Unbinned likelihood approach

to telescope-calorimeter alignment

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LUXE ECAL-P meeting
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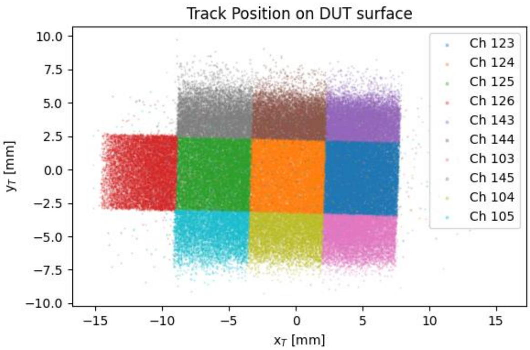
Introduction

I started to think more about the problem after Michal's talk in Warsaw:



We have clear correlation between extrapolated track position and calorimeter hit position.

⇒ It should be possible to extract telescope-calorimeter alignment parameters directly from this dependence.

We should be able to use most of collected data for this procedure (without a need to select event samples with special topology) resulting in high alignment precision...

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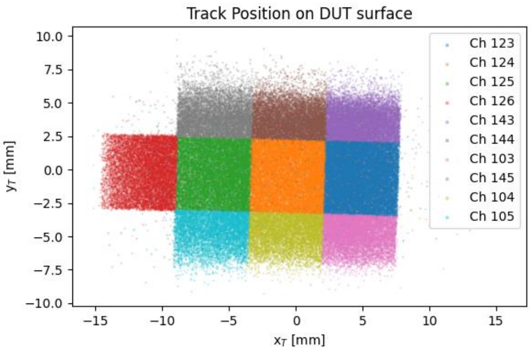

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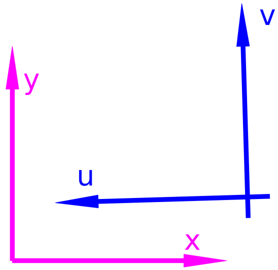
And having a week free from teaching duties...

Alignment parameters



We need to transform track position (x,y) in the telescope reference frame to position (u,v) in the calorimeter frame.

(u,v) correspond to cell column and row number (integer values for cell center)

General form of transformation:

$$\begin{aligned} u &= -a_x\,(\;\; x\cos\phi + y\sin\phi) + u_0 \\ v &= \;\; a_y\,(-x\sin\phi + y\cos\phi) + v_0 \end{aligned}$$

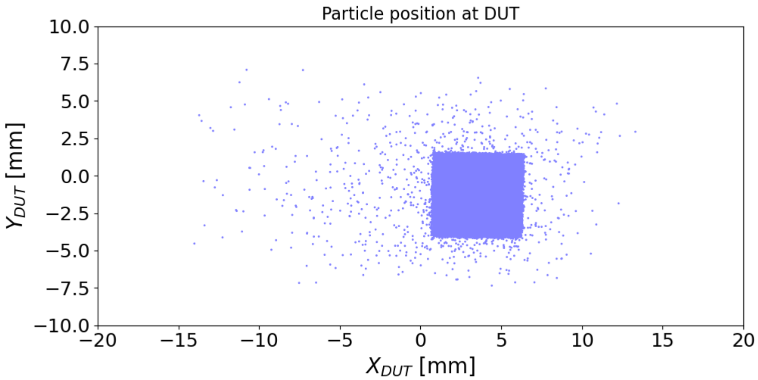
note horizontal coordinate sign flip

Main parameters: u_0, v_0 and ϕ

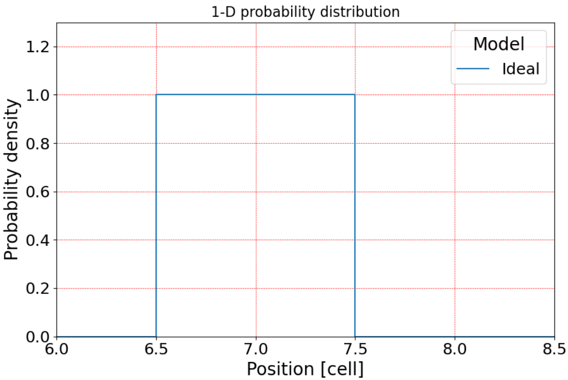
Parameters a_x and a_y should reflect sensor cell size/pitch, but we can also include them in the fit...

