School on Statistics Tools 2012

Introduction to Financìal Risk Management

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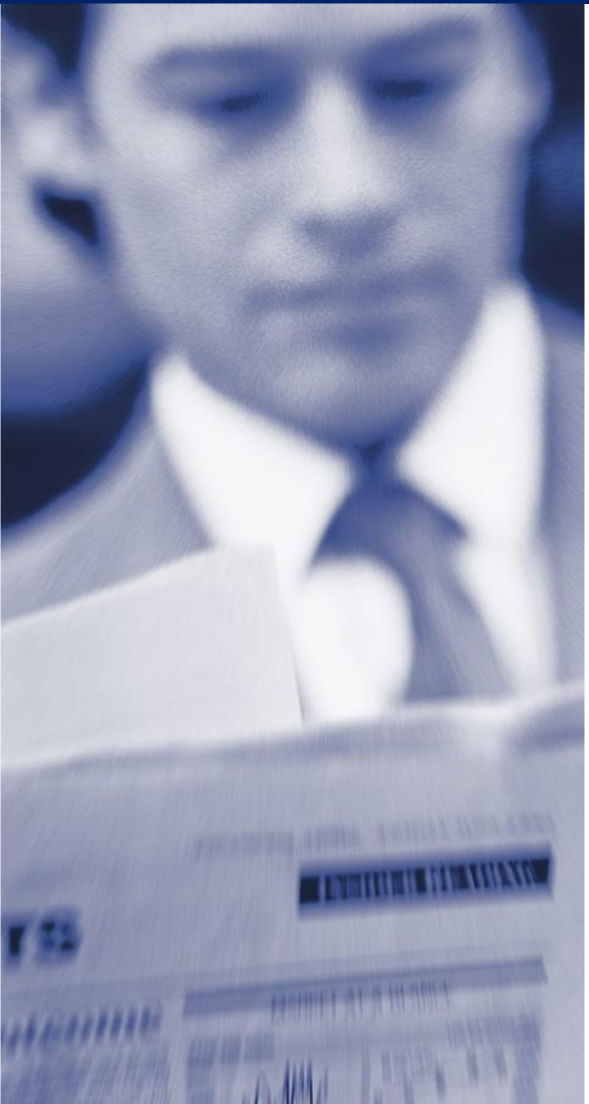
A. Defining d-fine

A. Defining d-fine

d-fine in a Nutshell

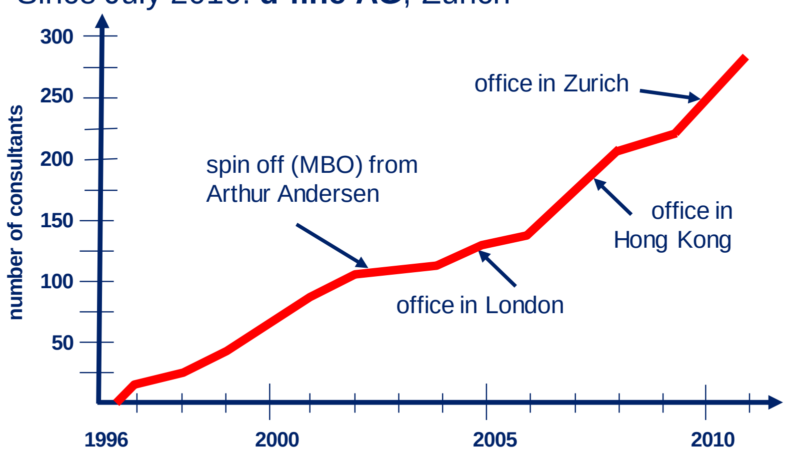
- With more than 350 **professionals** and offices in Frankfurt, Munich, London, Zurich, and Hong Kong, d-fine is one of the **leading European consulting** firms in the area of risk management
  - In Germany, d-fine ranks among the top 25 management consultancies (according to Lündendonk List 2011)
- d-fine is built around
  - **quantification and modeling**
  - **risk management**
  - **finance and reporting**
  - **IT integration**
- We help our clients to design and implement their **risk management**, **ALM**, **credit**, **trading**, and **reporting** systems
  - from analysis and design to industry-strength solutions
  - from mathematical modeling to business process implementations
  - from retail and corporate loans to exotic derivatives
  - from capital allocation and funds transfer pricing to risk-adjusted portfolio management

d-fine in a Nutshell



- Successful in business since 1996
  - Founded as a speciality consulting service of **Arthur Andersen** Germany
  - Continuous and **constant organic growth**
  - Hundreds of **successful projects** on all scales
  - **Long standing cooperation** with universities and software providers

- d-fine as a separate legal entity
  - Since July 2002 – **d-fine GmbH**, Frankfurt and Munich
  - Since Nov 2004 – **d-fine Ltd**, London
  - Since October 2007: **d-fine (HK) Ltd**, Hong Kong
  - Since July 2010: **d-fine AG**, Zurich



A. Defining d-fine

Our International Team

- Strong technical and mathematical **skills**
  - approx. 50% physicists, 35% mathematicians, 15% IT and MBA
  - approx. 65% PhD level degrees, 35% master level degrees
- Well-balanced **profiles**
  - Analytical abilities
  - Technology expertise
  - Social and management skills
- Continuous and intensive **training** and **research**
  - Training cooperation with University of Oxford, FSFM Frankfurt (M.Sc. in Mathematical Finance), Warwick Business School (Corporate Finance), Mannheim Business School (MBA)
  - State of the art know-how through internal research and training, research cooperation with universities, close contact with regulatory authorities, and regular attendance at conferences and seminars

see
our www library

- Extensive financial industry and implementation **know-how**
  - Wealth of experience gained through successful delivery of numerous and varied projects

Our Service Focus

Quantitative Finance

Development of pricing models

- d-fine C++ library for complex and structured products

Development of risk management models

- Market, credit, and operational risks
- ALM and liquidity management
- P&L methods
- Economic capital

Independent model validation

- Test and validation of pricing models
- Test and validation of IRB rating models, VaR models, and PFE & EPE models

System validation

Processes and Governance

Processes and procedures

- Design
- Implementation
- Documentation
- Optimisation

Regulatory requirements

- Basel II and CRD: Capital requirements and best practice risk management
- Preparation for Basel III
- UCITS III / IV
- Solvency II

Accounting

- IFRS and multi-GAAP
- Portfolio hedges (IAS 39)
- Valuation adjustments

IT Integration

Implementation of trading and risk systems

- Business requirements
- System selection
- Design and parameterisation
- Data feeds and interfaces
- Add-ons, analytics and reporting
- Testing
- Project management

Development of bespoke software

- C/C++, .NET, C#, Java, VBA, …

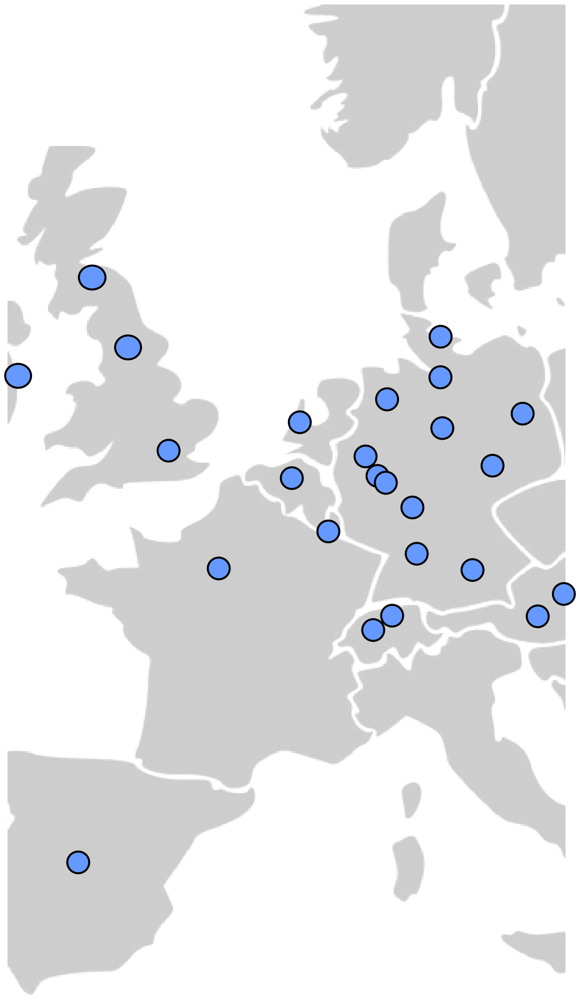
Data management

- Data warehouses
- Market databases
- Data quality management

Our clients profit from our integrated approach and the synergy effects from our experience in financial models, conceptual design, risk management, and IT integration.

