## Higgs Physics, New Physics and a tiny bit of Statistics

#### Philip Bechtle



7th - 8th March 2012



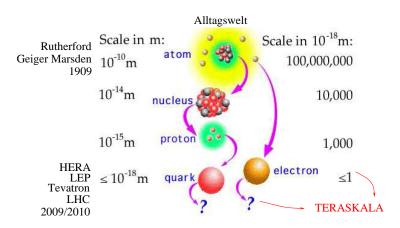
- 1 On the way to the Terascale
  - The Search for the System behind Matter
- 2 Higgs Physics
  - Theory
  - Limits
  - Discovery and Measurements?
- Other New Physics?
  - SUSY: The missing link at the Terascale?
  - One Possibility to Measure Features of SUSY
  - Other New Physics than SUSY



- 1 On the way to the Terascale
  - The Search for the System behind Matter
- 2 Higgs Physic
  - Theory
  - Limits
  - Discovery and Measurements?
- Other New Physics?
  - SUSY: The missing link at the Terascale?
  - One Possibility to Measure Features of SUSY
  - Other New Physics than SUSY



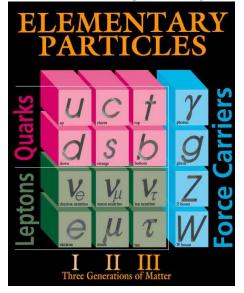
#### The Search for the Fundamental Buildung Blocks



A new era with the LHC, almost exactly 100 years after the first look into the atom

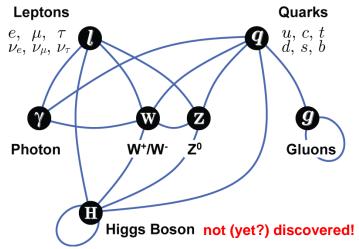


#### You know them very well by now . . .





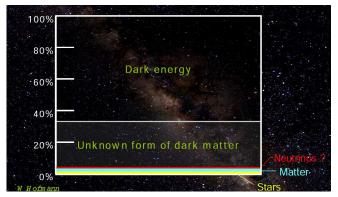
#### ... but this is a better picture of the SM particles:





## Why we know we missed something fundamental

Experimentally known: The SM is incomplete!



It it isn't dark, it doesn't matter

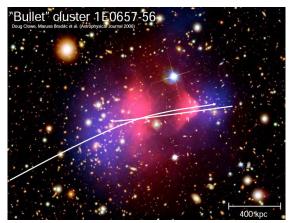
 We do not experimentally know any particle or field which could explain dark matter or dark energy



Bechtle:

## Why we know we missed something fundamental

Experimentally known: The SM is incomplete!



 We do not experimentally know any particle or field which could explain dark matter or dark energy

#### The Search for the System behind Matter

Introduction to Terascale Physics School 2012



Bechtle:

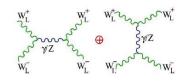


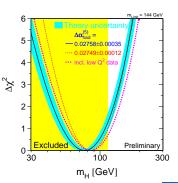
Why is the electromagnetic force of this tiny magnet so much stronger than all the gravity of the whole planet?



#### Why there must be New Physics at the Terascale

- We expect new physics at the Terascale  $\approx 1\,\mathrm{TeV}$
- For theoretical reasons:
  - Without the Higgs: SM WW scattering violates unitarity at  $\sqrt{s} \approx 1 \, \mathrm{TeV}$
  - Very severe fine-tuning problem between  $m_h$  and  $m_{GUT}$ : Need new physics below  $\approx 1\,\mathrm{TeV}$
- For experimental reasons:
  - Blue-band-plot shows that something like the Higgs must be there!
     Otherwise, all precision data would be wrong by orders of magnitude!
  - Dark matter







## Teilchenphysik ist auch Philosophie

Nicht von Beginn an enthüllten die Götter uns Sterblichen alles; Aber im Laufe der Zeit finden wir, suchend, das Bess're. Diese Vermutung ist wohl, ich denke, der Wahrheit recht ähnlich. Sichere Wahrheit erkannte kein Mensch und wird keiner erkennen Über die Götter und alle die Dinge, von denen ich spreche, Selbst wenn es einem einst glückt, die vollkommene Wahrheit zu künden,

Wissen kann er es nie: Es ist alles durchwebt von Vermutung.

XENOPHANES VON KOLOPHON, ca. 500 v.u.Z.



- $oldsymbol{0}$  On the way to the Terascale
  - The Search for the System behind Matter
- 2 Higgs Physics
  - Theory
  - Limits
  - Discovery and Measurements?
- Other New Physics?
  - SUSY: The missing link at the Terascale?
  - One Possibility to Measure Features of SUSY
  - Other New Physics than SUSY



• Global Gauge Invariance: Require that  $\mathcal{L}$  (i.e. the equation of motion) is invariant under the transformation:

$$\psi(x) \to e^{i\alpha}\psi(x)$$

with  $\alpha$  being the same everywhere.



• Global Gauge Invariance:

Require that  $\mathcal{L}$  (i.e. the equation of motion) is invariant under the transformation:

$$\psi(x) \to e^{i\alpha}\psi(x)$$

with  $\alpha$  being the same everywhere. But given relativity, why should we use the same gauge here and behind the moon at the same time?



• Global Gauge Invariance: Require that  $\mathcal{L}$  (i.e. the equation of motion) is invariant under the transformation:

$$\psi(x) \to e^{i\alpha} \psi(x)$$

with  $\alpha$  being the same everywhere. But given relativity, why should we use the same gauge here and behind the moon at the same time?

Local Gauge Invariance:
 Require that L is invariant under local transformations:

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$



• Global Gauge Invariance: Require that  $\mathcal{L}$  (i.e. the equation of motion) is invariant under the transformation:

$$\psi(x) \to e^{i\alpha} \psi(x)$$

with  $\alpha$  being the same everywhere. But given relativity, why should we use the same gauge here and behind the moon at the same time?

Local Gauge Invariance:
 Require that L is invariant under local transformations:

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

This principle is the foundation of the SM



QED is a local abelian U(1) gauge symmetry

Using our knowledge about the Lagrangian, we construct the Lagrangian which gives us the equation of motion of the Dirac equation  $((i\partial_{\mu}\gamma^{\mu}-m)\psi=0)$ :

$$\mathcal{L}_{\text{free}} = \bar{\psi} (i\partial \!\!\!/ - m) \psi$$

using  $\partial \!\!\!/ = \partial_{\mu} \gamma^{\mu}.$ 

Make the theory gauge invariant under local U(1) transformations:

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

What is the transformation behaviour of the free Lagrangian?



Introduction to Terascale Physics School 2012

#### Introduction: QED

QED is a local abelian U(1) gauge symmetry

Using our knowledge about the Lagrangian, we construct the Lagrangian which gives us the equation of motion of the Dirac equation  $((i\partial_{\mu}\gamma^{\mu}-m)\psi=0)$ :

$$\mathcal{L}_{\text{free}} = \bar{\psi}(i\partial \!\!\!/ - m)\psi$$

using  $\partial = \partial_{\mu} \gamma^{\mu}$ .

Make the theory gauge invariant under local U(1) transformations:

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

What is the transformation behaviour of the free Lagrangian?

$$\mathcal{L}_{\mathrm{free}} o \mathcal{L}_{\mathrm{free}} - \bar{\psi} \gamma_{\mu} \psi (\partial^{\mu} \alpha(\mathsf{x}))$$



Bechtle:

QED is a local abelian U(1) gauge symmetry

Using our knowledge about the Lagrangian, we construct the Lagrangian which gives us the equation of motion of the Dirac equation  $((i\partial_{\mu}\gamma^{\mu}-m)\psi=0)$ :

$$\mathcal{L}_{\text{free}} = \bar{\psi} (i \partial \!\!\!/ - m) \psi$$

using  $\partial = \partial_{\mu} \gamma^{\mu}$ .

Make the theory gauge invariant under local U(1) transformations:

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

What is the transformation behaviour of the free Lagrangian?

$$\mathcal{L}_{\mathrm{free}} o \mathcal{L}_{\mathrm{free}} - \bar{\psi} \gamma_{\mu} \psi(\partial^{\mu} \alpha(\mathsf{x}))$$

That's not invariant!