Likelihood definition

Example position distribution for events with cell #94 hit in plane #0, run 1492.
For the perfect detector performance, we should just see a square



Likelihood definition



example plot for $i = 7$

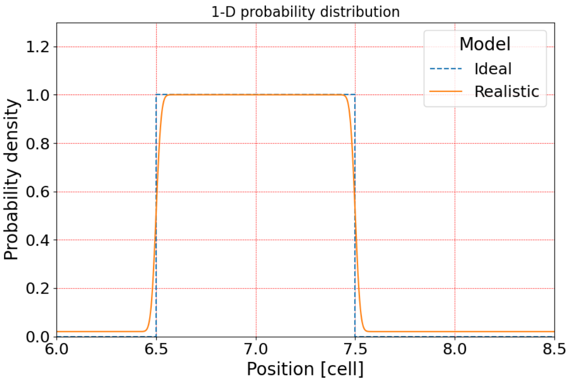
1-D probability density model

For the ideal detector:

$$p(v) = \begin{cases} 1 & \mathit{for} \quad |u - i| < 0.5 \\ 0 & \quad\quad\; |u - i| \geq 0.5 \end{cases}$$

where i is the column number of the cell

Likelihood definition



example plot for $i = 7$

1-D probability density model

Realistic detector, assuming finite telescope position resolution (σ) and noise (p_0):

$$p(v) \quad = \quad p_1 \cdot SF\left(\frac{|u - i| - \frac{1}{2}}{\sigma}\right) + p_0$$

where $SF()$ is the "survival function" of the normal distribution (from SciPy library)

σ can be an additional model parameter

p_1 and p_0 should rather be fixed

Unbinned likelihood

Assuming no correlation between x and y, 2-D likelihood can be written as:

$$p(u,v) \quad = \quad p_1 \cdot SF\left(\frac{|u-i| - \frac{1}{2}}{\sigma_x}\right) \cdot SF\left(\frac{|v-j| - \frac{1}{2}}{\sigma_y}\right) + p_0$$

for calorimeter hit in cell (i, j).

Unbinned likelihood

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For a set of events $(k = 1 \ldots N)$ unbinned likelihood can be defined as:

$$\mathcal{L} \quad = \quad \prod_k p(u_k, v_k)$$

where (u_k, v_k) track positions in calorimeter frame, depend on the alignment parameters.

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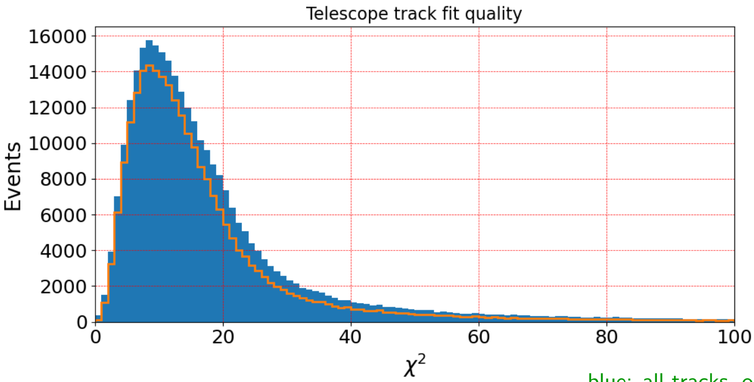
These parameters can be then obtained by maximizing $\mathcal{L}$ or minimizing the function:

$$-2 \cdot \ell \;=\; -2 \cdot \ln \mathcal{L} \approx \chi^2$$

I just tried 'minimize' method from SciPy 'optimize' library and it worked...

