# Search for RTR from a periodic multilayer target with degraded periodicity at the Mainz Microtron MAMI

## Experiment and simulation of target degradation

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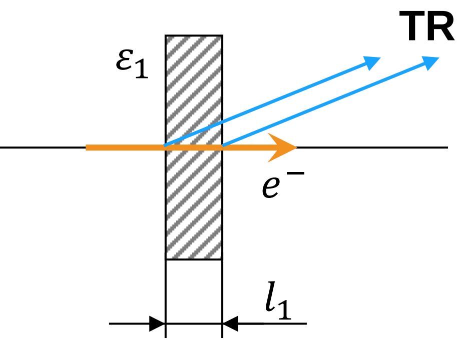
# Motivation

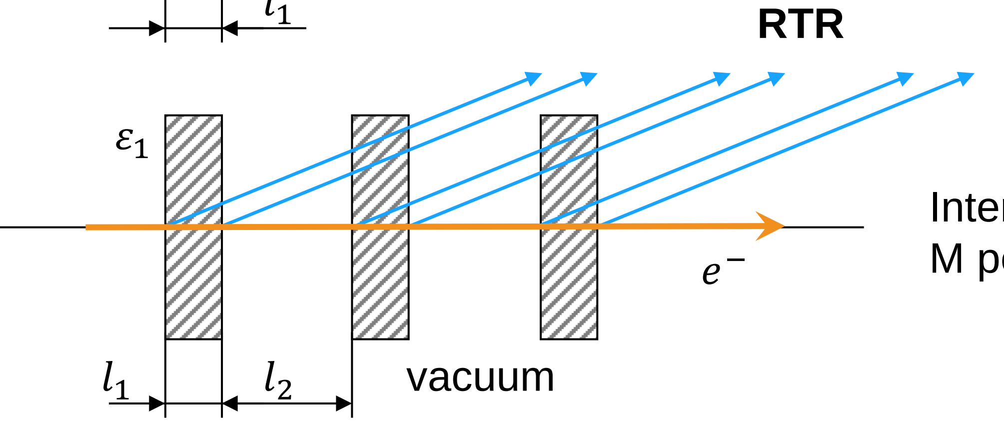
1. The main goal of this experiment is a **direct test of the Ter-Mikaelian model** of transition radiation (TR) from a periodic multilayer target
2. In this model, the interference condition for a fixed observation angle θ leads to **two branches of resonant solutions** (a“+” and “-”) for the resonant transition radiation (RTR) photon energy.
3. When the observation angle θ is increased:
   - the resonant peaks corresponding to the “-” solution are shifted to the **hard (high-energy) part of the spectrum**,
   - while the peaks of the “+” solutions move to the **soft (low-energy) part of the spectrum**.
4. For our Kapton multilayer radiator at MAMI, the model predicts pronounced resonant peaks at ~8 and ~27 keV at angles close to $\theta \approx 1/\gamma$.

# Why resonant transition radiation from multilayer targets?

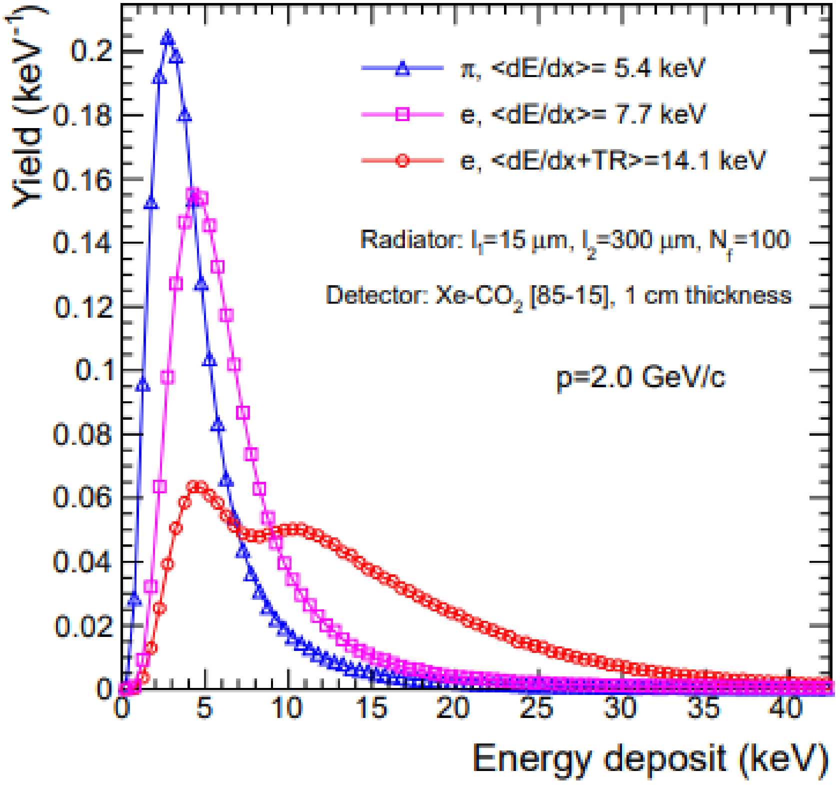
1. TR from a single interface is relatively weak and has a broad photon spectrum.
2. A periodic multilayer radiator with M periods can produce resonant of X-ray at specific photon energies.
3. In the ideal case of a perfectly periodic radiator (neglecting photon attenuation), the intensity at resonance scales as $Y \sim M^2$, leading to narrow, intense peaks in the spectrum.




Interference of TR from the two boundaries of the foil → $\mathbf{F_2}$




Interference of TR from M periods → $\mathbf{F_3}$



# RTR spectral-angular distribution

## for a multilayer radiator

The full **spectral–angular distribution of RTR** from a periodic multilayer target can be written in the form:

$$\frac{d^2W}{d\omega\, d\Omega} = \left(\frac{d^2W}{d\omega\, d\Omega}\right)_1 F_2(\omega,\theta,l_1)\;\; F_3(\omega,\theta,l_1,l_2,M)$$

where

- $\left(\frac{d^2W}{d\omega\, d\Omega}\right)_1 = \frac{\alpha\,\theta^2(\omega_p/\omega)^4}{\pi^2\omega(1/\gamma^2+\theta^2)^2\;\;(1/\gamma^2+\theta^2+(\omega_p/\omega)^2)}$ is the TR **from a single interface**,
- $F_2 = 4\;\sin^2(l_1/Z_1)$ is the interference of TR **from the two boundaries** of a foil,
- $F_3 = \sin^2(MX)/\sin^2(X)$ describes interference o TR **from M periods** of the multilayer structure,
- $Z_i = \frac{4\,\beta\,c}{\omega\,(1/\gamma^2+\theta^2+(\omega_i/\omega)^2)},\quad (\mathrm{i} = 1, 2)$ are the **formation lengths** in the two media,
- $X = l_1/Z_1 + l_2/Z_2$ is the **phase** per period
- The resonance (spectral peak) condition → $X = k\,\pi,$ $k$ - integer

# Interference of TR in a periodic multilayer

## with and without attenuation

- For a radiator with ***M* periods**, each period contributes an amplitude with a phase $X(E_\gamma, \theta)$
- The interference factor for an **ideal periodic structure (no attenuation)** is

$$F_3 = \left|1 + e^{iX} + e^{2\,iX} + \cdots e^{(M-1)\,iX}\right|^2 = \frac{\sin^2(M\,X)}{\sin^2(X)}$$