Bechtle:



QED is a local abelian U(1) gauge symmetry

Using our knowledge about the Lagrangian, we construct the Lagrangian which gives us the equation of motion of the Dirac equation  $((i\partial_{\mu}\gamma^{\mu}-m)\psi=0)$ :

$$\mathcal{L}_{\text{free}} = \bar{\psi} (i \partial \!\!\!/ - m) \psi$$

using  $\partial \!\!\!/ = \partial_{\mu} \gamma^{\mu}$ .

Make the theory gauge invariant under local U(1) transformations:

$$\psi(x) \to e^{i\alpha(x)}\psi(x)$$

What is the transformation behaviour of the free Lagrangian?

$$\mathcal{L}_{\text{free}} \to \mathcal{L}_{\text{free}} - \bar{\psi} \gamma_{\mu} \psi(\partial^{\mu} \alpha(\mathbf{x}))$$

That's not invariant!

But luckily it's also not QED . . .



In order to save QED under the transformation  $U(x) = e^{-1\alpha(x)}$ , add a gauge field obeying:

$$A_{\mu}(x) \rightarrow U^{-1}A_{\mu}U + \frac{1}{q}U^{-1}\partial_{\mu}U = A_{\mu}(x) - \frac{1}{q}\partial_{\mu}\alpha(x)$$

A miracle has occured: we introduced not only a gauge field, but also a charge q. Also, we would have needed the photon  $A_{\mu}$  anyway...

Now modify the derivative:

$$\partial_{\mu} \rightarrow \partial_{\mu} + iqA_{\mu}(x) = D_{\mu}$$

Let's write  $\mathcal L$  again with all possible Lorentz and gauge invariant terms:

$$\mathcal{L} = -rac{1}{4} F_{\mu
u} F^{\mu
u} + ar{\psi} (i\partial \!\!\!/ - m) \psi - q ar{\psi} A \!\!\!/ \psi$$

using

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$$



Let's check the transformational behaviour under local U(1) again:

$$\mathcal{L}
ightarrow\mathcal{L}'=-rac{1}{4}F'_{\mu
u}F'^{\mu
u}+ar{\psi}'(i\partial\!\!\!/-m)\psi'-qar{\psi}'A\!\!\!/\psi'$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial \!\!\!/ - m)\psi - \bar{\psi}\gamma_{\mu}\psi(\partial^{\mu}\alpha(x)) - q\bar{\psi}\gamma_{\mu}\psi A^{\mu} + \bar{\psi}\gamma_{\mu}\psi(\partial^{\mu}\alpha(x))$$
$$= \mathcal{L}$$

with

$$F'_{\mu\nu} = \partial_{\mu}(A_{\nu} - \frac{1}{q}\partial_{\nu}\alpha(x)) - \partial_{\nu}(A_{\mu} - \frac{1}{q}\partial_{\nu}\alpha(x))$$
$$= F_{\mu\nu} - \partial_{\mu}\frac{1}{q}\partial_{\nu}\alpha(x) + \partial_{\nu}\frac{1}{q}\partial_{\mu}\alpha(x) = F_{\mu\nu}$$

QED including a gauge field is invariant under local U(1)! Use this principle to construct the SM



## **QFD**: $SU(2)_I \times U(1)_Y$ Leptonic Sector

We choose the SU(2), doublett

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_{L} = \frac{1}{2} (1 - \gamma^{5}) \begin{pmatrix} \nu \\ e \end{pmatrix}, \quad \begin{matrix} I_{3} = +\frac{1}{2}, \ Q = 0, \ Y = -1 \\ I_{3} = -\frac{1}{2}, \ Q = -1, \ Y = -1 \end{matrix}$$

and the singlett

$$R = e_R = \frac{1}{2}(1 + \gamma^5)e$$
,  $I_3 = 0$ ,  $Q = -1$ ,  $Y = -2$ 

which transform SU(2), according to

$$L \rightarrow L' = e^{i\alpha^a \frac{\tau_a}{2}} L, \quad R \rightarrow R' = R$$

and under  $U(1)_{\vee}$  according to

Bechtle:

$$L \rightarrow L' = e^{i\beta^a \frac{Y}{2}} L, \quad R \rightarrow R' = e^{i\beta^a \frac{Y}{2}} R$$



# **QFD**: $SU(2)_L \times U(1)_Y$ Leptonic Sector

Now we construct the gauge fields  $W_{\mu}^{a}$  for  $SU(2)_{L}$  analogously to  $SU(3)_{C}$  before and  $B_{\mu}$  of  $U(1)_{Y}$  analously to the QED before. We get the covariant derivative

$$D_{\mu} = \partial_{\mu} + ig\frac{\tau_{a}}{2}W_{\mu}^{a} + ig'\frac{Y}{2}B_{\mu}.$$

Using this, we can construct the first part of the QFD Lagrangian

$$\mathcal{L}_{\mathrm{QFD}}^{1} = -\frac{1}{4}W_{\mu\nu}^{a}W_{a}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\overline{L}\not\!D L + i\overline{R}\not\!D R,$$

with

$$W_{\mu\nu}^{a} = \partial_{\mu}W_{\nu}^{a} - \partial_{\nu}W_{\mu}^{a} - g\epsilon_{bc}^{a}W_{\mu}^{b}W_{\nu}^{c}$$
$$B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}.$$



**QFD:** 
$$SU(2)_L \times U(1)_Y$$
 Masses

Mass of the gauge bosons
 Now we would like to add gauge boson masses:

$$\frac{1}{2}M^2B^{\mu}B_{\mu}$$

However, this is not invariant under SU(2):

$$\rightarrow \frac{1}{2}M^2(B^{\mu} - \frac{1}{g'}\partial^{\mu}\alpha(x))(B_{\mu} - \frac{1}{g'}\partial_{\mu}\alpha(x))$$



# **QFD:** $SU(2)_L \times U(1)_Y$ Masses

Mass of the gauge bosons
 Now we would like to add gauge boson masses:

$$\frac{1}{2}M^2B^{\mu}B_{\mu}$$

However, this is not invariant under SU(2):

$$ightarrow rac{1}{2}M^2(B^\mu - rac{1}{g'}\partial^\mu lpha(x))(B_\mu - rac{1}{g'}\partial_\mu lpha(x))$$

Mass of the fermions

$$-mar{e}e = -mar{e}\left(rac{1}{2}(1-\gamma^5) + rac{1}{2}(1+\gamma^5)
ight)e$$

$$= -m(ar{e}_Re_L + ar{e}_Le_R)$$

But only  $e_L$  and not  $e_R$  is transforming under SU(2)!



# **QFD**: $SU(2)_I \times U(1)_Y$ Masses

 Mass of the gauge bosons Now we would like to add gauge boson masses:

$$\frac{1}{2}M^2B^{\mu}B_{\mu}$$

However, this is not invariant under SU(2):

$$ightarrow rac{1}{2}M^2(B^\mu - rac{1}{g'}\partial^\mu lpha(x))(B_\mu - rac{1}{g'}\partial_\mu lpha(x))$$

Mass of the fermions

$$-mar{e}e = -mar{e}\left(rac{1}{2}(1-\gamma^5) + rac{1}{2}(1+\gamma^5)
ight)e$$

$$= -m(ar{e}_Re_L + ar{e}_Le_R)$$

But only  $e_L$  and not  $e_R$  is transforming under SU(2)!

In order to allow masses for the gauge bosons, we introduce the Higgs doublett into the theory:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \ Y = +1 \quad \text{ which is gauged like } \quad \Phi = e^{i\frac{\sigma_a\alpha^a}{2\nu}}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \nu + \eta \end{pmatrix}$$

We obtain  $v=\sqrt{-\mu^2/\lambda}$  as vacuum expectation value of the field in the potential

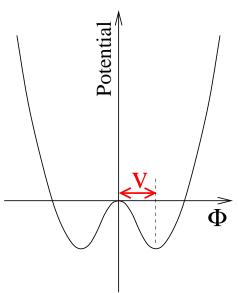
$$V(\Phi) = \frac{\mu^2}{2} \Phi^+ \Phi + \frac{\lambda}{4} (\Phi^+ \Phi)^2$$

with  $\lambda>0$  and  $\mu^2<0$ , such that there is spontaneous symmetry breaking (the ground state does not obey the symmetries of the theory).  $\phi^+$  has to be gauged to 0 in order to render the charge operator  $Q=I_3+\frac{Y}{2}$  unbroken. Otherwise the photon acquires mass.



