Charting the Higgs sector: SMEFT, HEFT and the geometry of scalar theories

Ilaria Brivio

Università & INFN Bologna

Swiss National Science Foundation

COMETA

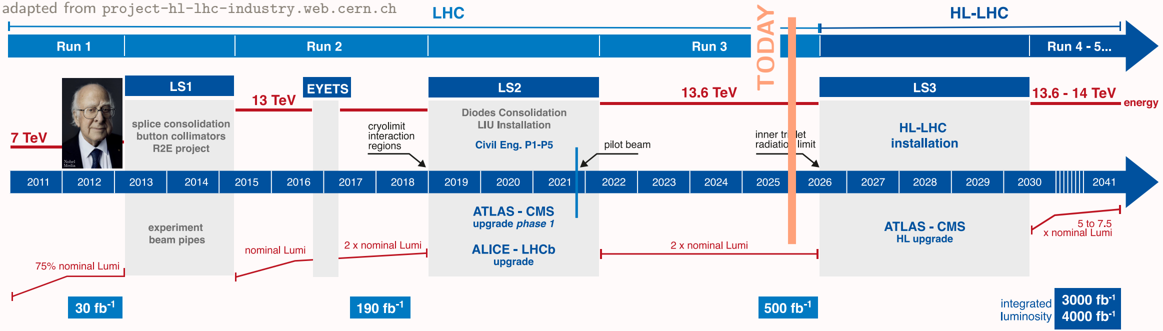
INFN Istituto Nazionale di Fisica Nucleare

ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA

Introduction

Searching for new physics at the LHC

adapted from project-hl-lhc-industry.web.cern.ch



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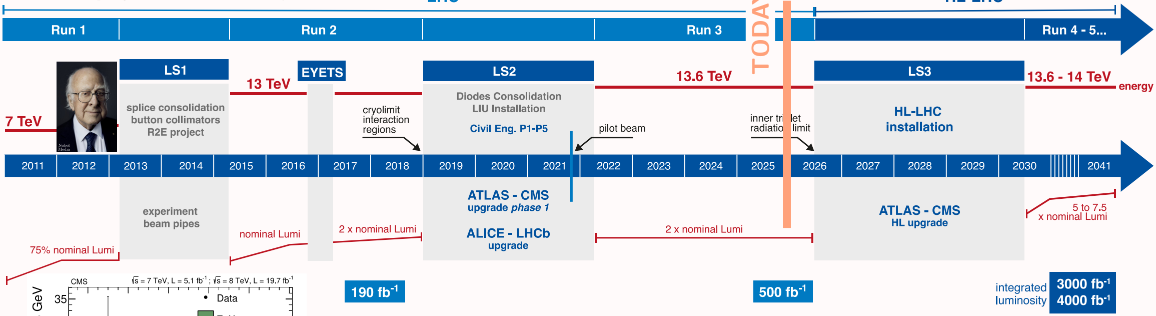

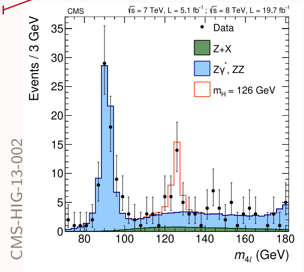

resonant searches

Searching for new physics at the LHC

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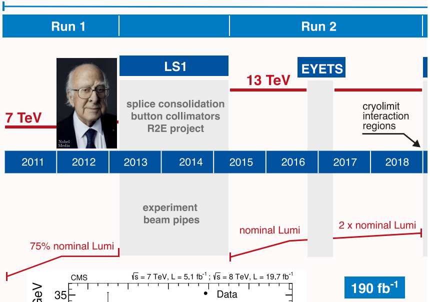

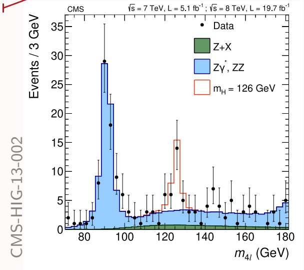

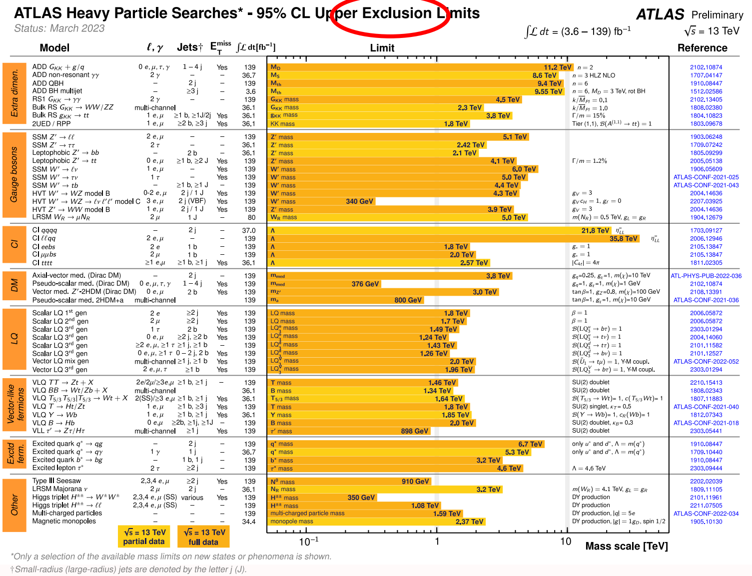

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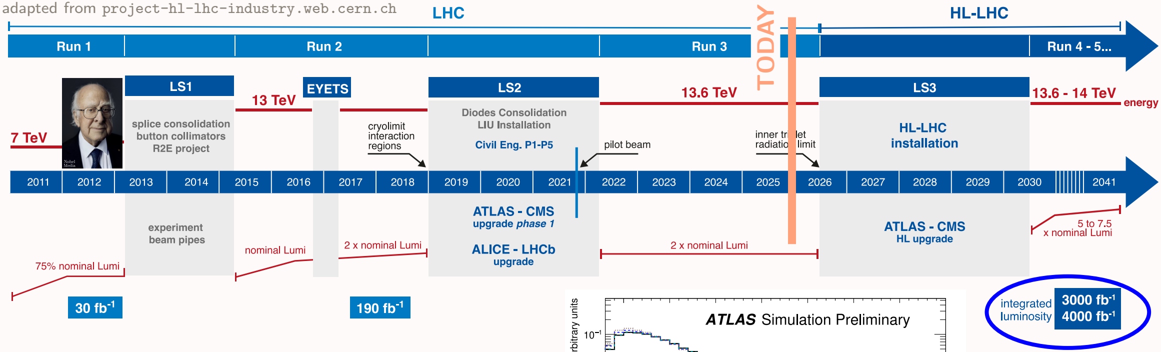
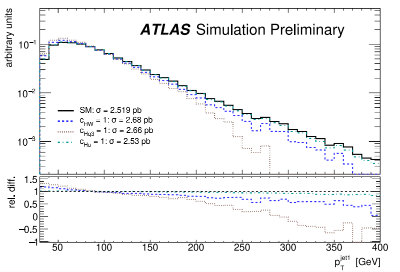


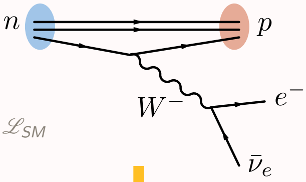


$\sim$90% of LHC data still to be collected!
$\rightarrow$ **% precision** will be possible

non-resonant searches

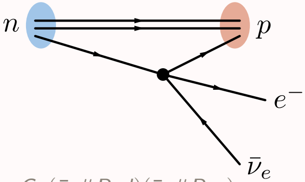
Effective Field Theories: general principle

