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NUMERICAL QUALITY FACTOR STATISTICS OF COATED SRF CAVITY

DESY-TEMF December 2025

2025-12-11

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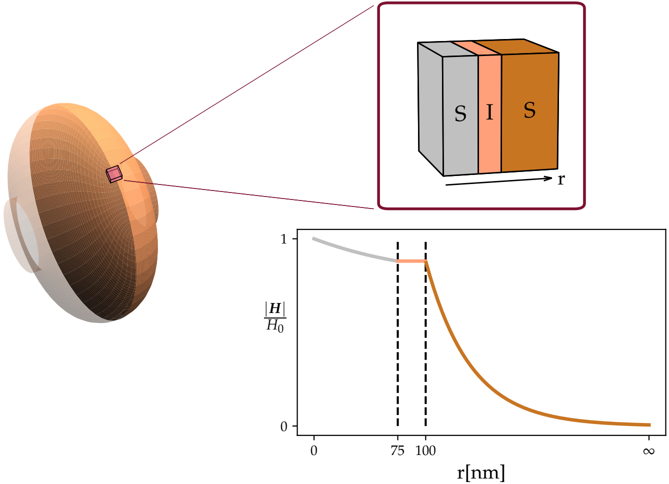
MOTIVATION & OUTLINE

Section 1



Observation: SRF benefits happen in thin layer

- Shield substrate from accelerating fields.
- Substrate with higher H_c
- Substrate with better thermal properties.

Valente-Feliciano A.-M., Superconducting RF materials other than bulk niobium: a review, *Supercond. Sci. Technol.* (2016)

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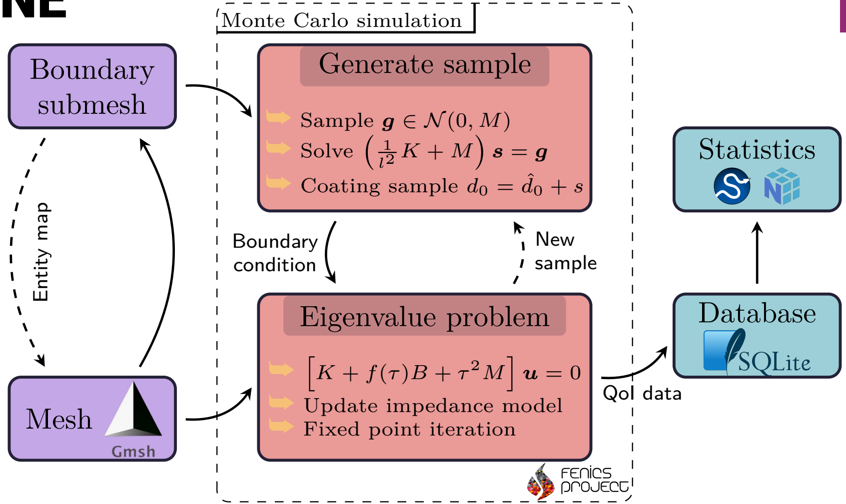
OUTLINE



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SURFACE IMPEDANCE

Section 2

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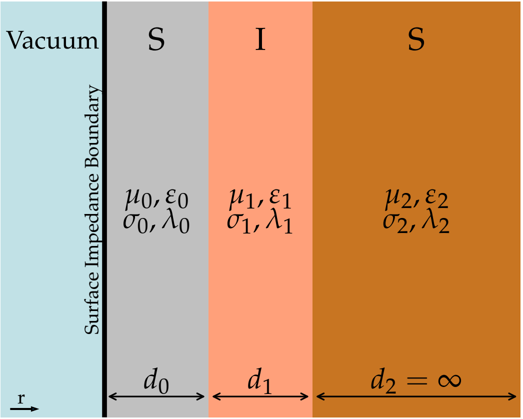

Insulator: $\alpha^2 = \mu\epsilon\omega^2$

Normal Conductor: $\alpha^2 = \mu\epsilon\omega^2 - j\mu\sigma\omega$

Superconductor: $\alpha^2 = \mu\epsilon\omega^2 - j\mu\sigma\omega - \frac{1}{\lambda^2}$

