

Full Summary of FSR Studies

Josh Newell, Ludovica Aperio Bella, Filippo Dattola, Linghua Guo



Original idea and motivation

See Maarten slides : [here](#)

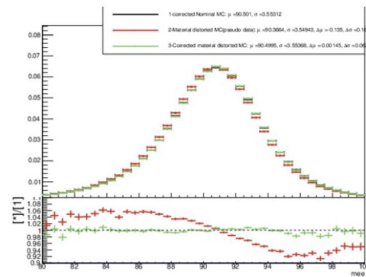
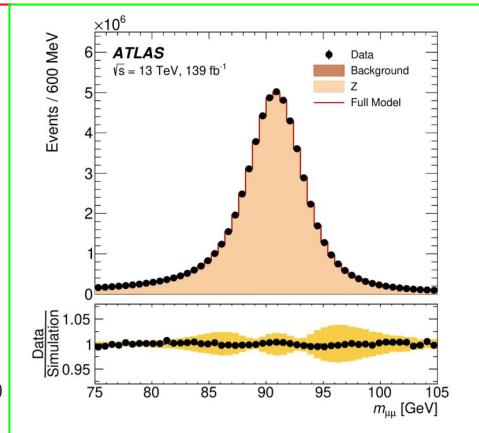
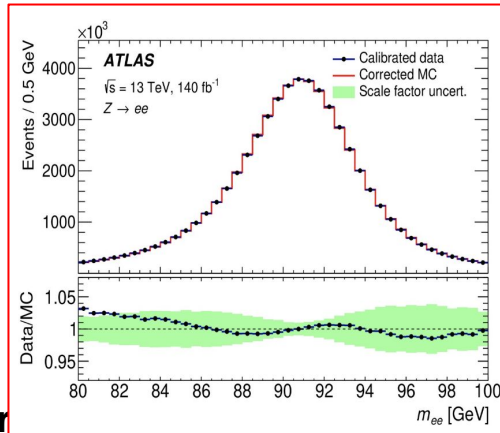
Motivation:

- Following calibration of $Z \rightarrow ee$ data and MC, comparison of mass lineshapes shows an excess of energy tails at low invariant mass (>1%-level)
- Ongoing since Run1 (throughout Run2, sliding windows and supercluster reco) this generates energy scale systematics (from fit window variations) that limit the overall calibration precision.
- **Muons** behave better

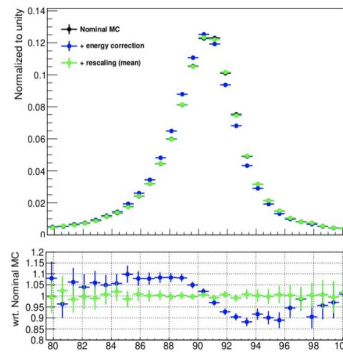
Mainly affecting W&Z analysis since mass peaks are very close – other E/gamma calibration systematics become dominant at higher/lower energy scales

Possible causes studied and excluded over the years:

- Intercalibration of the Presampler and the accordion layers, even S3
- Readout non-linearity
- Lateral shower shapes
- Passive Material variation



(a) Config A



Hypothesis Outline

Zee invariant mass lineshape is affected by the differences in behaviour of QED FSR between data and MC

Most QED FSR is not included in MVA (BDT) stage (step 3) of energy calibration:

MVA calibration only trained on single electrons with Bremsstrahlung

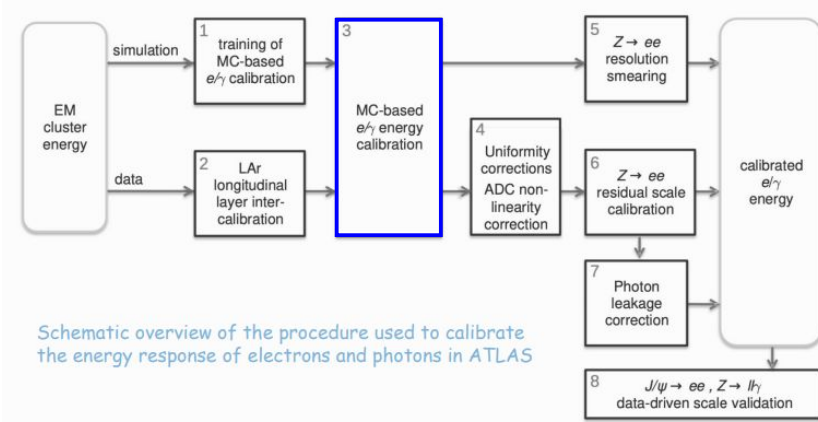
Ideally train MVA with FSR but computationally expensive

$$E_{\text{MVA}} \approx E_0 + E_1 + E_2 + E_3 \text{ (sum of energies in presampler and accordion)}$$

Behaviour of FSR in simulation can affect distribution of energy in calorimeter layers, thus causing changes relative to variables used in MVA

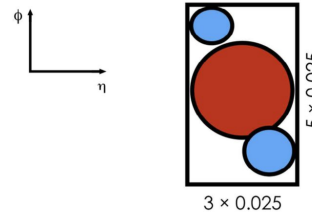
Can introduce effects that cause the MVA to over/under correct the reconstructed energy:

- Situation 1 - A slightly higher pT/harder FSR deposited within the cluster can modify E_0 , E_1 , E_2 , E_1/E_2
- Situation 2 - FSR is too hard (dR too large) to be within the cluster:
 - This is lateral leakage, we completely lose FSR information in this case
 - MVA would not correct the energy, and consider this as a lower energy electron



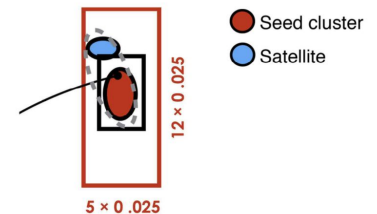
All e^\pm, γ :

Add all clusters within 3×5 window around seed cluster.



Electrons only:

Seed, secondary cluster match the same track.



Aim to study effect of FSR properties (angular separation and FSR pT) on kinematic variables of $Z \rightarrow ee$ events

Methodology Overview: Event Classification

In order to study the effects of QED FSR on Zee events, need to differentiate each event / electron based on QED FSR properties:

Classify events in terms of the region of angular separation that contains the highest total pT resulting from QED FSR

Initially, the angular separation was classified in terms of concentric rings, centered on the reco electron

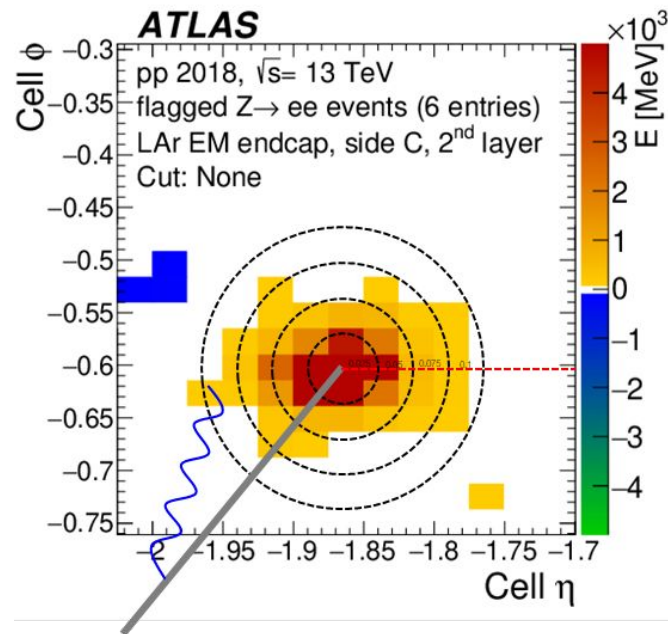
FSR photons initially matched to leading/subleading electron using minimum ΔR (i.e. find ΔR between photon and each electron, then match to electron with smallest ΔR)