Fermi Theory of β decay



$\mathscr{L} = \mathscr{L}_{SM}$

$q^2 < m_N^2 \ll m_W^2$



$\mathscr{L} = -G_F(\bar{u}\gamma^\mu P_L d)(\bar{\nu}\gamma^\mu P_L e)$

E

Λ

Full theory

$\rightarrow$ renormalizable: $[\mathscr{L}] = 4$

TAYLOR SERIES in ($\mu/\Lambda \ll 1$)

μ

Effective Field Theory

$$\mathscr{L}_{EFT} = \mathscr{L}_4 + \frac{1}{\Lambda}\mathscr{L}_5 + \frac{1}{\Lambda^2}\mathscr{L}_6 + \frac{1}{\Lambda^3}\mathscr{L}_7 \dots$$

Appelquist, Carazzone 1975

$\rightarrow$ heavy DOFs are removed: cannot be produced at $E \ll M$
$\rightarrow$ local, analytic, higher-dimensional terms added to $\mathscr{L}$

Effective Field Theories: general principle

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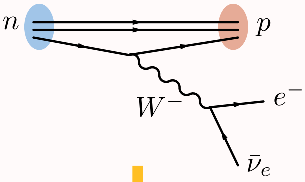
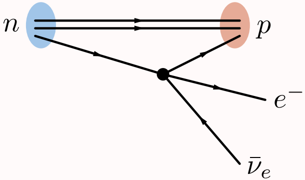


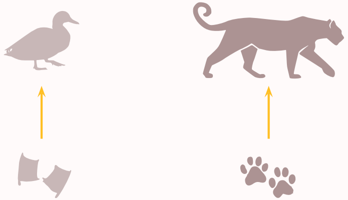
$q^2 < m_N^2 \ll m_W^2$



Bottom-up paradigm

measuring EFT parameters **reveals properties** of full theory

$\rightarrow$ *complement* direct searches, reach into higher energies



EFT fully specified by **fields+symmetries at E** $= \mu$

$\rightarrow$ no reference to underlying model

$\rightarrow$ free couplings that can be measured!

The Standard Model Effective Field Theory – SMEFT

promoting the Standard Model to an EFT → add **higher-dimensional** terms made of SM **fields** and respecting the SM **symmetries**

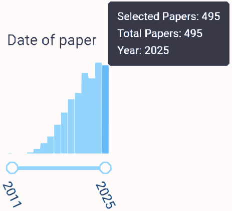
$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \frac{1}{\Lambda^3}\mathcal{L}_7 + \frac{1}{\Lambda^4}\mathcal{L}_8 + \dots \qquad \mathcal{L}_d = \sum_i C_i \mathcal{O}_i^{(d)}$$

C_i = Wilson coefficients

$\mathcal{O}_i^{(d)}$ = gauge-invariant operators forming a basis: a complete, non-redundant set

Buchmüller,Wyler 1986

- a complete catalogue of all allowed beyond-SM effects, organized by expected size
- extensively developed in the past decade. largely adopted in LHC searches
- can be used as **a common framework** for LHC *and* other experiments



SMEFT at $d=6$: Warsaw basis

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC}\widetilde G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK}\widetilde W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu^I\varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

Grzadkowski,Iskrzynski,Misiak,Rosiek 1008.4884

SMEFT at $d = 6$: Warsaw basis

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$		

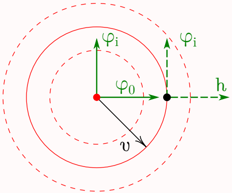
Grzadkowski,Iskrzynski,Misiak,Rosiek 1008.4884

Two EFT ways to extend the SM!

SMEFT

Higgs doublet:

$$\boxed{H} = \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_2 + i\varphi_1\\ \varphi_0 - i\varphi_3\end{pmatrix} = \frac{v+h}{\sqrt{2}}\mathbf{U}\begin{pmatrix}0\\1\end{pmatrix}$$

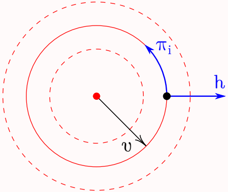


Higgs EFT (EWχL)

Goldstone chiral field:

$$\boxed{\mathbf{U}} = \exp\left(\frac{i\vec{\sigma}\cdot\vec{\pi}}{v}\right)$$

Physical Higgs singlet: $\boxed{h}$



The Higgs EFT

HEFT Lagrangian

"standard" LO

Feruglio 9301281, Grinstein,Trott 0704.1505, Buchalla,Catà 1203.6510, Alonso+ 1212.3305, IB+ 1311.1823,1604.06801, Buchalla+ 1307.5017,1511.00988. . .

$$
\begin{aligned}
\mathscr{L}_0 = & -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& \boxed{+\frac{1}{2}\partial_\mu h \partial^\mu h - \lambda v^4 \mathcal{V}(h) - \frac{v^2}{4}\mathrm{Tr}\left(\mathbf{V}_\mu \mathbf{V}^\mu\right)\mathcal{F}_C(h) + c_T \frac{v^2}{4}\mathrm{Tr}\left(\mathbf{T}\mathbf{V}_\mu\right)^2 \mathcal{F}_T(h)} \\
& + i\overline{Q}_L \not{D} Q_L + i\overline{Q}_R \not{D} Q_R + i\overline{L}_L \not{D} L_L + i\overline{L}_R \not{D} L_R \\
& - \frac{v}{\sqrt{2}}\left(\overline{Q}_L \mathbf{U} \mathcal{Y}_Q(h) Q_R + \mathrm{h.c.}\right) - \frac{v}{\sqrt{2}}\left(\overline{L}_L \mathbf{U} \mathcal{Y}_L(h) L_R + \mathrm{h.c.}\right)
\end{aligned}
$$

where

$\mathbf{V}_\mu = (D_\mu \mathbf{U})\mathbf{U}^\dagger \xrightarrow{\text{unit. gauge}} W^\pm_\mu, Z_\mu$

$\mathbf{T} = \mathbf{U}\sigma^3\mathbf{U}^\dagger \to$ custodial break

$\mathcal{F}_i(h), \mathcal{V}(h)$ all of the form $1 + a_i \frac{h}{v} + b_i \frac{h^2}{v^2} + \dots$

$\mathcal{Y}_Q(h) = \mathrm{diag}\left(Y_u \mathcal{F}_u(h), Y_d \mathcal{F}_d(h)\right)$

Why HEFT?

historically: descendant of technicolor, non-linear σ models

- LO $\sim \kappa$ framework $\rightarrow$ could capture large anomalous h interactions
- would hold if h was not "the Higgs boson" $\rightarrow$ probe of h nature

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more recent use: interesting way to chart around $d = 6$ SMEFT, more focus on higher orders

is SMEFT valid at the LHC?

Λ unknown + LHC measurements often not precise enough $\Rightarrow \Lambda \sim$ TeV allowed

HEFT is **more general** than SMEFT!

$\exists$ BSM models that cannot be approximated by SMEFT (more later) Cohen,Craig,Lu,Sutherland 2008.08597

HEFT higher orders?

- organization criterion for **HEFT operators** not obvious!

U $\rightarrow$ sort in derivatives vs. $\psi, X_{\mu\nu}$ $\rightarrow$ sort in canonical dimension ?

