DIFFERENTIAL CROSS SECTION OF FAST CHARGED PARTICLE SCATTERING BY PERIODIC ATOMIC PLANES IN THE EIKONAL APPROXIMATION

Viktoriia Omelchenko

National Science Center “Kharkiv Institute of Physics and Technology”

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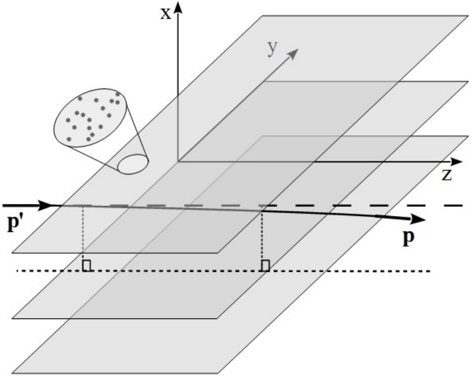
Scattering Problem

$$x_j = aj;$$

$$g(y_n) = \begin{cases} \frac{1}{L_y}, & -\frac{L_y}{2} \leq y_n \leq \frac{L_y}{2}; \\ 0, & |y_n| > \frac{L_y}{2}. \end{cases}$$

N_x planes of N_p atoms in each of them;
Plane sizes: L_y, L_z;
$N_p = n_{yz} L_y L_z$



Differential Scattering Cross Section in Eikonal Approximation

$$\frac{d^2\sigma}{dq_x dq_y} = |a(\vec{q}_\perp)|^2,$$

$$a(\vec{q}_\perp) = \int_{\mathbb{R}^2} d^2\rho e^{\frac{i}{\hbar}\vec{q}\vec{\rho}} \left\{ 1 - exp\left[\frac{i}{\hbar}\chi_0^{(N)}(\vec{\rho})\right]\right\},$$

where $\vec{\rho} = (x, y)$, $\vec{q}_\perp = (q_x, q_y)$.

$$\chi_0^{(N)} = -e\int_{-\infty}^{\infty} dz U^{(N)}(\vec{\rho}, z),$$

$$\chi_0^{(N)}(\vec{\rho}) = \sum_{n=1}^{N} \chi_0(\vec{\rho} - \vec{\rho}_n),$$

$$\chi_0(\vec{\rho} - \vec{\rho}_n) = -e\int_{-\infty}^{\infty} dz u(\vec{\rho} - \vec{\rho}_n, z).$$

Differential Scattering Cross Section in Eikonal Approximation

$$\frac{d\sigma}{d^2 q_\perp} = \frac{1}{4\pi^2} \int_{\mathbb{R}^4} d^2\rho d^2\rho' e^{i\vec{q}_\perp(\vec{\rho}-\vec{\rho}')} \times$$

$$\times \langle 1 - e^{i\chi_0^{(N)}(\vec{\rho})} - e^{-i\chi_0^{(N)}(\vec{\rho}')} + e^{i[\chi_0^{(N)}(\vec{\rho})-\chi_0^{(N)}(\vec{\rho}')]} \rangle .$$

$$\langle e^{i[\chi_0^{(N)}(\vec{\rho})-\chi_0^{(N)}(\vec{\rho}')]} \rangle = \exp\left\{ N_p \left[i \sum_{j=1}^{N_x} \langle \chi_{(j)} - \chi'_{(j)} \rangle - \right.\right.$$

$$\left.\left. - \frac{1}{2} \sum_{j=1}^{N_x} \langle \left(\chi_{(j)} - \chi'_{(j)} \right)^2 \rangle + \frac{1}{2} \sum_{j=1}^{N_x} \langle \chi_{(j)} - \chi'_{(j)} \rangle^2 + \ldots \right] \right\} ,$$

where $\chi_{(j)} = \chi_0(x - aj, y - y_0)$, $\chi'_{(j)} = \chi_0(x' - aj, y' - y_0)$, the brackets $\langle \ldots \rangle$ denote averaging.

Differential Scattering Cross Section in Eikonal Approximation

$$\frac{d\sigma}{d^2q_\perp} = \frac{1}{4\pi^2}\int_{\mathbb{R}^4} d^2\rho d^2\rho' \; e^{i\vec{q}_\perp(\vec{\rho}-\vec{\rho}')}\left(1 - e^F - e^{F'} + e^F e^{F'} e^G\right),$$

