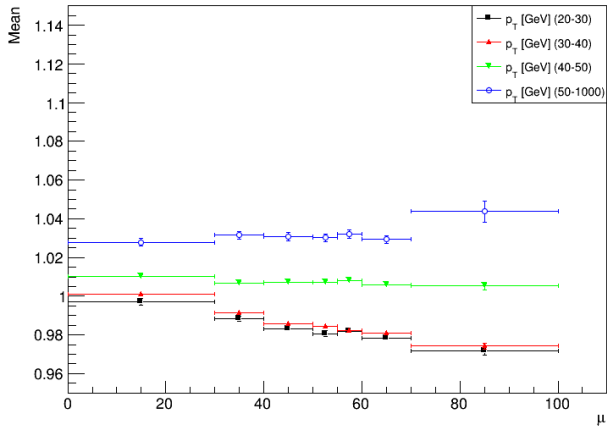
Mean η 3



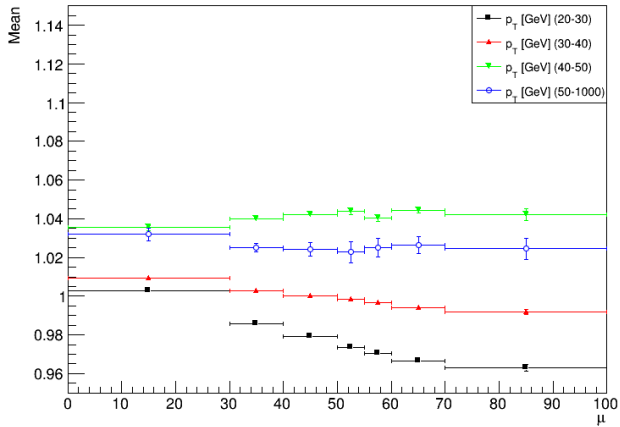
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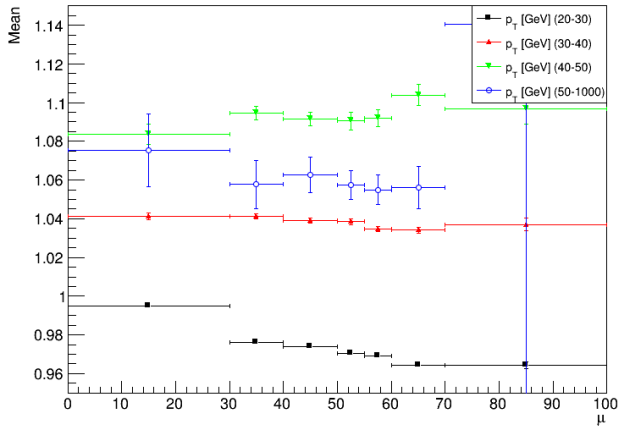
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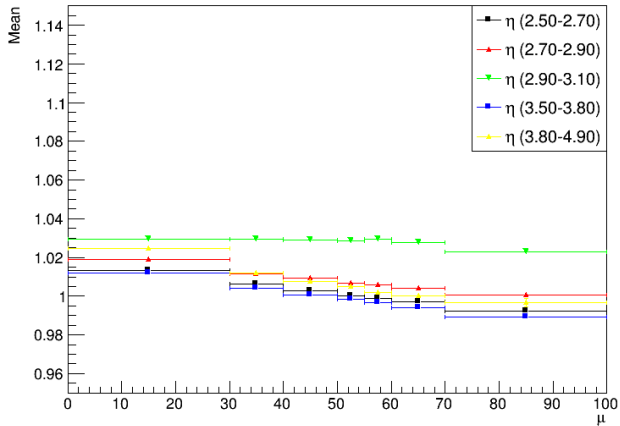
Mean η 6