Buchalla,Catá,(Celis),Krause 1307.5017,1312.5624,1603.03062
Gavela,Jenkins,Manohar,Merlo 1601.07551, IB,Gonzalez-Fraile,Gonzalez-Garcia,Merlo 1604.06801

conventional LO + NLO agreed upon $\rightarrow$ NLO bases Buchalla+ 1307.5017, IB+ 1604.06801, Sun+ 2210.14939

some arguments *ad hoc*. not systematized to higher orders

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- some calculations in the literature truncate at linear order in NLO coefficients.
  some do not truncate at all. . .

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- rules for sorting **HEFT diagrams** also unclear

  - some calculations in the literature truncate at linear order in NLO coefficients. some do not truncate at all...

- ...even the **matching to UV models** seems ambiguous Dawson+ 2305.07689,2311.16897

HEFT power counting revisited

dimensionally, EFT calculations will give

IB,Gröber,Schmid 2511.23410

$$\sigma \sim \underbrace{p^{-2}(4\pi)^3}_{\text{dimensional term}} \ \underbrace{\left(\frac{p}{\Lambda}\right)^{N^p_\Lambda} \left(\frac{4\pi v}{\Lambda}\right)^{N_v} \left(\frac{g}{4\pi}\right)^{N_g} \left(\frac{y}{4\pi}\right)^{N_y} \left(\frac{\lambda}{16\pi^2}\right)^{N_\lambda} (C_i)^{N_{C_i}}}_{\text{dimensionless suppression factors}} \dots$$

counting rules must be such that

- they count a linear combination of N^p_Λ, N_v, $N_g \dots$
- they preserve $m_{SM} = (g\,v, y\,v, \sqrt{\lambda}\,v) \sim p$

e.g. **SMEFT** counts $N^p_\Lambda + N_v$

HEFT power counting revisited

dimensionally, EFT calculations will give

IB,Gröber,Schmid 2511.23410

$$\sigma \sim \underbrace{p^{-2}(4\pi)^3}_{\text{dimensional term}} \underbrace{\left(\frac{p}{\Lambda}\right)^{N_\Lambda^p} \left(\frac{4\pi v}{\Lambda}\right)^{N_v} \left(\frac{g}{4\pi}\right)^{N_g} \left(\frac{y}{4\pi}\right)^{N_y} \left(\frac{\lambda}{16\pi^2}\right)^{N_\lambda} (C_i)^{N_{C_i}} \dots}_{\text{dimensionless suppression factors}}$$

counting rules must be such that

- they count a linear combination of N_Λ^p, N_v, $N_g \dots$
- they preserve $m_{SM} = (g\,v, y\,v, \sqrt{\lambda}\,v) \sim p$

e.g. **SMEFT** counts $N_\Lambda^p + N_v$

HEFT <u>cannot</u> expand in $(4\pi v/\Lambda) \geqslant 1$ if it is defined with $(h/\mathbf{v}), (\pi^I/\mathbf{v})$

$\Rightarrow$ it must count $N_{\mathrm{HEFT}} = N_\Lambda^p + N_g + N_{g'} + N_y + 2N_\lambda$

HEFT power counting revisited

$$N_{\rm HEFT} = N_\Lambda^p + N_g + N_{g'} + N_y + 2N_\lambda$$

IB,Gröber,Schmid 2511.23410

- **operators** can be sorted by $N_\chi = \frac{f}{2} + q - 2 + N_{g,g',g_s,y} + 2N_\lambda$

  f = # fermions
  q = # $D_\mu, \partial_\mu, \mathbf{V}_\mu, X_{\mu\nu}$

- **diagrams** have order $N_{\rm HEFT} = 2L + n - 2 + \sum_{i,\in \text{vert.}} N_{\chi,i} - N_{g_s}$

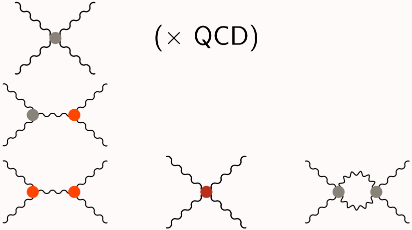
  L = # loops
  n = # external legs

  $\otimes N_{g_s}$

2 (× QCD)

3

4



$N_\chi = 0,\ 1,\ 2$

- **diagram products** $\mathcal{M}_a\mathcal{M}_b^\dagger$ have order $N_{\rm HEFT,a} + N_{\rm HEFT,b}$

HEFT operator classification

$$\mathscr{L}_{\rm HEFT} = \mathscr{L}_0 + \mathscr{L}_1 + \mathscr{L}_2 + \ldots$$

Class	**Example**	N_χ	"standard" order
$\mathscr{L}_0$	$\langle \mathbf{T}\mathbf{V}^\mu\rangle^2 \mathcal{F}_C(h)$	0	LO
X^2	$gg' B_{\mu\nu}\langle \mathbf{T} W^{\mu\nu}\rangle \mathcal{F}(h)$	0 +2	NLO
$D\psi^2$	$(\bar{Q}_L \gamma^\mu \mathbf{V}_\mu Q_L)$	0	NLO
$X\psi^2$	$g' y (\bar{Q}_L \sigma^{\mu\nu} \mathbf{U} Q_R) B_{\mu\nu}$	0 +2	NLO
ψ^4	$(\bar{Q}_L \mathbf{U} Q_R)^2$	0	NLO
D^2X	$g\langle W_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]\rangle$	1 +1	NLO
X^3	$g\varepsilon^{IJK} W_\nu^{I\mu} W_\rho^{J\nu} W_\mu^{K\rho}$	1 +1	NLO/NNLO
$D^2\psi^2$	$y(\bar{Q}_L \mathbf{U} Q_R)\langle \mathbf{V}_\mu \mathbf{V}^\mu\rangle$	1 +1	NLO
D^4	$\langle \mathbf{V}_\mu \mathbf{V}^\mu\rangle^2 \mathcal{F}(h)$	2	NLO

HEFT vs SMEFT: "order-by-order" example

Éboli,Gonzalez-Garcia,Martines 2112.11468. also: IB+ 1311.1823,1604.06801
related study: Isidori,(Manohar),Trott 1305.0663,1307.4051

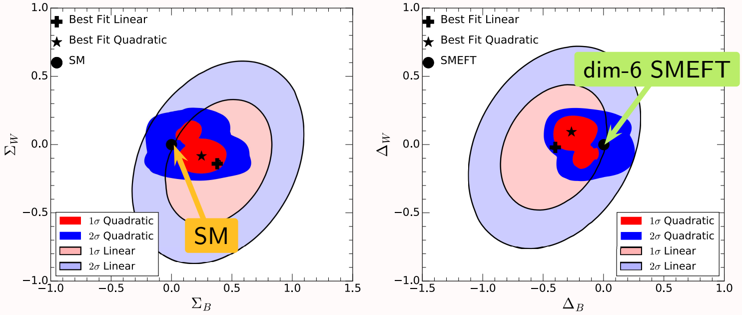
$$\frac{c_{BH}}{\Lambda^2} D_\mu H^\dagger g' B_{\mu\nu} D^\mu H$$

vs

TGC

$$c_2\, g' B_{\mu\nu} \langle \mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]\rangle \mathcal{F}(h)$$
$$+$$
$$c_4\, g' B_{\mu\nu} \langle \mathbf{T}\mathbf{V}^\mu\rangle \partial^\nu \mathcal{F}(h)$$

hVV



$$\Sigma_B = 2c_2 + c_4 \sim c_{BH}$$
$$\Delta_B = 2c_2 - c_4 \qquad \longrightarrow \qquad \sim \text{dim-8}$$

☞ comparing to dim-6: HEFT signature is $\Delta_{B,W} \neq 0$
comparing to dim-8: HEFT signature is $|\Delta_{B,W}| \gtrsim (v^4/\Lambda^4)$?