Introduction to Terascale Physics School 2012

## **Higgs Potential**





Bechtle:

QFD: 
$$SU(2)_L \times U(1)_Y$$
 EWSB

Using the global  $SU(2)_L$  gauge transformation from before

$$L \to L' = e^{-i\frac{\sigma^2 \alpha_a}{2\nu}} L \Rightarrow \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + \eta \end{pmatrix}$$

we obtain the following expression for the mass sector of the QFD:

$$\mathcal{L}_{\mathrm{QFD}}^2 = -\sqrt{2}f(\overline{L}\Phi R + \overline{R}\Phi^+ L) + |D_\mu\Phi|^2 - V(\Phi)$$



Using the global  $SU(2)_L$  gauge transformation from before

$$L 
ightarrow L' = e^{-irac{\sigma^2 lpha_a}{2\nu}} L \, \Rightarrow \, \Phi = rac{1}{\sqrt{2}} \left( egin{array}{c} 0 \\ v + \eta \end{array} 
ight)$$

we obtain the following expression for the mass sector of the QFD:

$$\mathcal{L}_{\mathrm{QFD}}^2 = -\sqrt{2}f\big(\overline{L}\Phi R + \overline{R}\Phi^+ L\big) + |D_\mu\Phi|^2 - V(\Phi)$$

From where do we get the fermion masses?

$$-\sqrt{2}f(\overline{L}\Phi R+\overline{R}\Phi^+L)$$

acts as a mass term with the Yukawa coupling parameter f determining the mass of the fermion.



The gauge boson masses are coming from

$$|D_{\mu}\Phi|^2 = \frac{1}{8}g^2v^2(W_{\mu\nu}^a)^2 + \frac{1}{8}g'^2v^2B_{\mu}B^{\mu} - \frac{1}{4}gg'v^2B^{\mu}W_{\mu}^3$$

using

$$(W_{\mu}^{1})^{2} + (W_{\mu}^{2})^{2} = (W_{\mu}^{1} + iW_{\mu}^{2})(W_{\mu}^{1} - iW_{\mu}^{2}) = 2W_{\mu}^{+}W_{\mu}^{-}$$

introducing the charged currents. That yields

$$\frac{1}{4}g^2v^2W_{\mu}^{+}W_{\mu}^{-} + \frac{1}{8}v^2(B^{\mu},W_{\mu}^{3})\begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix}\begin{pmatrix} B^{\mu} \\ W^{3\mu} \end{pmatrix}$$

We have the mass term on the  $W^{\pm}$  already. Let's diagonalize the mass matrix of the hypercharge field  $B_{\mu}$  and the third component of the  $SU(2)_L$  gauge field  $W^3_{\mu}$ :

$$\begin{pmatrix} A_{\mu} \\ Z_{\mu}^{0} \end{pmatrix} = \begin{pmatrix} \cos\theta_{W} & \sin\theta_{W} \\ -\sin\theta_{W} & \cos\theta_{W} \end{pmatrix} \begin{pmatrix} B^{\mu} \\ W^{3\mu} \end{pmatrix}$$

Now another miracle has occured: The photon field  $A_{\mu}$  drops out of EWSB.

we have now introduced the Weinberg angle

$$\sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

From the diagonalization of the mass matrix for  $W_{\mu}^3$  and  $B_{\mu}$ 

$$A_{\mu} = rac{1}{\sqrt{g^2 + {g'}^2}} (g' W_{\mu}^3 + g B_{\mu}), \quad m_A^2 = 0$$

$$Z_{\mu}^{0}=rac{1}{\sqrt{\sigma^{2}+{\sigma^{\prime}}^{2}}}(gW_{\mu}^{3}-g^{\prime}B_{\mu}), \quad m_{Z^{0}}^{2}=rac{(g^{2}+{g^{\prime}}^{2})v^{2}}{4}$$



We also obtain the charged current and its coupling to the  $W_{\mu}^{+}$  as

$$\frac{g}{2\sqrt{2}}(\bar{\nu}_L\gamma^\mu e_L W_\mu^+ + h.c.)$$

In addition, as the first tested firm prediction of this theory, the neutral currents have been introduced ('74 November revolution: Gargamelle):

$$\frac{\sqrt{g^2 + {g'}^2}}{4} (\overline{L} \gamma^{\mu} \tau_3 L - 2 \frac{{g'}^2}{g^2 + {g'}^2} \overline{e} \gamma^{\mu} e) Z_{\mu}^0, \qquad \frac{g g'}{\sqrt{g^2 + {g'}^2}} \overline{e} \gamma^{\mu} e A_{\mu}$$

where

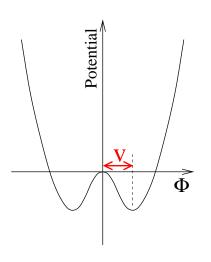
$$q_e = \frac{gg'}{\sqrt{g^2 + g'^2}}$$

is the electromagnetic charge and  $e = e_I + e_R$ 

This formalism has to be written for all three lepton families  $\ell=e,\mu_{\text{two-first hom}}$ 



# **QFD**: $SU(2)_L \times U(1)_Y$ Properties of the Higgs



Bechtle:

- The heavier the particle, the stronger the Higgs coupling to it (or the other way around!)
- The position of the minimum of the potential

$$V(\Phi) = \frac{\mu^2}{2} \Phi^+ \Phi + \frac{\lambda}{4} (\Phi^+ \Phi)^2$$

is known: Compare

$$\frac{\rm g}{2\sqrt{2}}\bar{\nu}_{\rm L}\gamma^{\mu}{\rm e}_{\rm L}W_{\mu}^+$$

with V-A theory:  $\mathcal{L}_{\it eff}^{V-A} \sim -\frac{G_F}{2} \dots$ 

$$\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} = \frac{G_F}{2} \Rightarrow v = 246 \,\text{GeV}$$

# **QFD:** $SU(2)_L \times U(1)_Y$ Remarks

There are a few non-trivial observations about EWSB in the SM:

ullet It is not trivial that the photon field  $A_{\mu}$  fullfills

$$m_A = 0$$
 $q_e \bar{e} \gamma^\mu e A_\mu$ 

(i.e. no coupling to the neutrino and the same coupling to the left and right fields) at the same time!

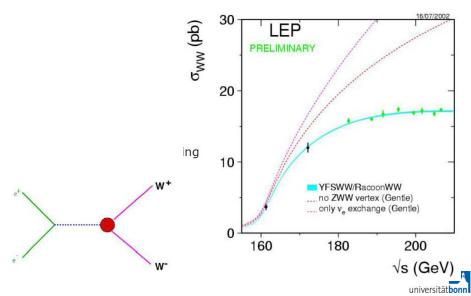
All three elements of

$$\frac{M_W}{M_Z} = \cos \theta_W$$

can be measured independently  $\Rightarrow$  precision tests

- The Higgs has been introduced to give mass to the gauge bosons, but it
  offers an elegant way to introduce masses of the fermions, too.
- There is a self-interaction among the gauge bosons in the  $-\frac{1}{4}W_{\mu\nu}^aW_a^{\mu\nu}$  term. This just pops out of the theory, it was not constructed as the gauge boson fermion interactions. Does Nature obey the SM also in this unforeseen field  $\Rightarrow$  precision tests

# **Self Interaction of Gauge Bosons**



## Graphical Representation of how Mass is Created

The Higgs mechanism is like a boring cocktail party:







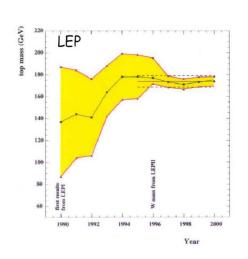
"famousness"  $g_f$  of a particle determines its mass:

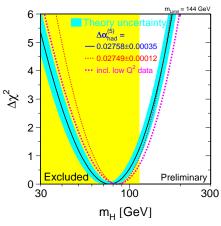
$$\frac{1}{\not q} + \frac{1}{\not q} \left( \frac{g_f v}{\sqrt{2}} \right) \frac{1}{\not q} + \dots = \frac{1}{\not q} \sum_{n=0}^{\infty} \left[ \left( \frac{g_f v}{\sqrt{2}} \right) \frac{1}{\not q} \right]^n = \frac{1}{\not q - \left( \frac{g_f v}{\sqrt{2}} \right)}$$



#### **Precision Tests of Loop Corrections**

 $e^+e^-$  machines can see effects of virtual particles

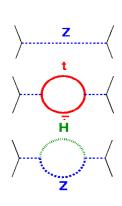






#### **Precision Tests of Loop Corrections**

 $e^+e^-$  machines can see effects of virtual particles

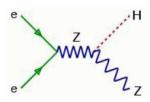


	Measurement	Fit	O <sup>meas</sup> -O <sup>fit</sup>  /σ <sup>meas</sup> 0 1 2 3
$\Delta \alpha_{had}^{(5)}(m_Z)$	$0.02758 \pm 0.00035$	0.02768	
m <sub>z</sub> [GeV]	$91.1875 \pm 0.0021$	91.1875	
Γ <sub>Z</sub> [GeV]	$2.4952 \pm 0.0023$	2.4957	<u> </u>
$\sigma_{had}^{0}$ [nb]	$41.540 \pm 0.037$	41.477	<u> </u>
	$20.767 \pm 0.025$	20.744	<u> </u>
$A_{fb}^{0,I}$	$0.01714 \pm 0.00095$	0.01645	_
$A_{l}(P_{\tau})$	$0.1465 \pm 0.0032$	0.1481	<u>-</u>
R <sub>b</sub>	$0.21629 \pm 0.00066$	0.21586	<u> </u>
R <sub>c</sub>	$0.1721 \pm 0.0030$	0.1722	
A <sub>fb</sub> <sup>0,b</sup> A <sub>fb</sub> <sup>0,c</sup>	$0.0992 \pm 0.0016$	0.1038	
A <sub>fb</sub> <sup>0,c</sup>	$0.0707 \pm 0.0035$	0.0742	
A <sub>b</sub>	$0.923 \pm 0.020$	0.935	<u> </u>
A <sub>c</sub>	$0.670 \pm 0.027$	0.668	<b>)</b>
A <sub>I</sub> (SLD)	$0.1513 \pm 0.0021$	0.1481	
$\sin^2 \theta_{eff}^{lept}(Q_{fb})$	$0.2324 \pm 0.0012$	0.2314	<u> </u>
m <sub>w</sub> [GeV]	$80.398 \pm 0.025$	80.374	
$\Gamma_{W}$ [GeV]	$2.140 \pm 0.060$	2.091	<u> </u>
m, [GeV]	$170.9 \pm 1.8$	171.3	<u> </u>
)			0 1 2 3

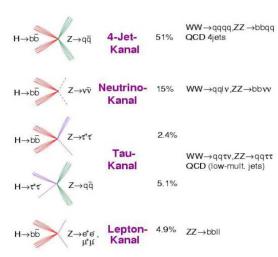
$$M_Z^2 = M_Z^{2 \text{ 0th order}} (1 + \mathcal{O}(m_t^2) + \mathcal{O}(\ln m_h^2) + \cdots)$$



# **Hunting for the Higgs: Signatures**



- The different Higgs decays and the different Z desays together define the signatures:
- For  $m_h < 115 \,\mathrm{GeV}$ : More than 80 % of all decays
- Typcal selection efficiencies: 50 %

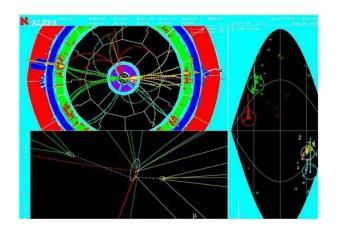




Introduction to Terascale Physics School 2012

## A Higgs Candidate

• A nice Higgs candidate from ALEPH ( $m_h = 115 \, \mathrm{GeV}$ ):

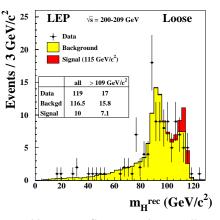


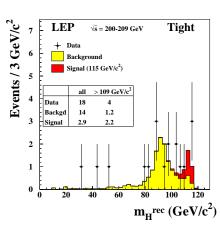


Bechtle:

#### Do we see a Higgs mass peak?

• Are there many of these candidates?





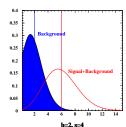
Introduction to Terascale Physics School 2012

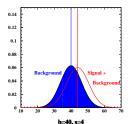
How significant is the small excess? Need advanced statistical analysis?

## How to Calculate the Sensitivity?

- If hypothesis exists with  $d \approx s+b$  on a significant level: Higgs found
- ullet If not: Calculate, how improbable a certain hypothesis ullet is ullet exclusion
- First example: Add all s, b, d of all channels (Counting Experiment)
- If  $s\neq 0$  only in one channel: this degrades sensitivity

Poisson-distributions for s=4,b=2 Poisson-distributions for s=4,b=40





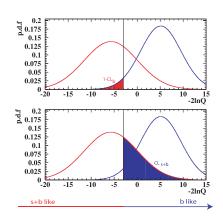
Not the most sensitive method . . . Instead: Define test statistics

$$-2 \ln Q = -2 \ln \frac{P_d(s+b)}{P_d(b)} = -2 \sum_{i} s_i + 2 \sum_{i} d_i \ln \left(1 + \frac{s_i}{b_i}\right)$$

## How to Calculate the Sensitivity?

- For optimal sensitivity, do just not add the total channel contents
   but use the information of full (mass) distributions
- Define the test statistics Q as a likelihood ratio  $P_d(s+b)/P_d(b)$
- Define 1 CL<sub>b</sub>: Probability of a b-experiment to give a less background like result than the observed one
- Define CL<sub>s+b</sub>: Probability of a s+b-experiment to give a more background like result than the observed one

Higgs. Searches. Statistics



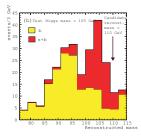
Conservative limit:

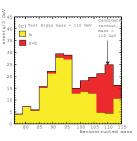
$$\mathrm{CL}_s = \mathrm{CL}_{s+b}/\mathrm{CL}_b$$

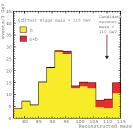


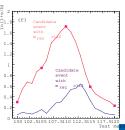
#### How to Calculate the Sensitivity?

- Don't know  $m_{\rm H_1}$ ,  $m_{\rm H_2}$ ,  $\sigma_{\mathrm{H}_i\mathrm{Z}},\ldots$  a priori
- Finite Detector Resolution  $\rightarrow$ 
  - $m_{\rm rec} \neq m_{\rm H}$
- → Test result of all searches under different hypotheses of  $m_{\rm H_1}$ ,  $m_{\rm H_2}$ ,  $\sigma_{\mathrm{H}_i\mathrm{Z}},\dots$



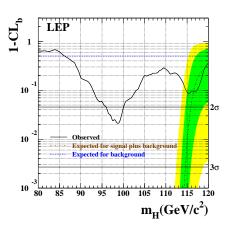








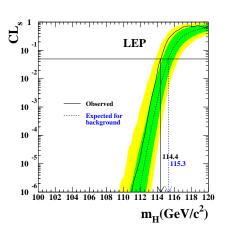
#### Is there a Significant Excess?