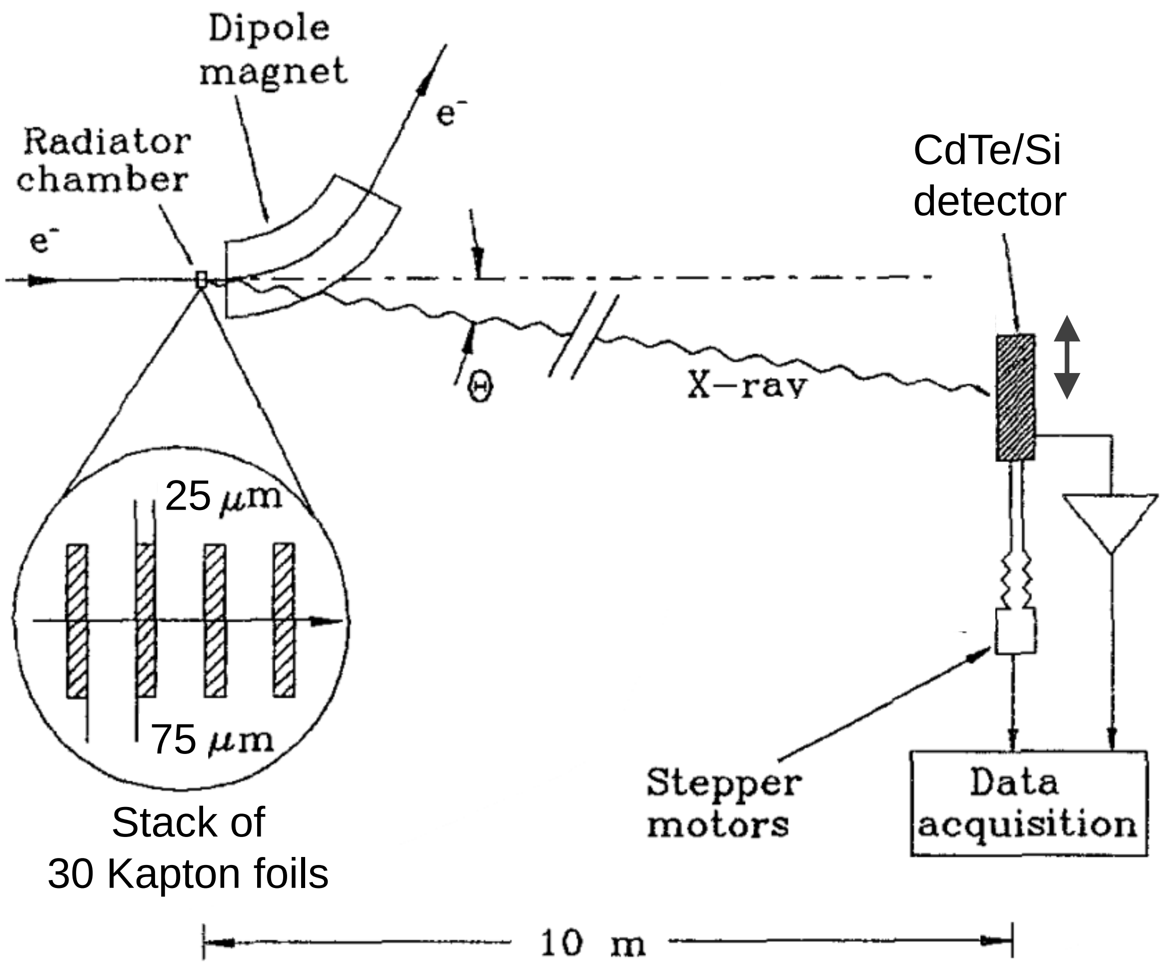
- At the **resonance condition** $X \approx k\pi$, the numerator scales as $\sim\boldsymbol{M^2}$
- For our radiator with $\boldsymbol{M = 30}$, $\boldsymbol{E_\gamma} \approx \boldsymbol{27}$ keV, and observation angle $\theta \approx 1/\gamma$, the ideal interference factor at resonance is approximately $\boldsymbol{F_3^{ideal}} \approx \boldsymbol{M^2} \approx \boldsymbol{900}$
- **Taking attenuation into account**, each period is suppressed by a factor exp(-σ)

$$F_3 = \left|1 + e^{-\sigma+iX} + e^{2\,(-\sigma+iX)} + \cdots e^{(M-1)\,(-\sigma+iX)}\right|^2 = \frac{1 + e^{-\mu\sigma} - 2e^{(-\mu\sigma/2)} + \cos(M\,X)}{1 - e^{-\sigma} - 2e^{(-\sigma/2)} + \cos(X)}$$

where $\sigma = \mu\, l_1$ if gap is vacuum

# Experimental setup at the Mainz Microtron MAMI

## Kapton multilayer radiator and experimental support kindly provided by Werner Lauth (MAMI)




$E_e = 855\ \text{MeV} \Rightarrow \gamma^{-1} \approx 1/1673 \approx 6 \cdot 10^{-4}$

**Radiator**: periodic **Kapton** target

- Manufactured in 1994 (>30 years old)
- Kapton foil thickness $(l_1)$: 25 μm
- Spacing between foils $(l_2)$: 75 μm
- Number of periods **M**: 30

**Detectors**: **CdTe** and **Si,** window **D=3 mm (?)**

**Angular resolution:** $\Delta\theta \approx \frac{3\ mm}{10\ m} \approx 3 \cdot 10^{-4}\ rad < \gamma^{-1}$

**Observation angles**: $\theta = (0.5 - 3)\,\gamma^{-1}$

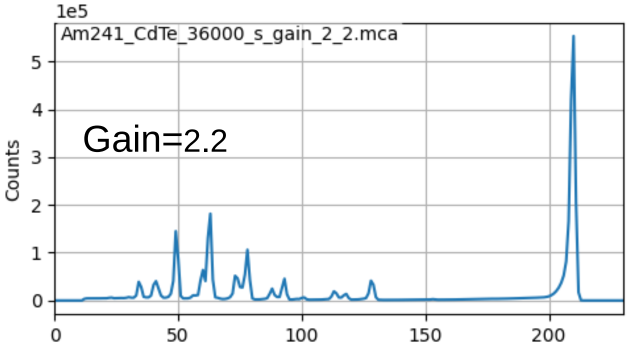
Experimental setup at MAMI. Layout adapted and slightly modified from
H. Backe, S. Gampert, A. Grendel, H.J. Hartmann, W. Lauth et al, *Resonant transition radiation in the X-Ray region from a low emittance 855-MeV electron beam*, Z. Phys **A349**, 87-92 (1994)