Rings: 0, 0.025, 0.05, 0.075, 0.1, 0.15, 0.2, 0.25, 0.3, 0.5, 1.0, 1.5, 2, 10

Near seed cluster

Possible within supercluster

Classification was improved in later iterations:



Methodology Overview: Event Classification

As mentioned: MVA calibration only trained on single electrons with Bremsstrahlung

Bremsstrahlung only emitted in ϕ direction due to direction of electron track bending from magnetic field

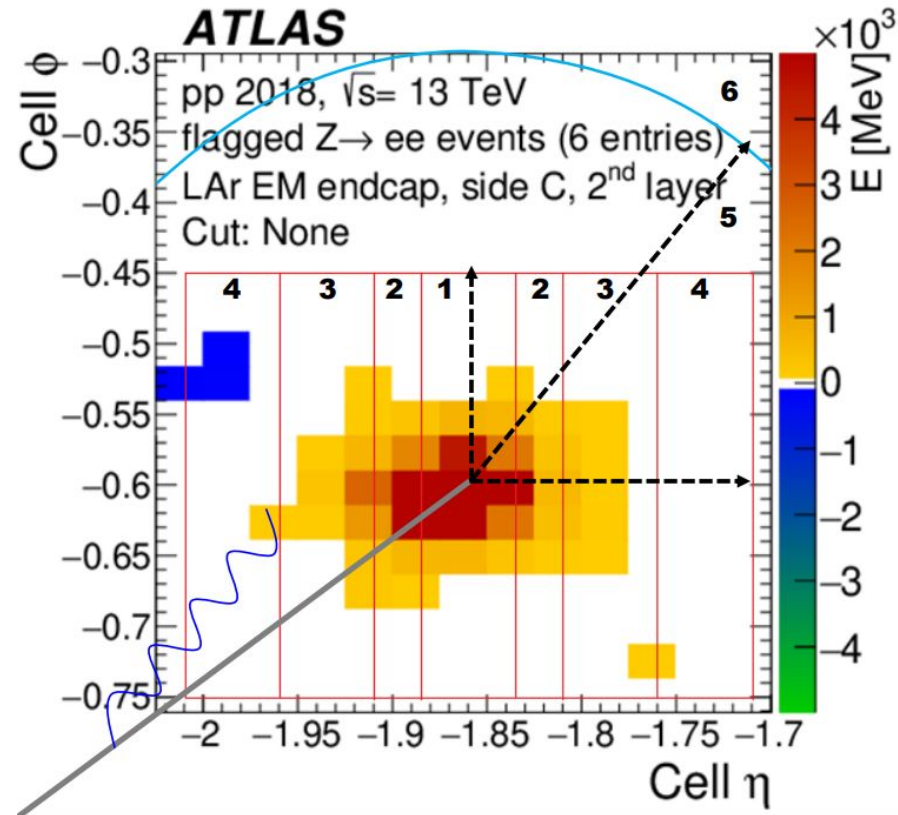
To focus on QED FSR independent of Bremsstrahlung, ring categories were replaced by a series of segments divided in $\Delta\eta$ (with constant $\Delta\phi$)

Segments divided in regions of $\Delta\eta$ and $\Delta\phi$ ($\Delta\eta$ - $|0.025|$, $|0.025 - 0.05|$, $|0.05 - 0.1|$, $|0.1 - 0.15|$; $\Delta\phi$ - $|0.15|$) up to segment 4.

Segment 5 is defined by a ΔR cone of 0.3

Segment 6 is defined by the region outside of the ΔR cone of 0.3

Electron-photon matching was also improved in later iterations:



Methodology Overview: Electron-photon Matching

Photons matched to one electron that are emitted outside of the $\Delta R > 0.3$ cone are generally categorised in segment 6

However, FSR emitted at a high enough angle may be matched to the wrong electron according to truth-level.

Only events where both electrons have no FSR can be guaranteed to be in this category

Otherwise there may be a small chance that an event where one electron in “no FSR” and one electron in segment 6 ($\Delta R > 0.3$) are mislabelled

One way to combat this is to consider the probability of photon emission relative to emission angle and electron + photon energies according to QED:

$$\frac{1}{2E_e E_\gamma (1 - \cos \theta)}$$

This should avoid cases where far FSR is matched to a particular electron despite it being more likely to be emitted by the other

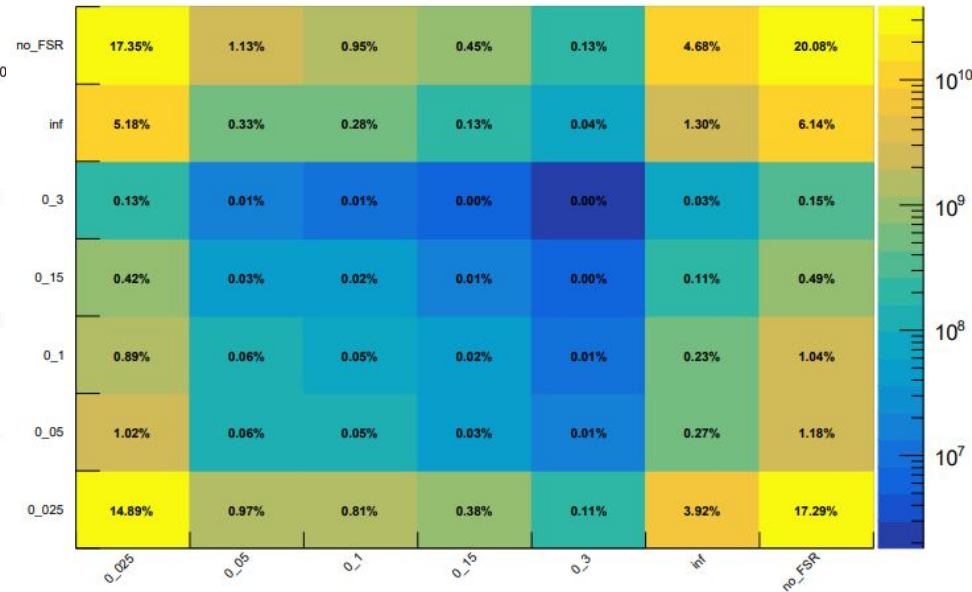
Another method to consider is looking at the born and bare energies of the electron as the sum of photon energies and electron bare energy should equal the born energy

FSR segment Grid

Old Matching



Improved Matching



Improved photon-electron matching overall has a small effect on the distribution of electrons across FSR segment categories