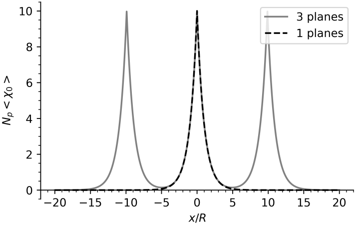
where $F(x) = iN_p\left\{\sum_{j=1}^{N_x}\langle\chi_{(j)}\rangle - \frac{1}{2}\sum_{j=1}^{N_x}\langle\chi^2_{(j)}\rangle\right\}$, $F'(x') = F^*(x')$,
$G(x,x',\Delta y) = 2N_p\sum_{j=1}^{N_x}\langle\chi_{(j)}\chi'_{(j)}\rangle$, $\tilde{G}(x,x') = G(x,x',\Delta y = 0)$

$$\frac{d\sigma}{dq_x} = \frac{L_y}{2\pi}\left\{I_0 - I_1 - I_2 + I_3\right\},$$

where $I_0 = \int_{\mathbb{R}^2} dxdx' \; e^{iq_x(x-x')}\left(1 - e^F\right)\left(1 - e^{F'}\right)$,
$I_1 = \int_{\mathbb{R}^2} dxdx' \; e^{iq_x(x-x')}\left(1 - e^F\right)\left(1 - e^{F'}\right)\left(1 - e^{\tilde{G}}\right)$,
$I_2 = \int_{\mathbb{R}^2} dxdx' \; e^{iq_x(x-x')}\left(1 - e^{\tilde{G}}\right)$,
$I_3 = 2Re\int_{\mathbb{R}^2} dxdx' \; e^{iq_x(x-x')}\left(1 - e^F\right)\left(1 - e^{\tilde{G}}\right)$.

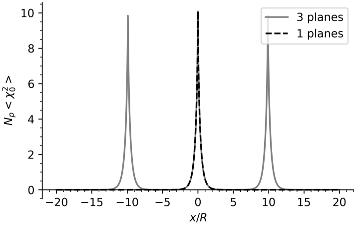
Scattering on Periodic Planes of Atoms

For screened Coulomb potential $u(\vec{r}) = \frac{Ze}{r}\exp(-\frac{r}{R})$:



$$N_p\langle\chi_0\rangle = A\exp\left[-\frac{|x|}{R}\right],$$

$$A = 2Z\alpha n_{yz} R L_z.$$



$$N_p\langle\chi_0\chi_0^{'}\rangle\Big|_{\Delta y=0} = \pi^2 Z\alpha A\{1-$$

$$-\xi\left[L_{-1}(\xi)K_0(\xi) + L_0(\xi)K_1(\xi)\right]\},$$

where $\xi = \frac{|x|+|x'|}{R}$, $K_m(\xi)$ is modified Bessel function of the second kind, $L_m(\xi)$ is modified Struve function.

$$N_p\langle(\chi_0)^2\rangle = \pi^2 Z\alpha A\left\{1 - \frac{2|x|}{R}\left[L_{-1}\left(\frac{2|x|}{R}\right)K_0\left(\frac{2|x|}{R}\right) + L_0\left(\frac{2|x|}{R}\right)K_1\left(\frac{2|x|}{R}\right)\right]\right\}$$

Differential Cross Section of Scattering on Isolated "Substructures"

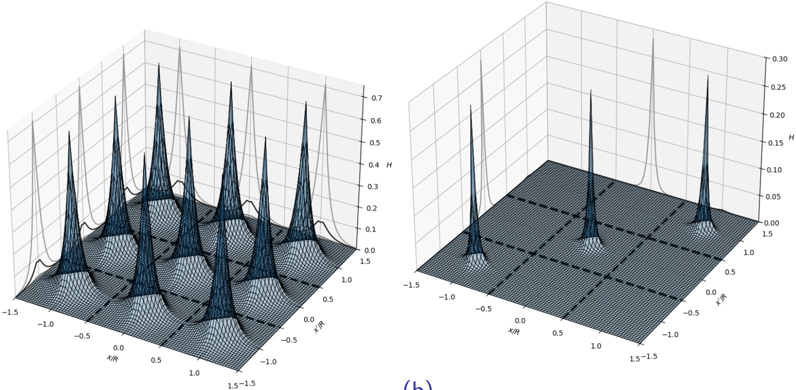
$$H\left(\left\{\sum_m f_{(s),m}(X)\right\}_{s=1}^{M}\right) \approx H\left(f_{(1),m}(X), ..., f_{(M),m}(X)\right)\Big|_{X\in Y_m}.$$

$$\int_{\mathbb{R}^{dimX}} dX\ e^{iQX} H\left(\left\{\sum_m f_{(s),m}(X)\right\}_{s=1}^{M}\right) \approx$$

$$\approx \left(\sum_m e^{iQX_m}\ \tilde{1}_{H(Y_m)}\right)\left(\int_{\mathbb{R}^{dimX}} dX\ e^{iQX} H\left(\left\{f_{(s)}^{(1)}(X)\right\}_{s=1}^{M}\right)\right),$$

where $\left\{f_{(s)}^{(1)}(X)\right\}_{s=1}^{M}$ corresponds to a set of functions $\left\{f_{(s),m}(X)\right\}_{s=1}^{M}$ for a single isolated structure