- $(1 CL_b)$  is a measure of the 'background-likeness' of an experiment. If  $(1 - CL_b)$  is e.g. 5%, then the probability of this outcome to be caused by a fluctuation of the background is 5 %
- No excess above  $3\sigma$
- Be aware of the 'look-elsewhere' effect1



## No Significant Excess: What's the Limit?



- $CL_s$  is a measure of how signal-like the outcome of an experiment is. If  $CL_s$  is small, it is very unlikely that there is a signal. Hence, a 95 % CL corresponds to  $CL_s = 0.05$
- Final word from LEP on the SM Higgs:

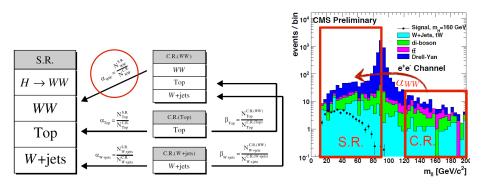
$$m_h > 114.4 \,\mathrm{GeV}$$



# Now we do things in a more complicated way...

Theory

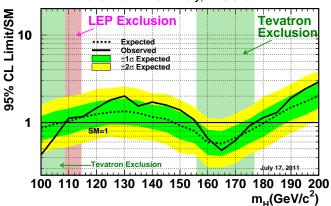
• The big thing since LEP: Ged rid of partly bayesian techniques by fitting the systematic uncertainties to the data during limit setting at each toy MC



#### **Current Situation**

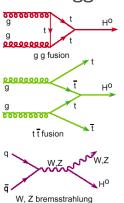
#### Tevatron:

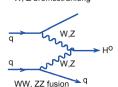
Tevatron Run II Preliminary, L ≤ 8.6 fb<sup>-1</sup>

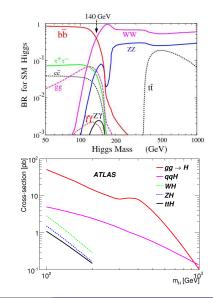


check http://indico.in2p3.fr/conferenceDisplay.py?confId=6001 and http://moriond.in2p3.fr/QCD/2012/qcd.html for updates universitätbonr

# Higgs Production and Decay at the LHC









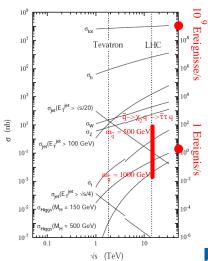
#### The ATLAS Experiment

ATLAS and CMS: First direct experimental access to the Terascale



Diameter 25 m Length 46 m Weight 7000 t

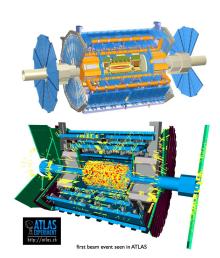
pprox 100 Million readout channels pprox 3000 km cables

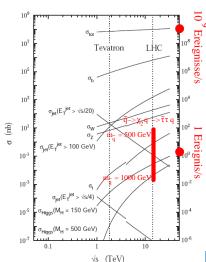


Higgs, Searches, Statistics

#### The ATLAS Experiment

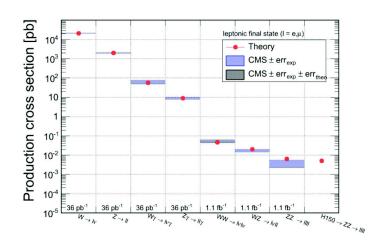
• ATLAS and CMS: First direct experimental access to the Terascale







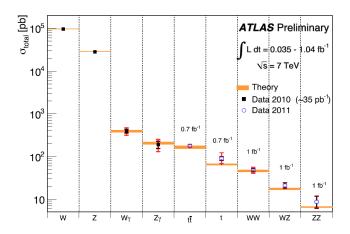
#### Very quick summary of CMS and ATLAS





Introduction to Terascale Physics School 2012

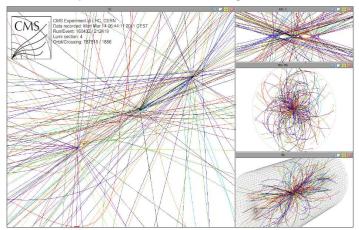
#### Very quick summary of CMS and ATLAS





Bechtle:

#### Impressive Luminosity at LHC



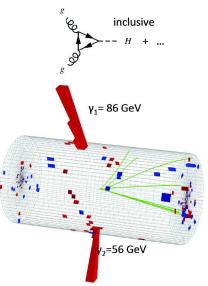
On average, 2011 data have 6 pile-up events per BX

Event shown above has 13 reconstructed vertices

Around  $int \mathcal{L} = 5 \, \mathrm{fb}^{-1}$  per experiment on tape,  $\mathcal{L}^{peak} 5 \times 10^{33}$ 



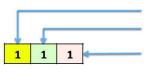
# The most sensitive search at very low masses



- Inclusive production
- Two isolated photons
- Best  $\Delta m \approx 1\%$
- Entirely data-driven analysis, use sidebands
- Background from real 'SM' di-photons and from fakes (e or  $\pi$  with missing tracks)

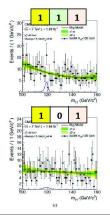


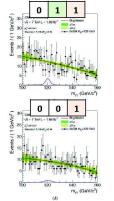
## Different classes for $h \to \gamma \gamma$

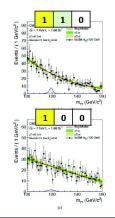


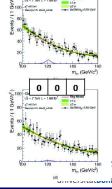
Both photons of high quality? Both photons in barrel?

Di-photon p<sub>T</sub>>40 GeV? (this bit is useful for fermiophobic Higgs only)



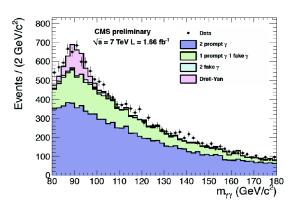






Bechtle:

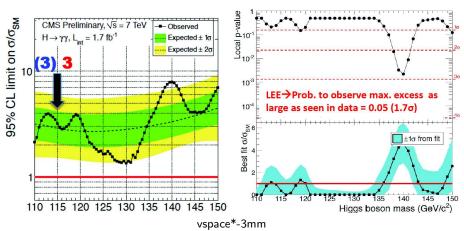
# $h \to \gamma \gamma$ Results



- MC just for illustration, not used
- Very good statistics already acquired
- Some interesting spikes... but can we have so many Higgses?



# $h \rightarrow \gamma \gamma$ Results

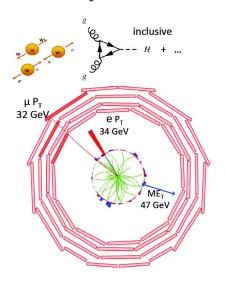


- Set limit around 3 times the SM cross section times BR
- Two small spikes at 113 and 120 compatible with Higgs and no Higgs
- Spike at 140 much too big for SM Higgs! Higgs, Searches, Statistics



Higgs. Searches. Statistics

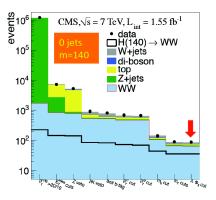
#### **Very wide sensitivity:** $h \to WW \to \ell \nu \bar{\ell} \bar{\nu}$

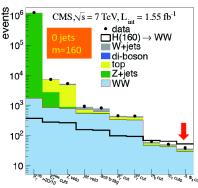


- Covers big region in m<sub>h</sub>
- mass resolution only  $\Delta m \approx 20\%$
- Trigger on two isolated leptons
- Require  $E_T^{miss}$ , small  $\Delta \phi$ , small  $m_l I$
- Use transverse mass  $m_T = \sqrt(2p_T^{\ell\ell}E_T^{miss}(1-\cos\theta))$
- Split up in different regions accoring to njets, lepton flavour, due to different backgrounds
- Backgrounds: tt̄, W+jets, WZ, WW, Drell-Yan



#### **WW Properties**



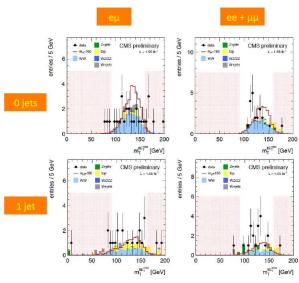


- Remarkable agreement cut by cut
- Would have seen a 160 GeV SM Higgs since long!