A. Defining d-fine

Our Clients



- **Large, medium sized, and specialised banks**
- **Insurances, asset managers, hedge funds**
- **International industry corporations**

Our client list (abridged):

ABN Amro (NL)
ampegaGerling
Barclays Capital (UK)
BayernLB
BBVA (ES)
Commerzbank
COMINVEST
CQS Management (UK)
Credit Suisse (CH)
Daimler
DBS (SG)
DekaBank
Deutsche Bank
Deutsche Bundesbank
DG Hyp
Deutsche Postbank
DVB Bank
DZ Bank
E.ON
Eurohypo
Fortis (BE)

Hannover Rück
HBOS (UK)
HSBC (UK)
HSH Nordbank
HVB
KfW
Landesbank Baden-Württemberg
Landesbank Berlin
MEAG
NRW.Bank
Raiffeisen International (AT)
RZB (AT)
Schiffsbank
Siemens Financial Services
Sparkasse KölnBonn
Standard Bank (UK)
Toyota
UBS (UK, CH)
Union Investment
VW Financial Services (DE, UK)
WestLB

B. Introduction

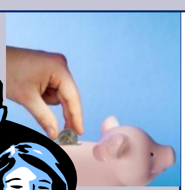
What’s a Bank good for?

Main Bank Activities

Credit business From simple retail credits to complex funding models	**Payment services**: Current accounts, all kinds of money transfer	**Investment services** Sales and management of investment products	**Corporate finance** Supporting corporates raising funds in capital markets (security issuance, equity,...)

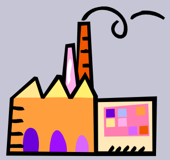
Credit business

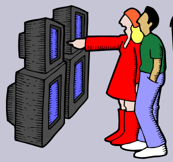
Depositor



Bank

Debtor

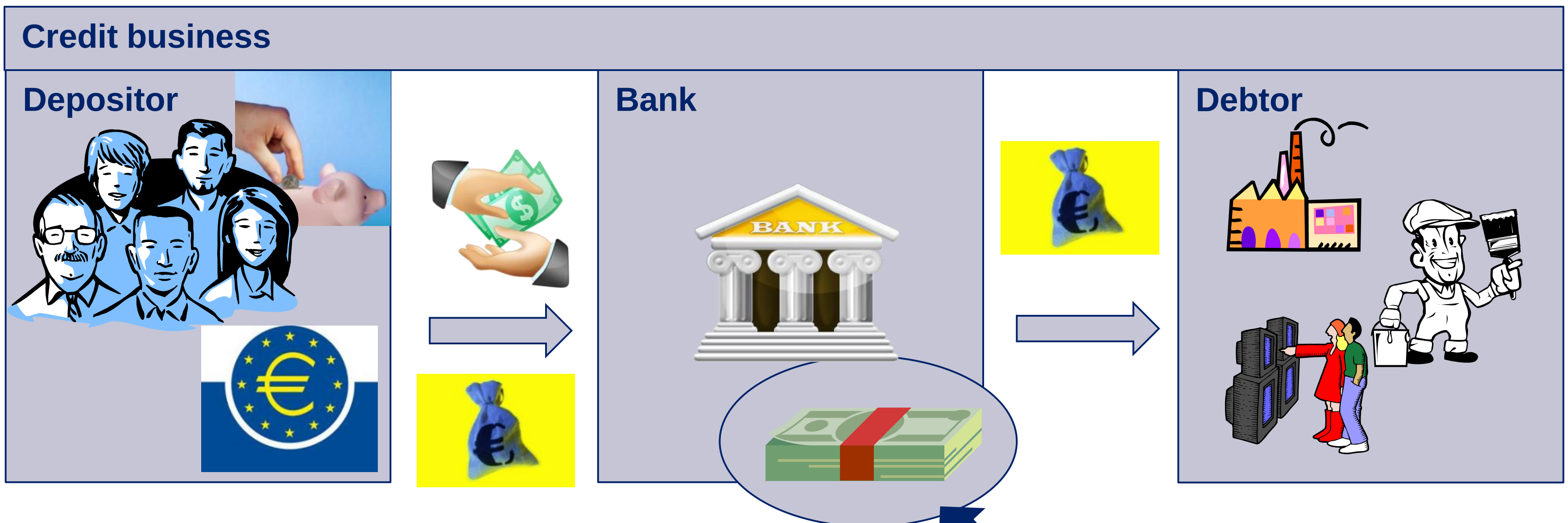




What’s a Bank good for?

Economic role of banks

- Transformation of maturities, e.g. short term deposits in long term loans
- Lot-size transformation, e.g. small deposits in larger loans
- Transformation of risk, e.g. risky loans in savings deposits
- Transfer of monetary policy (interest rates)



- Banks are essential for the economy
- Their business is **not riskless**
- **They must have sufficient capital to cover their risks. But how much?**

Risk and Regulation

Basel Committee on Banking Supervision (founded 1974)

- Members: Central banks and supervisors (founded by G10, now global membership)
- Goal: Development of high consistent standards for banking and supervision
- Activity: Quarterly meetings in Basel (Bank for International Settlements)
- Results: Development of legally non-binding guidelines and recommendations through discussions with banks and regulatory authorities

Policy Development Group
Risk management/-modelling, liquidity, trading book issues, research, (supervision of) capital resources, cross-border-topics

Standards Implementation Group
Validation,
operational risk

Accounting Task Force
Conceptional issues, audit, financial standards in praxis

Basel Consultative Group
Dialogue with non-membership authorities on supervisory issues and initiatives of the committee

Banking regulation is strongly influenced by the Basel Committee

Risk and Regulation

International (Basel Committee on Banking Supervision)

Basel II*

- Pillar 1: Minimum regulatory capital requirements for credit-, market- and operational risk
- Pillar 2: Supervisory review of capital adequacy
- Pillar 3: Market discipline and disclosure

Europe

Capital Requirements Directive (CRD)

EU Banking Directive (2006/48/EC)

- Principles of the banking supervision
- Minimum capital requirements for credit and operational risk

Capital Adequacy Directive (2006/49/EC)

- Minimum capital requirements for market risk
- Detailed rules for credit- und operational risk

Germany

German Banking Act – Kreditwesengesetz (KWG)

- § 10: Minimum capital requirements for credit, market and operational risk → SolvV
- § 25a: Organizational obligations → MaRisk
- § 44: Disclosure obligations / audit

Solvency Regulation (SolvV)

- Specifications of the capital requirements
- Implementation of pillar 1 of Base II

regulatory capital

Minimum requirements for risk managm.(MaRisk)

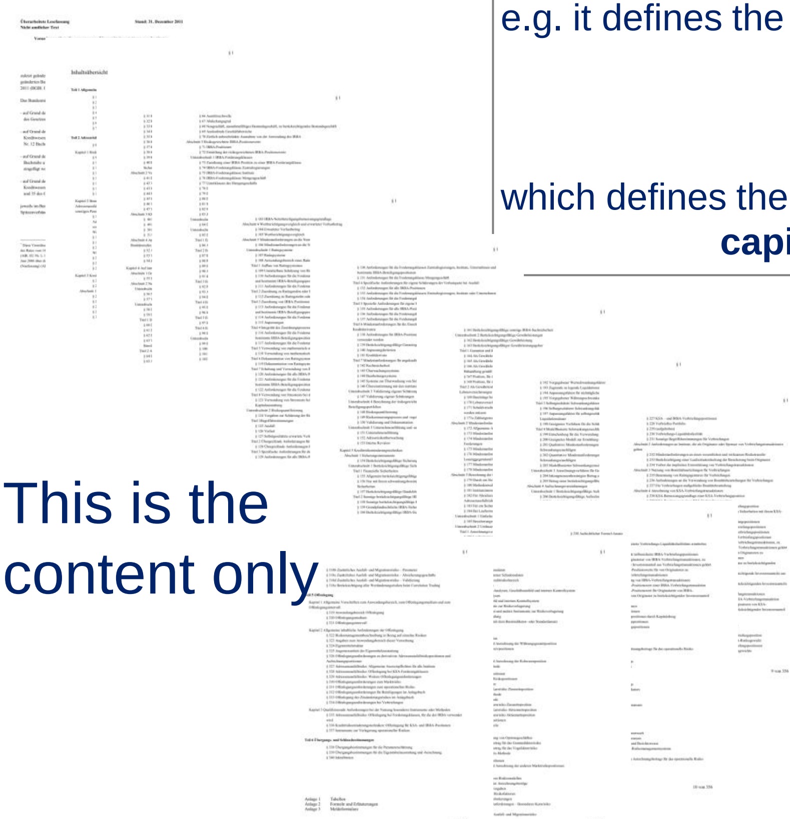
- Quantitative requirement for risk management
- Organizational requirements

economic capital

* Basel III is in the implementation stage

Risk and Regulation

- The requirements are quite detailed
- The SovV alone consists of 356 pages plus appendices