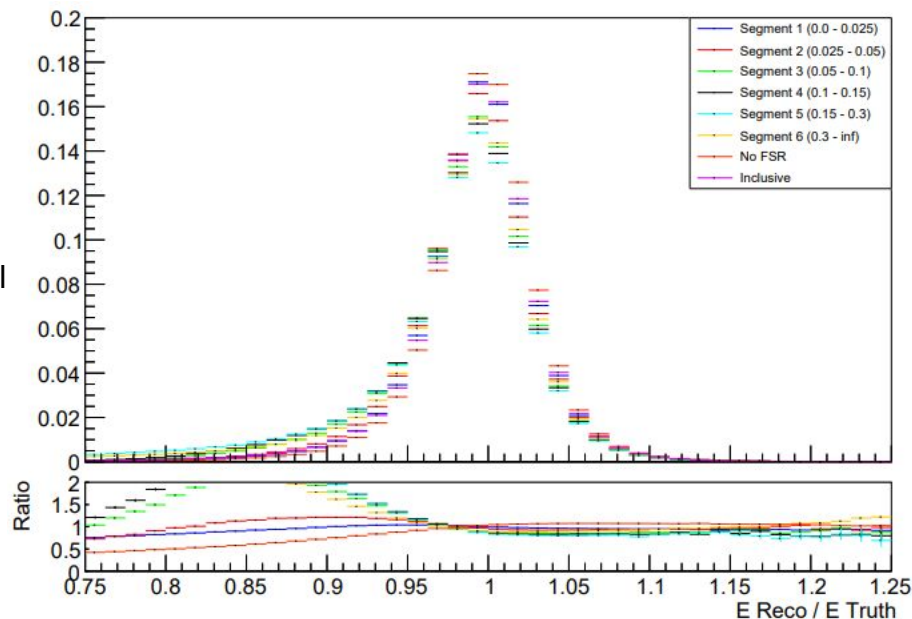
For the leading electron (x-axis) there is a per-mille level reduction in the percentage of electrons classified as far FSR, with a corresponding increase in the proportion of no FSR events

The opposite case holds for the subleading electron, where far FSR increases in exchange for no FSR

E Reco / E Truth

Ratio of reconstructed energy to truth energy shows differences arising from the electron reconstruction process

- If electron is reconstructed perfectly, $E_{\text{reco}}/E_{\text{truth}}$ is always 1
- Since energy reconstruction is performed only in a small area around electron (3x5 cells), some energy is lost through hard FSR at higher dR
- Results show regions further from electron (segments 3 to 6) deviate from 1, with large (>100%) tails at low $e_{\text{reco}}/e_{\text{truth}}$, i.e. reconstruction causes energy is lost (as expected)



E Reco / E Truth: Ratio vs No FSR

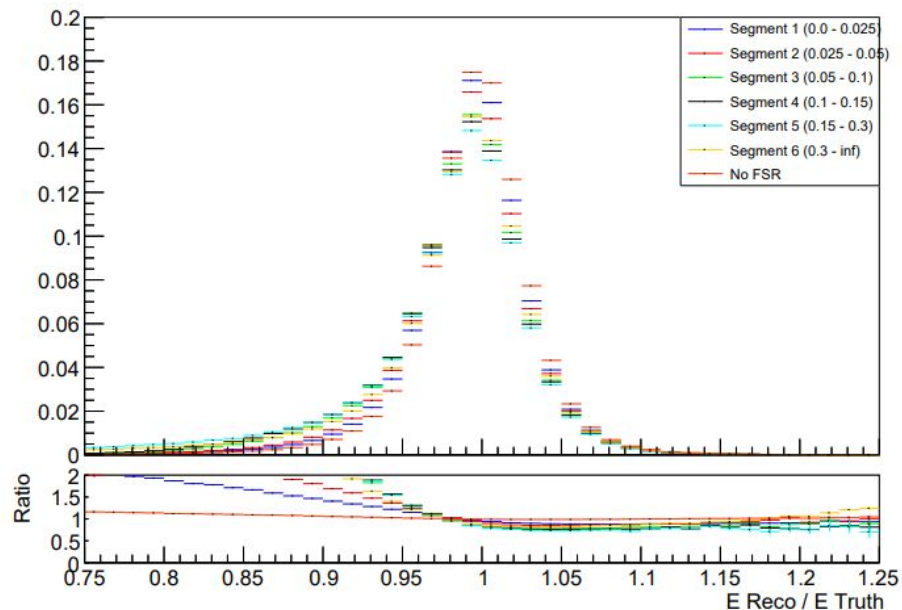
Ratio of reconstructed energy to truth energy shows differences arising from the electron reconstruction process

Since MVA is trained on events with no FSR, born events (events with no FSR) in the MC sample are expected to be the most accurately reconstructed events.

By comparing categories with pure born events (both electrons with no FSR), we can see the impact of FSR in different regions

Current plot shows inclusive categories (events that contain at least one electron in a given segment) compared with exclusive no FSR distribution.

- Plotting the ratio against exclusive no FSR category shows all categories have large tails below 1 aside from the inclusive no FSR category

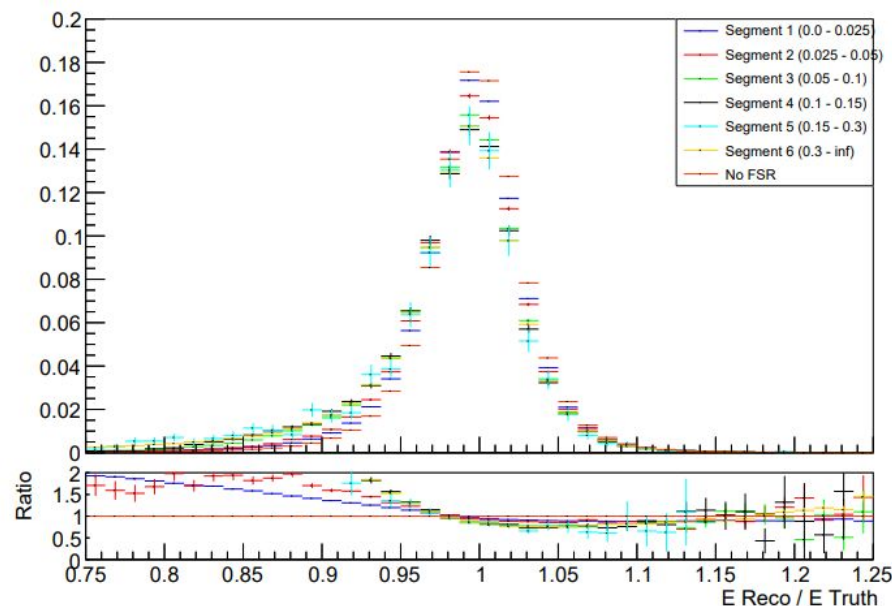


E Reco / E Truth: Ratio vs no FSR - Exclusive Categories

Ratio of reconstructed energy to truth energy shows differences arising from the electron reconstruction process

Exclusive selection is defined as events in which **BOTH** electrons are in the same segment and is useful for studying the effects of each category without bias from other categories

