Workshop on the DFG project "Relativistic Electron Beam Dynamics in Crystals and Related Electrodynamic Processes"

Radiation spectra and energy losses of relativistic electrons in straight and bent crystals

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The testing of our programs for numerical simulation of radiation spectra

When relativistic electrons pass through a crystal at small angles to the crystallographic axes a coherent effect arises in the electron radiation on the atoms of the crystal lattice. This leads to a significant increase in the spectral density of radiation in the spectrum. The total radiative losses of an electron in a crystal can also differ significantly from the case of bremsstrahlung in an amorphous target. We perform theoretical calculations of the effect of radiation losses on the passage of an electron beam through straight ad bent crystals.

The programs for numerical calculation of radiation spectra during the passage of electrons through a straight oriented crystal, which we previously used in studying the emission spectra at 200 MeV have been upgraded.

[1] V.B. Ganenko, A.A. Greenenko, N.F. Shul'ga, V.I. Truten. Simulation of the GeV electrons coherent radiation in oriented crystals. «Journal of Kharkiv University», physical series «Nuclei, Particles, Fields», issue 3 /31, No.744. (2006) p. 89-93.

[2] V.B. Ganenko et al., Mechanisms of 200 MeV electron radiation in diamond crystal in the axial orientation NIM B 424. (2018) p. 17–25.

The testing of our programs for numerical simulation of radiation spectra

These programs were used by us in studying the radiation spectra during the passage of electrons through an oriented crystal under axial crystal orientation <100>.
Let us first consider a comparison of our results of modeling the radiation spectra for straight and bent crystals with the results of various works.

Comparison of our results of modeling radiation spectra for a straight crystal with other works

[3] ***Channeling and radiation of the 855 MeV electrons enhanced by the re-channeling in a periodically bent diamond crystal.***

Andrei V. Korol1, Victor G. Bezchastnov, Andrey V. Solov'yov.

Figure 1 shows a comparison of our results with the results of paper [3].

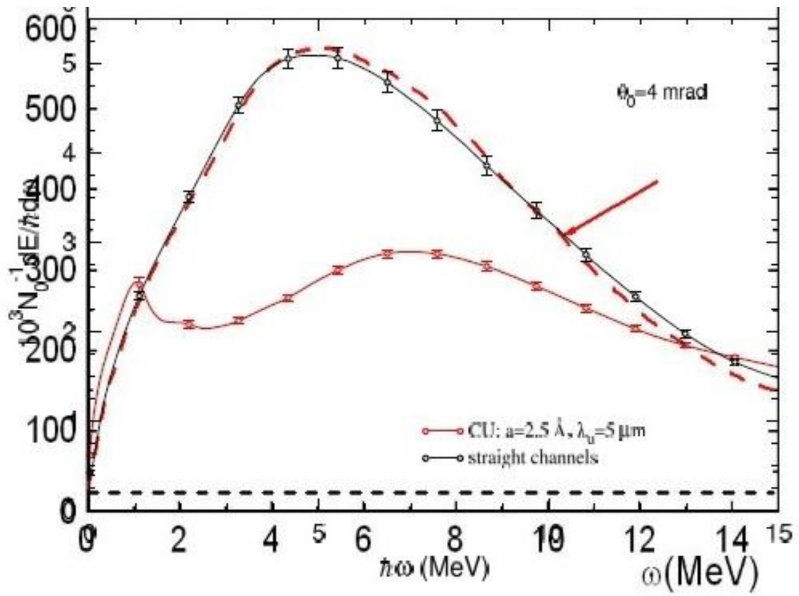


Fig. 1. Emission spectra produced by the 855 MeV electrons in the crystalline undulator (red curves) and a straight crystal (black curves). The emission aperture θ_0=4 mrad. Our results are shown by the red dashed curves and red arrow.

The figure 1 shows good agreement between our results and the results of paper [3], (presented in Fig. 5) which indicates that our programs works correctly.

Now consider a comparison of our results on modeling the radiation spectra for both straight and bent crystal using for example the paper

[4] ***Radiation Emission by Electrons Channeling in Bent Silicon Crystals.***
R.G. Polozkov, V.K. Ivanov, G.B. Sushko, A.V. Korol.
arXiv:1403.6040v1 [physics.acc-ph] 24 Mar 2014

Comparison of our results of numerical simulations of the emission spectra during the passage of electrons through straight and bent Si (110) crystals with axial orientation <100> with the results in Fig. 6 of this work is presented in Fig. 2.

