From resummed calculations to exclusive events with Geneva

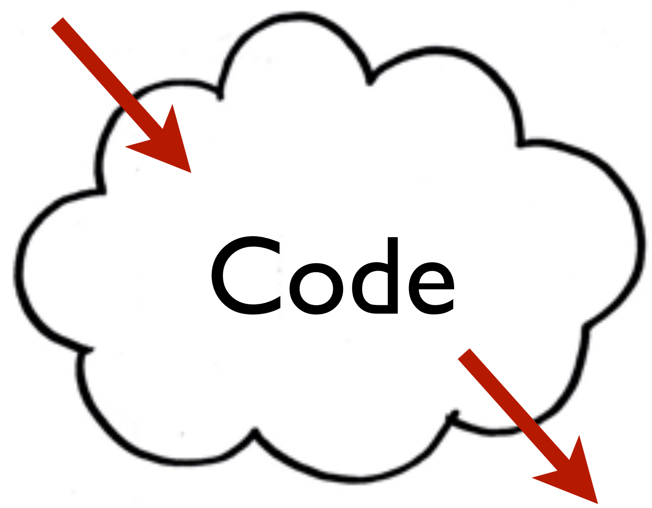
Simone Alioli, Christian Bauer,
Calvin Berggren, Andrew Hornig,
Frank Tackmann, CV,
Jonathan Walsh, Saba Zuberi

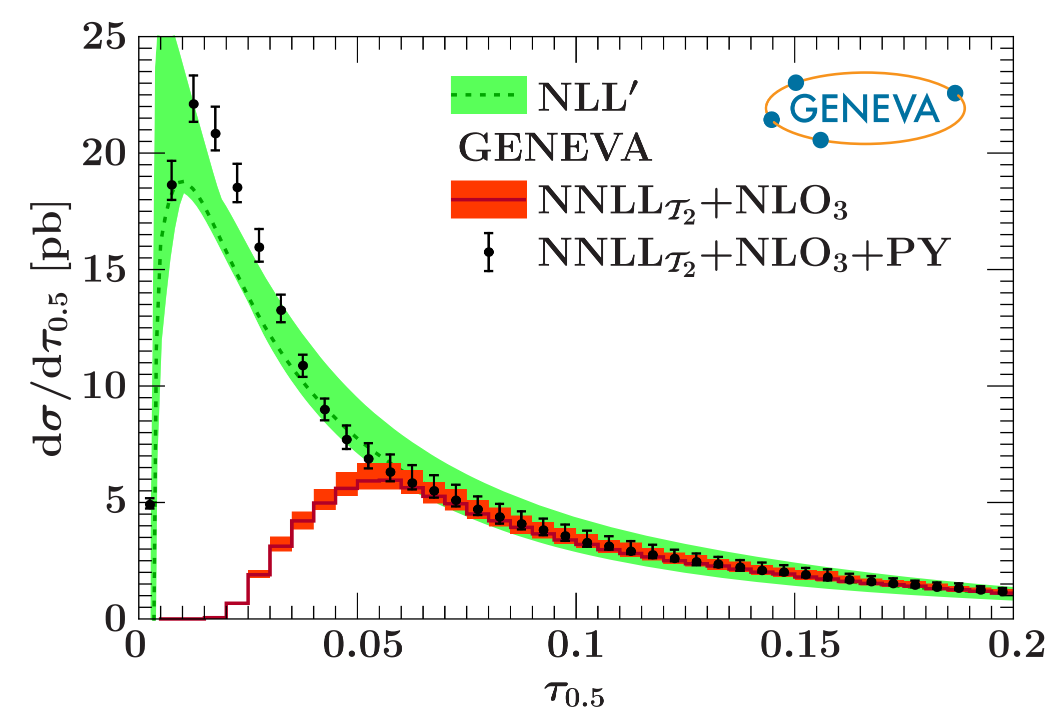
Christopher Vermilion
Lawrence Berkeley National Laboratory

Event Generators and Resummation
DESY
May 30, 2012

Geneva: a full-fledged event generator incorporating state-of-the art calculations.

$$\frac{d\sigma_{\geq 2}}{d\tau} = \sigma_2(\tau_{\rm cut})\,\delta(\tau) + \frac{d\sigma_{\geq 3}}{d\tau}\theta(\tau > \tau_{\rm cut})$$



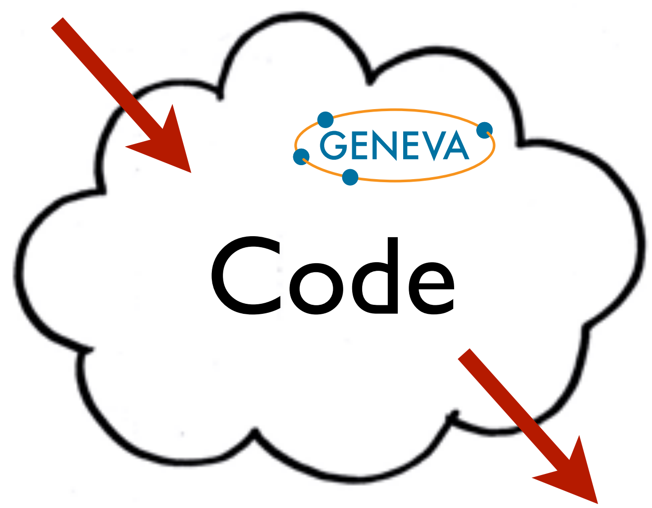


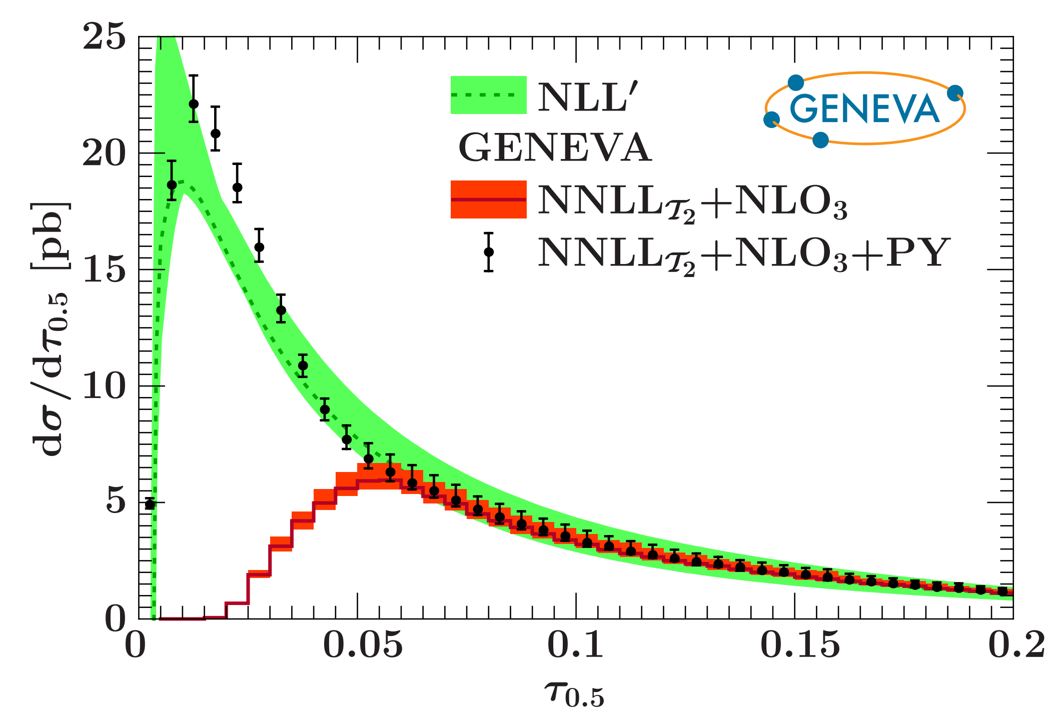
Basic task is to turn best available partonic calculations into collider predictions.

Physics drives code.

Geneva: a full-fledged event generator incorporating state-of-the art calculations.

$$\frac{d\sigma_{\geq 2}}{d\tau} = \sigma_2(\tau_{\mathrm{cut}})\,\delta(\tau) + \frac{d\sigma_{\geq 3}}{d\tau}\theta(\tau > \tau_{\mathrm{cut}})$$



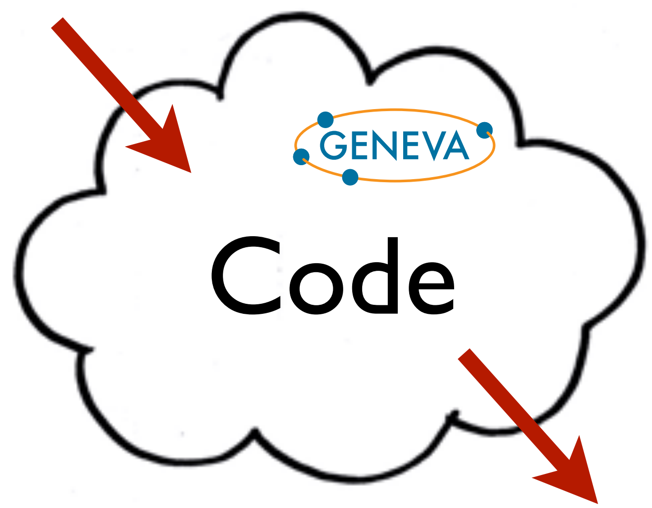


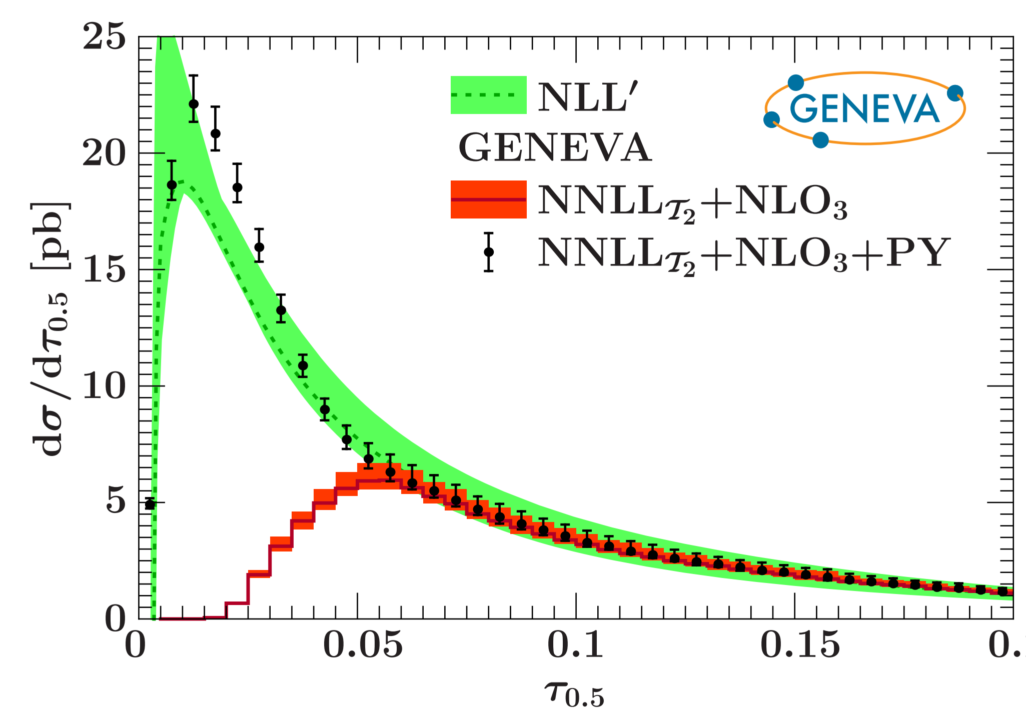
Basic task is to turn best available partonic calculations into collider predictions.

Physics drives code.

Geneva: a full-fledged event generator incorporating state-of-the art calculations.

$$\frac{\mathrm{d}\sigma_{\geq 2}}{\mathrm{d}\tau} = \sigma_2(\tau_{\mathrm{cut}})\,\delta(\tau) + \frac{\mathrm{d}\sigma_{\geq 3}}{\mathrm{d}\tau}\theta(\tau > \tau_{\mathrm{cut}})$$





Basic task is to turn best available partonic calculations into collider predictions.

Physics drives code.

Overview

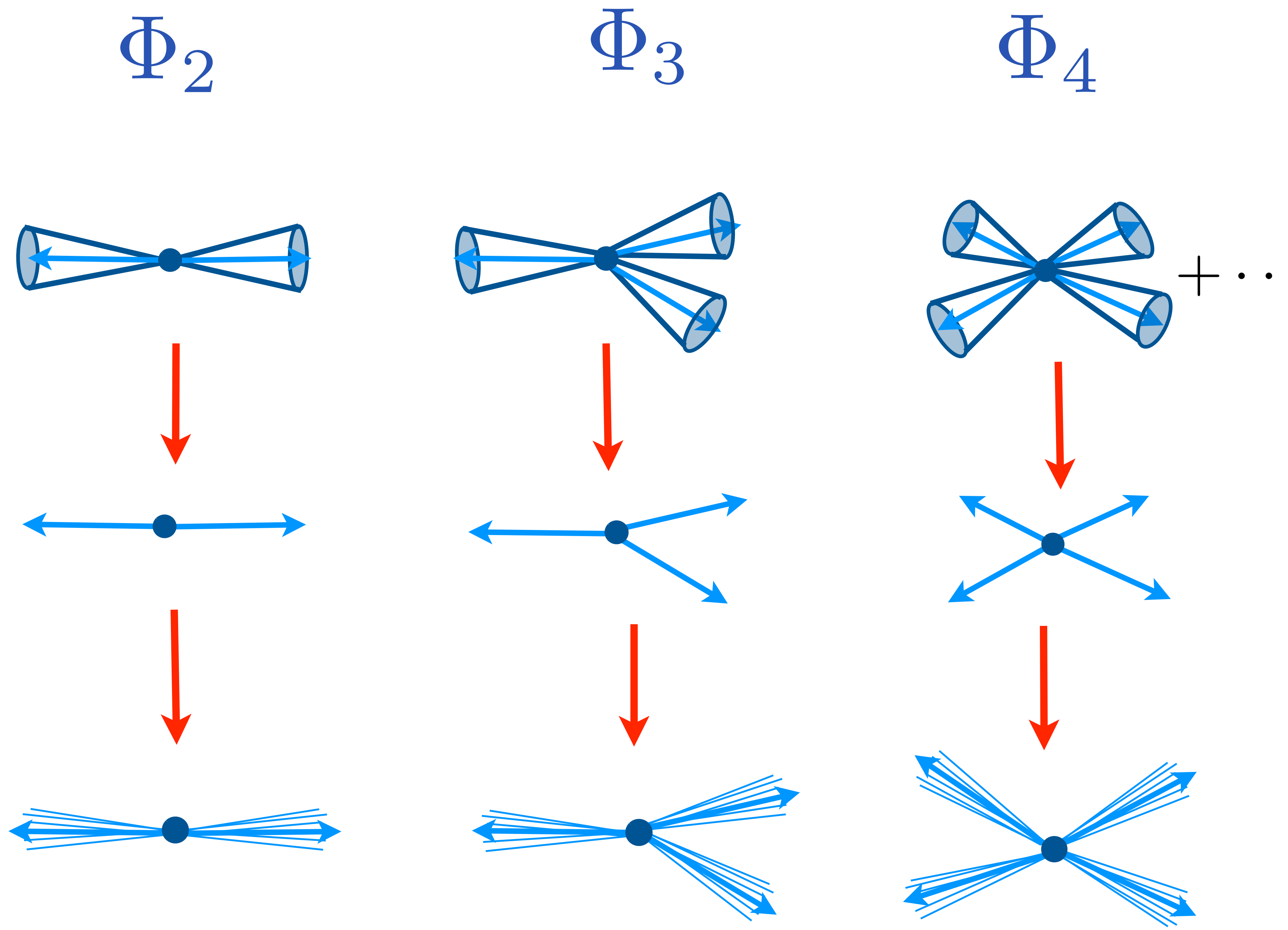
Will mostly focus on e^+e^- for simplicity, return to pp at end

(See Christian's talk)