Differential Cross Section of Scattering on Isolated "Substructures"



(a) $H = \left(1 - e^{F}\right)\left(1 - e^{F'}\right)$

(b) $H = \left(1 - e^{F}\right)\left(1 - e^{F'}\right)\left(1 - e^{\tilde{G}}\right)$

Figure: Dependence of H on x and x' for $A = 1$ for 3 atomic planes

Differential Cross Section of Scattering on Isolated "Substructures"

$$\frac{d\sigma}{dq_x} = \frac{L_y}{2\pi}\left\{D_{N_x} I_0^{(1)} + N_x\left(-I_1^{(1)} - I_2^{(1)} + I_3^{(1)}\right)\right\},$$

where $D_{N_x} = \left|\sum_{k=1}^{N_x} e^{iq_x x_k}\right|^2,$

$I_0^{(1)} = \int_{\mathbb{R}^2} dxdx'\ e^{iq_x(x-x')}\left(1 - e^{F^{(1)}}\right)\left(1 - e^{F'^{\ (1)}}\right),$

$I_1^{(1)} = \int_{\mathbb{R}^2} dxdx'\ e^{iq_x(x-x')}\left(1 - e^{F^{(1)}}\right)\left(1 - e^{F'^{\ (1)}}\right)\left(1 - e^{\tilde{G}^{(1)}}\right),$

$I_2^{(1)} = \int_{\mathbb{R}^2} dxdx'\ e^{iq_x(x-x')}\left(1 - e^{\tilde{G}^{(1)}}\right),$

$I_3^{(1)} = 2Re\int_{\mathbb{R}^2} dxdx'\ e^{iq_x(x-x')}\left(1 - e^{F^{(1)}}\right)\left(1 - e^{\tilde{G}^{(1)}}\right)$

Calcultions

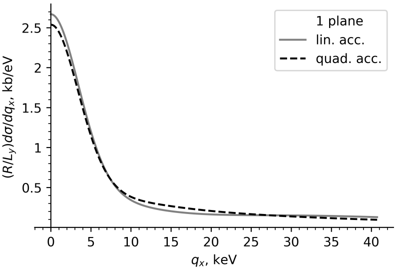
$$A = 10$$

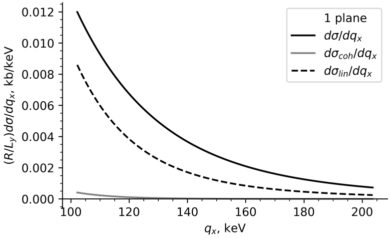
For parameters of (110) Si planes:

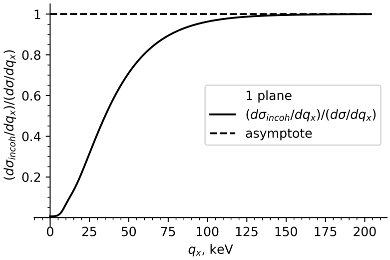
$$L_z[\mu\text{m}] = 0.026\ A \implies L_z = 0.26\ \mu\text{m}$$

$$\varepsilon[\text{GeV}] \gg 20\ (L_z[\mu\text{m}])^2 \implies \varepsilon \gg 1.35\ \text{GeV}$$