Fields in each layer:

$$E_x^{(k)}(r) = C_{2k}e^{j\alpha_k r} + C_{2k+1}e^{-j\alpha_k r}$$

$$H_y^{(k)}(r) = -\frac{\alpha_k}{\mu_k\omega}\left[C_{2k}e^{j\alpha_k r} - C_{2k+1}e^{-j\alpha_k r}\right]$$

Leontovich boundary condition:

$$\boldsymbol{n} \times \boldsymbol{E} = Z(\omega)(\boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{H}))$$

Yuferev S. *et al.*, Surface impedance boundary conditions: a comprehensive approach, *CRC press* (2018)

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GAUSSIAN FIELD COATING

Section 3

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Zero-mean Gaussian random field can be obtained by solving stochastic PDE

Let $s : \mathbb{R}^d \to \mathbb{R}$ be a Gaussian random field, i.e. $s \sim \mathrm{GP}(0, c_s(\boldsymbol{x}, \boldsymbol{y}))$, with Matérn kernel

$$c_s(\boldsymbol{x}, \boldsymbol{y}) = \frac{\sigma^2}{2^{\nu-1}\Gamma(\nu)} \left(\frac{\sqrt{2\nu}}{l} \|\boldsymbol{x} - \boldsymbol{y}\| \right)^{\nu} K_{\nu} \left(\frac{\sqrt{2\nu}}{l} \|\boldsymbol{x} - \boldsymbol{y}\| \right)$$

then $s(\boldsymbol{x})$ is the solution of the stochastic PDE

$$(\kappa^2 - \nabla^2)^{\beta} s(\boldsymbol{x}) = \frac{1}{\tau} g(\boldsymbol{x})$$

where $g(\boldsymbol{x}) \sim \mathrm{GP}(0, \delta(\boldsymbol{x} - \boldsymbol{y}))$ and $\kappa(\nu, l)$, $\beta(\nu, d)$, $\tau(\sigma, \nu, d, l)$.

Koh K. *et al.*, Stochastic PDE representation of random fields for large-scale Gaussian process regression and statistical finite element analysis, *Comput. Methods Appl. Mech. Engrg.* (2023)

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Zero-mean Gaussian random field can be obtained by solving stochastic PDE

For $d = 3$ and $\nu = 1/2$ this simplifies to

$$c_s(\boldsymbol{x},\boldsymbol{y}) = \sigma^2 \exp\left(-\frac{\|\boldsymbol{x}-\boldsymbol{y}\|}{l}\right), \quad \left(\frac{1}{l^2} - \nabla^2\right) s(\boldsymbol{x}) = \sqrt{\frac{8\pi\sigma^2}{l}} g(\boldsymbol{x})$$

Using a finite element approximation we obtain a discrete linear system

$$s(\boldsymbol{x}) \approx \sum_{i=1}^{N} s_i \phi_i(\boldsymbol{x}), \quad g(\boldsymbol{x}) \approx \sum_{i=1}^{N} g_i \phi_i(\boldsymbol{x}) \quad \Rightarrow \quad \left(\frac{1}{l^2} M + K\right) \boldsymbol{s} = \boldsymbol{g}$$

where M is the mass matrix, K the stiffness matrix and $\boldsymbol{g} \sim \mathcal{N}(\boldsymbol{0}, M)$

Koh K. *et al.*, Stochastic PDE representation of random fields for large-scale Gaussian process regression and statistical finite element analysis, *Comput. Methods Appl. Mech. Engrg.* (2023)