- Calculate jet cross section

- Assign weight to parton event

- Use Parton Shower to fill jet

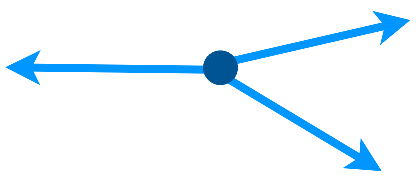


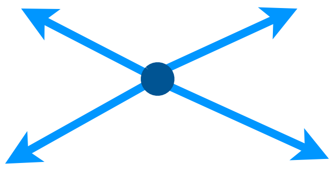
Slides credit: Saba Zuberi

Defining Φ_N

Φ_2 Φ_3 Φ_4

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_N}\ \mathrm{LO}:$$

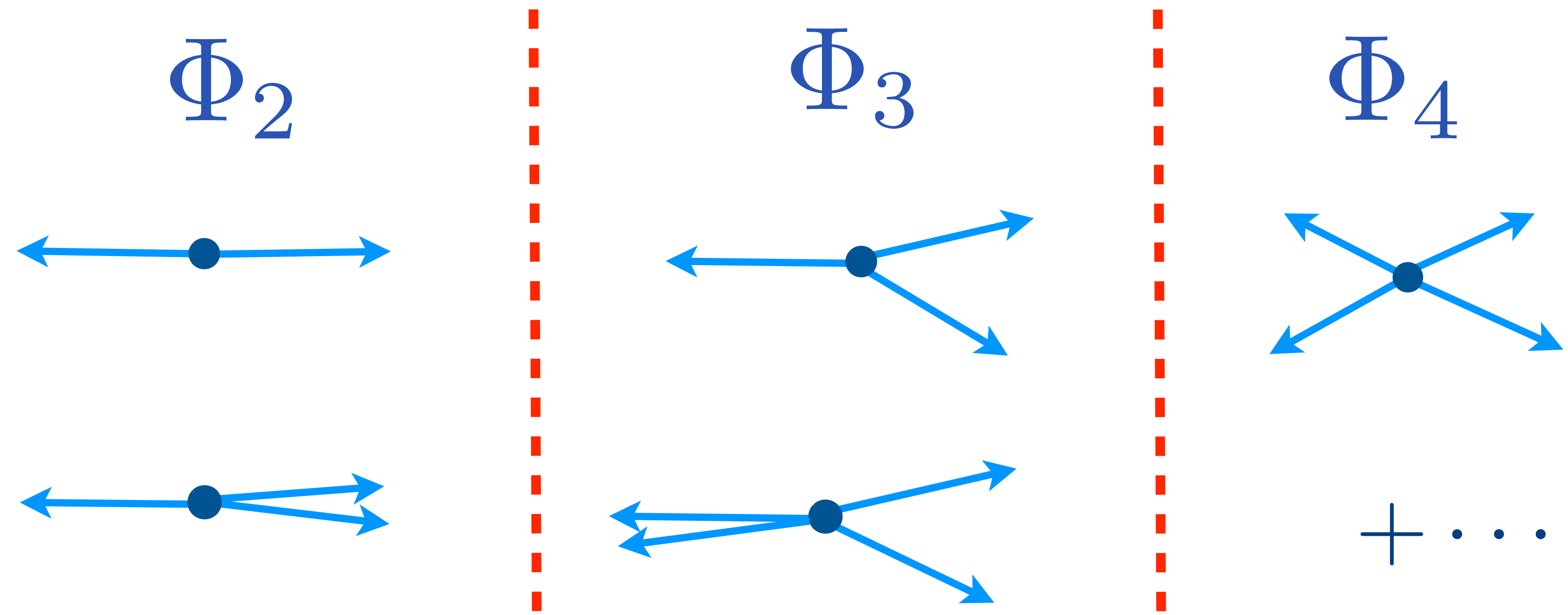




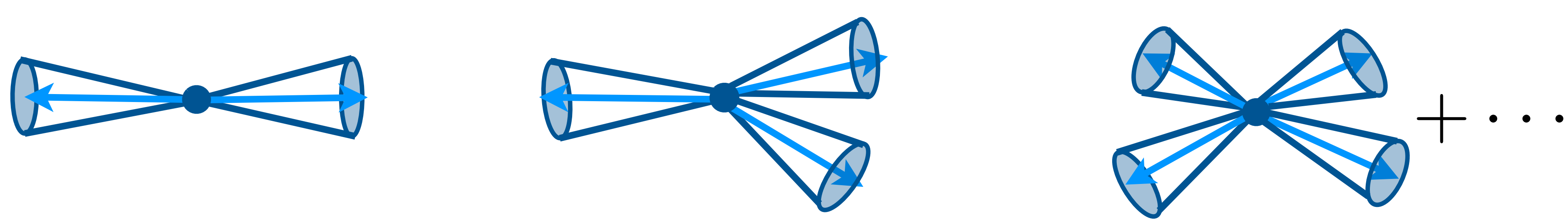
Defining Φ_N

$$\frac{d\sigma}{d\Phi_N} \ \text{LO}:$$

$$\frac{d\sigma}{d\Phi_N} \ \text{NLO}:$$

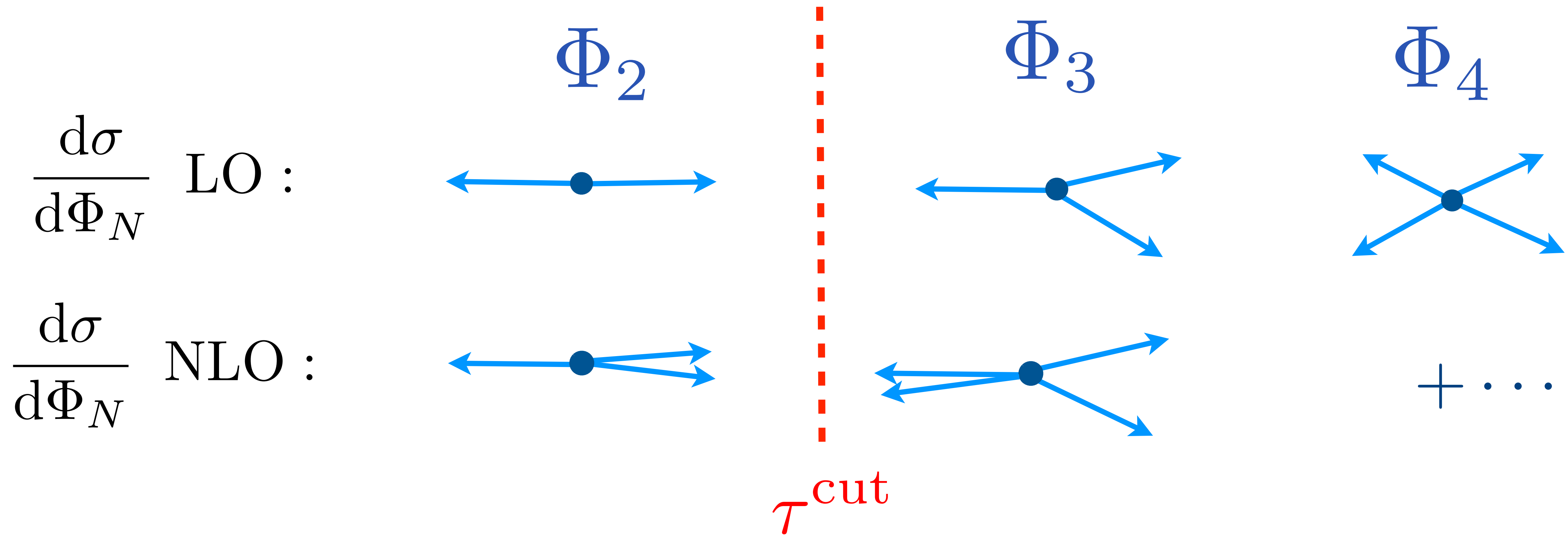


- Need **resolution variable** to slice phase space and **define** Φ_N beyond LO



- Must be IR safe and be able to carry out resummation.

Defining Φ_N



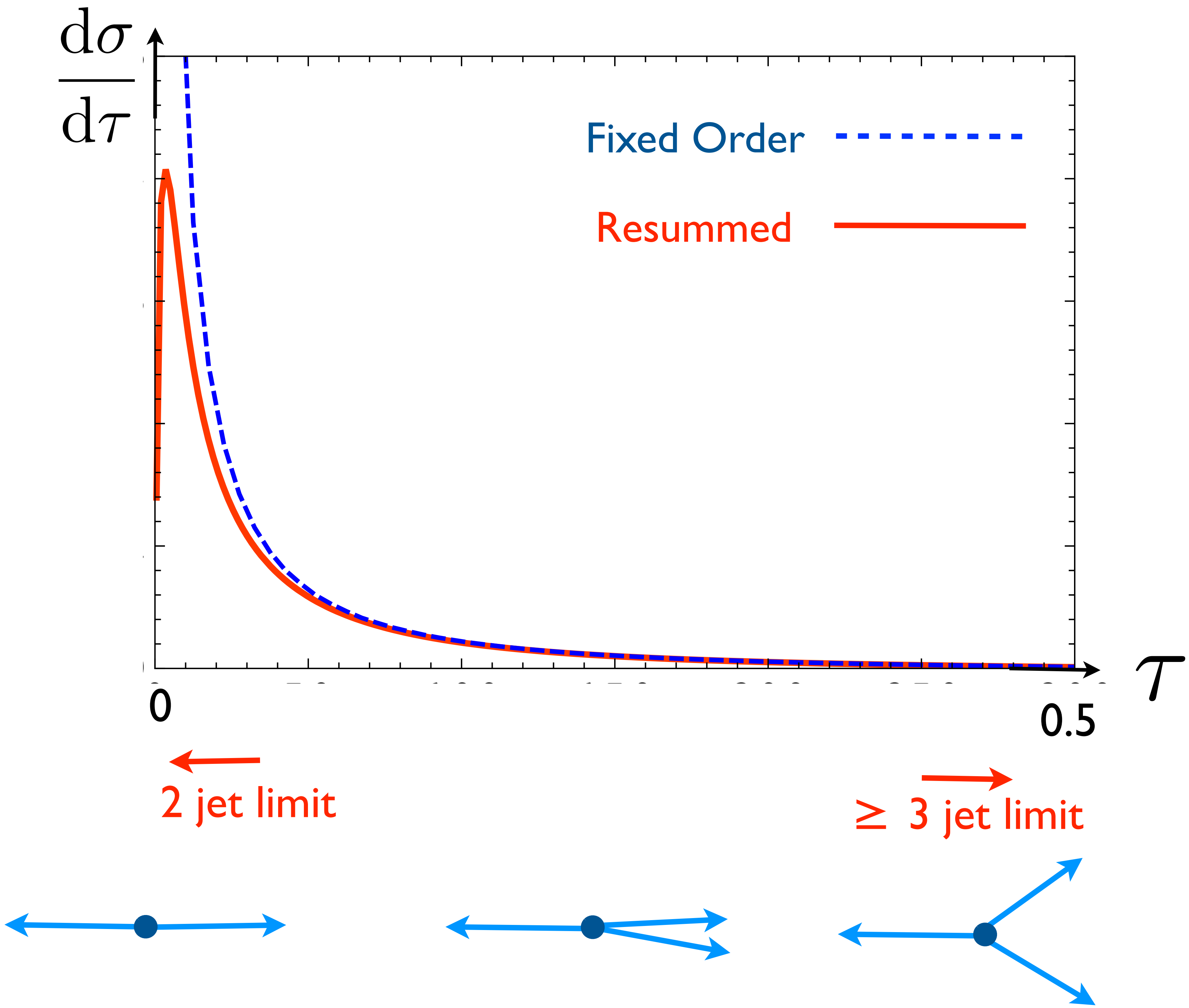
- Use thrust as resolution variable

$$\tau = 1 - T = 1 - \max_{\vec{n}} \sum_k \frac{|\vec{n} \cdot \vec{p}_k|}{E_{cm}}$$

- 2 body $\frac{d\sigma_2}{d\Phi_2}\Theta(\tau < \tau^{\text{cut}})$ and $\geq$ 3 body $\frac{d\sigma_{\geq 3}}{d\Phi_3}\Theta(\tau > \tau^{\text{cut}})$

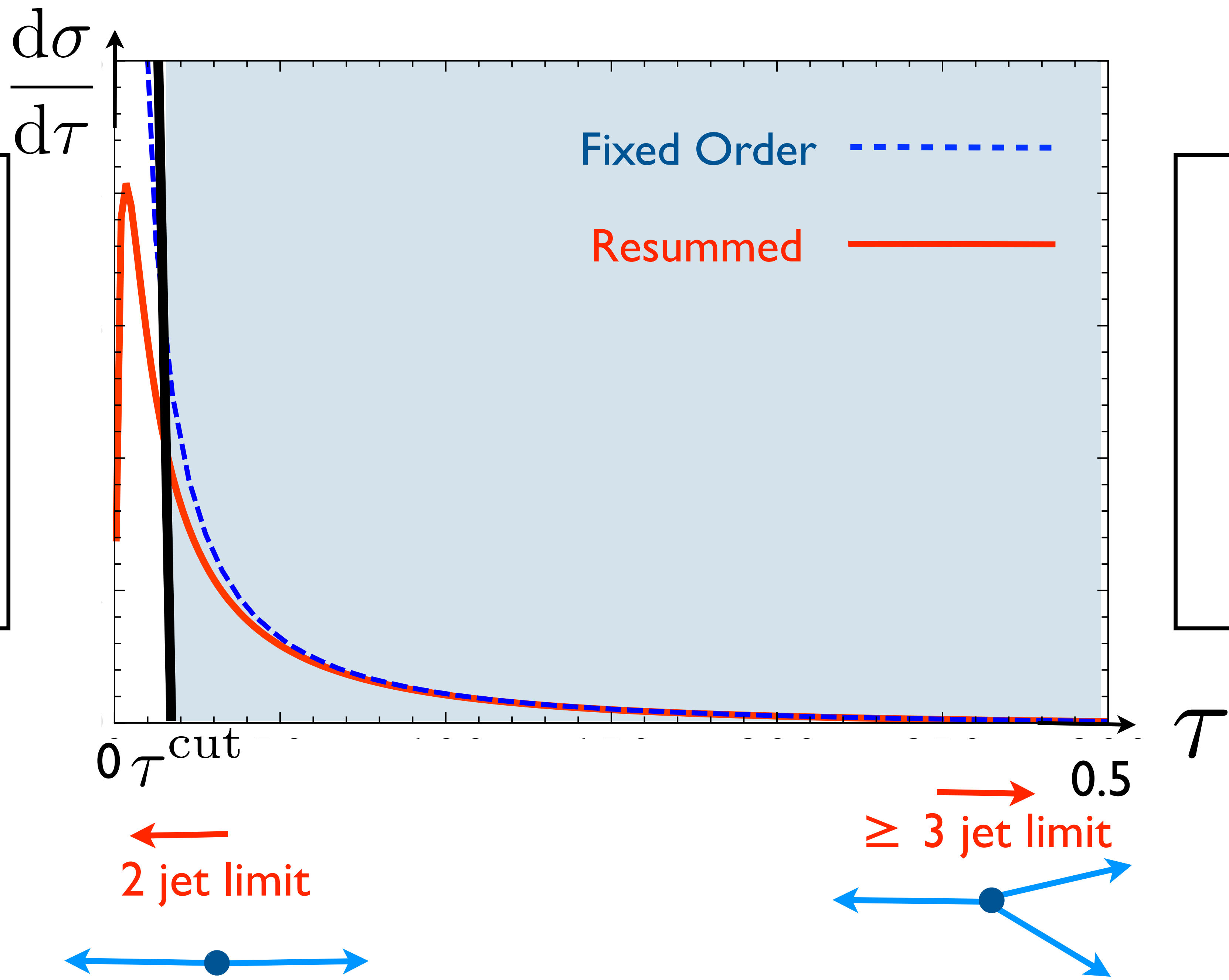
- How do we calculate these weights?

Resolution Variable



Resolution Variable

$$d\sigma_2(\tau^{\mathrm{cut}}) \downarrow \frac{d\sigma_2}{d\Phi_2}(\tau^{\mathrm{cut}})$$



$$\frac{d\sigma_{\geq 3}}{d\tau} \downarrow \frac{d\sigma_{\geq 3}}{d\Phi_3}$$

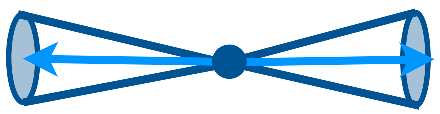
$$\Phi_3 = \{\Phi_2, \tau,\ 2 \to 3\,\mathrm{splitting}\}$$
$$\Phi_2 = \{\ \mathrm{thrust\ axis}\}$$

Master Formula

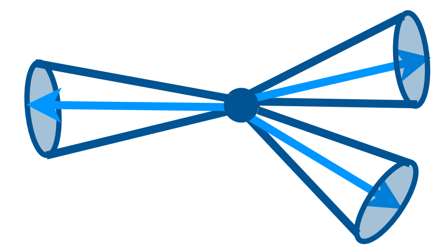
- Combine description in Peak, Transition and Tail regions.
  2 jet NNLL+NLO$_2$ and 3 jet NNLL+NLO$_3$

$$\mathcal{T}_2 = 2\,Q\,\tau = 1000\,\tau\,\mathrm{GeV}$$

- Distribute events according to:



2-body events



$\geq$ 3-body events

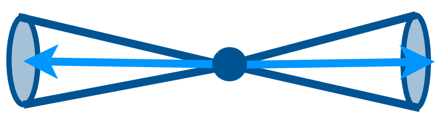
$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}(\mathcal{T}_2^{\mathrm{cut}}) = \int_0^{\mathcal{T}_2^{\mathrm{cut}}} \mathrm{d}\mathcal{T}_2 \frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2\,\mathrm{d}\mathcal{T}_2} \quad + \quad \frac{\mathrm{d}\sigma_{\geq 3}}{\mathrm{d}\Phi_3} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_2} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}\;\mathcal{T}_2}\Big|_{\mathrm{exp}} \right) \frac{\mathrm{d}\sigma_{\geq 3}^{\mathrm{NLO}}}{\mathrm{d}\Phi_3}\,\theta(\mathcal{T}_2 > \mathcal{T}_{\mathrm{cut}})$$

Master Formula

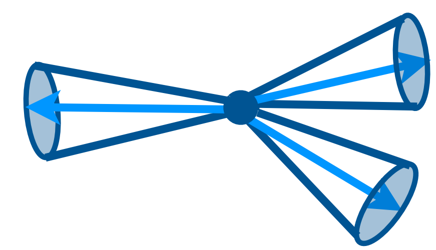
- Combine description in Peak, Transition and Tail regions.
  2 jet NNLL+NLO_2 and 3 jet NNLL+NLO_3

$$\mathcal{T}_2 = 2\,Q\,\tau = 1000\,\tau\,\mathrm{GeV}$$

- Distribute events according to:



2-body events



$\geq$ 3-body events

$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2}(\mathcal{T}_2^{\mathrm{cut}}) = \int_0^{\mathcal{T}_2^{\mathrm{cut}}} \mathrm{d}\mathcal{T}_2 \frac{\mathrm{d}\sigma_2}{\mathrm{d}\Phi_2\,\mathrm{d}\mathcal{T}_2} \quad + \quad \frac{\mathrm{d}\sigma_{\geq 3}}{\mathrm{d}\Phi_3} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_2} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}\,\mathcal{T}_2}\Big|_{\mathrm{exp}}\right) \frac{\mathrm{d}\sigma_{\geq 3}^{\mathrm{NLO}}}{\mathrm{d}\Phi_3}\,\theta(\mathcal{T}_2 > \mathcal{T}_{\mathrm{cut}})$$

Normalization $\mathrm{d}\sigma_2(\mathcal{T}_2^{\mathrm{cut}})$ at NNLL + NLO_2 resummed.

Constant for all 2-body events

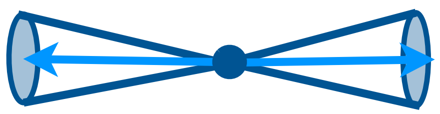
Shape from parton shower

Master Formula

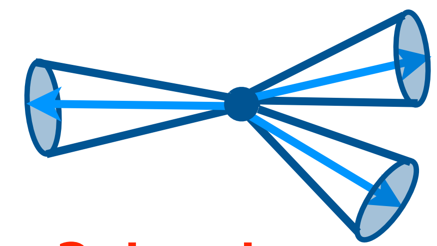
- Combine description in Peak, Transition and Tail regions.
  2 jet NNLL+NLO_2 and 3 jet NNLL+NLO_3

$$\mathcal{T}_2 = 2\,Q\,\tau = 1000\,\tau\,\mathrm{GeV}$$

- Distribute events according to:



2-body events



$\geq$ 3-body events

$$\frac{d\sigma_2}{d\Phi_2}(\mathcal{T}_2^{\mathrm{cut}}) = \int_0^{\mathcal{T}_2^{\mathrm{cut}}} d\mathcal{T}_2 \frac{d\sigma_2}{d\Phi_2\, d\mathcal{T}_2} + \frac{d\sigma_{\geq 3}}{d\Phi_3} = \left(\frac{d\sigma}{d\mathcal{T}_2} \Big/ \frac{d\sigma}{d\,\mathcal{T}_2}\Big|_{\mathrm{exp}} \right) \frac{d\sigma_{\geq 3}^{\mathrm{NLO}}}{d\Phi_3}\, \theta(\mathcal{T}_2 > \mathcal{T}_{\mathrm{cut}})$$

Has full Φ_3 dependence

Resummed to NNLL

Expanded to $\mathcal{O}(\alpha_s^2)$

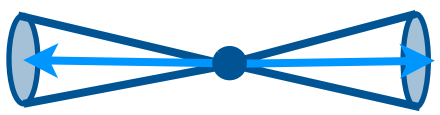
Full FO contribution at NLO_3

Master Formula

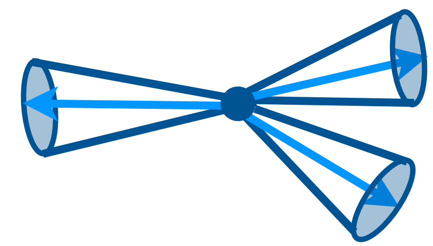
- Combine description in Peak, Transition and Tail regions.
  2 jet NNLL+NLO$_2$ and 3 jet NNLL+NLO$_3$

$\mathcal{T}_2 = 2\,Q\,\tau = 1000\,\tau\,\mathrm{GeV}$

- Distribute events according to:



2-body events



$\geq$ 3-body events

$$\frac{d\sigma_2}{d\Phi_2}(\mathcal{T}_2^{\mathrm{cut}}) = \int_0^{\mathcal{T}_2^{\mathrm{cut}}} d\mathcal{T}_2 \frac{d\sigma_2}{d\Phi_2\, d\mathcal{T}_2} \; + \; \frac{d\sigma_{\geq 3}}{d\Phi_3} = \left(\frac{d\sigma}{d\mathcal{T}_2} \Big/ \frac{d\sigma}{d\;\mathcal{T}_2}\Big|_{\mathrm{exp}} \right) \frac{d\sigma^{\mathrm{NLO}}_{\geq 3}}{d\Phi_3}\, \theta(\mathcal{T}_2 > \mathcal{T}_{\mathrm{cut}})$$

Tail Region: $\mathcal{T}_2$

Resummation turns off.