Data sample

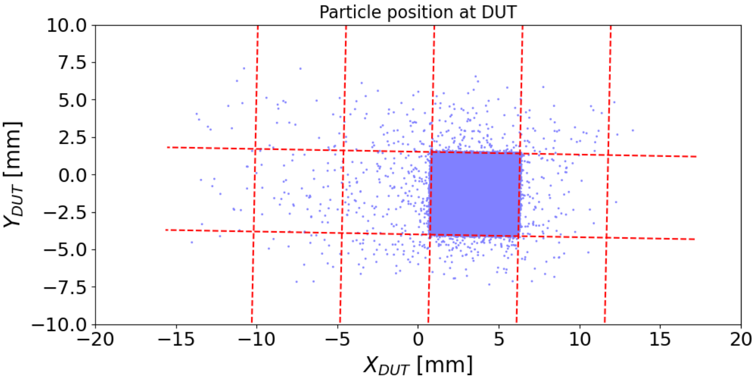
Presented results were obtained for data from Run 1492, selecting events with only one telescope track, fitted to all planes, with $\chi^2 < 20$:



blue: all tracks, orange: 6 plane tracks

Single cell fit

Example fit (plane 0, cell 94)



Alignment from 3 parameter fit:

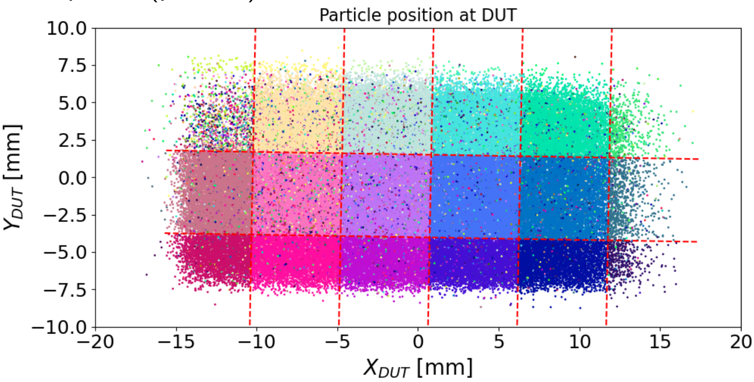
$$u_0 = 18.64495 \pm 0.00081$$
$$v_0 = 7.22515 \pm 0.00147$$
$$\phi = -0.01953 \pm 0.00192$$

position in "cell units"

Multi-cell fit

For higher accuracy of the fit, and to fit more model parameters, we need to include more cells

Example fit (plane 0)



Alignment from 3 parameter fit:

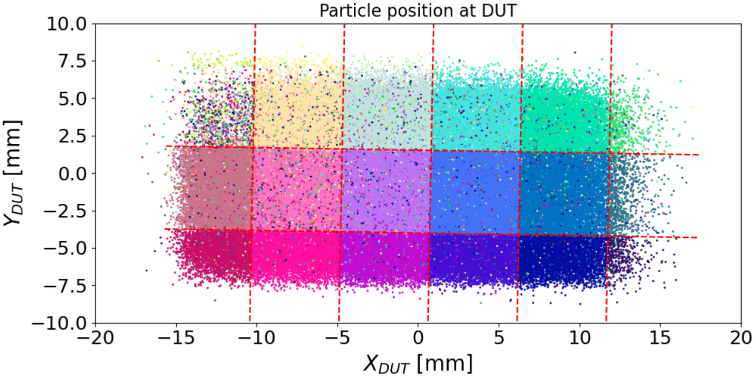
$$u_0 = 18.64438 \pm 0.00027$$
$$v_0 = 7.22621 \pm 0.00027$$
$$\phi = -0.01758 \pm 0.00021$$

σ_ϕ reduced by order of magnitude!