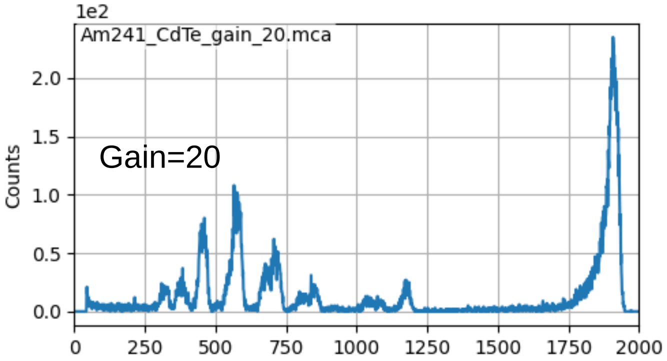
# Detector calibrations

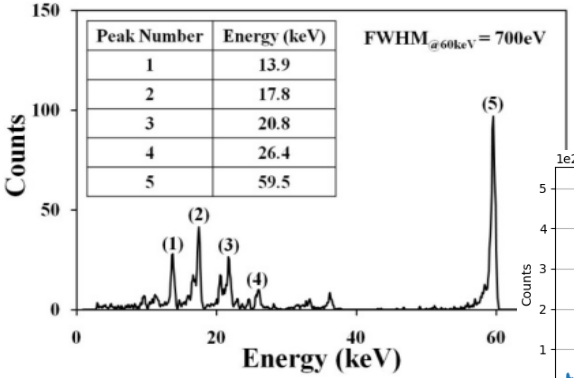
## CdTe and Si detectors with different gains

**Calibrated**

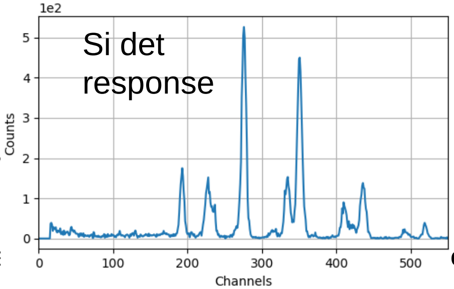
**CdTe** **Si**

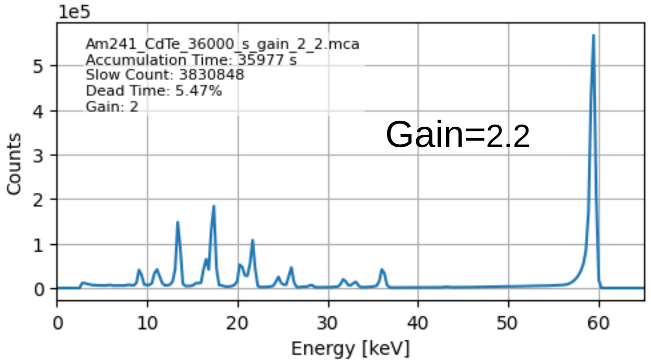


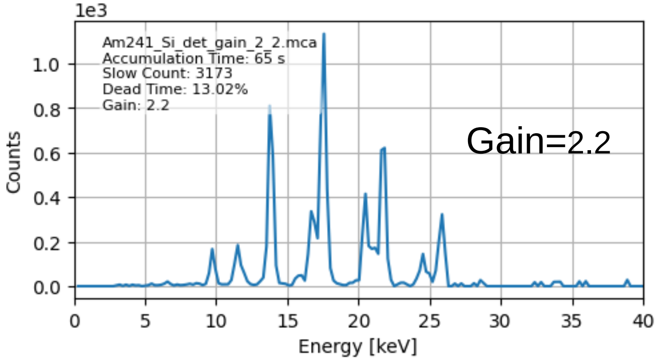



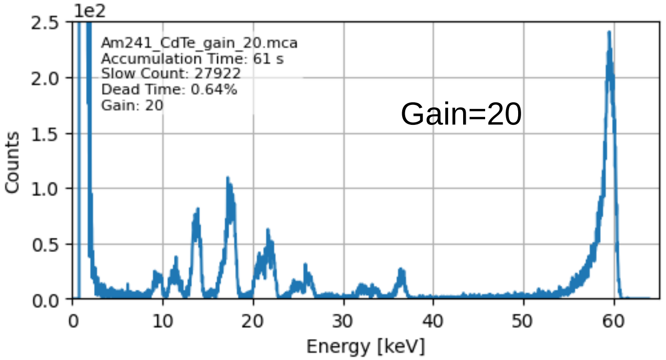



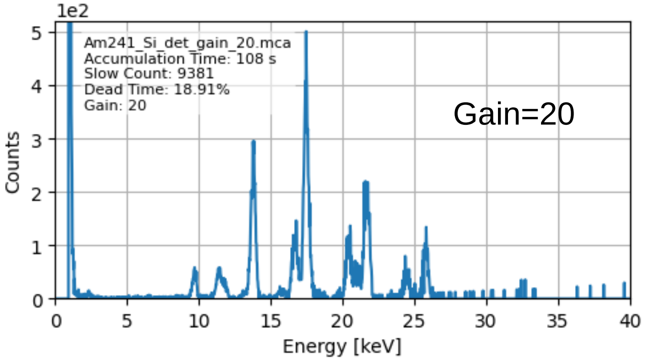

| Peak Number | Energy (keV) |
|---|---|
| 1 | 13.9 |
| 2 | 17.8 |
| 3 | 20.8 |
| 4 | 26.4 |
| 5 | 59.5 |