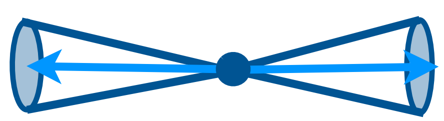
Ratio starts at $\mathcal{O}(\alpha_s^3)$

Master Formula

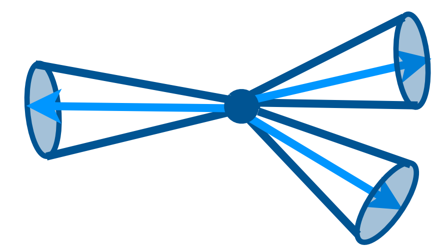
- Combine description in Peak, Transition and Tail regions.
  2 jet NNLL+NLO$_2$ and 3 jet NNLL+NLO$_3$

$$\mathcal{T}_2 = 2\,Q\,\tau = 1000\,\tau\,\mathrm{GeV}$$

- Distribute events according to:



2-body events



$\geq$ 3-body events

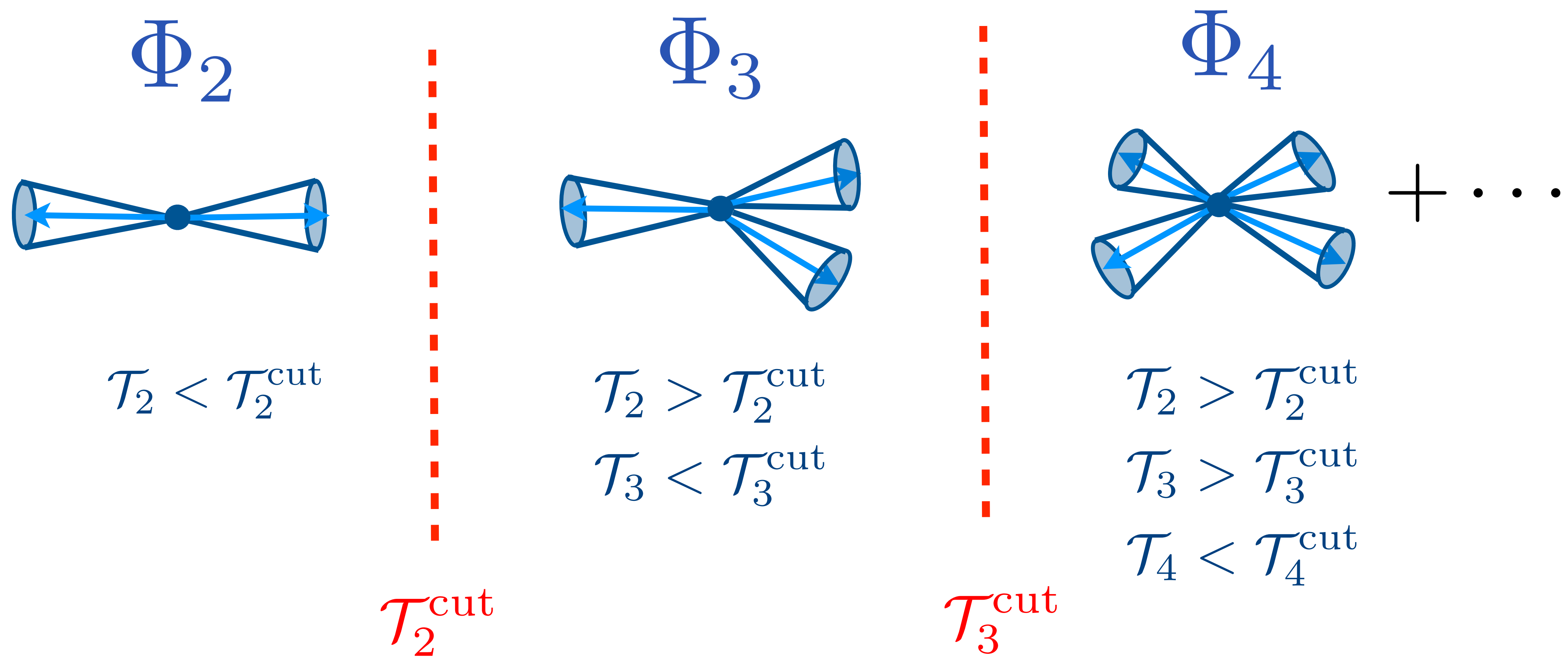
$$\frac{d\sigma_2}{d\Phi_2}(\mathcal{T}_2^{\rm cut}) = \int_0^{\mathcal{T}_2^{\rm cut}} d\mathcal{T}_2\, \frac{d\sigma_2}{d\Phi_2\, d\mathcal{T}_2} + \frac{d\sigma_{\geq 3}}{d\Phi_3} = \left(\frac{d\sigma}{d\mathcal{T}_2} \Big/ \frac{d\sigma}{d\,\mathcal{T}_2}\Big|_{\rm exp} \right) \frac{d\sigma_{\geq 3}^{\rm NLO}}{d\Phi_3}\, \theta(\mathcal{T}_2 > \mathcal{T}_{\rm cut})$$

Peak Region $\mathcal{T}_2$:

Resummation important

Ratio starts at N^{3}LL

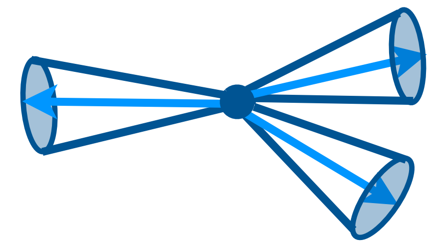
Resolution Variable at Higher Orders



NLO₃ Calculation

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_2} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}\,\mathcal{T}_2}\Big|_{\exp}\right)\frac{\mathrm{d}\sigma^{\rm NLO}_{\geq 3}}{\mathrm{d}\Phi_3}$$

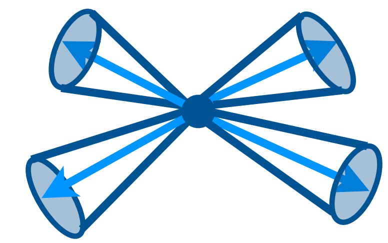
$$\frac{\mathrm{d}\sigma^{\rm NLO}_{\geq 3}}{\mathrm{d}\Phi_3} = \frac{\mathrm{d}\sigma^{\rm NLO}_{3}}{\mathrm{d}\Phi_3} + \frac{\mathrm{d}\sigma^{\rm LO}_{\geq 4}}{\mathrm{d}\Phi_4}$$ =



$\mathcal{T}_2 > \mathcal{T}_2^{\rm cut}$

$\mathcal{T}_3 < \mathcal{T}_3^{\rm cut}$

\+



$\mathcal{T}_2 > \mathcal{T}_2^{\rm cut}$

$\mathcal{T}_3 > \mathcal{T}_3^{\rm cut}$

$+\cdots$

- 3 jet contribution at NLO involves integrating over 4 body phase space.

$$\frac{\mathrm{d}\sigma^{\rm NLO}_{3}}{\mathrm{d}\Phi_3} = B_3(\Phi_3) + V_3(\Phi_3) + \int \mathrm{d}\Phi_4 B_4(\Phi_4)\delta[\Phi_3 - \Phi_3^{\rm NLO}(\Phi_4)]\theta(\mathcal{T}_3^{\rm cut} - \mathcal{T}_3)$$

- Implementation of NLO QCD computations known. Use FKS subtraction method as in POWHEG BOX, MadFKS. [Frixione, Kunszt, Signer]
This is done on the fly.

Regulating divergences with FKS subtractions

- **To perform the fully exclusive NLO calculation we adopt the FKS subtraction procedure**

$$d\sigma_{\texttt{NLO}} = \left\{ B(\Phi_n) + V(\Phi_n) + \Big[\underbrace{\overbrace{R(\Phi_n, \Phi_{\text{rad}})}^{\text{IRdivergent}} - \overbrace{C(\Phi_n, \Phi_{\text{rad}})}^{\text{IRdivergent}}}_{\text{finite}} \Big] d\Phi_{\text{rad}} \right\} d\Phi_n$$

$$V(\Phi_n) = \underbrace{\overbrace{V_b(\Phi_n)}^{\text{IRdivergent}} + \overbrace{\int C(\Phi_n, \Phi_{\text{rad}})\, d\Phi_{\text{rad}}}^{\text{IRdivergent}}}_{\text{finite}}$$

- **Full separation of divergencies into non-overlapping singular regions.**
- **Subtraction counterterms C obtained from $()_+$ distributions and written in terms of eikonal and collinear factorization formulae for matrix elements.**
- **The general case for any process, with any number of legs can be worked once and for all (cfr. `POWHEG BOX` and `MADFKS`)**
- **Great reduction in number of independents subtraction terms needed brings to higher computational efficency.**
- Φ_n **kinematics is fixed $=$ no complicated mappings $\Phi_{n+1} \to \Phi_n$. However, e.g. $\mathcal{T}_0(\Phi_2) \neq \mathcal{T}_0(\Phi_1)$ suggests $\mathcal{T}_1^{\text{cut}} \ll \mathcal{T}_0^{\text{cut}}$ to reduce mismatch.**

Procedure for calculating $d\sigma_{NLO}/d\phi_N$:

$$d\sigma_{\texttt{NLO}} = \left\{ B(\Phi_n) + V(\Phi_n) + \left[\overbrace{R(\Phi_n, \Phi_{\rm rad})}^{\rm IR divergent} - \overbrace{C(\Phi_n, \Phi_{\rm rad})}^{\rm IR divergent} \right]_{\rm finite} d\Phi_{\rm rad} \right\} d\Phi_n$$

$$V(\Phi_n) = \underbrace{\overbrace{V_b(\Phi_n)}^{\rm IR divergent} + \overbrace{\int C(\Phi_n, \Phi_{\rm rad})\, d\Phi_{\rm rad}}^{\rm IR divergent}}_{\rm finite}$$

1. Generate N-parton phase space point ϕ_N
   1.1. Check that it is in N-jet region
2. Calculate $B(\phi_N)$, $V(\phi_N)$ terms
3. Do "R-S" integral -- Monte Carlo integral over ϕ_{rad}
   3.1. Combine ϕ_N with ϕ_{rad} to get ϕ_{N+1}
   3.2. If ϕ_{N+1} is in N-jet region, include $R(\phi_{N+1})$ term
   3.3. Subtract $C(\phi_{N+1})$ (FKS counterterm)

Interfacing with Parton Shower

- Goal: add exclusivity w/o changing underlying partonic phase space
- Schematically, $\phi_N(\phi_{showered}) \sim \phi_N$
  - projection of showered phase space to ϕ_N gets back to original point
  - *up to some tunable level of precision*
- Need to **define** this projection, in particular $\mathcal{T}_M(\phi_N)$
  -- more than one choice:
  - Exact (minimized) definition in principle possible, but computationally expensive
  - Can instead use a **recursive** definition: on an *M*-body configuration, cluster the two particles that would share a "jet" when minimizing *(M-1)*-jettiness
    - Defines a clustering algorithm (metric is E_{ij} - p_{ij}); simply iterate

Interfacing with Parton Shower

- **Veto and restart shower if :**
  - 2 partons : $\mathcal{T}_2^{FR}(\Phi_{\rm PY}) > \mathcal{T}_2^{\rm cut}(1+\lambda/2)$
  - 3 partons: $\mathcal{T}_{\rm clust} > \mathcal{T}_3^{\rm cut}$ or $|\mathcal{T}_2^{\rm FR}(\Phi_{\rm PY}) - \mathcal{T}_2^{\rm FR}(\Phi_3)| > \lambda \mathcal{T}_2^{\rm FR}(\Phi_3)$
  - 4 partons: $\mathcal{T}_{\rm clust} > \mathcal{T}_3^{\rm QCD}(\Phi_4)$ or $|\mathcal{T}_2^{\rm FR}(\Phi_{\rm PY}) - \mathcal{T}_2^{\rm FR}(\Phi_4)| > \lambda \mathcal{T}_2^{\rm FR}(\Phi_3)$

    closest QCD allowed pair

  - λ controls the amount $\mathcal{T}_2$ of the underlying configuration is allowed to change. Currently run with λ=0.1
  - $\mathcal{T}_{\rm clust} = \max_i 2(E_i - |p_i|)$ is a proxy for where we would start a $\mathcal{T}_N$ ordered shower.

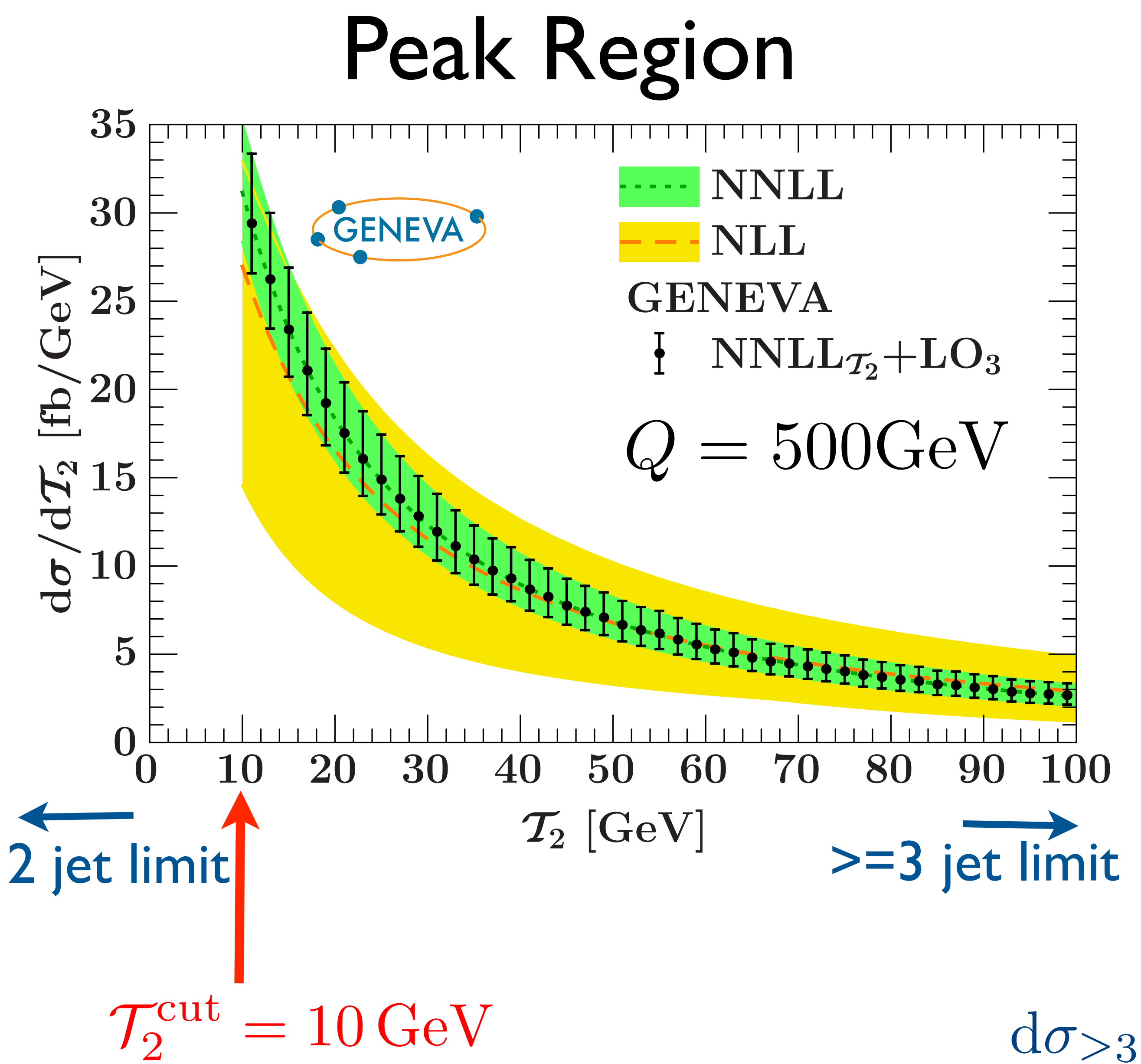
No events are thrown out!
No unweighting!