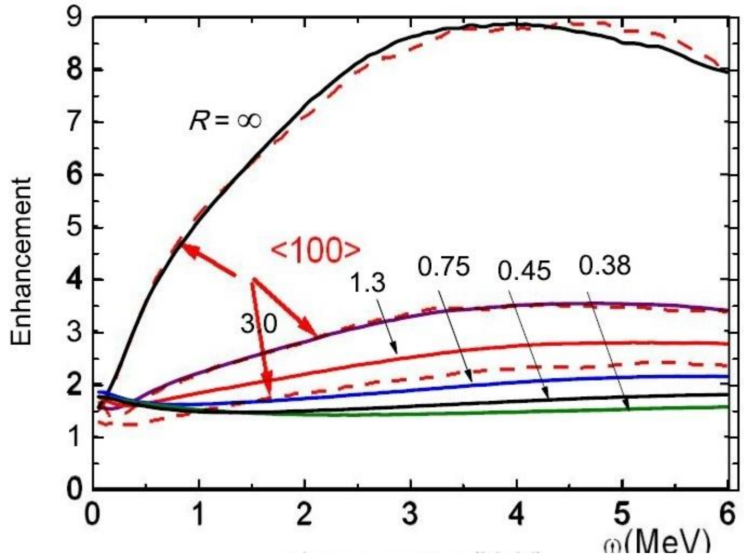
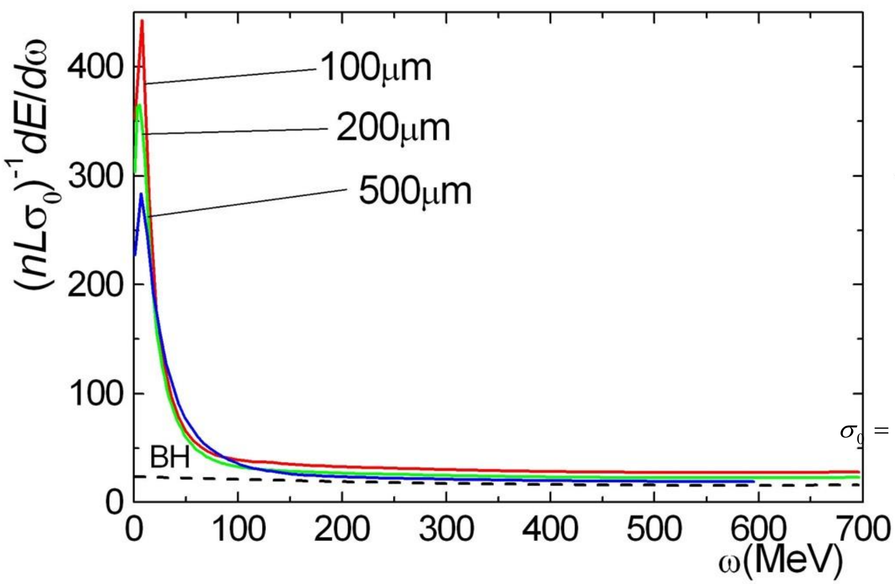


Fig. 2. Emission spectra for 855 MeV electrons channeled in L=75 μm in straight and bent Si (110) crystals. The numbers indicate the values of the bending radius in cm. The data refer to the aperture θ_0=2.4 mrad.
Our results are shown with red dashed lines and red arrows.

All this results shows that our program of numerical simulations of the emission spectra in crystals work correctly for both straight and bent crystals.

The results of computer simulation of radiation spectra during the passage of electrons through straight and bent oriented Si crystals with the axial orientation <100> for different energies, for different thicknesses



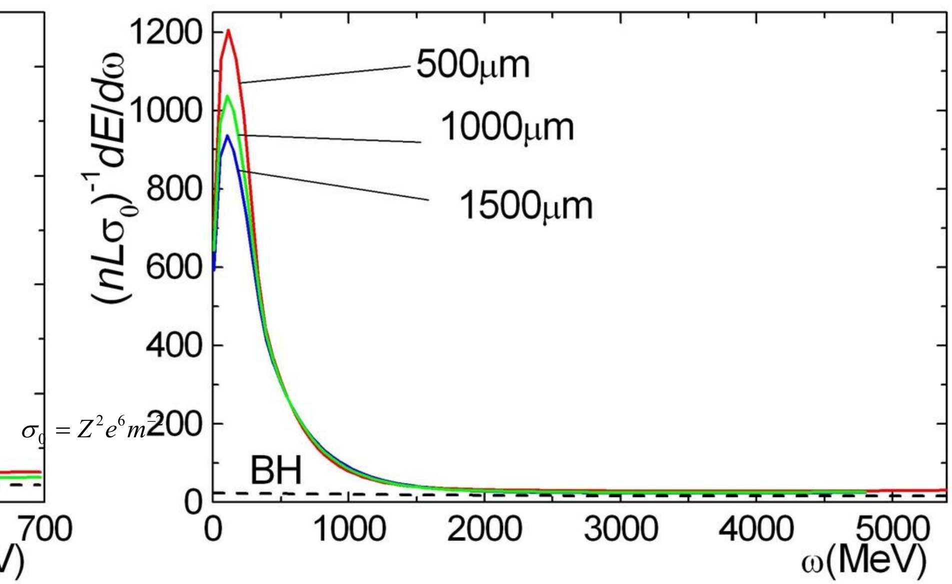


Fig. 3. E=855 MeV, straight crystals

Fig. 4. E=6000 MeV, straight crystals

The incidence angle on the crystals is ψ=0,
dashed lines BH are the Bethe-Heitler spectra,
L is the target thickness, *n* is the atomic density, $\sigma_0 = Z^2 e^6 m^{-2}$