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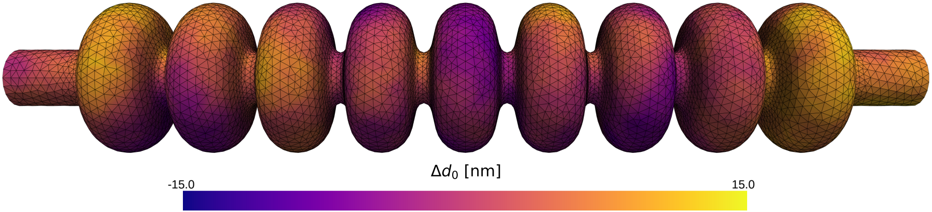
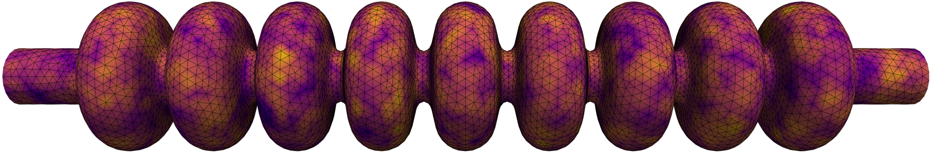
Correlation length = 0.1 m



Correlation length = 0.01 m



Note: TESLA geometry from Wanzenberg R.

Wanzenberg, R., Monopole, Dipole and Quadrupole Passbands of the TESLA-cell Cavity, *DESY* (2001)

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EIGENVALUE PROBLEM

Section 4

$$\nabla \times (\nabla \times \boldsymbol{E}) = \left(\frac{\omega}{c}\right)^2 \boldsymbol{E} \qquad \text{on } \Omega,$$

$$\boldsymbol{n} \times \boldsymbol{E} = Z(\omega)\left[\boldsymbol{n} \times (\boldsymbol{n} \times \boldsymbol{H})\right] \qquad \text{on } \partial\Omega$$

yields weak formulation

$$\frac{1}{\kappa^2}\int_\Omega \mathrm{d}V\ [(\nabla \times \boldsymbol{u}) \cdot (\nabla \times \boldsymbol{v})] + \frac{\tau}{\kappa}\frac{Z_0}{Z(\kappa,\tau)}\int_{\partial\Omega} \mathrm{d}S\ [(\boldsymbol{n} \times \boldsymbol{u}) \cdot (\boldsymbol{n} \times \boldsymbol{v})] + \tau^2 \int_\Omega \mathrm{d}V\ [\boldsymbol{u} \cdot \boldsymbol{v}] = 0$$

where

$$\kappa = \frac{2\pi f}{c} \text{ and } \tau = -\frac{1}{2Q} + j \quad \Rightarrow \quad \omega = 2\pi f\left(1 + \frac{j}{2Q}\right) = -jc\kappa\tau$$

and hence non-linear complex eigenvalue problem of the form

$$\left[K + f(\tau)B + \tau^2 M\right] \boldsymbol{u} = 0$$

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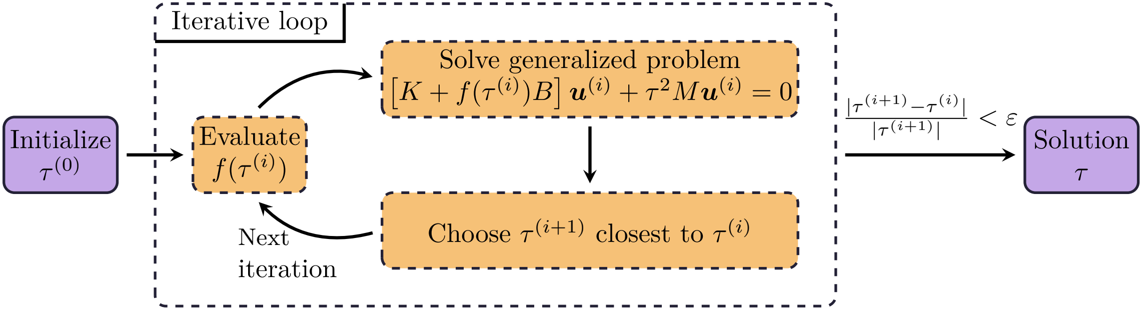
Solve non-linear eigenvalue problem with fixed point iteration