- Exclusive selection shows similar trends to normal selection definition (no FSR category is 1 by definition)
- Segments 1 and 2 show around the same level of deviation from 1 at lower E_reco/E_truth ratio
- Outer segments show significant tails at low E_reco/E_truth ratio
- Low stats affect categories for segments further from electron



Invariant Mass

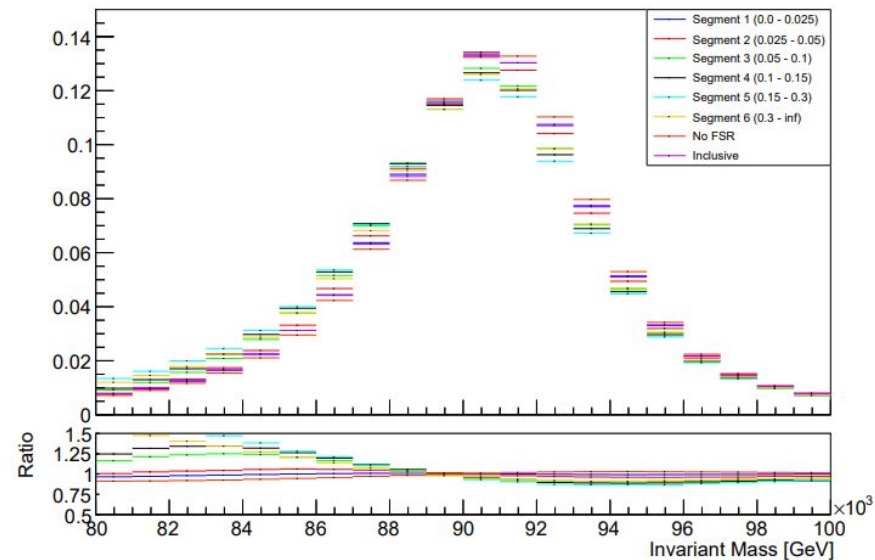
Studying invariant mass for each category shows potential effects of mismodelling on the distributions

Since invariant mass can only be defined by both electrons in an event (rather than on an electron-by-electron basis) the distribution for a given category is filled by an event if that event contains **AT LEAST ONE** electron in that category

This means the sum of the distributions for the categories is not equal to the inclusive distribution (Full MC sample)

- Innermost segments (blue and red) behave closest to inclusive since energy is contained within cluster (or very close by)
- Outer segments show larger deviation from inclusive and therefore higher chance for mismodelling by MVA since this region covers up to the edge of the satellite region (potential for lateral leakage in segment 3)
- No FSR category shows shift away from lower invariant mass

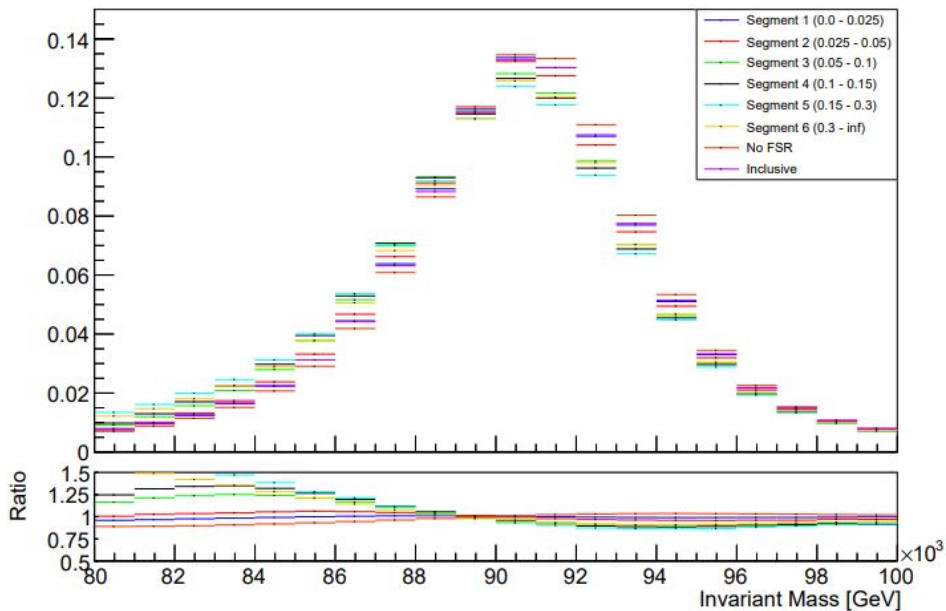
Tails similar to data/MC distributions seem to be reproduced by events with FSR further from electron



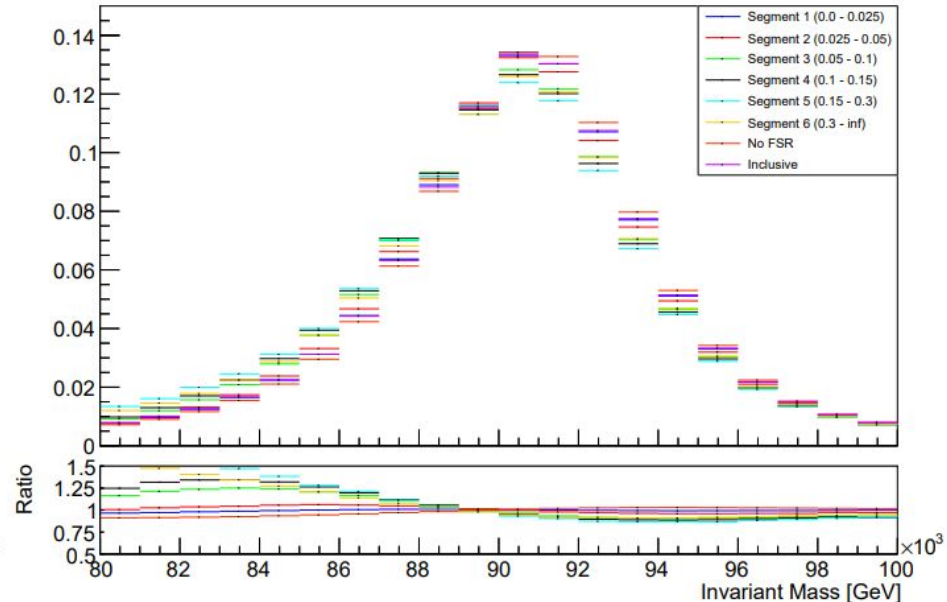
Above plot uses improved photon-matching matching model for far FSR (in $\Delta R > 0.3$ region)

Invariant Mass: FSR matching comparison

Old Matching



Improved Matching



Improved photon-electron matching for far FSR shows a small reduction in the low-energy tail in the segment 6 distribution while the tail increases in the no FSR category

I.e. more events classified as having at least one electron with no FSR (most likely from photons being matched to other electron that already has higher-energy photons in other segments)