The results of computer simulation of radiation spectra during the passage of electrons through straight and bent oriented Si crystals with the axial orientation <100> for different thicknesses and for different bending in (110) plane.

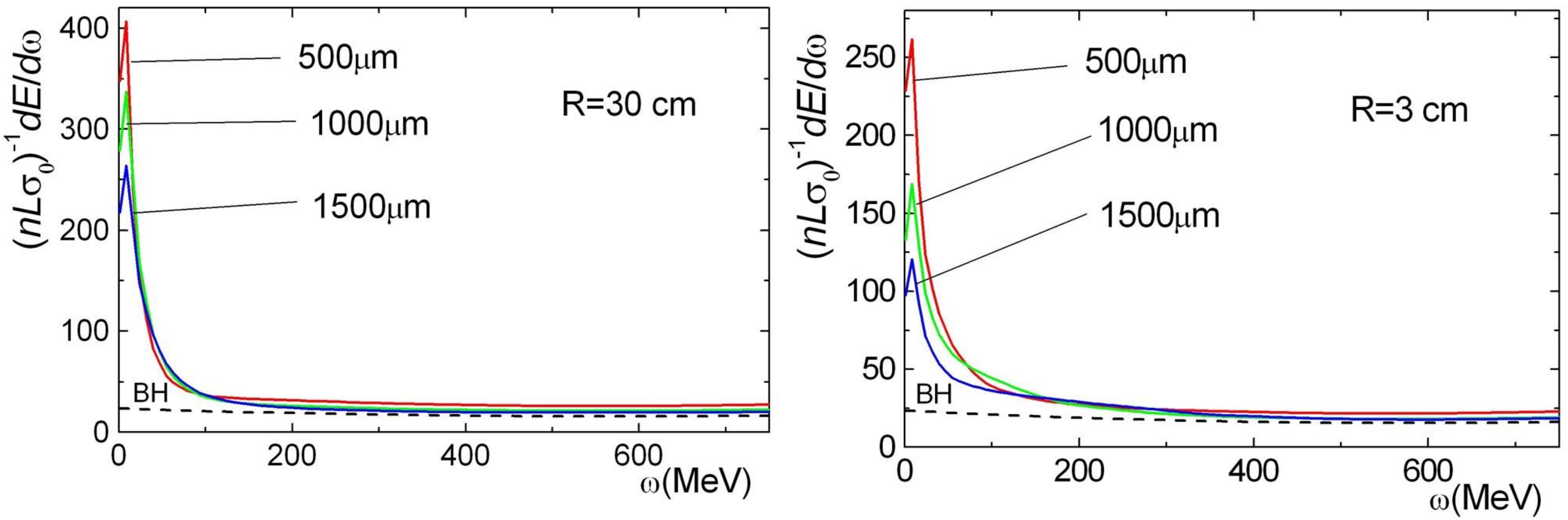


Fig. 5. E=855 MeV, bent Si (110) crystals, R - bending radius cm.

The results of computer simulation of radiation spectra during the passage of electrons through straight and bent oriented Si crystals with the axial orientation <100> for different thicknesses and for different bending in (110) plane.