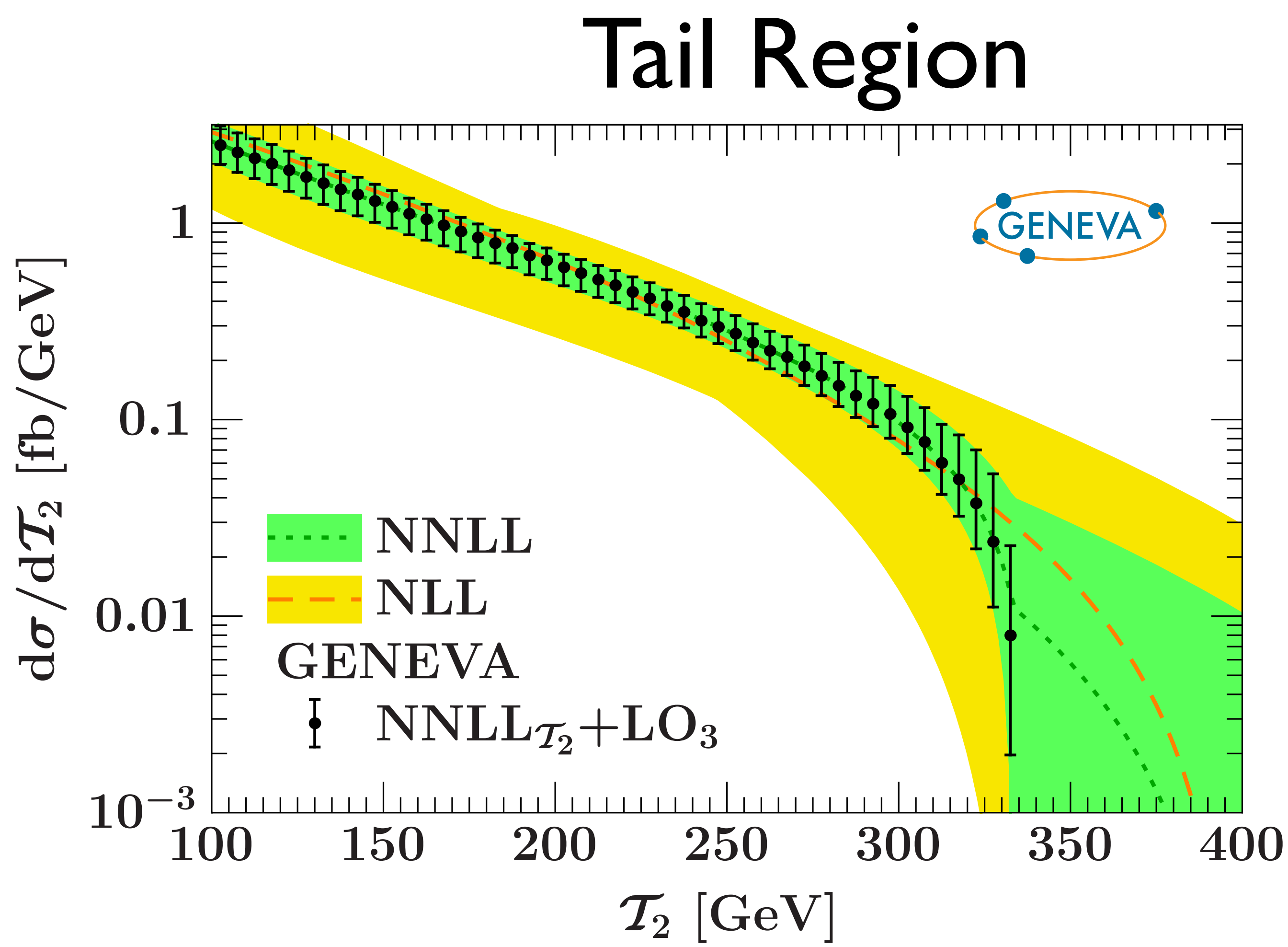
Cross Checks

Resolution Variable

Validating $\mathcal{T}_2$: NNLL+LO$_3$

- **Geneva exactly reproduces NNLL input.**
  Error bars are the event-by-event scale uncertainties.

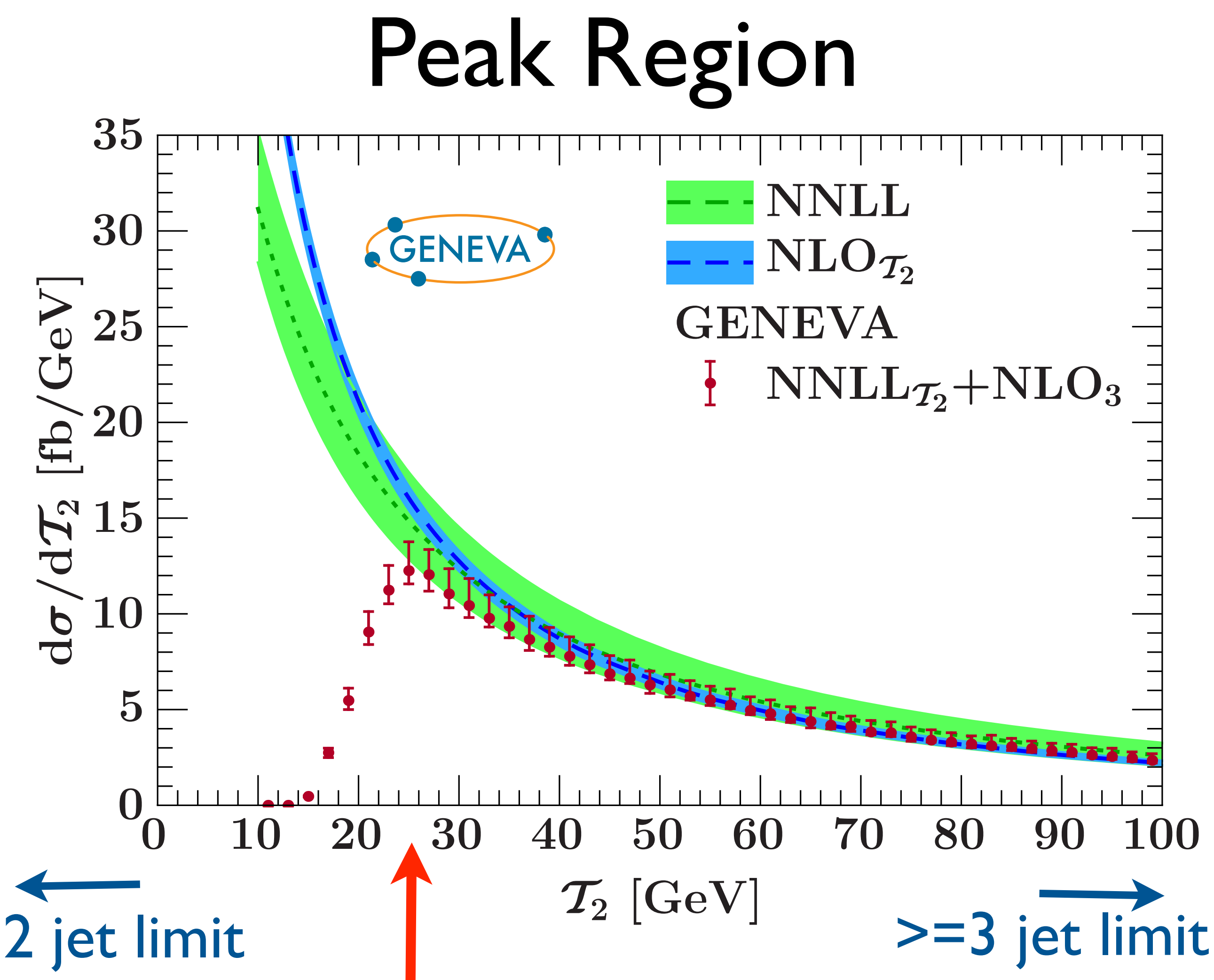


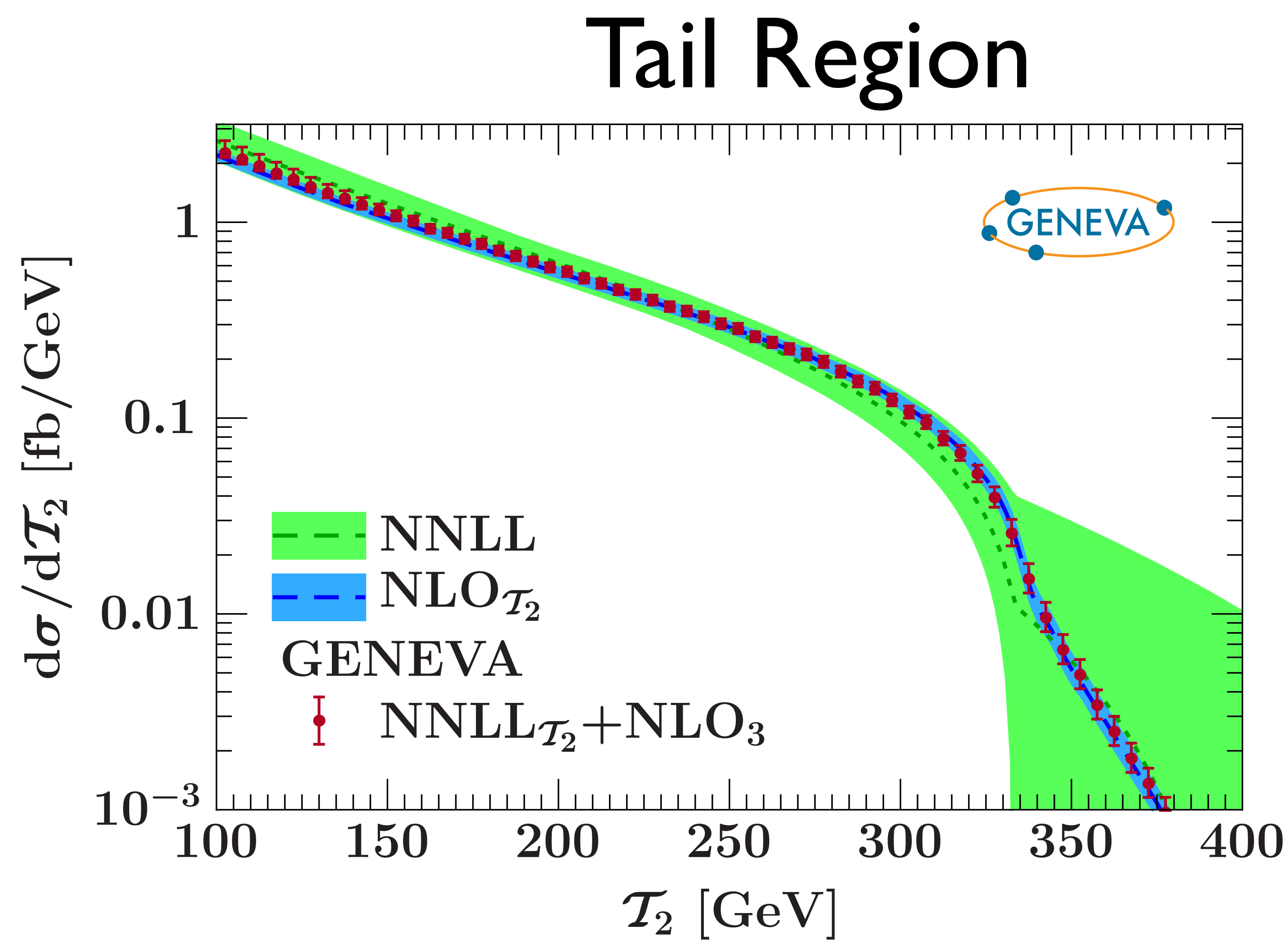


$$\frac{d\sigma_{\geq 3}}{d\Phi_3} = \left(\frac{d\sigma}{d\mathcal{T}_2} \Big/ \frac{d\sigma}{d\;\mathcal{T}_2}\Big|_{\rm exp} \right) \frac{d\sigma_{\geq 3}^{\rm LO}}{d\Phi_3}\, \theta(\mathcal{T}_2 > \mathcal{T}_{\rm cut})$$

Resolution Variable $\mathcal{T}_2$: NNLL+NLO3

- **Compare Geneva to best available prediction in each region.**





$\mathcal{T}_2^{\mathrm{cut}} = 20\,\mathrm{GeV}, \mathcal{T}_3^{\mathrm{cut}} = 5\,\mathrm{GeV}$

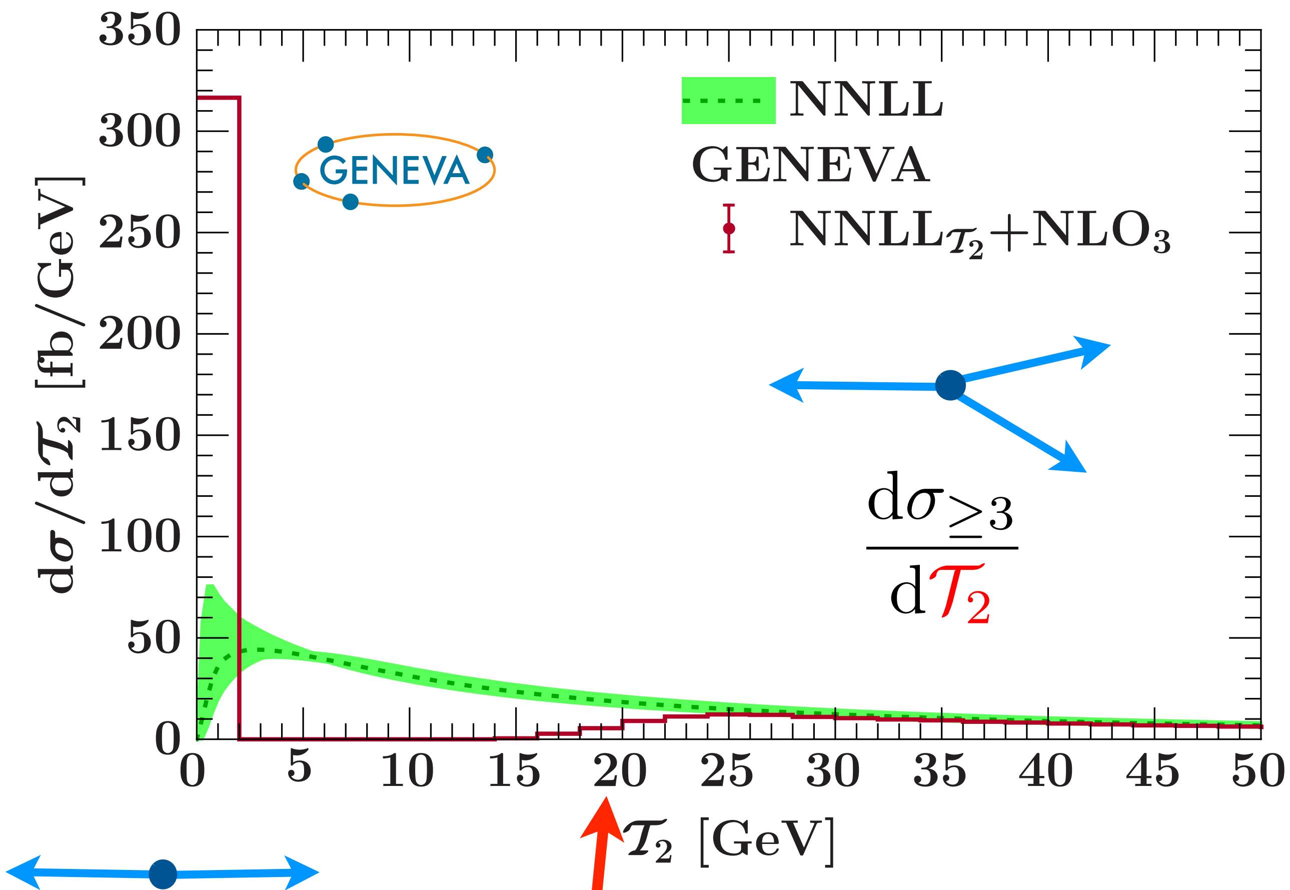
Smooth cut off implemented

$$\frac{d\sigma_{\geq 3}}{d\Phi_3} = \left(\frac{d\sigma}{d\mathcal{T}_2} \Big/ \frac{d\sigma}{d\,\mathcal{T}_2}\Big|_{\mathrm{exp}} \right) \frac{d\sigma_{\geq 3}^{\mathrm{NLO}}}{d\Phi_3}\, \theta(\mathcal{T}_2 > \mathcal{T}_{\mathrm{cut}})$$

Filling out Jets with Parton Shower

- **Effect of $\mathcal{T}_2^{\mathrm{cut}}$ removed smoothly after showering.**

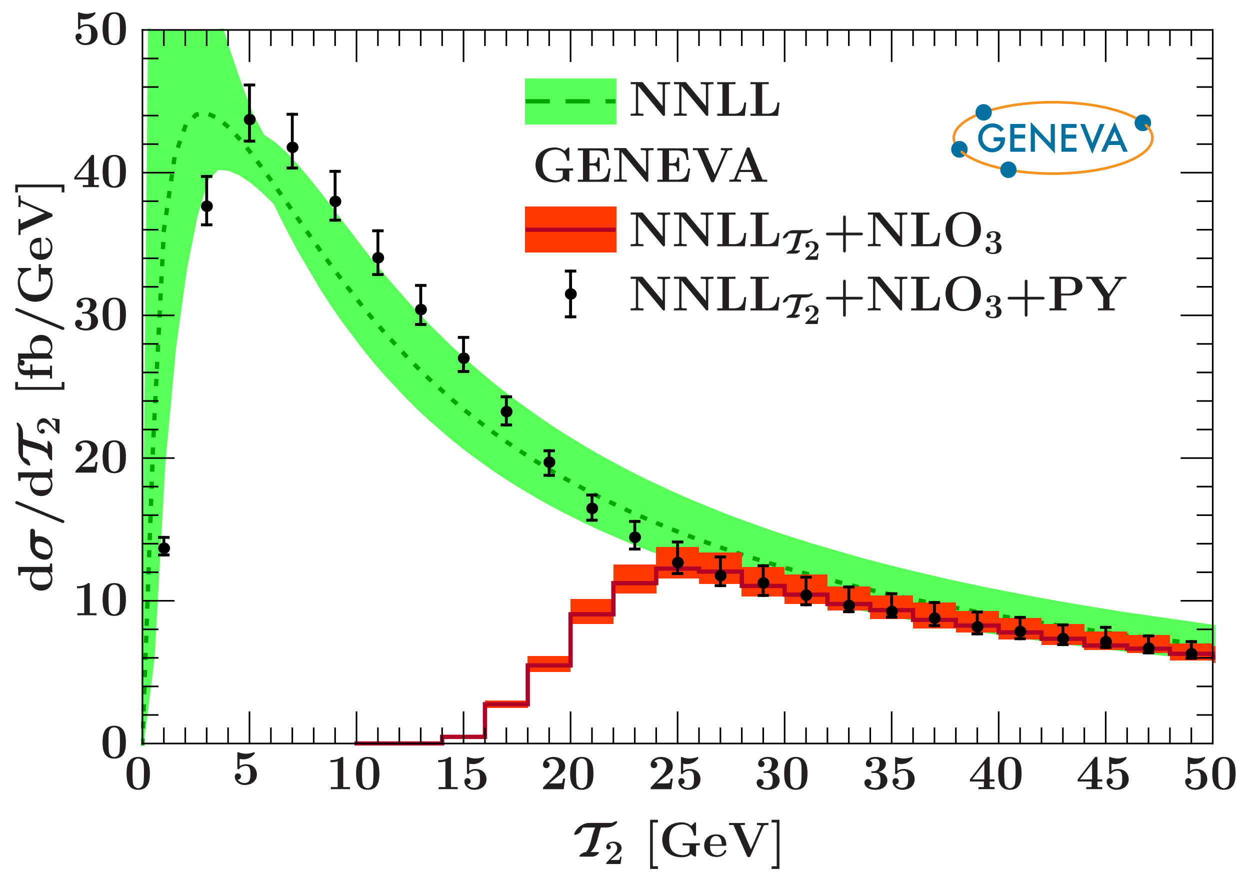
Peak Region without Pythia



$d\sigma_2(\mathcal{T}_2^{\mathrm{cut}})$

$\mathcal{T}_2^{\mathrm{cut}} = 20\,\mathrm{GeV}$,
$\mathcal{T}_3^{\mathrm{cut}} = 5\,\mathrm{GeV}$
Smooth cut off implemented

Peak Region with Pythia



***Convention**:
Red dots and histogram = NNLL+NLO$_3$
Black dots = NNLL+NLO$_3$+**PY**

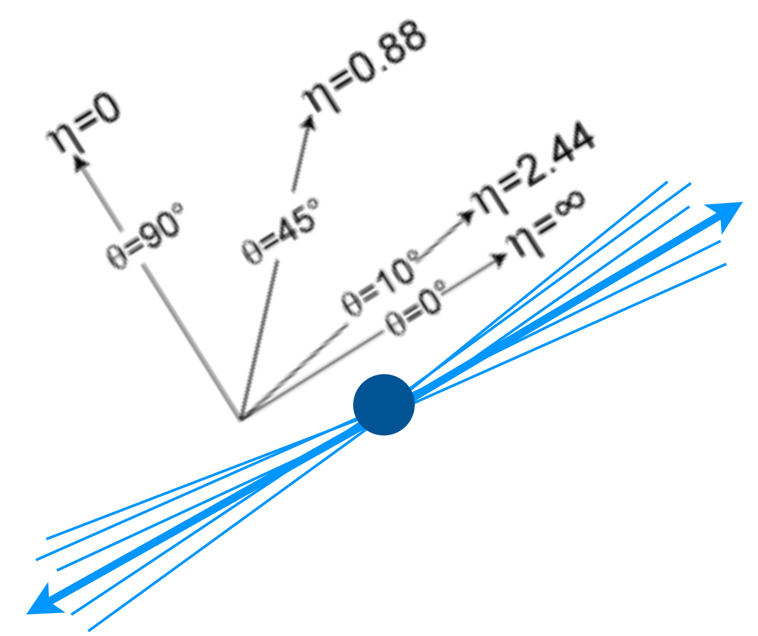
Results

Other Observables

Angularities

- Class of observables

$$\tau_a = \frac{1}{Q}\sum_{i\in X}|\mathbf{p_i^T}|\mathbf{e}^{-|\eta_\mathbf{i}|(\mathbf{1}-\mathbf{a})}$$



- Larger a weights particles closer to thrust axis more strongly.