Note: use $\boldsymbol{u}^{(i)}$ for the initial search space of iteration $i + 1$

Hernandez V. *et al.*, SLEPc: A scalable and flexible toolkit for the solution of eigenvalue problems, *ACM Trans. Math. Software* (2005)

RESULTS

Section 5

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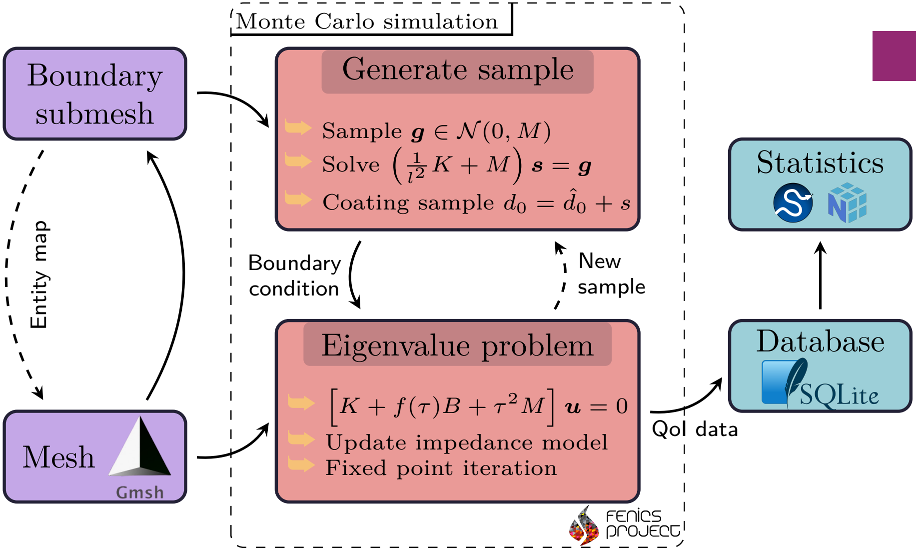

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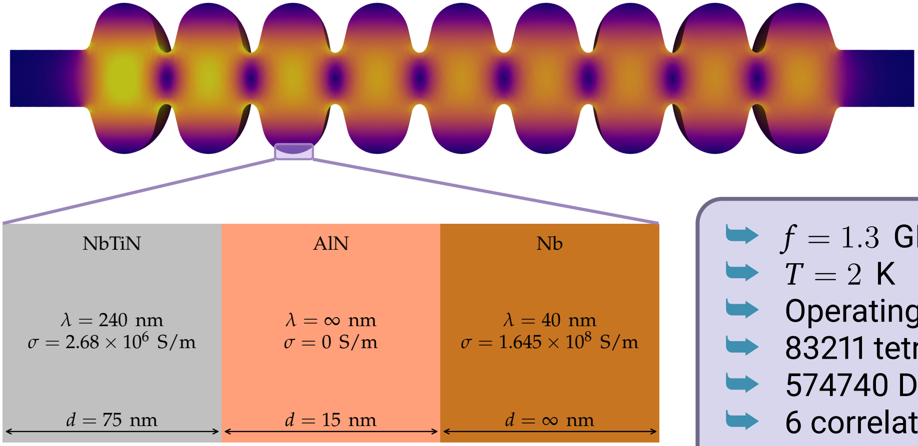

$$Z = 596.932\,\text{n}\Omega + 1.225j\,\text{m}\Omega$$

- $f = 1.3$ GHz
- $T = 2$ K
- Operating π-mode
- 83211 tetrahedra
- 574740 DOFs
- 6 correlation lengths l
- 1024 samples per l

Keckert S., Characterization of Nb3Sn and multilayer thin films for SRF apllications, *Universität Siegen* (2020)