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e.g. it defines the risk weight for a lending to an AAA rated company company

KSA-Risikogewicht Unternehmen kurzfristige Bonitätsbeurteilung

Bonitätsstufe	1	2	3	4	5	6
KSA-Risikogewicht	20 %	50 %	100 %	150 %	150 %	150 %

which defines the capital requirement (in the standardized approach):

capital requirement = 8% * debt * 20%

... or for a country like Germany

2. Wird ihre Erfüllung von

a) der Bundesrepublik Deutschland, einem rechtlich unselbständigen Sondervermögen der Bundesrepublik Deutschland, der Deutschen Bundesbank oder

b) von einer Zentralregierung oder einer Zentralnotenbank eines anderen Staates des Europäischen Wirtschaftsraums

geschuldet und ist sie in der Landeswährung des Staates geschuldet und refinanziert, darf ein KSA-Risikogewicht von 0 Prozent verwendet werden.

Risk and Regulation

Risk types to be covered

Regulatory capital

capital requirement by the regulator

credit risk

- losses due to defaults of counterparties

market risk

- losses due to changes in market prices

operational risk

- losses resulting from inadequate or failed internal processes, people and systems or from external events

Economic capital

capital measure for the internal risk management

credit risk	market risk	operational risk

potential further risk categories	
business risk	smaller profit as expected
investment risk	loss resulting from value changes in the investment portfolio
insurance risk	loss due to unexpected high insurance claims
reputation risk	losses due to negative external perceptions
tax risk	losses due to unexpected tax changes
……	

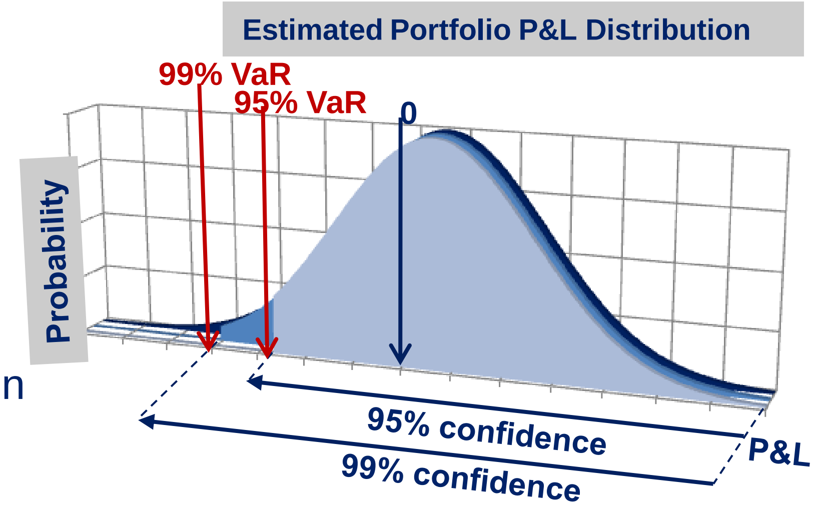
Includes all sources for potential unexpected losses

Risk Measure

Value-at-Risk (VaR)

The Value-at-Risk of a portfolio over a certain time horizon T is the maximum loss which will not be exceeded with a given probability (one sided confidence level).

- VaR is measured in monetary units (e.g. €).
- VaR is the risk measure mainly used in conjunction with regulatory and economic capital



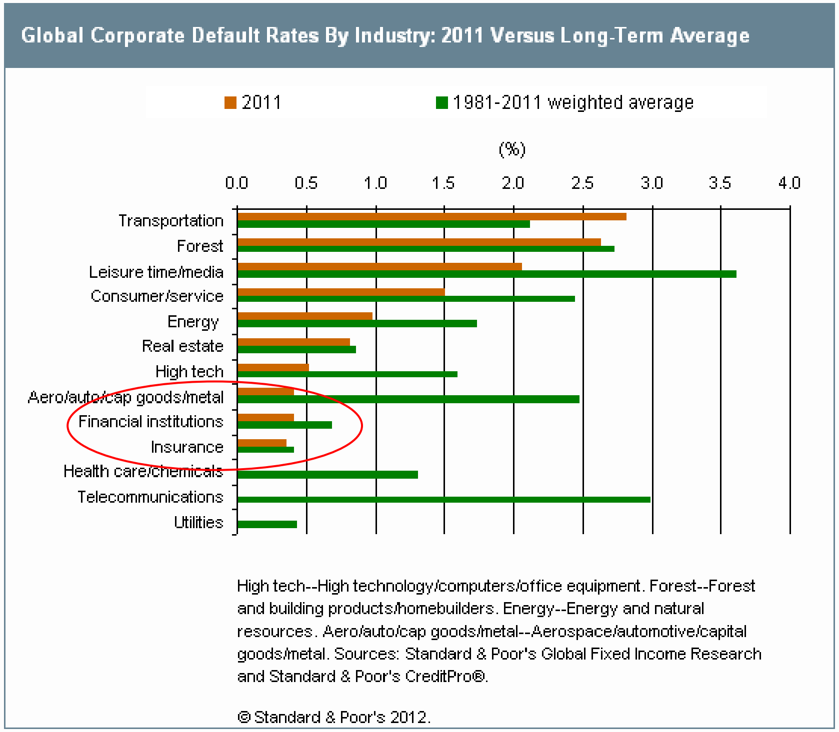
VaR quantifies potential portfolio losses based on historical data

Quantitative Requirements

	confidence level	holding period
Regulatory capital		
credit risk (IRB approach)	99,9 %	one year
operational risk (Advanced Measurement Approach)	99,9 %	one year
market risk (internal model)	99 %	10 day
Economic capital		
typical value	99,9 %	one year

corresponds to only one default per year per thousand banks

It is essential that the risk determination is done independently from the trading department



C. Modeling Risk

C. Modeling Risk

- The modeling of risk is a wide area

- Only a few examples are given in the following
  - **Risk aggregation**
  - Modeling market risk
  - Model validation

Risk Aggregation

Economic Capital

Aggregation

Market Risk	Credit Risk	Operational Risk	Business	....

P&L forecast probability densities

- **How to aggregate the different distributions technically?**
- **How to model the dependence structure?**

a) Assume a multivariate normal distribution:

$$\mathrm{VaR}_{\text{total}} = \sum_{i,j} \mathrm{VaR}_i \cdot \mathrm{Corr}_{ij} \cdot \mathrm{VaR}_j$$

☑ **easy to use**

b) Copula approach

☑ **arbitrary marginal distributions**

☑ **tail dependence**

Risk Aggregation

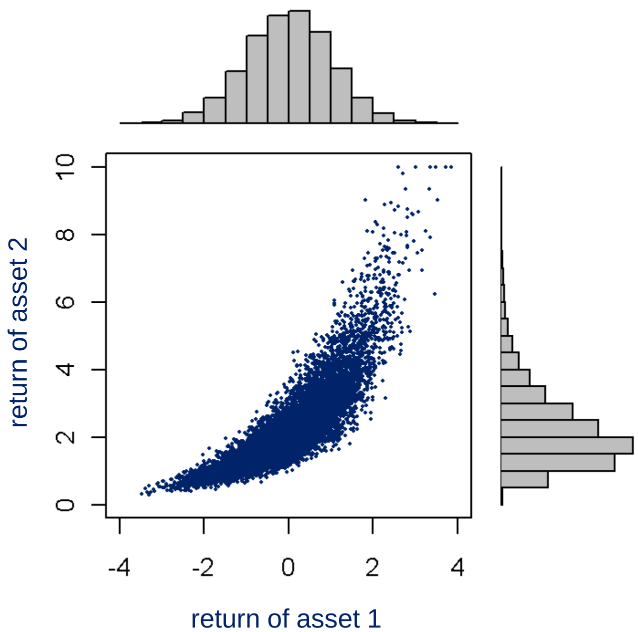
What is a copula?