SMEFT and HEFT can be mapped

$$H = \frac{v+h}{\sqrt{2}}\mathbf{U}\begin{pmatrix}0\\1\end{pmatrix}$$

$$\mathbf{U} = \frac{\begin{pmatrix}\tilde{H} & H\end{pmatrix}}{\sqrt{H^\dagger H}}\,, \qquad h = \sqrt{2H^\dagger H} - v$$

- all HEFT effects have a SMEFT-equivalent, at high enough d (?)

- most pheno differences due to mismatched suppression patterns
  HEFT takes some effects to **lower** orders, e.g.
  - custodial violation: $\mathrm{Tr}[\mathbf{T}\mathbf{V}_\mu] = \frac{(H^\dagger \overleftrightarrow{D}_\mu H)}{(H^\dagger H)^2}$
  - uncorrelated h couplings: $\mathcal{F}(h) \sim \sum_{n=0}^{\infty} c_n \left(\frac{H^\dagger H}{\Lambda^2}\right)^n = \sum_{n=0}^{\infty} c_n \frac{(v+h)^{2n}}{\Lambda^{2n}}$

$\Rightarrow$ HEFT is more covergent

is there anything more fundamental?

SMEFT vs HEFT: geometry

Generalized goal: invariance under field redefinitions

focus on **scalars only**

$$\frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_2 + i\varphi_1 \\ \varphi_0 - i\varphi_3\end{pmatrix} \quad \text{or} \quad h, \exp\left[\frac{i\pi^I\sigma^I}{v}\right] \qquad \longrightarrow \qquad \vec{\phi} = \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix}$$

on-shell amplitudes, cross-sections are **invariant under field redefinitions**

e.g. $\phi_1 \mapsto \phi_1 + \phi_2^2$, $\phi_1 \mapsto \phi_1 e^{i\phi_2}$ ...

☞ look for a formalism that maintains this manifestly at Feynman rules / Lagrangian levels

Generalized goal: invariance under field redefinitions

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$$\frac{1}{\sqrt{2}}\begin{pmatrix}\varphi_2 + i\varphi_1\\ \varphi_0 - i\varphi_3\end{pmatrix} \quad \text{or} \quad h, \exp\left[\frac{i\pi^I\sigma^I}{v}\right] \quad \longrightarrow \quad \vec{\phi} = \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix}$$

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☞ look for a formalism that maintains this manifestly at Feynman rules / Lagrangian levels

- geometry originally introduced in the 60–80s. has applications e.g. in quantum gravity
  Meetz 1969, (Ecker),Honerkamp 1971,1972, Tataru 1975, Alvarez-Gaumé+ 1981, Vilkovisky 1984, Gaillard 1986...
- first considered ∼10 yrs ago for a "universal" formulation of SMEFT/HEFT that can be studied independently of field representation Alonso,Jenkins,Manohar 1511.00724, 1605.03602
- in the last 5 years it has found several applications in the context of EFTs
  Pilaftis+ 2006.05831,2406.13594 Cohen+ 2008.08597,2108.03240,2202.06965,2312.06748, 2410.21378,2504.12371,2509.20449 Cheung+ 2111.03045,2202.06972 Alonso+ 2109.13290,2207.02050,2307.14301 Helset+ 2210.08000,2212.03253 Craig+ 2305.09722 Assi+ 2307.03187,2504.18537 Jenkins+ 2310.19883,2308.06315 Derda+ 2403.12142 Craig,Lee 2307.15742 IB+ 2308.00017,2509.20482 Li+ 2411.04173 Aigner+ 2503.09785, Bonnefoy+ 2508.18165 ...

Geometrical description for scattering amplitudes

interactions with **2 derivatives** define a **metric** on the manifold of field configurations (field space)

$$\mathscr{L} = \frac{1}{2}\,\partial_\mu\phi^i\,\partial^\mu\phi^j\,g_{ij}(\phi) + \ldots \qquad \longrightarrow \qquad g = g_{ij}(u)\,du^i du^j$$

On-shell amplitudes. for massless/soft scalars

Cheung+ 2111.03045, Helset+ 2210.08000

$$\begin{aligned}
\mathcal{A}_{ijk} &= 0\\
\mathcal{A}_{ijkl} &= s_{ij}\,R_{ikjl} + s_{ik}\,R_{ijkl}\\
\mathcal{A}_{ijklm} &= s_{ij}\,\nabla_l R_{ikjm} + s_{ik}\,\nabla_l R_{ijkm} + s_{il}\nabla_k R_{ijlm} + s_{kl}\,\nabla_i R_{mkjl} + (s_{jl}+s_{il})\,\nabla_i R_{mjkl}
\end{aligned}$$

$R, \nabla R\ldots$ are evaluated at the vacuum $\phi^i \equiv 0$, $\quad s_{ij} = (p_i + p_j)^2$

example: $\mathscr{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1\left[1 + a_2\frac{\phi_2}{\Lambda}\right] + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2\left[1 + b_{11}\frac{\phi_1^2}{2\Lambda^2} + b_{111}\frac{\phi_1^3}{3!\Lambda^3}\right]$

$$R_{1212} = \left(a_2{}^2 - 2b_{11}\right)/4\Lambda^2 \qquad \nabla_1 R_{1212} = -b_{111}/2\Lambda^3 \qquad \nabla_2 R_{1212} = -a_2{}^3/2\Lambda^3$$

What do we learn?

- $\mathcal{A}$ are **tensors** under diffeomorphism field redefinitions   Cohen+ 2202.06965,2312.06748, Alminawi 2510.24866

  true even with masses, up to terms that vanish at the vacuum ($\frac{\partial\Gamma}{\partial\phi}$) or for on-shell external legs ($\frac{\partial^2\Gamma}{\partial\phi\partial\phi}$)

- if ϕ^i are mass eigenstates, $\mathcal{A}_{i_1\ldots i_n}$ with fixed $i_1 \ldots i_n$ indices is **invariant**

  if they are not, we can move to a mass eigenstate basis $\phi^i \to \phi'^a$
  by LSZ, a physical on-shell amplitude is $\mathcal{A}'_{a_1\ldots a_n} = U^{i_1}{}_{a_1}\ldots U^{i_n}{}_{a_n}\mathcal{A}_{i_1\ldots i_n}$ with $U^i{}_a = \delta\phi^i/\delta\phi'^a$

- $R, \nabla R\ldots$ at $\phi^i = 0$ are **invariant, measurable** quantities

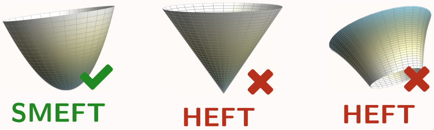
- the expressions of $\mathcal{A}_{i_1\ldots i_n}$ in terms of Riemanns are **universal**: computed once for all theories

- a theory can be characterized by the geometry of its field space

SMEFT vs HEFT: fundamental differences

$$\mathbf{U} = \frac{(\tilde{H} \quad H)}{\sqrt{H^\dagger H}} \Longrightarrow \text{HEFT can be converted to SMEFT } \textbf{only if} \text{ the theory is well-behaved at } H^\dagger H = 0$$