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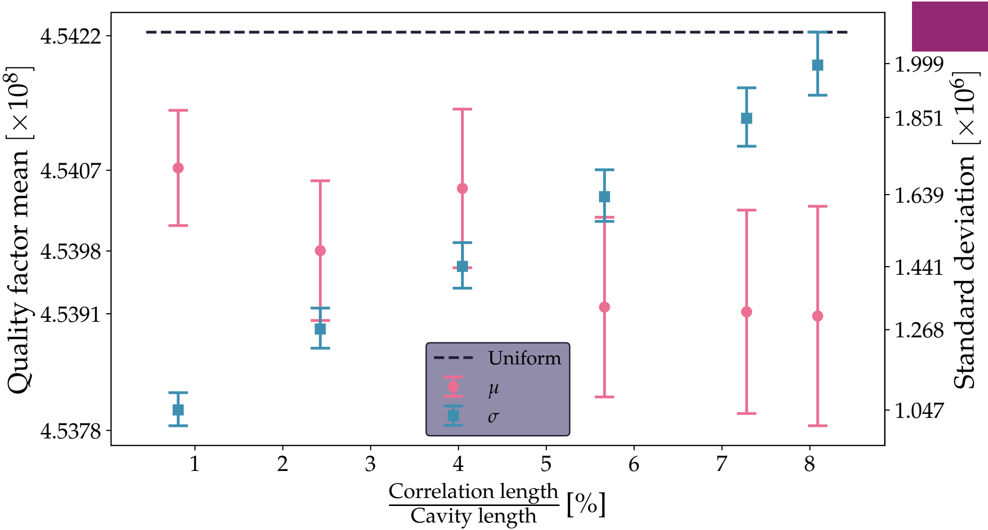

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Levene's test

H_0: The variances of two populations are equal

$l[m]$	0.01	0.03	0.05	0.07	0.09
0.03	5×10^{-8}				
0.05	2×10^{-20}	8×10^{-5}			
0.07	3×10^{-37}	1×10^{-14}	1×10^{-4}		
0.09	5×10^{-53}	8×10^{-27}	9×10^{-13}	5×10^{-4}	
0.1	2×10^{-62}	4×10^{-35}	2×10^{-19}	5×10^{-8}	0.040

Table: p-values for Levene's test between each pair of correlation length datasets. The significance level is $\alpha = 0.05$, red colored cells reject H_0 and green colored cells accept H_0.

Welch's t-test (unequal variance t-test)

H_0: The means of two populations are equal, the variance may be unequal

$l[m]$	0.01	0.03	0.05	0.07	0.09
0.03	0.073				
0.05	0.682	0.248			
0.07	0.011	0.330	0.052		
0.09	0.016	0.331	0.061	0.946	
0.1	0.019	0.324	0.065	0.956	0.806

Table: p-values Welch's t-test between each pair of correlation length datasets. The significance level is $\alpha = 0.05$, red colored cells reject H_0 and green colored cells accept H_0.

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Two-sample Kolmogorov-Smirnov test

H_0: Two samples are drawn from the same continuous distribution

$l[m]$	0.01	0.03	0.05	0.07	0.09
0.03	7×10^{-3}				
0.05	1×10^{-3}	4×10^{-2}			
0.07	7×10^{-9}	3×10^{-3}	9×10^{-3}		
0.09	1×10^{-11}	3×10^{-5}	4×10^{-4}	0.588	
0.1	5×10^{-13}	9×10^{-7}	5×10^{-5}	0.116	0.806

Table: p-values for two-sample Kolmogorov-Smirnov test between each pair of correlation length datasets. The significance level is $\alpha = 0.05$, red colored cells reject H_0 and green colored cells accept H_0.

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CONCLUSION & OUTLOOK

Section 6

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Conclusion

- Impact on Q-factor due to inhomogeneity is minor
- Differences between different correlation lengths is numerically visible. Sample means are mostly similar, sample variances are statistically different
- Homogeneous quality factor is higher than all inhomogeneous samples

Outlook

- Repeat with different materials
- Repeat with tuned cavity and consider field flatness
- Assume thicker coating in areas of high induced current to emulate ”worst case”
- Use more realistic coating distribution instead of Gaussian field