Higgs, Searches, Statistics

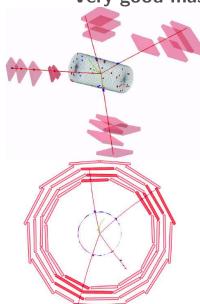


#### $h \rightarrow WW$ Properties





#### **Very good mass res.:** $h \rightarrow ZZ \rightarrow 4\ell$



Bechtle:

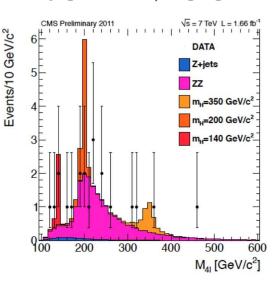
- Inclusive Producton
- 4 isolated leptons  $4e, 4\mu, 2e2\mu$
- no impact parameter
- final discriminant: m<sub>4ℓ</sub>
- $\Delta m \approx 1\%$
- ZZ and  $t\bar{t}$ , Z+jets backgrounds

Introduction to Terascale Physics School 2012

• Also look at  $2\ell 2\nu$ 



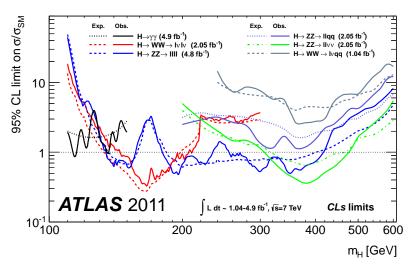
#### Very good data/bkg agreement in $h \rightarrow ZZ \rightarrow 4\ell$



- 21 obs, 21.2 expected
- Note: Low background and very good  $\Delta m$ : Very single candidate will make big impact on limit/discovery!
- Therefore, observed limits still strongly changing with each update

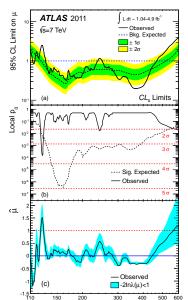


#### Interplay of the Searches in the SM



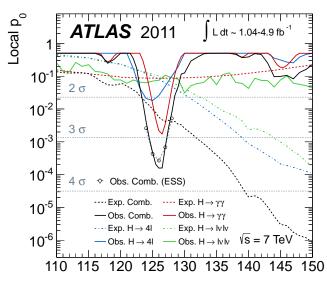


#### **ATLAS SM Combinations**





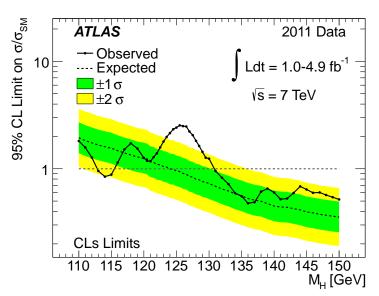
#### **ATLAS SM Combinations**





Bechtle:

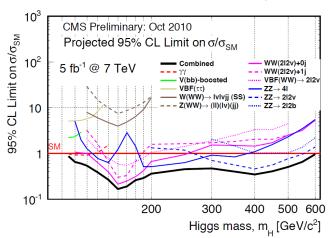
#### ATLAS SM Combinations





Bechtle:

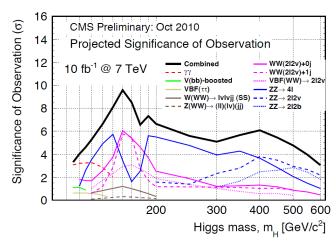
# CMS Projections for 2011/12



- Could cover the full SM range in 2012
- At least in a LHC combination . . .



# CMS Projections for 2011/12

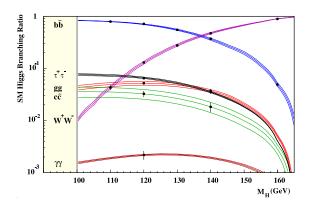


- Could cover the full SM range in 2012
- At least in a LHC combination . . .



# **Precision Tests of the Higgs Mechanism**

Once absolute Higgs cross-section can be measured at a LC: Make an absolute and precise measurement of the coupling  $g_f$  of each particle to the Higgs:



and make a detailed comparison with the SM

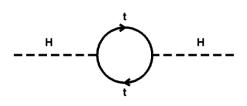


- On the way to the Terascale
  - The Search for the System behind Matter
- 2 Higgs Physic
  - Theory
  - Limits
  - Discovery and Measurements?
- Other New Physics?
  - SUSY: The missing link at the Terascale?
  - One Possibility to Measure Features of SUSY
  - Other New Physics than SUSY



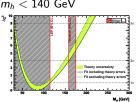
# Supersymmetry

• Even if we find the Higgs, we still have a problem . . .



$$\Delta m_h \sim \Lambda^2$$
  
natural  $m_h = M_{Planck}^2$   
Finetuning:

From indirect measurements:

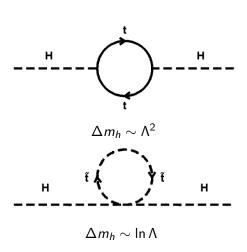


$$m_{h,obs} = \underbrace{10^{2\cdot 19}\,\mathrm{GeV}}_{\mathrm{nat.\ mass}} - \underbrace{(1-\epsilon)10^{2\cdot 19}\,\mathrm{GeV}}_{Renormalisation} \approx 100\,\mathrm{GeV}$$



## Supersymmetry

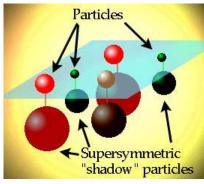
• Even if we find the Higgs, we still have a problem . . .



- From indirect measurements:  $m_h < 140 \text{ GeV}$
- To prevent quadratic divergencies: Introduce shadow world: One SUSY partner for each SM d.o.f.
- Nice addition for free: If R-parity conserved, automatically the Lightest SUSY Particle (LSP) is a stable DM candidate
- But: Where are all those states?

# Supersymmetry

• Even if we find the Higgs, we still have a problem . . .



In any case:  $m_{Hlike} < 1 \, \mathrm{TeV}$   $m_{SUSY} \leq \mathcal{O}(\mathrm{TeV})$   $\Rightarrow \mathsf{Terascala}$ 

- From indirect measurements:
   m<sub>h</sub> < 140 GeV</li>
- To prevent quadratic divergencies: Introduce shadow world:
   One SUSY partner for each SM d.o.f.
- Nice addition for free: If R-parity conserved, automatically the Lightest SUSY Particle (LSP) is a stable DM candidate
- But: Where are all those states?
- SUSY breaking introduces a lot of additional parameters
   Understand model: Measure parameters!

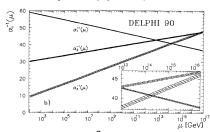


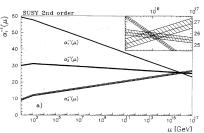
# Why try (trust?) SUSY?

#### Wim de Boer et al. (1991):

It was shown that the evolution of the coupling constants within the minimal Standard Model with one Higgs doublet does not lead to Grand Unification, but if one adds five additional Higgs doublets, unification can be obtained at a scale below  $2 \cdot 10^{14}$  GeV. However, such a low scale is excluded by the limits on the proton lifetime.

On the contrary, the minimal supersymmetric extension of the Standard Model leads to unification at a scale of  $10^{16.9\pm0.3}$  GeV. Such a large unification scale is compatible with the present limits on the proton lifetime of about  $10^{32}$  years. Note that the Planck mass  $(10^{19}$  GeV) is well above the unification scale of  $10^{16}$  GeV, so presumably quantum gravity does not influence our results.





"Prediction" of  $\sin^2 \theta_W$ :

$$\sin^2 \theta_W^{SUSY} = 0.2335(17),$$

 $\sin^2 \theta_{W}^{exp} = 0.2315(02)$ 



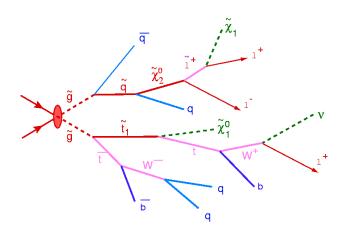
# A Warning: Apparent Finetuning





Bechtle:

## What do we hope to find?

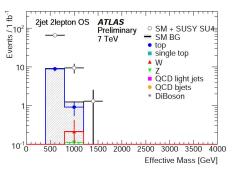


Need everything: MET, Jets, B-Jets, elektrons, myons, taus



Bechtle:

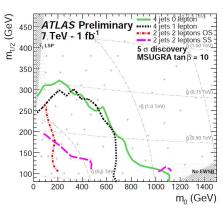
- inclusive spectra: probably fastest way to discover SUSY-like physics
- Challenging because very good detector understanding with relatively little data needed (ca.  $\mathcal{L} \approx 1\,\mathrm{fb}^{-1}$ )



$$M_{eff} = \sum_{i} p_{T,i} + E_{Tmiss}$$
  
ATLAS MC 1 fb<sup>-1</sup> @ 7 TeV



- inclusive spectra: probably fastest way to discover SUSY-like physics
- Challenging because very good detector understanding with relatively little data needed (ca.  $\mathcal{L} \approx 1 \, \mathrm{fb}^{-1}$ )
- Is it really SUSY? Or something else?
- Which particles, which masses, which decay chains?
- Quantum numbers, couplings?