- Given a n-dimensional **multivariate cumulative probability distribution $F(x_1,\ldots,x_n)$** and the corresponding **marginal distributions $F_1(x_1),\ldots,F_n(x_n)$** the (elliptical) **copula $C:[0,1]^n \to R$** describes the dependency structure of the random variables $x_1,\ldots,x_n$ according to

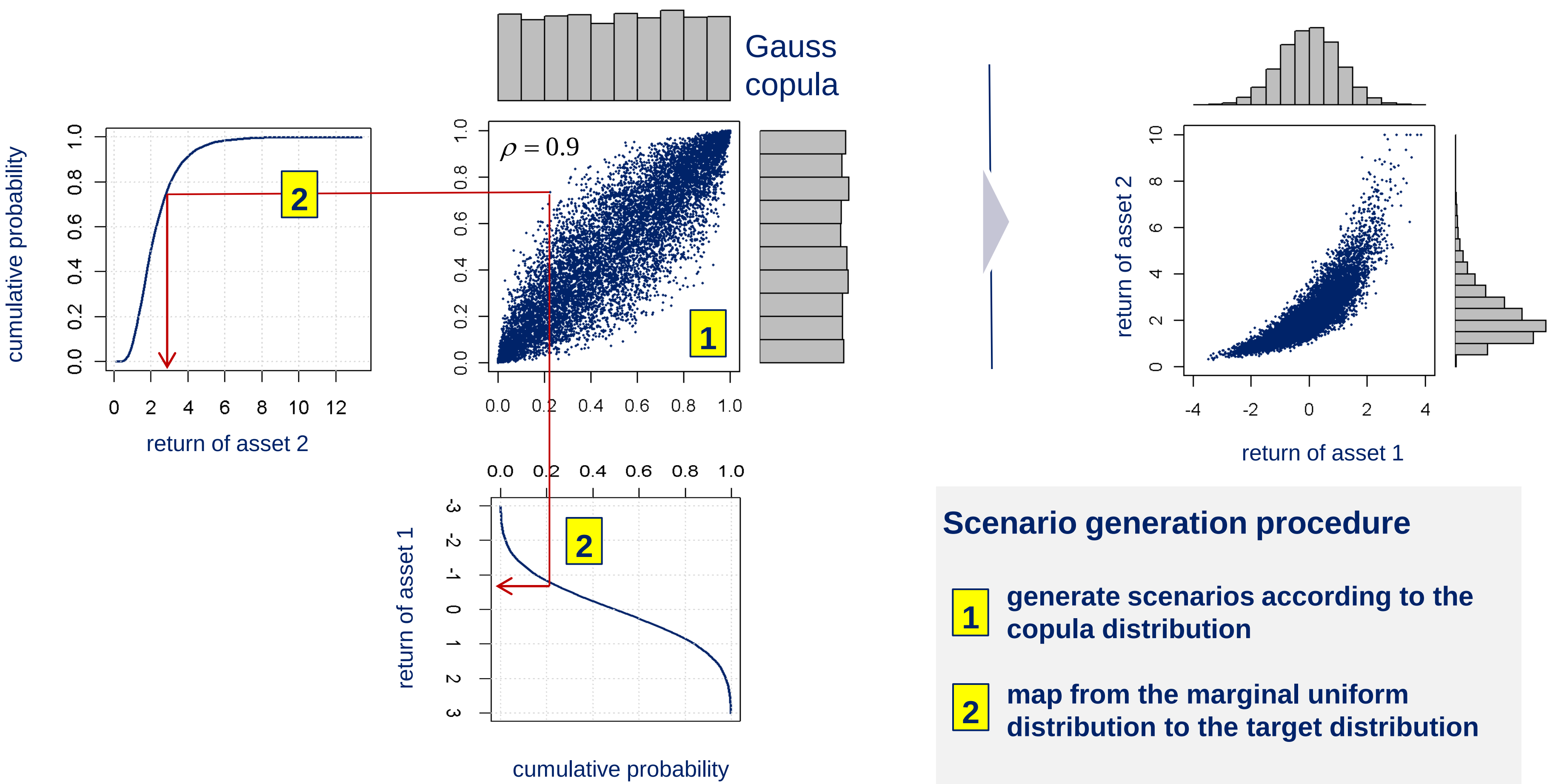
$$C(u_1,\ldots,u_n) = F[F_1^{-1}(u_1),\ldots,F_n^{-1}(u_n)]$$

where $u_i = F_i(x_i)$ are uniformly distributed.

- The purpose of the copula concept is to model the full dependency structure of random variables beyond a correlation matrix approach.



Risk Aggregation



Scenario generation procedure

1 **generate scenarios according to the copula distribution**

2 **map from the marginal uniform distribution to the target distribution**

Risk Aggregation

	Elliptical		Archimedean		
	$C(u_1,\dots,u_n)=F[F_1^{-1}(u_1),\dots,F_n^{-1}(u_n)]$		$C(u_1,\dots,u_n)=\varphi^{-1}[\varphi(u_1)+\dots+\varphi(u_n)]$		
Characteristics	**Gauss**	**Student-t**	**Gumbel**	**Frank**	**Clayton**
Correlation matrix	✓	✓	🚫	🚫	🚫
Further parameters	**0**	**1**	**1**	**1**	**1**
Symmetry	Symmetric	Symmetric	Asym.	Asym.	Asym.
Fat tails	🚫	✓	✓	🚫	✓

hard to calibrate

Student-t copula is popular because…
1) it allows studying effect of fat tails on VaR,
2) its parameters can be estimated (correlation matrix + 1 additional parameter).

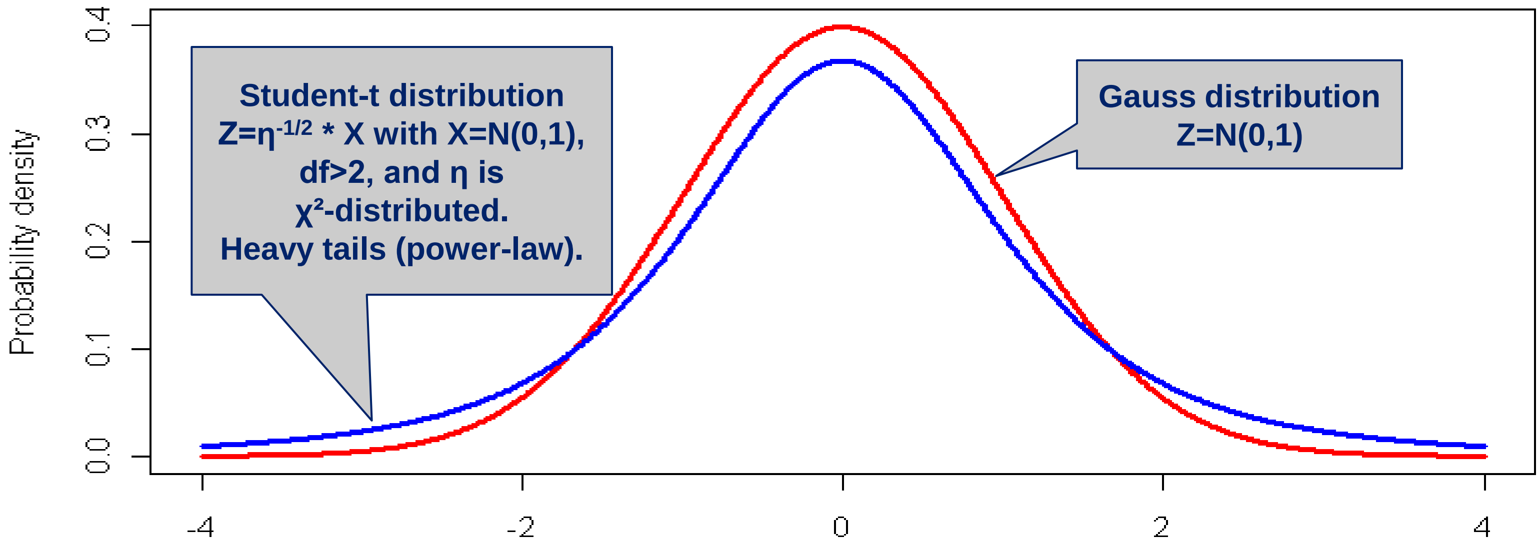
C. Modeling Risk

Risk Aggregation

Student-t copula is extension of Gauss copula with additional parameter df. Keep in mind the two limits