Jet Broadening

$$a = 1$$

$$B = \tau_1/2$$

Thrust

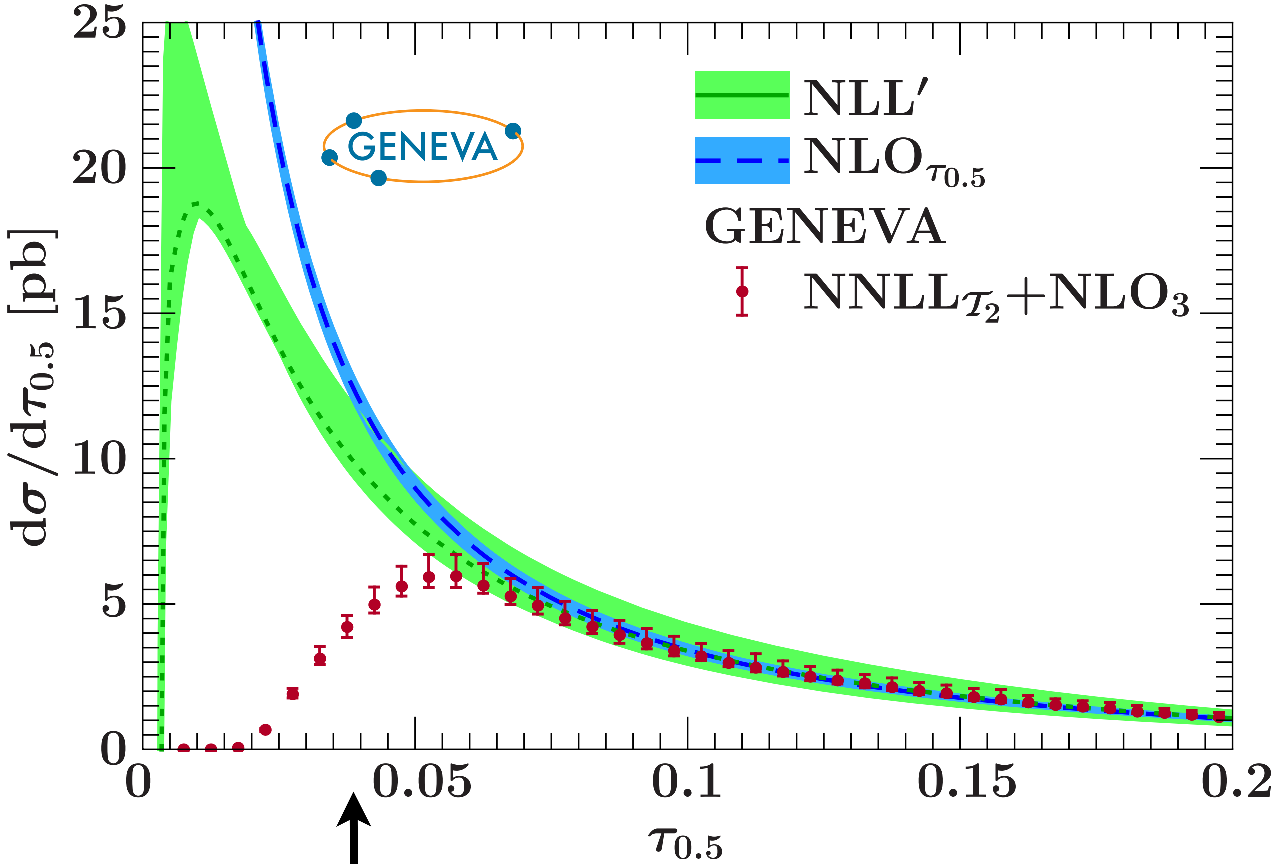
$$a = 0$$

$$\mathcal{T}_2 = 2\,Q\,\tau_0$$

$\tau_{0.5}$ Angularity

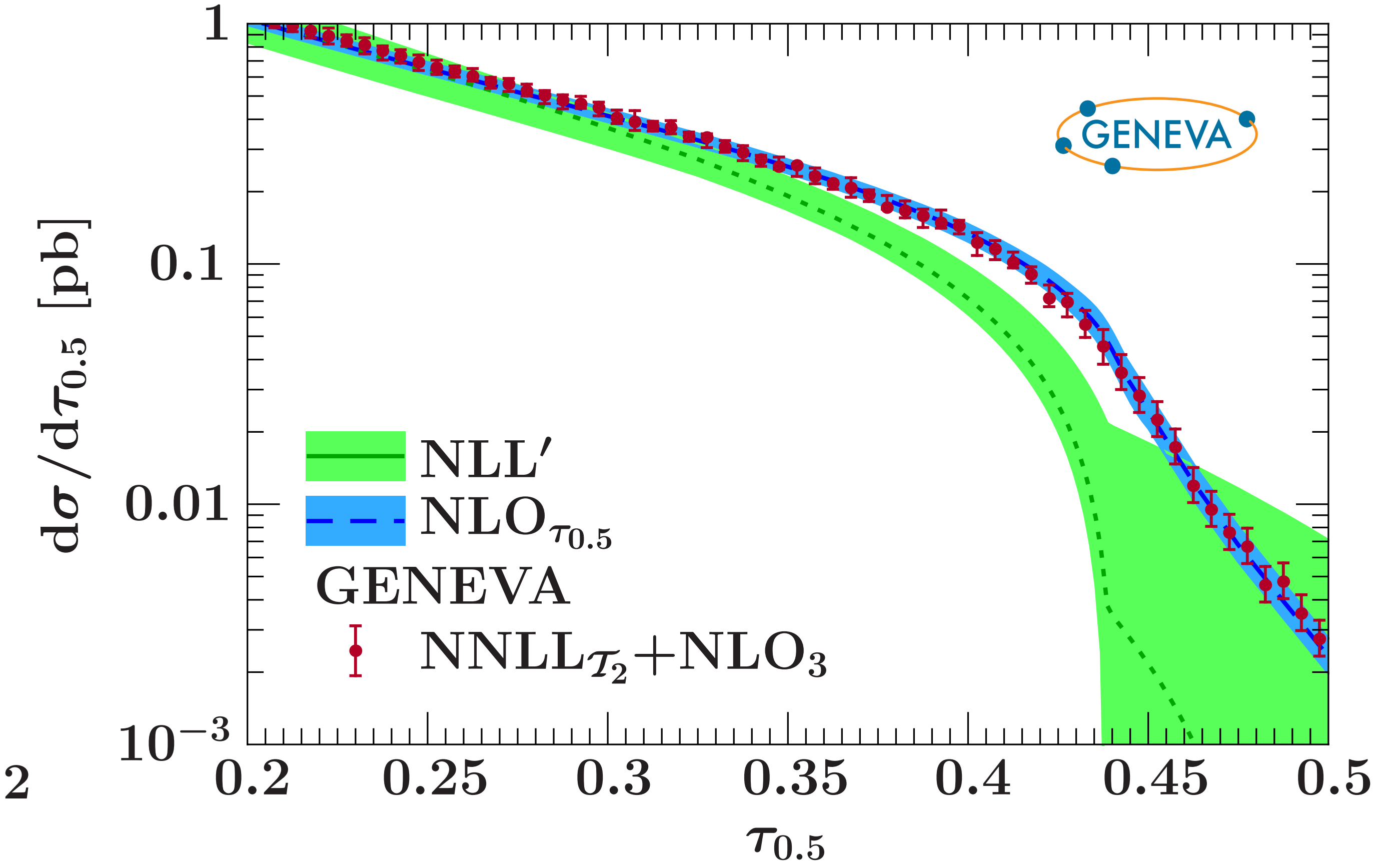
- **Away from tail, results consistent with NLL' resummation of** $\tau_{0.5}$

Peak Region



Effect of $\mathcal{T}_2^{\rm cut}$

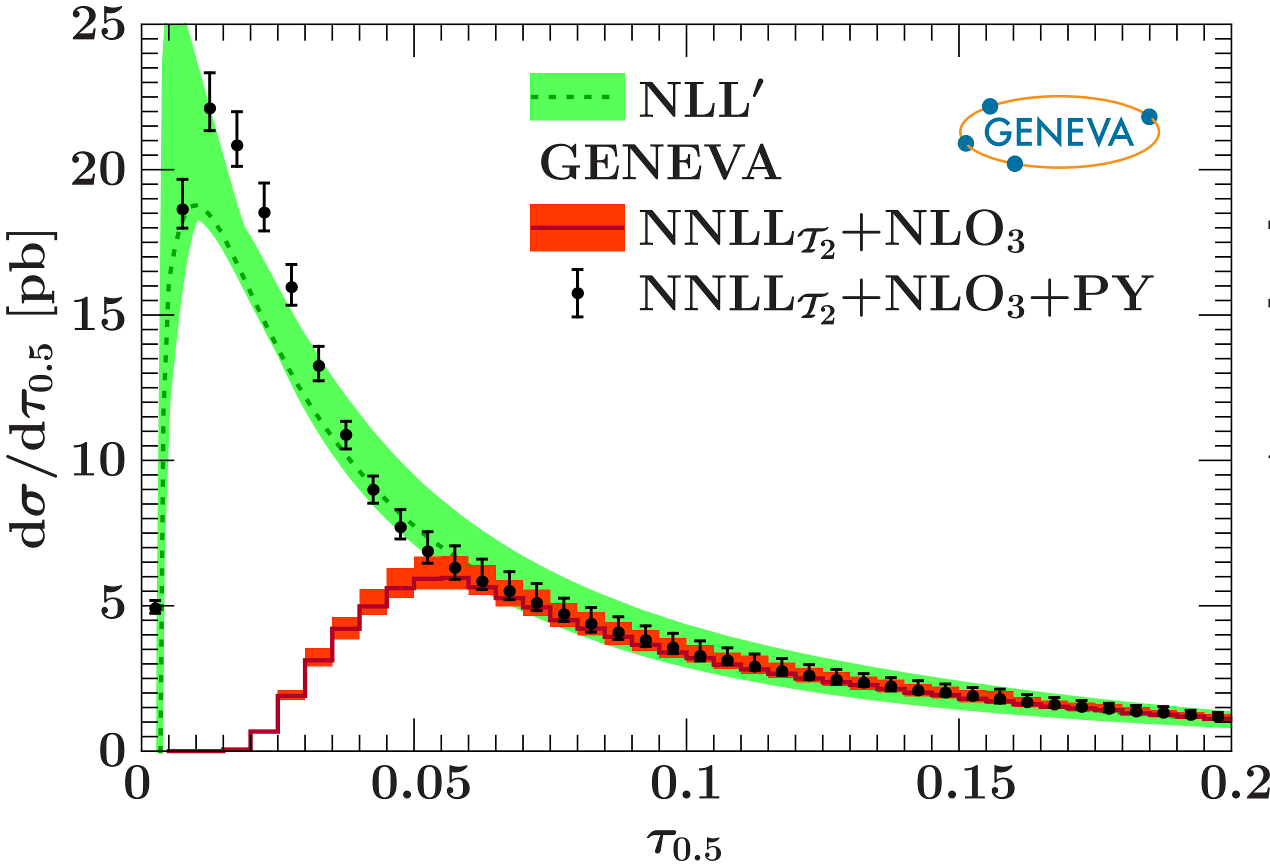
Tail Region



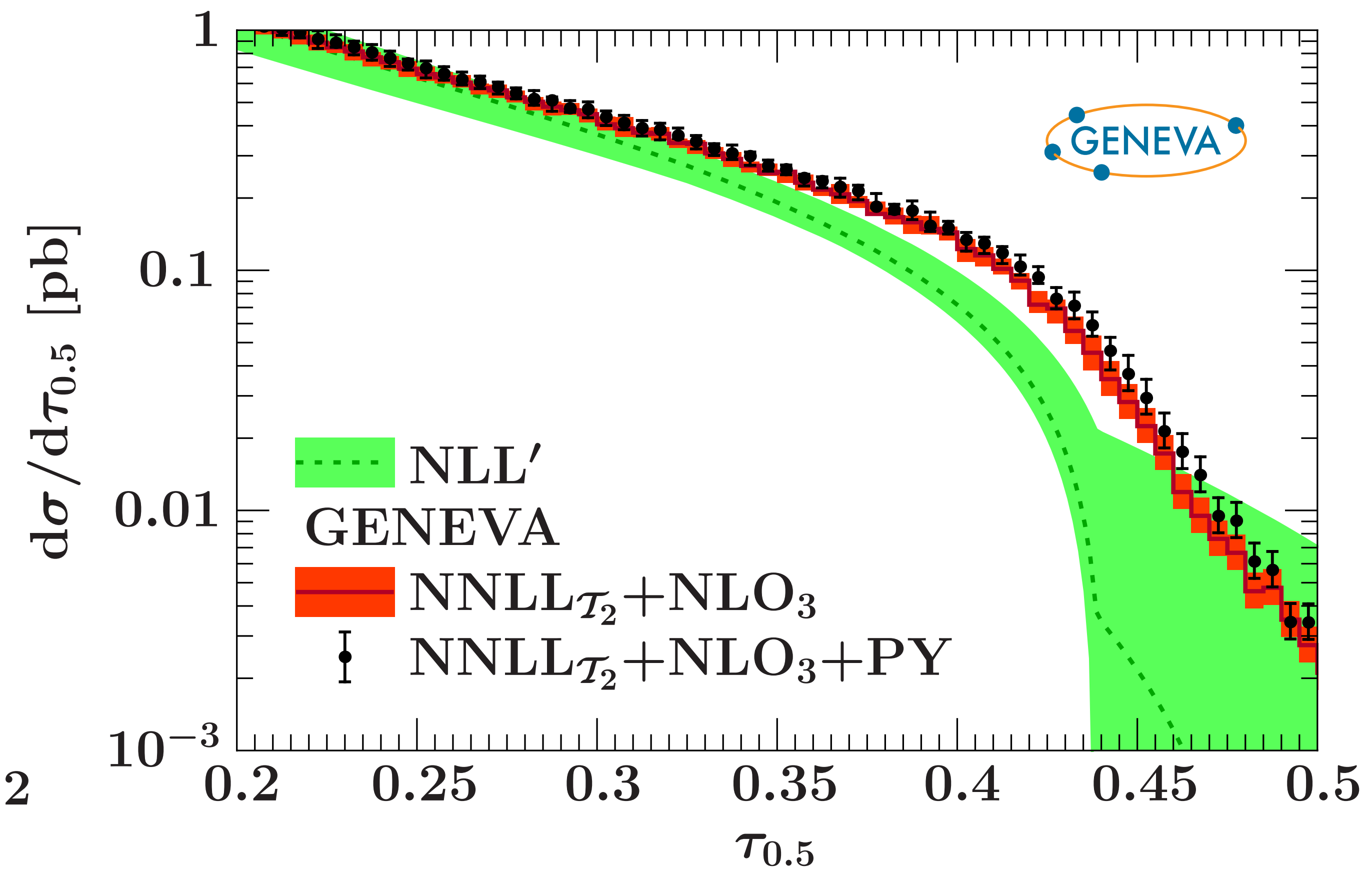
$\tau_{0.5}$ Angularity with Parton Shower

- **Adding the shower smoothly matches analytic NLL' resummation in peak and does not change the distribution in the tail.**