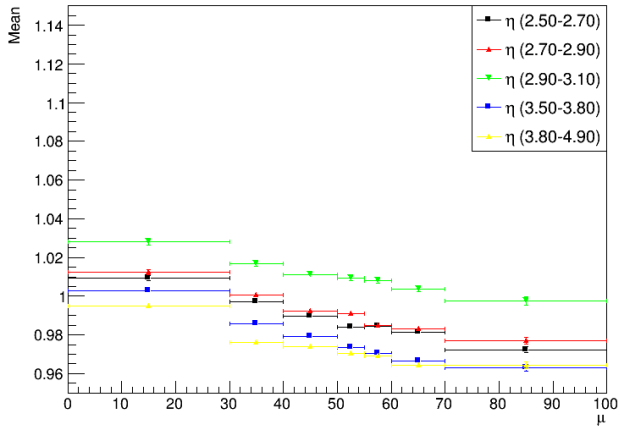
Mean η 7



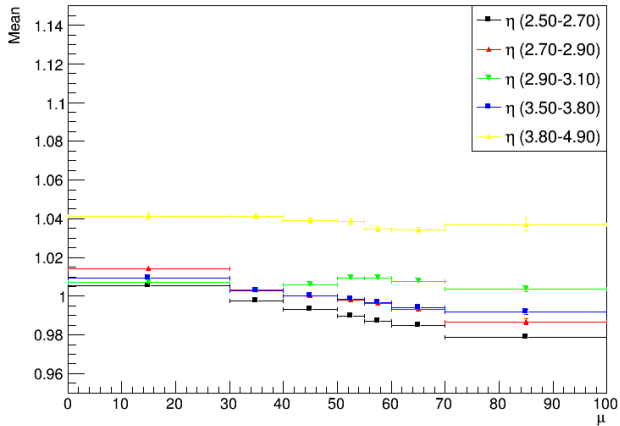
Mean p_T 0



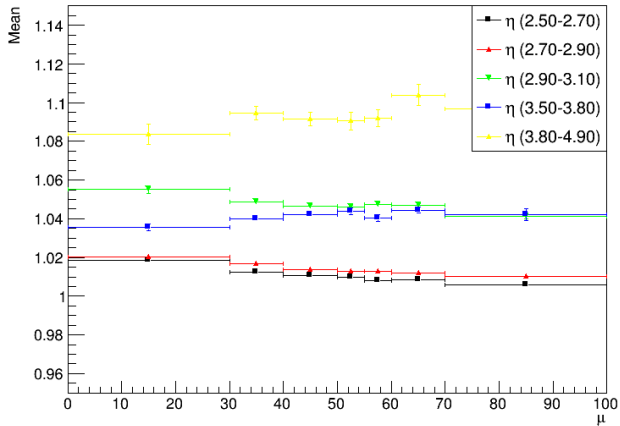
Mean p_T 1



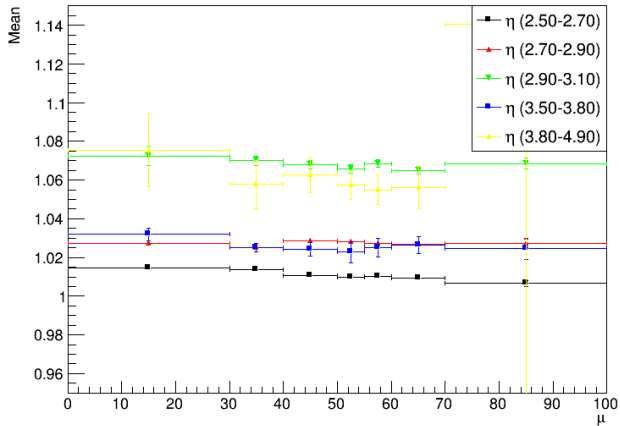
Mean p_T 2



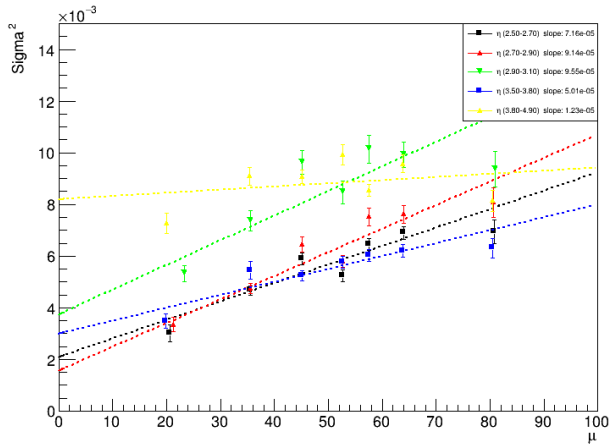
Mean p_T 3



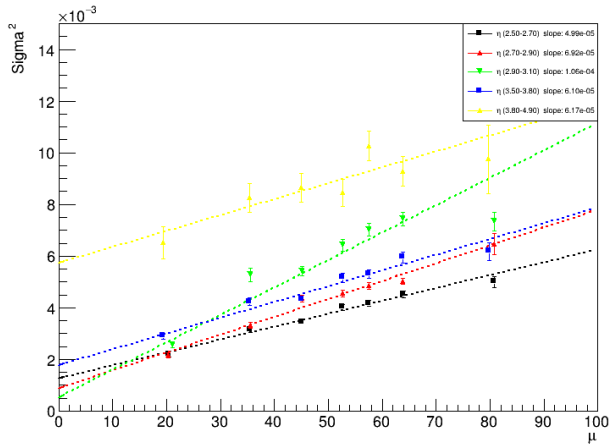