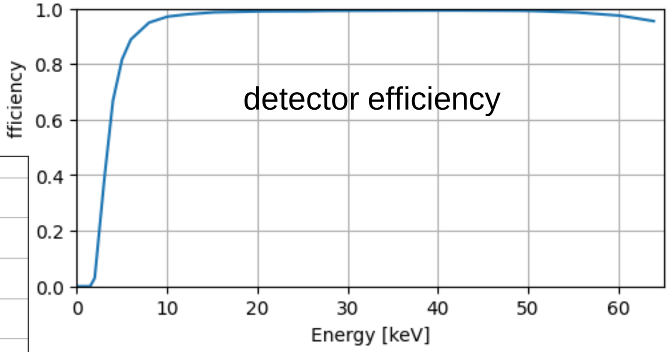


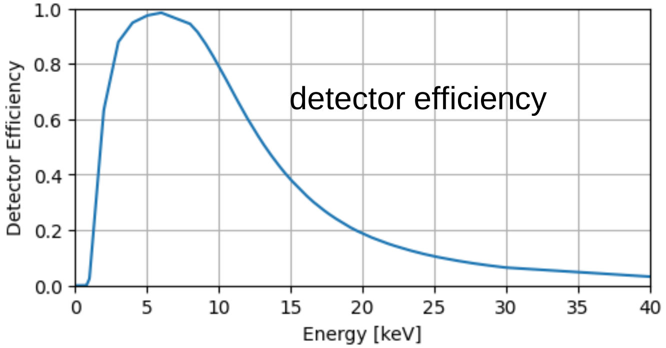



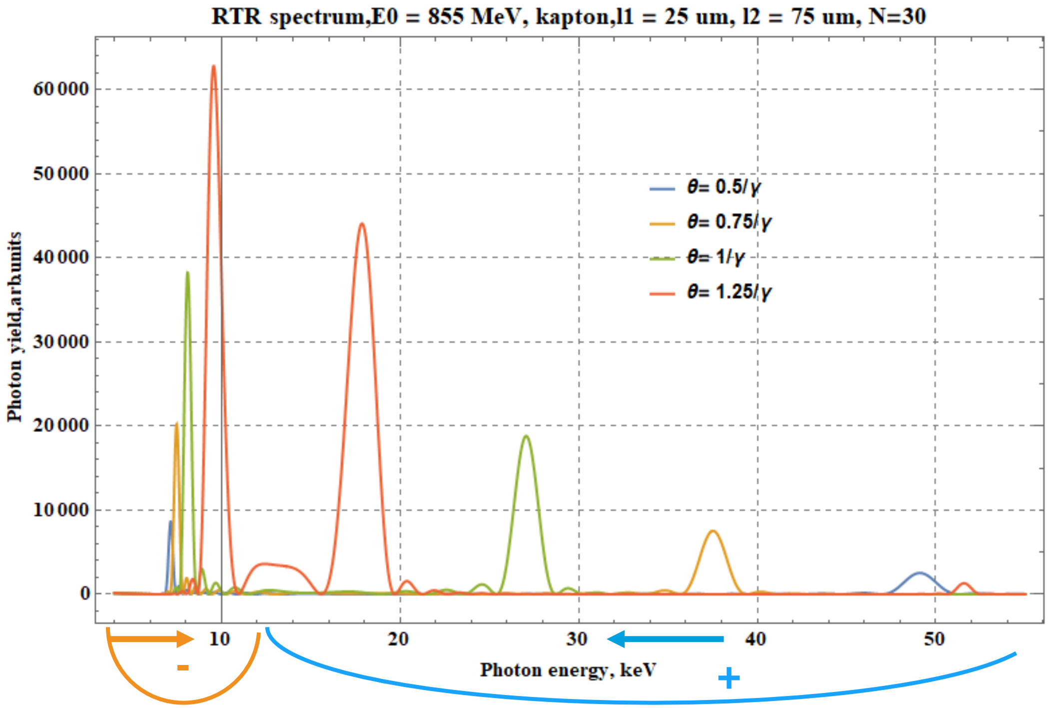



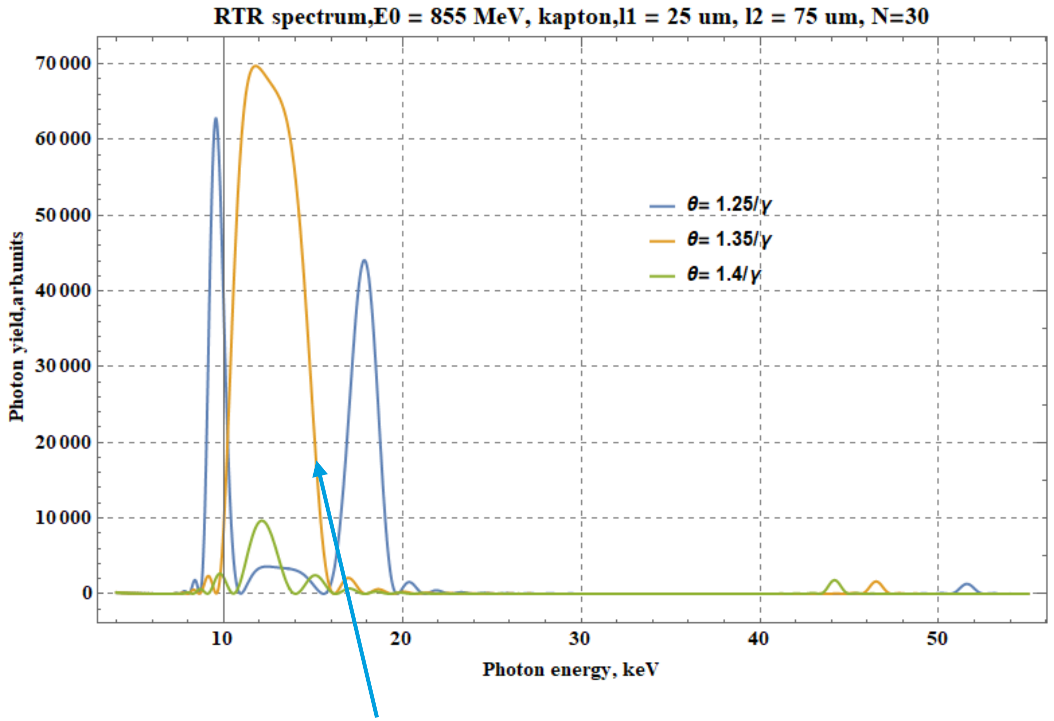



# Calculated RTR spectra for this experiment

## Ter-Mikaelian model, ideal periodic radiator

RTR spectra calculated for a Kapton radiator with
M=30, l1=25 µm, l2=75 µm, Ee=855 MeV



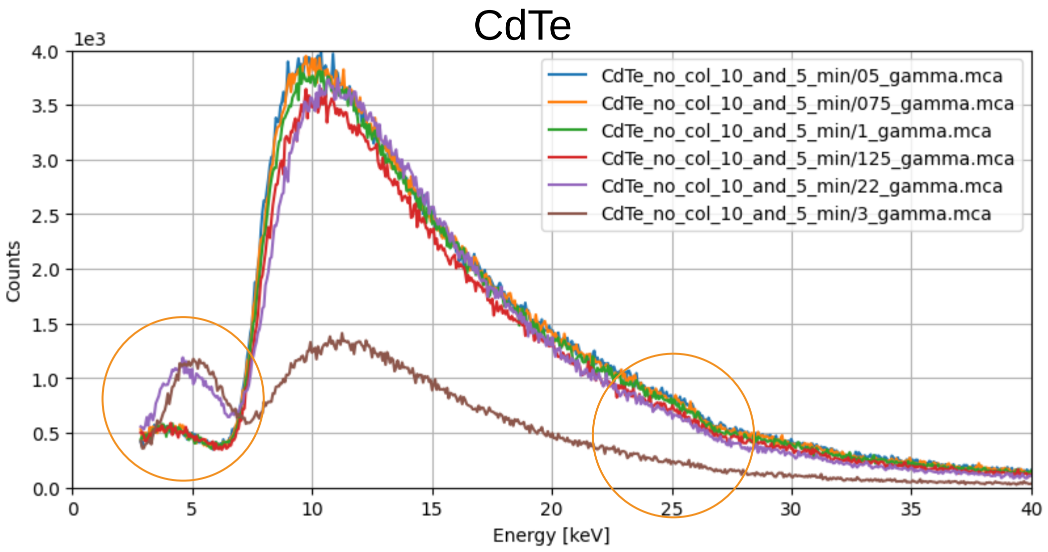

RTR peaks with photon energies **<12 keV belong to the "-"** T-M solution

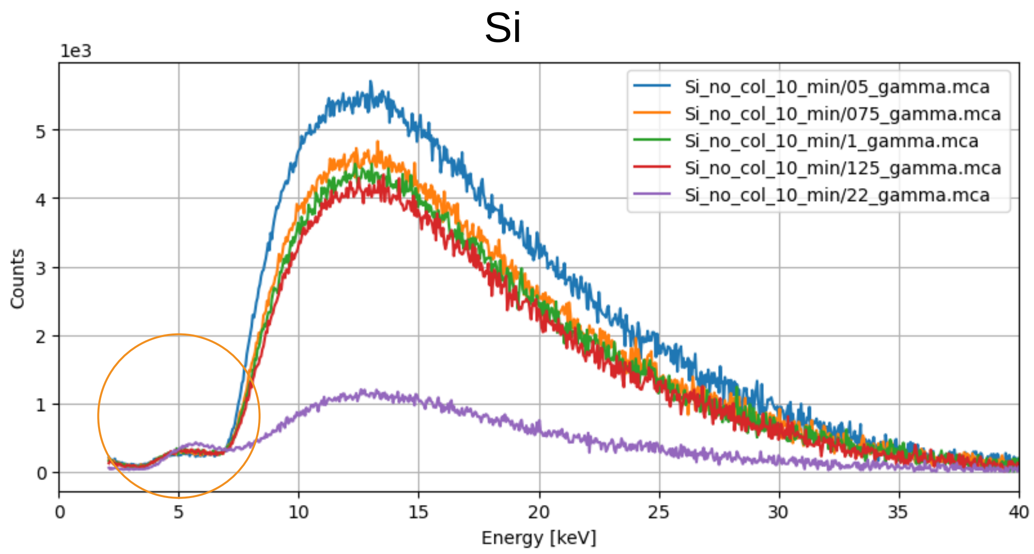
RTR peaks with photon energies **>12 keV belong to the "+"** solution