- **df=∞: Gauss copula recovered.**
- **df=2: Limit where tails become heaviest.**

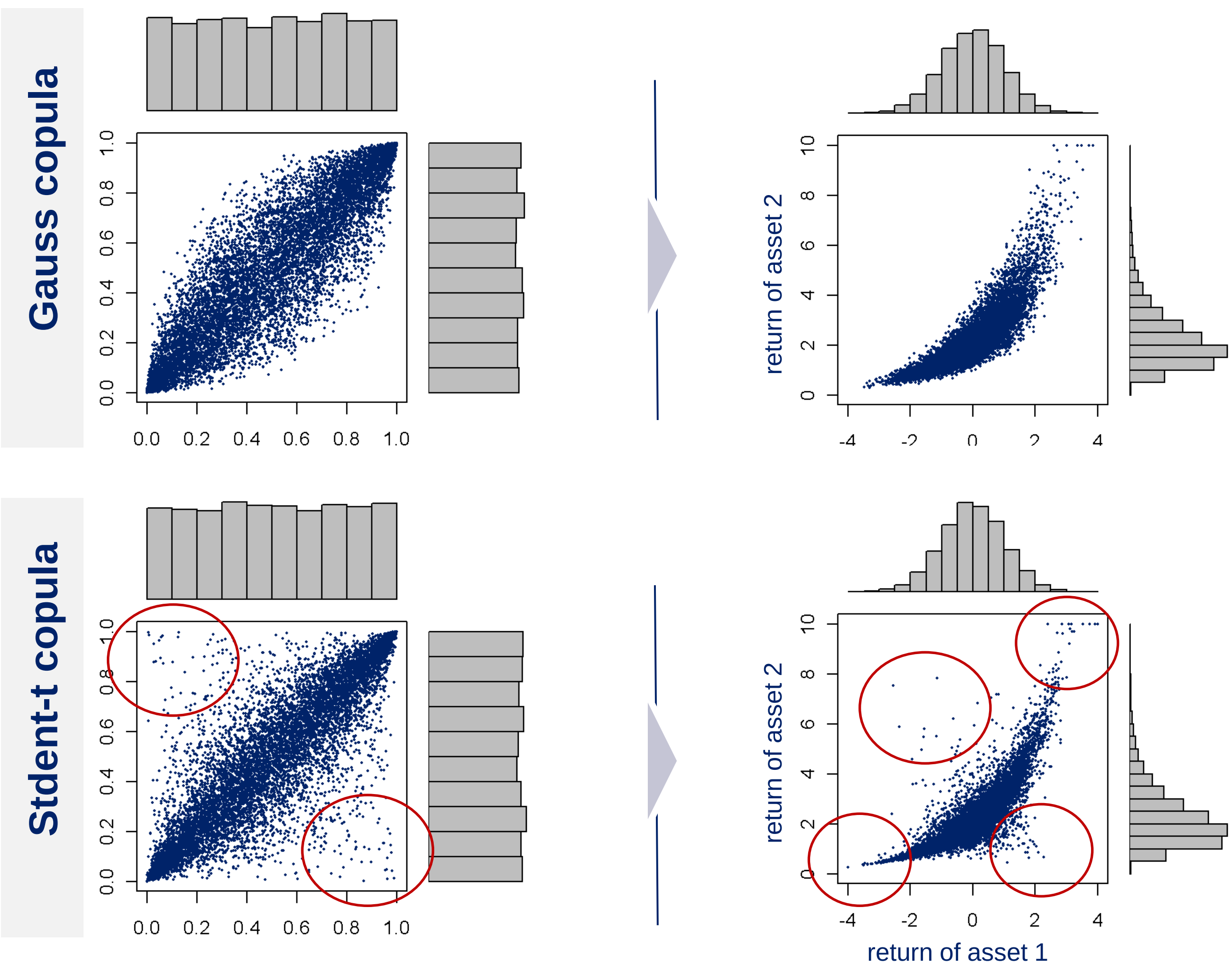
Additional parameter df controls tail dependence.



Student-t copula comes along with additional parameter df, quantifying heavy-tailedness, which needs to be fixed

C. Modeling Risk

Risk Aggregation

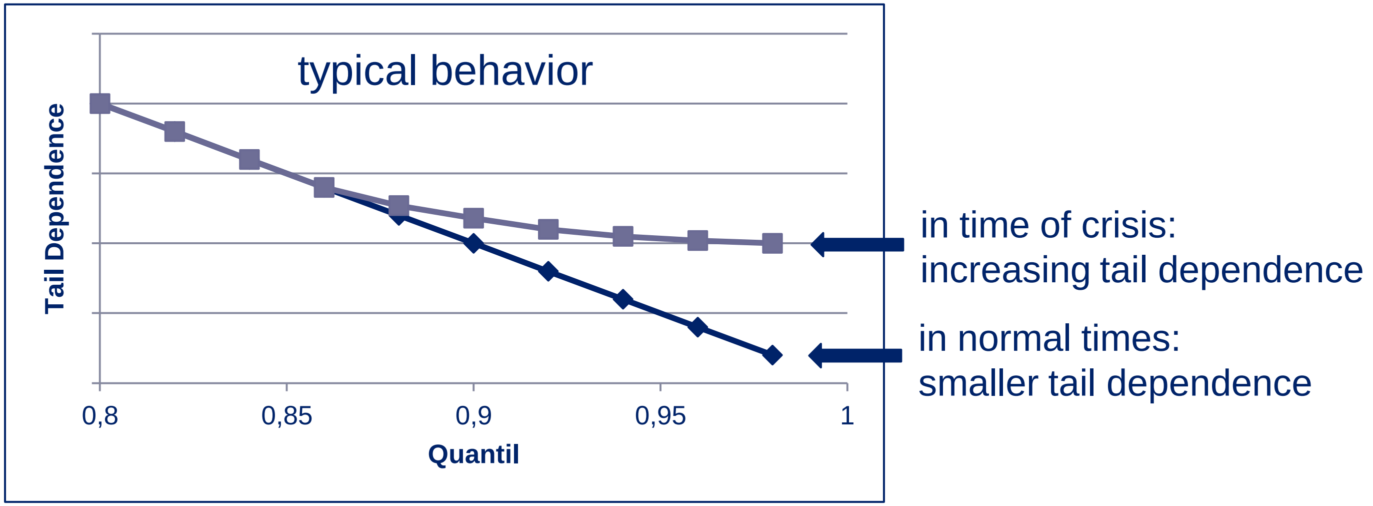


- **marginal distributions are the same**
- **more extreme events with a t-copula**

Risk Aggregation

Tail Dependence:

- Consider pair of risk factor time series $x_1(t)$ and $x_2(t)$ with marginal distributions $F_1(x_1)$, $F_2(x_2)$.
- For given quantile q compute tail dependence $P(F_1(x_1)>q \mid F_2(x_2)>q)$.



- Tail dependence occurs in the Student-t approach for df finite
- Calibration of tail dependence using time series

C. Modeling Risk

- The modeling of risk is a wide area
- Only a few examples are given in the following
  - Risk aggregation
  - **Modeling market risk**
  - Model validation

Financial Instruments

There is **no unique classification** of financial instruments. Every classification depends on its purpose and may therefore refer to different aspects of the financial instruments (e.g. accounting categories, financial mathematics aspects).

Here is an example of very simple **classification** according to the **main market risk drivers**:

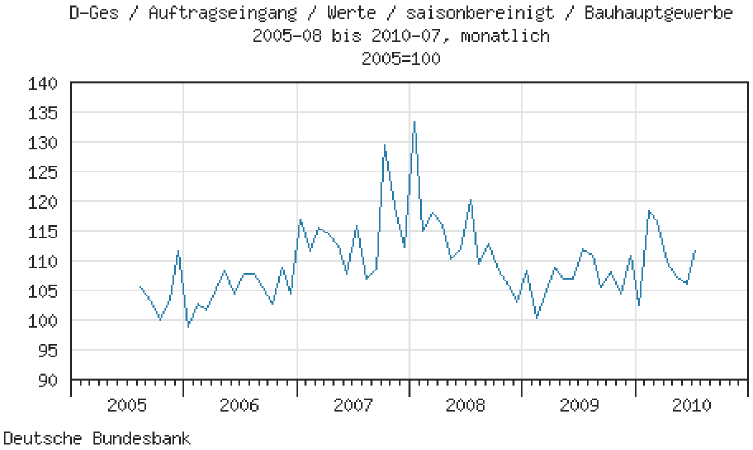
Interest rate products	Equity products	FX products	Commodity Futures/ Forwards	Credit products
• Corporate Bonds • Government Bonds • Swaps • Loans • Bond Options • Swaptions • Caps/Floors • IR Futures • …	• Shares • Equity Options • Index Options • Equity Certificates • Basket Options • ...	• FX Cash • FX Forwards • FX Options	• Crude Oil • Gas Oil • Base Metals • Precious Metals • Coal • Power • …	• Credit Default Swaps • Credit Spread Options • Securitisations • Credit Index Products • Credit Basket Derivatives • ...