Peak Region



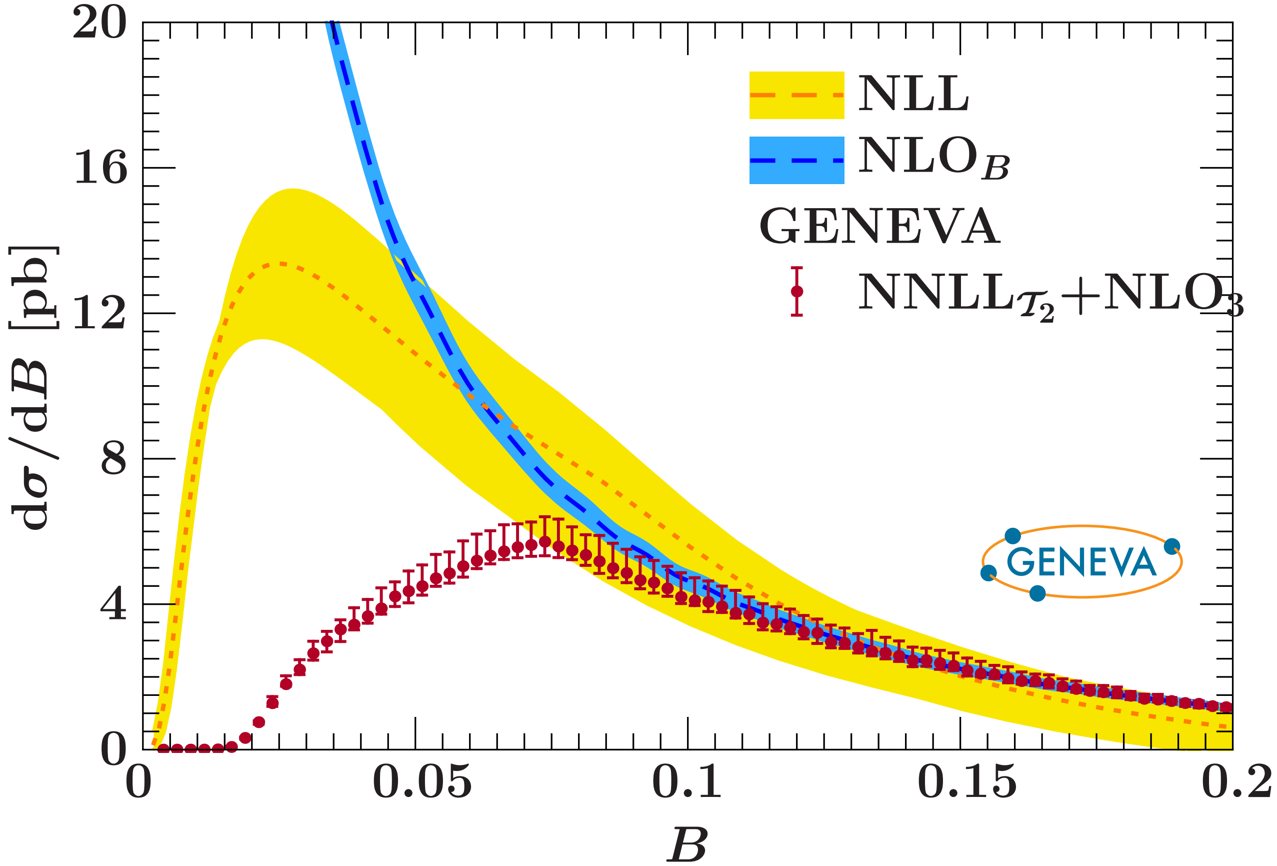
Tail Region



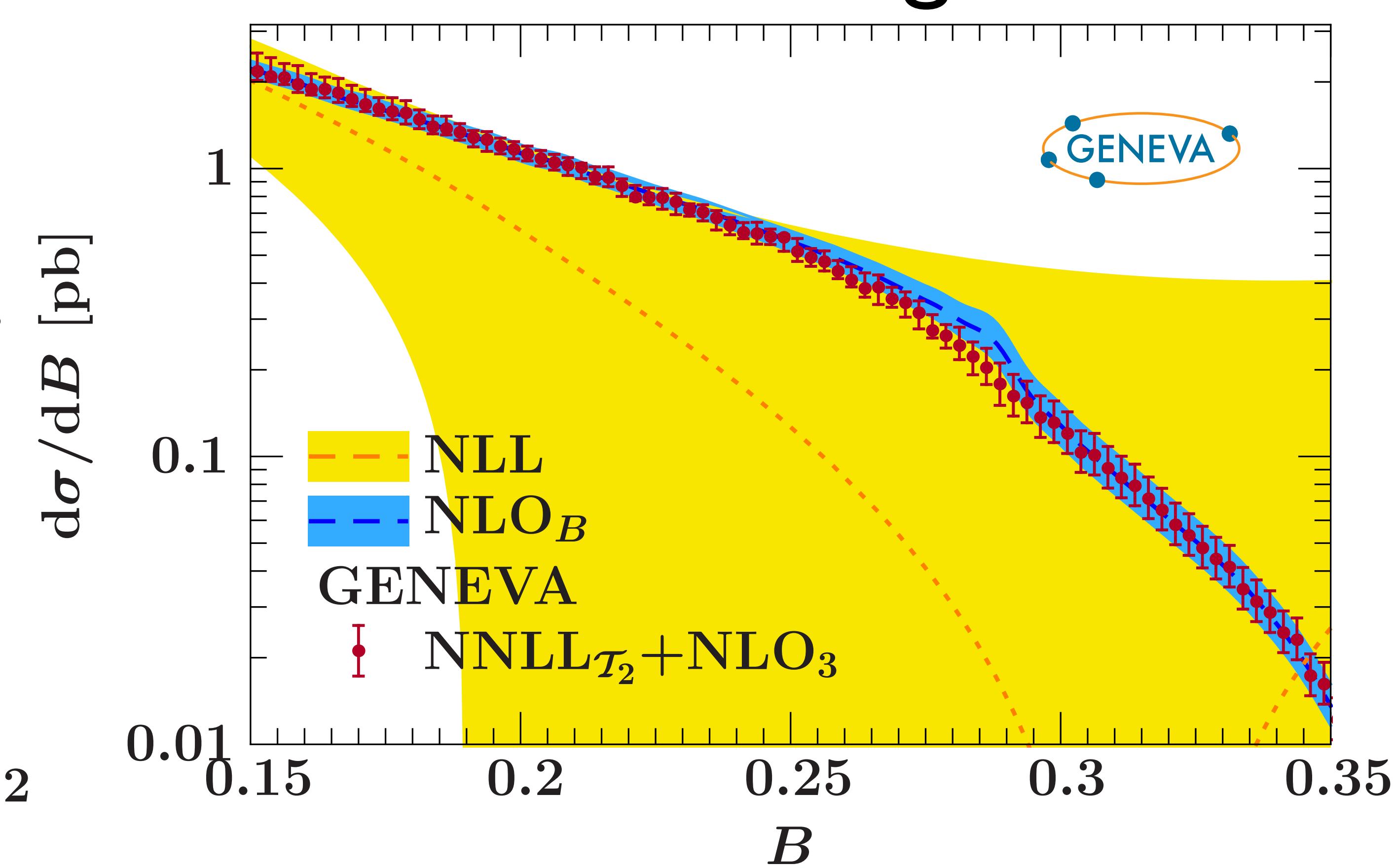
τ_1 Jet Broadening

- **QCD resummation structure of jet broadening is very different to thrust. Geneva is consistent with analytic NLL resummation of B= τ_1 /2**

Peak Region



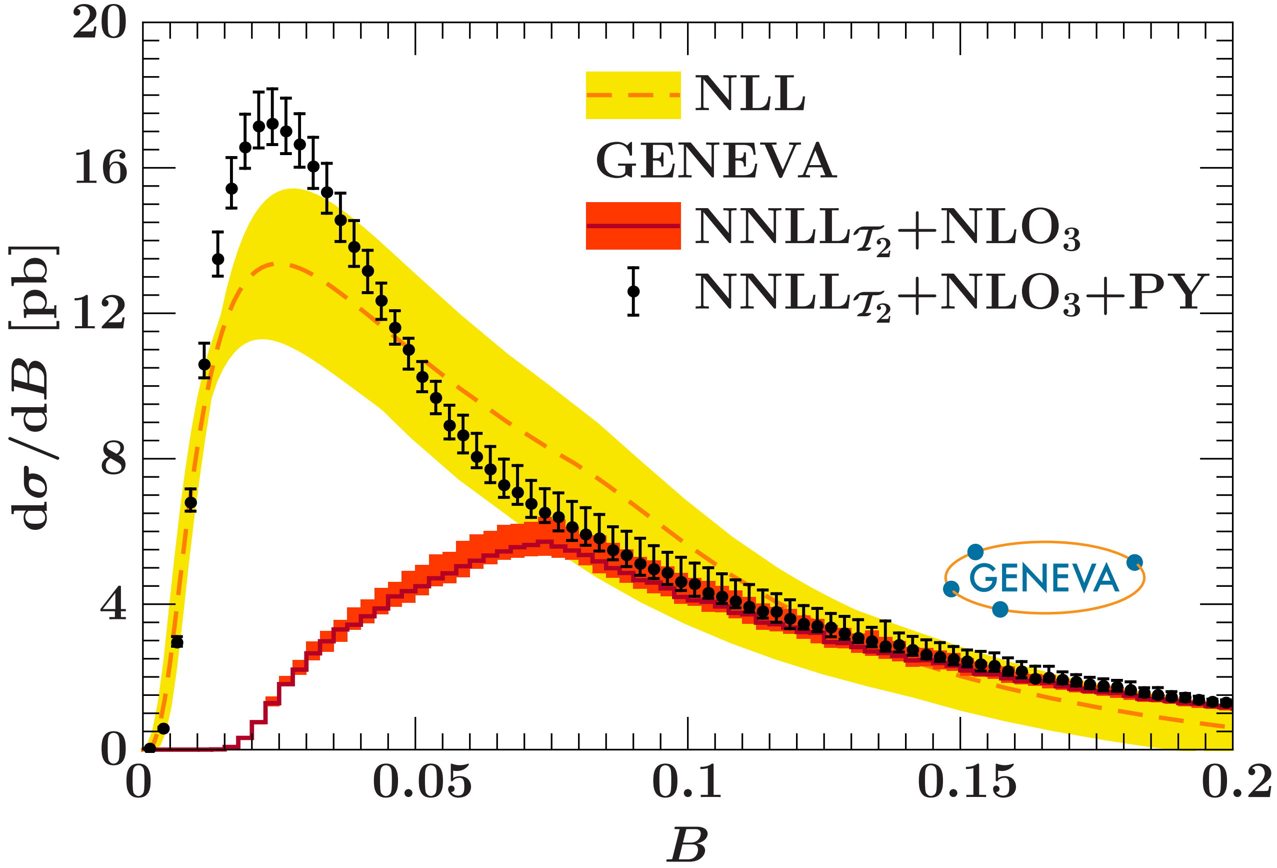
Tail Region



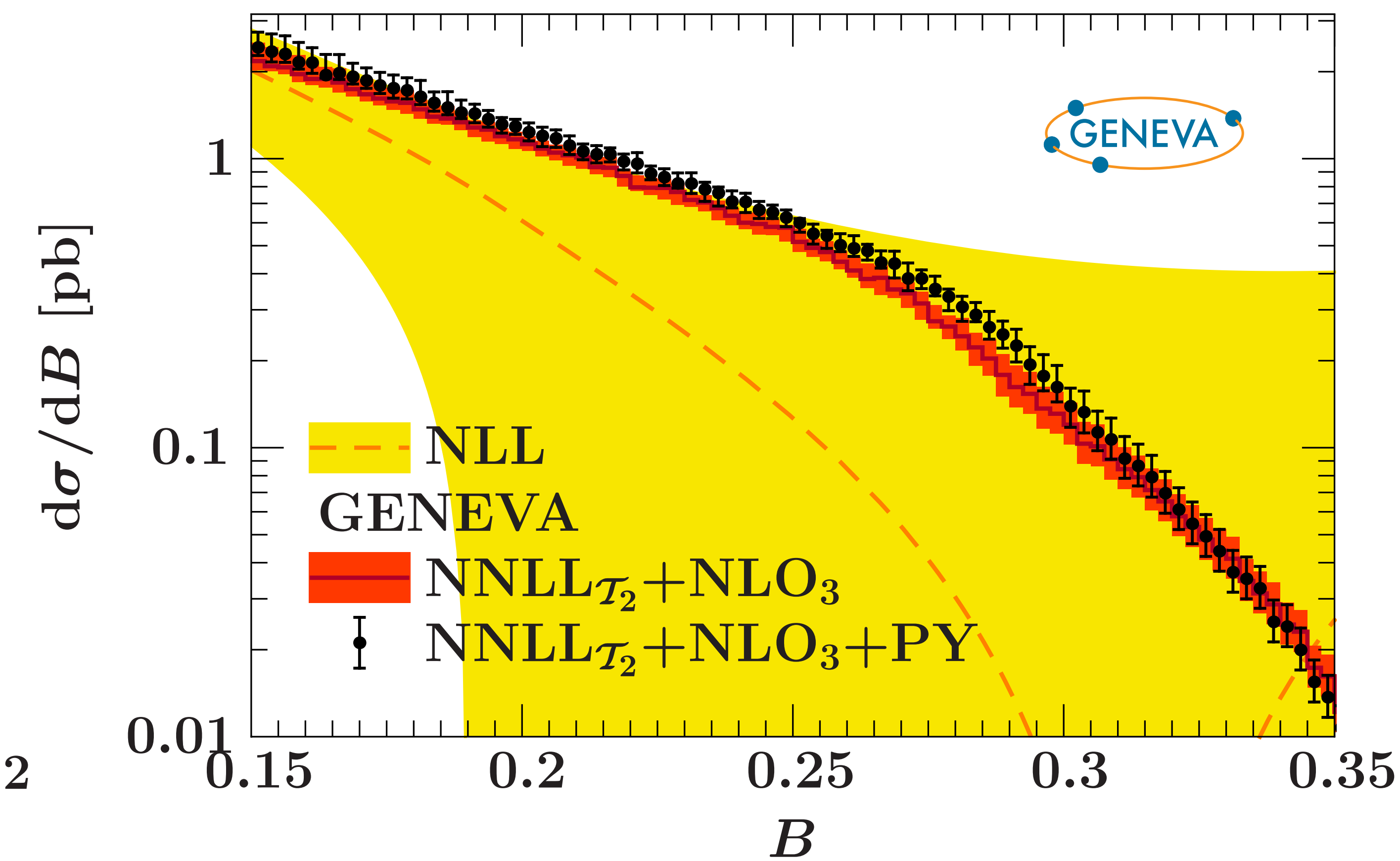
τ_1 Jet Broadening with Parton Shower

- **Adding shower removes $\mathcal{T}_2^{\rm cut}$ dependence smoothly**

Peak Region



Tail Region



Conclusions / Outlook (e^+e^-)

- e^+ e^- important milestone for Geneva:

  - Validates approach for combining jet multiplicities.

  - Demonstrates implementation of

    - Resummation of resolution variable,
    - Fixed order calculation,
    - Interfacing with parton shower.

  - Clear next steps:

    - Resummation of $\mathcal{T}_3^{\rm cut}$, comparison to data

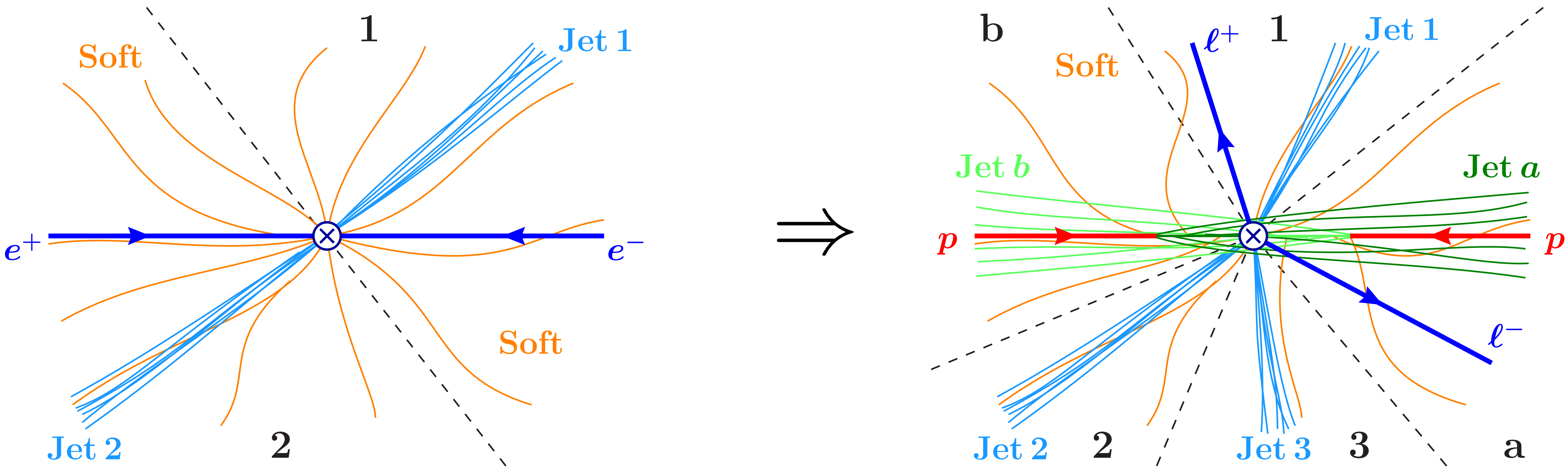
On to pp collisions!

- **Geneva prescription to separate N-jet bin σ_N and $\geq N+1$, $\sigma_{\geq N+1}$ analogous to e^+e^-**

$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{\mathrm{tot}}}{\mathrm{d}\Phi_N dx_a dx_b} &= \int_0^{\mathcal{T}_N^{\mathrm{cut}}} \mathrm{d}\mathcal{T}_N \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_N dx_a dx_b\, \mathrm{d}\mathcal{T}_N} + \\
&\int \mathrm{d}\Phi_{N+1} \frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_{N+1} dx_a dx_b} \delta\left(\bar{\Phi}_N - \Phi_N(\Phi_{N+1})\right) \theta\left(\mathcal{T}_N^{\mathrm{cut}} - \mathcal{T}_N(\Phi_{N+1})\right) \\
&= \underbrace{\frac{\mathrm{d}\sigma_N(\mathcal{T}_N^{\mathrm{cut}})}{\mathrm{d}\Phi_N}}_{Resummed\ matched\ to\ NLO} + \underbrace{\int_{\mathcal{T}_N^{\mathrm{cut}}} \frac{\mathrm{d}\sigma_{\geq N+1}}{\mathrm{d}\Phi_{N+1}}}_{\text{“}Resummation\ improved\text{“}\ NLO}
\end{aligned}
$$

- **Needs resummed for $\mathcal{T}_N < \mathcal{T}_N^{\mathrm{cut}}$ and resummation improved NLO for $\mathcal{T}_N > \mathcal{T}_N^{\mathrm{cut}}$**

Slide credit: Simone Alioli



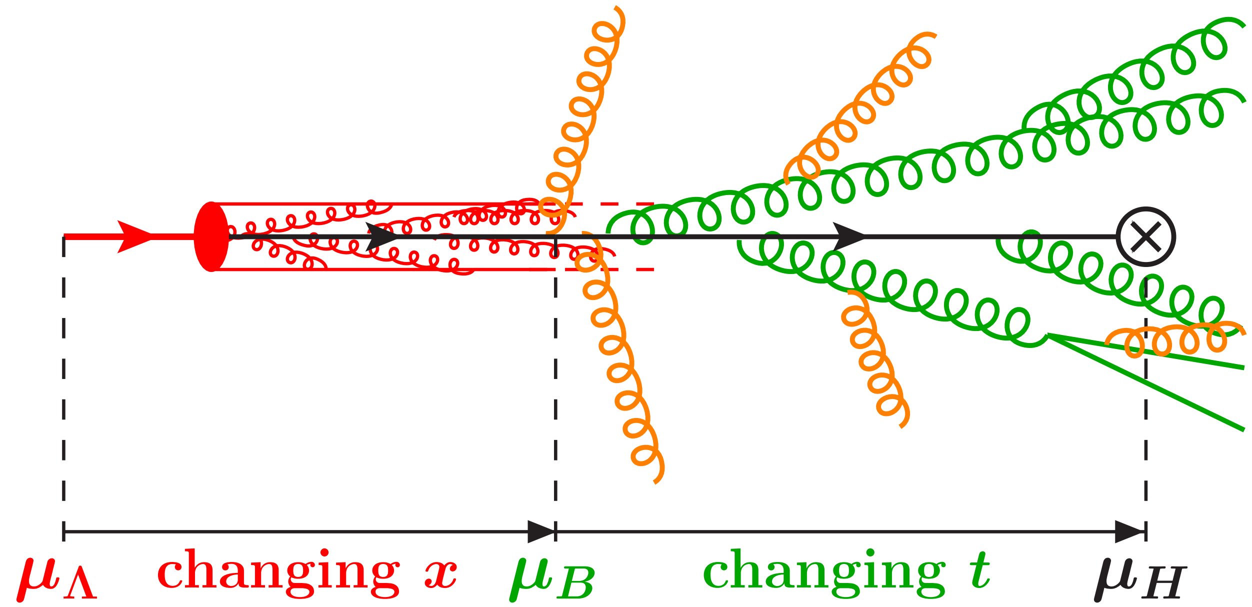
- N-jettiness resolution parameter, straightforward extension for beams directions

$$\mathcal{T}_N = \frac{2}{Q^2}\sum_k \min\{q_1 \cdot p_k, \ldots, q_N \cdot p_k\} \quad \Rightarrow \quad \mathcal{T}_N = \frac{2}{Q^2}\sum_k \min\{q_a \cdot p_k, q_b \cdot p_k, q_1 \cdot p_k, \ldots, q_N \cdot p_k\}$$

- N-jettiness has good factorization properties. IR safe and resummable at all orders.