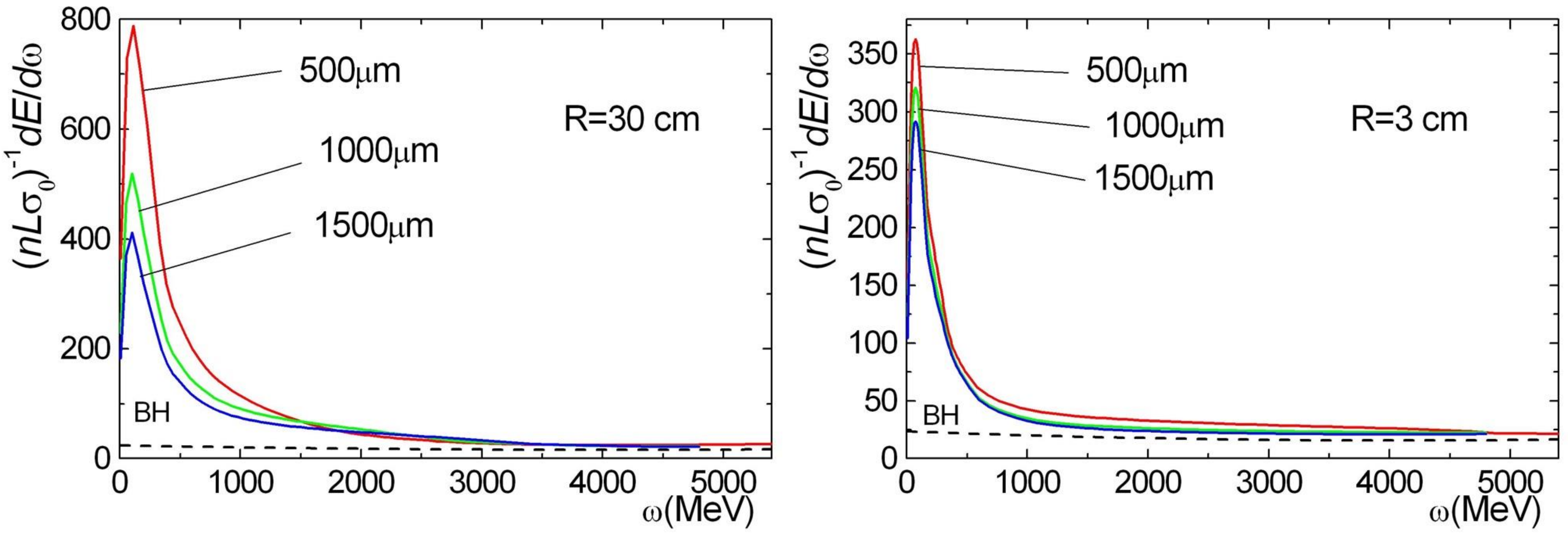


Fig. 6. E=6000 MeV, bent Si (110) crystals, R - bending radius cm.

Energy losss due to radiation during the passage electrons through a straight and bent oriented crystal

The particle energy losses for amorphous target ΔE_{am} as thickness function is divided by formula

$$\Delta E_{am}=E_0 \cdot \exp(-t/L_{rad}), \quad (1)$$

where E_0 is the initial particle energy, t is the target thickness and L_{rad} is the radiation length.

Energy losses, that remains (absorbs) in amorphous target E_{Losam} is equal

$$E_{Losam} = E_0 - \Delta E_{am} = E_0 \cdot (1-\exp(-t/L_{rad})). \quad (2)$$

In the case of small thickness t

$$(1-\exp(-t/L_{rad})) = t/L_{rad}. \quad (3)$$

As a result we have for energy losses in small amorphous target:

$$E_{Losam} = E_0 \cdot t/L_{rad}. \quad (4)$$

The radiation in crystals is grater than the radiation in amorphous target, that's why the particle energy losses in crystals E_{Loscr} mast by greater than E_{Losam} in amorphous target by the amount of excess of the total radiation in crystals E_{totcr} over the total radiation in amorphous E_{totam}.

The total radiation in crystals E_{totcr} and in amorphous E_{totam} can be obtained by integration of spectra that are in figures from fig. 3 to fig. 6.

From equation (4) it follows that the energy losses for crystals E_{Loscr} are being carried out according to formula:

$$E_{Loscr} = E_{Losam} \cdot (E_{totcr} / E_{totam}) = E0 * (t/L_{rad}) \cdot (E_{totcr} / E_{totam}), \quad (5)$$

where E_{totam} – integral of radiation in amorphous spectra,
E_{totcr} – integral of radiation in crystals spectra, L_{rad} =10 cm for Si.

Energy loses due to radiation during the passage electrons through straight and bent oriented crystals

The obtained results of calculations are shown in the table

Energy MeV	thickness μm	straight		bending radius 30 cm		bending radius 3 cm	
		E_{totcr}/E_{totam}	E_{Loscr} MeV	E_{totcr}/E_{totam}	E_{Loscr} MeV	E_{totcr}/E_{totam}	E_{Loscr} MeV
855	100	2.3	**1.97**	2.25	**1.92**	1.9	**1.62**
855	200	2.	**3.42**	2.	**3.42**	2.79	**2.79**
855	500	1.8	7.7	1.8	7.7	1.5	**6.4**
6000	500	5.	**150**	4.87	**146**	2.44	73
6000	1000	5.	**300**	3.81	**229**	2.12	**127**
6000	1500	4.85	**430**	3.28	**295**	1.97	**177**

From obtained results we can see that the radiation losses at the passage of an electron beam through crystals with crystal thickness increasing are becomes larger and for bent crystals are less than in straight crystals, and this decrement becomes less with bending radius lessening.

Thank You for Your Attention