For observation angle $\theta \approx 1.35/\gamma$ the **"+"** and **"-"** solutions **merge,** so they give a **single resonant peak**

# Measured spectra by CdTe and Si detectors

## detector efficiencies taken into account



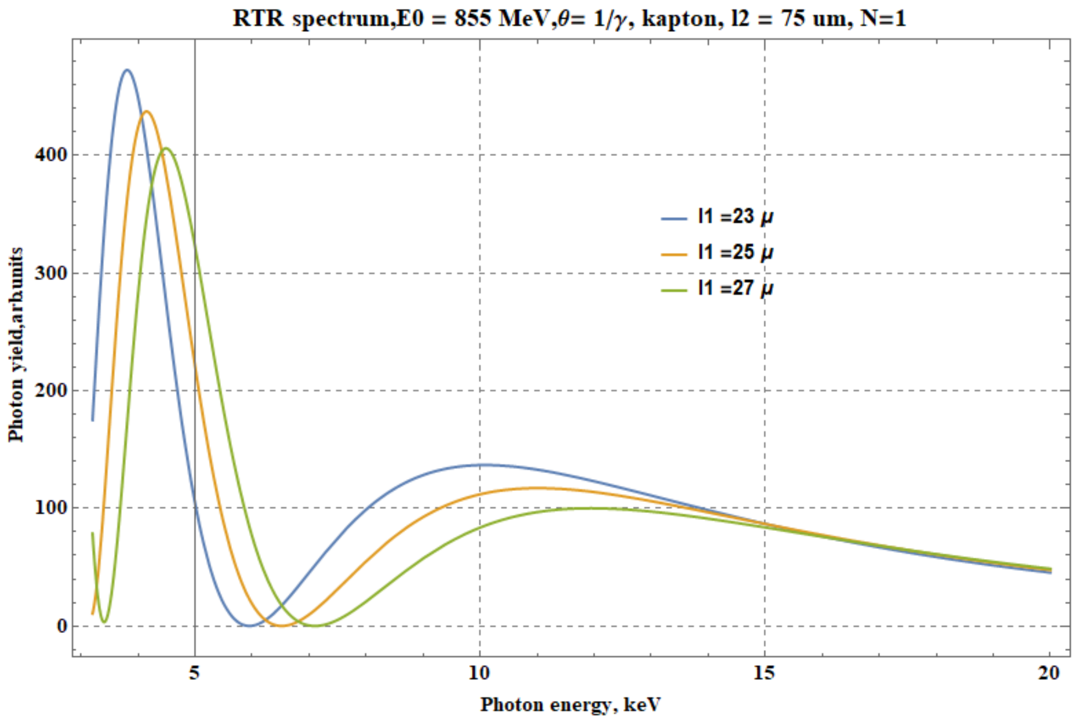




1. **Energy spectra measured** by CdTe and Si detectors for several observation angles between **0.5** $\boldsymbol{\gamma^{-1}}$ and **3.0** $\boldsymbol{\gamma^{-1}}$
2. **No sharp resonant peaks** are observed where the model predicts them
3. **Pronounced features** around **5** and **25 keV**
4. Broad high-energy tail similar to **bremsstrahlung**
5. All spectra are corrected for the energy-dependent detector efficiencies of CdTe and Si.

# TR spectrum from a single Kapton foil

## Calculation

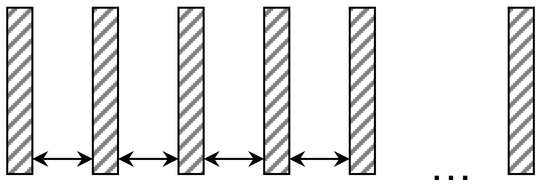
- Three foil thicknesses are compared: l1=23 μm, 25 μm, and 27 μm.
- No narrow resonant peaks appear for a single foil.
- Strong, narrow RTR peaks must come from interference in a multilayer.
- **Hypothesis**: if the periodicity is degraded, the spectrum may become closer to this single-foil shape.



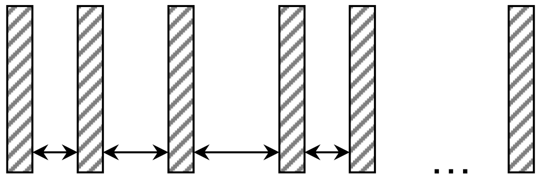
# Why are the expected peaks missing?

## Hypothesis of degraded periodicity

Ideal periodicity



Degraded periodicity



The 30 year old radiator had been used before and may have suffered:

- mechanical deformations,
- thermal cycling,
- partial damage or bending.

As a result, the radiator **may no longer be perfectly periodic.**

Instead of a **single phase** $X(E_\gamma, \theta)$ per period, we have **period-dependent phases**:

$$X_k = X_0\big(E_\gamma, \theta\big)(1 + \nu_k),$$

where $\boldsymbol{\nu_k}$ are random fluctuations in interval $-\mathbf{d} \leq \boldsymbol{\nu_k} \leq +\mathbf{d}$.