Beam-Thrust resummation via SCET

$\mathcal{T}_B \equiv \mathcal{T}_0$



- **The factorized beam thrust formula reads**

$$\frac{d\sigma^s}{dx_a dx_b\, d\mathcal{T}_B} = \sigma_B \cdot H(\mu_H) \otimes U_H(\mu_H, \mu) \cdot B(x_a, \mu_{B_a}) \otimes U_B(\mu_{B_a}, \mu)$$
$$\otimes B(x_b, \mu_{B_b}) \otimes U_B(\mu_{B_b}, \mu) \otimes S(\mathcal{T}_B, \mu_S) \otimes U_S(\mu_S, \mu)$$

- **Beam functions connected to PDF via OPE in SCET**

$$B_i(t, x; \mu_B) = \sum_k \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ik}\left(t, \frac{x}{\xi}; \mu_B\right) f_k(\xi; \mu_B)$$

- **Soft S, Beam B and Jet J functions are all known and available in the literature at NNLL, together with the Hard H function. They are all calculable in SCET.**
- **Evolution kernels U are obtained by RGE running at NNLL.**

One subtlety:

$$\frac{d\sigma_{\geq N+1}}{d\Phi_{N+1}} = \left(\frac{d\sigma}{d\mathcal{T}_N} \Big/ \frac{d\sigma}{d\mathcal{T}_N}\Big|_{\text{exp}} \right) \frac{d\sigma^{\text{FO}}}{d\Phi_{N+1}}$$

All three components involve PDFs!

-> Need to integrate over PDFs in numerator *and* denominator of resummation ratio. Essentially an integral over beam functions, which can be implemented via lookup tables.

Status of Geneva for hadronic collisions

- ✓ Analytic resummation of Beam-thrust.
- ✓ Implemented general subtraction method, for a generic QCD NLO calculation.
- ✓ Interface to Madgraph for automatic generation of tree level amplitudes
- ✓ Interface to automatic virtuals following Les Houches accord.
- Needs to be completed:
  - Ongoing validation of W, Z and W, Z plus a jet production at NLO.
  - Interface to parton showering is next major step. Strategies and experience in validating FSR will be directly applicable to ISR too.
- Stay tuned and expect first results soon ...

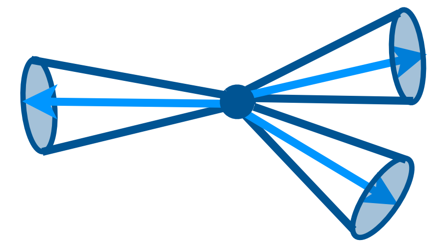
Thank you!

Bonus slides

Bonus slides

NLO$_3$ Calculation

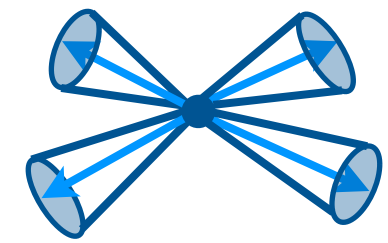
$$\frac{d\sigma^{NLO}_{\geq 3}}{d\Phi_3} = \frac{d\sigma^{NLO}_{3}}{d\Phi_3} + \frac{d\sigma^{LO}_{\geq 4}}{d\Phi_4} =$$



$\mathcal{T}_2 > \mathcal{T}_2^{cut}$

$\mathcal{T}_3 < \mathcal{T}_3^{cut}$

$+$



$\mathcal{T}_2 > \mathcal{T}_2^{cut}$

$\mathcal{T}_3 > \mathcal{T}_3^{cut}$

$+ \cdots$

- Evaluating the integral over 4 body phase space involves a mapping:

$$\Phi_4 \leftrightarrow \Phi_3^{NLO} \times \Phi_{rad}$$

[Frixione, Kunszt, Signer]

$$\frac{d\sigma_3^{NLO}}{d\Phi_3} \supset \int d\Phi_4 B_4(\Phi_4)\delta[\Phi_3 - \Phi_3^{NLO}(\Phi_4)]\theta(\mathcal{T}_3^{cut} - \mathcal{T}_3)$$

$$\int d\Phi_{rad} B_4\big(\Phi_4(\Phi_3, \Phi_{rad})\big)\theta(\mathcal{T}_3^{cut} - \mathcal{T}_3)$$

- This mapping defines $\Phi_3^{NLO} = \{\Phi_2, \mathcal{T}^{Map}{}_2, \text{jet splitting angles}\}$
- Recall: Must resum resolution variable $\mathcal{T}_2$. Requires $\mathcal{T}_3^{cut} \ll \mathcal{T}_2$

A Subtlety: Tau_2 Definitions

- Given a 3 parton configuration. Non-trivial to add a splitting while holding Tau_2 fixed. $$\Phi_4 \leftrightarrow \Phi_3^{\mathrm{NLO}} \times \Phi_{\mathrm{rad}}$$

- FKS Map gives Φ_3^{NLO} with Tau_2^{FKS} which differs from Tau_2 calculated exactly on Φ_4 configuration.

- Invert a modified definition of Tau_2 : Fully Recursive Tau_2^{FR} defined by a metric $d_{ij} = |\mathbf{p}_i| + |\mathbf{p}_j| - |\mathbf{p}_{i+}\mathbf{p}_j|$

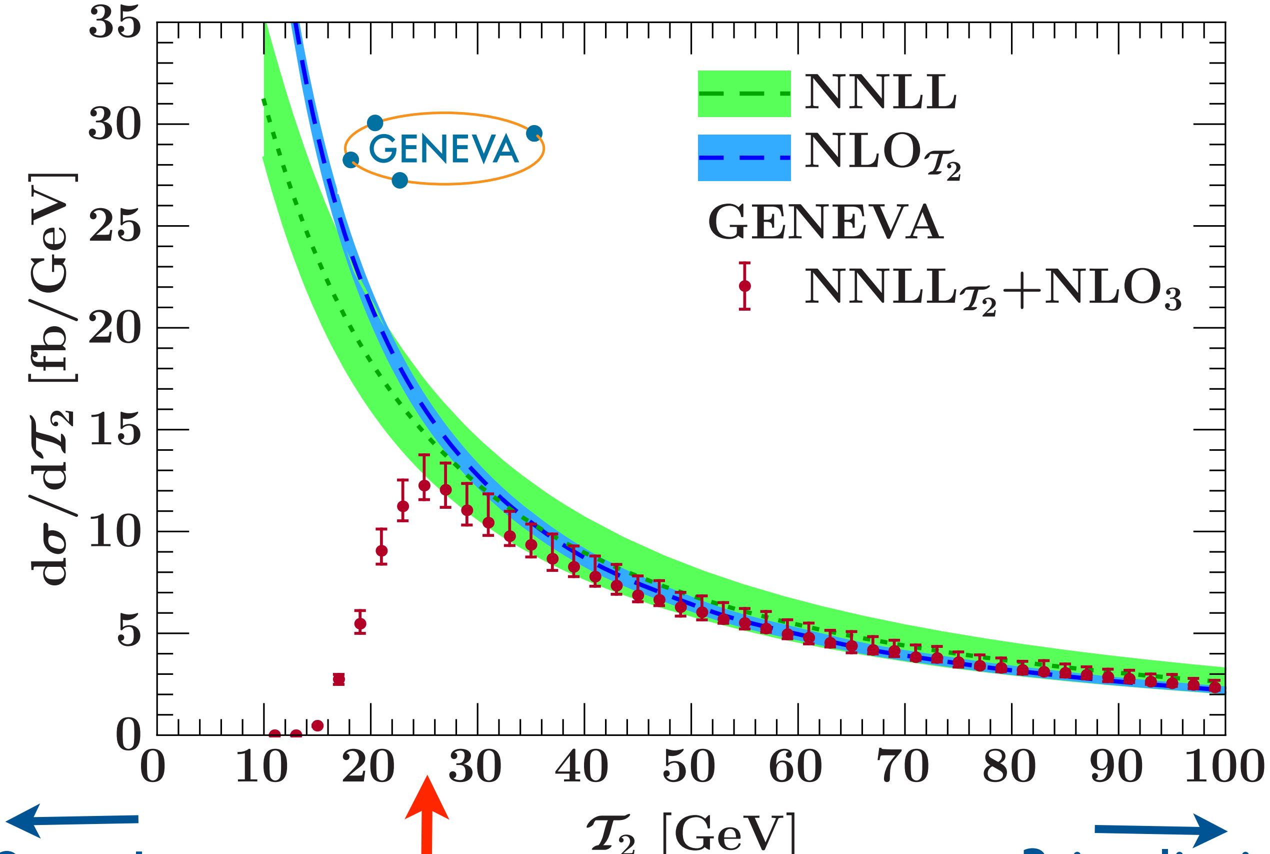
- Same up to $O(\alpha_s^2)$ as Tau_2 numerically. Size of power corrections dramatically smaller.

- Implement mapping that keeps Tau_2^{FR} fixed in FKS subtraction.

Resolution Variable $\mathcal{T}_2$: NNLL+NLO3

- **Compare Geneva to best available prediction in each region.**

Peak Region



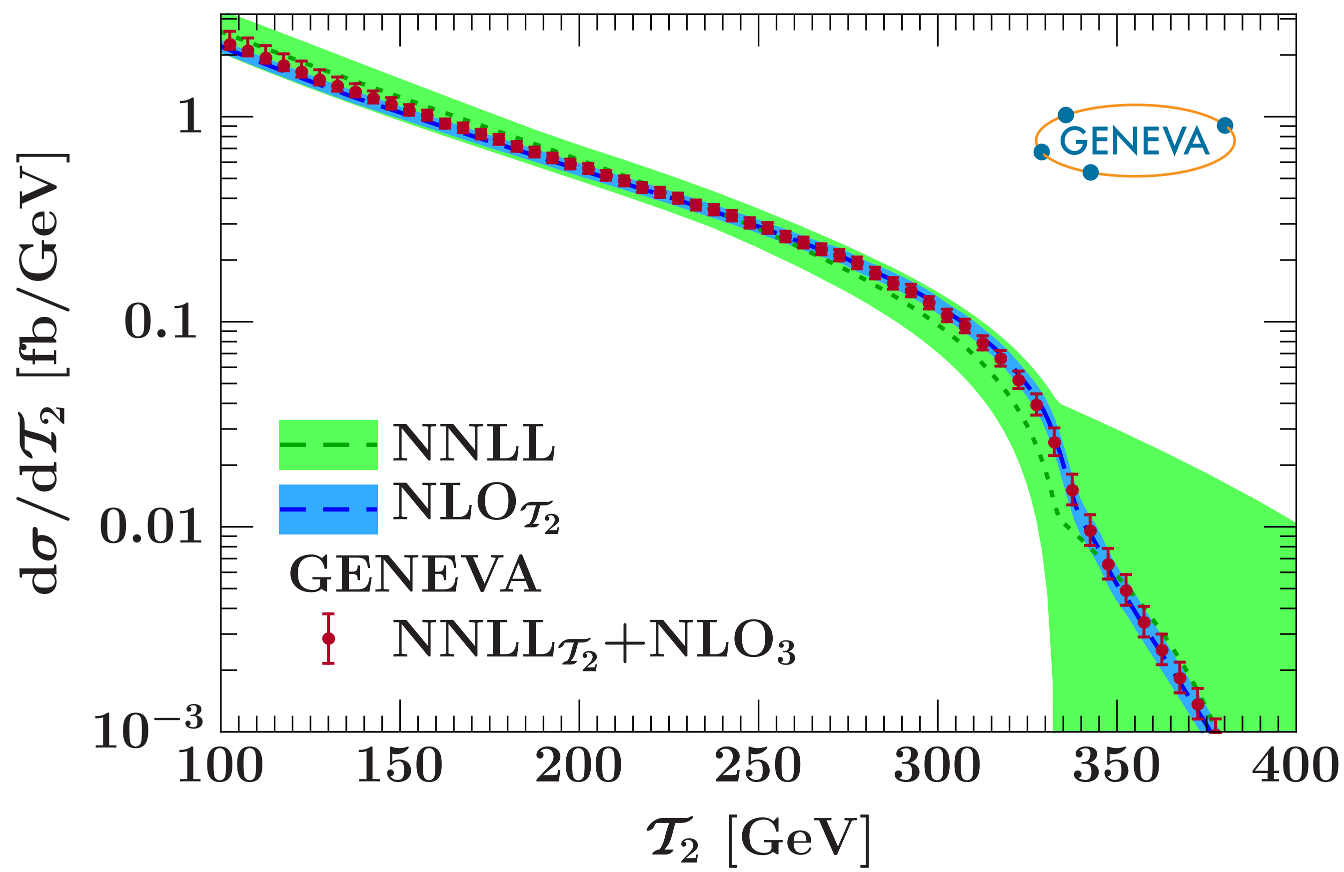
2 jet limit

\>=3 jet limit

$\mathcal{T}_2^{\mathrm{cut}} = 20\,\mathrm{GeV}, \mathcal{T}_3^{\mathrm{cut}} = 5\,\mathrm{GeV}$

Smooth cut off implemented

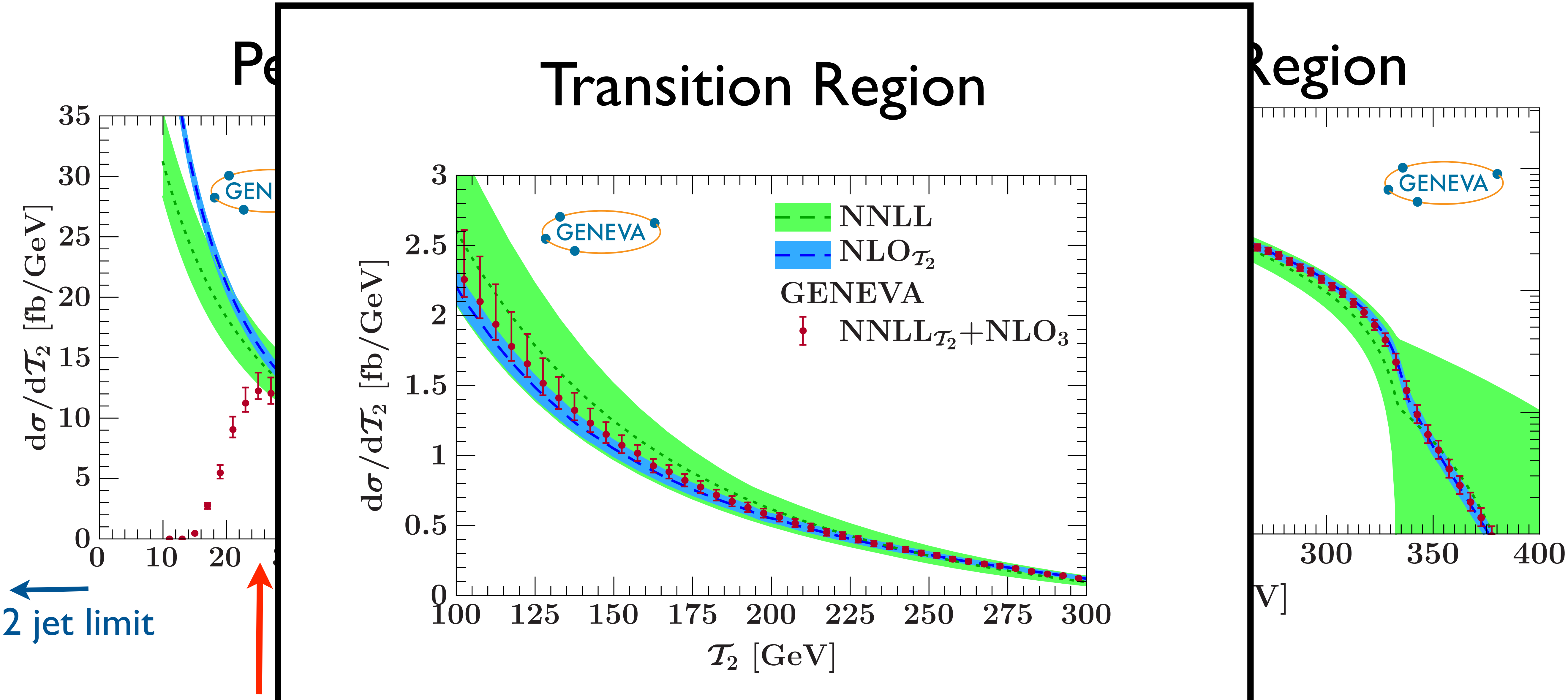
Tail Region



$$\frac{d\sigma_{\geq 3}}{d\Phi_3} = \left(\frac{d\sigma}{d\mathcal{T}_2} \Big/ \frac{d\sigma}{d\,\mathcal{T}_2}\Big|_{\mathrm{exp}} \right) \frac{d\sigma_{\geq 3}^{\mathrm{NLO}}}{d\Phi_3}\, \theta(\mathcal{T}_2 > \mathcal{T}_{\mathrm{cut}})$$

Resolution Variable $\mathcal{T}_2$: NNLL+NLO$_3$

- **Compare Geneva to best available prediction in each region.**



$\mathcal{T}_2^{\mathrm{cut}} = 20\,\mathrm{GeV}, \mathcal{T}_3^{\mathrm{cut}} = 5\,\mathrm{GeV}$