Alonso+ 1511.00724,1605.03602, Falkowski,Rattazzi 1902.05936, Cohen+ 2008.08597



SMEFT vs HEFT: fundamental differences

$\mathbf{U} = \frac{(\tilde{H} \quad H)}{\sqrt{H^\dagger H}} \Longrightarrow$ HEFT can be converted to SMEFT **only if** the theory is well-behaved at $H^\dagger H = 0$

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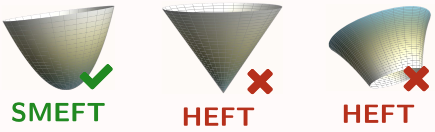


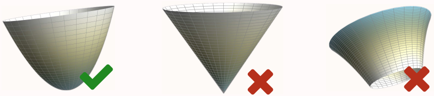
if there are **BSM sources of EWSB**, we cannot reach $H^\dagger H = 0$ in the EFT

e.g. 2HDM away from alignment

SMEFT vs HEFT: fundamental differences

$$\mathbf{U} = \frac{(\tilde{H} \quad H)}{\sqrt{H^\dagger H}} \Longrightarrow$$ HEFT can be converted to SMEFT **only if** the theory is well-behaved at $H^\dagger H = 0$

Alonso+ 1511.00724,1605.03602, Falkowski,Rattazzi 1902.05936, Cohen+ 2008.08597



SMEFT HEFT HEFT

the $H^\dagger H$ point is singular if there are **loryons**

Banta,Cohen,Craig, Lu,Sutherland 2110.02967
Crawford,Sutherland 2409.18177

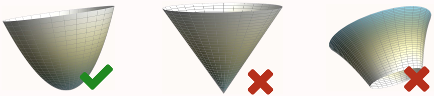
=

BSM states that are integrated out, but received $\geqslant 50\%$ of their mass from EWSB

SMEFT vs HEFT: fundamental differences

$$\mathbf{U} = \frac{(\tilde{H} \quad H)}{\sqrt{H^\dagger H}} \implies$$ HEFT can be converted to SMEFT **only if** the theory is well-behaved at $H^\dagger H = 0$

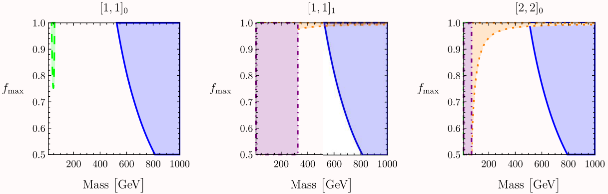
Alonso+ 1511.00724,1605.03602, Falkowski,Rattazzi 1902.05936, Cohen+ 2008.08597



SMEFT ✔ HEFT ✘ HEFT ✘

the $H^\dagger H$ point is singular if there are **loryons**

Banta,Cohen,Craig, Lu,Sutherland 2110.02967
Crawford,Sutherland 2409.18177



Geometry: limitations & challenges

- interactions with $\partial^{\geqslant 4}$ or $\partial^0 \equiv V(\phi)$ don't have a geometric meaning

  "functional geometry" Helset+ 2210.08000,2202.06972, Cohen+ 2202.06965,2312.06748,2410.21378,2509.20449
  $\rightarrow$ allow p-dependent metric

  different spaces: Lagrange spaces Craig+ 2305.09722
  fibre/jet bundles Craig,Lee 2307.15742, Alminawi,IB,Davighi 2308.00017,2509.20842

- invariance under **derivative field redefinitions** is not captured, e.g. $\phi_1 \rightarrow \phi_1 + \Box\phi_2/\Lambda^2$

- **computing** non-trivial, scales badly with number of external legs

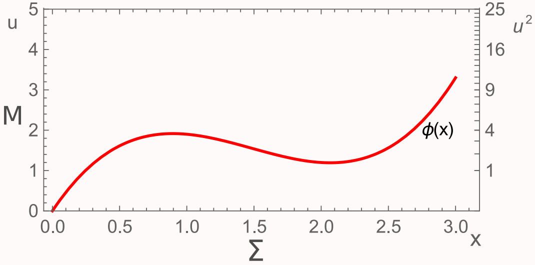
  traditional Feynman rules are not tensors.
  tensors in amplitudes emerge from intricate cancellations

Fibre bundle picture

main aim: include derivatives $\partial_\mu \phi(x) \rightarrow$ keep ϕ dependence on x manifest!

natural structure: **fibre bundle**

Alminawi,IB,Davighi 2308.00017



Fibre bundle picture

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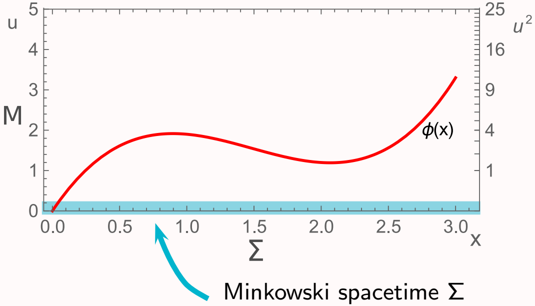
Alminawi,IB,Davighi 2308.00017



Minkowski spacetime Σ

w/ coord x^μ, metric η

$\equiv$ **base space**

Fibre bundle picture

main aim: include derivatives $\partial_\mu \phi(x) \to$ keep ϕ dependence on x manifest!

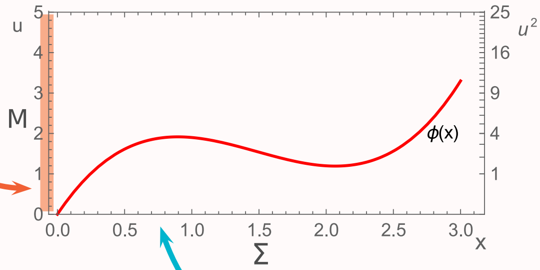
natural structure: **fibre bundle**

Alminawi,IB,Davighi 2308.00017

field config. space M

w/ coord u^i

$\equiv$ **fibre**



Minkowski spacetime Σ

w/ coord x^μ, metric η

$\equiv$ **base space**

Fibre bundle picture

main aim: include derivatives $\partial_\mu \phi(x) \to$ keep ϕ dependence on x manifest!

natural structure: **fibre bundle** (E, Σ, π)

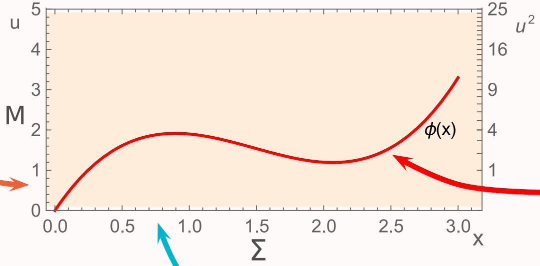
Alminawi,IB,Davighi 2308.00017

locally: $E_x = \Sigma_x \times M$

field config. space M

w/ coord u^i

$\equiv$ **fibre**



field $\phi^i(x) : \Sigma \to M$

$\equiv$ **section** of E over Σ

$\neq u^i$

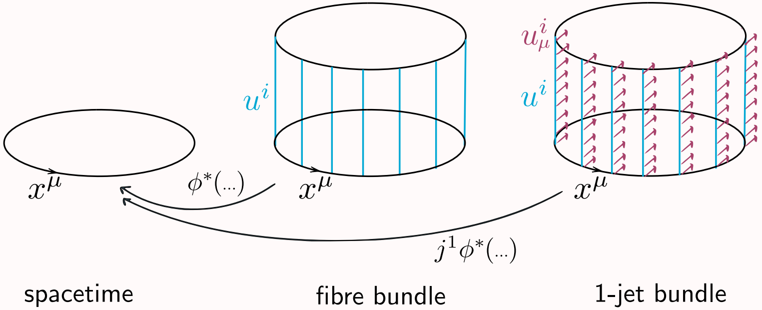
Minkowski spacetime Σ

w/ coord x^μ, metric η

$\equiv$ **base space**

Jet bundles

Alminawi,IB,Davighi 2308.00017
Saunders 1989. see also Craig,Lee 2307.15742



$j^r\phi$ = r-jet of ϕ = equiv. class of sections identical up to r-th derivative