Invariant Mass: Ratio vs No FSR

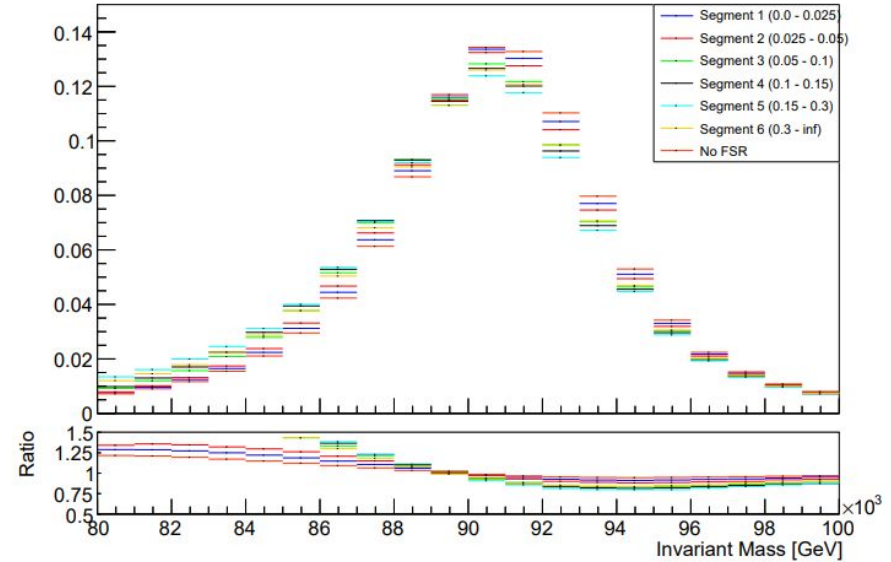
Invariant mass distributions for each category compared with exclusive no FSR distribution

Since MVA is trained on events with no FSR, born events (events with no FSR) in the MC sample are expected to be the most accurately reconstructed events.

By comparing categories with pure born events (both electrons with no FSR), we can see the impact of FSR in different regions

Current plot shows inclusive categories (events that contain at least one electron in a given segment) compared with exclusive no FSR distribution.

- As expected, no FSR category (events with at least one electron with no FSR) shows best agreement with exclusive no FSR - not perfect agreement due to electrons that still have FSR
- Segments 1 and 2 have slightly worse agreement - shows events not perfectly reconstructed
- Further segments show much worse agreement, indicating poor reconstruction with further FSR, as expected

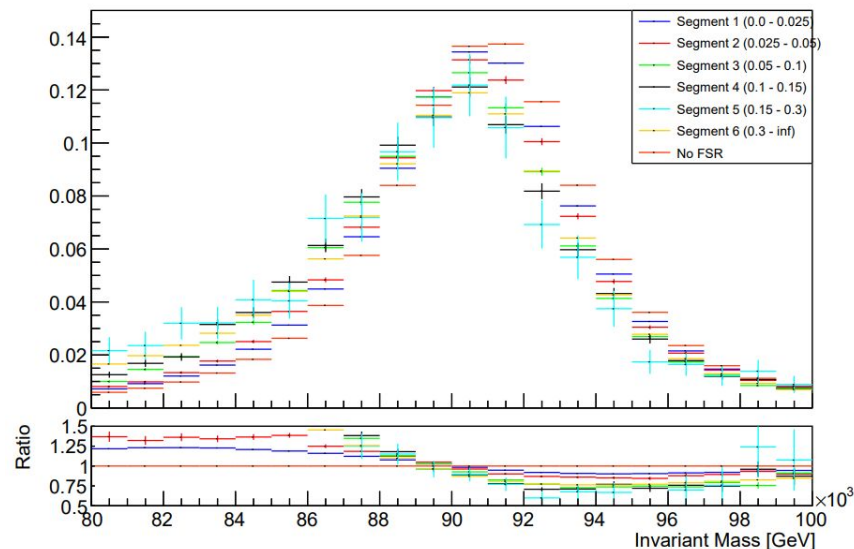


Invariant Mass: Ratio vs no FSR - Exclusive Categories

Invariant mass distributions for each category compared with exclusive no FSR distribution

Current plot shows exclusive categories (events that contain at least one electron in a given segment) compared with no FSR distribution.

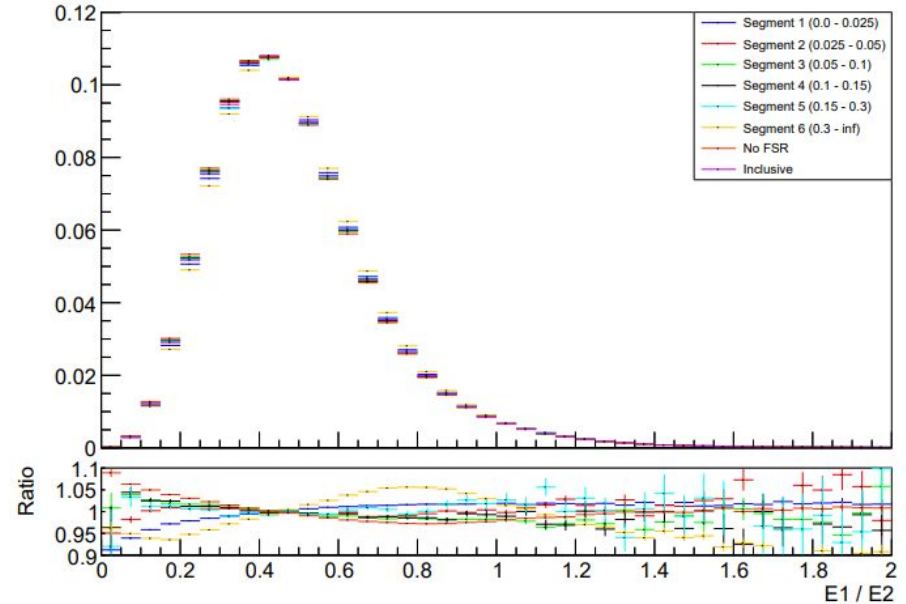
- No FSR category shows perfect agreement at 1
- Segments 1 and 2 are noticeably out of agreement with no FSR - shows events not perfectly reconstructed
- Further segments show much worse agreement, indicating poor reconstruction with further FSR, as expected



E1 / E2

Ratio of energies deposited in layers 1 and 2 of the calorimeter

- One of the most important variables for MVA calibration since: $E_{MVA} \approx E_0 + E_1 + E_2 + E_3$
- Depends both on electron and FSR photon energy and whether FSR is within the cluster
- Ratio usually $\approx \frac{1}{2}$, i.e. twice as much energy deposited in layer 2 than layer 1
- Plots shows that electrons in segments 1 and 2 behave roughly as expected but have a sharp decrease (compared to inclusive) below around 0.4
- Further segments show slight increase at lower ratios and decrease at higher ratios
- Segment 6 shows large increase between 0.6 and 1.0, suggesting more energy is deposited in layer 1