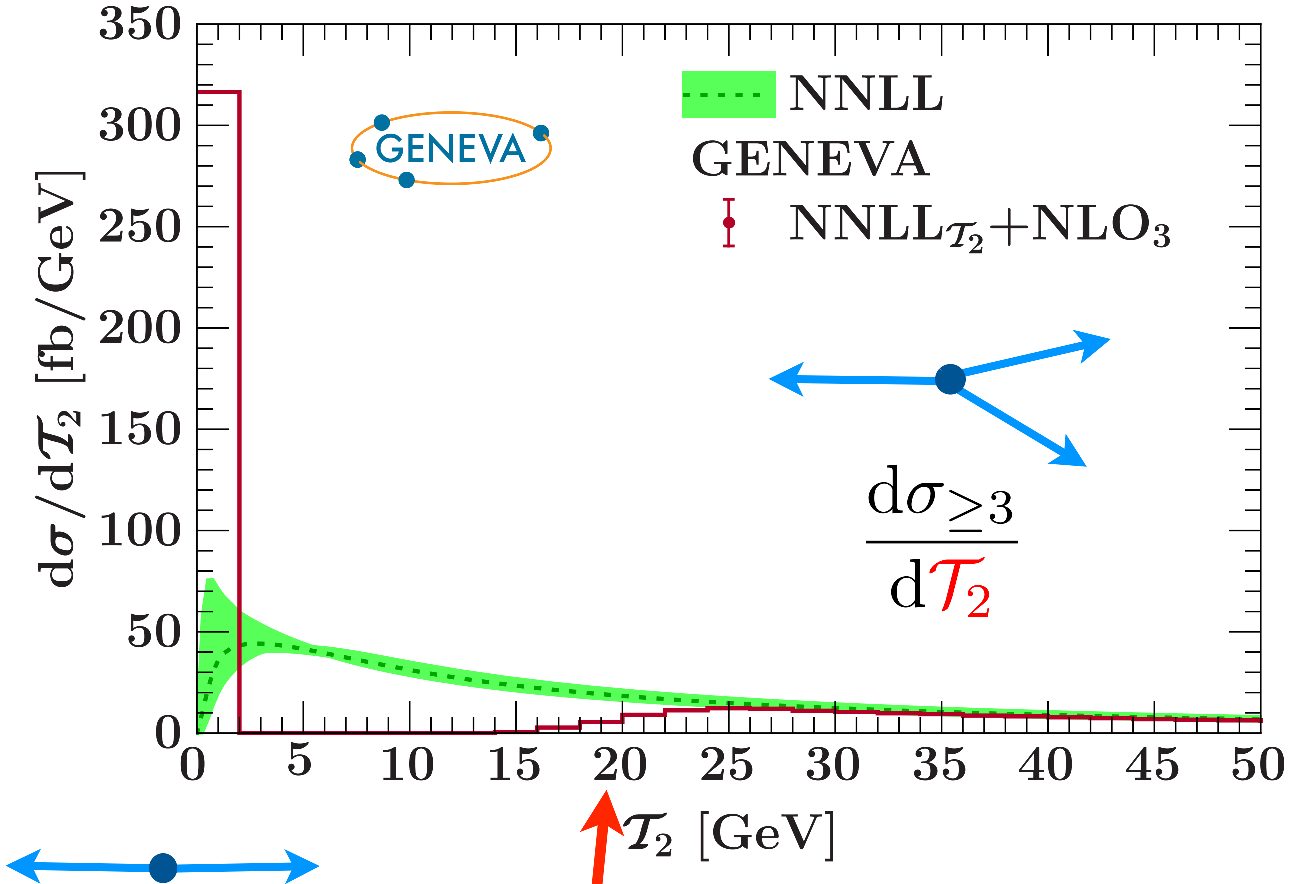
Smooth cut off implemented

$$\frac{\mathrm{d}\sigma_{\geq 3}}{\mathrm{d}\Phi_3} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\mathcal{T}_2} \Big/ \frac{\mathrm{d}\sigma}{\mathrm{d}\ \mathcal{T}_2}\Big|_{\mathrm{exp}} \right) \frac{\mathrm{d}\sigma_{\geq 3}^{\mathrm{NLO}}}{\mathrm{d}\Phi_3}\ \theta(\mathcal{T}_2 > \mathcal{T}_{\mathrm{cut}})$$

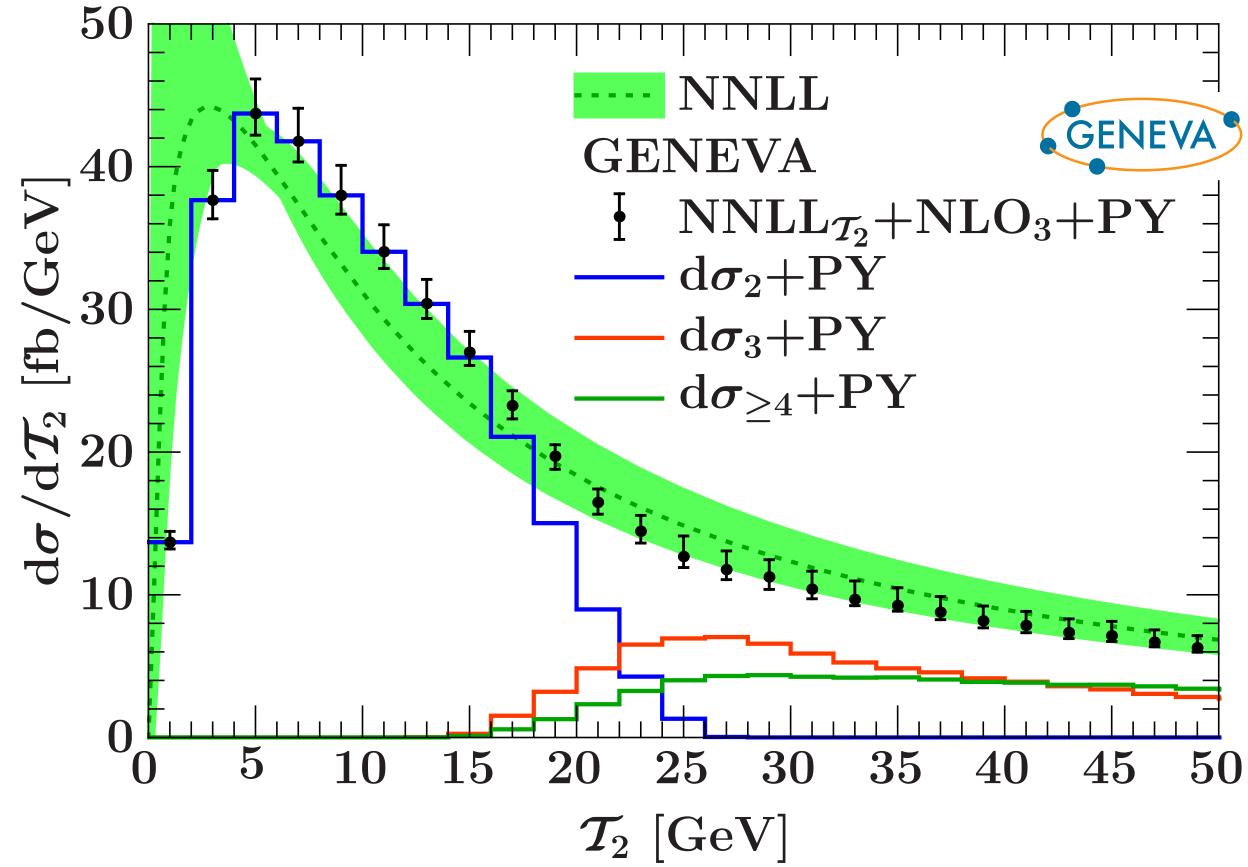
Filling out Jets with Parton Shower

- **Effect of $\mathcal{T}_2^{\mathrm{cut}}$ removed smoothly after showering.**

Peak Region without Pythia



Peak Region with Pythia



$d\sigma_2(\mathcal{T}_2^{\mathrm{cut}})$

$\mathcal{T}_2^{\mathrm{cut}} = 20\,\mathrm{GeV},$

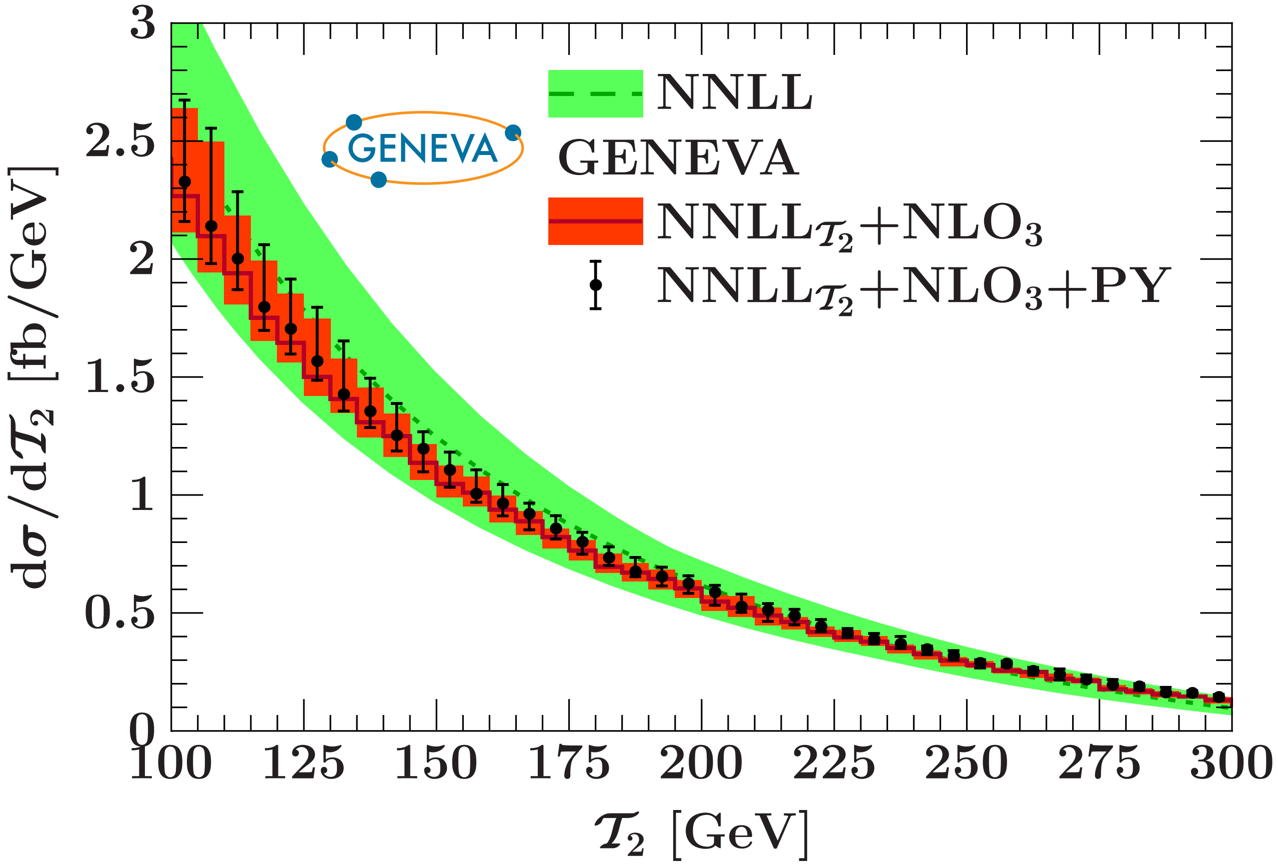
$\mathcal{T}_3^{\mathrm{cut}} = 5\,\mathrm{GeV}$

Smooth cut off implemented

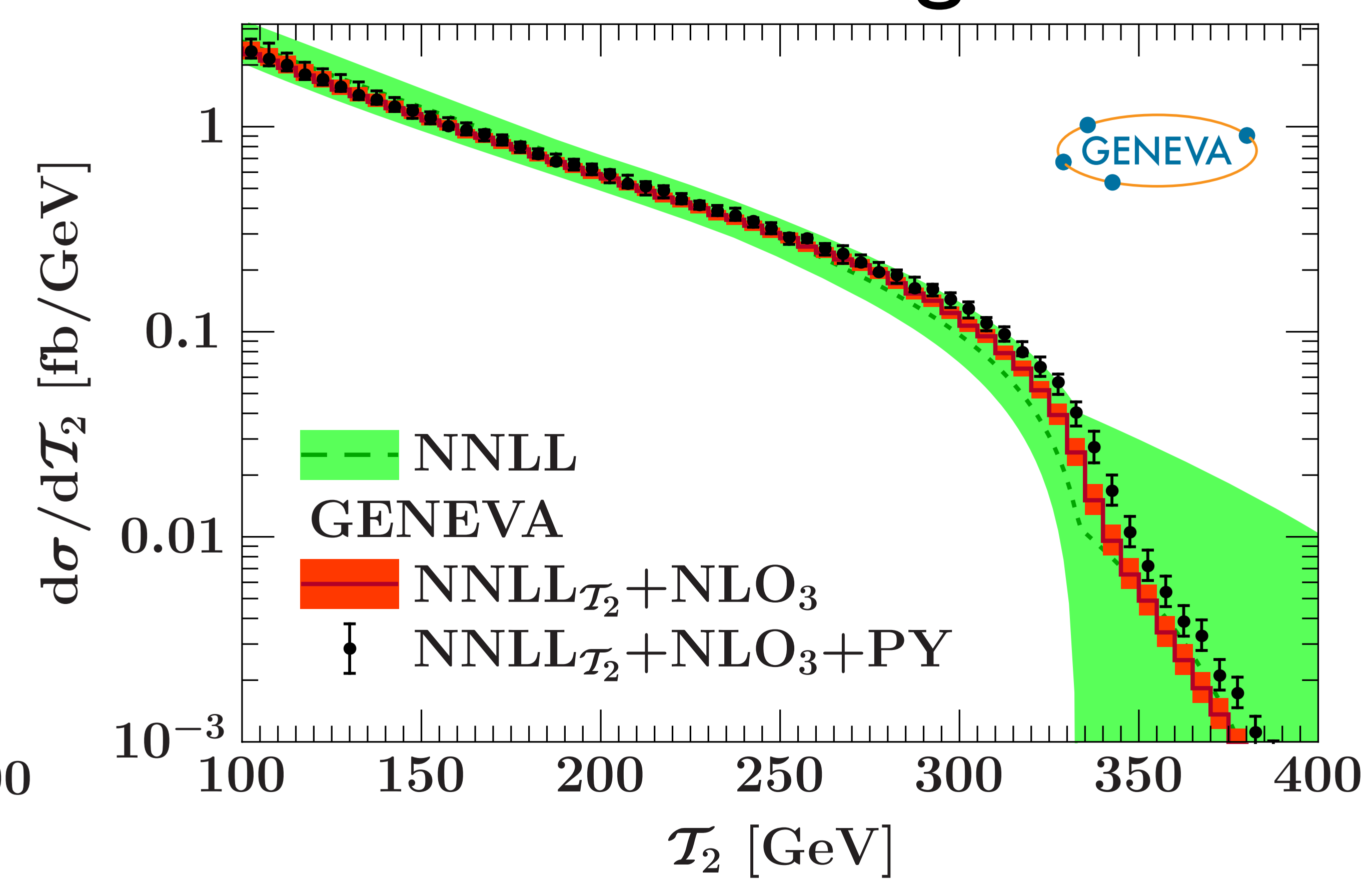
Resolution Variable $\mathcal{T}_2$ with Parton Shower

- **Compare Geneva results before and after showering**

Transition Region



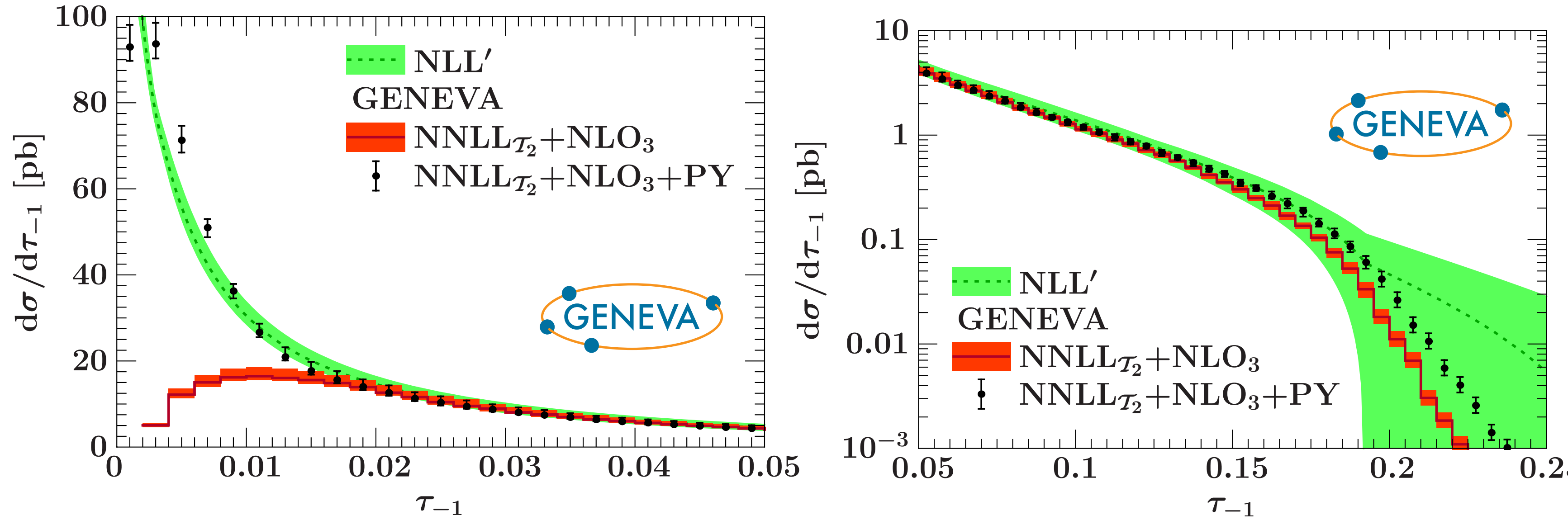
Tail Region



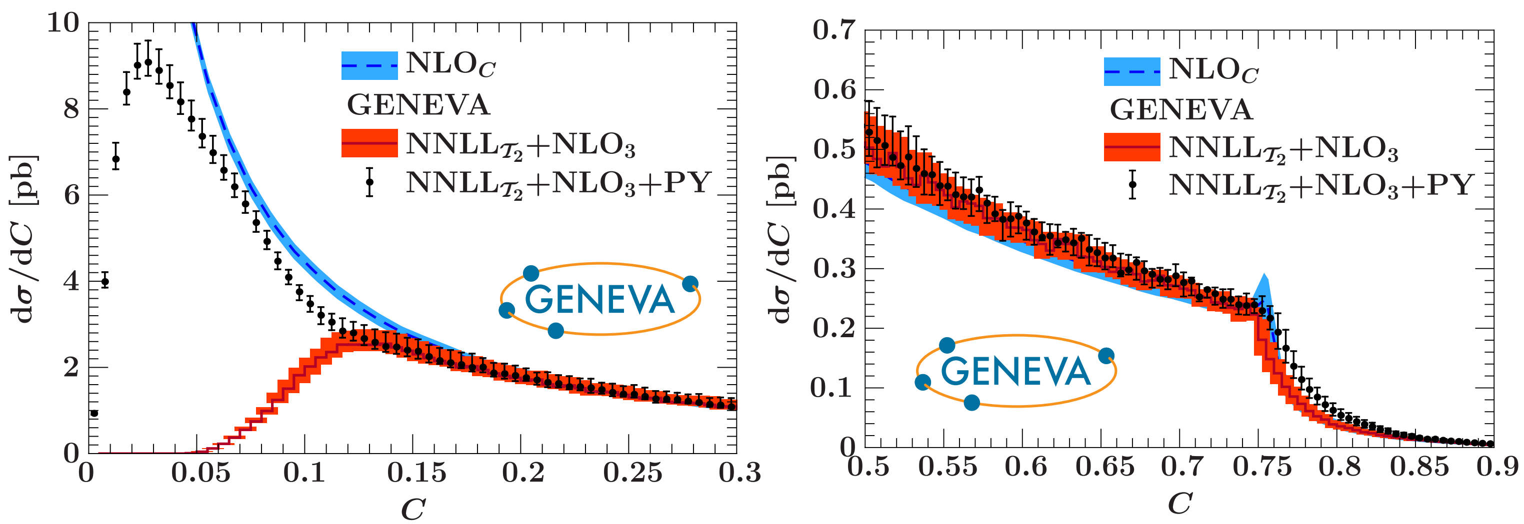
- Small shift in tail region above 3 body end point (333 GeV).

Other Variables

τ_{-1}



C parameter



cosθ