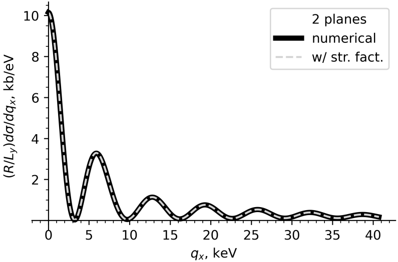
Scattering on a Single Plane of Atoms

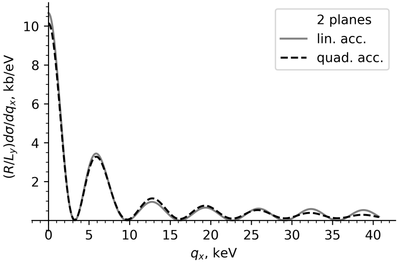




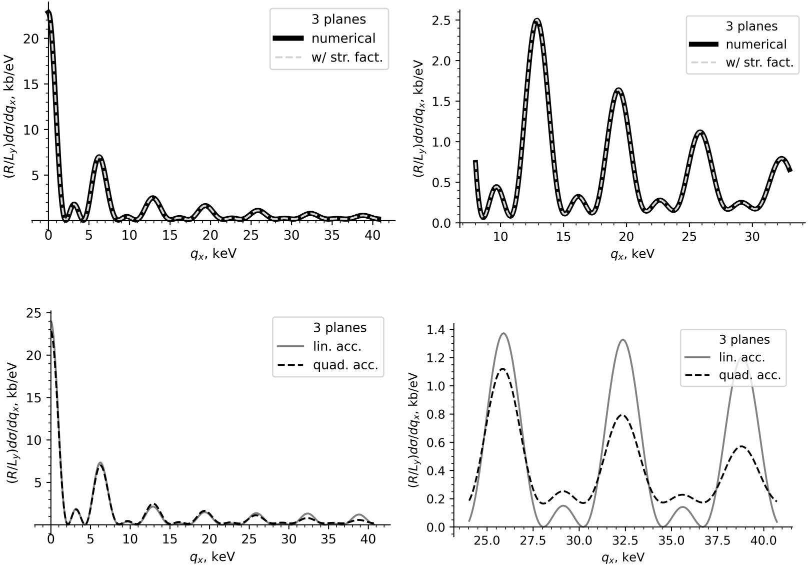


Differential Cross Section of Scattering on 2 Planes of Atoms

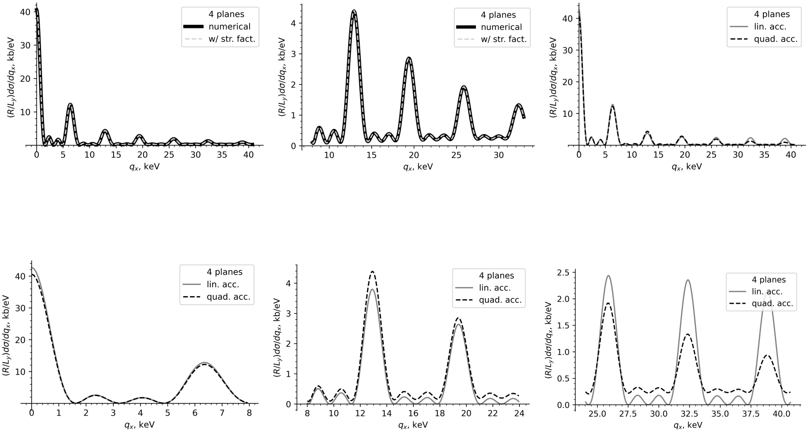




Differential Cross Section of Scattering on 3 Planes



Differential Cross Section of Scattering on 4 Planes



Comparison between eikonal and Born approximations

Born approximation:

$$\frac{d\sigma_{Born}^{(N_x)}}{dq_x} = N_p \left\{ N_x \frac{d\sigma_{Born}^{atom}}{dq_x} + 2\pi n_{yz} L_z D_{N_x} e^{-q_x^2 \langle u_x^2 \rangle} \frac{d^2\sigma_{Born}^{atom}}{dq_x dq_y}\bigg|_{q_y=0} \right\}.$$

In terms of planes:

$$\frac{d\sigma_{Born}^{(N_x)}}{dq_x} = N_x \frac{d\sigma_{B,incoh}^{(1)}}{dq_x} + D_{N_x} \frac{d\sigma_{B,coh}^{(1)}}{dq_x},$$

$$\frac{d\sigma_{B,coh}^{(1)}}{dq_x} = 2\pi n_{yz} N_p L_z e^{-q_x^2 \langle u_x^2 \rangle} \frac{d^2\sigma_{Born}^{atom}}{dq_x dq_y}\bigg|_{q_y=0}, \quad \frac{d\sigma_{B,incoh}^{(1)}}{dq_x} = N_p \frac{d\sigma_{Born}^{atom}}{dq_x}.$$

Eikonal approximation:

$$\frac{d\sigma}{dq_x} = \frac{L_y}{2\pi} \left\{ D_{N_x} I_0^{(1)} + N_x \left(-I_1^{(1)} - I_2^{(1)} + I_3^{(1)} \right) \right\}.$$

For $N_x \gg 1$: $D_{N_x} \sim N_x \sum_{\tilde{g}_j} \delta(q_x - \tilde{g}_j)$, where $\tilde{g}_j = \frac{2\pi j}{a}$

Conclusions

- We obtained the differential cross section in the eikonal approximation for a fast charged particle scattering on sets of periodic planes of atoms numerically and using structure factor.
- Cross sections obtained with both methods agree well.
- Suggested approach allows considering scattering on targets of complicated structure in a relatively easy way using less computing power and time comparing to the numerical approach.
- Obtained cross sections are sensitive to number of planes in the target.
- The case of large number of planes in the target was also considered.

For more information:

1. V.D. Omelchenko. On fast charged particles scattering on periodic planes of atoms. Preprint arXiv:2504.18351 [hep-th]. 2025. https://doi.org/10.48550/arXiv.2504.18351

2. V.D. Omelchenko. On fast charged particle scattering by periodic atomic planes: quadratic potential corrections. Preprint arXiv:2511.17667 [quant-ph]. 2025. https://doi.org/10.48550/arXiv.2511.17667

THANK YOU FOR ATTENTION

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