Metric $\to$ Lagrangian

field space $\quad g_{ij}(\phi) \quad \mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij}(\phi)$

Metric → Lagrangian

field space $\longrightarrow$ fibre bundle

field space: $g_{ij}(\phi)$ $\qquad \mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij}(\phi)$

Poincaré invariance

fibre bundle: $\begin{array}{cc} x^\mu \\ u^i \end{array}\begin{pmatrix} g_{\mu\nu} & \cancel{g_{\mu j}} \\ \cancel{g_{\nu i}} & g_{ij} \end{pmatrix}$ $\qquad \mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij}(\phi) + \boxed{\frac{1}{2}\eta^{\rho\sigma}g_{\rho\sigma}(\phi)}$

$= -V(\phi)$

Metric $\rightarrow$ Lagrangian

field space $\qquad g_{ij}(\phi) \qquad \mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij}(\phi)$

$\downarrow$

Poincaré invariance

fibre bundle
$$\begin{array}{c c} x^\mu & \\ u^i & \end{array}\begin{pmatrix} g_{\mu\nu} & \cancel{g_{\mu j}} \\ \cancel{g_{\nu i}} & g_{ij} \end{pmatrix} \qquad \mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij}(\phi) + \boxed{\frac{1}{2}\eta^{\rho\sigma}g_{\rho\sigma}(\phi)}$$

$$= -V(\phi)$$

$\downarrow$

1-jet bundle
$$\begin{array}{c} x^\mu \\ u^i \\ u^i_\mu \end{array}\begin{pmatrix} g_{\mu\nu} & g_{\mu j} & g^\nu_{\mu j} \\ g_{\nu i} & g_{ij} & g^\nu_{ij} \\ g^\mu_{\nu i} & g^\mu_{ij} & g^{\mu\nu}_{ij} \end{pmatrix}$$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij} + \frac{1}{2}\eta^{\rho\sigma}g_{\rho\sigma} + \boxed{\partial^\mu\phi^i g_{\mu i}}$$
$$\boxed{+ (\partial^\mu\partial_\nu\phi^j)g^\nu_{\mu j} + (\partial_\rho\phi^i)(\partial^\rho\partial_\nu\phi^j)g^\nu_{ij} + \frac{1}{2}(\partial_\rho\partial_\mu\phi^i)(\partial^\rho\partial_\nu\phi^j)g^{\mu\nu}_{ij}}$$

redundant
∂^4 basis

Metric $\rightarrow$ Lagrangian

field space $\quad g_{ij}(\phi) \quad \mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij}(\phi)$

$\downarrow$

Poincaré invariance

fibre bundle

$$\begin{array}{c c} x^\mu \\ u^i \end{array} \begin{pmatrix} g_{\mu\nu} & \cancel{g_{\mu j}} \\ \cancel{g_{\nu i}} & g_{ij} \end{pmatrix} \qquad \mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij}(\phi) + \boxed{\frac{1}{2}\eta^{\rho\sigma}g_{\rho\sigma}(\phi)}$$

$= -V(\phi)$

$\downarrow$

1-jet bundle

$$\begin{array}{c} x^\mu \\ u^i \\ u^i_\mu \end{array} \begin{pmatrix} g_{\mu\nu} & g_{\mu j} & g^\nu_{\mu j} \\ g_{\nu i} & g_{ij} & g^\nu_{ij} \\ g^\mu_{\nu i} & g^\mu_{ij} & g^{\mu\nu}_{ij} \end{pmatrix}$$

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\nu\phi^j g_{ij} + \frac{1}{2}\eta^{\rho\sigma}g_{\rho\sigma} + \partial^\mu\phi^i g_{\mu i}$$
$$+ (\partial^\mu\partial_\nu\phi^j)g^\nu_{\mu j} + (\partial_\rho\phi^i)(\partial^\rho\partial_\nu\phi^j)g^\nu_{ij}$$
$$+ \frac{1}{2}(\partial_\rho\partial_\mu\phi^i)(\partial^\rho\partial_\nu\phi^j)g^{\mu\nu}_{ij}$$

redundant ∂^4 basis

$\downarrow$

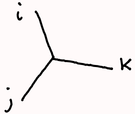
r-jet bundle $\longrightarrow$ complete $\mathscr{L}$ with up to $2(r+1)$ derivatives

On-shell amplitudes from fibre bundle

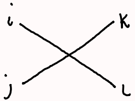
Alminawi,IB,Davighi 2509.20482

- geometry allows to easily derive **covariant Feynman rules** Finn,Karamitsos,Pilaftsis 1910.06661 via a generalization of normal coordinates

$$\Delta^{ij}(p^2) = i\left(p^2 g_{ij} + \Lambda^4 R^{\mu}{}_{i\mu j}/2\right)^{-1}$$



$$\mathcal{R}_{ijk} = i\frac{\Lambda^4}{2}\nabla_i R^{\mu}{}_{j\mu k}$$



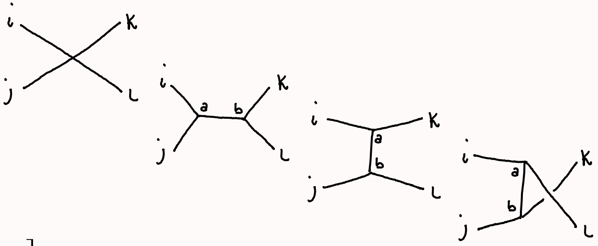
$$\mathcal{R}_{ijkl} = i\sum_{S_4}\left[\frac{\Lambda^4}{2}\nabla_i\nabla_j R^{\mu}{}_{k\mu l} - 2\Lambda^4 R^{\mu}{}_{i\nu j} R^{\nu}{}_{k\mu l} - 2s_{ij}R_{iklj}\right]$$

On-shell amplitudes from fibre bundle

Alminawi,IB,Davighi 2509.20482

- geometry allows to easily derive **covariant Feynman rules** Finn,Karamitsos,Pilaftsis 1910.06661
  via a generalization of normal coordinates
- they can be connected together with propagators as usual

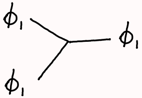
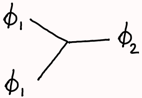
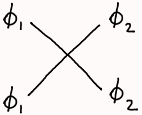
$$
\begin{aligned}
\mathcal{A}_{ijkl} = & \mathcal{R}_{ijkl} \\
& + \mathcal{R}_{ija}\, \Delta^{ab}(s)\, \mathcal{R}_{bkl} \\
& + \mathcal{R}_{ika}\, \Delta^{ab}(t)\, \mathcal{R}_{bjl} \\
& + \mathcal{R}_{ila}\, \Delta^{ab}(u)\, \mathcal{R}_{bkl} \\
= & \mathcal{R}_{ijkl} + \frac{1}{4!}\sum_{S_4}\left[3\, \mathcal{R}_{ija}\, \Delta^{ab}(s_{ij})\, \mathcal{R}_{bkl}\right]
\end{aligned}
$$