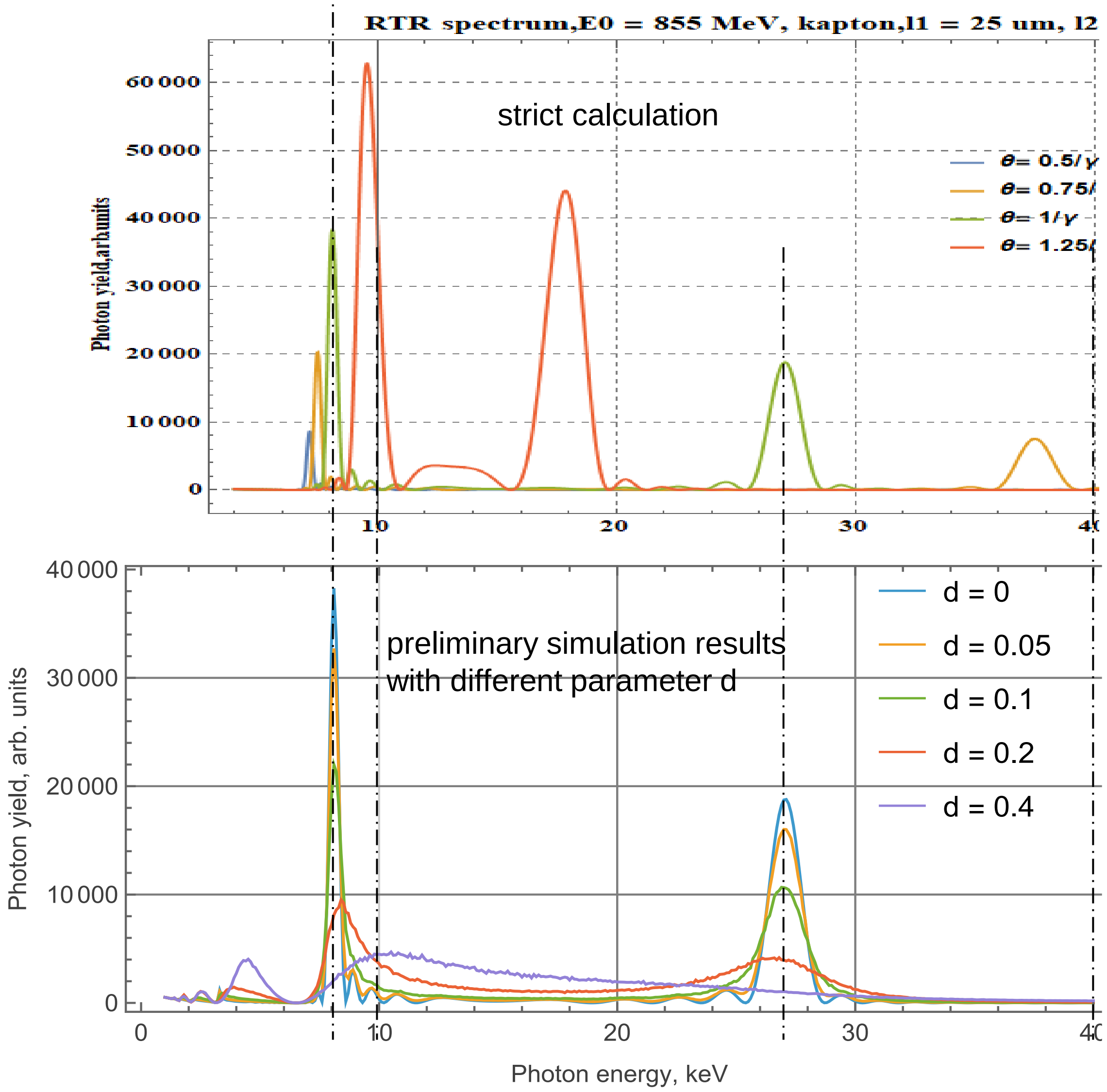
$$F_3 = \left|1 + e^{-\sigma+iX} + e^{2\,(-\sigma+iX)} + \cdots e^{(M-1)\,(-\sigma+iX)}\right|^2$$

$$F_{rand}(d) = \left|1 + e^{-\sigma+iX_1} + e^{2\,(-\sigma+iX_2)} + \cdots e^{(M-1)\,(-\sigma+iX_k)}\right|^2$$

Even relatively small fluctuations of the phases $\boldsymbol{X_k}$ **can destroy the coherent M$^2$ enhancement** and turn a sharp peaks into a **broad spectrum**.

# Random-phase interference factor (preliminary sim results)

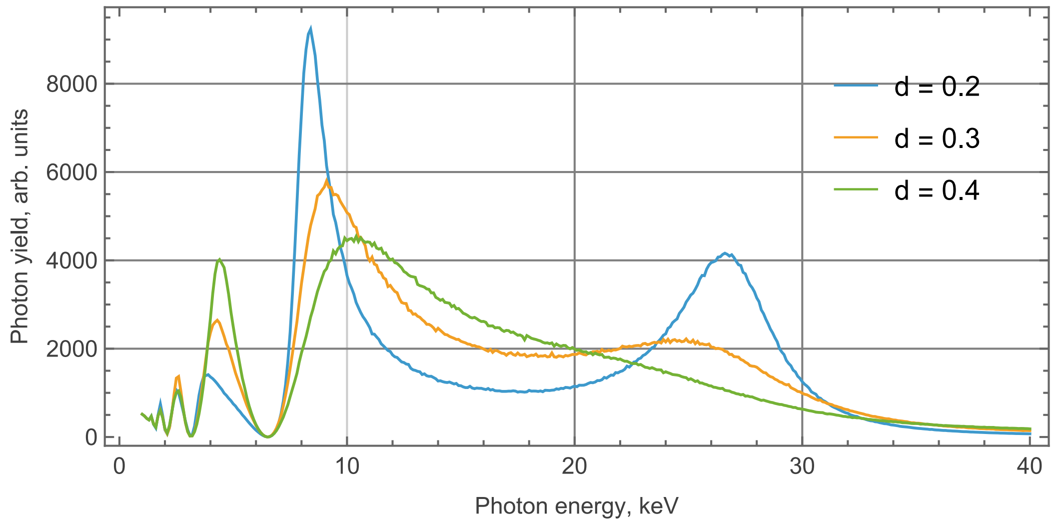
## Simulation results for M=30, $\theta = 1/\gamma$, and different degradation parameters d vs calculation




- **Random-phase factor** $F_{rand}(E_\gamma, d)$ was simulated for M=30 and $\theta = 1/\gamma$
- In the RTR spectrum, the strict periodic factor $F_3(E_\gamma)$ was **replaced** by the random-phase factor $F_{rand}(E_\gamma, d)$.
- For each degradation parameter **d,** the photon spectrum was calculated as

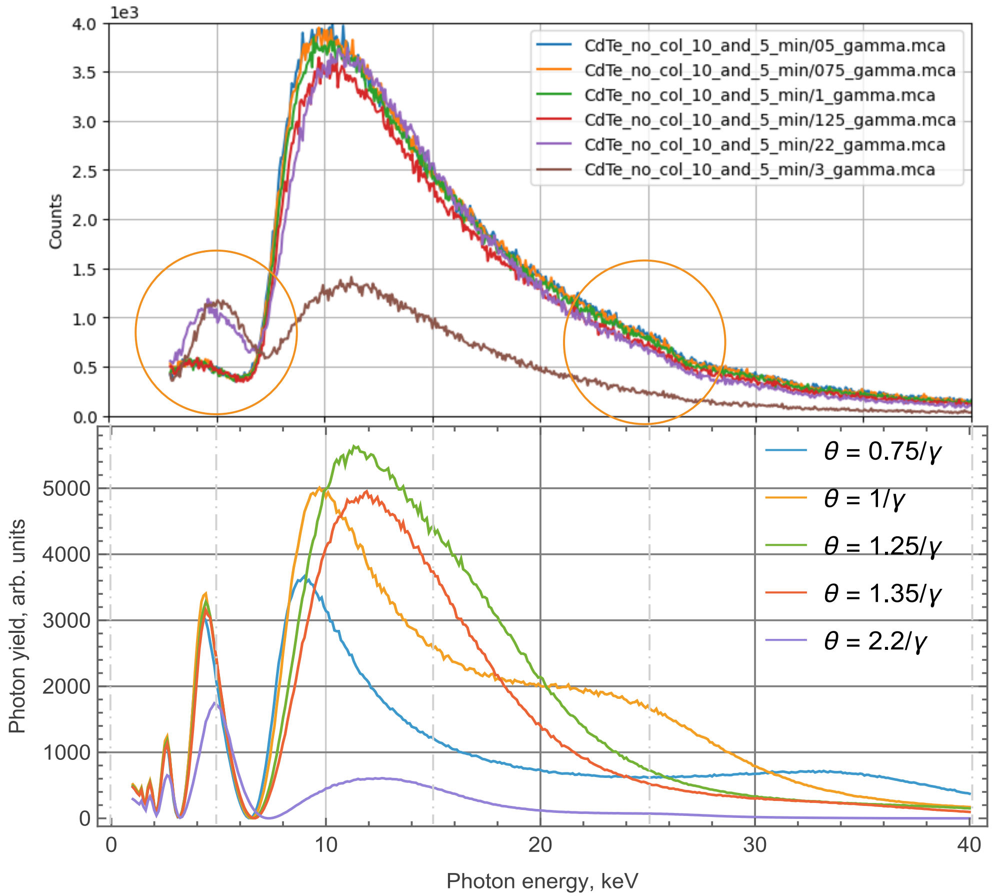
$$\frac{dN_{\mathrm{rand}}}{dE_\gamma}(E_\gamma, d) = N_{\mathrm{TR}}^{(1)}(E_\gamma, \theta)\, F_2(E_\gamma)\, F_{\mathrm{rand}}(E_\gamma, d).$$