Different financial instruments are sensitive to different aspects of market risk.

Market Risk of Financial Instruments – Motivation

The **current (market) value** of a financial instrument is subject to the current **market conditions**. Therefore the **value** of any financial product will usually **fluctuate** from one day to another (and also during the day) due to changes in the market conditions.

Market conditions can be quantified by (or from) quantities that can be observed in the market, e.g., exchange rates or equity courses.

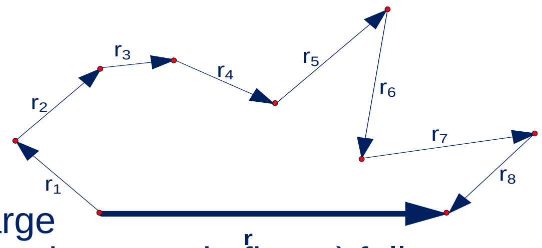


The quantities that quantify market conditions reflect the **market's current opinion**, which in turn depend on more or less **unpredictable economic variables** (e.g. unemployment rate, company news, GDPs, …) and **psychology** (the reactions of the market to any news are also more or less unpredictable).

Therefore the quantities quantifying the market conditions need to be modeled as **stochastic variables**.

Value of financial instruments depends on (stochastic) market conditions.

Risk Factors – Stochastic Processes (1/2)



Random Walk

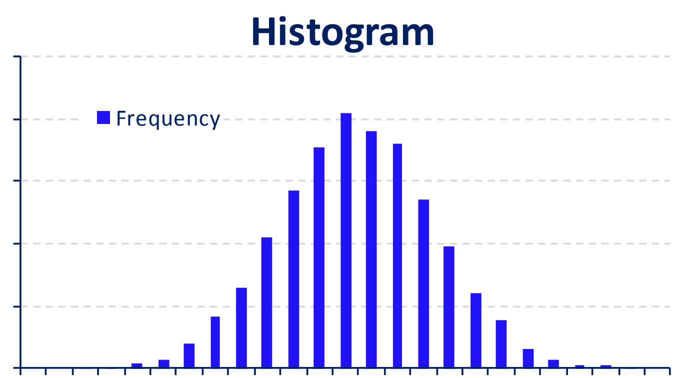
For the number of steps n (n=8 in figure) sufficiently large the distribution of r_{log} (end-to-end vector **r** in figure) follows a **normal distribution**:

$$p(r_{\log}) = \frac{1}{\sqrt{2\pi \operatorname{var}(r_{\log})}} \exp\left[\frac{(r_{\log} - \mathrm{E}[r_{\log}])^2}{2\operatorname{var}(r_{\log})}\right]$$

$$\mathrm{E}[r_{\log}] = 0 \quad , \quad \operatorname{var}(r_{\log}) = n \cdot \mathrm{E}[r_i^2]$$

The **number of steps** in the random walk n is proportional to the **time Δt** that passes during the random walk.

This also holds for **absolute returns** (relative returns do not follow a random walk but one can still assume that they are normally distributed).



Random walk is used as process model for risk factors.

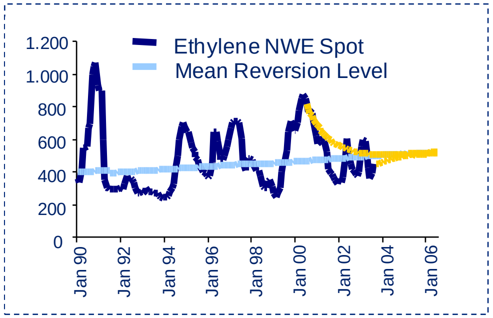
Risk Factors – Stochastic Processes (2/2)

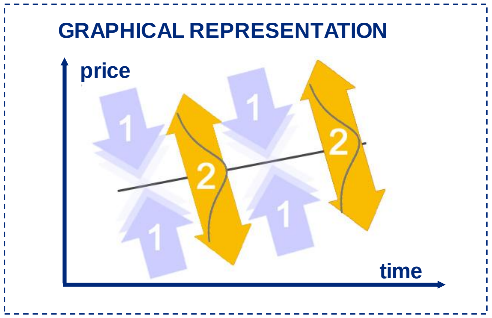
The simple process model can be generalized to a **general diffusion process** with drift and variance being functions of *t* and *ρ*:

$$d\ln(\rho_t) = a(t,\rho_t)\cdot dt + b(t,\rho_t)\cdot dW$$

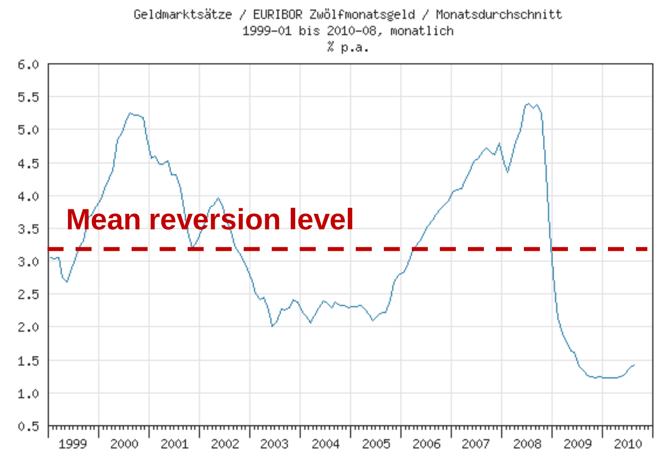
(**lognormal model**)

In this way the model can also be used for risk factors that show different (e.g. mean reverting) behavior like commodity prices or FX rates.





Example **Vasiceck model**:
(Ornstein-Uhlenbeck process)



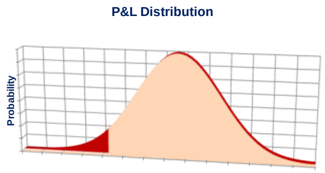
$$d\rho_t = (\mu - \nu\rho_t)\cdot dt + \sigma\cdot dW$$

(**normal model**)

Mean reversion force

The simple diffusion model can be generalized to more complex processes.

VaR Models – Monte Carlo Method (1/2)



The calculation of the Portfolio P&L distribution on the risk horizon is the basis of each VaR model.

- **Monte Carlo Simulation – Idea**
  - Instead of an analytic approximation, as in the case of the parametric VaR , the P&L distribution is generated by revaluation of the portfolio under a large number of risk factor scenarios.
  - The risk factor scenarios are generated from the assumed risk factors distributions / stochastic processes.

- **Example (log-normal distribution):**

timestep **volatility**

drift **random number**

relative risk factor change

$$r_{\log}(t,\delta t) = \mu\delta t + \sigma\sqrt{\delta t}\,X$$

new risk factor value

$$S(t+\delta t) - S(t) = \delta S(t) \approx S(t)\left(e^{r_{\log}} - 1\right)$$

old value **relative change**

Basic idea: generate P&L distribution from large number of scenarios.