\# topologies Alminawi 2510.24866

Example: 4-point on-shell amplitude from fibre bundle geometry

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1\left[1 + a_2\frac{\phi_2}{\Lambda}\right] + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \left[\frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2 + \frac{v_{111}}{3!}\Lambda\phi_1^3 + \frac{v_{1122}}{4}\phi_1^2\phi_2^2\right]$$

Usual FR		Covariant FR
$-i\Lambda\, v_{111}$		$-i\Lambda v_{111}$
$-i\dfrac{a_2}{2\Lambda}\left[s_{11} - 2m_1^2\right]$		$-i\dfrac{a_2}{2\Lambda}\left[2m_1^2 - m_2^2\right]$
$-i\,v_{1122}$		$-i\,v_{1122} - i\dfrac{{a_2}^2}{2\Lambda^2}\left[4m_1^2 - m_2^2 - \dfrac{s_{11}}{2}\right]$

Example: 4-point on-shell amplitude from fibre bundle geometry

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1\left[1 + a_2\frac{\phi_2}{\Lambda}\right] + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \left[\frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2 + \frac{v_{111}}{3!}\Lambda\phi_1^3 + \frac{v_{1122}}{4}\phi_1^2\phi_2^2\right]$$

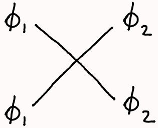
$$\begin{aligned}\mathcal{A}_{1122} = \\ & - v_{1122} \\ & + \frac{{a_2}^2}{4\Lambda^2}\left(s - 8m_1^2 + 2m_2^2\right) \\ & - \frac{{a_2}^2}{\Lambda^2}\left(m_1^2 - \frac{m_2^2}{2}\right)^2\left[\frac{1}{t - m_1^2} + \frac{1}{u - m_1^2}\right]\end{aligned}$$

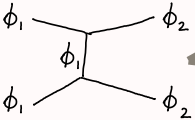
Example: 4-point on-shell amplitude from fibre bundle geometry

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1\left[1 + a_2\frac{\phi_2}{\Lambda}\right] + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 - \left[\frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2 + \frac{v_{111}}{3!}\Lambda\phi_1^3 + \frac{v_{1122}}{4}\phi_1^2\phi_2^2\right]$$

Usual diagrams

Covariant diagrams



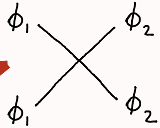


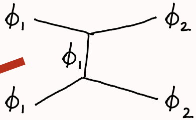
$$\mathcal{A}_{1122} =$$

$$- v_{1122}$$

$$+ \frac{{a_2}^2}{4\Lambda^2}\left(s - 8m_1^2 + 2m_2^2\right)$$

$$- \frac{{a_2}^2}{\Lambda^2}\left(m_1^2 - \frac{m_2^2}{2}\right)^2\left[\frac{1}{t - m_1^2} + \frac{1}{u - m_1^2}\right]$$





Summary

- **HEFT** is an EFT extension of the SM obtained treating separately Goldstones and Higgs boson
- attracting increasing attention recently, as a framework to generalize or stress-test SMEFT
- it is established that **SM $\subseteq$ SMEFT $\subseteq$ HEFT**

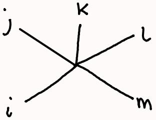
  $\to$ $\exists$ models that can match to HEFT but **not** SMEFT [e.g. loryons, 2HDM outside alignment]
  $\to$ HEFT can be **more convergent** than SMEFT when both allowed [e.g. composite H]
- HEFT power counting revisited recently to systematize operators and amplitude series
  $\to$ crucial as most HEFT signatures rely on different suppression assignments
- geometrical methods being developed as formalisms to study scattering amplitudes independently of parameterization $\to$ useful for "all-orders" SMEFT-HEFT comparisons

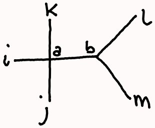
  $\to$ **fibre/jet bundle** formalism captures operators with $n \neq 2$ derivatives
  $\to$ scattering amplitudes easily computed with **covariant FRs** in fibre bundle
  $\to$ future: amplitudes with loops and in 1-jet. gauging, fermions

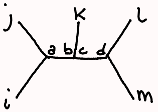
Backup slides

Alminawi,IB,Davighi 2509.20482

5-point on-shell amplitudes.







$$
\begin{aligned}
\mathcal{A}_{ijklm} &= \mathcal{R}_{ijklm} \\
&+ \frac{1}{5!}\sum_{S_5}\Big[10\,\mathcal{R}_{ijka}\,\Delta^{ab}(s_{45})\,\mathcal{R}_{blm} + \\
&\qquad +15\,\mathcal{R}_{ija}\,\Delta^{ab}(s_{ij})\,\mathcal{R}_{kbc}\,\Delta^{cd}(s_{lm})\,\mathcal{R}_{dlm}\Big]
\end{aligned}
$$

where

$$
\mathcal{R}_{ijklm} = \frac{i}{5!}\sum_{S_5}\left[\frac{\Lambda^4}{2}\nabla_i\nabla_j\nabla_k R^{\mu}{}_{l\mu m} - 7\Lambda^4 R^{\mu}{}_{i\nu j}\nabla_k R^{\nu}{}_{l\mu m} - 5 s_{ij}\,\nabla_m R_{ilkj}\right]
$$

6-point on-shell amplitudes.

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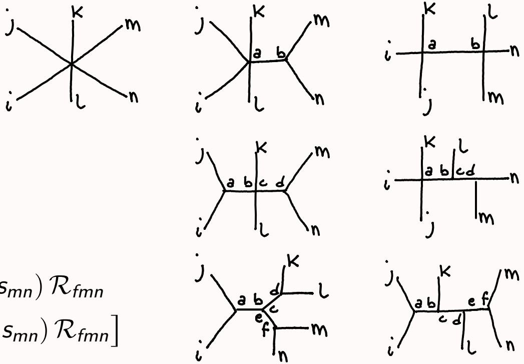
$$\mathcal{A}_{ijklmn} = \mathcal{R}_{ijklmn}$$
$$+\frac{1}{6!}\sum_{S_6}\Big[15\,\mathcal{R}_{ijkla}\,\Delta^{ab}(s_{mn})\,\mathcal{R}_{bmn}$$
$$+10\,\mathcal{R}_{ijka}\,\Delta^{ab}(s_{ijk})\,\mathcal{R}_{blmn}$$
$$+45\,\mathcal{R}_{ijab}\,\Delta^{ac}(s_{kl})\,\mathcal{R}_{ckl}\,\Delta^{bd}(s_{56})\,\mathcal{R}_{dmn}$$
$$+60\,\mathcal{R}_{ijka}\,\Delta^{ab}(s_{ijkl})\,\mathcal{R}_{bck}\,\Delta^{cd}(s_{mn})\,\mathcal{R}_{dmn}$$
$$+15\,\mathcal{R}_{ija}\,\Delta^{ab}(s_{ij})\,\mathcal{R}_{bcd}\,\Delta^{ce}(s_{kl})\,\mathcal{R}_{ekl}\,\Delta^{df}(s_{mn})\,\mathcal{R}_{fmn}$$
$$+90\,\mathcal{R}_{ija}\,\Delta^{ab}(s_{ij})\,\mathcal{R}_{bck}\,\Delta^{cd}(s_{ijk})\,\mathcal{R}_{del}\,\Delta^{ef}(s_{mn})\,\mathcal{R}_{fmn}\Big]$$



where
$$\mathcal{R}_{ijklmn} = \frac{i}{6!}\sum_{S_6}\Big[\frac{\Lambda^4}{2}\nabla_i\nabla_j\nabla_k\nabla_l R^{\mu}{}_{m\mu n} - 11\Lambda^4 R^{\mu}{}_{i\nu j}\nabla_k\nabla_l R^{\nu}{}_{m\mu n} - 7\Lambda^4\nabla_i R^{\mu}{}_{j\nu k}\nabla_l R^{\mu}{}_{m\nu n}$$
$$+ 8\Lambda^4 R^{\mu}{}_{i\nu j} R^{\nu}{}_{k\rho l} R^{\rho}{}_{m\mu n} - 9 s_{ij}\nabla_m\nabla_n R_{ilkj} - 8 s_{in} R_{ijk}{}^{q} R_{qlmn}\Big]$$