- For d=0.05, the resonant peaks are only slightly reduced compared to the ideal $F_{3N}(E_\gamma) = F_{rand}(E_\gamma, d = 0)$
- For d=0.4, the resonant structures are disappeared, the spectrum becomes broad and smooth very similar to the measured spectra



# Simulated spectra with degraded target vs experiment

## degradation parameter d=0.35

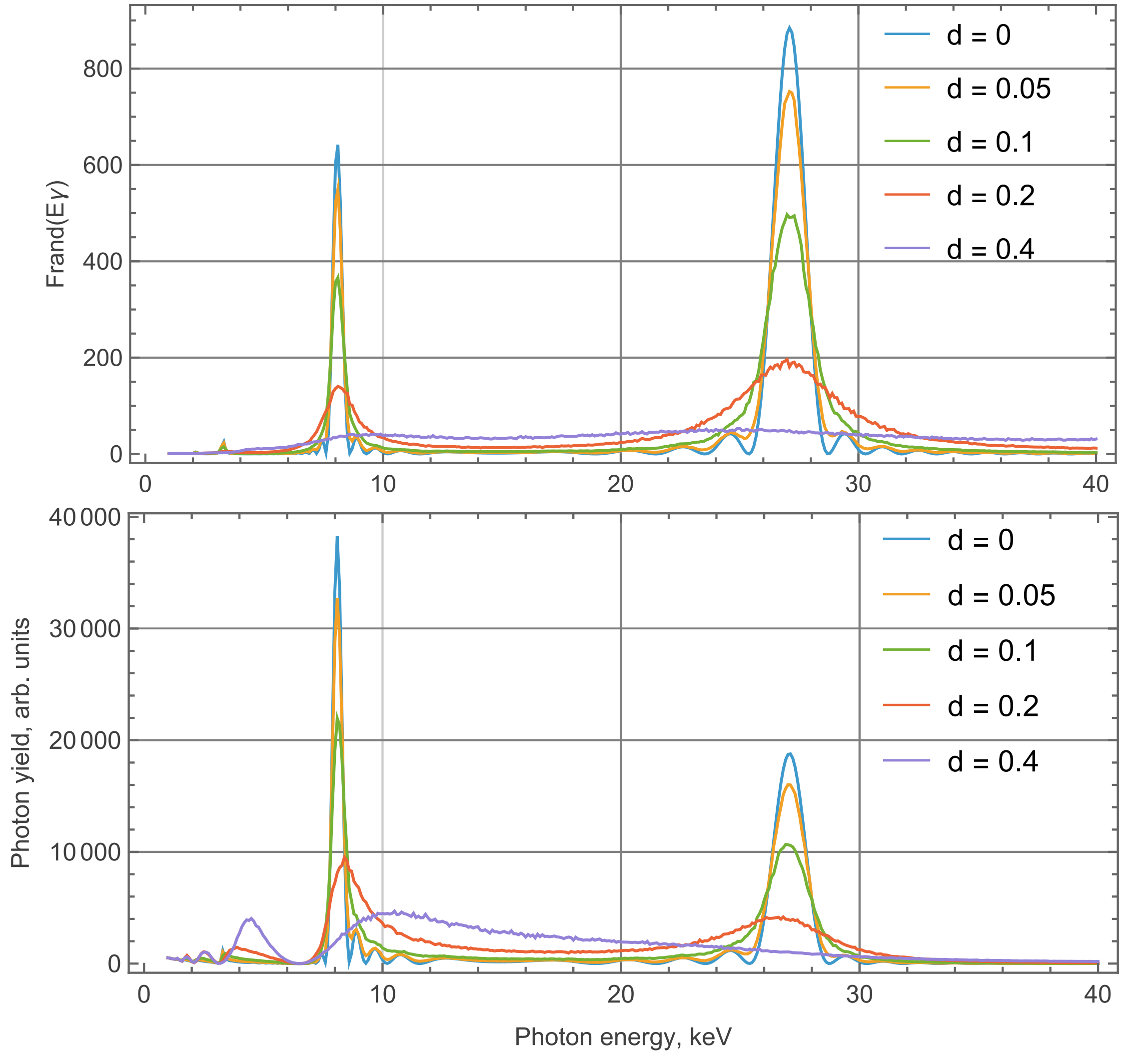


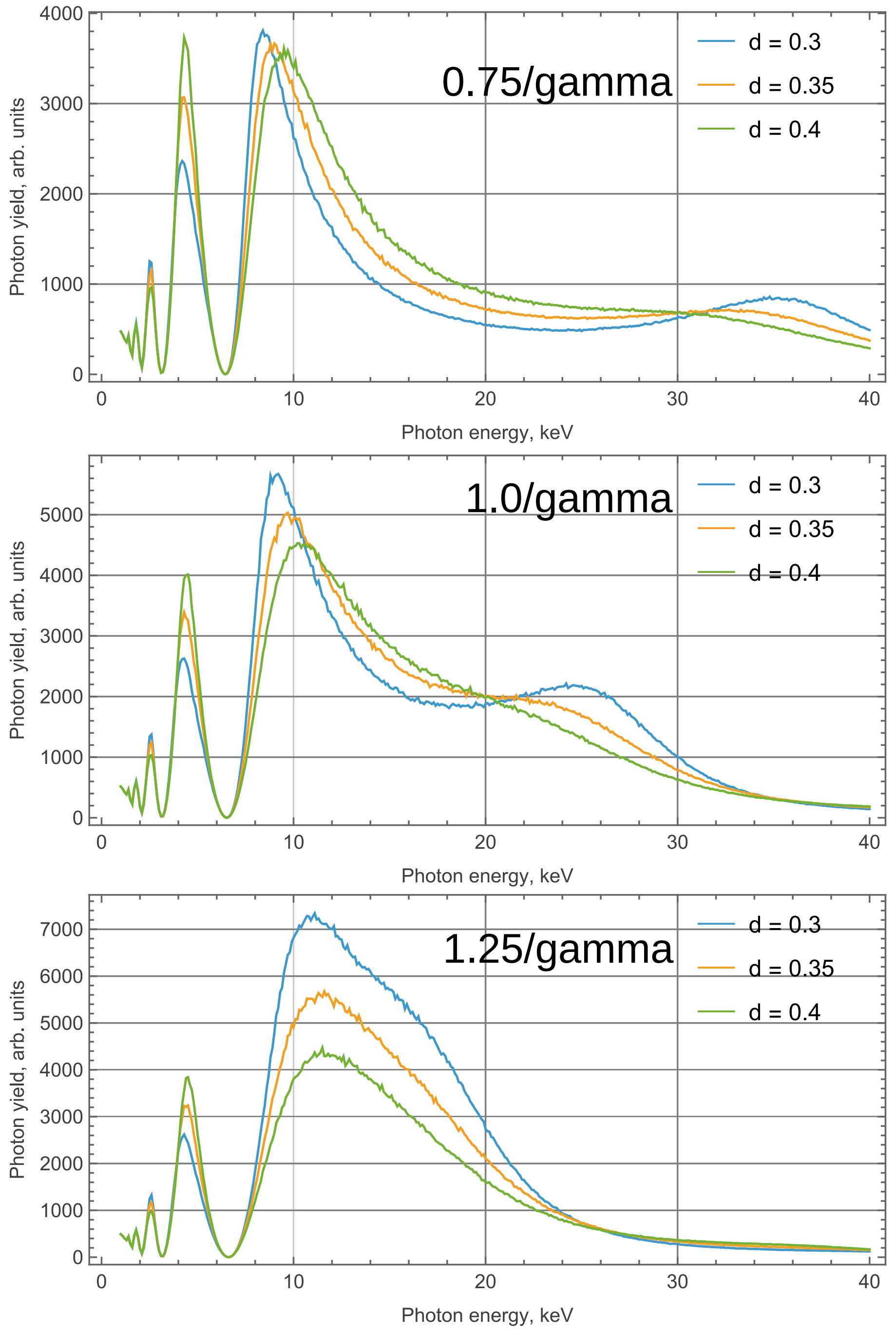

- **Preliminary, the simulation results with target degradation (d=0.35) qualitatively agrees** with our measured spectra: no sharp peaks, similar features at 5 and 25 keV.
- This may suggest that the actual Kapton radiator was degraded and far from an ideal periodic structure.

# Conclusion

1. We performed a **search for RTR** from a periodic Kapton radiator with M=30 periods at MAMI (855 MeV).
2. The experimental photon spectra show **no sharp resonant peaks** in the expected energy range but they clearly exhibit the **TR peak from a single foil**.
3. Using a **random-phase interference simulation** we found that:
   - phase fluctuations between periods with degradation parameter ***d*** **destroy the $M^2$ resonance,**
   - for degradation parameter d≈0.35, the **simulated spectra qualitatively agree with the measured spectra**,
   - this may imply a **significant degradation of the radiator periodicity**.
4. Our model provides the RTR spectrum simulation with taking into account an empirical degradation parameter ***d*** and can be used to describe future measurements with multilayer radiators.

# Thank you

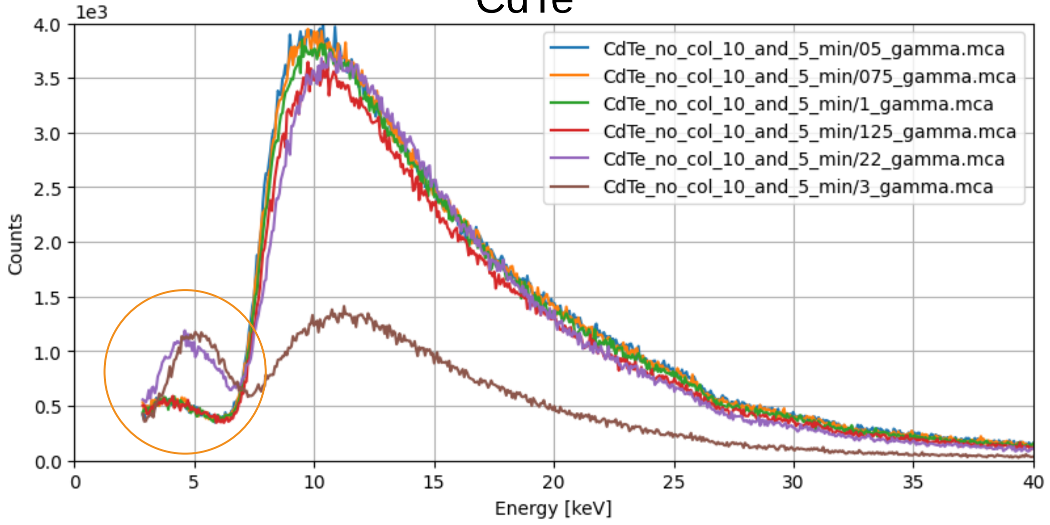

Frand(Eγ)
800
600
400
200
0
0
10
20
30
40
d = 0
d = 0.05
d = 0.1
d = 0.2
d = 0.4
Photon yield, arb. units
40 000
30 000
20 000
10 000
0
0
10
20
30
40
d = 0
d = 0.05
d = 0.1
d = 0.2
d = 0.4
Photon energy, keV

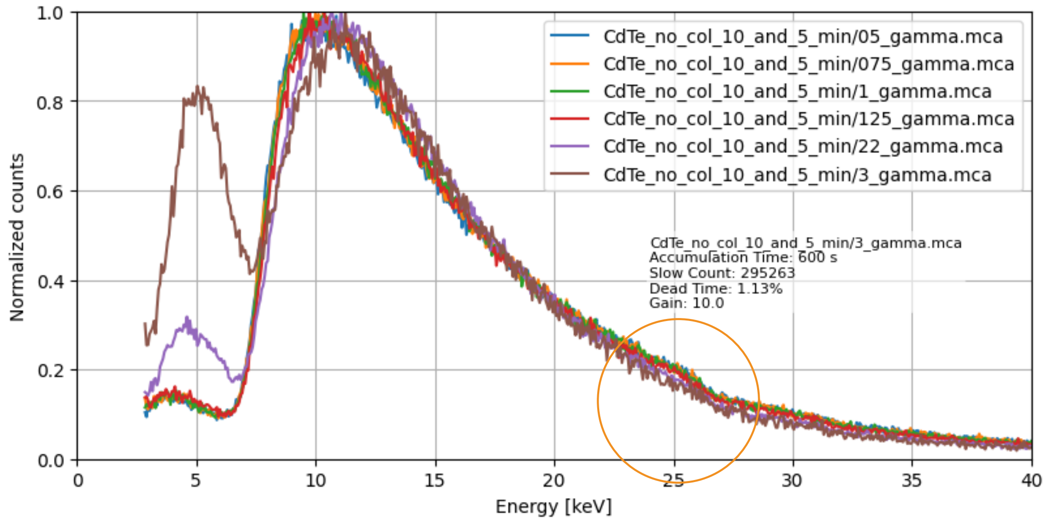

0.75/gamma
d = 0.3
d = 0.35
d = 0.4
Photon yield, arb. units
Photon energy, keV
1.0/gamma
d = 0.3
d = 0.35
d = 0.4
Photon yield, arb. units
Photon energy, keV
1.25/gamma
d = 0.3
d = 0.35
d = 0.4
Photon yield, arb. units
Photon energy, keV

# Measured spectra from CdTe and Si detectors

## detector efficiencies taken into account

CdTe




Si