Mean p_T 4



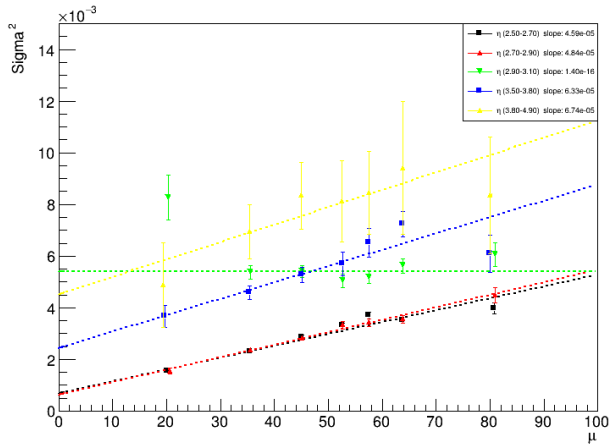
$\Sigma^2 p_T \uparrow$



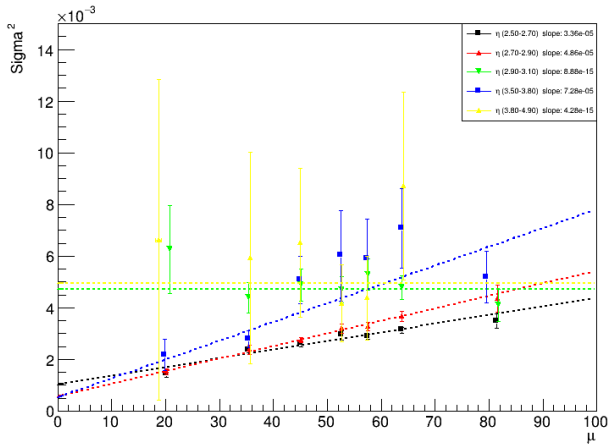
Σp_T^2



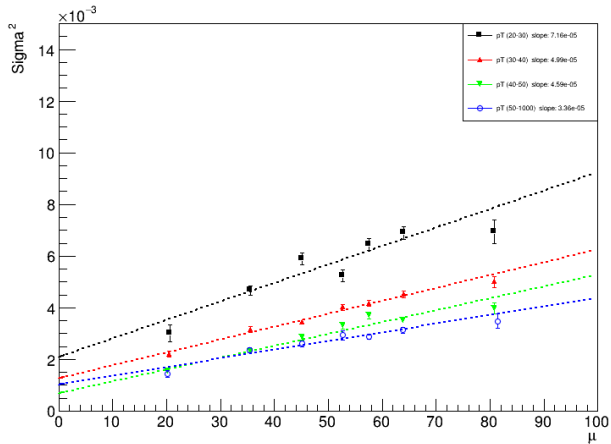
$\Sigma^2 p_T^3$



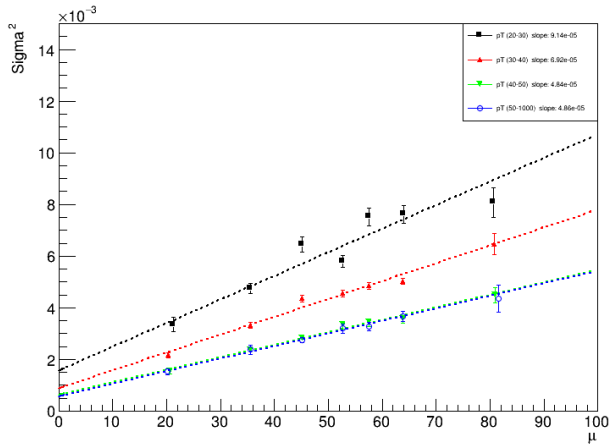
$\Sigma^2 p_T^4$



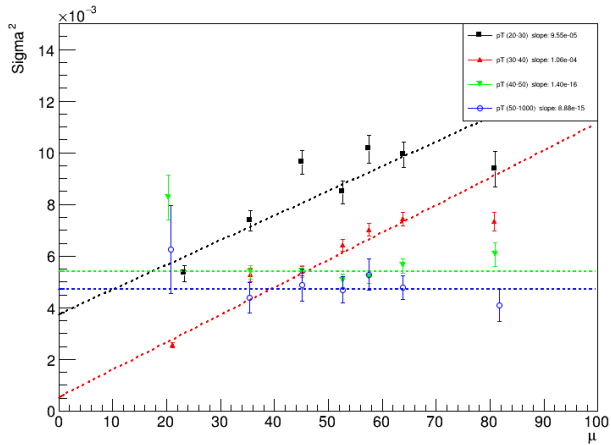