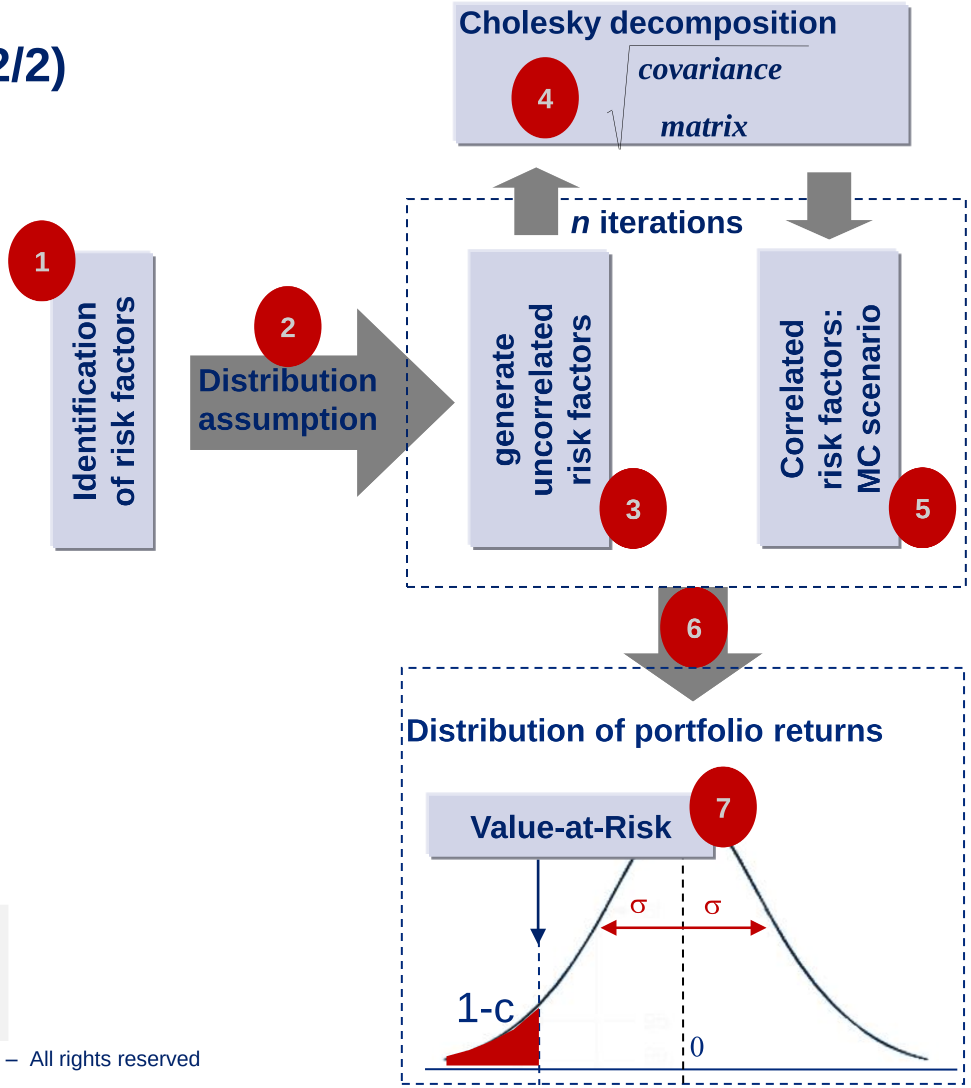
VaR Models – Monte Carlo Method (2/2)

Illustration of main Calculation Steps

1) Identification of all relevant risk factors
2) Distribution assumption(s) for risk factor returns over the time horizon from time series analyses
3) Generate random numbers for the uncorrelated risk factor returns according to the distribution assumption(s)
4) Transformation of uncorrelated risk factors into correlated risk factors via multiplication with Cholesky (or e.g. SVD) decomposition of covariance matrix: correlated risk factors returns form a Monte Carlo scenario.
5) Full (or sensitivity based) revaluation of portfolio under the generated scenario
6) Generation of a large number of scenarios leads to P&L distribution
7) Estimation of VaR as the desired quantile of this distribution

Other VaR methods:

- **Historical Simulation**
- **Variance-Covariance Method**



Market Risk

Comments

- Risk calculation must be run every day
- For large portfolios, a huge number of trade valuations is required for the VaR calculation
- In addition extreme events have to be examined (stresstests)
- Daily backtesting compares the forecast with the real profit and loss

Monte Carlo Simulation
Typical large German Bank

C. Modeling Risk

- The modeling of risk is a wide area

- Only a few examples are given in the following
  - Risk aggregation
  - Modeling market risk
  - **Model validation**

C. Modeling Risk

Model Validation

- When using a model one has to check the adequacy of the model
- Such model validation is also required by the regulators to be done on a regular basis

<u>Examples</u>

- Validation of market data used for the calibration of the models
- Validation of the theoretical valuation models
- Validation of risk factor models ⇨ e.g. **testing the distribution assumptions**
- Validation of model parameter ⇨ e.g. **consistency of the correlation matrix**
- Validation of scenarios generation
- **...**

Model Validation

Testing the distribution assumptions for market risk factors

Hypothesis	Test	Test statistics	
Autokorrelation first order = 0	Durbin-Watson	$d = \frac{\sum (r_t - r_{t-1})^2}{\sum r_r^2}$	• *d=0* corresponds to ρ=1 • *d=2* corresponds to ρ=0 • *d=4* corresponds to ρ=-1
normal distribution	Kolmogarov-Smirnov	$d = \sup_{r_i} \left\| \int_{-\infty}^{r_i} f_{empirisch}(r_i)\, dr - \int_{-\infty}^{r_i} \frac{1}{\sqrt{2\pi\sigma}} \cdot e^{-\frac{1}{2}\frac{(r-\bar{r})^2}{\sigma^2}}\, dr \right\|$	Robust, can be used for other distributions as well
normal distribution	Lilliefors		More powerful as Kolmogarov-Smirnov test. Takes into account the effects of volatility and mean estimation.
normal distribution	Shapiro-Wilk	$d = \frac{\left(\sum_i a_i \cdot r_{i-ter\ Rangplatz}\right)^2}{\sum_i (r_i - \bar{r})^2}$	More powerful as the Lilliefors test.
normal distribution	Jarque-Bera	$d = \frac{n}{6} \cdot \left[S^2 + \frac{(K-3)^2}{4} \right]$	Based on kurtosis (K) and skewness (S). Especially sensitive to Heavy Tails.

C. Modeling Risk

Model Validation

Plausibility checks for correlations

- Running a Monte Carlo simulation for market risk requires to know the correlations between all risk factors
- This is a huge matrix, e.g. 5.000 risk factors ⇨ about 12.5 mio. correlation coefficients

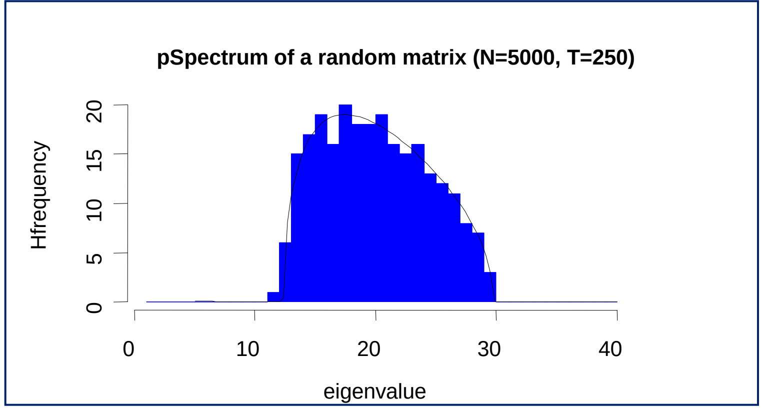
- **How many principal components carry correlation information?**
- **What is noise?**

Model Validation

Plausibility checks for correlations

Random matrices were introduced in nuclear physics by Eugene Wigner in 1955 "Characteristic vectors of bordered matrices with infinite dimensions". *Ann. Of Math.* 62 (3)

- Laloux et al. (Phys. Rev. Lett 83(7):1467, Risk 12(3):69) proposed to determine this from random matrix theory
- For a random matrix of Wishart-type $\Sigma = \frac{1}{T} \cdot R \cdot R'$ where the returns are uncorrelated the eigenvalue spectrum has an upper and lower limit (for N and T $\rightarrow \infty$)
- **Eigenvalues above the upper boundary carry correlation information**



- Note:
  - the correlation matrix is decomposed by eigenvectors in the following way: $\Sigma = \sum_{k=1}^{N} \lambda_k \cdot v_k \cdot v_k{}'$
  - an eigenvalue of the correlation matrix is the variance for the combination of risk factors given by the corresponding eigenvector