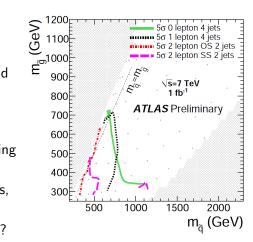


$$M_{eff} = \sum_{i} p_{T,i} + E_{Tmiss}$$
  
ATLAS MC 1 fb<sup>-1</sup> @ 7 TeV





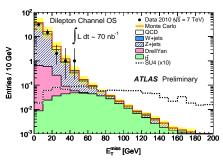
- inclusive spectra: probably fastest way to discover SUSY-like physics
- Challenging because very good detector understanding with relatively little data needed (ca.  $\mathcal{L} \approx 1 \, \mathrm{fb}^{-1}$ )
- Is it really SUSY? Or something else?
- Which particles, which masses, which decay chains?
- Quantum numbers, couplings?



$$M_{eff} = \sum_{i} p_{T,i} + E_{Tmiss}$$
  
ATLAS MC 1 fb<sup>-1</sup> @ 7 TeV



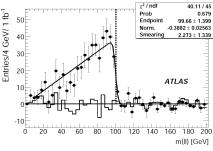
- inclusive spectra: probably fastest way to discover SUSY-like physics
- Challenging because very good detector understanding with relatively little data needed (ca.  $\mathcal{L} \approx 1\,\mathrm{fb}^{-1}$ )
- Is it really SUSY? Or something else?
- Which particles, which masses, which decay chains?
- Quantum numbers, couplings?



ATLAS data @7 TeV only 70 nb!



- inclusive spectra: probably fastest way to discover SUSY-like physics
- Challenging because very good detector understanding with relatively little data needed (ca.  $\mathcal{L} \approx 1\,\mathrm{fb}^{-1}$ )
- Is it really SUSY? Or something else?
- Which particles, which masses, which decay chains?
- Quantum numbers, couplings?



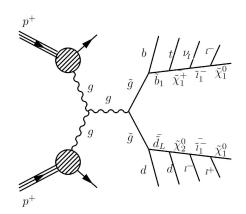
### ATLAS MC $1\,\mathrm{fb}^{-1}$ @ 14 $\mathrm{TeV}$

kinematic edges

 $\Rightarrow$  mass information



# Typical SUSY Process in the Expected Region



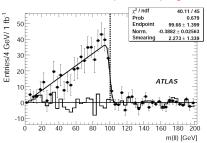
- Cannot detect LSP
- Only SM-particles visible:
  - Leptons (3)
  - Jets (at least 4)
  - missing transverse momentum
- Observable: Inavriant mass  $m_{\ell^+\ell^-}^2(m_{ ilde{\chi}_2^0}^2,m_{ ilde{\ell}_1}^2,m_{ ilde{\chi}_1^0}^2)$

- Cannot reconstruct any sparticle mass directly
- ullet Observable  $m_{\ell\ell}^2$  depends on sparticle masses  $(m_{ ilde{\chi}_2^0}^2, m_{ ilde{\ell}_1}^2, m_{ ilde{\chi}_1^0}^2)$
- Combinatoric background from second decay chain



#### Measurement of the invariant $\ell\ell$ mass

#### ATLAS 14TeV MC, update in progress!



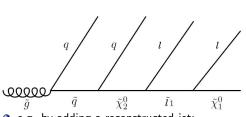
$$m_0 = 100 \text{ GeV}$$
  
 $m_{12} = 300 \text{ GeV}$   
 $A_0 = -300 \text{ GeV}$   
 $\tan \beta = 6$ 

- Select events with  $E_{Tmiss}$ , hard jets and at least 2  $\ell$
- Sharp edge in the m<sub>ℓℓ</sub> spektrum smeared due to finite resolution
   ⇒ E.g. calibrate inflection point to edge
- Use data itself to subtract background:
   OS SS or OSSF QSDF



# More Mass Edges

 $\bullet$  One observables  $m^2_{\ell\ell}$  epends on 3 sparticle masses  $(m^2_{\tilde\chi^0_2},m^2_{\tilde\ell_1},m^2_{\tilde\chi^0_1})$ 



- e.g. by adding a reconstructed jet:
  - 1 additional sparticle mass
  - But 3 additional observables!
- ullet  $\ell_{\it near}$  and  $\ell_{\it far}$  cannot be resolved o
  - $q\ell_{high}$  and  $q\ell_{low}$  edges
- Di-leptonic final states: → 4(5) observables and 4 sparticle masses → distinct solution(s)

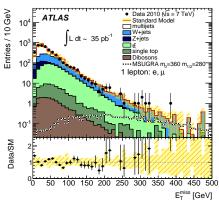
#### Mass edges:

$$\begin{split} & m_{\ell^+\ell^-}^2(m_{\tilde{\chi}_2^0}^2, m_{\tilde{\ell}_1}^2, m_{\tilde{\chi}_1^0}^2) \\ & m_{q\ell^+\ell^-}^2(m_{\tilde{q}}^2, m_{\tilde{\chi}_2^0}^2, m_{\tilde{\ell}_1}^2, m_{\tilde{\chi}_1^0}^2) \\ & m_{q\ell_{near}}^2(m_{\tilde{q}}^2, m_{\tilde{\chi}_2^0}^2, m_{\tilde{\ell}_1}^2, m_{\tilde{\chi}_1^0}^2) \\ & m_{q\ell_{far}}^2(m_{\tilde{q}}^2, m_{\tilde{\chi}_2^0}^2, m_{\tilde{\ell}_1}^2, m_{\tilde{\chi}_1^0}^2) \\ & m_{q\ell_{low}}^2 = \min[(m_{q\ell_{near}}^2), (m_{q\ell_{far}}^2)] \\ & m_{q\ell_{high}}^2 = \max[(m_{q\ell_{near}}^2), (m_{q\ell_{far}}^2)] \end{split}$$

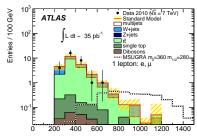


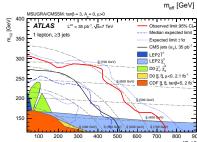
#### **Examples for the current situation**

Search for at least 3 hard jets, isolated leptons and  $E_T^{miss}$ 



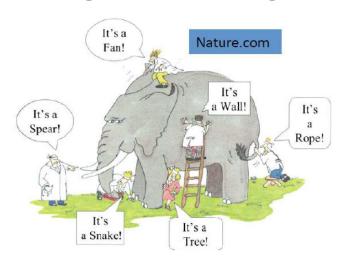
Look at 
$$M_{eff} = \sum_{i} p_{T,i} + E_{Tmiss}$$







### How to figure out what we might see?



thanks to I. Fleck universitätbonn

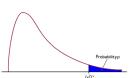
Bechtle:

# Statistical Evaluation of Agreement

- Assume a set of N measurements  $O_i$  with uncertainties  $\sigma_i$
- Assume theory with N predictions  $T_i(P_i)$  and M parameters  $P_i$
- The statistically most sensitive quantity (derived from max. Likelihood thechnique) for approximately gaussian errors  $\sigma_i$  is

$$\chi^{2} = \sum_{i=0}^{i < N} \frac{(O_{i} - T_{i}(P_{j}))^{2}}{\sigma_{i}^{2}}$$

- Vary  $P_j$  until  $\chi^2$  is minimal: Best fit point  $P_i^{opt}$
- Derive two important quantities:
  - Does the theory describe the data? Given by  $\mathcal{P}_{\chi^2}(\chi^2_{min}, ndf)$
  - How wide can I vary the parameters  $P_i$  (with respect to  $P_i^{opt}$ ) without loosing agreement too much?