$\Sigma^2 \eta_1$



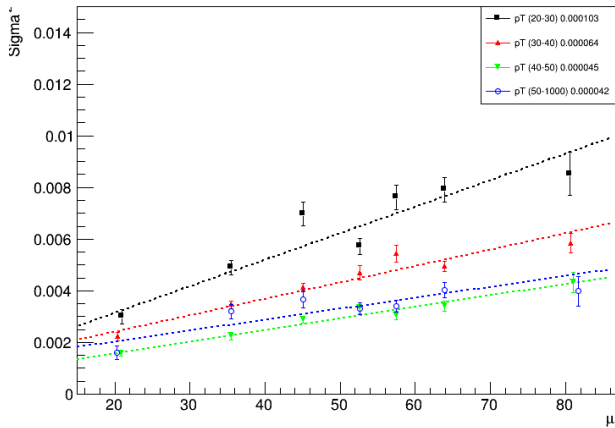
$\Sigma^2 \eta$



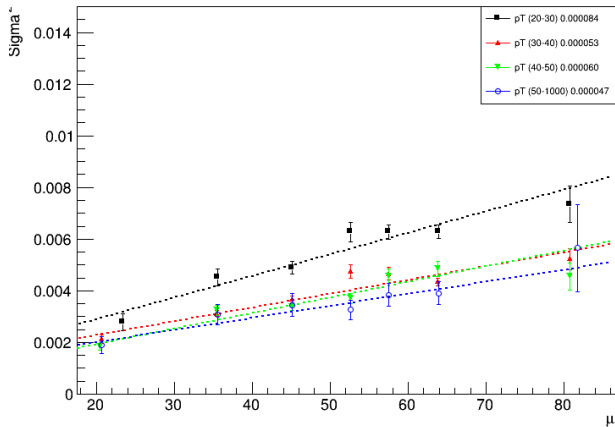
$\Sigma^2 \eta$ 3



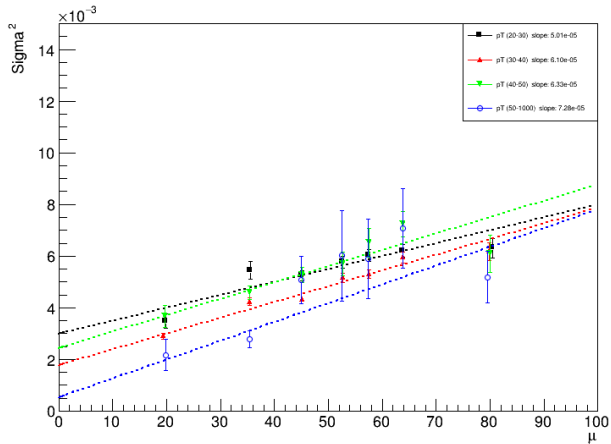
$\Sigma^2 \eta$



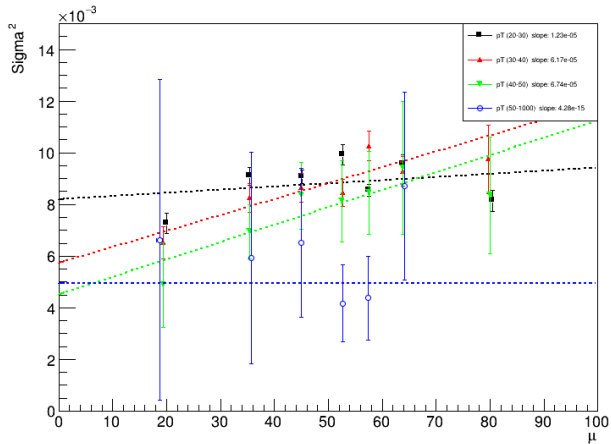
$\Sigma^2 \eta$ 5