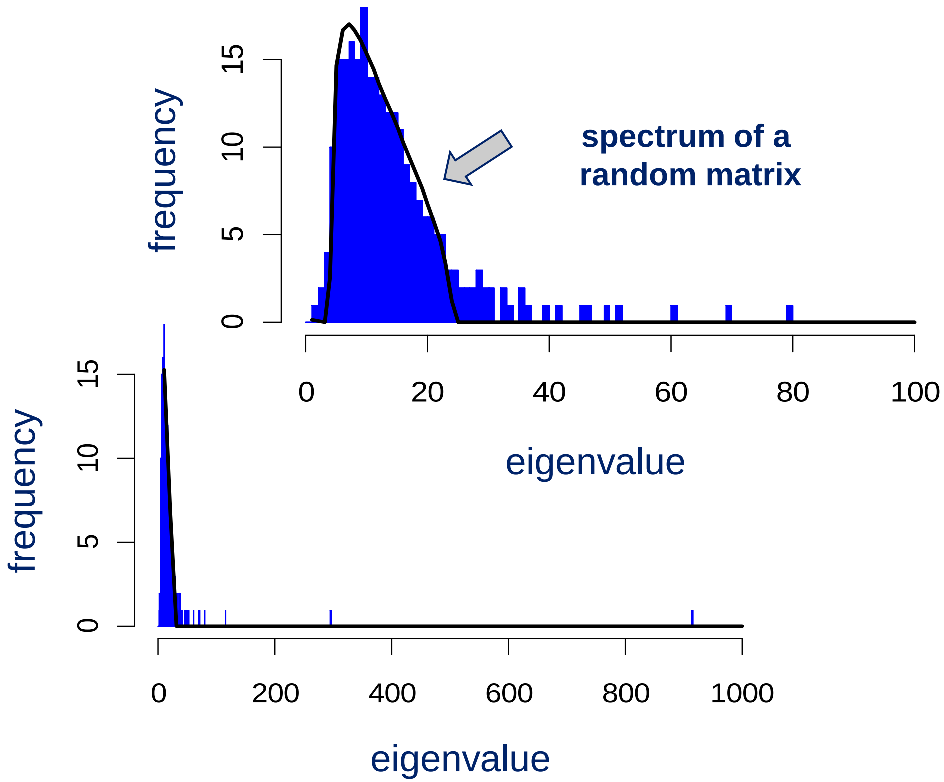
$$Var\left(\sum_{i=1}^{N} v_{ki} \cdot r_i\right) = v_k' \cdot \Sigma \cdot v_k = \lambda_k \cdot v_k' \cdot v_k = \lambda_k$$

Model Validation

Case study

- 5039 risk factors
  - 4529 equities
  - 404 interest rates
  - 76 inflation rates
  - 30 exchange rates

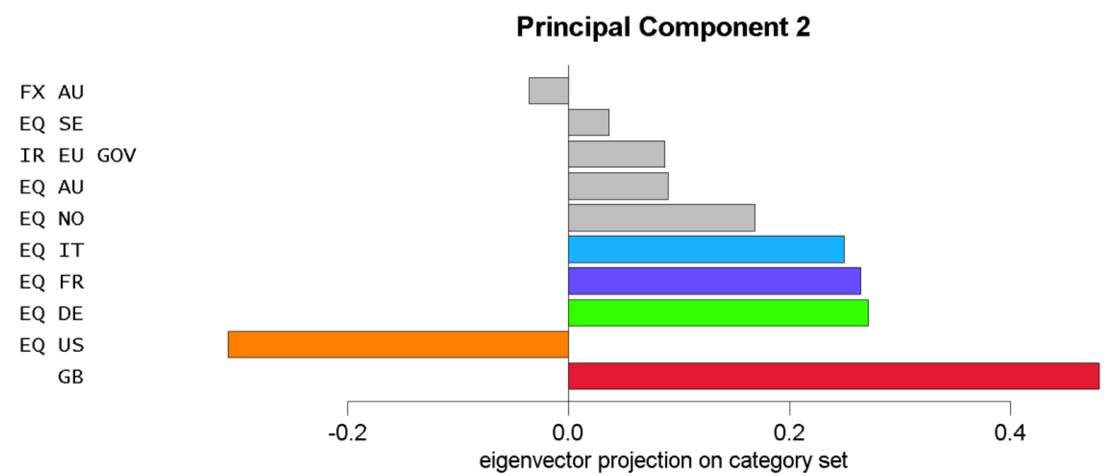
- time series from July 2008 to July 2009
- 255 eigenvalues > 0
- 54 % of the covariance is in the area of the random spectrum
- 28 eigenvalues lie above the upper limit of the random spectrum

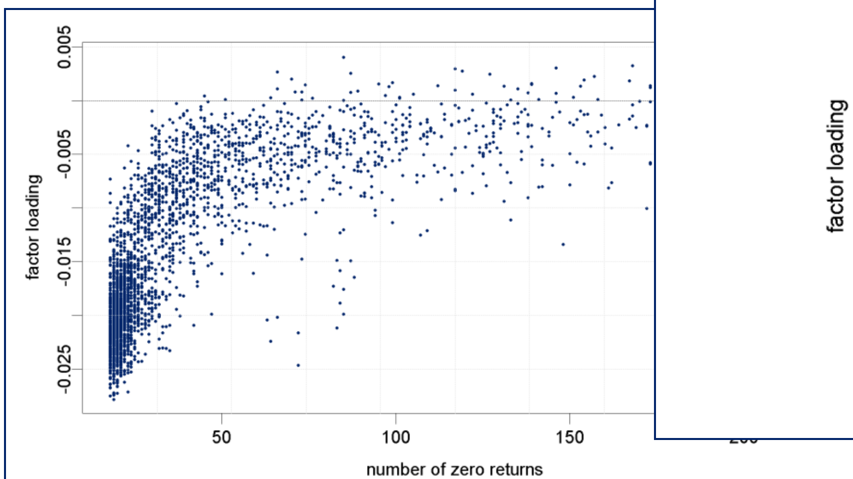


Model Validation

Example

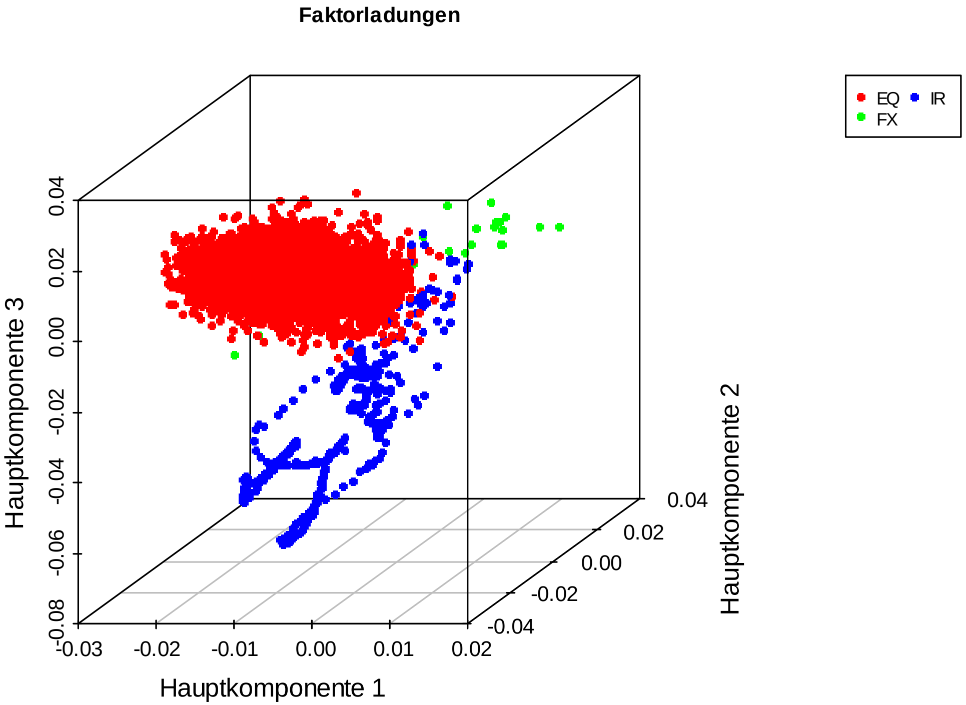
- The second principal component shows an anti-correlation between US and Europe
- A data quality issue is the observed tail to zero factor loadings. This is due to missing returns





Model Validation

3D-plots of the factor loadings can be useful for visual inspections



Conclusions

- Some examples on quantitative models in finance were addressed
- There is a LOT MORE, e.g. we have not talked about the important areas of
  - financial engineering / theoretical valuation
  - credit risk modeling
  - insurance risk
  - .....

- If you are interested in more
  ... have a look at http://www.d-fine.de
  ... or visit our office in Frankfurt on April,19th ("Ein Tag vor Ort", DPG)

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