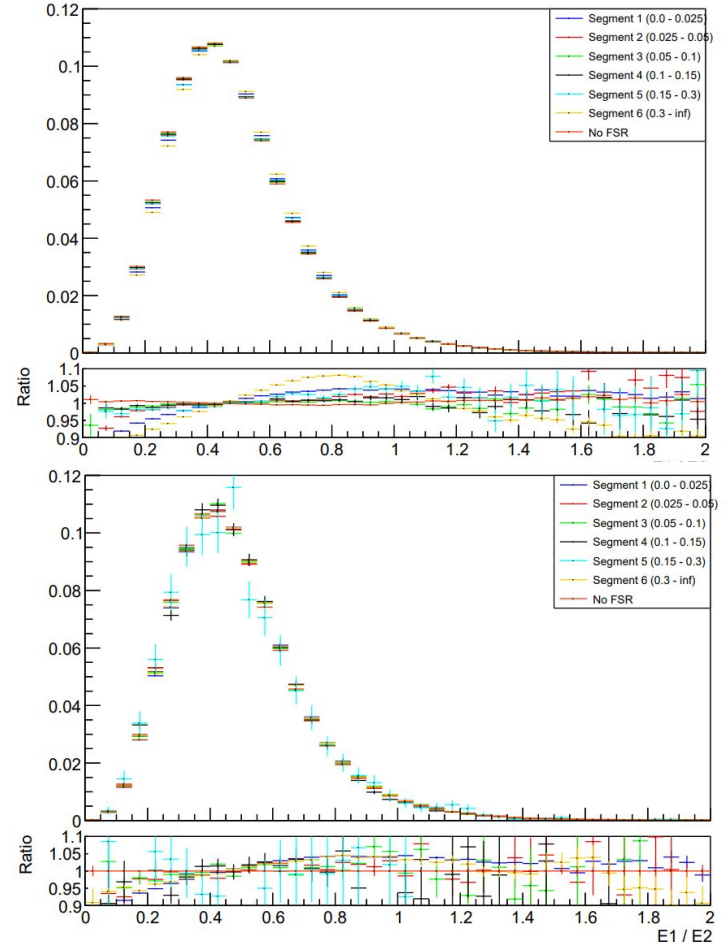
E1 / E2: Ratio vs no FSR

Ratio of energies deposited in layers 1 and 2 of the calorimeter

Current plot shows inclusive categories (top) and exclusive categories (bottom) compared with exclusive no FSR distribution.

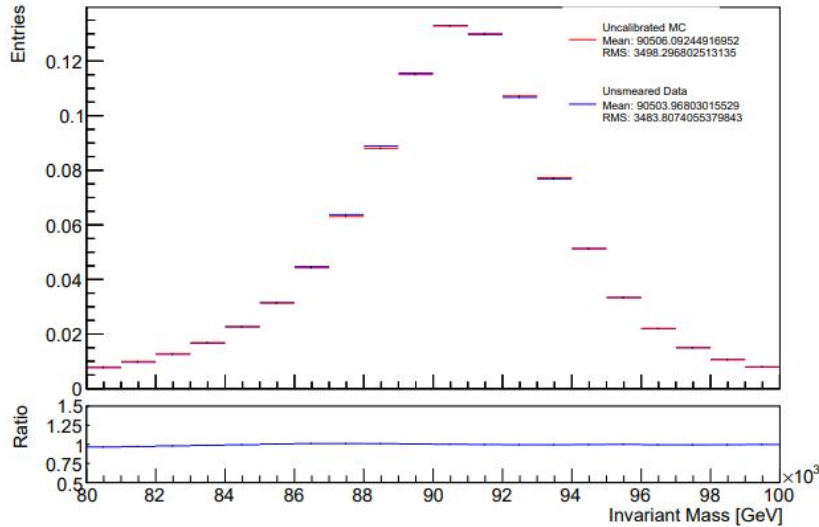
- As expected, inclusive no FSR category shows best agreement with exclusive no FSR.
- Far FSR category (segment 6, $dR > 0.3$) shows distinct peak around 0.8
- Segment 1 also shows clear peak, while other segments have high statistical uncertainty

Higher E1/E2 ratio indicates more energy deposited in Layer 1 of the calorimeter, caused by incident electrons having lower energy (as expected in segment 6)

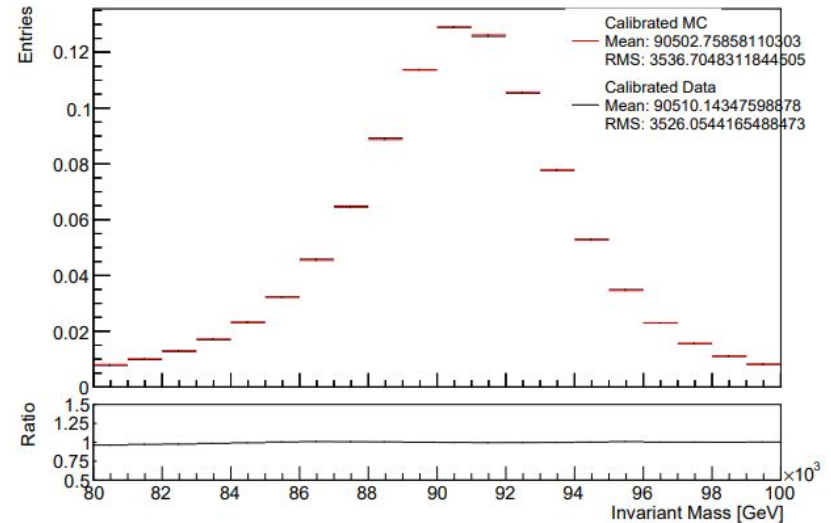


Pseudodata Calibration

Pre-insitu



Post-insitu



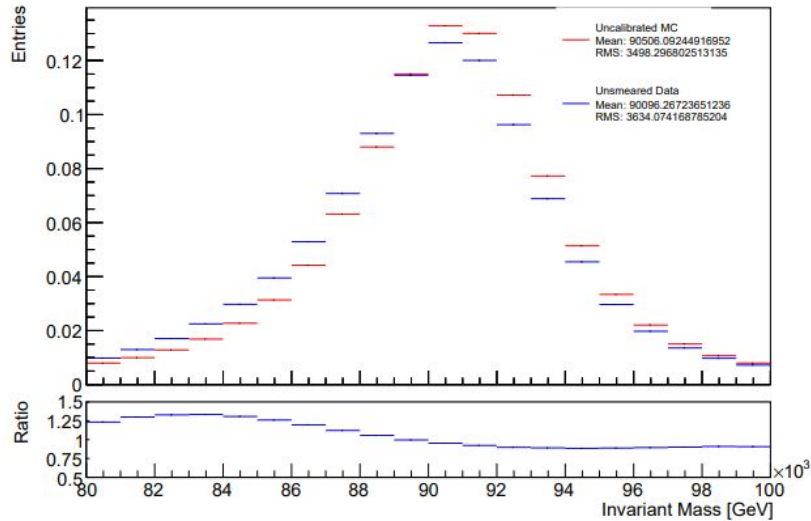
Full MC sample categorised into “pseudodata” samples based on segments (if events contains AT LEAST one electron in segment “X”, it is placed in category “X”)

Pseudodata (after 1% smearing injection) is calibrated against the full MC sample using the normal in-situ calibration method (Example shown for **Segment 1: 0.0 - 0.025**)