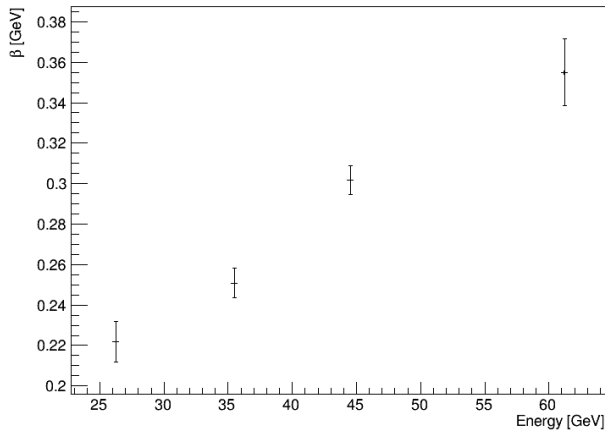
$\Sigma^2 \eta_6$



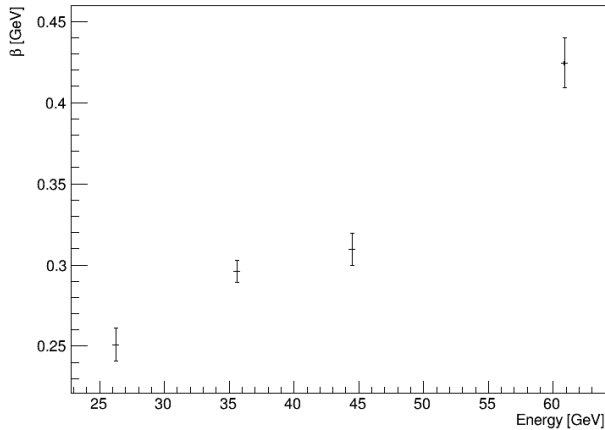
$\Sigma^2 \eta$ 7



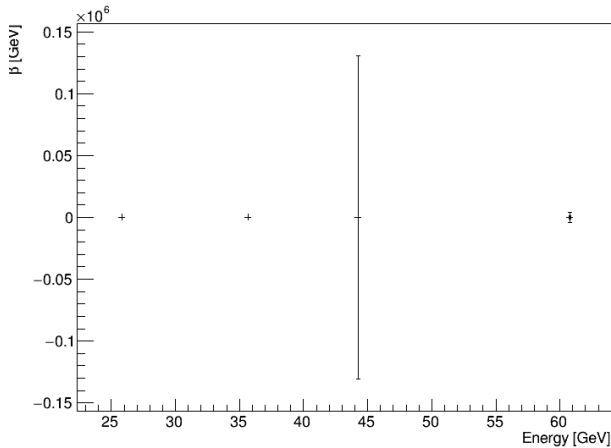
Beta vs $p_t \eta$ 1



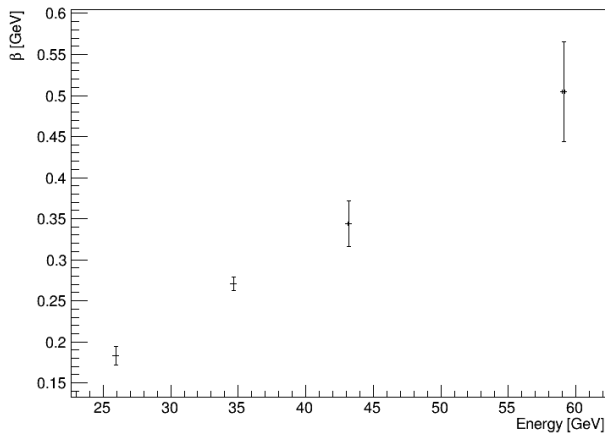
Beta vs $p_t \eta^2$



Beta vs $p_t \eta$



Beta vs $p_t \eta$



Beta vs $p_t \eta$