Loops

- on-shell amplitudes of **any loop order** can be written as **covariant tensors**

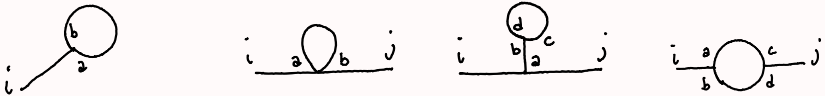
Cohen+ 2312.06748
Alonso,West 2207.02050

- vacuum and on-shell conditions become $\boxed{\dfrac{\delta\Gamma^{(L)}}{\delta\phi_i} = 0}$, $\boxed{\dfrac{\delta^2\Gamma^{(L)}}{\delta\phi^i\delta\phi^j} = 0}$

- verified that covariant FR technology works for 1- and 2-point functions at 1-loop

$$\mathcal{A}_i^{(1)} = \int \frac{d^dk}{(2\pi)^d}\,\mathcal{R}_{iab}\,\Delta^{ab}(k^2)$$

$$\mathcal{A}_{ij}^{(1)} = \frac{1}{2}\int \frac{d^dk}{(2\pi)^d}\,\mathcal{R}_{ijab}\,\Delta^{ab}(k^2) + \mathcal{R}_{ija}\,\Delta^{ab}(0)\,\mathcal{R}_{bcd}\,\Delta^{cd}(k^2) + \mathcal{R}_{iab}\,\Delta^{ac}(k^2)\,\Delta^{bd}((p+k)^2)\,\mathcal{R}_{cdj}$$



- in principle should generalize for any n-point [in progress]

HEFT bosonic basis in $\mathscr{L}_1$

39 operators (vs **15** in dim-6 Warsaw basis, **89** in dim-8 Murphy basis) Sun,Xiao,Yu 2206.07722

$$
\begin{array}{lll}
\langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle^2 \mathcal{F}(h) & \langle \mathbf{V}_\mu \mathbf{V}_\nu \rangle^2 \mathcal{F}(h) & \partial_\mu \partial_\nu \mathcal{F}(h) \partial^\mu \partial^\nu \mathcal{F}(h) \\
\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}_\nu \rangle \langle \mathbf{V}^\mu \mathbf{V}^\nu \rangle \mathcal{F}(h) & \langle \mathbf{T}\mathbf{V}_\mu \rangle^2 \langle \mathbf{V}_\nu \mathbf{V}^\nu \rangle \mathcal{F}(h) & \langle \mathbf{T}\mathbf{V}_\mu \rangle^4 \mathcal{F}(h) \\
\langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{V}_\nu \mathbf{V}^\nu \rangle \partial^\mu \mathcal{F}(h) & \langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{V}^\mu \mathbf{V}^\nu \rangle \partial_\nu \mathcal{F}(h) & \langle \mathbf{T}\mathbf{V}_\mu \mathbf{V}_\nu \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \partial^\nu \mathcal{F}(h) \\
\langle \mathbf{T}\mathbf{V}_\mu \rangle^2 \langle \mathbf{T}\mathbf{V}_\nu \rangle \partial^\nu \mathcal{F}(h) & \langle \mathbf{T}\mathbf{V}_\mu \rangle \langle \mathbf{T}\mathbf{V}_\nu \rangle \partial^\mu \partial^\nu \mathcal{F}(h) & \langle \mathbf{T}\mathbf{V}_\mu \rangle^2 \partial_\nu \mathcal{F}(h) \partial^\nu \mathcal{F}(h) \\
\langle \mathbf{V}_\mu \mathbf{V}_\nu \rangle \partial^\mu \partial^\nu \mathcal{F}(h) & \langle \mathbf{V}_\mu \mathbf{V}^\mu \rangle \partial^\nu \mathcal{F}(h) \partial_\nu \mathcal{F}(h) & \langle \mathbf{T}\mathbf{V}_\mu \rangle \partial_\nu \mathcal{F}(h) \partial^\mu \partial^\nu \mathcal{F}(h) \\
\langle \tilde{W}_{\mu\nu} \mathbf{V}^\mu \rangle \langle \mathbf{T}\mathbf{V}^\nu \rangle \mathcal{F}(h) & \langle \mathbf{T}[\tilde{W}_{\mu\nu}, \mathbf{V}^\nu] \rangle \langle \mathbf{T}\mathbf{V}^\mu \rangle \mathcal{F}(h) & \langle W_{\mu\nu} \mathbf{V}^\mu \rangle \langle \mathbf{T}\mathbf{V}^\nu \rangle \mathcal{F}(h) \\
\langle W_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h) & B_{\mu\nu} \langle \mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h) & \langle W_{\mu\nu} \mathbf{T} \rangle \langle \mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h) \\
\langle \tilde{W}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h) & \tilde{B}_{\mu\nu} \langle \mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu] \rangle \mathcal{F}(h) & \langle W_{\mu\nu} \mathbf{T} \rangle \langle \tilde{W}^{\mu\nu} \mathbf{T} \rangle \mathcal{F}(h) \\
B_{\mu\nu} \langle W^{\mu\nu} \mathbf{T} \rangle \mathcal{F}(h) & \tilde{B}_{\mu\nu} \langle W^{\mu\nu} \mathbf{T} \rangle \mathcal{F}(h) & \langle W_{\mu\nu} \mathbf{T} \rangle^2 \mathcal{F}(h) \\
B_{\mu\nu} B^{\mu\nu} \mathcal{F}(h) & W_{\mu\nu} W^{\mu\nu} \mathcal{F}(h) & G_{\mu\nu} G^{\mu\nu} \mathcal{F}(h) \\
B_{\mu\nu} \tilde{B}^{\mu\nu} \mathcal{F}(h) & W_{\mu\nu} \tilde{W}^{\mu\nu} \mathcal{F}(h) & G_{\mu\nu} \tilde{G}^{\mu\nu} \mathcal{F}(h) \\
f_{abc} G^a_{\mu\nu} G^{b\nu\rho} G^{c\nu}_\rho \mathcal{F}(h) & \varepsilon_{ijk} W^i_{\mu\nu} W^{j\nu\rho} W^{k\nu}_\rho \mathcal{F}(h) & \varepsilon_{ijk} B_{\mu\nu} W^{i\nu\rho} W^{j\nu}_\rho \mathbf{T}^k \mathcal{F}(h) \\
f_{abc} \tilde{G}^a_{\mu\nu} G^{b\nu\rho} G^{c\nu}_\rho \mathcal{F}(h) & \varepsilon_{ijk} \tilde{W}^i_{\mu\nu} W^{j\nu\rho} W^{k\nu}_\rho \mathcal{F}(h) & \varepsilon_{ijk} \tilde{B}_{\mu\nu} W^{i\nu\rho} W^{j\nu}_\rho \mathbf{T}^k \mathcal{F}(h)
\end{array}
$$