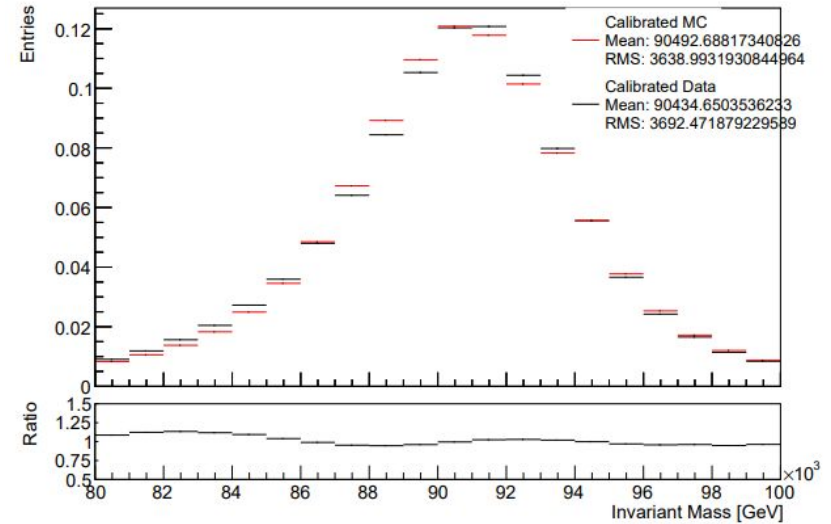
Calibration does not fully remove tail

Pseudodata Calibration

Pre-insitu



Post-insitu



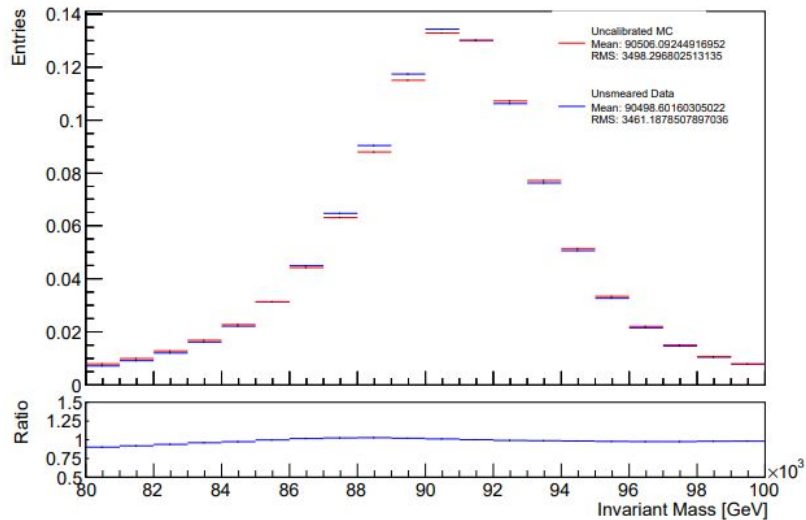
Full MC sample categorised into “pseudodata” samples based on segments (if events contains AT LEAST one electron in segment “X”, it is placed in category “X”)

Pseudodata (after 1% smearing injection) is calibrated against the full MC sample using the normal in-situ calibration method (Example shown for **Segment 4: 0.1 - 0.15**)

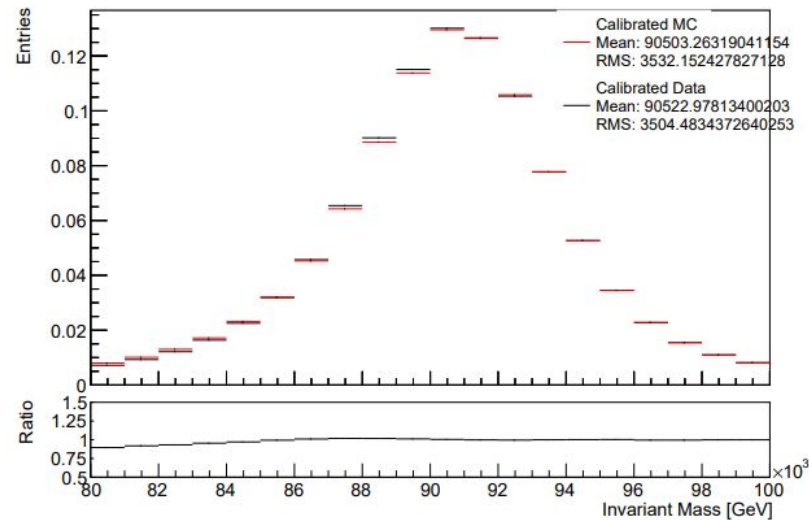
In further segments, calibration reduces does not fully remove tails - repeated for exclusive segments

Pseudodata Calibration - Exclusive Categories

Pre-insitu



Post-insitu



Full MC sample categorised into “pseudodata” samples based on segments (if events contains AT LEAST one electron in segment “X”, it is placed in category “X”)

Pseudodata (after 1% smearing injection) is calibrated against the full MC sample using the normal in-situ calibration method (Example shown for **Segment 1: 0.0 - 0.025**)

Calibration reduces does not fully remove tails - repeated for exclusive segments

Pseudodata Reweighting (original)

Investigating effects of changing proportion of events from each segment

Full MC sample is first calibrated against high-mu dataset

Calibrated MC events are sorted into FSR segments as before

New invariant mass distributions produced by independently scaling fraction of events from each segment:

- E.g. events containing at least one electron in segment “X” (in this case no FSR) scaled by 0.9 (red) or 1.1 (purple) (events with both electrons in the same segment are scaled by $0.9 * 0.9$ or $1.1 * 1.1$).

Reweighted distributions are then compared with data using χ^2 test to find combination of weights that gives lowest χ^2 value

Initial method used a range of weights (e.g. from 0.8 - 1.2 in 0.05 increments)

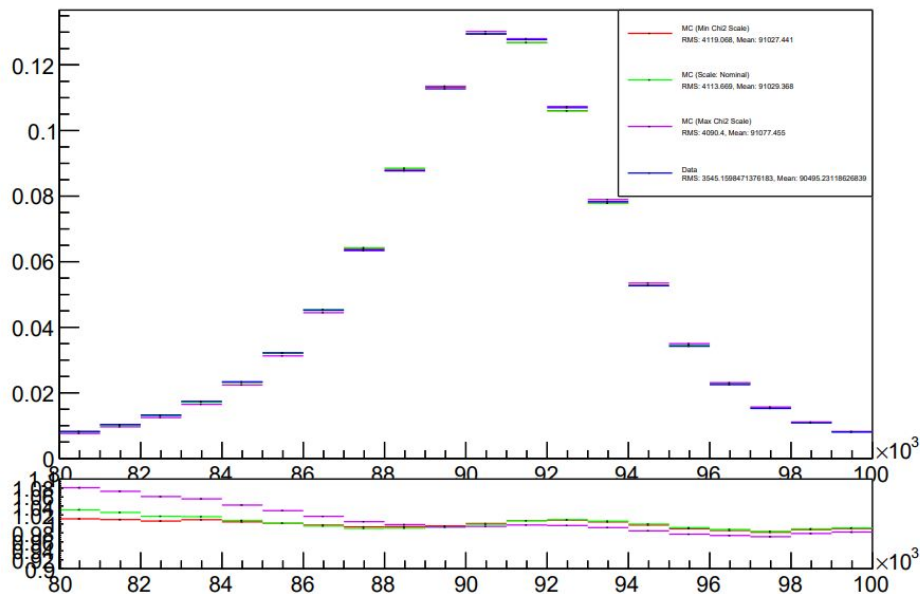
All possible combinations of weights were iterated through across the 7 FSR categories: i.e. all weight combinations from the range:

[0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 0.8] to **[1.2, 1.2, 1.2, 1.2, 1.2, 1.2, 1.2]**

Each combination was tested against the data to find the one with the lowest χ^2

Minimum χ^2 result given by: **[0.8, 0.8, 0.8, 0.85, 1.2, 1.2, 1.1]**

Maximum χ^2 result given by: **[0.8, 0.8, 0.8, 0.8, 0.8, 0.8, 1.2]**



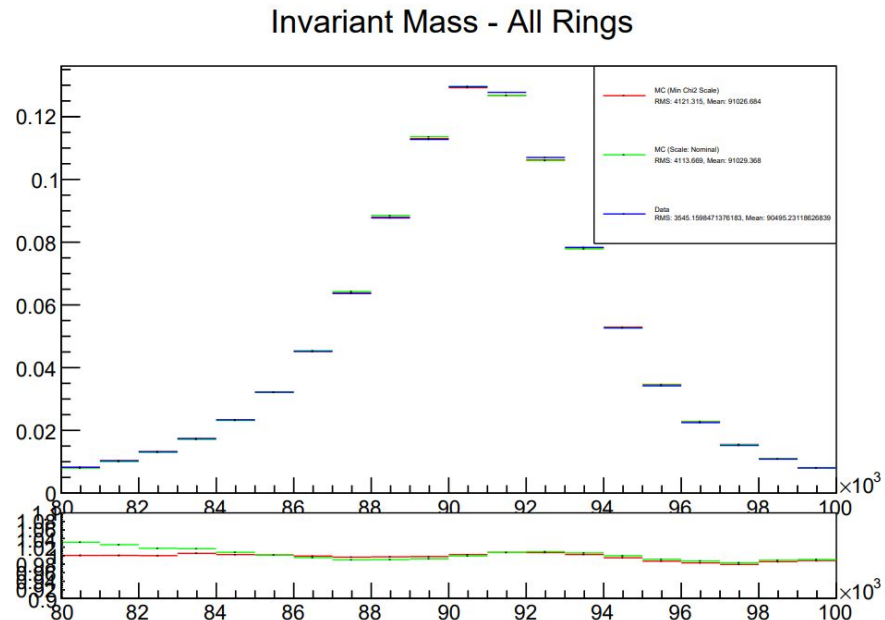
Inefficient and not a “true” minimum

Pseudodata Reweighting (Optimised)

Continuing investigation of effects of changing proportion of events from each segment

To improve the fitting of the weighted distribution to the data, `scipy.optimize.minimize()` was used:

- Start with some initial “hypothesis” e.g. all MC segment weights = 1.0
- Give some function to test goodness of fit (Chi squared and negative log likelihood both tested)
- `minimize()` handles optimisation of weights to give minimum chi squared / negative log likelihood



Pseudodata Reweighting Results (Optimised)

Continuing investigation of effects of changing proportion of events from each segment

After some initial, strange results (fits not converging, especially when using the negative log likelihood method) started normalising the full, un-weighted MC to the data before fitting

Results show that chi squared and negative log likelihood give similar weights:

Chi Squared - Pre-normalised data and MC:

Fitted weights: [0.57496483, 0.94606719, 0.95579646, 0.98644989, 1.00940794, 1.31974361, 1.21897932]

Chi_2: 731.6769263536437

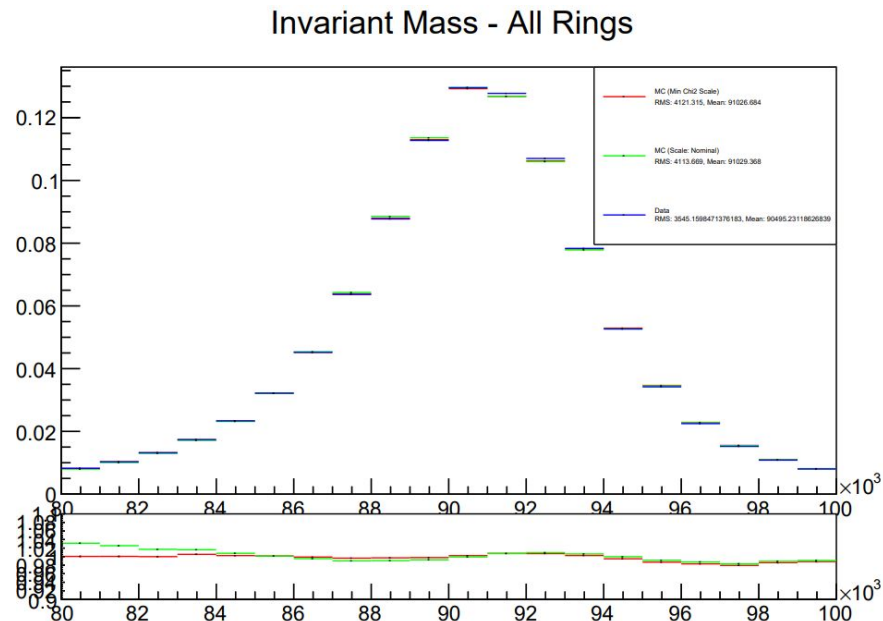
Chi_2_NDF: 38.50931191334967

Neg Log Likelihood - Pre-normalised data and MC:

Fitted weights: [0.5769822, 0.9570898, 0.94606284, 0.98994506, 1.01519951, 1.33165995, 1.22575883]

Chi_2: 731.642818746387

Chi_2_NDF: 38.507516776125634



Results shown for neg log likelihood fit

Conclusion

Low-energy tails in $Z \rightarrow ee$ invariant mass distributions in data vs MC prompted investigation into effects of QED FSR on Z mass lineshape

- Classification of events by angular separation and total pT of FSR photons was refined to account for Bremsstrahlung and far/wide-angle FSR that could be mis-matched to wrong electron
- Distributions of kinematic variables (invariant mass, $E_{\text{reco}}/E_{\text{truth}}$ ratio and E_1/E_2 ratio) show different behaviours depending on classification of electron/event. Electrons with further FSR, in general, deviate further from inclusive/nominal distributions
 - $E_{\text{reco}}/E_{\text{truth}}$ ratios suggest segments 1 and 2 are reconstructed best, when compared to born events. Further segments are in general reconstructed more poorly
 - Invariant mass distributions (in both inclusive and exclusive) again show a trend where further segments show increasing disagreement with born events - lower energy tails more present in events with further FSR
 - E_1/E_2 ratio is unclear for some categories but far FSR (segment 6, $dR > 0.3$) shows a clear increase in E_1/E_2 . Caused by lower energy electrons which deposit more energy in first layer
- Calibration of invariant mass distributions against inclusive does not remove tails (i.e. not solvable by applying currently-used methods)
- Used weighted sum of categorised distributions to attempt to match as close as possible to data
 - Initial method was rudimentary, only limited range and number of different weights
 - Improved using scipy optimisation algorithm to find true minimum χ^2 / negative log likelihood
 - Closest match to data given by roughly halving events in segment 1, while increasing far/noFSR by 20-30%