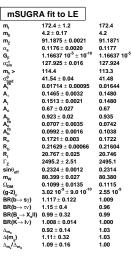


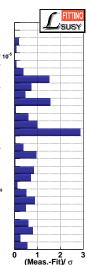
Higgs. Searches. Statistics



m, [GeV]

# **Experimental Status of SUSY**



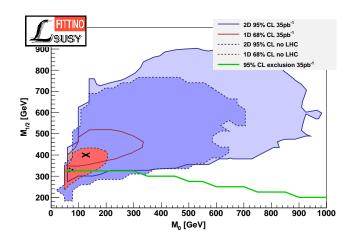


- Fit SM+mSUGRA to measured observables
- $\chi^2 = 20.6$  at 23 d.o.f.  $\Rightarrow$  $\mathcal{P}$ -Value = 60.5 %
- Best fit for sign $\mu = +1$  und

Parameter	Value and Uncertainty
aneta	$13.2 \pm 7.2$
$M_{12}$	$331.5 \pm 86.6$
$M_0$	$76.2^{+79.8}_{-29.2}$
$A_0$	$383.1 \pm 647.0$
$\alpha_s$	$0.1177 \pm 0.0020$
$lpha_{\it em}$	$127.924 \pm 0.014$
$m_Z$	$91.1871 \pm 0.0020$
$m_t$	$172.4 \pm 1.1$
$G_F$	$1.16637 \cdot 10^{-5} \pm 1 \cdot 10^{-10}$

Bechtle:

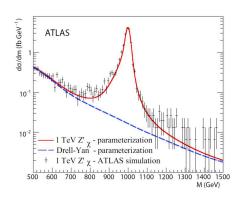
#### How does this compare to the current limits?



No stretch between indirect expectations and direct observations yet Stretch would show if no discovery at around  $\mathcal{L}^{int} = 7 \, \mathrm{fb}^{-1}$ 



## Finally: Other New Phyics



- GUT theories (new, larger gauge groups SO(10) etc.) or extra dimensions tend to predict new bosons
- They mix with SM gauge bosons
- Similar interactions (but different mass and couplings) than  $Z^0$  and  $W^\pm$
- No public results from CMS or ATLAS yet . . .



# Summary

- From precision experiments (LEP, SLC, W-mass at Tevatron, etc.) we know that somethig new has to be around the corner
- From theory (unitarity bound) we know: Whatever it is, it has to be below 1 TeV:

#### New physics below or at the Terascale

- For the first time, we are directly probing the Terascale at ATLAS and CMS
- New physics below or at the Teracale could be
  - Just the SM Higgs (leaves many questions open! Dark Matter!)
  - A (or several) Higgs and something else (typical for SUSY)
  - New strong interactions, new dimensions, many other things up to now unthought of!

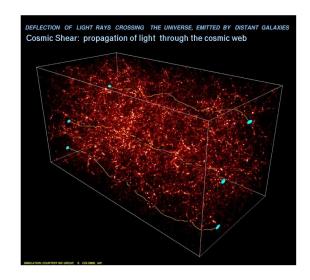
Very interesting times ahead for you!



# **Backup Slides**

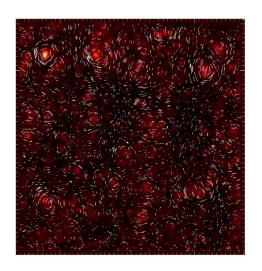


# Weak Lensing



Bechtle:

# Weak Lensing





Bechtle:

# Prerequisites: $\gamma_{\mu}, \partial^{\mu}$ and the $\dagger$

The notation is a little bit confusing sometimes, so let's try to sort things a little bit:

Fermions are represented by 4-dimensional spinors:

$$\psi(p) = \sqrt{p_0 + m} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\vec{p}}{p_0 + m} \chi_s \end{pmatrix}, \quad \chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The 4  $\times$  4  $\gamma$  matrices are acting on the 4 dimensions of ythe spinors.

An index  $(\gamma_{\mu}, A_{\mu} \text{ or } F_{\mu\nu})$  always denotes a 4-dimensional Lorentz vector. This 4-dimensional space is independent of the 4-dimensional spinor space.

 $\partial^{\mu}$  denotes a partial derivative for  $x^0, x^1, x^2, x^3$  respecively.

Einstein convention:

4-vector:  $x^{\mu}$  scalar:  $x^{\mu}y_{\mu}$  matrix:  $x^{\mu}v^{\nu}$ 



# Prerequisites: $\gamma_{\mu}, \partial^{\mu}$ and the †

Dirac matrices (each matrix acting on a 4-dim spinor):

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Hermitean adjoint:  $\psi^{\dagger}$ :  $a_{ii}=a_{ii}^{*}$ , Dirac adjoint:  $\bar{\psi}=\psi^{\dagger}\gamma^{0}$ 



# The Lagrangian

Require that the action S remains invariant under small changes of the fiends  $\phi$ :

$$\frac{\delta \mathcal{S}}{\delta \varphi_i} = 0$$

S is determined by the Lagrangian (classically:  $\mathcal{L} = T - V$ )

$$\mathcal{S}[\varphi_i] = \int \mathcal{L}[\varphi_i(s)] \,\mathrm{d}^n s,$$

where  $s_{\alpha}$  denotes the parameters of the system.

The equations of motion of the system can then be derived from the Euler-Lagrange equation:

$$\partial_{\mu}\left(rac{\partial \mathcal{L}}{\partial(\partial_{\mu}arphi)}
ight)-rac{\partial \mathcal{L}}{\partialarphi}=0$$



# The Lagrangian

Classical Example in three-dimensional space:

$$L(\vec{x},\dot{\vec{x}}) = \frac{1}{2} m \dot{\vec{x}}^2 - V(\vec{x}).$$

Then, the Euler-Lagrange equation is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = 0$$

with i = 1, 2, 3. The derivation yields:

Bechtle:

$$\frac{\partial L}{\partial x_i} = -\frac{\partial V}{\partial x_i}$$

$$\frac{\partial L}{\partial \dot{x}_i} = \frac{\partial}{\partial \dot{x}_i} \left( \frac{1}{2} m \dot{\vec{x}}^2 \right) = \frac{1}{2} m \frac{\partial}{\partial \dot{x}_i} (\dot{x}_i \dot{x}_i) = m \dot{x}_i$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_i} \right) = m \ddot{x}_i$$

From the Euler-Lagrange-equation we get the equation of motion:



79

# **Some Mathematics:** SU(2)

For the special unitary group SU(2), the generators are proportional to the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The generators of the group are  $\tau_i = \frac{1}{2}\sigma_i$ . The Pauli matrices obey

$$[\sigma_i, \sigma_j] = 2i \,\varepsilon_{ijk} \,\sigma_k$$
  
$$\{\sigma_i, \sigma_j\} = 2\delta_{ij} \cdot I$$

Example for an SU(2) transformation:

$$\psi(x) \to e^{i\tau_i\alpha^i(x)}\psi(x)$$

SU(2) and SU(3) are not abelian, i.e. the generators of the group do not commute.

# **Some Mathematics:** SU(3)

The analog of the Pauli matrices for SU(3) are the Gell-Mann matrices:

$$\begin{split} \lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ \lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \end{split}$$

The generators of SU(3) are defined as T by the relation

$$T_a = \frac{\lambda_a}{2}$$
.



# **Some Mathematics:** SU(3)

The generators T obey the relations

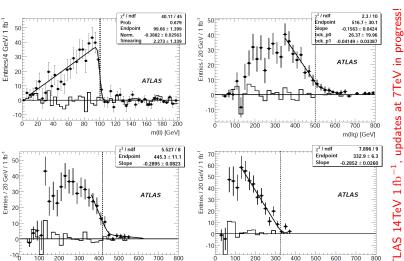
$$[T_a, T_b] = i \sum_{c=1}^{8} f_{abc} T_c$$

where f is called structure constant and has a value given by

$$f^{123} = 1$$
 $f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}$ 
 $f^{458} = f^{678} = \frac{\sqrt{3}}{2}$ 
 $\operatorname{tr}(T_a) = 0$ 



#### Expected mass spectra





m(lq) [GeV]

m(lq) [GeV]