Multi-cell fit

For higher accuracy of the fit, and to fit more model parameters, we need to include more cells

Example fit (plane 0)



Alignment from 5 parameter fit:

$$
\begin{aligned}
u_0 &= 18.64439 \pm 0.00027 \\
v_0 &= 7.22587 \pm 0.00030 \\
\phi &= -0.01760 \pm 0.00021 \\
a_x^{-1} &= 5.51901 \pm 0.00131 \text{ mm} \\
a_y^{-1} &= 5.52846 \pm 0.00286 \text{ mm}
\end{aligned}
$$

matches the expected pitch

Multi-cell fit

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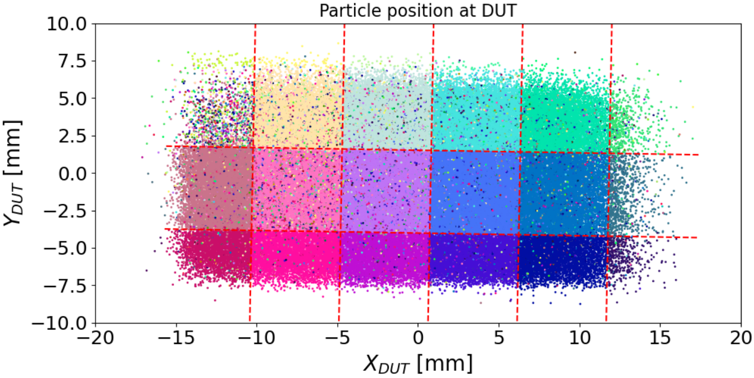
Example fit (plane 0)



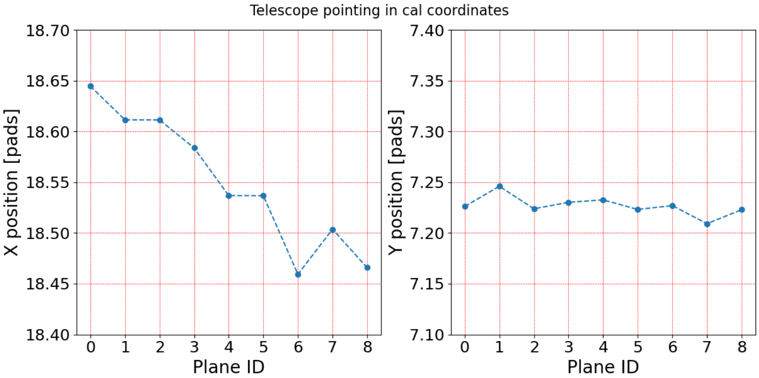
Alignment from 7 parameter fit:

$$u_0 = 18.64460 \pm 0.00022$$
$$v_0 = 7.22609 \pm 0.00025$$
$$\phi = -0.01748 \pm 0.00017$$
$$a_x^{-1} = 5.51887 \pm 0.00108 \text{ mm}$$
$$a_y^{-1} = 5.52968 \pm 0.00246 \text{ mm}$$
$$\sigma_x = a_x \cdot 76.86 \pm 1.37 \ \mu\text{m}$$
$$\sigma_y = a_y \cdot 76.54 \pm 1.66 \ \mu\text{m}$$

independent telescope precision estimate

Multi-cell fit with 7 parameters

Summary of results for all available telescope planes



Horizontal dependence ($\sim$1 mm shift) corresponds to about 0.7° rotation

Multi-cell fit with 7 parameters

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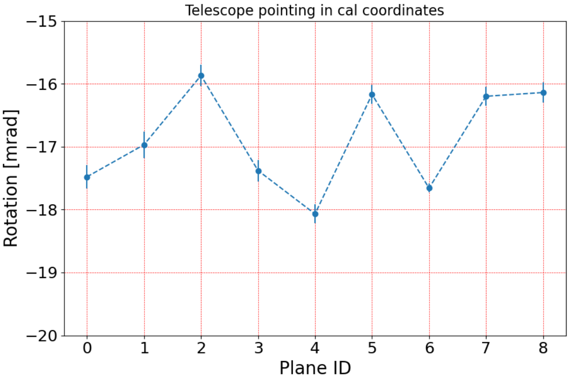

Consistent results between planes ($\phi \approx 1^\circ$).

Mechanical precision: $\frac{10\mu m}{50mm} = 0.2$ mrad

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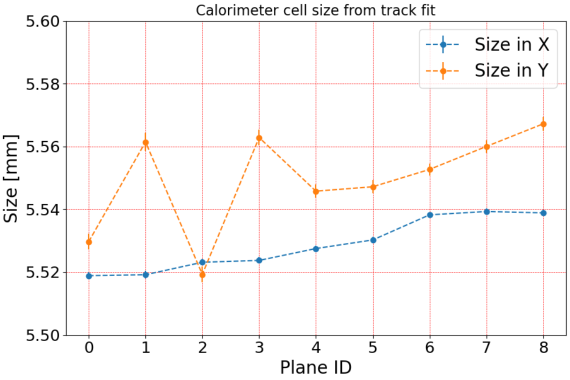

Clear dependency on the plane ID: beam divergence?

Nominal pixel pitch is 5.53 mm.

Multi-cell fit with 7 parameters

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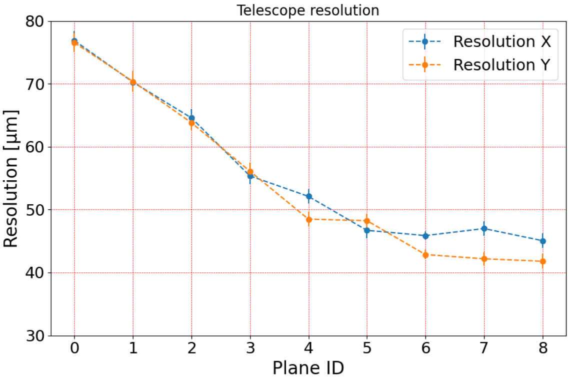

Beam enters from the right ?!

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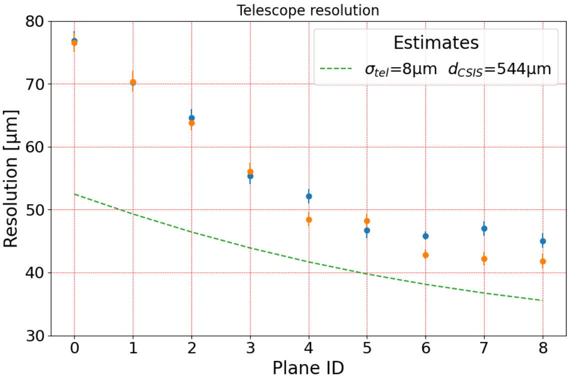

Beam enters from the right ?! Does not match expected dependence ?!

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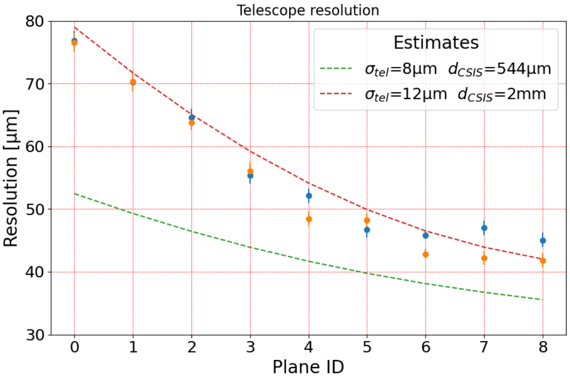

Beam enters from the right ?! Telescope performance worse and more multiple scattering ?!

Conclusions

With large statistics of collected data, unbinned likelihood method can be used to extract alignment parameters with high precision.

First tests of the method very promising, but the model is still simplified...

Many features of the model still need to be studied/improved:

- gap between cells
- gap between sensors
  $\Rightarrow$ we could also use two sets of alignment parameters for two sensors!

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- gap between cells
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  $\Rightarrow$ we could also use two sets of alignment parameters for two sensors!
- "signal" normalization (should it depend on a_x and a_y ?)
- "background" normalization (can it be included as fit parameter?)
- data selection (removal of noisy cells?)
- fit convergence (fails for data of poor quality)